What can we learn about learning through human-machine interactions?

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Developing new representations of space:
A clinical context

Ordinary space is Euclidean

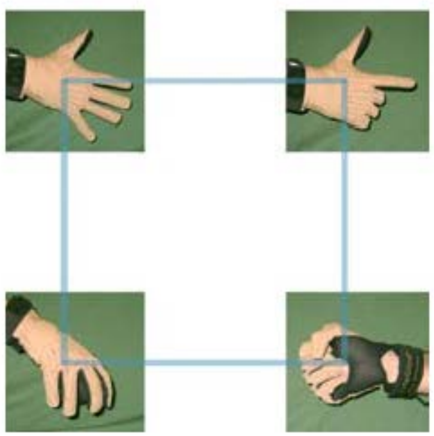
- The shortest path between two points is a straight line
- Direction and extent are independent of each other
- All directions are equivalent
- Rotations/reflections/translations (Rigid transformations) do not change the Euclidean measure of size (Sum of squares or "L_2 norm")

A working hypothesis

Neither the visual nor the motor signals spaces are Euclidean.

The structure of space in perception and movement is learned through practice and can be remapped

A different kind of reaching

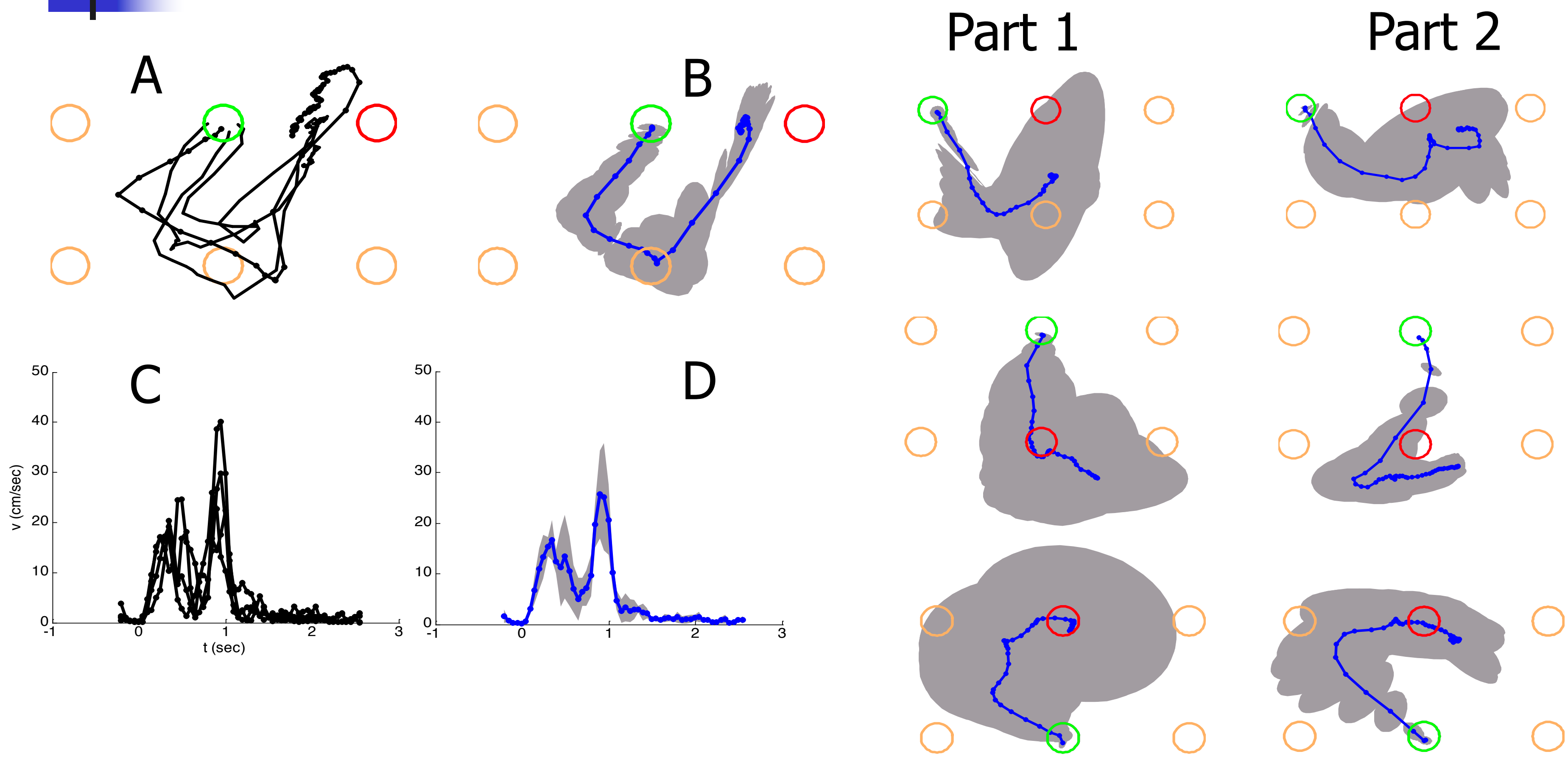


19 SIGNALS
(Hand Configuration)

2 Screen Coordinates

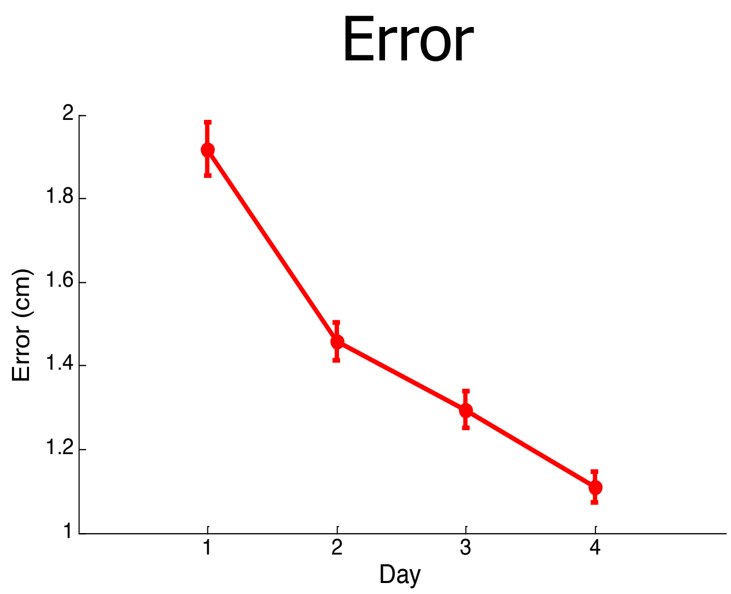
Mosier, et Al. J. of Neurophysiology (2005).

Some examples

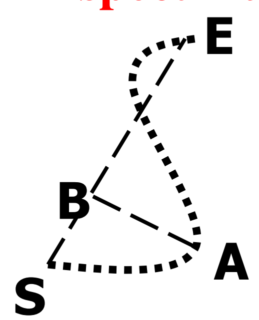


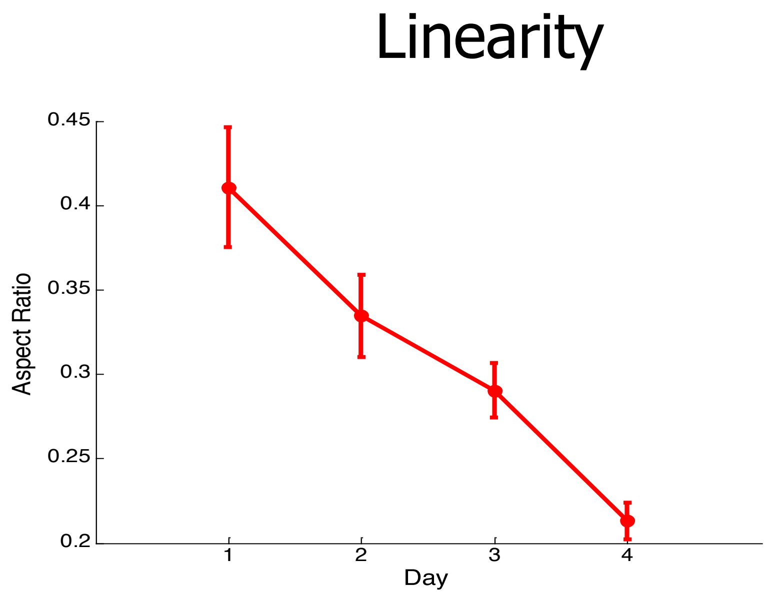
Learning Results -1

Over multiple sessions cursor motions became more precise and straighter



Aspect-Ratio: AB/SE

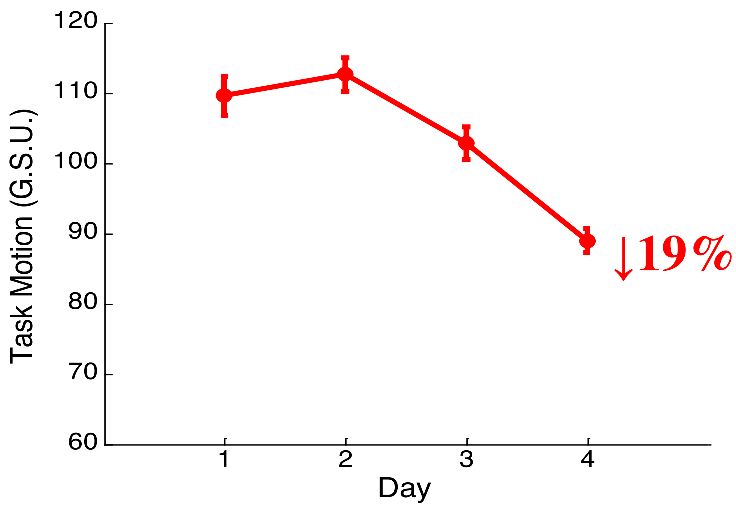




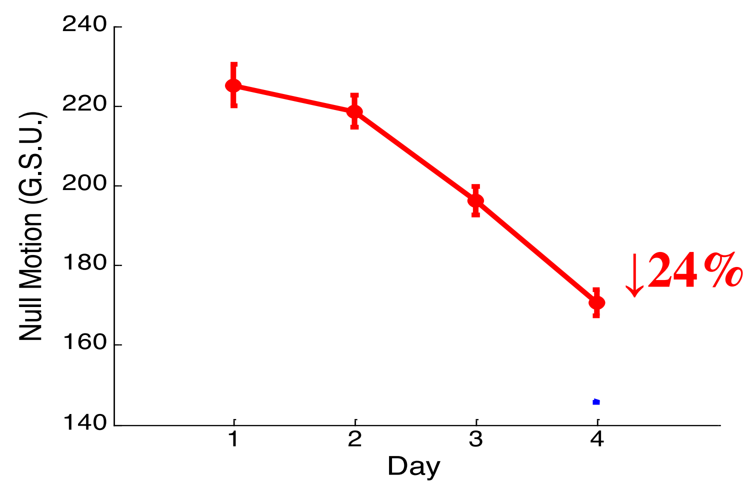
Learning Results-2

Subjects learned to "waste" less unnecessary motion.

