Dissipative relativistic dense hadro-partonic fluids with strangeness production

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Motivation

Long, long ago, but in this Galaxy the paper appeared
Entropy production in relativistic hydrodynamics
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This is just to begin with

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  - at forward rapidity
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There is a need for

Hydro model with dynamical chemical reactions and multi-fluid included

However - Flows

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The "mean-free-path" criterion is replaced by the requirement that gradients around some local equilibrium are small when compared to system temperature.

Does the requirement of small gradients limit the applicability of fluid dynamics to the near-equilibrium regime – this is THE question.

However - Flows

It's possible to generalize the notion of fluid dynamics to systems far from local equilibrium, where normalized gradients are not only of order unity, but even large.

e.g. M. P. Heller and M. Spalinski, Phys. Rev. Lett. "Hydrodynamics Beyond the Gradient Expansion: Resurgence and Resummation", 115, 072501 (2015); P. Romatschke, "Relativistic Fluid Dynamics Far From Local Equilibrium", Phys. Rev. Lett. 120, 012301 (2018)

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Numerical experiments indicate that fluid dynamics can match exact results even if the gradient corrections (normalized by temperature) are of order unity.

e.g. P. M. Chesler and L. G. Yaffe, Phys. Rev. D 82, 026006 (2010); M. P. Heller, R. A. Janik, and P. Witaszczyk, Phys. Rev. Lett. 108, 201602 (2012); B. Wu and P. Romatschke, Int. J. Mod. Phys. C 22, 1317 (2011).

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e.g. G. Aad et al. (ATLAS Collaboration), Phys. Rev. Lett. 116, 172301 (2016); V. Khachatryan et al. (CMS Collaboration), Phys. Rev. Lett. 116, 172302 (2016); P. Bozek, Eur. Phys. J. C 71, 1530 (2011); K. Werner, I. Karpenko, and T. Pierog, Phys. Rev. Lett. 106, 122004 (2011); R. D. Weller and P. Romatschke, Phys. Lett. B 774, 351 (2017).

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However - Strangeness Production

Strangeness production in plasma is dominated by gluons

$$g + g \to s + \bar{s}$$

with small contributions of quarks annihilation

$$u + \bar{u} \to s + \bar{s}\,; \qquad d + \bar{d} \to s + \bar{s}$$

In the local rest frame, we obtain:

$$\partial_t \rho_i \approx \mathcal{J}_i \equiv \sum_j [G_{i\leftarrow j}\rho_j - L_{j\leftarrow i}\rho_i]$$

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However - Strangenes Production

Due to the high temperature involved in the QGP phase, the thermal production of $\bar{s}s$ becomes possible. Another source of enhancement of pairs comes from the process of Pauli blocking of the light quarks. As more and more light quarks are produced in the collision, they fill up the available low energy levels and it becomes favourable to create pairs. The production of anti-strange and multi-strange baryons will be enhanced as well.

Even if an enhancement of strangeness occurs in the QGP, there are some difficulties in quantifying the magnitude of this enhancement. The lifetime of the QGP phase is unknown, it is impossible to compute the actual values of particle production. An enhancement is expected to occur in A+A collisions compared to scaled p+p collisions (secondary collisions). Therefore, the unanswered theoretical question is what is the normal enhancement expected in A+A collisions. This can only be extracted from experimental results.

