Finite volume effects on the QCD phase diagram

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Outline

- System size: motivations and implementations
- Effects on the phase diagram in various models
- Results in the ELSM
- Summary

What are the typical sizes?

- Typical size of the fireball in heavy ion collisions is a few fm.
- Neutron stars and compact stars built up from strongly interacting matter (with extra structure) with a size $\sim$ 10 km.
- Several models with finite (different) size.
- In field theoretical calculations (LSM, NJL, DS, etc) usually the size is infinite.

Why does it matter?

- It can be seen that the properties of the system can change significantly.
- One example: In the phase diagram of QCD the CEP (and the first order region) might disappear.

Volume dependence

Studying finite size effects.

- General consideration in statistical physics: Finite size scaling theory (FSS).
- Specially for strongly interacting systems: Studying the volume effects in our models.

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Volume dependence of phase transitions (thermodyn. singularities)

- No singularities in finite system.
- Scaling of thermodynamical quantities with the finite size.
- The direction of change may depend on the boundary condition.
- Physics may change qualitatively and quantitatively.

J. Cardy: Finite-Size Scaling (1988)

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Models with **finite volume**: Straightforward.

Models with **infinite volume**: How to mimic the volume effect?

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It is usual to have constraint in momentum space. →

- Discretization: $\int dp \to \sum_n$
- Low momentum cutoff: $\int_0^\infty dp \to \int_\lambda^\infty dp$

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Tested also in HRG model:

Karsch, Morita and Redlich: Phys. Rev. C 93, no.3, 034907 (2016)

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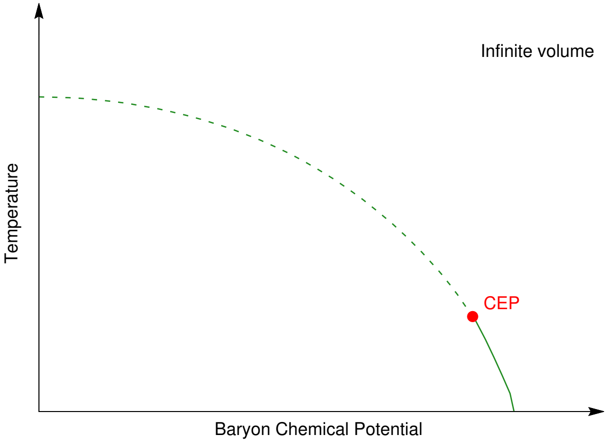
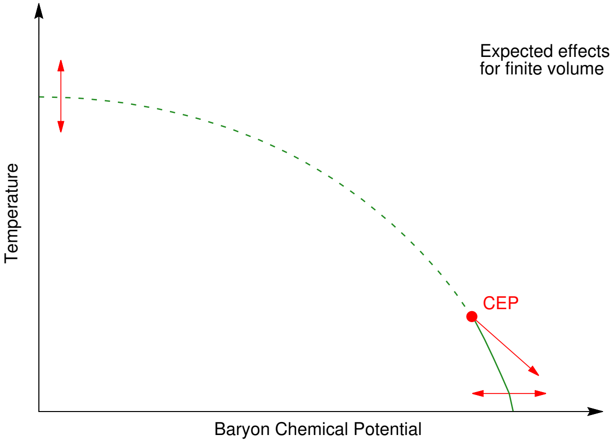
Other effects – eg. surface – are not taken into account.


Infinite volume
Temperature
CEP
Baryon Chemical Potential

Expected changes in the chiral phase diagram



There are already results in HRG, (P)NJL, (P)LSM, DS, etc. calculations.
For example for the phase diagram:

LSM

Palhares, Fraga and Kodama,
J. Phys. G **38**, 085101 (2011)

PNJL

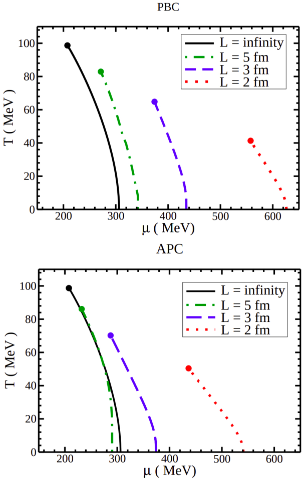
Bhattacharyya, Deb, Ghosh, Ray and Sur,
Phys. Rev. D **87**, no.5, 054009 (2013)