POTENT SPACE



NULL SPACE

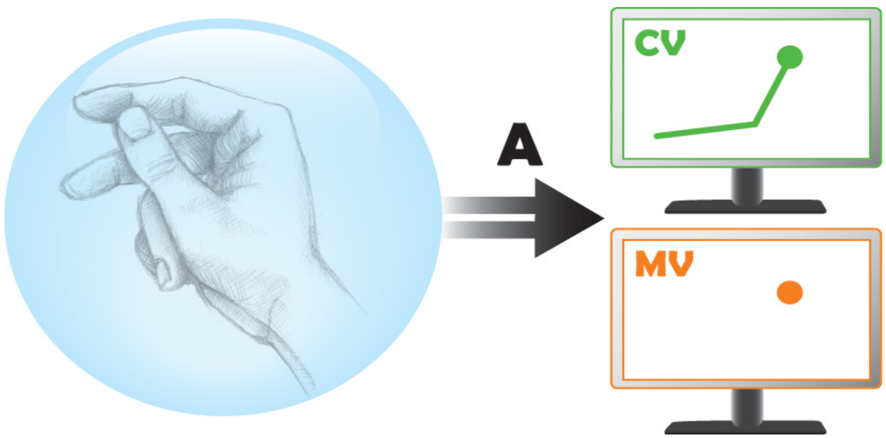


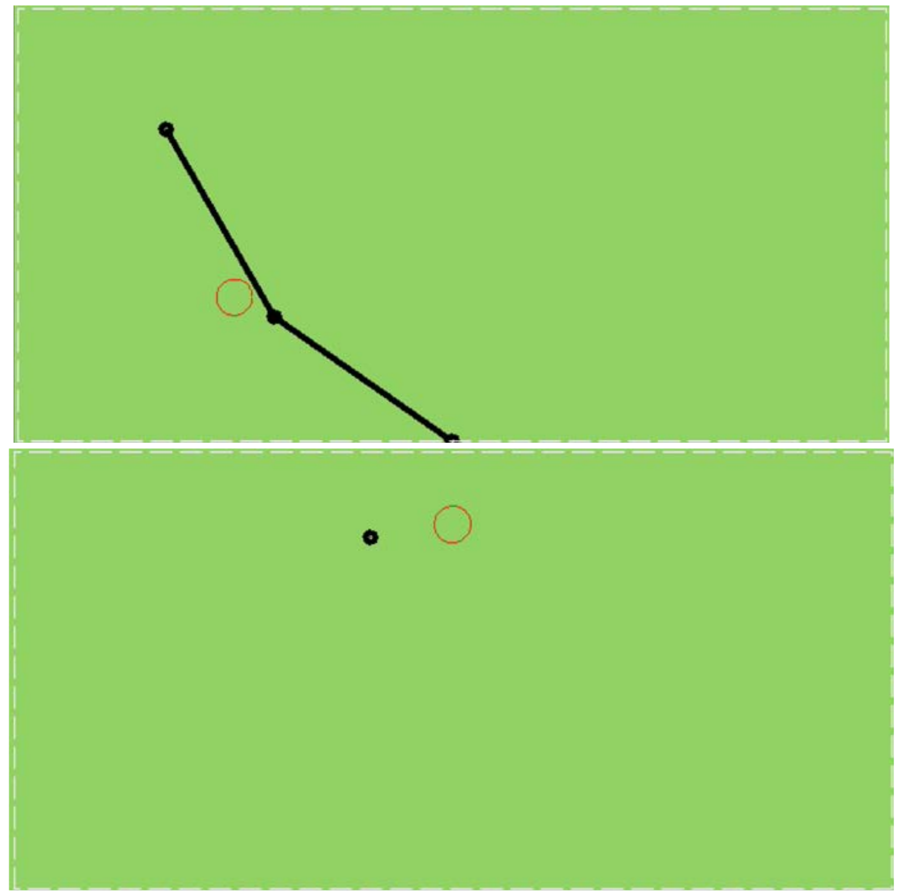
Learning curved geometries

STATIC-NONLINEAR MAP

$$\theta = A \cdot h \xrightarrow{\zeta(\theta,\hat{s})} (x, y)$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\zeta(\theta,\hat{s})=\begin{bmatrix}\cos(\theta_1) & \cos(\theta_1+\theta_2) & 1 & 0\\ \sin(\theta_1) & \sin(\theta_1+\theta_2) & 0 & 1\end{bmatrix}\cdot\hat{s}$$

