BBN fusion reactions in damped electron-positron plasma

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This work is base on linear respond method that is developed by
M.Formanek, C.Grayson, J.Rafelski and B.Müller

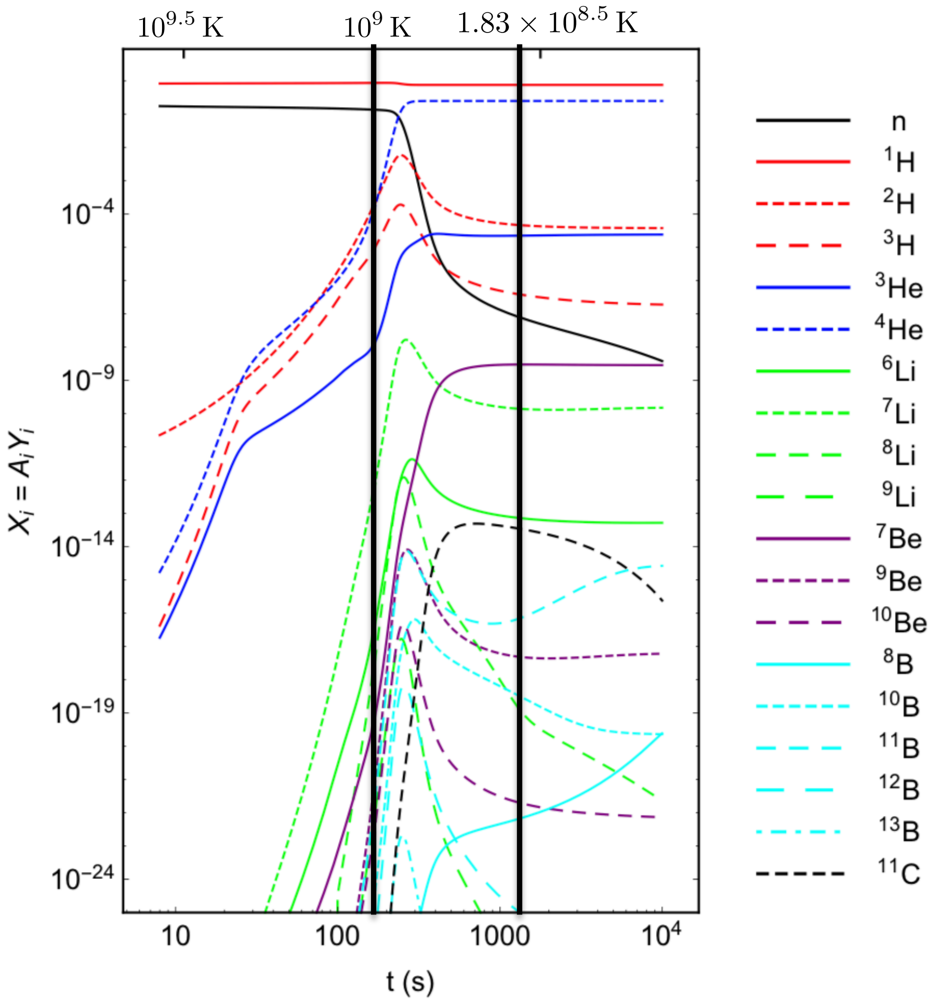
M. Formanek, C. Grayson, J. Rafelski and B.Muller,``Current-conserving relativistic linear response for collisional plasmas,'' Annals Phys. 434, 168605 (2021)

Unstable Beryllium-8 in BBN

The Big Bang Nucleosynthesis produce the primordial abundances of the light elements in early universe around the typical temperature $\boxed{10^9\mathrm{K} = 86.17\mathrm{keV}}$ $\quad 50\mathrm{keV} = 1.83 \times 10^{8.5}\mathrm{K}$

C.Pitrou, A.Coc, J.P.Uzan and E.Vangioni, ``Precision big bang nucleosynthesis with improved Helium-4 predictions,'' Phys. Rept. 754, 1-66 (2018)

$$T_{\mathrm{BBN}} = 86.17 - 50\mathrm{keV}$$



- The unbound nature of ^{8}Be creates a bottleneck in nucleosynthesis which prevent the production of heavy elements in BBN

$$^8Be \rightarrow {}^4He + {}^4He \qquad \tau = 8.19 \times 10^{-17}\mathrm{sec.}$$

$$\boxed{\mathrm{Release\ energy} = 91.84\mathrm{keV}}$$

- Since temperature has the same magnitude, it is necessary to explore if a small change in the nuclear EM potential due to the dynamical damped screening could impact the BBN results.

X.Yao, T.Mehen and B. Müller, ``Dynamical screening of alpha-alpha resonant scattering and thermal nuclear scattering rate in a plasma,'' Phys. Rev. D 95, no.11, 116002 (2017)

R.T.Scherrer and R.J.Scherrer, ``Big bang nucleosynthesis with stable ^{8}Be and the primordial lithium problem,'' Phys. Rev. D 96, no.8, 083507 (2017)