QM model FRG

Tripolt, Braun, Klein and Schaefer,
Phys. Rev. D **90**, no.5, 054012 (2014)

DS approach

Bernhardt, Fischer, Isserstedt and Schaefer,
Phys. Rev. D **104**, no.7, 074035 (2021)



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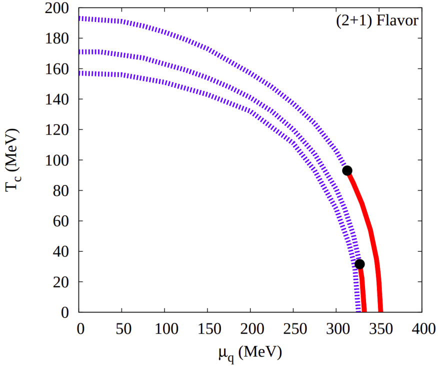

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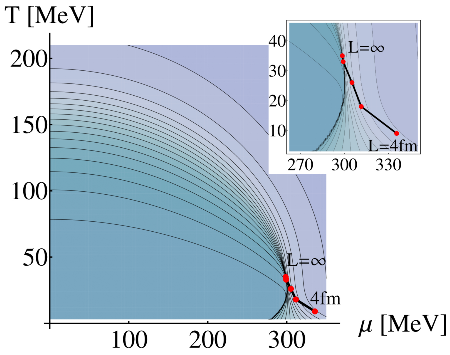

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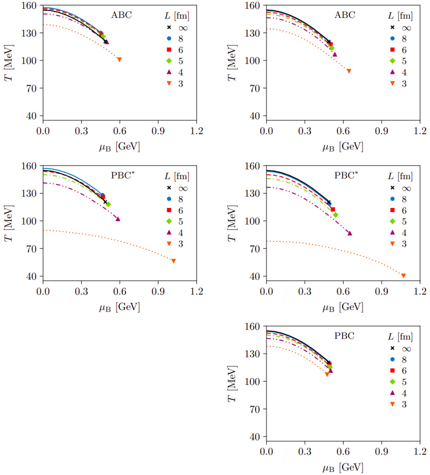

Vector and axial vector meson **E**xtended Polyakov **L**inear **S**igma **M**odel.
Effective model to study the phase diagram of strongly interacting matter at finite T and μ.
Phys. Rev. D 93, no. 11, 114014 (2016)

- **L**inear **S**igma **M**odel: "simple" quark-meson model
- **E**xtended: Vector and Axial vector nonets (besides to Scalar and Pseudoscalar)
  Isospin symmetric case: 16 mesonic degrees of freedom.
- Polyakov: Polyakov loop variables give 2 order parameters Φ, $\bar{\Phi}$.
- The mesonic Lagrangian $\mathcal{L}_m$ with chiral symmetry

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \rightarrow SU(2)_I \times U(1)_V$$

broken explicitly (and spontaneously) and with the axial anomaly taken into account

- $\mathcal{L}_m$ contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
  - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
  - Each meson-meson terms upto 4th order that are allowed by the chiral symmetry.
- Constituent quarks ($N_f = 2+1$) in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu \partial_\mu - g_F(S - i\gamma_5 P) - g_V \gamma^\mu (V_\mu + \gamma_5 A_\mu)\right)\psi \tag{1}$$

  In the 2016 version $g_V = 0$ was used. **Phys. Rev. D 104, 056013 (2021)**

- SSB with nonzero vev. for scalar-isoscalar sector $\phi_N,\ \phi_S$.
  $\Rightarrow m_{u,d} = \frac{g_F}{2}\phi_N,\ m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.
- Mean field level effective potential $\rightarrow$ the meson masses and the thermodynamics are calculated from this.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \mathrm{diag}(m_u, m_d, m_s)\right)\psi$$

  Functional integration over the fermionic fields.
  The momentum integrals are renormalized.