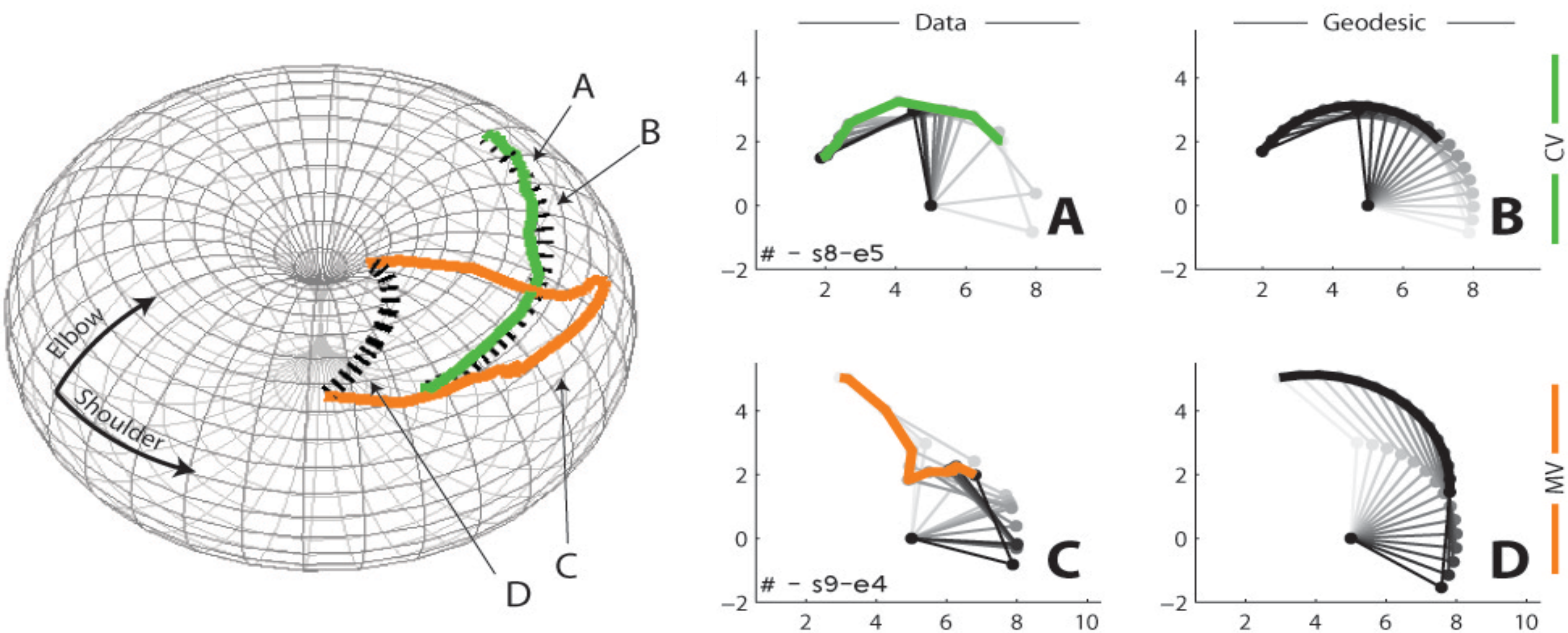




Danziger, Z., & Mussa-Ivaldi, F. A. (2012). *Journal of Neuroscience*.

Example trajectories

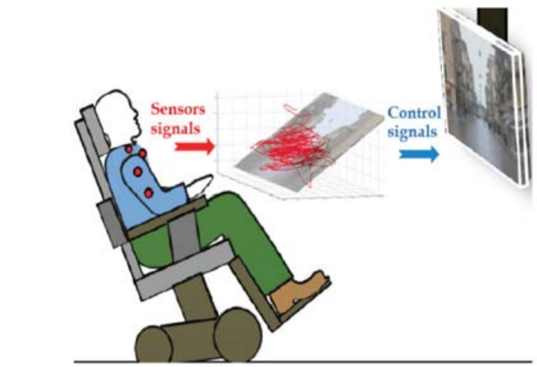
Examples in the configuration space



The Body-Machine Interface

Construct a mapping (*H*) from high dimensional "body vector" (*q*) to low dimensional "control vector" (*p*)

$$p = H \cdot q$$



Subjects engage in a self-paced "body dance".

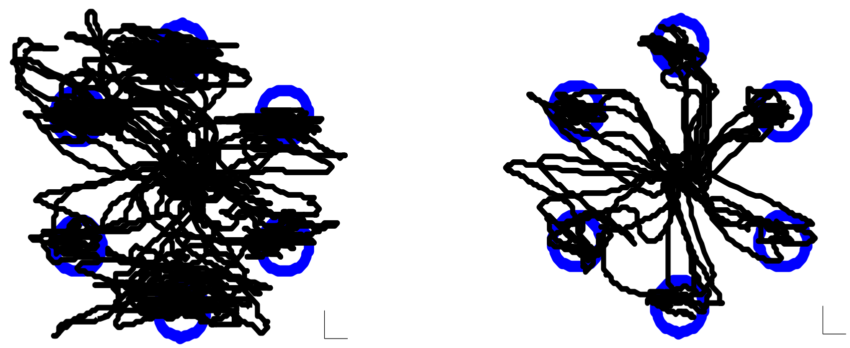
PCA extracts the 2D subspace of highest signal variance.

Through practice subjects learn to form one or more "right inverses" of *H*

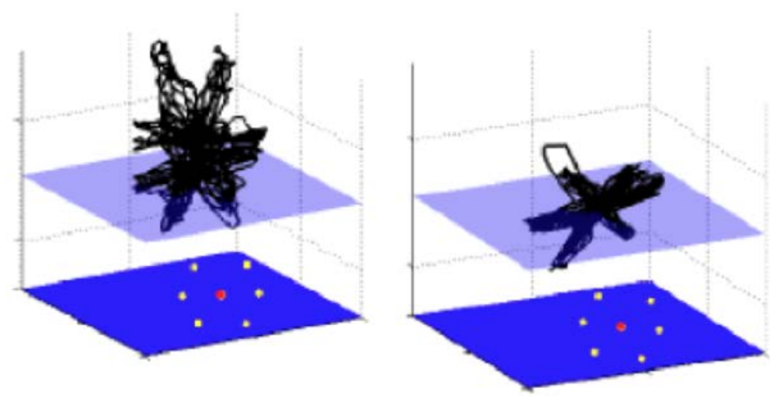
$$q = G \cdot \hat{p} \quad \text{with} \quad H \cdot G = I_2$$