Dissipative hydro primer

The energy-momentum tensor for an imperfect fluid

$$T^{\mu\nu} \equiv (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} + \delta T^{\mu\nu} = T^{\nu\mu} \quad ,$$

The evolution of the system is described by:

$$\partial_\mu T^{\mu\nu} = f^\nu \quad ,$$

Continuity equations

$$\partial_\mu \rho_i^\mu \equiv \partial_\mu \rho_i u^\mu = 0 \quad ,$$

Dissipative terms to vanish in the local rest frame

$$T^{00}_{(0)} = \epsilon \quad \longrightarrow \quad \delta T^{00}_{(0)} = 0 \quad \longrightarrow \quad u_\mu u_\nu \delta T^{\mu\nu} = 0 \quad ,$$

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Diffusion, particle production, chemical reactions

In order to incorporate diffusion and particle production or chemical reactions, we now generalize

$$\partial_\mu \tilde{\rho}_i^\mu \equiv \partial_\mu(\rho_i^\mu + \triangle^{\mu\nu}\partial_\nu \mathcal{R}_i) = \mathcal{J}_i \quad ,$$

$$\triangle_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu$$

In the local rest frame:

$$\mathcal{J}_i = \sum_j [G_{i\leftarrow j}\rho_j - L_{j\leftarrow i}\rho_i] \quad ,$$

The diffusion term $\propto \partial\mathcal{R}_i$ will be determined consistently with the Second Law of Thermodynamics

Entropy production

Since the entropy of the QGP phase at hadronization determines the observable particle multiplicity, it is crucial to understand its production.

The entropy production in a dissipative relativistic fluid is given by:

$$T\partial_\mu s^\mu = -\frac{\kappa}{T}\mathcal{Q}_\mu \triangle^{\mu\nu}\mathcal{Q}_\nu + \frac{\eta}{2}\mathcal{W}_\alpha^\beta \triangle_\beta^\gamma \mathcal{W}_\gamma^\delta \triangle_\delta^\alpha + \zeta(\partial_\mu u^\mu)^2 \quad ,$$

$$u^2 = 1(\hbar = c = k_B = 1)$$

$$\mathcal{Q}_\mu \equiv \partial_\mu T - Tu^\nu \partial_\nu u_\mu \quad \text{, heat-flow four-vector}$$

$$\mathcal{W}_{\mu\nu} \equiv \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3} g_{\mu\nu}\partial_\gamma u^\gamma \quad \text{, shear tensor}$$

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Entropy Production

The (local) equilibrium relations,

$$-PV = \Omega(T, V, \mu_i) = U - TS - \sum_i \mu_i N_i \quad ,$$

Duhem-Gibbs relation for the densities

$$Ts = \epsilon + P - \sum_i \mu_i \rho_i \quad .$$

The First Law of Thermodynamics imply:

$$0 = T\partial_\mu \tilde{s}^\mu + \sum_i \mu_i(\mathcal{J}_i - \partial_\mu \triangle^{\mu\nu}\partial_\nu \mathcal{R}_i) + u_\nu \partial_\mu \delta T^{\mu\nu} \quad ,$$

with an auxiliary entropy four-current, $\tilde{s}^\mu \equiv s u^\mu$, and $\rho_i^\mu \equiv \rho_i u^\mu$

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Entropy four-current

The proper entropy four-current:

$$s^\mu \equiv \tilde{s}^\mu + T^{-1} u_\nu \delta T^{\mu\nu} + \sum_i \mathcal{L}_i \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i \quad .$$

An imperfect fluid evolving under the influence of conservative external forces

$$\begin{aligned} T\partial_\mu s^\mu \quad = \quad & -\sum_i \mu_i \mathcal{J}_i - T^{-1}(\partial_\mu T) u_\nu \delta T^{\mu\nu} + (\partial_\mu u_\nu)\delta T^{\mu\nu} \\ & + \sum_i (\mu_i + T\mathcal{L}_i)\partial_\mu \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i + T\sum_i (\partial_\mu \mathcal{L}_i) \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i \quad , \end{aligned}$$

