# Using Quantum Computer to Simulate Systems in Plasma

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F.Ringer and XY, 2106.08394

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(appear tonight)

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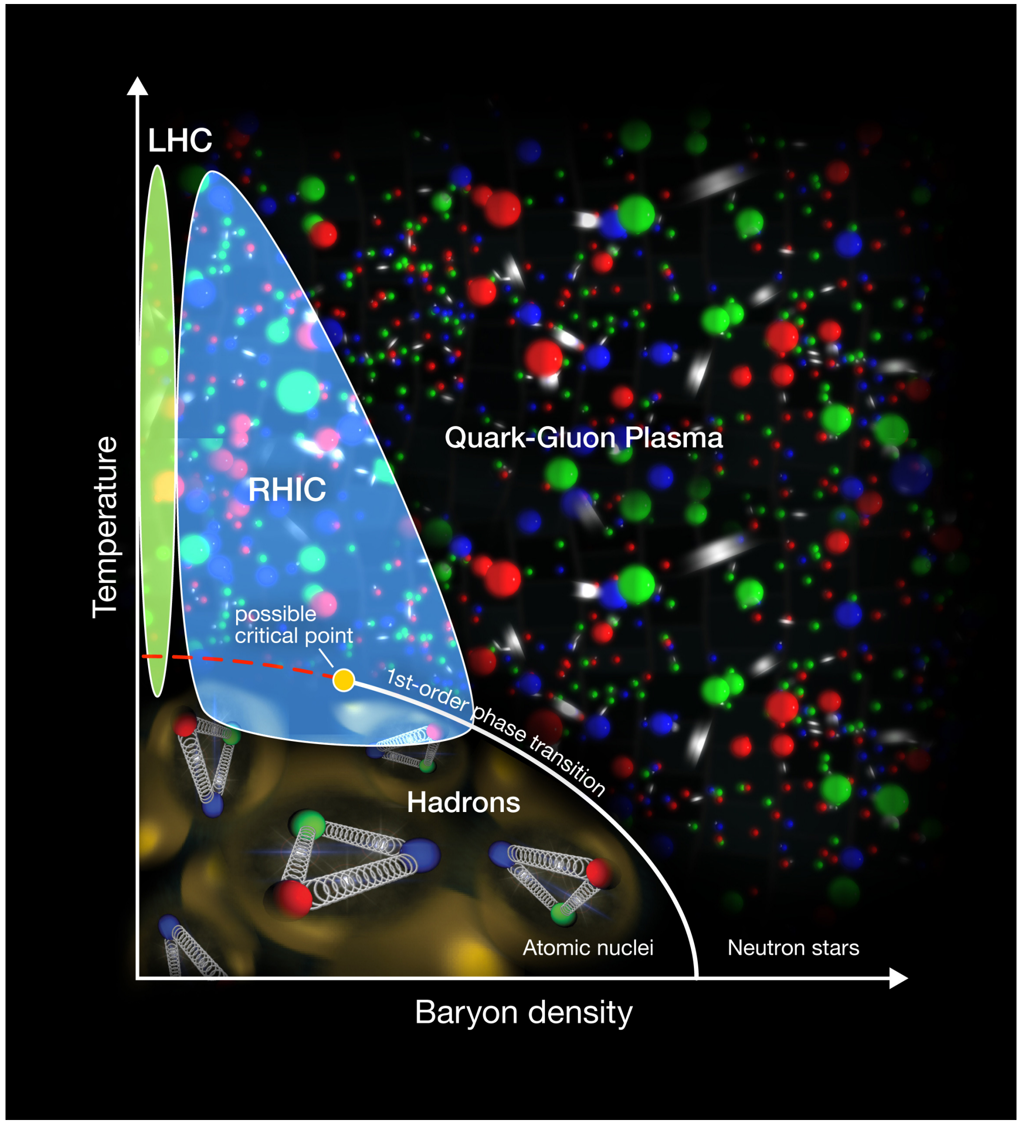
# Why Quantum Computing

- Usefulness of Euclidean lattice QCD at finite temperature: QCD matter crossover at zero chemical potential, transport coefficients (e.g. heavy quark diffusion)

- Challenges of Euclidean lattice QCD, sign problem (signal-to-noise ratio): fermion at finite chemical potential, real-time observables




- Use quantum computers to simulate quantum systems

*"Simulating physics with computers"* R. P. Feynman, 1982

- Quantum computing devices are developing quickly: superconducting circuits (IBM Q, Google, rigetti), trapped ion (Ion Q)

*"Quantum supremacy using a programmable superconducting processor" Google AI Quantum, 2019*

# Example 1: Schwinger Model in Thermal Bath

- **U(1) gauge theory in 1+1D** $\mathcal{L} = \overline{\psi}(iD^\mu \gamma_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

$$\gamma^0 = \sigma_z$$
$$\gamma^1 = -i\sigma_y$$

- **Hamiltonian formulation in axial gauge** $A_0 = 0$

**Staggered fermion** **Jordan-Wigner transform**

J. B. Kogut and L. Susskind
Phys. Rev. D11(1975) 395–408

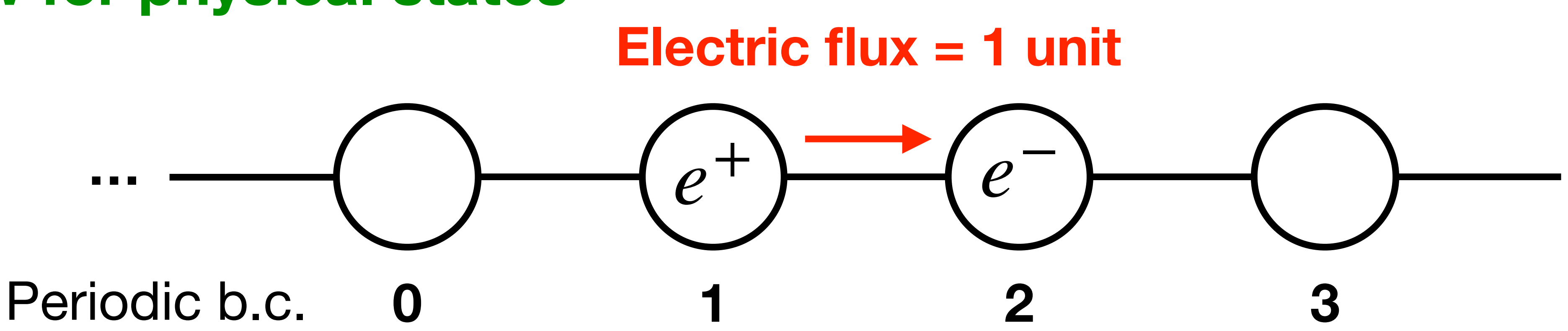
**E field and gauge link solved by ladder** $[E(n), U(n+1,n)] = eU(n+1,n)$

$E(n) \rightarrow \ell_n$, $U(n+1,n) \rightarrow L_n^+$

$$H_S = \frac{1}{2a}\sum_n \Big(\sigma^+(n)L_n^-\sigma^-(n+1) + \sigma^+(n)L_{n-1}^+\sigma^-(n-1)\Big) + \frac{m}{2}\sum_n (-1)^n\big(\sigma_z(n)+1\big) + \frac{ae^2}{2}\sum_n \ell_n^2$$

- **Impose Gauss law for physical states**

$$\partial_1 E = e\psi^\dagger\psi$$




Even sites: fermion, odd sites: anti-fermion

# Schwinger Model Coupled w/ Thermal Scalars

- **Hamiltonians** $H = H_S + H_E + H_I$

$$H_E = \int dx \left[ \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right]$$

$$H_I = \lambda \int dx\, \phi(x)\overline{\psi}(x)\psi(x) = \int dx\, O_E(x)O_S(x)$$