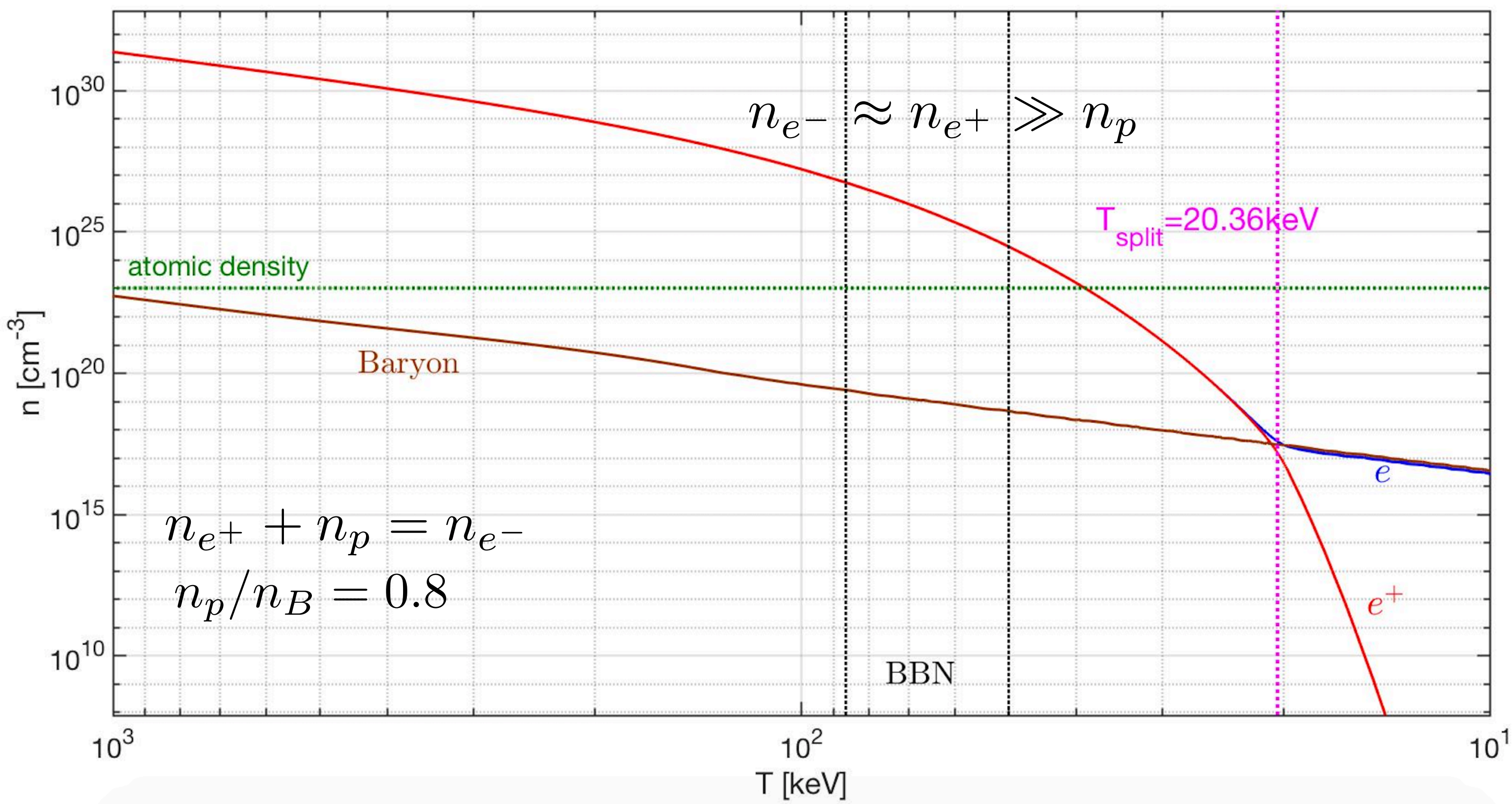
A.Coc, P.Descouvemont, K.A.Olive, J.P.Uzan and E.Vangioni, ``The variation of fundamental constants and the role of A=5 and A=8 nuclei on primordial nucleosynthesis,'' Phys. Rev. D86, 043529 (2012)

Question: Can the plasma effect help stabilize ^{8}Be against decay into two ^{4}He nuclei during the BBN?

BBN in rich Electron/Positron Plasma

We will show that when the temperature is T=86.71keV we have a lot of electrons and positrons in cosmic plasma with $n_{e^\pm}/n_B = 10^7$

$$T_{\mathrm{BBN}} = 86.17 - 50\mathrm{keV}$$



In this case the BBN happened in the electron-positron rich plasma, and plasma effect needs to be accounted for in the precision study of the final abundances of light elements produced in BBN.

Linear response for damped collisional plasmas

M. Formanek, C. Grayson, J. Rafelski and B. Müller, ``Current-conserving relativistic linear response for collisional plasmas,'' Annals Phys. 434, 168605 (2021)

The theoretical description of plasma is based on transport theory, i.e., the general relativistic Boltzmann equation is used to derive equations describing the relativistic plasma

$$(p \cdot \partial) f(x,p) + q F^{\mu\nu} p_\nu \frac{\partial f(x,p)}{\partial p^\mu} = (p \cdot u) C(x,p)$$

$$\boxed{C(x,p) = \kappa \left(f_{\rm eq}(p) \frac{n(x)}{n_{\rm eq}} - f(x,p) \right)}$$

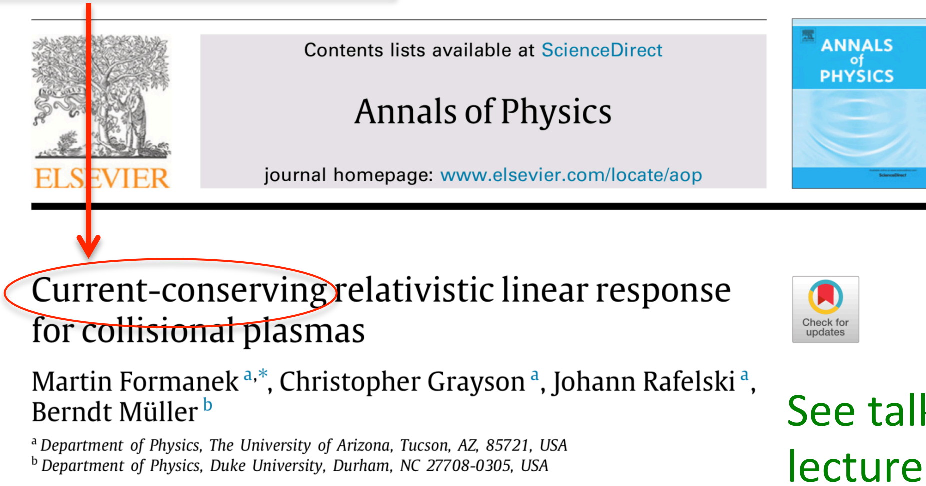
$$f_{\rm eq}(p) \equiv \frac{1}{\exp(p \cdot u / T) + 1}$$

Damped collision

$\kappa = 1/\text{relaxation time}$

For infinite medium of electron/positron

$$e^\pm + \gamma \longrightarrow e^\pm + \gamma,$$
$$e^\pm + e^\pm \longrightarrow e^\pm + e^\pm,$$
$$e^+ + e^- \longrightarrow e^+ + e^-.$$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Current-conserving relativistic linear response for collisional plasmas

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See talk by M. Formanek's lecture for detail overview

Prior application by C.Grayson's lecture: EM field in relativistic Quark-Gluon Plasma

C. Grayson, M. Formanek, J. Rafelski and B.Mueller, ``Dynamic Magnetic Response of Quark-Gluon Plasma to Electromagnetic Fields'', arXiv:2204.14186 [hep-ph]

From statistic to damped dynamic screening

The electron cloud surrounding the charge of an ion screens other nuclear charges far from the own radius and reduces the Coulomb barrier.

