Multiphoton Processes in the Field of a ‘Pedestal’.

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Varró S, Multiphoton Processes in the Field of a ‘Pedestal’. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

Outline of the talk:

- **Introduction. ‘Pedestals’ of amplified light pulses. Amplified Spontaneous Emission (ASE).**

- **Bridges between strong field physics and quantum optics.**

- **The ‘n-factorial increase’ of multiphoton excitation probabilities. Asymmetry of absorption and emission for Bose distribution.**

- **Note on the RABBITT [reconstruction of attosecond beating by interference of two-photon transitions]**

- **Summary.**

Introduction. ‘Pedestals’ of amplified light pulses. Amplified Spontaneous Emission (ASE).

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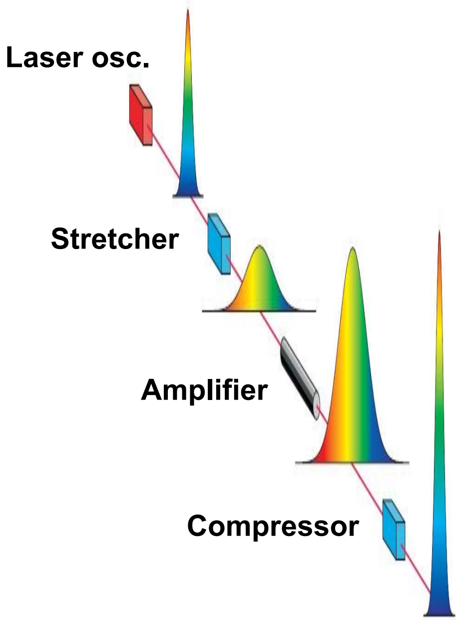
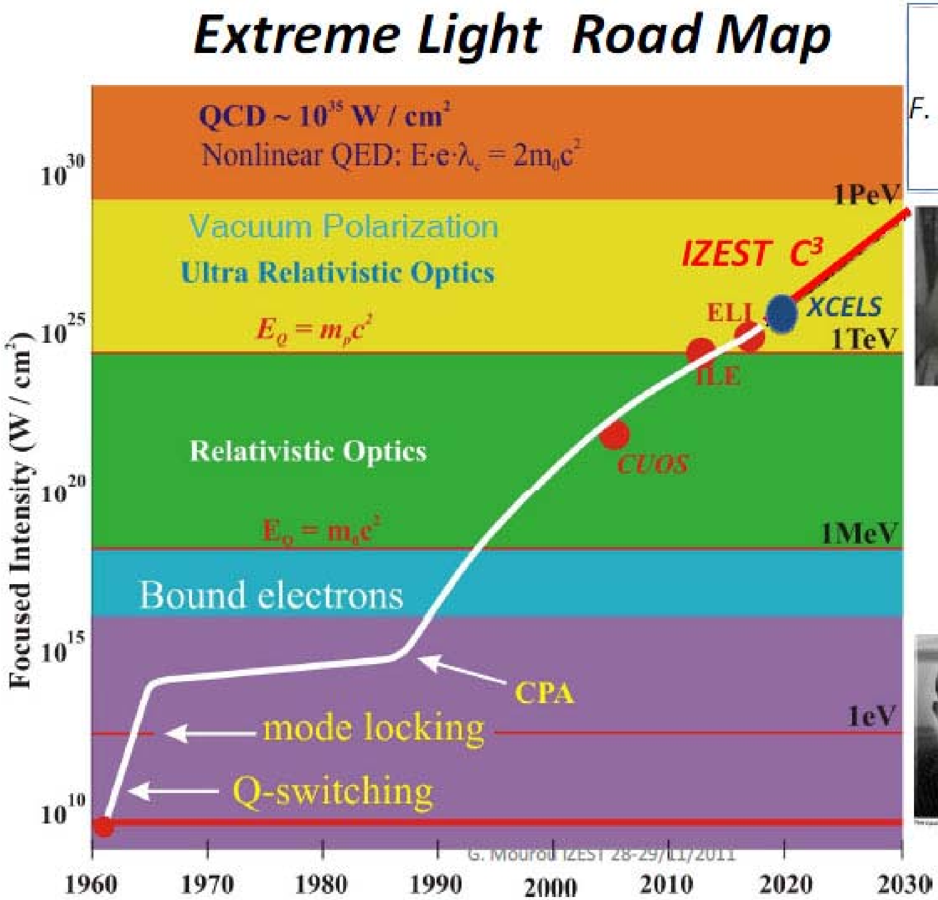
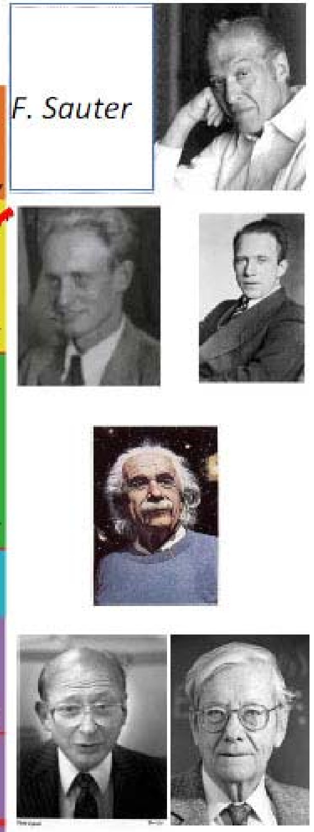
Development of very large intensity lasers based on the 'CPA technique'. [D. Strickland and G. Mourou. The Nobel Prize in Physics 2018.]

CPA: Chirped Pulse Amplification [1985]







Right Figure taken from Gérard Mourou's presentation at 'IZEST' [International Zeta-Exawatt Science and Technology.] Launching Workshop, Ecole Polytechnique, 28-29 November, 2011. Paris, France.

The contrast of ultrashrt light pulses. The question of the photon statistics of the amplified, very-high-intensity, ultrashort light pulses. The ‘pedestal’.

‘mirrorless laser’

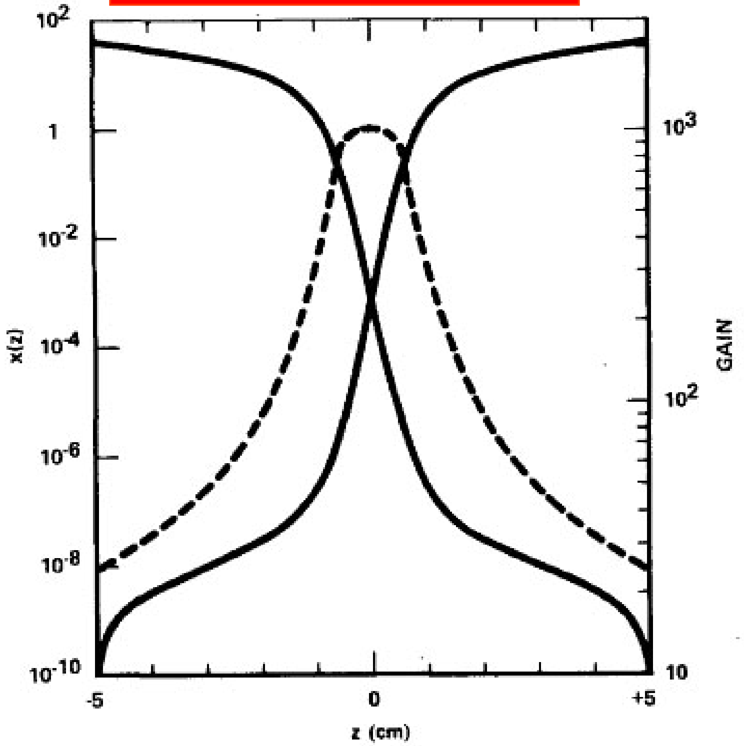
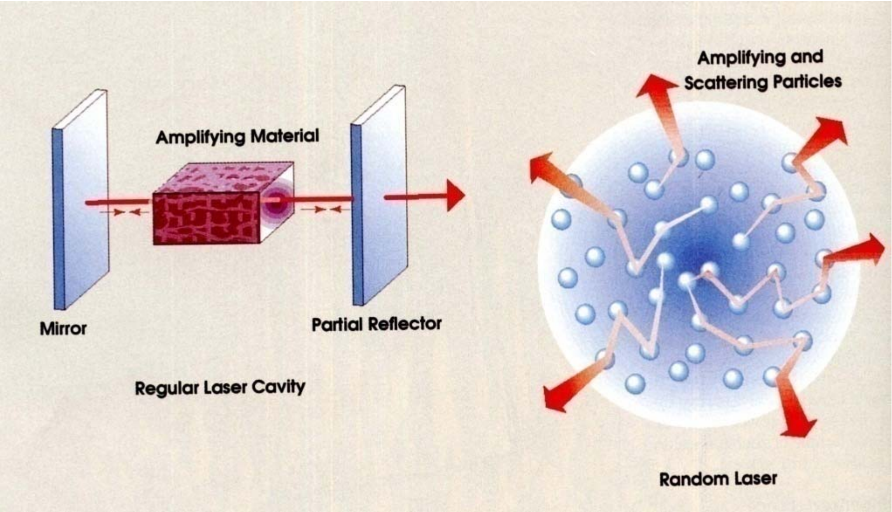


FIG. 2. Intensity of the right and left traveling waves as a function of position in a mirrorless laser amplifier with $x_0 = 10^{-8}$ and $l = 10$ cm (solid lines). Saturated gain distribution (dashed line).