Two aspects of Learning

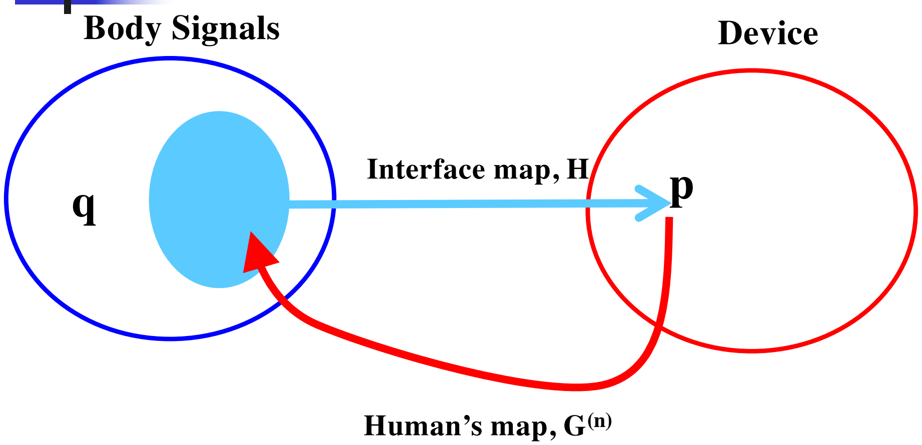
LEARNING THE TASK



LEARNING THE MAP



Learning an inverse model



dim(Body) > dim (Device)
dim(q) > dim(p)

$$p^{(n)} = H\, q^{(n)}$$
$$q^{(n)} = G^{(n)} \hat{p}^{(n)}$$

- The human operator must learn to choose an action from a set of a-priori "equivalent" actions
- The human must learn a right-inverse of H, i.e. an inverse model out of a multitude of choices
- The inverse model $G^{(n)}$ is the "state of learning"

Dimensions

- In the following examples:

$$\dim(q) = 8$$

$$\dim(p) = 2$$

$$\dim(H) = 2 \times 8$$

$$\dim(G) = 8 \times 2$$

State-space approach

Standard formulation of state-space dynamics

$$\begin{cases} x^{(n+1)} = Ax^{(n)} + Bu^{(n)} \\ y^{(n)} = Cx^{(n)} + Du^{(n)} \end{cases}$$

State Equation

Output Equation

In our case

$$\begin{cases} G^{(n+1)} = G^{(n)} + B^{(n)} \cdot u^{(n)} \\ p^{(n)} = H \cdot G^{(n)} \cdot \hat{p}^{(n)} \end{cases}$$

State Equation (describes the update of G, based on the observed error)

Output Equation (describes the response to the presentation of a target based on the current model)

Error

$$u^{(n)} \equiv p^{(n)} - \hat{p}^{(n)}$$

Learning of the inverse model as a gradient-descent (analogous to a mechanical system)

Reaching error

$$E^{(n)2} = \frac{1}{2}\left\|u^{(n)}\right\|^2 = \frac{1}{2}\left(p^{(n)} - \hat{p}^{(n)}\right)^T \cdot \left(p^{(n)} - \hat{p}^{(n)}\right)$$

This is a matrix

Error

$$\boxed{\frac{\partial E^{(n)2}}{\partial G^{(n)}}} = (p^{(n)} - \hat{p}^{(n)})^T \cdot \frac{\partial p}{\partial G^{(n)}} = H^T \cdot u^{(n)} \otimes \hat{p}^{(n)T}$$

Forward Map

Target

$$G^{(n+1)} = G^{(n)} + \eta \cdot H^T \cdot u^{(n)} \otimes \hat{p}^{(n)T}$$

This regulates the learning rate

The gradient depends on H. Implausible to assume that the learner "knows" H a priori.

Learning the forward model

- Compare the observed cursor with the predicted cursor (sensation error).
- Gradient descent on the sensation error

$p^{(n)}$ Observed $\qquad H_{EST}^{(n)} \cdot q^{(n)}$ Predicted

$\varepsilon^{(n)2} = \left\| p^{(n)} - H_{EST}^{(n)} q^{(n)} \right\|^2$ Sensation Error

$$\frac{\partial \varepsilon^2}{\partial H_{EST}^{(n)}} = 2\varepsilon^{(n)} \otimes q^{(n)T} = 2\left(p^{(n)} - H_{EST}^{(n)} q^{(n)}\right) \otimes q^{(n)T}$$

