Hb-graphs and their applications to the ranking of information in an information space

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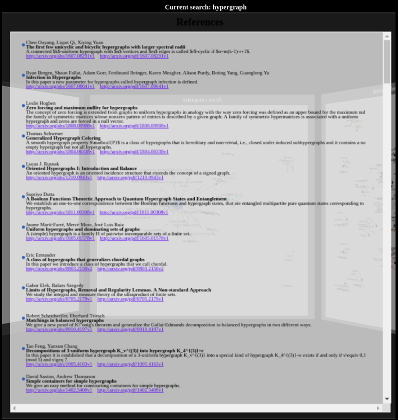
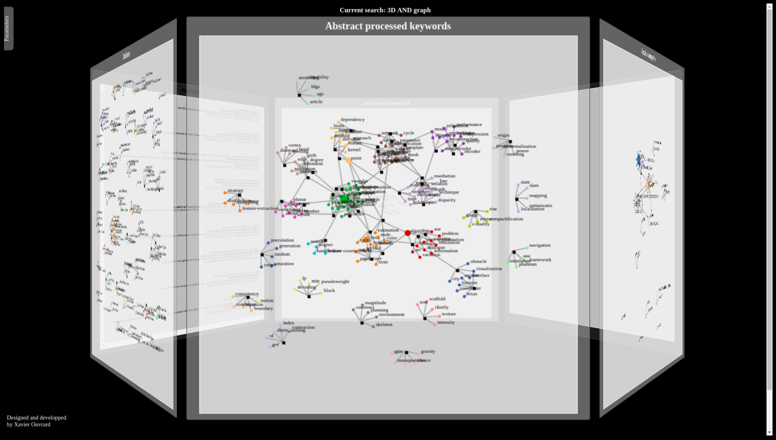
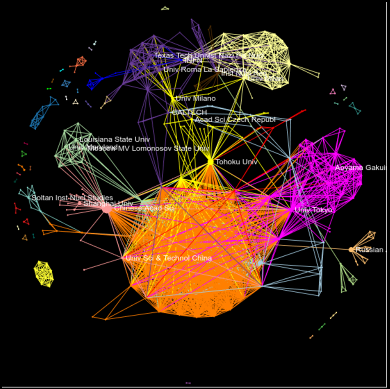
The example of search on a Scientific Publication Database



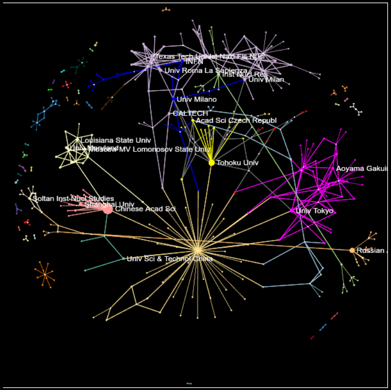
Searching on a Scientific Publication Database



Visualizing facets I



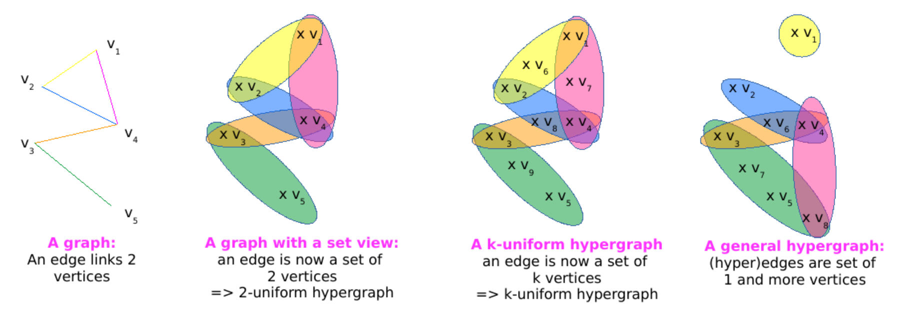
Clique view



Extra-node view

Figures from Ouvrard et al. [2017]

From graphs to hypergraphs



Motivation for a new structure I

- **Background question:**

How to coarsen a hyper(bag)-graph?

- Task to be solved:
  - spot out the **important structures** of a hypergraph
- Important for
  - **spraying** the information shown
  - give **focus on important information, at vertex and edge level**
- Approach taken: **diffusion approach**

Motivation for a new structure II

At **which level** the **diffusion process** occurs?