Figure 2. copied from Casperson L W, Threshold charatcteristics of mirrorless lasers. *Journal of Applied Physics*, Vol. 48, No.1, January 1977, pp 256-262.

Remark. Amplified spontaneous emission.
[Intereseting: ‘Random light jets, cosmic lasers’.]



Photon statistics of an amplified coherent signal: approximately ‘thermo-coherent’

$$p(n) = \frac{\bar{n}_{ASE}^{\,n}}{(1+\bar{n}_{ASE})^{n+1}} \exp\left(-\frac{\bar{n}_S}{1+\bar{n}_{ASE}}\right) L_n\left(-\frac{\bar{n}_S/\bar{n}_{ASE}}{1+\bar{n}_{ASE}}\right)$$

Theory by Mollow and Glauber (1967) on parametric amplifiers. Or, see the book: Saleh B E A and Teich M C, *Fundamentals of Photonics*. (John Wiley & Sons, Inc., New York, 1991). p. 493.

Varró S, Multiphoton Processes in the Field of a ‘Pedestal’. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

The contrast of ultrashrt light pulses. The problem of the 'ASE pedestal'. Pulse cleaning. Plasma mirror (1990).

Enhanced Absorption and ASE Pedestal Suppression in the Generation of Ultrashort-Pulse Solid-Density Plasmas

H.C. Kapteyn[1], *M.M. Murnane*[1], *A. Szoke*[2], *A. Hawryluk*[2], *and R.W. Falcone*[1]

[1]Department of Physics, University of California at Berkeley, Berkeley, CA 94720, USA

[2]Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

Abstract. Two techniques are demonstrated which will facilitate the production of solid-density ultrashort-pulse plasmas at very high (>10^{19} W/cm^2) intensities. Grating targets are shown to dramatically enhance the energy absorption of an incident 150 fs laser pulse over that of a flat target, from <20% to >90%. Also, the technique of self-induced plasma shuttering is demonstrated to be an effective means of reducing the pedestal energy accompanying the ultrashort pulse. Thus, solid-density plasmas can now be produced even with a laser pulse of low contrast ratio.

122

Springer Series in Chemical Physics, Vol. 53 **Ultrafast Phenomena VII**
Editors: C.B. Harris · E.P. Ippen · G.A. Mourou · A.H. Zewail

Varró S, Multiphoton Processes in the Field of a 'Pedestal'. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

The problem of ASE pedestal. Pulse cleaning with plasma mirror.

[Attosecond pulse generation with plasma mirror.]

Contrast.

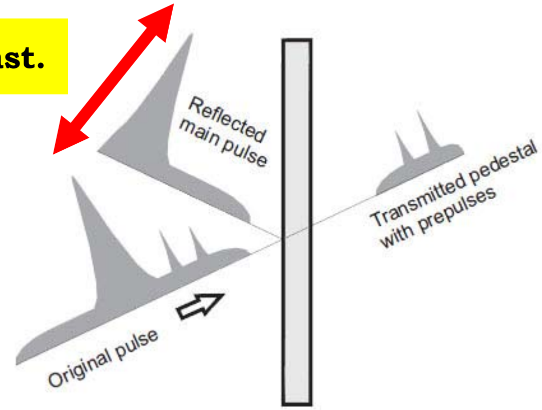


Figure 2.1: Plasma mirror concept. A laser pulse is focused onto a transparent bulk target. The fluence is adjusted so that the the leading pedestal and prepulses traverse the target while the striking edge of the main pulse ionizes it and almost instantaneously generates a highly reflective flat plasma layer on the target's surface. The main pulse and the post pulse cleaned from the pedestal and prepulses are specularly reflected off the plasma mirror.

Figure 2.1 copied from Wittmann Tibor, *Complete characterization of plasma mirrors and development of single-shot carrier-envelope phase meter*. PhD dissertation (University of Szeged, Szeged, 2009).

J. Phys. B: At. Mol. Opt. Phys. **50** (2017) 132002

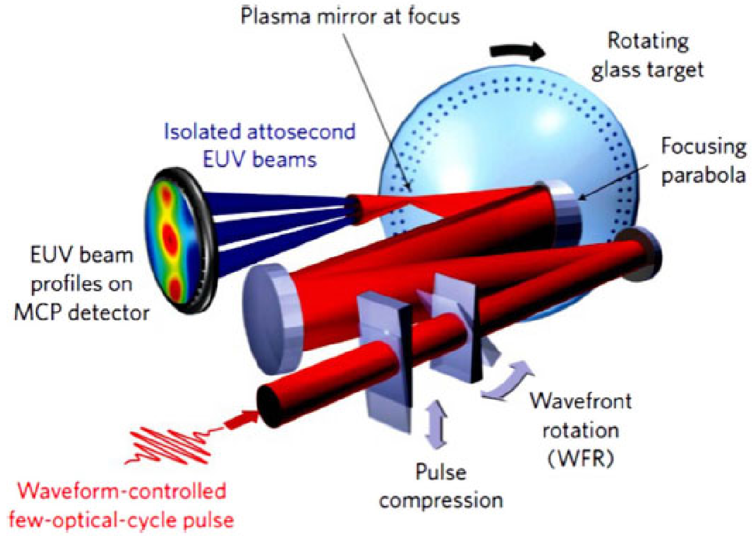


Figure 27. A waveform-controlled few-cycle IR laser pulse is focused onto a rotating optically polished target surface creating a plasma mirror. XUV radiation, in the form of an attosecond pulse train, is non-linearly generated in the direction of the reflected laser beam through the process of either coherent wake emission or relativistic oscillating mirror. Wavefront rotation (WFR) is intro-

Figure 27 copied from: Kühn S, Dumerge M, Kahaly S, *et al* 2017 ELI-ALPS facility: The next generation of attosecond sources. *J. Phys. B. At. Mol. Opt. Phys*. 50, 132002 (2017).

Varró S, Multiphoton Processes in the Field of a 'Pedestal'. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

Two examples for the amplification by 'parametric process'. [a two-stage OPCPA, and the ELI HR laser system, first phase. 2017.]

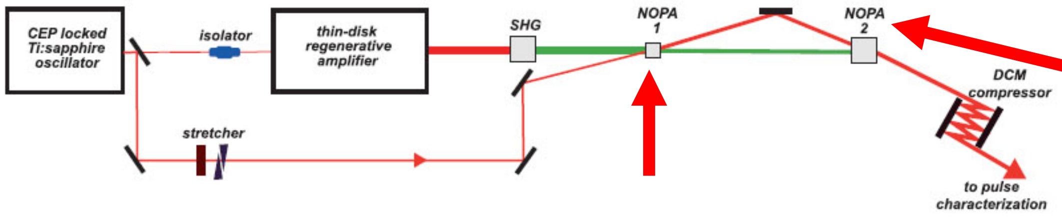


Fig. 1. Schematic setup of the noncollinear OPCPA. The abbreviations are explained in the

Parametric Amplifier

J. Phys. B: At. Mol. Opt. Phys. **50** (2017) 132002 Topical Review

Table 1. Parameters of the HR laser system in phase 1 and phase 2.