- Static screening

  In an isotropic and homogeneous plasma the Coulomb potential of a point-like particle with charge Ze at rest is modified into

$$\boxed{\phi(r) = \frac{Ze}{4\pi\epsilon_0 r} e^{-m_D r}}$$

The reduction of Coulomb barrier makes the penetration probability easier and enhance the thermonuclear reaction rates.

$$\text{enhancement} = \exp\left[\frac{Z_1 Z_2 e^2 m_D}{T}\right]$$

E. E.Salpeter, ``Electron screening and thermonuclear reactions,'' Austral. J. Phys 7 (1954), 373-388

- Dynamic screening

  When a test charge moves with a velocity that is enough to react with the background charge in plasma, the Coulomb potential is modified by the dynamical effect.

  Carraro, C., Schafer, A., & Koonin, S. E. "Dynamic screening of thermonuclear reactions", 1988, Astro- phys. J. , 331, 565. doi:10.1086/166582

  A. V. Gruzinov, "Dynamic screening and thermonuclear reaction rates," Astrophys. J. 496 (1998), 503 doi:10.1086/305349

  E. Hwang, D. Jang, K. Park, M. Kusakabe, T. Ka- jino, A. B. Balantekin, T. Maruyama, C. M. Ryu and M. K. Cheoun, "Dynamical screening effects on big bang nucleosynthesis," JCAP 11 (2021), 017 doi:10.1088/1475-7516/2021/11/017

- Damped dynamics screening

  We use the **linear response theory (M. Formanek's lecture) adapt by C. Grayson to describe the inter nuclear potential in electron/positron plasma**. We improve the prior efforts by evaluation and inclusion of the **collision damping rate due to scattering in the dense plasma medium**.

Property of electron/positron plasma in BBN

M. Formanek, C. Grayson, J. Rafelski and B. Müller, ``Current-conserving relativistic linear response for collisional plasmas,'' Annals Phys. 434, 168605 (2021)

To apply the linear response method to **infinite medium of nonrelativsitic electron/positron cosmic plasma**. We need to study the properties of electron/positron plasma in BBN:

$$f_{\text{eq}}(p) \equiv \frac{1}{\exp(p \cdot u/T) + 1}$$

$$C(x,p) = \kappa \left(f_{\text{eq}}(p) \frac{n(x)}{n_{\text{eq}}} - f(x,p) \right)$$

- Two component plasma with equal number of particle and antiparticle.

$$\mu \ll T, m_e$$

$$\Downarrow$$

We use this approximation in the temperature domain of interested for electron/positron plasma in early universe.

- Damping: Plasma constituents collide on time scale.

$$\kappa = 1/\text{relaxation time}$$

$$\Downarrow$$

This depends on the collisional property of the electron/positron plasma in early universe.

Three principles we use to determine the electron chemical potential

Our objective is to explore the dense electron/positron plasma in early universe under the hypothesis **charge neutrality** and **entropy conservation**

2. Total comoving entropy is conserved. At $T \leq T_f$ the dominant contributors to entropy are photons, $e^{\pm}$, and neutrinos. In addition, after neutrino freezeout, neutrino comoving entropy is independently conserved. This implies that the combined comoving entropy in γ, $e^{\pm}$ is also conserved for $T_\gamma \leq T_f$. On the other hand, comoving baryon number is also conserved, hence the ratio n_B/s is conserved, and we have

$$\frac{n_B}{s} = \frac{n_B}{\sum_i s_i} = \text{constant}.$$

1. Charge neutrality of the Universe:

$$n_e - n_{\bar{e}} = n_p - n_{\bar{p}} \approx n_p,$$

where n_ℓ denotes the number density of particle type ℓ.

$$n_e - n_{\bar{e}} = \frac{n_p}{n_B}\left(\frac{n_B}{s_{\gamma,e,\bar{e}}}\right) s_{\gamma,e,\bar{e}}$$

$$n_e - n_{\bar{e}} = \frac{g_e}{2\pi^2}\left[\int_0^\infty \frac{p^2 dp}{\exp\left(\left(E - \mu_e\right)\right)/T_\gamma + 1} - \int_0^\infty \frac{p^2 dp}{\exp\left(\left(E + \mu_e\right)/T_\gamma\right) + 1}\right]$$

3. Neutrinos decouple (freeze out) at a temperature $T_f \simeq 2\text{MeV}$, after which they free stream through the Universe with an effective temperature

$$T_\nu(t) = T_f a(t_f)/a(t),$$

where $a(t)$ is the FLRW Universe scale factor.