- At **graph level** => Laplacian matrix ... linked to adjacency matrix
- At **hyper(bag)-graph level** => **incidence matrix**

Pitfalls: HH^T take us back to pairwise relationships => n-adicity not totally taken into account

- An **adjacency tensor** is needed:
  - Well defined for uniform hypergraphs
  - For general hypergraphs:
    - **Adjacency** has to be refined
    - Convenient adjacency tensor to ensure diffusion and Laplacian tensor

On adjacency in general hypergraphs

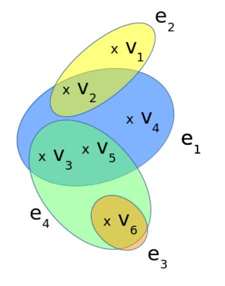
Refining adjacency in hypergraphs

- **Adjacency is more than pairwise notion**
- Two distinct concepts:
  - **e-adjacency**: vertices of a given hyperedge are said e-adjacent
  - **k-adjacency**: k given vertices are k-adjacent if it exists a hyperedge that hold them
- **$\overline{k}$-adjacency**: the maximal k-adjacency a hypergraph holds
- In k-uniform hypergraph:
  - $\overline{k}$-adjacency corresponds to k-adjacency
  - The e-adjacency corresponds to $\overline{k}$-adjacency
- In general hypergraphs:

$\overline{k}$-adjacency $\neq$ e-adjacency

Hypergraph $\mathcal{H} = (V, E)$

- $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
- $E = \{e_1, e_2, e_3, e_4\}$



- v_2 and v_3 are 2-adjacent
- v_2, v_3 and v_4 are 3-adjacent
- v_1 and v_2 are e-adjacent
- $\overline{k}$-adjacency corresponds to 4-adjacency

Tensor for general hypergraphs: the art of filling or how to obtain cubic form from a non cubic object

First way: cut everything in small pieces...

First way: ... and gather

First way: ... and gather

- Uniformisation process based on **hyperedge splitting**

=> push everything in a tensor of order $|r_{\mathcal{H}}|$ and size $|V|$.
=> **Banerjee & al**[2]: split the storage in the elements of the tensor that is not occupied
The ([Author's note]: **e-) adjacency hypermatrix** of $\mathcal{H}$ written
$\mathcal{A}_{\mathcal{H}} = \left(a_{i_1 \dots i_{k_{\max}}}\right)_{1 \leqslant i_1, \dots, i_{k_{\max}} \leqslant n}$ is such that for a hyperedge: $e = \{v_{l_1}, ..., v_{l_s}\}$ of cardinality $s \leqslant k_{\max}$.

$$a_{p_1 \dots p_{k_{\max}}} = \frac{s}{\alpha}, \text{ where } \alpha = \sum_{\substack{k_1, \dots, k_s \geqslant 1 \\ \sum k_i = k_{max}}} \frac{k_{\max}!}{k_1! \dots k_s!}$$

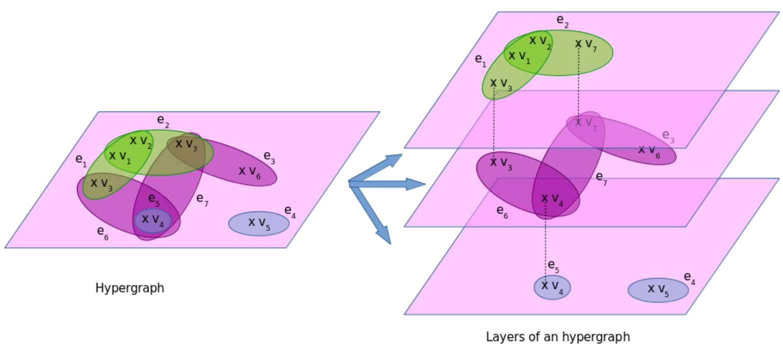
with p_1, ..., $p_{k_{\max}}$ chosen in all possible way from $\{l_1, ..., l_s\}$ with at least once from each element of $\{l_1, ..., l_s\}$.
=> **Sun & al**[3]: similarly the same, but not symmetric.

[2] Banerjee et al. [2017]
[3] Sun et al. [2018]

Second way: the art of filling

e-adjacency tensor of a general hypergraph I

- split the hypergraph in layers of uniformity:



e-adjacency tensor of a general hypergraph II

- hypergraph uniformisation process based on **hyperedge filling**:

Various filling options:

- Always add the **same vertex**

=> **straightforward approach**

- Add a **different vertex** per uniform hypergraph layer up to fulfillment

=> **silo approach**

- Add a vertex for each layer (previous approach)

=> **iterative approach**

In the first two approaches, **multisets** are required to keep interpretability

First two approaches require multisets

A parenthesis on multisets

Multisets:

Multiset: a universe and a multiplicity function $A_m = (A, m)$

Natural multiset: the range of the multiplicity function is a subset of $\mathbb{N}$.