Concurrent forward-inverse learning

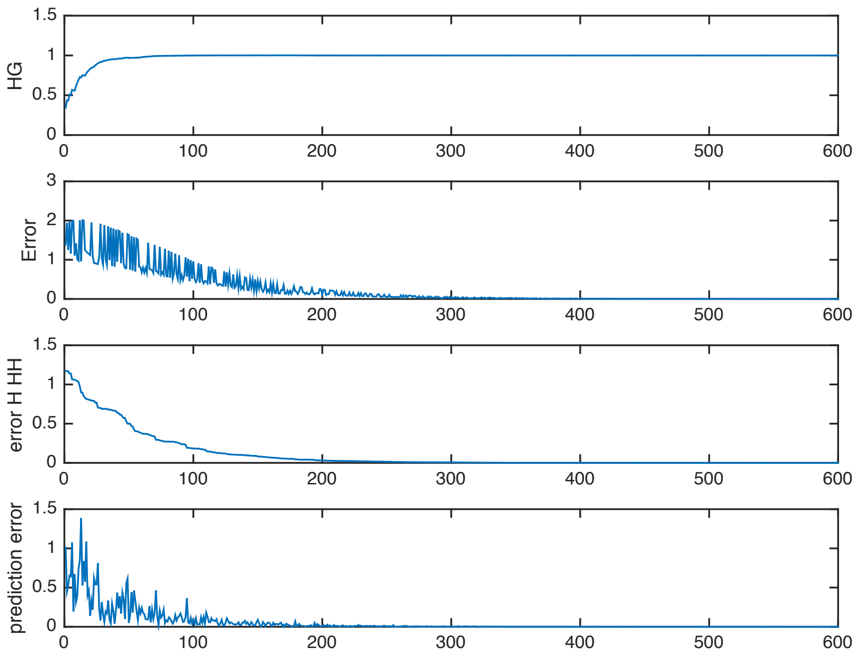
State: $\left[H_{EST}^{(n)}, G^{(n)} \right]$

$H_{EST}^{(n+1)} = H_{EST}^{(n)} + \delta \cdot (p^{(n)} - H_{EST}^{(n)} \hat{q}^{(n)}) \otimes \hat{q}^{(n)T}$ Forward Update

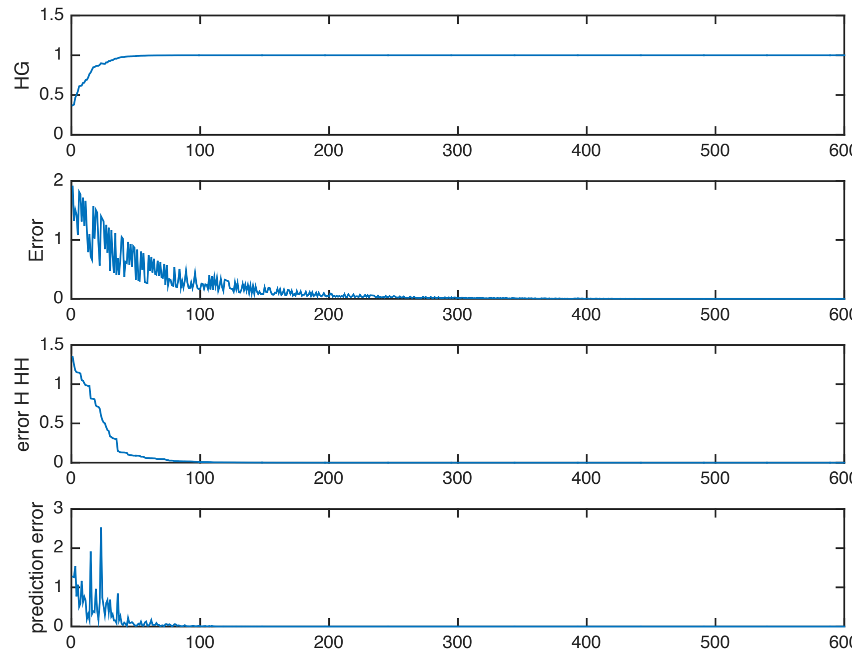
$G^{(n+1)} = G^{(n)} + \eta \cdot H_{EST}^{(n)} \cdot u^{(n)} \otimes \hat{p}^{(n)T}$ Inverse Update

$\hat{q}^{(n)} = G^{(n)} \hat{p}^{(n)} + \alpha \cdot n$ Noise

$n_i \sim N(0,1) \qquad i = (1, \ldots, \dim(q))$



$\alpha = 0.5$



$\alpha = 0.7$

Model Estimates

Target / Configuration histories

$$\hat{P}^{(n)} = \left[\hat{p}^{(n-T)}, \hat{p}^{(n-T+1)}, \ldots, \hat{p}^{(n)}\right]$$