BBN in rich Electron/Positron Plasma

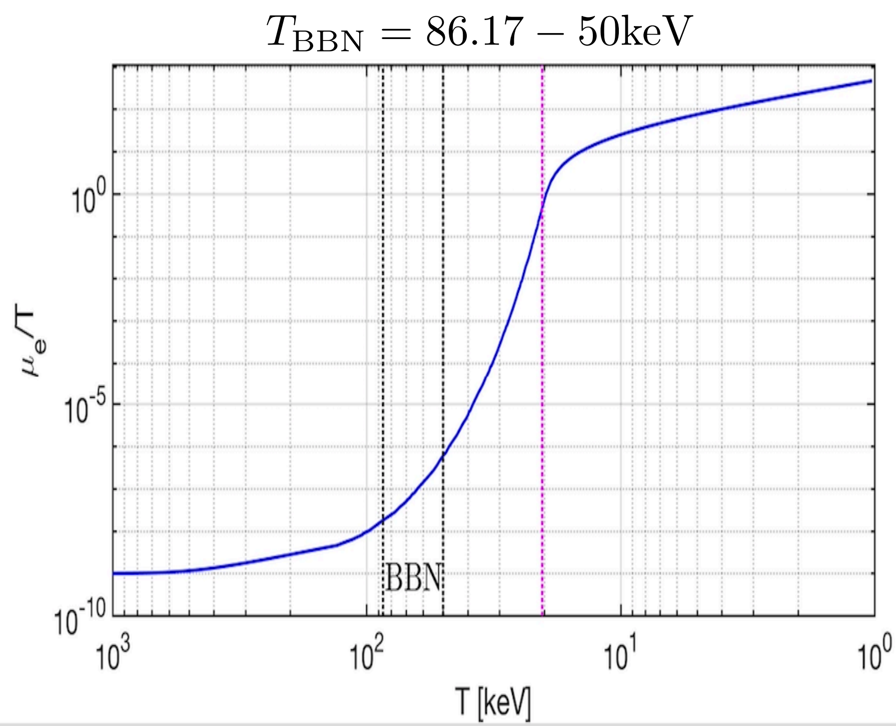
We found that when the temperature is T=86.71keV we have a lot of electrons and positrons in cosmic plasma with $n_{e^\pm}/n_B = 10^7$, and after temperature T_{split}=20.36keV the positron density decrease because of annihilation.

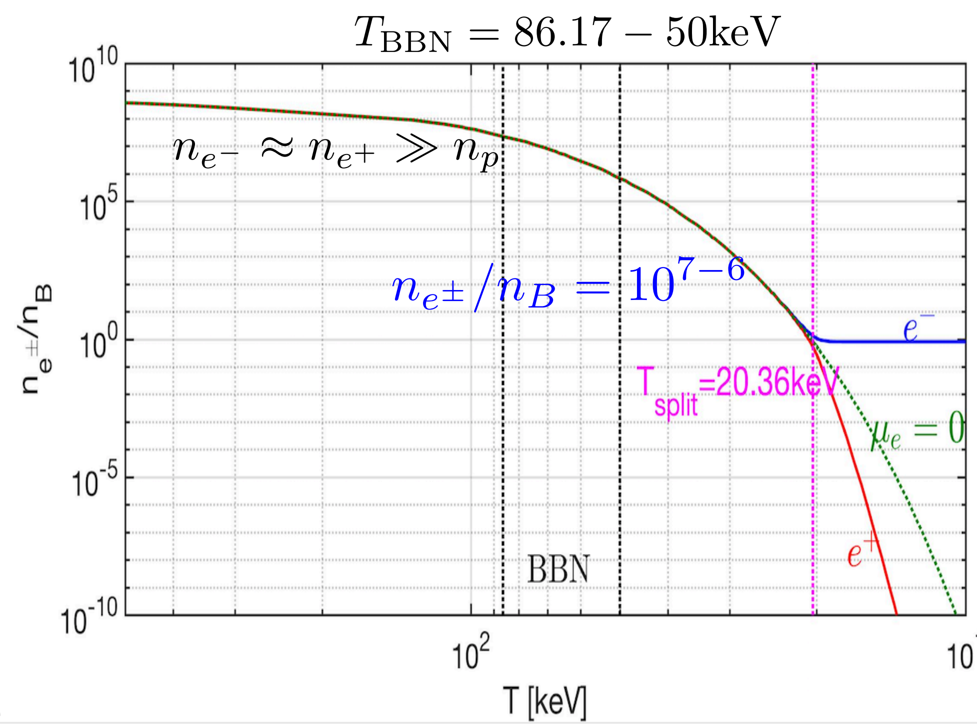
$$\left(\frac{n_B}{s_{\gamma,e,\bar{e}}}\right) = \left(\frac{n_B}{s_\gamma}\right)_{t_0} = \left(\frac{n_B}{n_\gamma}\right)_{t_0}\left(\frac{n_\gamma}{s_\gamma}\right)$$

$$n_e - n_{\overline{e}} = \frac{n_p}{n_B}\left(\frac{n_B}{s_{\gamma,e,\overline{e}}}\right) s_{\gamma,e,\overline{e}}$$

$(n_B/n_\gamma)_{t_0} = 6.05 \times 10^{-10}$ $\quad s_\gamma/n_\gamma = 3.602$

$n_p/n_B = 0.8$



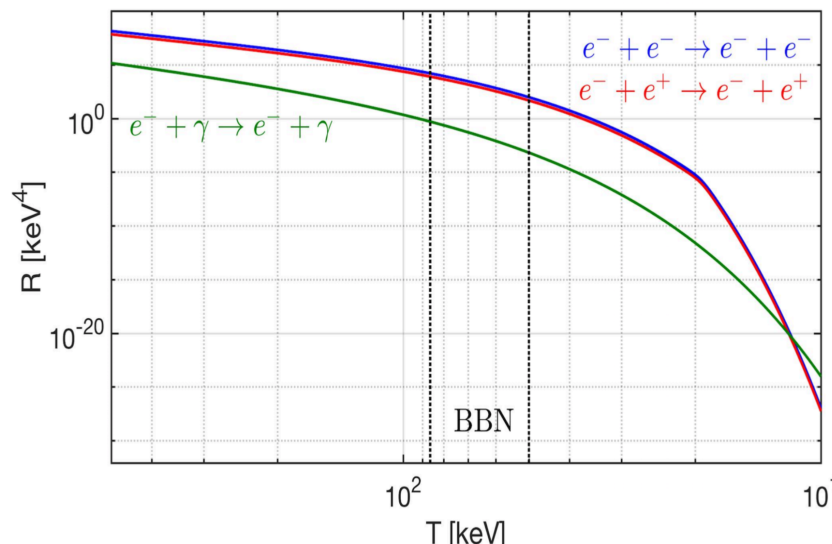


In this case the BBN happened in the electron-positron rich plasma, and we have equal number of electron and positron during the BBN temperature range.