- **Trace out environment —> time evolution of reduced density matrix**

$$\rho_S(t) = \mathrm{Tr}_E\Big[U(t,0)(\rho_S(t=0)\otimes\rho_E)U^\dagger(t,0)\Big]$$

- **Lindblad equation in quantum Brownian motion limit** XY, 2102.01736

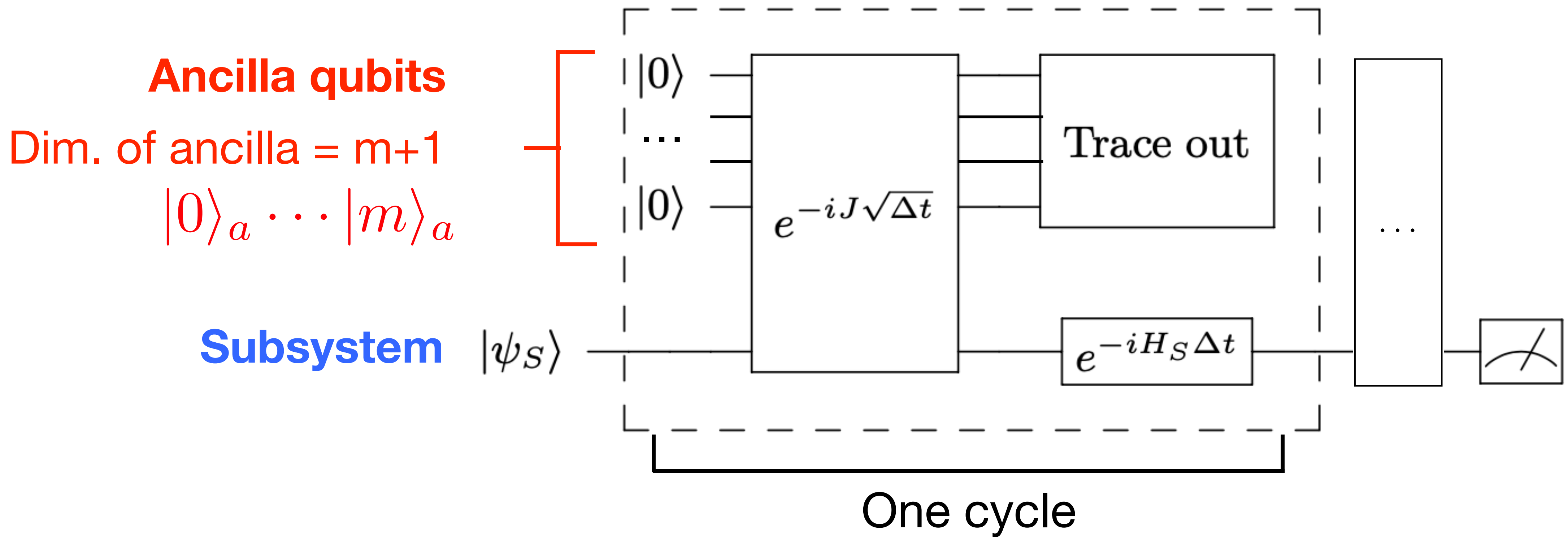
$$\frac{d\rho_S(t)}{dt} = -i\big[H_S, \rho_S(t)\big] + L\rho_S(t)L^\dagger - \frac{1}{2}\big\{L^\dagger L, \rho_S(t)\big\}$$

Only one Lindblad operator: $L = \sqrt{2aND(k_0=0, k=0)}\Big(O_S - \frac{1}{4T}\big[H_S, O_S\big]\Big)$

# Quantum Circuit for Non-unitary Evolution

- **Lindblad evolution = unitary evolution of subsystem coupled with ancilla with ancilla traced out**

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_{j=1}^{m} \left( L_j \rho_S L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho_S\} \right)$$




Reproduce Lindblad equation if expanded to linear order in $\Delta t$

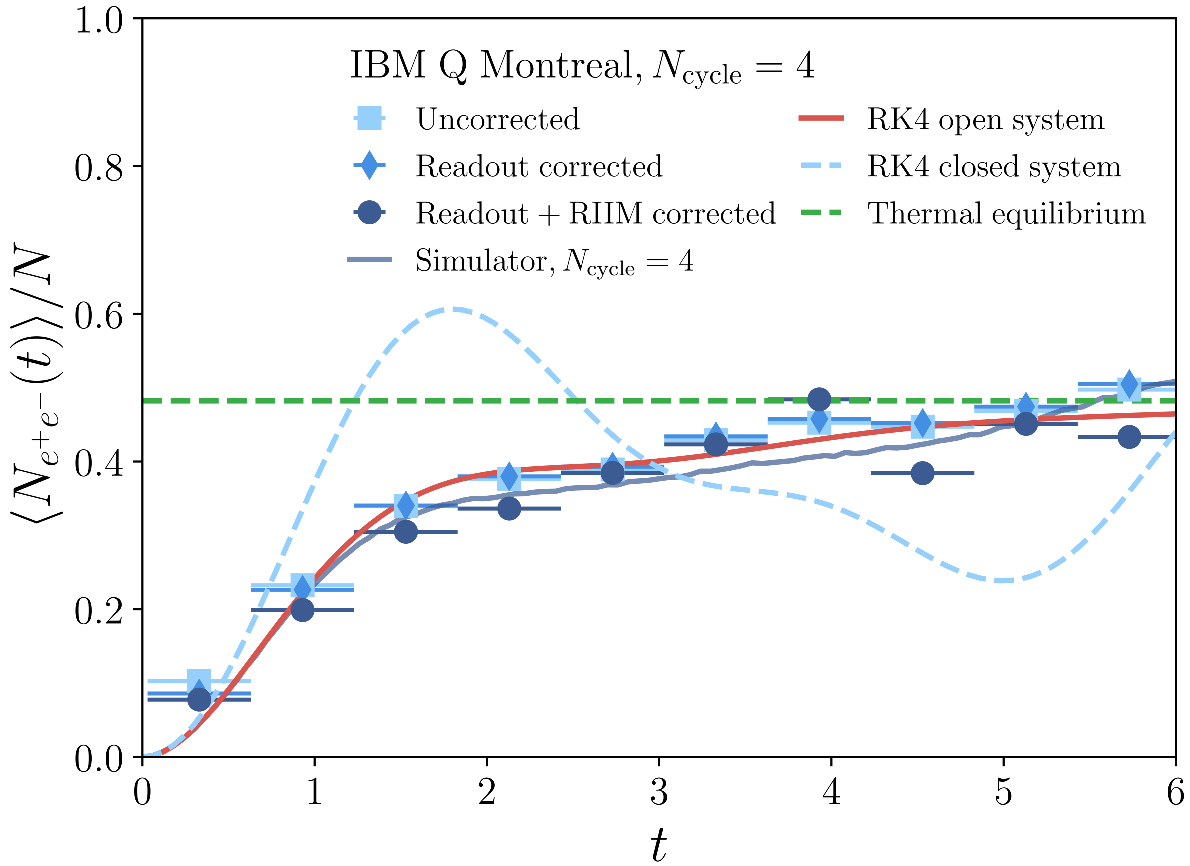
$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