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Second Law of Thermodynamics

$$\sum_i \mu_i \mathcal{J}_i \leq 0 \quad .$$

$$\delta T^{\mu\nu} = \kappa(\triangle^{\mu\gamma}u^{\nu} + \triangle^{\nu\gamma}u^{\mu})\mathcal{Q}_{\gamma} + \eta\triangle^{\mu\gamma}\triangle^{\nu\delta}\mathcal{W}_{\gamma\delta} + \zeta\triangle^{\mu\nu}\partial_{\gamma}u^{\gamma} \quad ,$$

$$\begin{aligned} \mathcal{Q}_{\mu} &\equiv \partial_{\mu}T - Tu^{\nu}\partial_{\nu}u_{\mu} \quad , \\ \mathcal{W}_{\mu\nu} &\equiv \partial_{\mu}u_{\nu} + \partial_{\nu}u_{\mu} - \frac{2}{3}g_{\mu\nu}\partial_{\gamma}u^{\gamma} \quad . \end{aligned}$$

$$\mathcal{L}_i \equiv -\frac{\mu_i}{T} \quad , \quad \mathcal{R}_i \equiv \sum_j \sigma_{ij}\frac{\mu_j}{T} \quad ,$$

Chemo- and Thermo-diffusion

The diffusion current

$$\vec{j}_i \equiv -\nabla\mathcal{R}_i = -\sum_j \frac{\sigma_{ij}}{T}(\nabla\mu_j - \frac{\mu_j}{T}\nabla T) \quad .$$

involves chemo- and thermo-diffusion contributions

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Entropy Production

All sources of entropy production

$$T\partial_\mu s^\mu = -\sum_i \mu_i \mathcal{J}_i - T\sum_{i,j} \sigma_{ij}(\partial_\mu \frac{\mu_i}{T})\triangle^{\mu\nu}(\partial_\nu \frac{\mu_j}{T})$$
$$-\frac{\kappa}{T}\mathcal{Q}_\mu \triangle^{\mu\nu}\mathcal{Q}_\nu + \frac{\eta}{2}\mathcal{W}_\alpha^\beta \triangle_\beta^\gamma \mathcal{W}_\gamma^\delta \triangle_\delta^\alpha + \zeta(\partial_\mu u^\mu)^2 \quad .$$

Population equation

Gain term

$$G_s \equiv \langle \sigma_s^{gg} v_{gg} \rangle (\bar{\rho}_g)^2 + \sum_{q=u,d} \langle \sigma_s^{q\bar{q}} v_{q\bar{q}} \rangle_{th} \bar{\rho}_q \bar{\rho}_{\bar{q}} \equiv \mathcal{G}$$

Loss term

$$L_s \equiv [\langle \sigma_{\mathrm{g}}^{s\bar{s}} v_{s\bar{s}} \rangle + \langle \sigma_q^{s\bar{s}} v_{s\bar{s}} \rangle] \rho_s^2 \equiv \mathcal{L} \rho_s^2$$

Source term for strangeness

$$\mathcal{J}_s = \partial_t \rho_s = \mathcal{G} - \mathcal{L} \rho_s^2$$

Continuity equation is replaced by

$$\partial_t \rho_s = \mathcal{G} - \mathcal{L} \rho_s^2$$

Event by event

Taking event by event averages $\langle\langle\cdot\rangle\rangle$

$$\partial_t \langle\langle \rho_s \rangle\rangle = \mathcal{G} - \mathcal{L} \langle\langle \rho_s^2 \rangle\rangle$$

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If $\langle\langle \rho_s \rangle\rangle^2 \gg \delta\rho_s^2$

$$\rho_s(\tau) = \sqrt{\frac{\mathcal{G}}{\mathcal{L}}} \tanh(\tau\sqrt{\mathcal{G}\mathcal{L}})$$

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In the local rest frame + x-dependence!

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Conclusions

- the entropy producing mechanisms of chemo- and thermo-diffusion together with the contribution of particle production is introduced
- we have an imperfect relativistic fluid with composition changing processes
- chemical equilibrium

$$J_i = 0$$

- thermal equilibrium

$$\partial_\mu s^\mu = 0$$

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Problems

- Potential instabilities due to the first-order approach
- Numerical implementation
- Additional dissipative processes: surface radiation