Collisional damping rate

In electron-positron plasma the major reactions between photons and electron/positron pairs are inverse Compton scattering, Møller scattering, and Bhabha scattering:

$$\boxed{\begin{aligned} e^{\pm} + \gamma &\longrightarrow e^{\pm} + \gamma \\ e^{\pm} + e^{\pm} &\longrightarrow e^{\pm} + e^{\pm} \\ e^{-} + e^{+} &\longrightarrow e^{-} + e^{+} \end{aligned}}$$



The general reaction rate per volume in two-body reaction in the Boltzmann approximation

$$R_{12\to 34} = \frac{g_1 g_2}{32\pi^4} \frac{T}{1 + I_{12}} \int_{s_{th}}^{\infty} ds\, \sigma(s) \frac{\lambda_2(s)}{\sqrt{s}} K_1(\sqrt{s}/T)$$

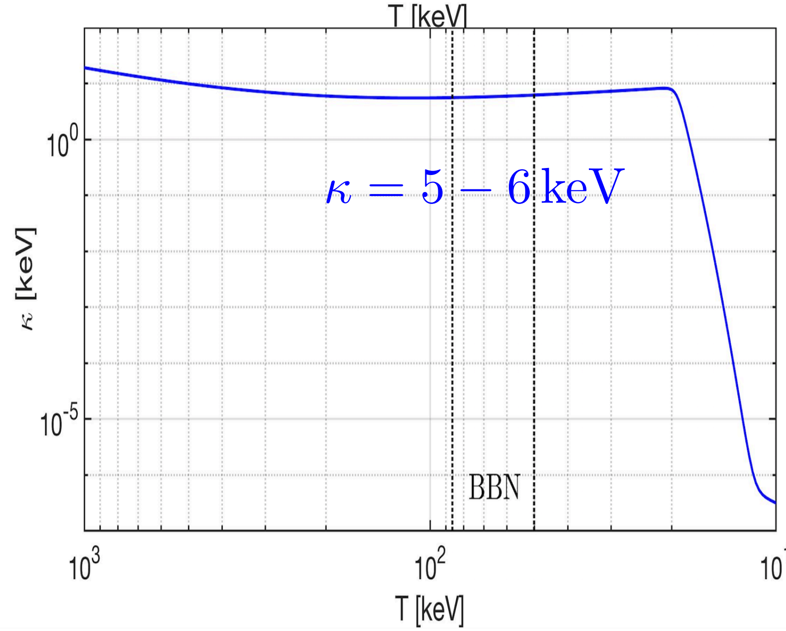
$$\lambda_2(s) = \left[2 - (m_1 + m_2)^2\right] \left[2 - (m_1 - m_2)^2\right]$$

Jean Letessier, Johann Rafelski, "Hadrons and Quark-Gluon Plasma"

The relaxation rate for the electron-positron plasma

$$\boxed{\kappa = \frac{(R_{e^-\gamma} + R_{e^+\gamma}) + (R_{e^-e^-} + R_{e^+e^+}) + R_{e^-e^+}}{2\sqrt{n_{e^-} n_{e^+}}}}$$

$$\sqrt{n_{e^-} n_{e^+}} = \frac{g_e}{2\pi^3} T^3 \left(\frac{m_e}{T}\right)^2 K_2(m_e/T)$$



Dynamic damped screening potential

The screened potential of nuclei can be obtained by solving Maxwell equations algebraically in Fourier space, in the presence of electron/positron medium.

C. Grayson, M. Formanek, J. Rafelski and B. Müller,, arXiv:2204.14186 [hep-ph]

M.Formanek, C.Grayson, J.Rafelski and B. Müller,Annals Phys 434, 168605 (2021)

$$\widetilde{\phi}(\omega, \boldsymbol{k}) = \frac{\widetilde{\rho}_{\text{ext}}(\omega, \boldsymbol{k})}{\varepsilon_{\|}(\omega, \boldsymbol{k})(\boldsymbol{k}^2 - \omega^2)}$$

Fourier transformed charge distribution of the traveling nuclei

Longitudinal permittivity

$$\boxed{\varepsilon_{\|}(\omega, \boldsymbol{k}) = \varepsilon_0 \left(\frac{\Pi_{\|}(\omega, \boldsymbol{k})}{\omega^2} + 1 \right)}$$

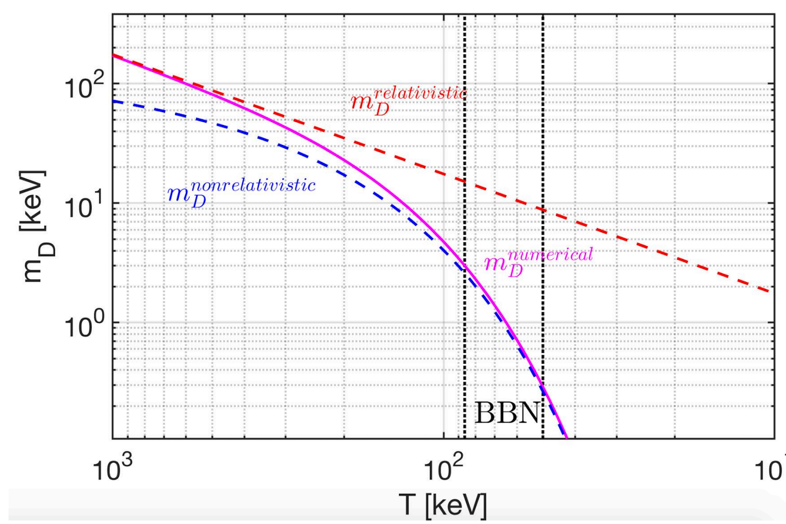
Using Linear response theory, the longitudinal polarization tensor for nonrelativistic electron-positron plasma

$$\Pi_{\|} = -m_D^2 \frac{\omega^2}{(\omega + i\kappa)^2} \frac{1}{1 - \frac{i\kappa}{\omega + i\kappa}\left(1 + \frac{Tk^2}{m(\omega + i\kappa)^2}\right)}$$

Debye mass:

$$m_D^2 = -\frac{2e^2}{\pi^2} \int_0^\infty d|\mathbf{p}||\mathbf{p}|^2 \frac{\partial f_{eq}}{\partial p_0}$$
$$= \frac{2e^2}{\pi^2} T^2 \int_{m_e/T}^\infty dx\, x \sqrt{x^2 - (m_e/T)^2} \frac{e^{-x}}{(1 + e^{-x})^2}$$

$$e^\pm + \gamma \longrightarrow e^\pm + \gamma,$$
$$e^\pm + e^\pm \longrightarrow e^\pm + e^\pm,$$
$$e^+ + e^- \longrightarrow e^+ + e^-.$$

