Markowitz portfolio model and market graphs

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MARKOWITZ PORTFOLIO SELECTION MODEL

(1) $\mathbf{x}^{\mathrm{T}} = [x_1, \ldots, x_n]$ portfolio, $\mathbf{R}^{\mathrm{T}} = [R_1, \ldots, R_n]$ reward
(2) $\mu^{\mathrm{T}} = [\mathrm{E}[R_1], \ldots, \mathrm{E}[R_n]]$ expedted returns
(3) Σ covariance matrix ($\sigma_1^2, \ldots, \sigma_n^2$ diagonal entries)
(4) $\mu^{\mathrm{T}}\mathbf{x} = \mathrm{E}[\mathbf{R}]$
(5) $\mathbf{x}^{\mathrm{T}}\Sigma\mathbf{x} = \mathrm{Var}[\mathbf{R}]$

$$\begin{array}{lll} \mu^{\mathrm{T}}\mathbf{x} \to \max & \mathbf{x}^{\mathrm{T}}\Sigma\mathbf{x} \to \min & \mu^{\mathrm{T}}\mathbf{x} - \lambda\mathbf{x}^{\mathrm{T}}\Sigma\mathbf{x} \to \max \\ \mathbf{x}^{\mathrm{T}}\Sigma\mathbf{x} \le s^2 & \mu^{\mathrm{T}}\mathbf{x} \ge m & \\ \mathbf{e}^{\mathrm{T}}\mathbf{x} = 1 & \mathbf{e}^{\mathrm{T}}\mathbf{x} = 1 & \mathbf{e}^{\mathrm{T}}\mathbf{x} = 1 \\ \mathbf{x} \ge \mathbf{0} & \mathbf{x} \ge \mathbf{0} & \mathbf{x} \ge \mathbf{0} \end{array}$$

(6) s^2 risk limit
(7) m guaranteed return
(8) λ aversion to risk

ROLE OF NORMS

Quadratic program reduces to linear program in the L_1, L_∞ cases.

$$\text{risk} = \left\{ \mathrm{E}\left[\left| \mathbf{R} - \mathrm{E}[\mathbf{R}] \right|^p \right] \right\}^{1/p}$$

norm	centrum	spread	association	model
L_1	median	quartile range	Spearman coefficient	modified
L_2	mean	variance	Pearson coefficient	Markowitz
L_∞	midrange	max–min	unnamed coefficient	modified

MARKET GRAPH

We define a graph $G = (V, E)$. Here V set of vertices, E set of edges. V associated with the financial instruments.
Two assets are connected by an edge if their correlation coefficient is below a fixed theshold value α.
We assign the expected rewards as weights to the vertices.

$$\text{portfolio} \quad \longleftrightarrow \quad \text{weighted clique}$$

$$\text{dominating set} \quad \longleftrightarrow \quad \text{stock indices}$$

DATA SCIENCE

The models can be applied in connection with not necessarily financial data.
Correlation coefficient can be replaced by other measures of associations.
We may use distance matrices, directed and bipartite graphs.

graph theory	data science
clique	association rule
dominating set	data condensing
node coloring	classification
independent set	clustering
clique partition	machine learning
matching	"twin" experiment
spanning tree	uncorrelated cases

SUMMARY

(1) The models become computationally more tractable. Quadratic programs, linear programs, graph algorithms.
(2) The range of applicability extended.
(3) Difficulties with the probabilistic interpretation.
(4) Graph models are exploratory tools.