Parameters	HR 1	HR 2 (targeted)
Center wavelength λ_c	1030 nm	1030 nm
Repetition rate	100 kHz	100 kHz
Average power	>100 W	>500 W
Pulse energy	>1 mJ	>5 mJ
Pulse duration (@λ_c)	<6.2 fs (<1.85 cycles)	<6.2 fs (<1.85 cycles)
Output energy stability	<0.8% (rms)	<0.8% (rms)
Beam quality (Strehl ratio)	>0.9	>0.9
CEP stability	<100 mrad (rms)	<100 mrad (rms)
Beam pointing instability	<2.5% (diffr. limited div.)	<2.5%
Trouble-free uninterrupted operation	>8 h	>8 h

All laser systems will be run from their own oscillator

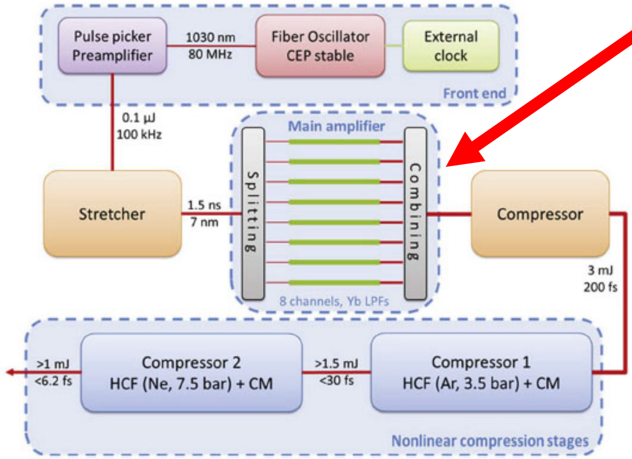


Figure 1. Scheme of the HR laser system with three major sub-systems: the front end, the main amplifier and the non-linear compression stages. (CM: chirped mirror stage).

Upper figure: Figure 1 copied from: Schultze M, Binhammer Th, Palmer G, et al, Multi-μJ, CEP-stabilized, two-cycle pulses from an OPCPA system with up to 500 kHz repetition rate. 20 December 2010 / Vol. 18, No. 26 / OPTICS EXPRESS 27291.
Lower figure: Table 1 and Figure 1 copied from: Kühn S, Dumerge M, Kahaly S, *et al* 2017 ELI-ALPS facility: The next generation of attosecond sources. *J. Phys. B. At. Mol. Opt. Phys*. 50, 132002 (2017).

Early and some recent bridges between strong-field physics and quantum optics. **The appearance of ‘non-classical states’; squeezing etc.**

René Magritte, The lost jockey (1926).

Varró S, Multiphoton Processes in the Field of a ‘Pedestal’. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

Volkov states. I. Classical radiation field.

Wolkow D M, Über eine Klasse von Lösungen der Diracschen Gleichung. *Zeitschrift für Physik* 94, 250-260 (1935). [Application to strong-field and multiphoton processes: from ~1960..]

$$[\gamma_\mu (i\partial - \varepsilon A_{rad})^\mu - \kappa]|\Psi\rangle = 0 \quad (\varepsilon\gamma_0 V|\Psi\rangle)$$

$$A_{rad}(\xi) = e_x A_0 f(\xi)$$

$$\xi = k_\mu x^\mu = \omega(t - z/c)$$

$$\Psi_{ps}^{(\pm)}(x) = \left[1 \pm \frac{\varepsilon(\gamma\cdot k)[\gamma\cdot A(\xi)]}{2k\cdot p}\right] u_{ps}^{(\pm)}$$

$$\times \exp\left[\mp i[p\cdot x + \int I_p^{(\pm)}(\xi)d\xi]\right]$$

$$I_p^{(\pm)}(\xi) = (1/2k\cdot p)[\pm 2\varepsilon p\cdot A(\xi) - \varepsilon^2 A^2(\xi)$$

$$\mathrm{e}^{-i\zeta\sin 2\omega_0 t} = \sum_k J_k(\zeta)\mathrm{e}^{-2in\omega_0 t}$$

Jacobi–Anger formula;

Multiphoton side-bands

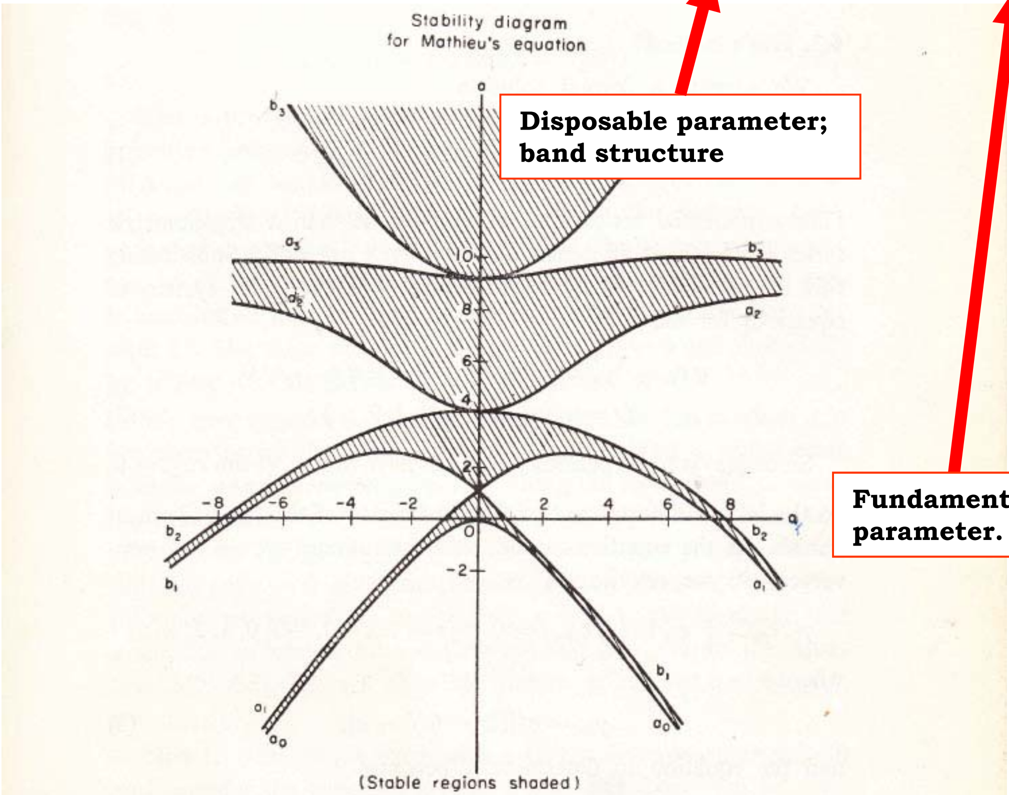
$$\mathrm{e}^{-iz\sin\omega_0 t} = \sum_n J_n(z)\mathrm{e}^{-in\omega_0 t}$$

Varró S, Multiphoton Processes in the Field of a 'Pedestal'. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

Classical radiation field. Mathieu–type solutions. $\xi = k_\mu x^\mu = \omega(t - n_m y / c)$

$$w'' + (\lambda - 2h^2 \cos 2z) w = 0$$

$$\lambda \propto k \cdot p, \quad h^2 \propto eA_0 p_\perp$$



Disposable parameter; band structure

Fundamental parameter.

[Figure taken from Arscott F M, *Periodic differential equations* (Pergamon Press, Oxford, 1964) p.123.] . Nikishov & Ritus (1967), Nikishov (1970), Narozhny & Nikishov (1974), **Dielectric. Becker (1977),** Fedorov, McIver … FEL theories. **PLASMA:**
[A] Varró S, New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium. *Laser Physics Letters* 10 (2013) 095301. [B] Varró S, A new class of exact solutions of the Klein-Gordon equation of a charged particle ... *LPL*11 (2014) 016001 . [C] Varró S, New exact solutions of the Klein-Gordon and Dirac equations of a charged particle propagating in a strong laser field in an underdense plasma. *Nucl. Instr. and Meth. in Phys.Res.*A 740 (2014) 280-283.

Squeezed states in quantum optics and strong-field physics. $|\xi;\alpha;n\rangle = \hat{S}(\xi)\hat{D}(\alpha)|n\rangle$

PARAMETRIC EXCITATION OF A QUANTUM OSCILLATOR

V. S. Popov, A. M. Perelomov

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}\omega^2(t)x^2\psi$$

An exact solution is obtained for the problem of a quantum oscillator with a time-dependent frequency $\omega(t)$ varying in an arbitrary manner. The quantum transition probabilities w_{mn} between the discrete spectrum states which are stationary for $t \to \pm\infty$ are calculated. The variation of the adiabatic invariant $I = E/\omega$ under the action of a variable frequency $\omega(t)$ is calculated for an arbitrary initial state.