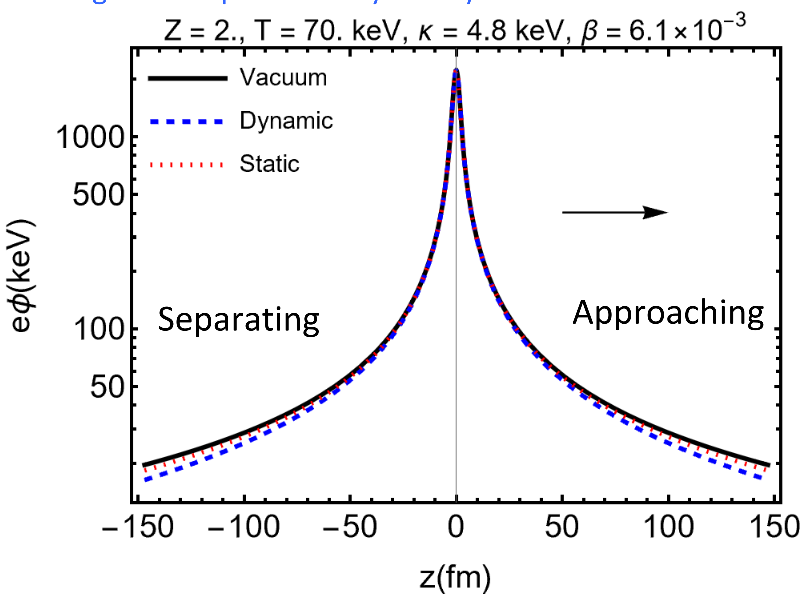


Example of thermalize ion dynamic response

We compute in linear respond theory the effective nuclear potential for an alpha nuclei(Z=2)

$m_\alpha = 3.727\text{GeV}$

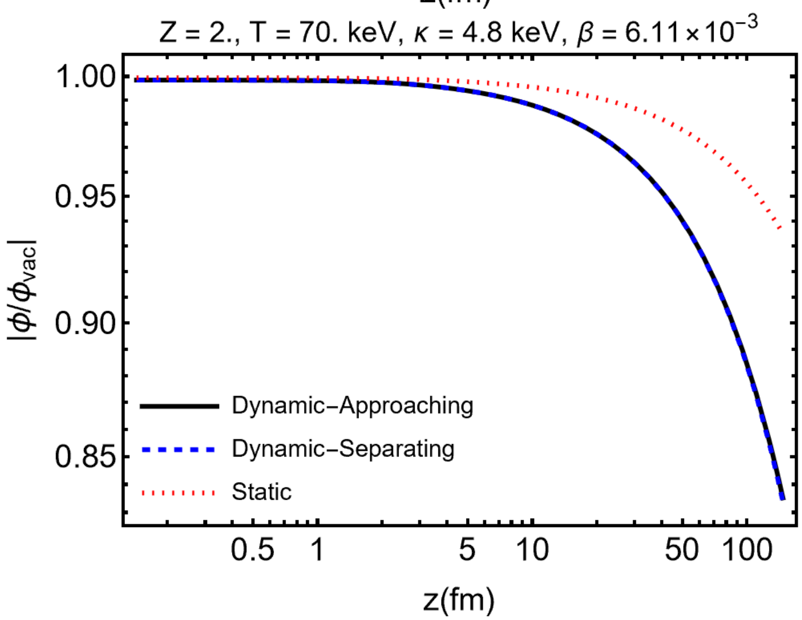
All figures are produced by C.Grayson

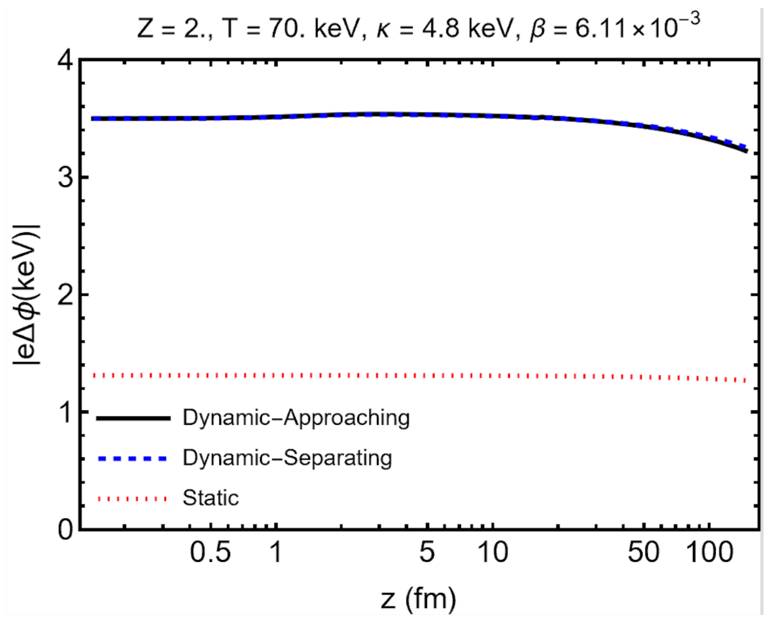


Thermal velocity of alpha particle: $\beta = \sqrt{\frac{2T}{m_\alpha}}$

Static screening limit: $(\omega \to 0)$

- The dynamic screening affect the nuclear potential for large distance.
- The dynamic screening reduce the vacuum nuclear potential about 3.5keV.