$$Q^{(n)} = \left[q^{(n-T)}, q^{(n-T+1)}, \ldots, q^{(n)}\right]$$

Forward Model

$$Q^{(n)} = G^{(n)} \hat{P}^{(n)}$$

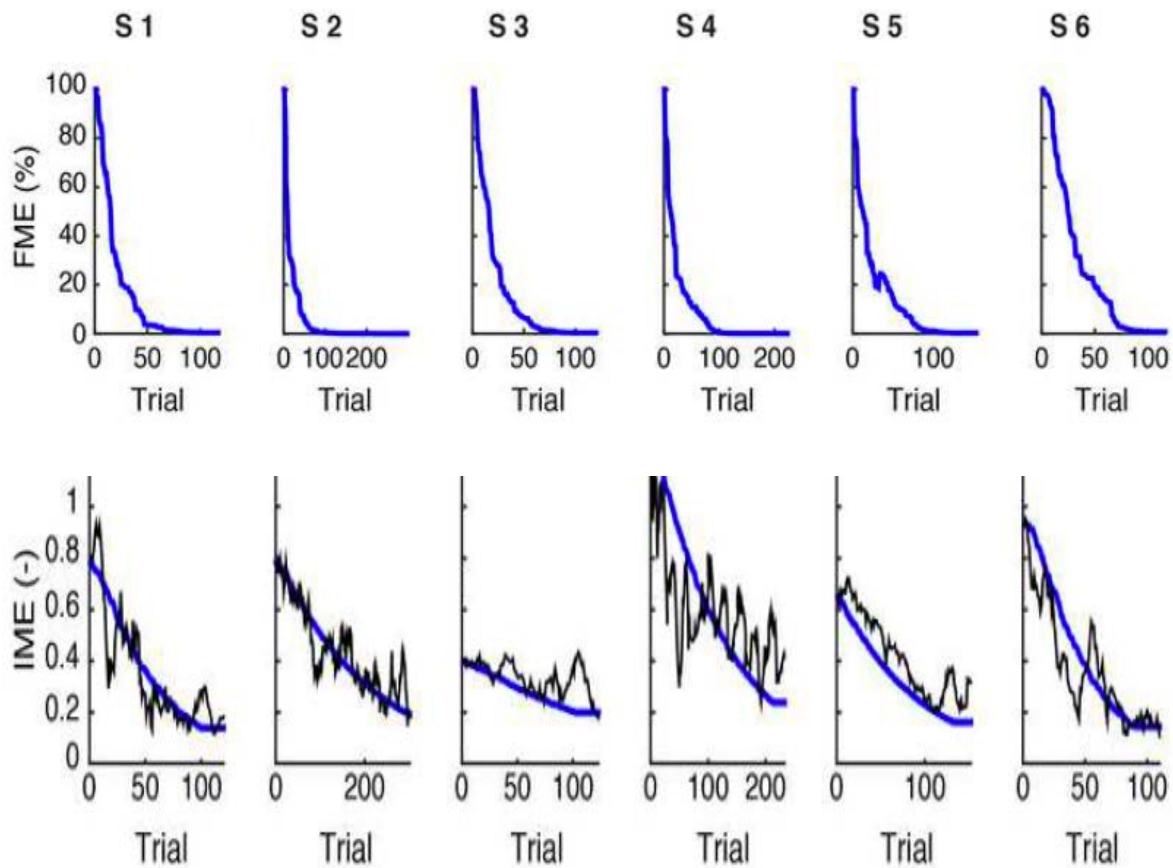
Inverse Model

$$Q^{(n)} = G^{(n)} \hat{P}^{(n)}$$

Maximum Likelihood Estimate

$$G^{(n)} \approx Q^{(n)} \hat{P}^{(n)T} \left(\hat{P}^{(n)} \hat{P}^{(n)T}\right)^{-1}$$

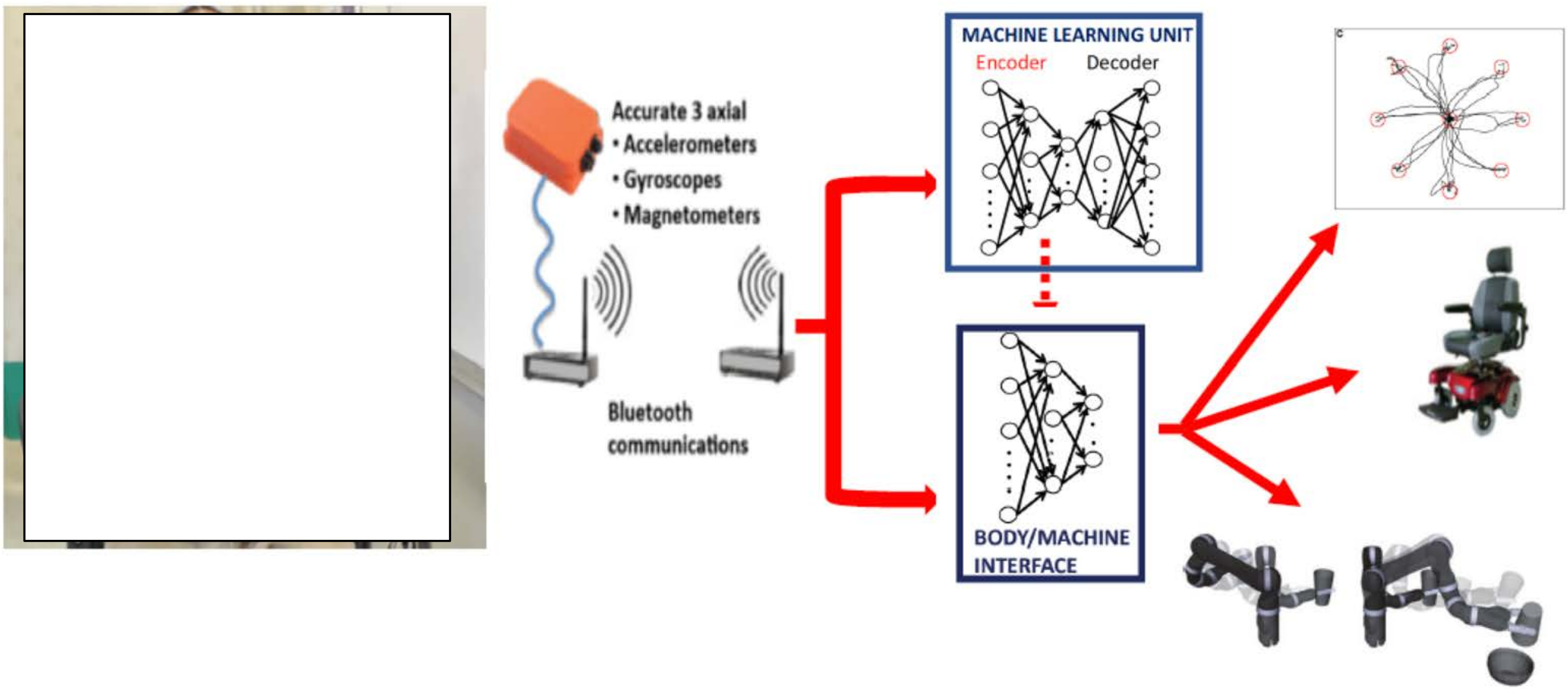
Model Validation: Forward and Inverse Model Errors