SOVIET PHYSICS JETP VOLUME 29, NUMBER 5 NOVEMBER, 1969

ELECTRON IN THE QUANTIZED FIELD OF A MONOCHROMATIC ELECTROMAGNETIC WAVE

I. BERSON

Institute of Physics, Latvian Academy of Sciences

Submitted November 16, 1968

Zh. Eksp. Teor. Fiz. 56, 1627–1633 (May, 1969)

The wave function is determined for a system consisting of an electron and the quantized field of a plane monochromatic electromagnetic wave interacting with it. Various representations of the obtained wave function are presented, and its relation to Volkov's solution is discussed.

$$\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - ieA_\mu\right) + m\right]\psi = 0,$$

$$A_\mu \propto e_\mu(\hat{a}\mathrm{e}^{-ik\cdot x} + \hat{a}^+\mathrm{e}^{ik\cdot x})$$

$$|\xi;\alpha;n\rangle = \hat{S}(\xi)\hat{D}(\alpha)|n\rangle$$

[1] Popov V S and Perelomov A M, Parametric excitation of a quantum oscillator. Zhurnal Eksp. Teor. Fiz. 56, 1375–1390 (1969). [2] Berson, I.Y. Electron in the quantized field of a monochromatic electromagnetic wave. Zhurnal Eksp. Teor. Fiz. 56, 1627–1633 (1969), (English Translation: Sov. Phys. JETP 1969, 29, 871.). [3] Fedorov M V and Kazakov A E 1973 An electron in a quantized plane wave and in a constant magnetic field Z. Phys. 261, 191-202 (1973).

[4] Bergou, J.; Varró, S. Nonlinear scattering processes in the presence of a quantized radiation field: I. Nonrelativistic treatment. J. Phys. A Math. Gen. 14, 1469–1482. (1981). (Bremsstrahlung) [5] Bergou, J.; Varró, S. Nonlinear scattering processes in the presence of a quantized radiation field: II. Relativistic treatment. J. Phys. A Math. Gen. 14, 2281–2303. (1981). (Compton scattering).
[5] Varró, S. Theoretical Study of the Interaction of Free Electrons with Intense Light. Ph.D. Thesis, University of Szeged, Szeged, Hungary, 1981. Hung. Phys. J. 31, 399–454. (1983) (In Hungarian).

Volkov states. II. Non-relativistic. Quantized radiation field.

$$H = \sum_i \frac{1}{2m}\left[\boldsymbol{p_i} - \frac{e}{c}\hat{\boldsymbol{A}}(\boldsymbol{r}_i) \right]^2 + \hbar\omega(\hat{A}^+\hat{A} + 1/2)$$

$$\left|\Psi_{E(\boldsymbol{P};n)}\right\rangle = \left|\boldsymbol{P}\right\rangle_e S_r D_\rho \left|n\right\rangle; \quad S_r \equiv e^{\frac{r}{2}(\hat{A}\hat{A}-\hat{A}^+\hat{A}^+)}; \quad D_\rho \equiv e^{\rho(\hat{A}^+-\hat{A})}$$

Squeezing always shows up in photon – free electron interactions due to the "A²" term in 'Q -Volkov state'.

'p.A-term': Displacement transitions (Bloch and Nordsieck 1937).

$$D(\sigma) = \exp(\sigma a^+ - \sigma^* a), \quad a \to D^+(\sigma) a D(\sigma) = a + \sigma, \quad \left\langle m \middle| D(\sigma) \middle| n \right\rangle = \sqrt{\frac{n!}{m!}}\sigma^{m-n} L_n^{m-n}(|\sigma|^2)\exp(-\tfrac{1}{2}|\sigma|^2)$$

'A²-term': Squeezing transitions (see Refs. [1], [2], 2021, 2022). Bogoliubov transformation.

$$S(\xi) = \exp[\tfrac{1}{2}(\xi\hat{a}^{+2} - \xi^*\hat{a}^2)], \qquad a \to b = S^+(\xi) a S(\xi) = a\cosh r + a^+ e^{i\varphi} a \sinh r,$$

$$\left\langle m \middle| S(\xi) \middle| n \right\rangle = \mathrm{e}^{-\eta/4}(2\mathrm{e}^{i\varphi}\tanh|\xi|)^\alpha \sqrt{\frac{n!}{m!}} \frac{\Gamma(\alpha+\frac{1}{2})}{\sqrt{\pi}} C_n^{\alpha+\frac{1}{2}}\left(\frac{1}{\cosh|\xi|}\right)$$

[1] Varró S, Quantum optical aspects of high-harmonic generation. Photonics 8, 269 (2021) [https://doi.org/10.3390/photonics8070269].

[2] Varró S, Coherent and incoherent superposition of transition matrix elements of the squeezing operator. New Journal of Phys., in press (2022). https://doi.org/10.1088/1367-2630/ac6b4d. arXiv: 2112.08430 [quant-ph].

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The role of classical stochastic and photon noise in multiphoton processes. Induced Bremsstrahlung.

A 'laser-assisted process': Induced (direct and inverse) multiphoton Bremsstrahlung.

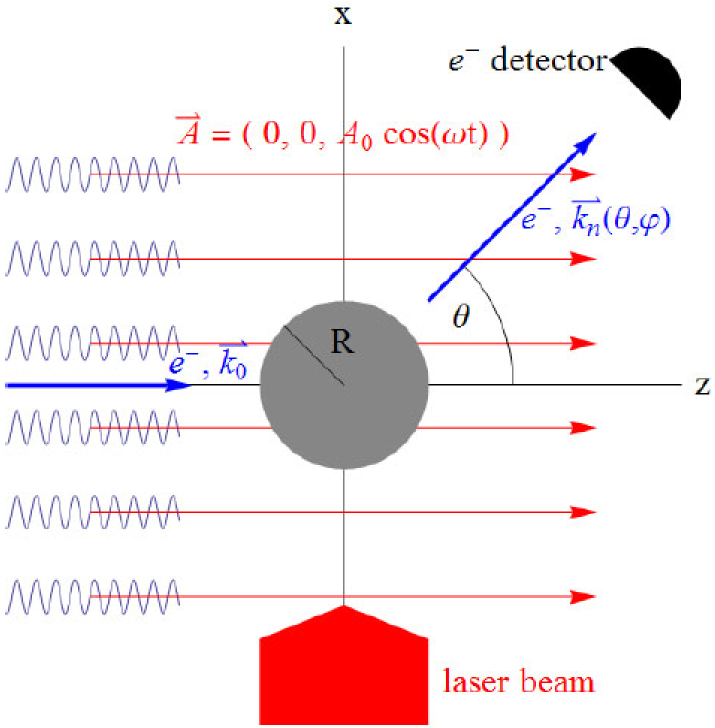


Figure 1: Geometry of the model for electron scattering on a nano-particle in the presence of a linearly polarized laser field, see the text for details.

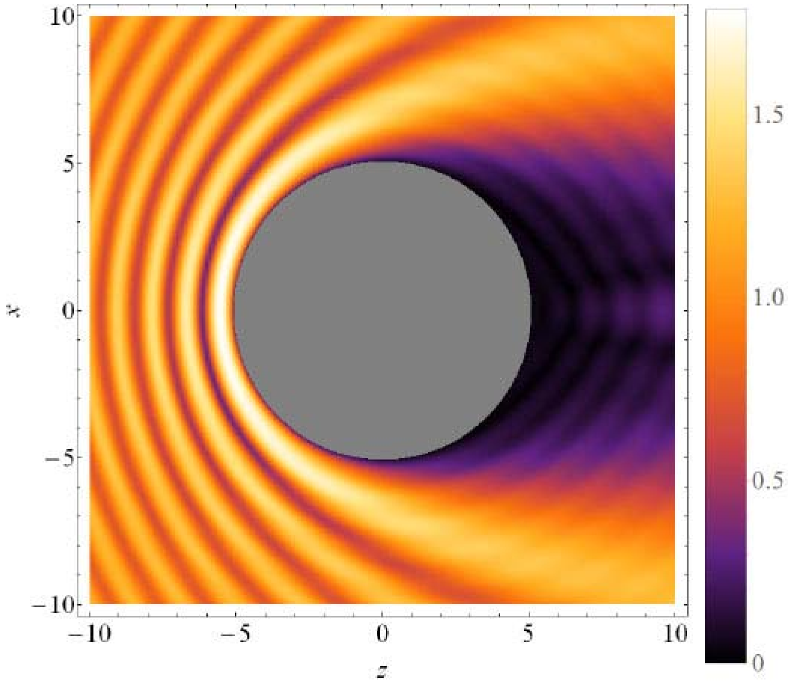


Figure 3: Density plot of the magnitude of the total electron wave-function, in the $x - z$ plane, around a hard sphere of 5 nm radius, assuming the weak field limit. Parameters: incoming electron energy $E_0 = 0.25$ eV, photon energy $\hbar\omega = 1.5$ eV.