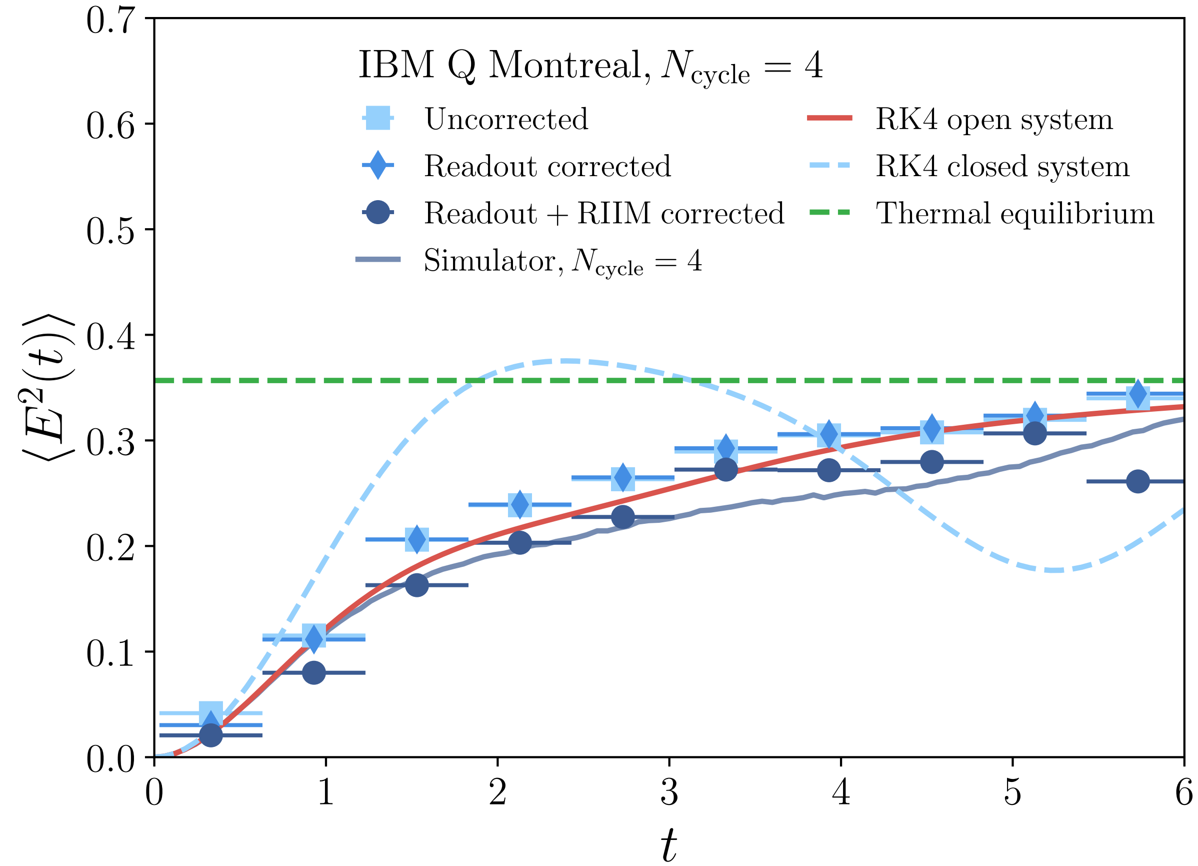
$$\rho(0) = |0\rangle_a \langle 0|_a \otimes \rho_S(0) = \begin{pmatrix} \rho_S(0) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

# Results from Real Quantum Devices

$N = 2$ spatial sites (4 fermion sites)

$$e = \frac{1}{a},\ m = \frac{0.1}{a},\ \beta = 0.1a,\ a = 1$$




Possible to run more cycles to reach closer to equilibrium

Provide a way to compute observables close to thermal equilibrium

W.A.de Jong, K.Lee, J.Mulligan, M.Ploskon
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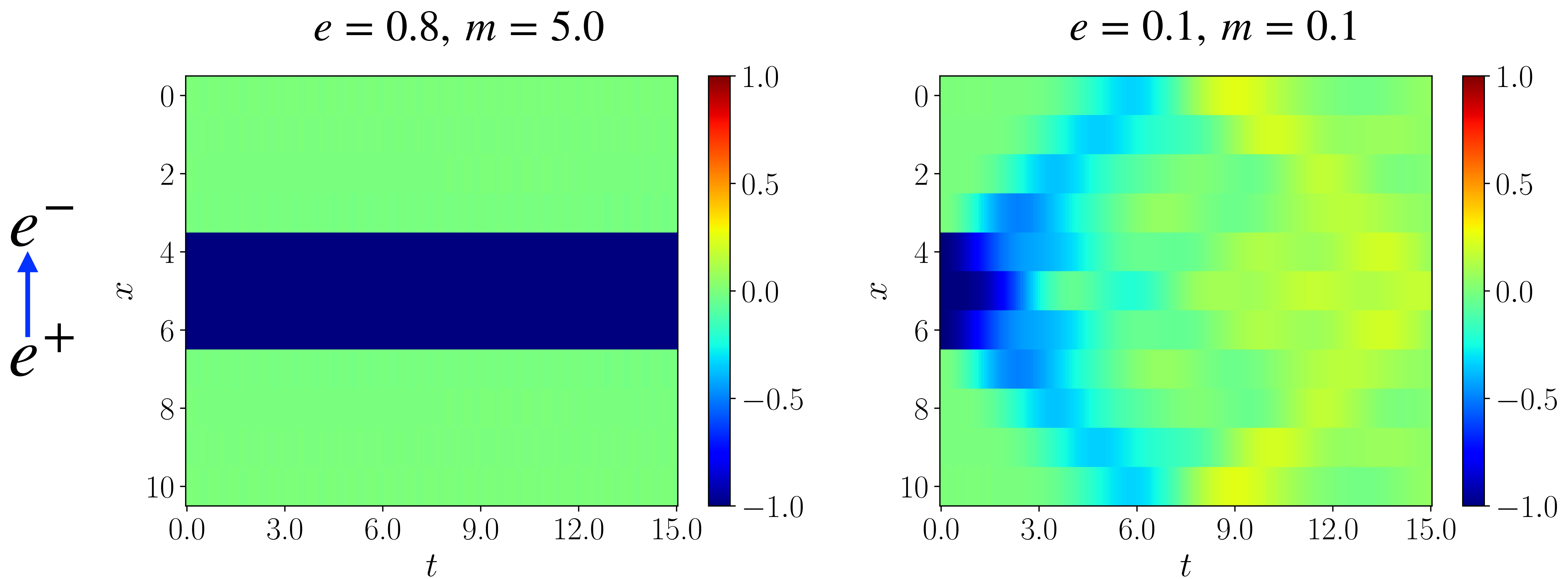
# String Breaking in Vacuum