- Polyakov loop potential.

$$\Omega(T, \mu_q) = U_{Cl} + \mathrm{tr}\int_K \log\left(iS_0^{-1}\right) + U(\Phi, \bar{\Phi}) \tag{2}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{3}$$

Curvature meson masses:

$$M^2_{ab} = \left.\frac{\partial^2\Omega}{\partial\varphi_a\partial\varphi_b}\right|_{\{\varphi_i\}=0} \tag{4}$$

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Functional integration over the fermionic fields.
The momentum integrals are renormalized. Low momentum cut!

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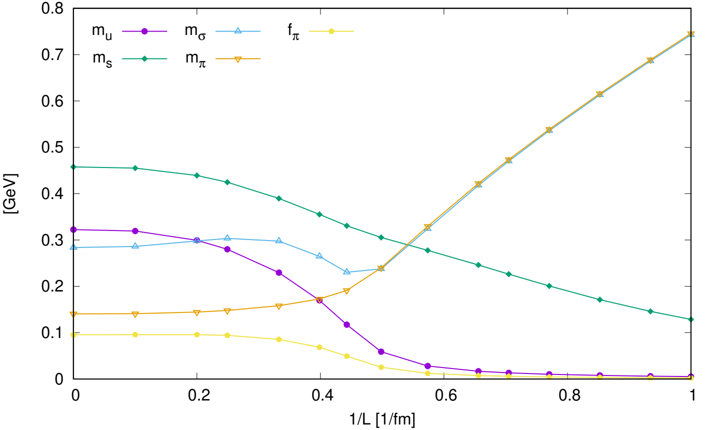
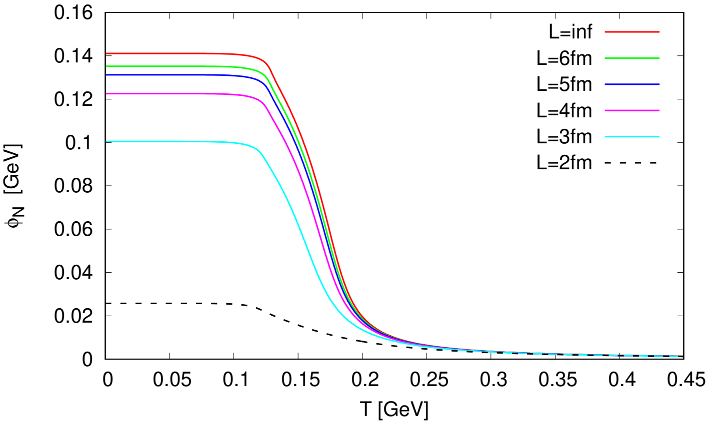
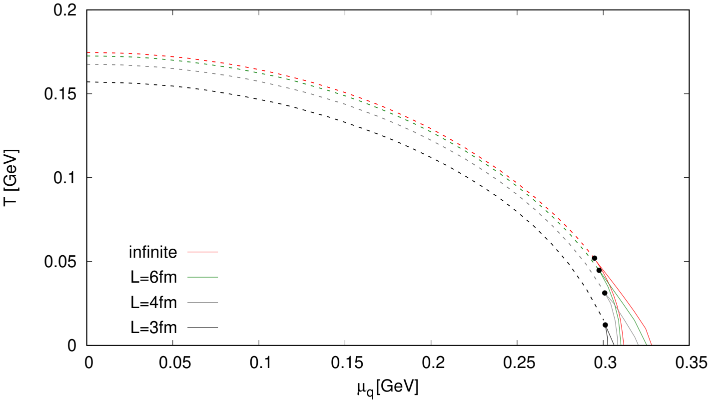
The order parameters $\phi_{N,S}$ scales with the size below ~ 5 fm.



The initial (vacuum) value of the order parameter ϕ_N drops rapidly under ~ 2.5 fm.