Figures copied from: Varró S, Czirják A and Szabó L, Laser-assisted electron scattering on a nano-sphere. ***Nuclear Instruments and Methods in Physics Research B*** **369, 29-33 (2016).**

Induced (direct and inverse) multiphoton Bremsstrahlung.

n-photon differential cross-sections:

$$\frac{d\sigma_n}{d\Omega} = \frac{p_f}{p_i} J_n^2(z) \frac{d\sigma_{Born}}{d\Omega}$$

Energy conservation:

$$E_f = \frac{\mathbf{p}_f^2}{2m} = \frac{\mathbf{p}_i^2}{2m} + n\hbar\omega$$

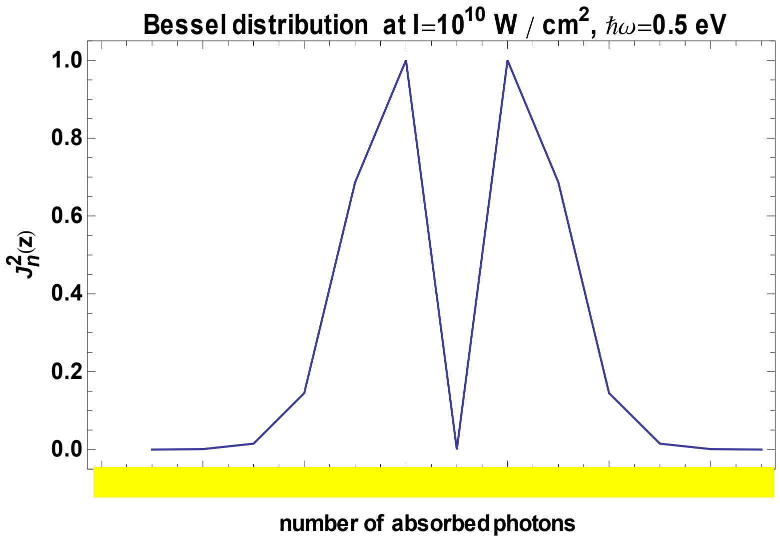
$$n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

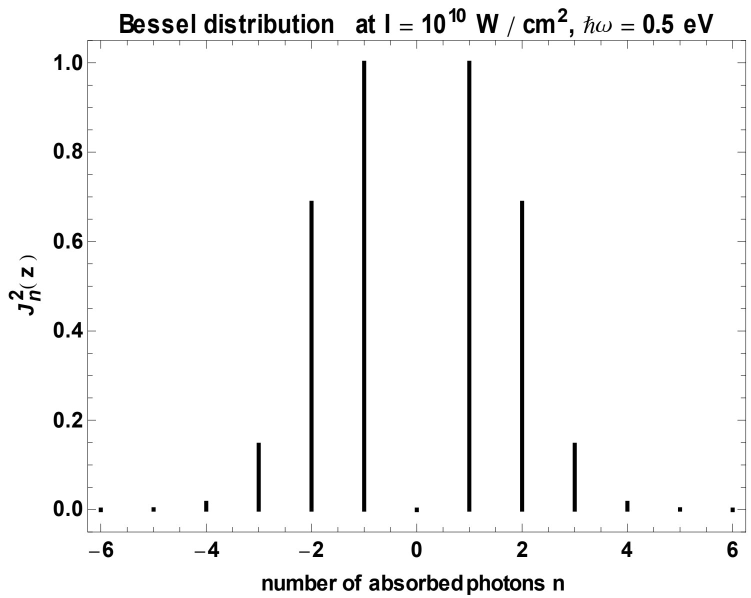
$$z = \mu_0 q / \hbar k = \mu_0 \sqrt{\frac{2mc^2}{\hbar\omega}\frac{E_i}{\hbar\omega}}\, \boldsymbol{\varepsilon} \cdot \left[\mathbf{n}_i - \mathbf{n}_f \left(1 + n\frac{\hbar\omega}{E_i} \right) \right]$$

Argument of the Bessel function.

$$\mu_0 = \frac{eF_0}{mc\omega} = 10^{-9}\sqrt{I_0[W/cm^2]} / \hbar\omega[eV]$$

Dimensionless intensity parameter („dimesionless vector potential”).





Incoherent sums of probabilities in black-body radiation or in ASE pedestal. [Planck – Bose weights.] Asymmetry in absorption and emission.

$$p_n = (1-b)b^n \qquad b = \frac{\overline{n}}{1+\overline{n}} = \mathrm{e}^{-h\nu/kT} \qquad \overline{n} = \frac{1}{\mathrm{e}^{h\nu/kT}-1}$$

Incoherent sums with Bose energy distribution of a component of black – body radiation.

$$\sum_{n=0}^{\infty} p_n \left|\langle n+k|D(\sigma)|n\rangle\right|^2 = b^{-k/2} I_k\left(2|\sigma|^2\sqrt{\overline{n}(1+\overline{n})}\right)\exp[-|\sigma|^2(1+2\overline{n})],$$

$b = \mathrm{e}^{-h\nu/kT}$

$$\sigma = (\beta/m\hbar\omega)^{1/2}(\boldsymbol{p}\cdot\boldsymbol{\varepsilon})$$

Where I is a modified Bessel function of first kind.

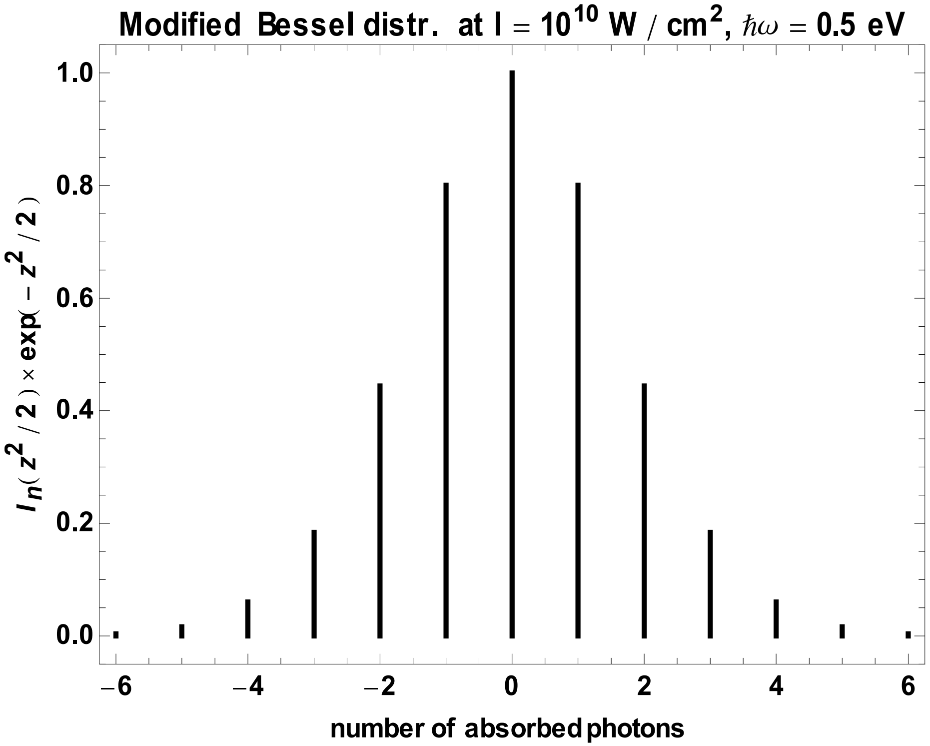
If hν << kT (Rayleigh – Jeans case), then b → 1, and we get back the semiclassical result with a 'chaotic field" (Zoller, 1980). (e.g. CO_2 'laser' or ASE pedestal).