- **Closed Schwinger model with open boundary condition**

Initialize a pair of electron and positron, time evolve

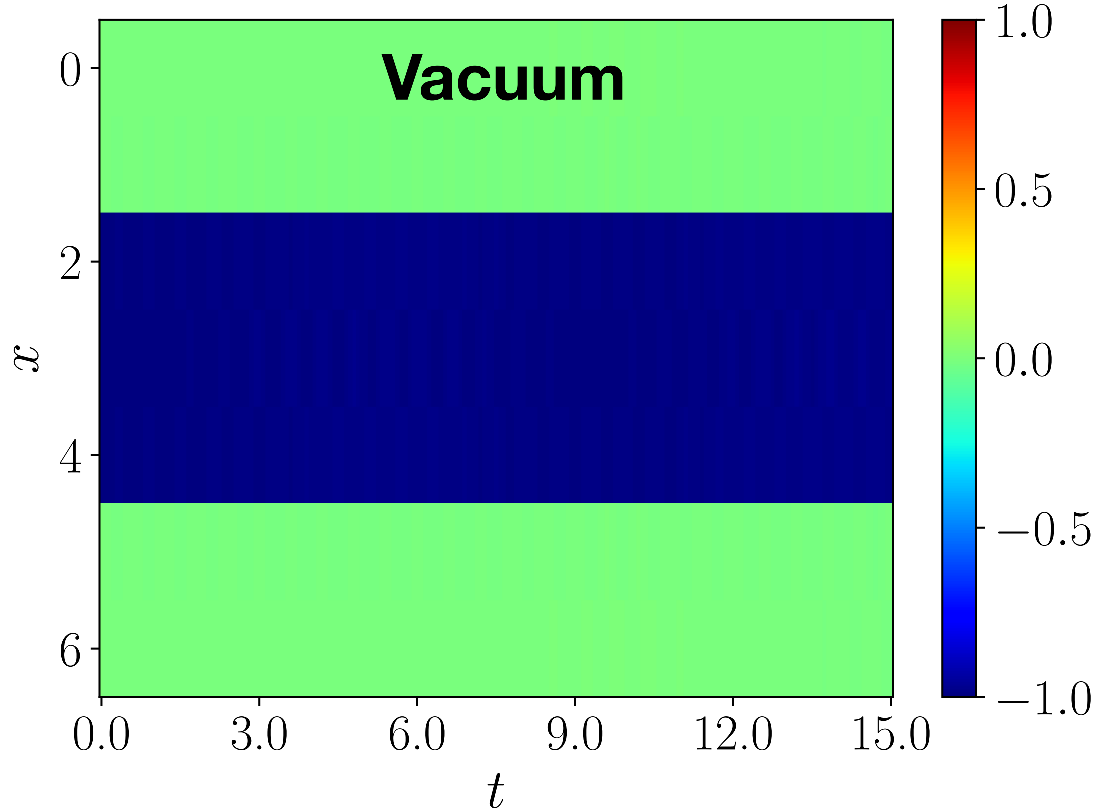
Calculate electric flux at each site (with vacuum contribution subtracted)