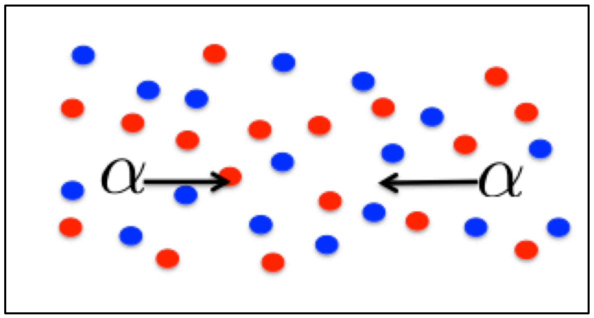


Linear response for nonrelativistic electron-positron plasma

The response of a relativistic plasma to electromagnetic fields in the framework of the Boltzmann equation incorporating a collision term in the relaxation rate approximation.

$$(p\cdot\partial)f(x,p)+qF^{\mu\nu}p_\nu\frac{\partial f(x,p)}{\partial p^\mu}=(p\cdot u)C(x,p)\,,\qquad C(x,p)=\kappa\left(f_{\rm eq}(p)\frac{n(x)}{n_{\rm eq}}-f(x,p)\right)$$

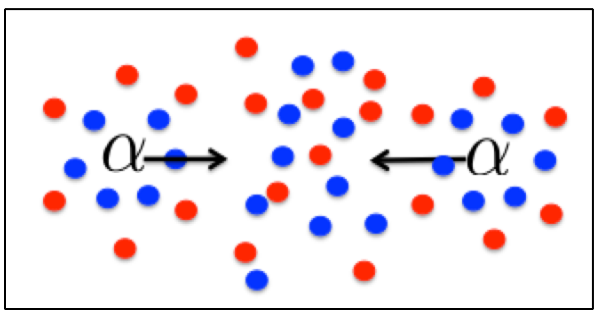
- Linear respond



The electron/positron medium is perturbed by alpha particle electromagnetically when it travel in the plasma.

$$f(x,p)=f_{\rm eq}(p)+\delta f(x,p)$$

- Beyond linear respond



The alpha particle is surround by the electron cloud and the electron cloud move with the ion in plasma together.

Linear respond does not separate electron and positron $\Rightarrow$ $\mu_e(x,t)$

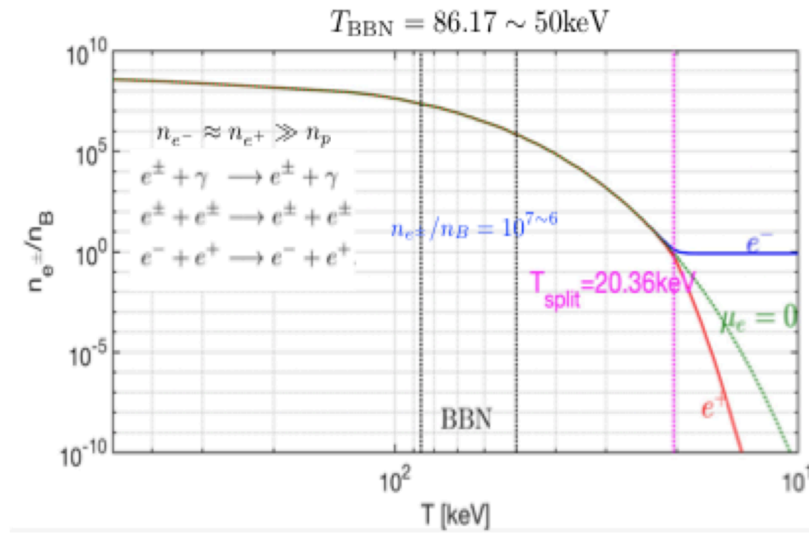
To study the damped dynamic screening in electron/positron plasma, more detail study of linear respond theory required.

Still working in progress!

Summary & Outlook

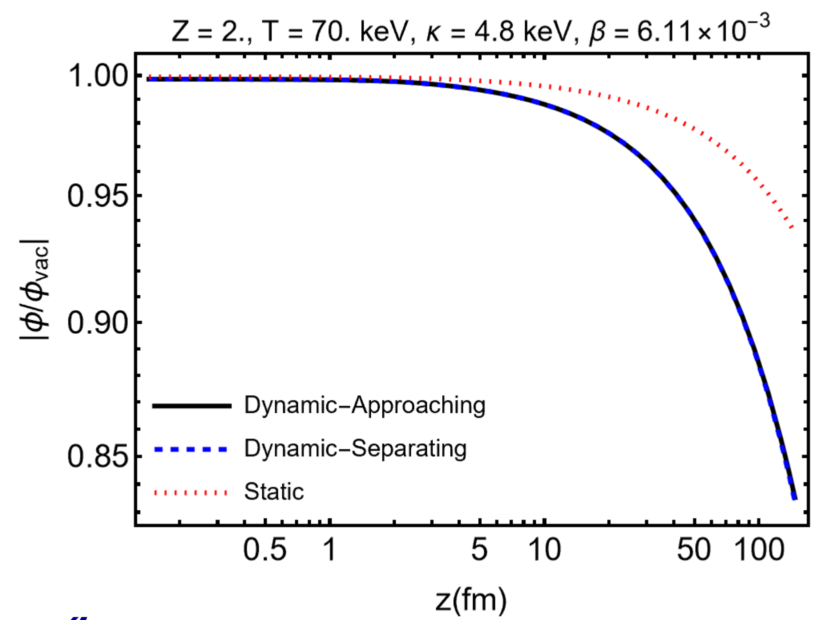
Summary:

- The presence of electron/positron during BBN would play a important role in the formation of the light elements in BBN.
- Using the linear response theory to describe the inter-nuclear potential in dense collisional electron-positron plasma.



Outlook:

- The presence of electron/positron plasma in early universe could affects the effective number of neutrino and the speed of expansion in the early universe.
- Refinement of the linear respond theory to study the dynamic screening effect is needed.
- Electron/positron screening of ^{8}Be in laboratory