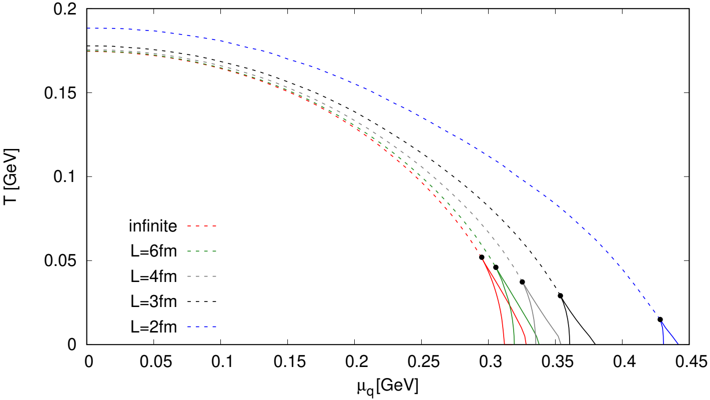

0.2
0.15
0.1
0.05
0
T [GeV]
infinite
L=6fm
L=4fm
L=3fm
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
μ_q[GeV]

- Finite volume effects on thermodynamics and the phase diagram of strong interaction can be studied with constraints in momentum space.
- The meson masses and other physical quantities start to significantly change with the system size (for ELSM under ~ 5 fm).
- The critical end point moves to lower temperature and higher chemical potential with the decreasing size.
- The CEP and the first order region disappear at a small finite size (~ 2.5 fm for ELSM).

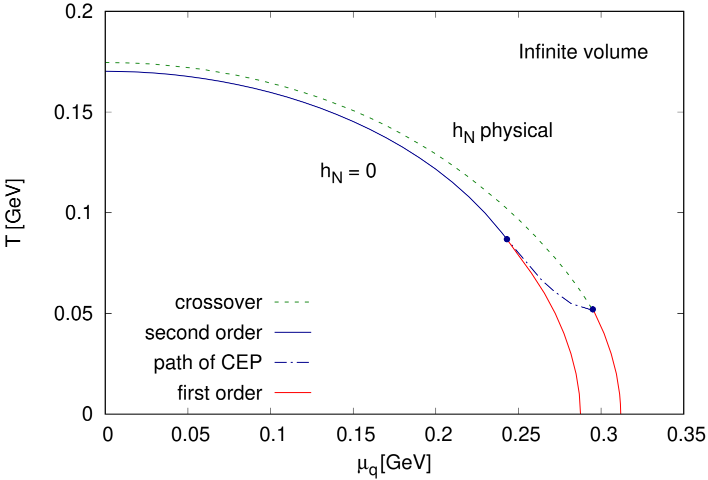
- Further study of the thermodynamics.
- Further study of the scaling of physical quantities.
- Open questions about the phase diagram.

Thank you!

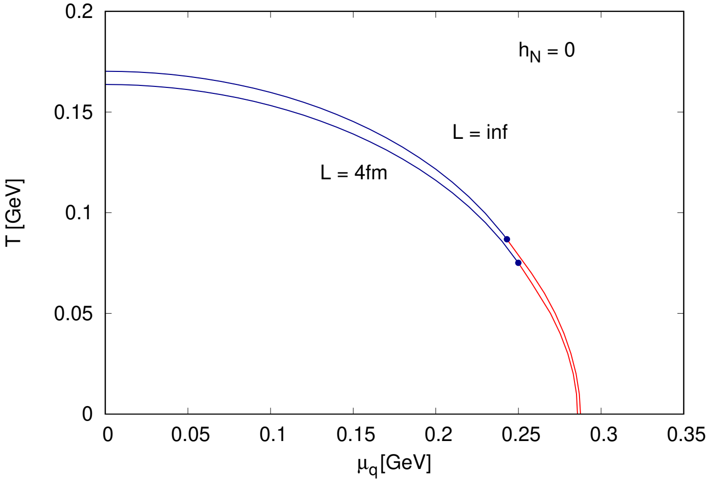
Backup: Phase diagram and critical end point – $\phi_{N/S}$ fixed



Backup: Phase diagram in the chiral limit



Backup: Finite volume effect in the chiral limit



Backup: Thermodynamics – pressure – energy density