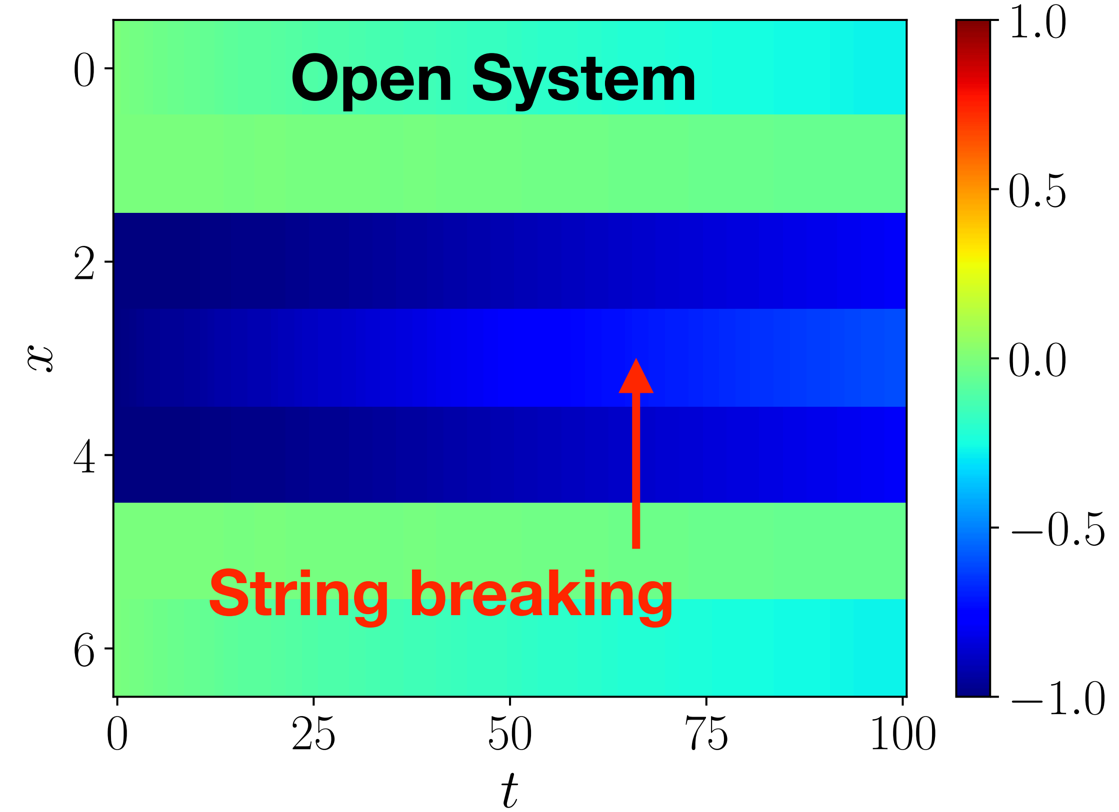

See also e.g. G. Magnifico, et al, 1909.04821

# String Breaking/Reconnection in Medium

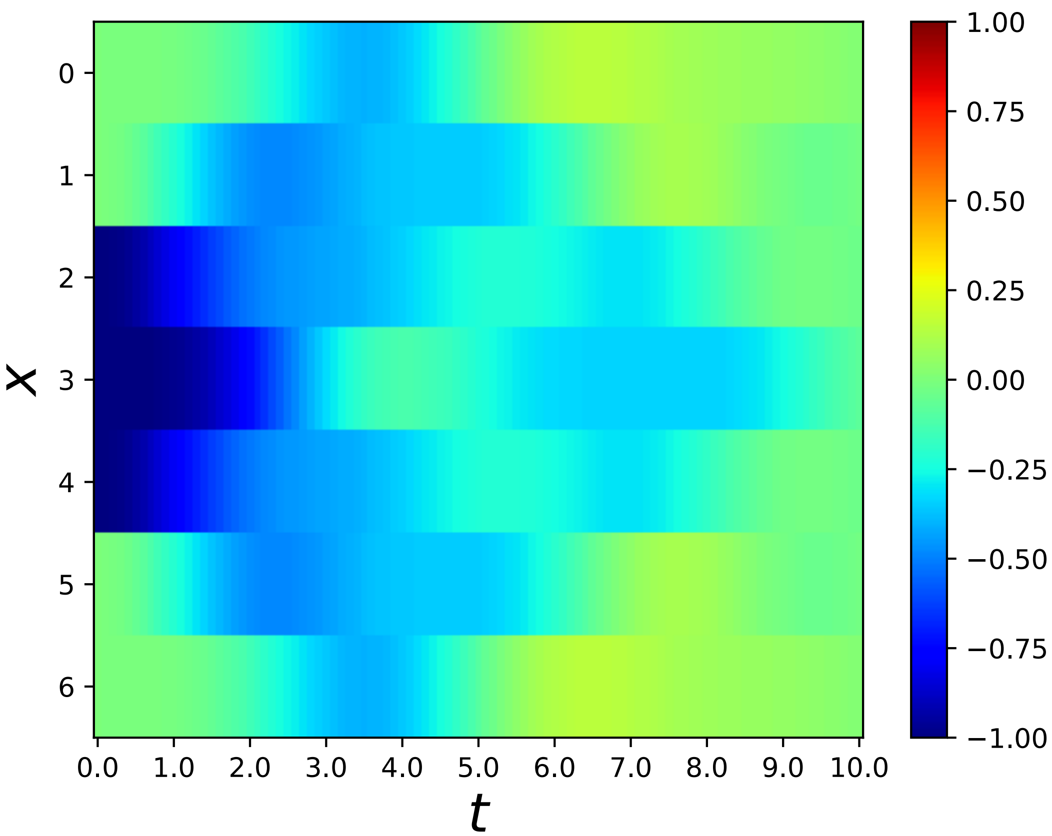



$$e = 0.8,\ m = 5.0$$

$$\beta = 0.1$$




String breaking



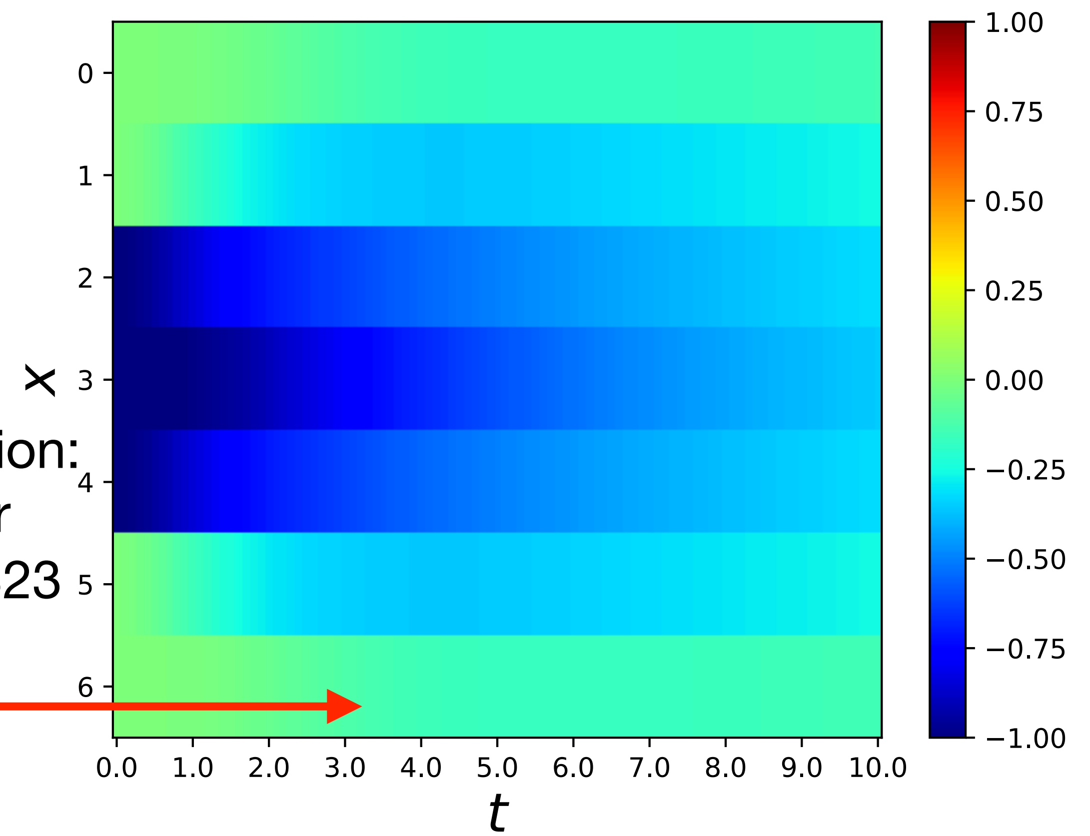

$$e = 0.36$$

$$m = -0.17$$

$$\beta = 0.5$$

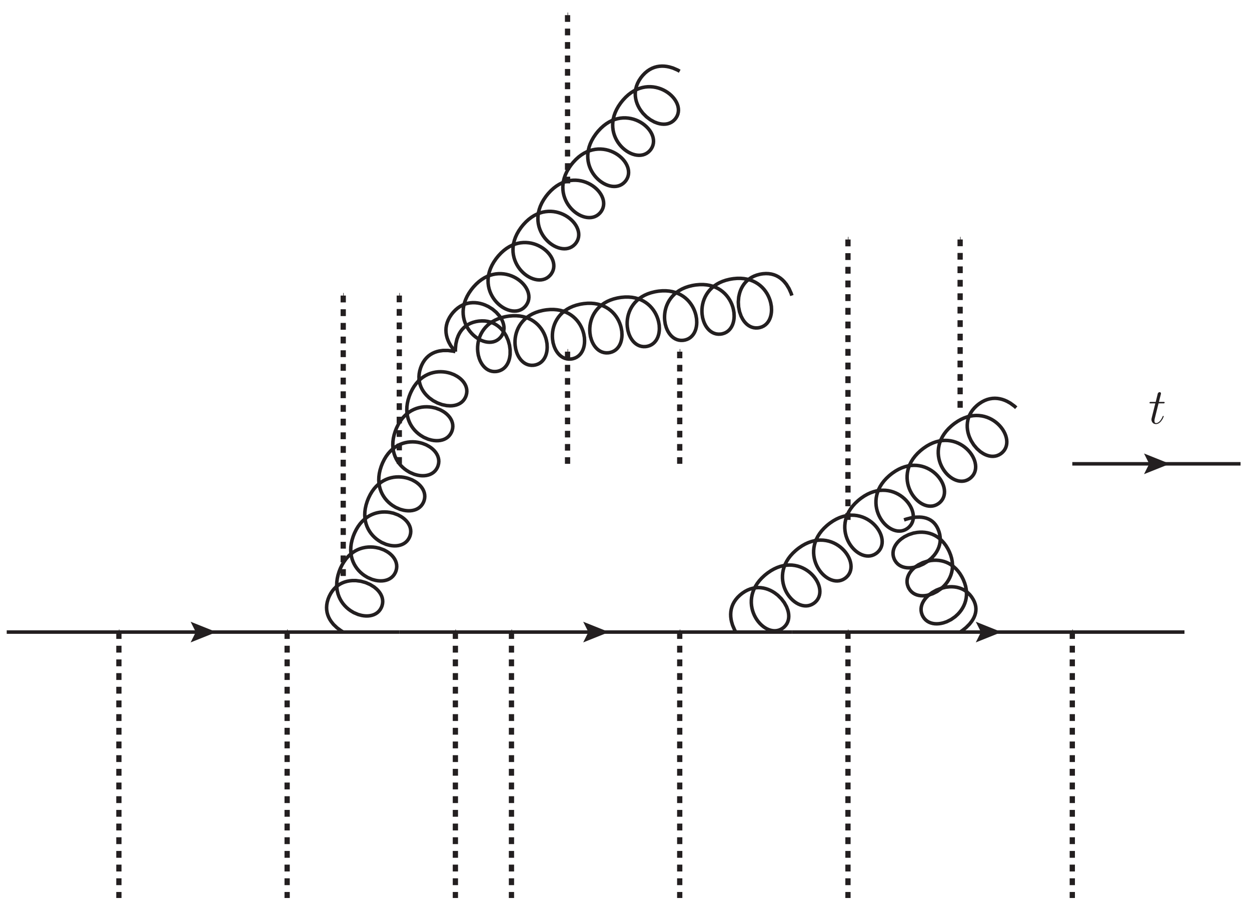
Quarkonium recombination:
R.L.Thews M.Schroedter
**J.Rafelski** hep-ph0007323