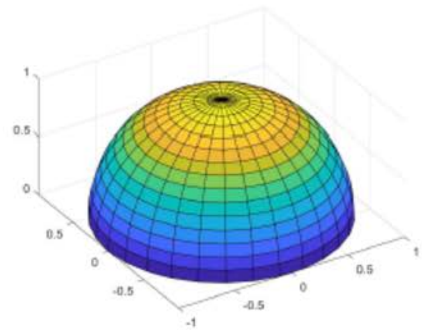
$$FME^{(n)} = \frac{\|H - \hat{H}^{(n)}\|}{\|H\|}$$

$$IME^{(n)} = \left\| HG^{(n)} - I_K \right\|$$

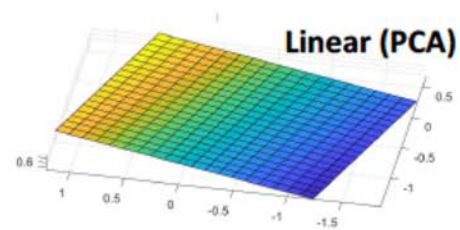
Nonlinear BMI



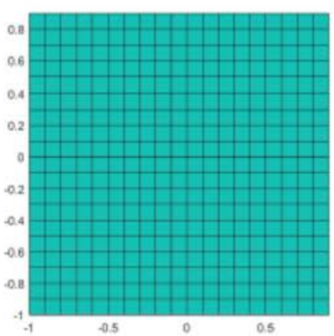
Representing ”curved” data



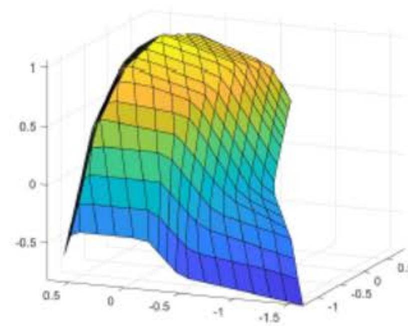
Sampled Surface



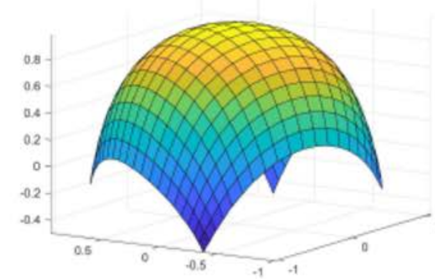
Decoded grid



Latent-space grid

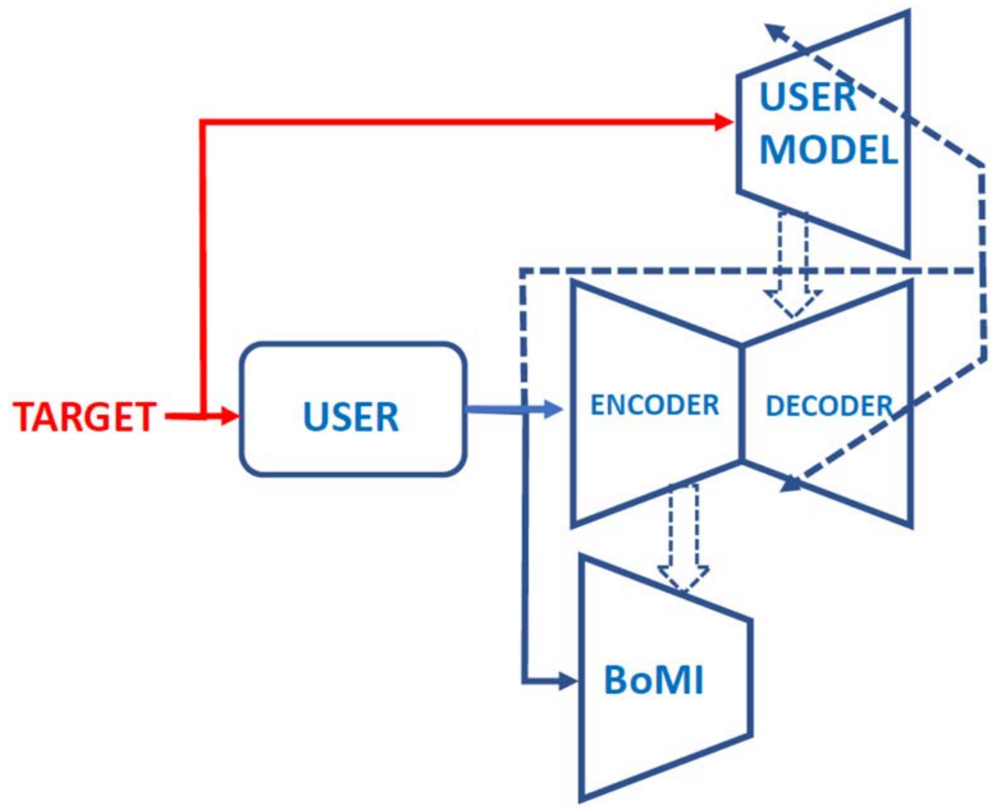


Rectifying Linear
(RELU)



Hyperbolic Tangent

Human/Machine Coadaptation



Credits

- People

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