$$\frac{1}{F_0^2}\int_0^{\infty} dFF\exp[-(F/F_0)^2]J_n^2[z_0(F/F_0)] = I_n(z_0^2/2)\exp(-z_0^2/2), \qquad z_0 = \alpha_0(\boldsymbol{p}\cdot\boldsymbol{\varepsilon})/\hbar$$

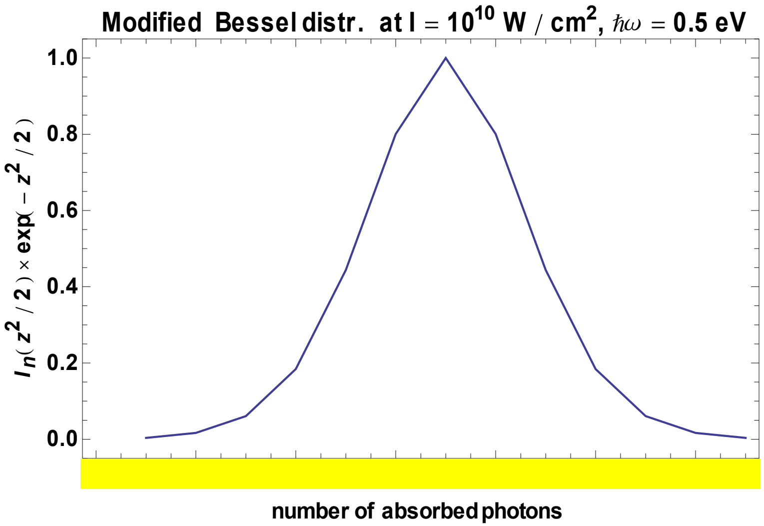
Where exp(– α)=exp(– E/kT) is the energy distribution of a 'chaotic' field. [It is a Gaussian distribution with respect to the field amplitude. In the Wien limit (by using classical stochastic field amplitudes in the calculation) the semi-classical description fails.]

Induced Multiphoton Bremsstrahlung in a „Pedestal" (Bose distribution from amplifier) [or a Black-Body Radiation Component]. 'n-factorial increase'.

Modified Bessel distribution from chaotic field.



Shape of the distribution.



$$I_n(z_0^2/2)\exp(-z_0^2/2), \quad z_0 = \alpha_0(\boldsymbol{p}\cdot\boldsymbol{\varepsilon})/\hbar$$

'n-factorial increase':

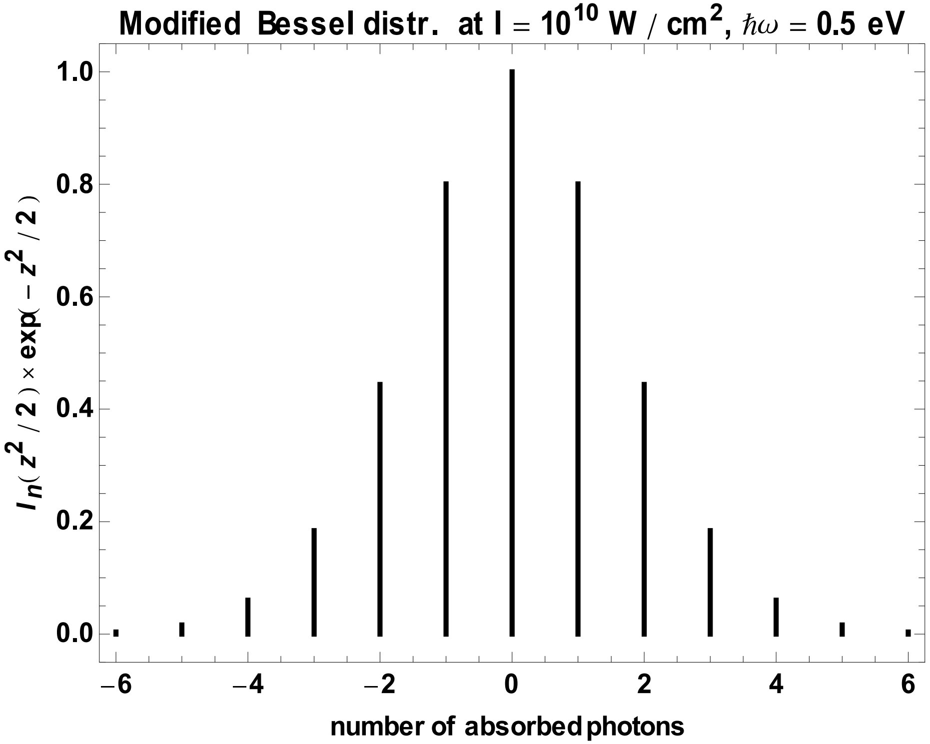
Coh. $$J_n^2(z_0) \approx \frac{1}{(n!)^2}(z_0^2/4)^n$$

Incoh. $$I_n(z_0^2/2) \approx \frac{1}{n!}(z_0^2/4)^n$$

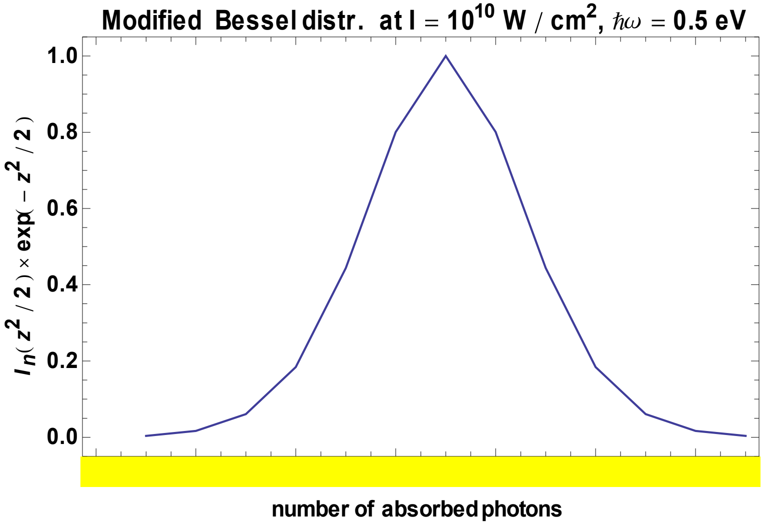
$$\frac{1}{F_0^2}\int_0^\infty dFF\exp[-(F/F_0)^2]J_n^2[z_0(F/F_0)] = I_n(z_0^2/2)\exp(-z_0^2/2)$$

Induced Multiphoton Bremsstrahlung in a „Pedestal” (Bose distribution from amplifier) [or a Black-Body Radiation Component].

Modified Bessel distribution from chaotic field.



Shape of the distribution.

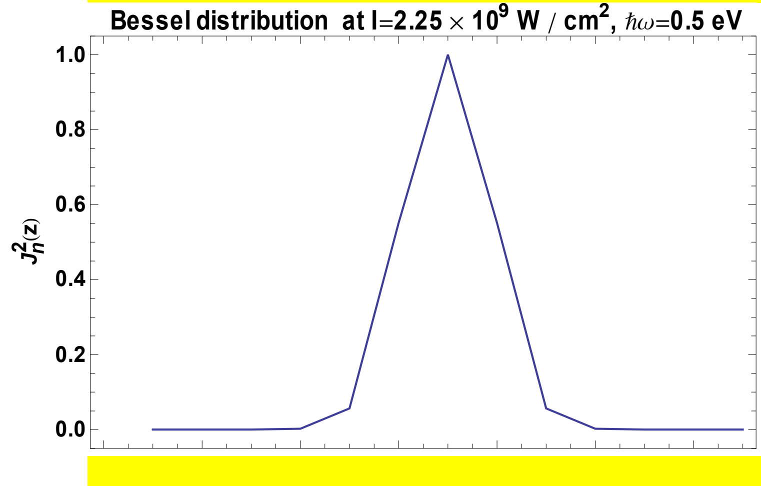


$$\sum_{n=0}^{\infty} p_n \left| \langle n+k | D(\sigma) | n \rangle \right|^2 = b^{-k/2} I_k \left(2 |\sigma|^2 \sqrt{\bar{n}(1+\bar{n})} \right) \exp[-|\sigma|^2 (1+2\bar{n})],$$