Reconnection



# Example 2: Quantum Simulation of Jet Quenching

- **In-medium radiation: Landau-Pomeranchuk-Migdal (LPM) effect**



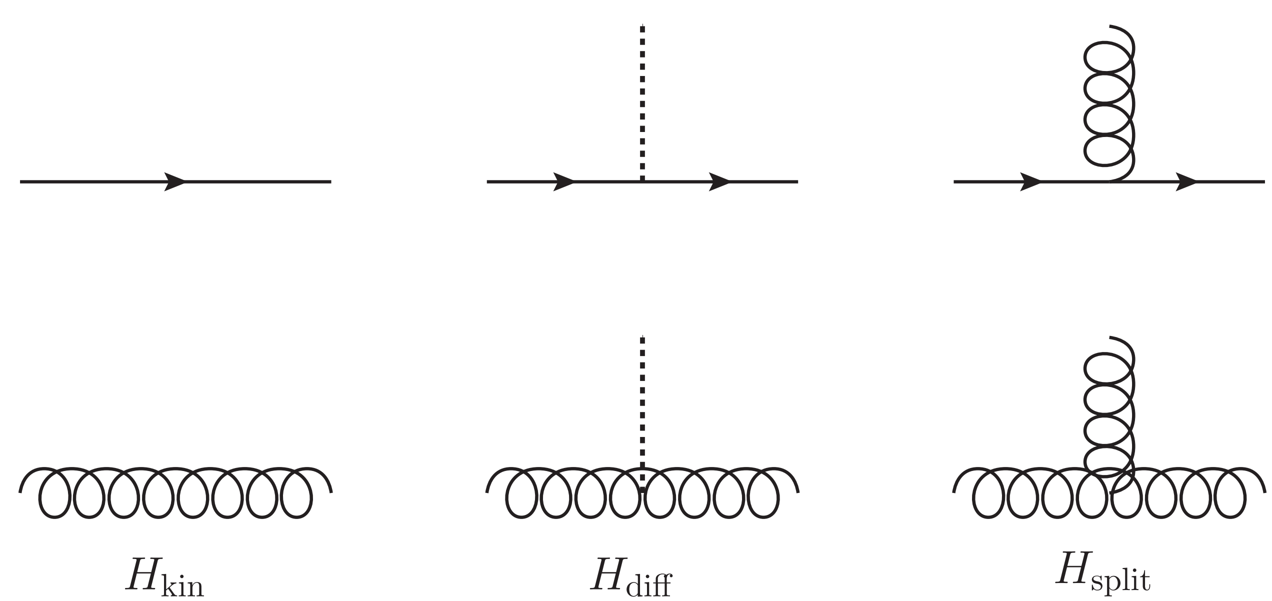

In-medium time evolution of high energy partons:
 kinetic evolution
 transverse momentum exchange
 parton splitting/recombination

Transverse momentum exchange changes kinetic energy randomly —> **random phase in time evolution** —> destructive interference suppresses radiation

- **Using light-front QCD Hamiltonian**

Diffusion Hamiltonian depends on classical background field $\bar{A}^{-a}(x^+, x_\perp)$

$$\left\langle \bar{A}^{-a}(x^+, k_\perp)\bar{A}^{-b}(y^+, -k_\perp)\right\rangle = \delta^{ab}\delta(x^+ - y^+)\gamma(k_\perp)$$



# Toy Model for LPM Effect

- **Scalar particles in 2+1D**

$$k^+ \in K^+_{\max}\{0.5, 1\}$$

$$k_\perp \in K^\perp_{\max}\{0, 1\}$$

$$|00\rangle : \quad k^+ = 0.5\,,\; k_\perp = 0$$

$$|01\rangle : \quad k^+ = 0.5\,,\; k_\perp = 1$$

$$|10\rangle : \quad k^+ = 1\,,\; k_\perp = 0$$

$$|11\rangle : \quad k^+ = 1\,,\; k_\perp = 1$$

- **LPM effect in cases with one initial particle, one splitting**

$$\Big|\underbrace{q_1}_{\text{separate 1- and 2-particle states}}\;\overbrace{q_2 q_3}^{\text{describe momenta of the 2nd particle}}\;\underbrace{q_4 q_5}_{\text{describe momenta of the 1st particle}}\Big\rangle$$

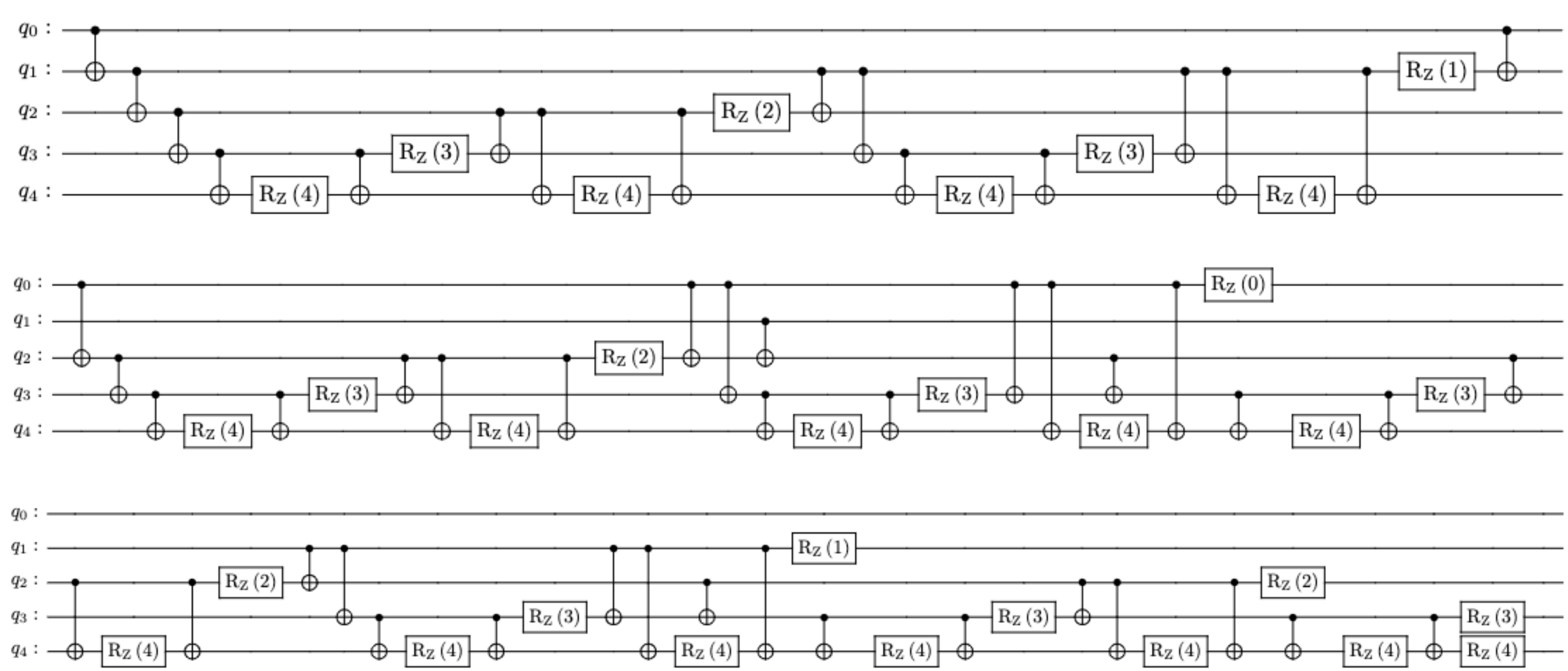
1-particle state $\quad |000q_4q_5\rangle$