In natural multisets: two views:

weighted set: $A_m = \left\{ x_1^{m_1}, \ldots, x_n^{m_n} \right\}$

collection of objects $\left\{ \left\{ \underbrace{x_1, \ldots, x_1}_{m_1 \text{ times}}, \ldots, \underbrace{x_n, \ldots, x_n}_{m_n \text{ times}} \right\} \right\}$

Hb-graphs I

Hb-graph $\mathcal{H} = (V, E)$: family of multisets $E = (e_i)_{i \in I}$, with $I = [\![p]\!]$- called **hb-edges** - where the hb-edges have:

- same universe $V = \{v_1, \ldots, v_n\}$, called **vertex set**.
- support a subset of V.
- each hb-edge has its own multiplicity function $m_e : V \to \mathbb{W}$ where $\mathbb{W} \subset \mathbb{R}^+$.

Incidence matrix of hb-graphs:

$$H = [m_j (v_i)]_{\substack{1 \leqslant i \leqslant n \\ 1 \leqslant j \leqslant p}}$$

.

Hb-graphs II

Different application of hb-graphs:

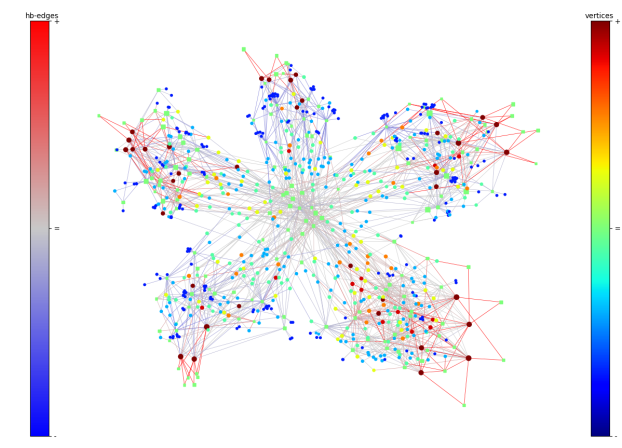
- General hypergraph e-adjacency tensor
- **Network of co-occurrences** and the hb-graph framework
- **Prime decomposition** and hb-graphs
- **Text and hb-graphs**:

=> bag of words to represent text are efficient modeling for information retrieval

- **Image and hb-graphs**:

=> bag of visual words are often used in image.

ML interest of hb-graphs



Exchange-based diffusion in hb-graphs:

- **Stochastic process**
- Allows **generalised random walk**
- Defines **a ranking** of vertices and hb-edges (akin to PageRank)
- Enables **coarsening** of hb-graphs and thus data landscape

Exchange-based diffusion algorithm

Given:

A hb-graph $\mathcal{H} = (V, E, w_e)$ with $|V| = n$ and $|E| = p$

Number of iterations: T

Initialisation:

For all $v_i \in V : \alpha_i := \frac{1}{n}$

For all $e_j \in E : \epsilon_j := 0$

DiffuseFromVerticesToHbEdges():

For $j := 1$ to p:

$\epsilon_j := 0$

For $v_i \in e_j^\star$:

$$\epsilon_j := \epsilon_j + \frac{m_j(v_i)\, w_e(e_j)}{d_{w,m}(v_i)} \alpha_i$$

DiffuseFromHbEdgesToVertices():

For $i := 1$ to n:

$\alpha_i := 0$

For e_j such that $v_i \in e_j^\star$:

$$\alpha_i := \alpha_i + \frac{m_j(v_i)}{\#_m e_j} \epsilon_j$$

Main():

Calculate for all $i : d_{w,m}(v_i)$ and for all $j : \#_m e_j$

For $t = 1$ to T:

DiffuseFromVerticesToHbEdges()

DiffuseFromHbEdgesToVertices()

Exchange-based diffusion algorithm

Given:

A hb-graph $\mathcal{H} = (V, E, w_e)$ with $|V| = n$ and $|E| = p$

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DiffuseFromHbEdgesToVertices():

For $i := 1$ to n:

$\alpha_i := 0$

For e_j such that $v_i \in e_j^\star$:

$$\alpha_i := \alpha_i + \frac{m_j(v_i)}{\#_m e_j} \epsilon_j$$

Main():

Calculate for all $i : d_{w,m}(v_i)$ and for all $j : \#_m e_j$

For $t = 1$ to T:

DiffuseFromVerticesToHbEdges()

DiffuseFromHbEdgesToVertices()

Time complexity:

$$O\left(T\left(d_{\mathcal{H}} n + r_{\mathcal{H}} p\right)\right)$$

where:

$$d_{\mathcal{H}} = \max_{v_i \in V} (d_i)$$

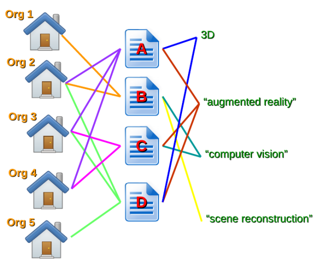
$$r_{\mathcal{H}} = \max_{e_j \in E} \left|e_j^\star\right|$$

Hb-graph m-uniformisation process and diffusion

Using a m-uniformisation process modifies the exchange-based diffusion:

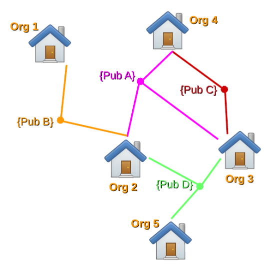
- **Explainable in the case of the hb-edge filling**
- Unclear how it is modified in the case of the hb-edge splitting

In a Scientific Publication Database

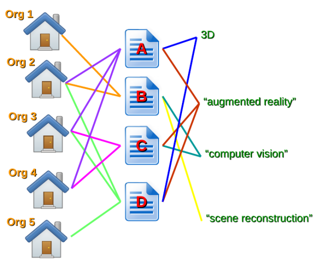


Reference: Publication, Facet: Organization

Pub A	Org 2, Org3, Org 4
Pub B	Org 1, Org 2
Pub C	Org 3, Org 4
Pub D	Org2, Org 3, Org 5

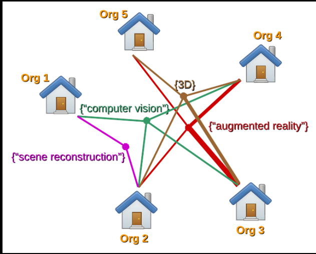


In a Scientific Publication Database

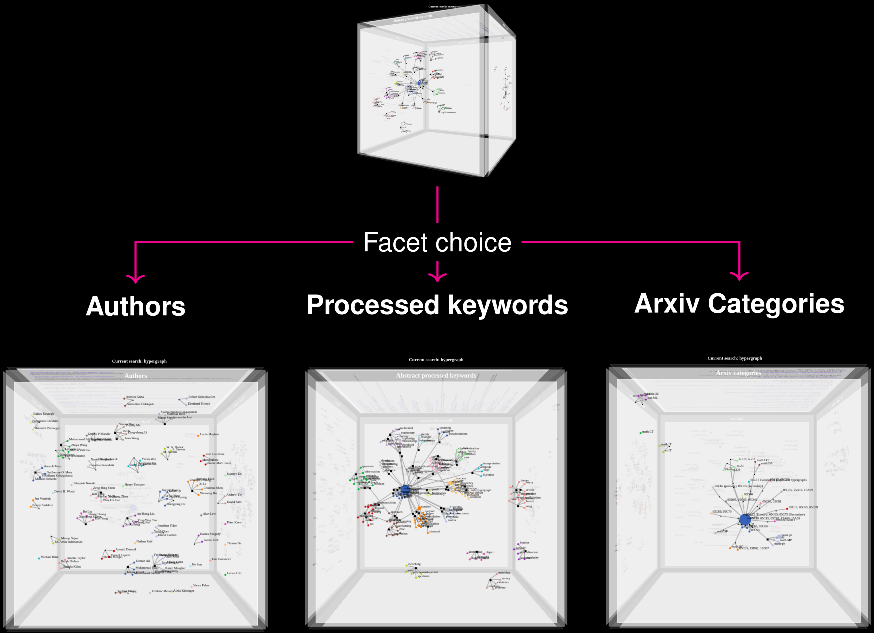


Reference: Keywords, Facet: Organization

scene reconstruction	$\{\{\text{Org } 1^1, \text{Org } 2^1\}\}$
computer vision	$\{\{\text{Org1}^1, \text{Org2}^1, \text{Org } 3^1, \text{Org } 4^1\}\}$
augmented reality	$\{\{\text{Org } 2^2, \text{Org } 3^3, \text{Org } 4^2, \text{Org } 5^1\}\}$
3D	$\{\{\text{Org } 2^2, \text{Org } 3^2, \text{Org } 4^1, \text{Org } 5^1\}\}$