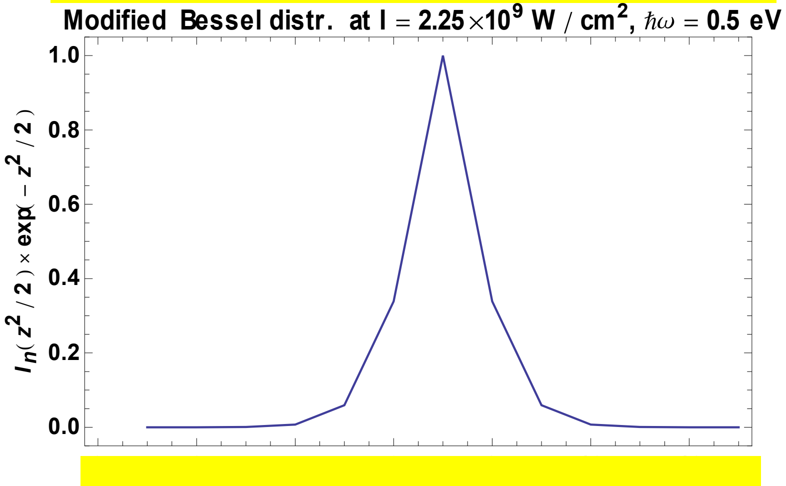
$$b = \frac{\bar{n}}{1+\bar{n}} = e^{-h\nu/kT_{eff}} \approx 1$$

Induced (direct and inverse) multiphoton Bremsstrahlung.

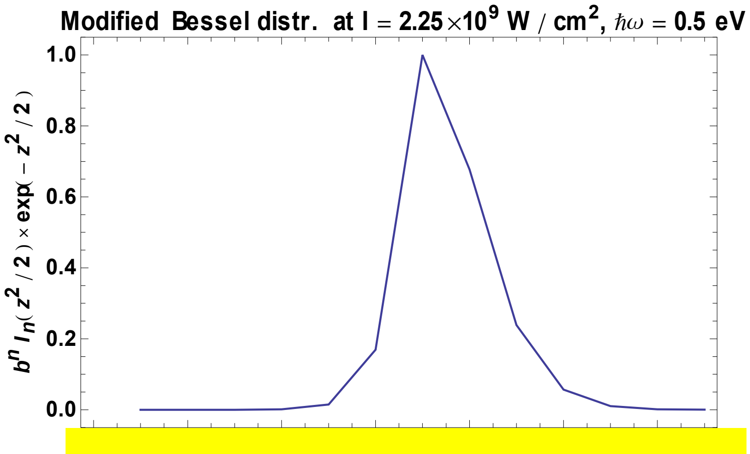
Bessel; coherent field.



Modified Bessel; chaotic field.



Exp*Modified Bessel; Planck-Bose field.



$$J_n^2[z_0(F/F_0)],$$

$$I_n(z_0^2/2)\exp(-z_0^2/2),$$

$$b^{-k/2} I_k\left(2|\sigma|^2\sqrt{\overline{n}(1+\overline{n})}\right)\exp[-|\sigma|^2(1+2\overline{n})],$$

The behaviour of the sidebands in a composite laser-assisting field. [In this case the assiting (original, IR) laser field is assumed to be a coherent superposition of the 'signal' and an (ASE) 'noise'.

Intensity distribution (c-number, stochastic):

$$p(w) = \frac{1}{\overline{w}_{ASE}} \exp\left(-\frac{w + w_S}{\overline{w}_{ASE}} \right) I_0 \left(\frac{2\sqrt{w_S w}}{\overline{w}_{ASE}} \right)$$

Semi-classical photoelectron counting.

$$p(n) = \frac{\overline{n}_{ASE}^{\,n}}{(1 + \overline{n}_{ASE})^{n+1}} \exp\left(-\frac{\overline{n}_S}{1 + \overline{n}_{ASE}} \right) L_n \left(-\frac{\overline{n}_S / \overline{n}_{ASE}}{1 + \overline{n}_{ASE}} \right)$$

The original coherent s-th sideband magnitudes go over into an interpolating expression, containing the (ASE) noise:

$$T^{\pm s}_{2N \pm s}(n,l,m,\boldsymbol{p}) \propto J_{\pm s}(\boldsymbol{\alpha}_L \cdot \boldsymbol{p} / \hbar) T_1(n,l,m,\boldsymbol{p})$$

The strengths of the sidebands in the composide field (coherent superposotion of signal plus noise).

$$\sum_{k=-\infty}^{\infty} J^2_{s-k}(z_0 \sqrt{1-x}) I_k(\tfrac{1}{2} z_0^2 x) e^{-\frac{1}{2} z_0^2 x}, \qquad x = \frac{\overline{w}_{ASE}}{w_S + \overline{w}_{ASE}}$$

x = 0: Coherent limit.

$$J_n^2(z_0) \approx \frac{1}{(n!)^2} (z_0^2 / 4)^n$$

x = 1: Chaotic limit. „n-factorial increase”

$$I_n(z_0^2 / 2) \approx \frac{1}{n!} (z_0^2 / 4)^n$$

Incoherent sums of squeezing probabilities in black-body radiation. [Planck – Bose weights.]

$$p_n = (1-b)b^n \qquad b = \frac{\overline{n}}{1+\overline{n}} = \mathrm{e}^{-h\nu/kT} \qquad \overline{n} = \frac{1}{\mathrm{e}^{h\nu/kT}-1}$$

Incoherent sums with Bose energy distribution of a component of black – body radiation.

$$\sum_{n=0}^{\infty} p_n \left| \langle n+2k | S(\xi) | n \rangle \right|^2 = \frac{1}{\pi\sqrt{\overline{n}(1+\overline{n})\sinh^2|\xi|}} b^{-k} Q_{k-\frac{1}{2}}(z), \qquad z = 1 + \frac{1}{2\overline{n}(1+\overline{n})\tanh^2|\xi|}$$

b^{-k} ← $\mathrm{e}^{-h\nu/kT}$

Where Q is a Legendre function of second kind.

If hν << kT (Rayleigh – Jeans case), then b → 1, and we get back the semiclassical result with a 'chaotic field" (e.g. CO_2 'laser' or ASE pedestal).

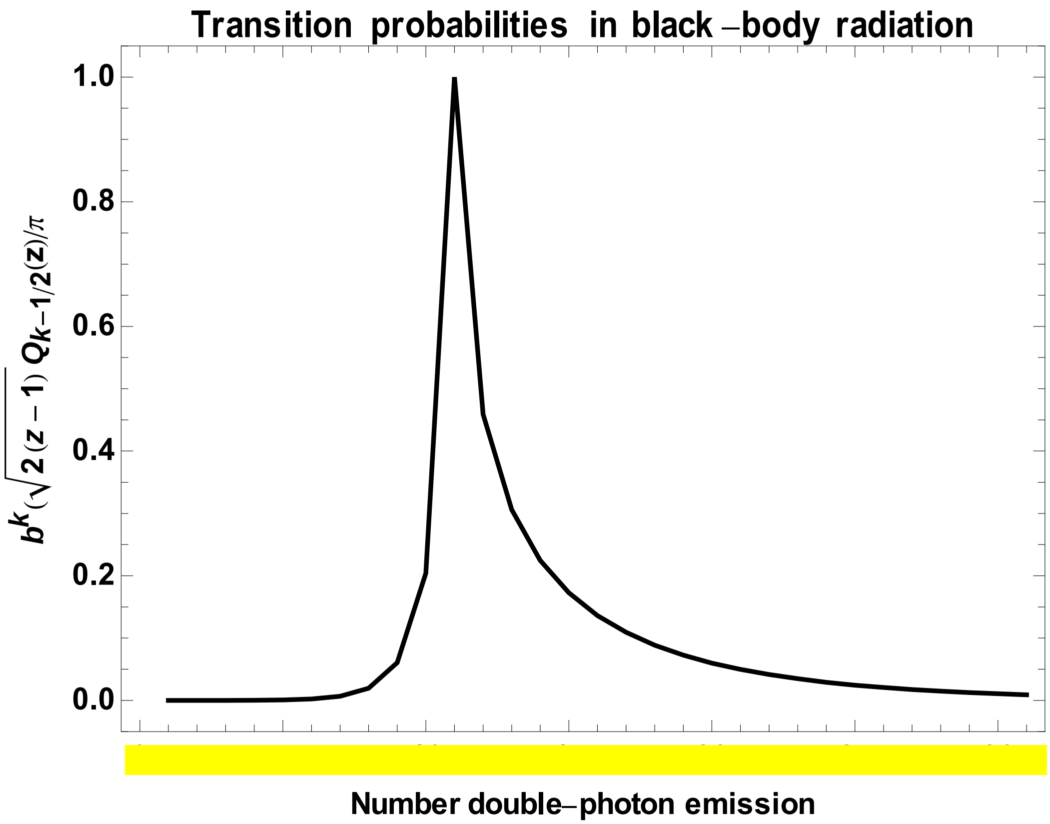
$$\frac{1}{F_0^2}\int_0^{\infty} dF F \exp[-(F/F_0)^2] J_k^2[u_0(F/F_0)^2] = \frac{z}{\pi} Q_{k-\frac{1}{2}}(1+\tfrac{1}{2}z^2), \qquad z = \frac{1}{u_0} = \frac{2\hbar\omega_0}{U_0}$$

Where exp(– α)=exp(– E/kT) is the energy distribution of a 'chaotic' field. [It is a Gaussian distribution with respect to the field amplitude. In the Wien limit (by using classical stochastic field amplitudes in the calculation) the semi-classical description fails.]