2-particle state $\quad |1q_2q_3q_4q_5\rangle$

# Kinetic Energy Hamiltonian

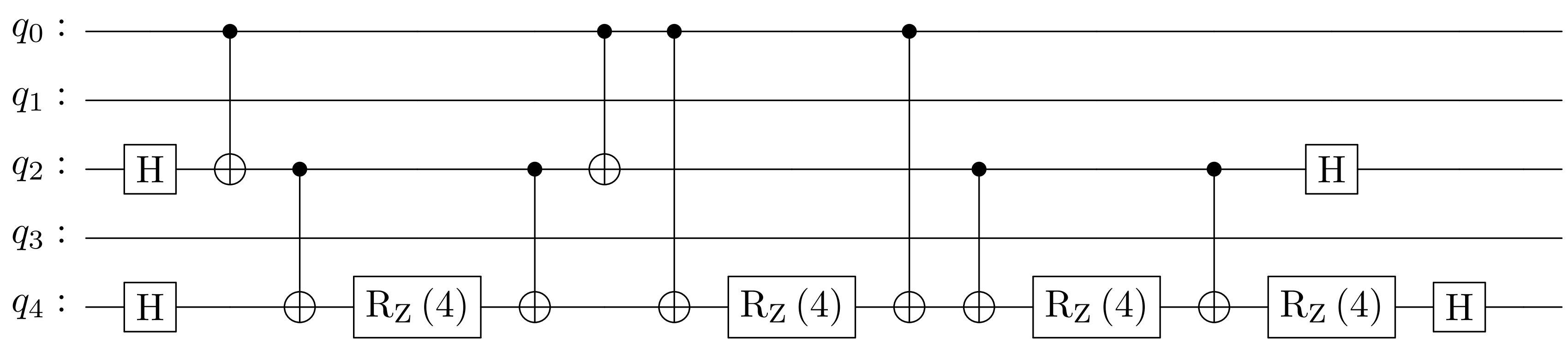
$$
\begin{aligned}
H_{\text{kin}} = \frac{(K^{\perp}_{\max})^2}{32K^{+}_{\max}}\Big( & -\sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^z \otimes \sigma_5^z + \sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^z - 3\sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z \otimes \sigma_5^z \\
& + 3\sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z + \sigma_1^z \otimes \sigma_2^z \otimes \sigma_4^z \otimes \sigma_5^z - \sigma_1^z \otimes \sigma_2^z \otimes \sigma_4^z + 3\sigma_1^z \otimes \sigma_2^z \otimes \sigma_5^z - 3\sigma_1^z \otimes \sigma_2^z \\
& - 3\sigma_1^z \otimes \sigma_3^z \otimes \sigma_4^z \otimes \sigma_5^z + 3\sigma_1^z \otimes \sigma_3^z \otimes \sigma_4^z - 9\sigma_1^z \otimes \sigma_3^z \otimes \sigma_5^z + 9\sigma_1^z \otimes \sigma_3^z - \sigma_1^z \otimes \sigma_4^z \otimes \sigma_5^z \\
& + \sigma_1^z \otimes \sigma_4^z - 3\sigma_1^z \otimes \sigma_5^z + 3\sigma_1^z + \sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^z \otimes \sigma_5^z - \sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^z + 3\sigma_2^z \otimes \sigma_3^z \otimes \sigma_5^z \\
& - 3\sigma_2^z \otimes \sigma_3^z - \sigma_2^z \otimes \sigma_4^z \otimes \sigma_5^z + \sigma_2^z \otimes \sigma_4^z - 3\sigma_2^z \otimes \sigma_5^z + 3\sigma_2^z + 3\sigma_3^z \otimes \sigma_4^z \otimes \sigma_5^z - 3\sigma_3^z \otimes \sigma_4^z \\
& + 9\sigma_3^z \otimes \sigma_5^z - 9\sigma_3^z - 7\sigma_4^z \otimes \sigma_5^z + 7\sigma_4^z - 21\sigma_5^z\Big)
\end{aligned}
$$

**Quantum circuit**



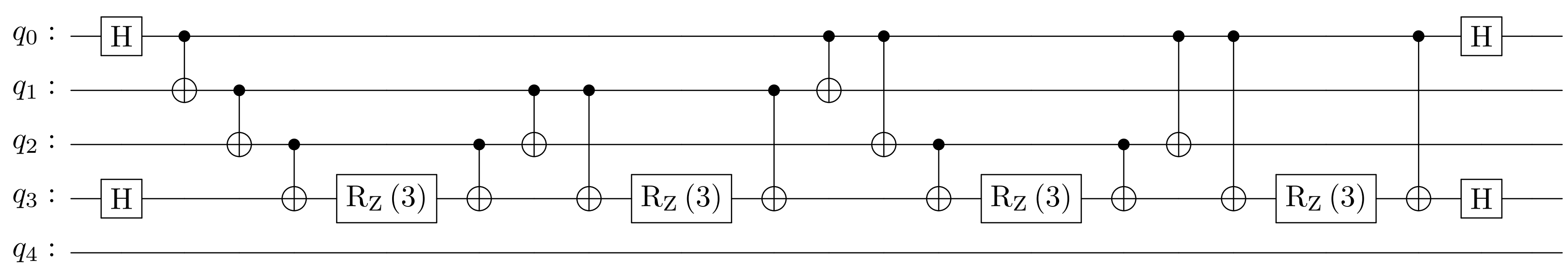
# Diffusion and Splitting Parts of Hamiltonian

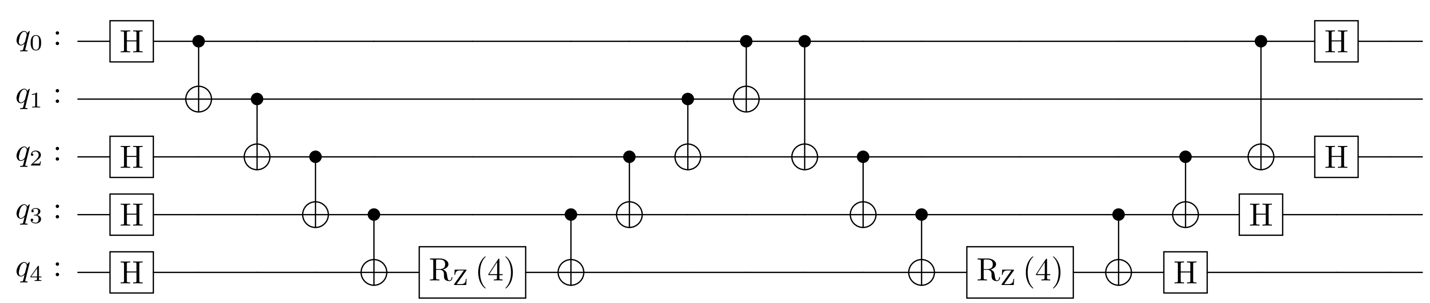
$$H_{\mathrm{diff}} = \frac{g_d}{2}\bar{A}^{-}(K_{\mathrm{max}}^{\perp})\Big( - \sigma_1^z \otimes \sigma_3^x \otimes \sigma_5^x + \sigma_1^z \otimes \sigma_5^x + \sigma_3^x \otimes \sigma_5^x + \sigma_5^x \Big)$$




$$H_{\mathrm{split1}} = \frac{g_s}{4}\Big(\sigma_1^x \otimes \sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^x + \sigma_1^x \otimes \sigma_2^z \otimes \sigma_4^x + \sigma_1^x \otimes \sigma_3^z \otimes \sigma_4^x + \sigma_1^x \otimes \sigma_4^x\Big)$$