Facets of the information space



Aggregating the information

Aggregating the information obtained on the different facets:

- **We have proposed a modified MC4, called WT-MC4**

=> allows ranking of references using weights on facets

- We also proposed a biased ranking of the information on facets, to put emphasize on some kind of features

A proposal

On each facet F_1,...,F_m=> a hb-graph $\mathcal{H}_k = (V_k, E_k)$ with $1 \leqslant k \leqslant m$
1. **Exchange-base diffusion on each facet hb-graph** => hb-edges $\epsilon_k(e_j)$
2. **Each hb-edge is linked to different physical references**: 1 to many. => ranking R_k of references r_i with possible ties = rank of the corresponding hb-edge.
3. Facets have **associated weights** (tunable)w_1,..., w_m such that $\sum\limits_{i \in [\![m]\!]} w_i = 1, w_i \geqslant 0.$
4. We start by computing a weighted majority matrix for each couple of references:

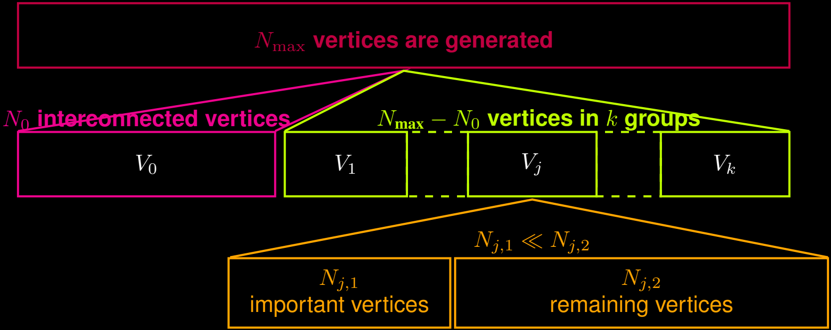
$$M(r_{i_1}, r_{i_2}) = \sum_{k \in [\![K]\!]} w_k \mathbf{1}_{R_k(r_{i_1}) < R_k(r_{i_2})} - \sum_{k \in [\![K]\!]} w_k \mathbf{1}_{R_k(r_{i_1}) > R_k(r_{i_2})}$$

5. We use a **modified MC4 of Dwork et al. [2001] with teleportation and weights**:
Current state reference: r_{current}.
- Choose a random number γ
- Choose an other reference r_{next}uniformly among all the references ranked.

If $\gamma > \gamma_0$:
    go to r_{next}
else:
    If $M(r_{\text{next}}, r_{\text{current}}) > 0$
        go to r_{next}
    else:
        stay in r_{current}.

Methodology of evaluation

- 2 parts in experimentation:
  - **generation of random hb-graphs** => 1 per facet



  - **a generated reference hb-graph is built out of the facets**
- **Perform diffusion on each facet separately (multi-diffusion)**
- Aggregation using the modified MC4 and comparison to Borda results

Results for our modified MC4 I

On the generated information space, we observe for **100 generated information spaces**, the following results for Kendall tau between Borda ranking and rankings obtained by:

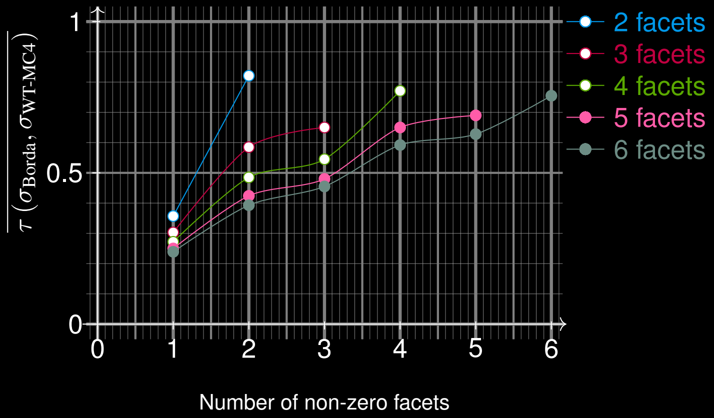
Current	$\overline{\tau\left(\sigma_{\text{Borda}}, \sigma_{\text{current}}\right)}$	$\sigma\left(\tau\left(\sigma_{\text{Borda}}, \sigma_{\text{current}}\right)\right)$
Facet 0	0.262	0.09
Facet 1	0.261	0.08
Facet 2	0.237	0.104
References	0.317	0.283
WT-MC4{'0': 0.33, '1': 0.33, '2': 0.33}	**0.649**	**0.116**
WT-MC4{'0': 0.5, '1': 0.5, '2': 0.0}	0.581	0.114
WT-MC4{'0': 0.5, '1': 0.0, '2': 0.5}	0.549	0.115
WT-MC4{'0': 0.0, '1': 0.5, '2': 0.5}	0.563	0.123
WT-MC4{'0': 0.0, '1': 0.0, '2': 1.0}	0.261	0.107
WT-MC4{'0': 0.0, '1': 1.0, '2': 0.0}	0.279	0.084
WT-MC4{'0': 1.0, '1': 0.0, '2': 0.0}	0.286	0.096

Results for our modified MC4 II

Average Kendall tau of the WT-MC4 ranking aggregation compared to the non-zero equal weight facet rankings (average on 1000 information spaces) depending on the number of facets having non-zero weights.

# non-zero weight facets ⇒	1		2		3		4		5		6	
# facets ⇓	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev
2	0.357	0.096	0.821	0.050								
3	0.303	0.094	0.585	0.127	0.650	0.112						
4	0.272	0.093	0.485	0.116	0.545	0.116	0.771	0.066				
5	0.250	0.091	0.424	0.108	0.480	0.109	0.650	0.094	0.690	0.085		
6	0.239	0.086	0.393	0.096	0.455	0.101	0.592	0.089	0.628	0.090	0.755	0.058

Results for our modified MC4 III



Results for our modified MC4 IV

- Putting weight on only one facet: ’authors’

Comparison of rankings obtained by Dwork_MC4 with weights:

$$\{\text{'authors'} : 1.0, \text{'keywords'} : 0, \text{'tags'} : 0\}$$

and rankings on the facet:

	Kendall’s Tau coefficient	**Scaled Spearman Footrule coeff**	**Jaccard Index @10 / @25**
authors	**0.819**	0.125	**0.818 / 0.923**
keywords	0.076	0.598	0.125 / 0.231
tags	-0.112	0.710	0.171 / 0.315

- **Same kind of results for putting all the weight on keywords or tags**

Results for our modified MC4 V

- Putting equal weights on each facet:

Comparison of rankings obtained by Dwork_MC4 with weights:

$$\left\{\text{'authors'} : \frac{1}{3}, \text{'keywords'} : \frac{1}{3}, \text{'tags'} : \frac{1}{3}\right\}$$

and other rankings:

	Kendall's Tau coefficient	**Scaled Spearman Footrule coeff**	**Jaccard Index @10 / @25**
authors	0.315	0.444	0.333 / 0.515
keywords	0.508	0.327	0.385 / 0.455
tags	0.228	0.612	0.242 / 0.449
Borda	**0.818**	**0.127**	**0.636 / 0.846**
Refs	-0.438	0.995	0.044 / 0.192

Results for our modified MC4 VI

- **Comparison of rankings obtained by Dwork_MC4** with weights:

$$\{\text{'authors'} : 0.5,\ \text{'keywords'} : 0,\ \text{'tags'} : 0.5\}$$

and other rankings:

	Kendall's Tau coefficient	**Scaled Spearman Footrule coeff**	**Jaccard Index @10 / @25**
authors	0.354	0.434	0.333 / 0.471
keywords	0.441	0.369	0.385 / 0.455
tags	0.264	0.620	0.242 / 0.449
Borda	**0.811**	**0.134**	**0.636 / 0.846**
Refs	-0.486	0.984	0.043 / 0.212

Perform visual queries

A demo is worth a thousand words...

Thank you for your attention!

Collaboration spotting

Leveraging insight into your data network by viewing co-occurrences while navigating across different perspectives.

The HbGraphDataEdron:

- Is part of the Collaboration Spotting family
- Collspotting Project leader: **Dr Jean-Marie Le Goff**
- Team members: Dimitrios Dardanis, Richard Forster, André Rattinger and Xavier Ouvrard

More information:

- http://collspotting.web.cern.ch
- https://www.infos-informatique.net
- xavier.ouvrard@cern.ch

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