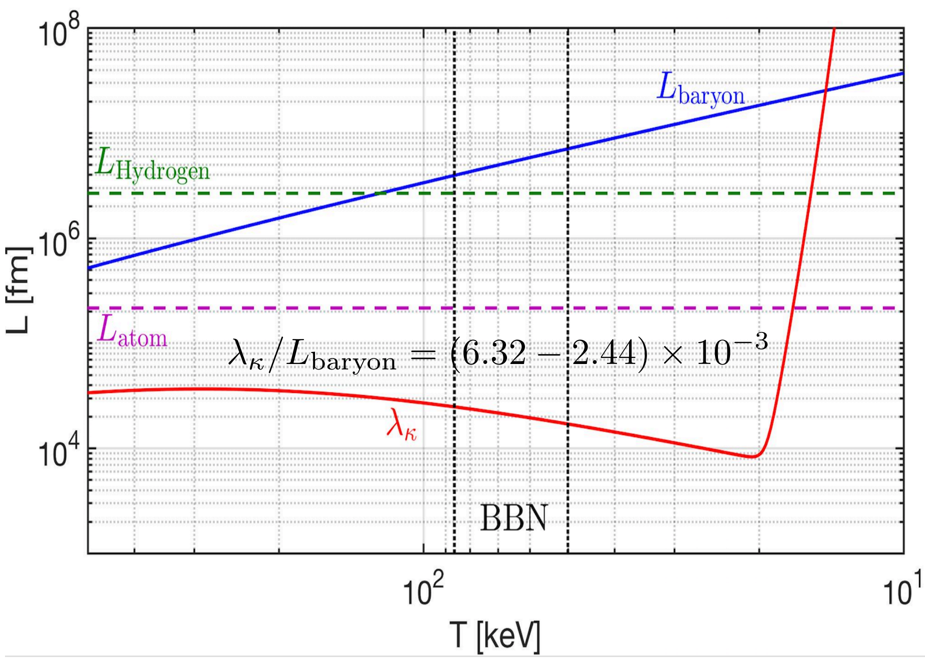


Thank You For Listening

Backup slides: Mean free path in plasma

To show large relaxation rate (the strong collision) for electron/positron travel in plasma, we introduce the mean free path for the electron/positron:

$$\lambda_\kappa = \frac{v_{e^\pm}}{\kappa} = \frac{1}{\kappa}\sqrt{\frac{2T}{m_e}}, \qquad L_{\rm baryon} = n_B^{-1/3}$$



- We see that for the BBN temperature range, the $\lambda_\kappa \ll L_{\rm baryon}$ which imply between two baryons, many electron/positron scattering processes can occur.
- Average spacing for normal matter

$$L_{\rm atom} = n_{\rm atom}^{-1/3}$$
$$n_{\rm atom} = 10^{-16} {\rm fm}^{-3} ({\rm STP\ condition})$$
$$L_{\rm Hydrogen} = n_{\rm Hydrogen}^{-1/3}$$
$$n_{\rm Hydrogen} = 0.53 \times 10^{-21} {\rm fm}^{-3} ({\rm STP\ condition})$$

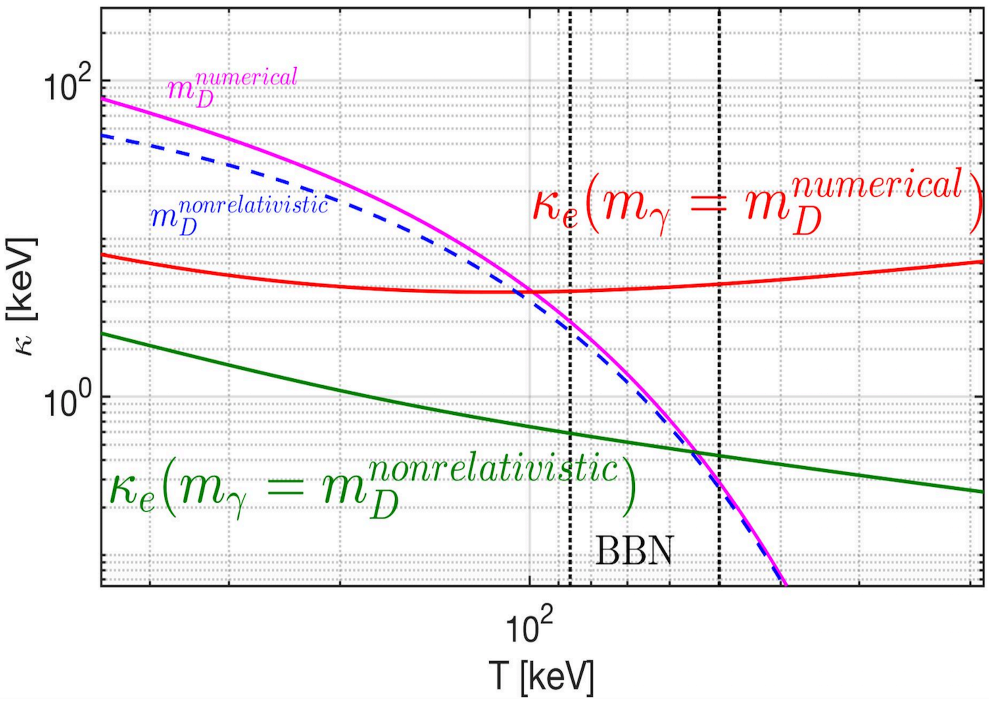
In this case, we have strong collision damping rate due to scattering in the dense electron/positron plasma.

Backup slides: The damping rate and Debye mass

In calculation we introduce the photon mass to avoid singularity in reaction matrix elements for Møller and Bhabha scattering

$$|M_{e^-e^-}|^2 = 64\pi^2\alpha^2\left[\frac{s^2+u^2+8m_e^2(t-m_e^2)}{2(t-m_\gamma^2)^2} + \frac{s^2+t^2+8m_e^2(u-m_e^2)}{2(u-m_\gamma^2)^2} + \frac{\left(s-2m_e^2\right)\left(s-6m_e^2\right)}{(t-m_\gamma^2)(u-m_\gamma^2)}\right]$$

$$|M_{e^-e^+}|^2 = 64\pi^2\alpha^2\left[\frac{s^2+u^2+8m_e^2(t-m_e^2)}{2(t-m_\gamma^2)^2} + \frac{u^2+t^2+8m_e^2(s-m_e^2)}{2(s-m_\gamma^2)^2} + \frac{\left(u-2m_e^2\right)\left(u-6m_e^2\right)}{(t-m_\gamma^2)(s-m_\gamma^2)}\right]$$



Debye mass:

$$m_D^2 = -\frac{2e^2}{\pi^2}\int_0^\infty d|\mathbf{p}||\mathbf{p}|^2\frac{\partial f_{eq}}{\partial p_0}$$
$$= \frac{2e^2}{\pi^2}T^2\int_{m_e/T}^\infty dx\, x\sqrt{x^2-(m_e/T)^2}\,\frac{e^{-x}}{(1+e^{-x})^2}$$

Backup slides: Example of thermalize ion dynamic response

We compute in linear respond theory the effective nuclear potential for an alpha nuclei(Z=2)