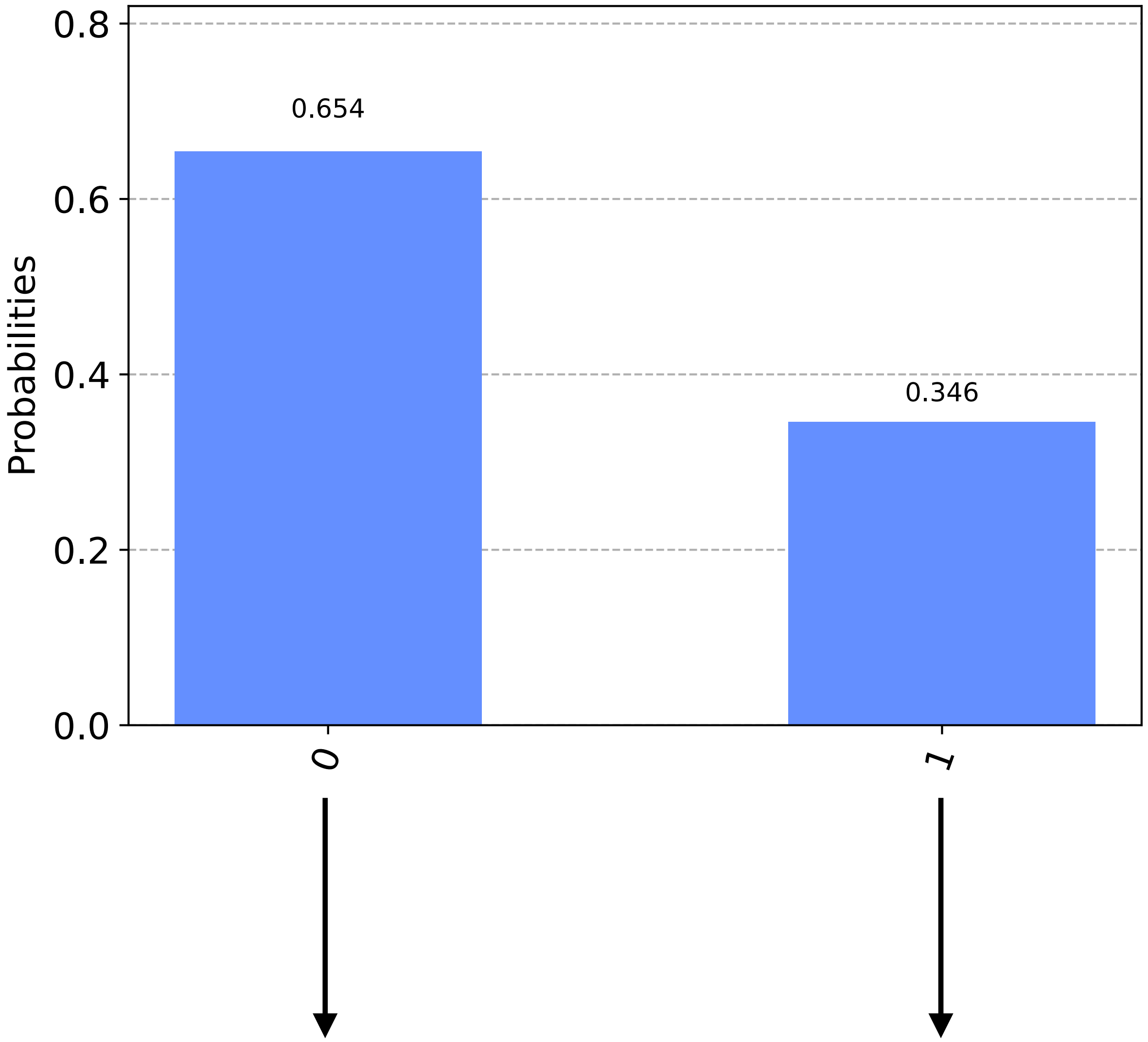
$$H_{\mathrm{split2}} = \frac{g_s}{2}\Big(\sigma_1^x \otimes \sigma_2^z \otimes \sigma_3^x \otimes \sigma_4^x \otimes \sigma_5^x + \sigma_1^x \otimes \sigma_3^x \otimes \sigma_4^x \otimes \sigma_5^x\Big)$$



# LPM Effect in Toy Model

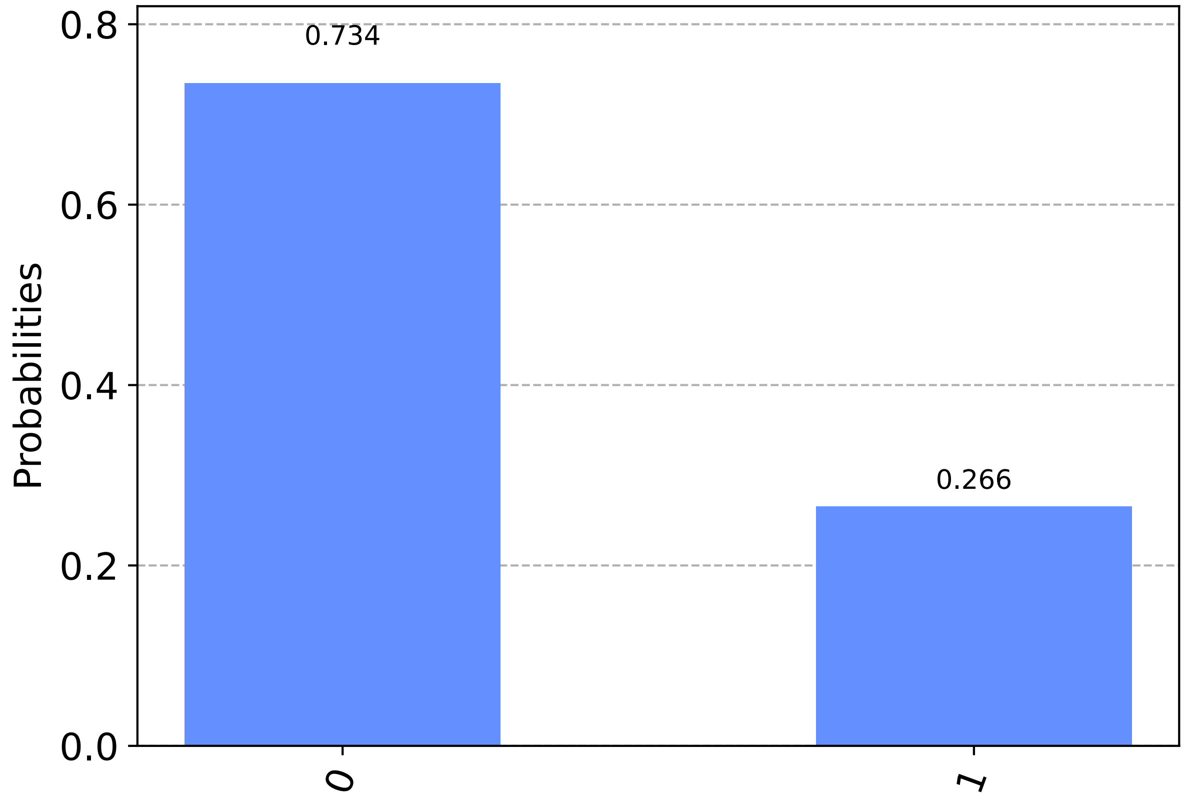
**Radiation probability in vacuum**




1-particle final state

2-particle final state

**Radiation probability in the medium**




500 quantum trajectories are averaged, each trajectory has a different set of classical background fields sampled

# Summary

- Quantum simulation for systems in thermal plasma
  - Schwinger model in thermal scalar bath, similar to quarkonium in quark-gluon plasma
  - Jet quenching in quark-gluon plasma, LPM effect seen in toy model
    with a few hundred fault-tolerant qubits, we can simulate LPM effect for more than two splittings and go beyond scope of state-of-the-art analyses