Varró S, Multiphoton Processes in the Field of a 'Pedestal'. [Seminar of ELI-ALPS [18 March 2022., ELI-ALPS, Szeged, Hungary].

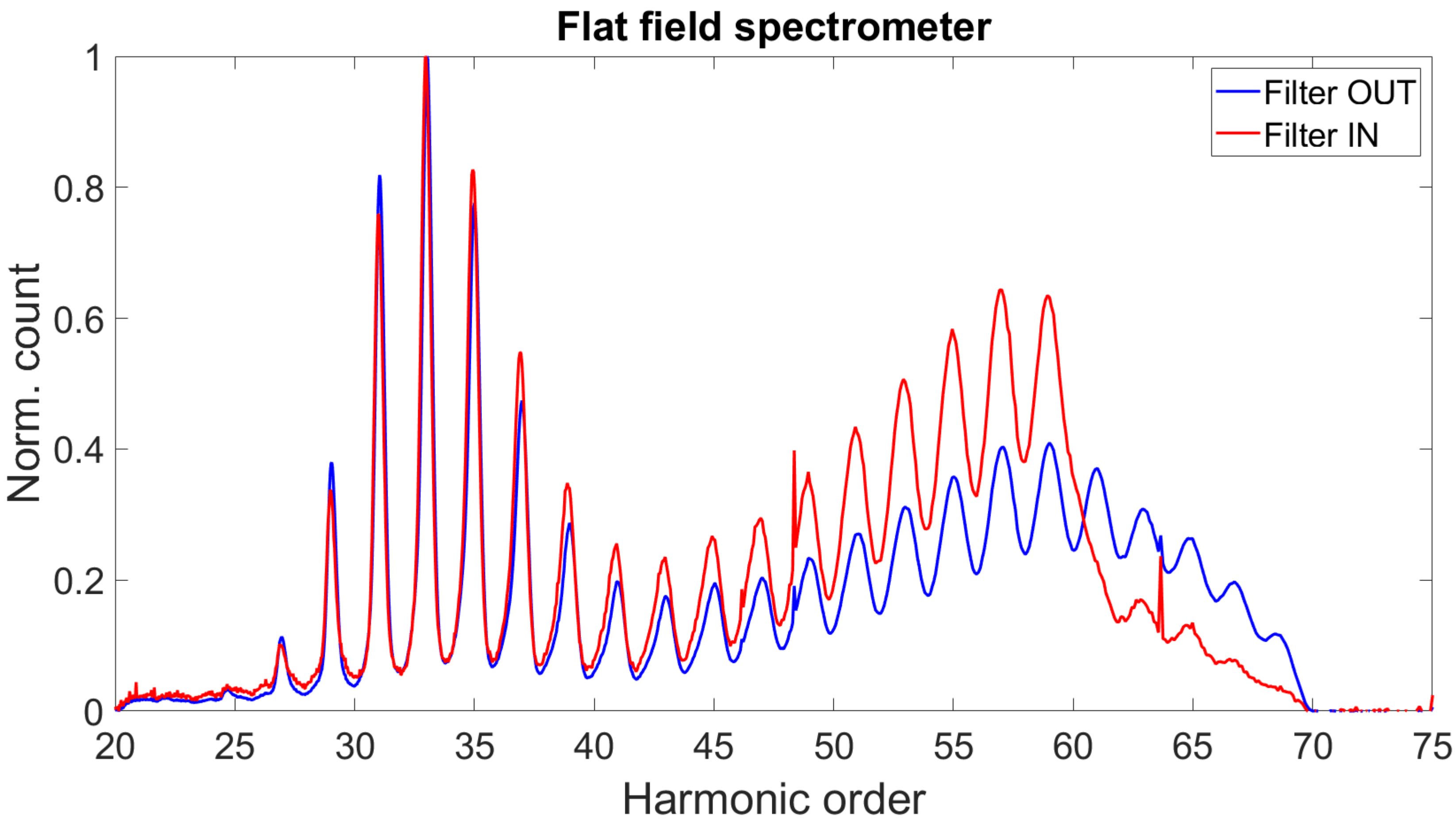
Incoherent sums of probabilities in the field of a 'pedestal'.
[(Planck –) Bose weights.]

$$\sum_{n=0}^{\infty} p_n \left|\langle n+2k \left| S(\xi) \right| n \rangle\right|^2 = \frac{1}{\pi\sqrt{\bar{n}(1+\bar{n})\sinh^2|\xi|}} b^{-k} Q_{k-\frac{1}{2}}(z),$$



$$(-15 \leq k \leq +15)$$

Note on the RABBITT [reconstruction of attosecond beating by interference of two-photon transitions]



ELI-ALPS. 20 June 2019. LTA4; HR1, 1030 nm.

Note on the RABBITT [reconstruction of attosecond beating by interference of two-photon transitions]

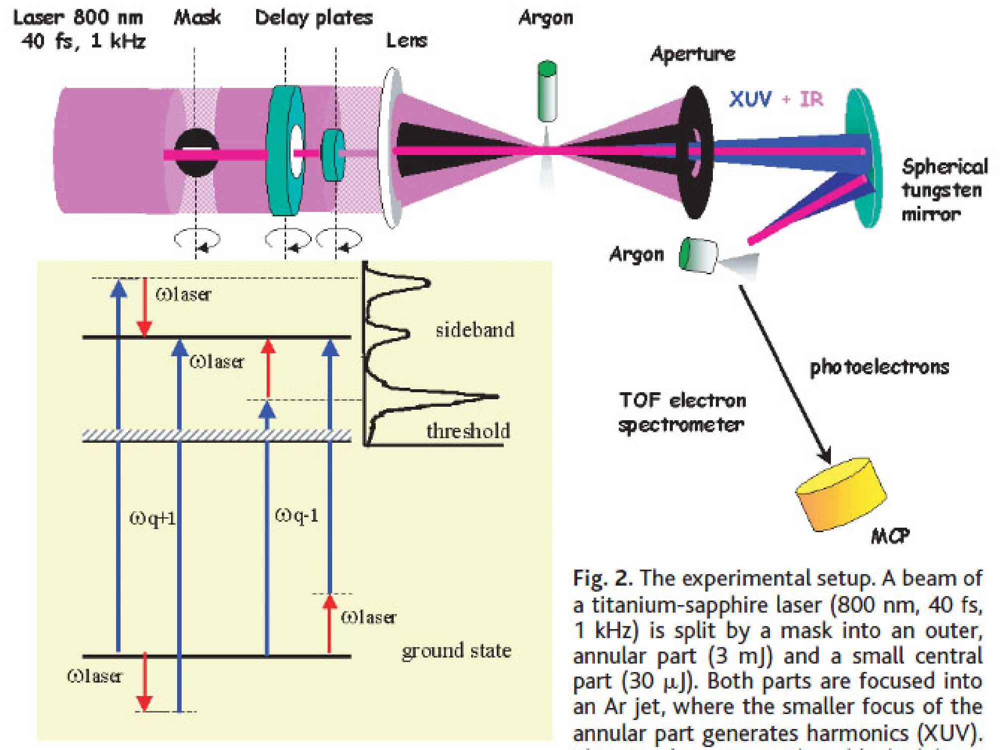
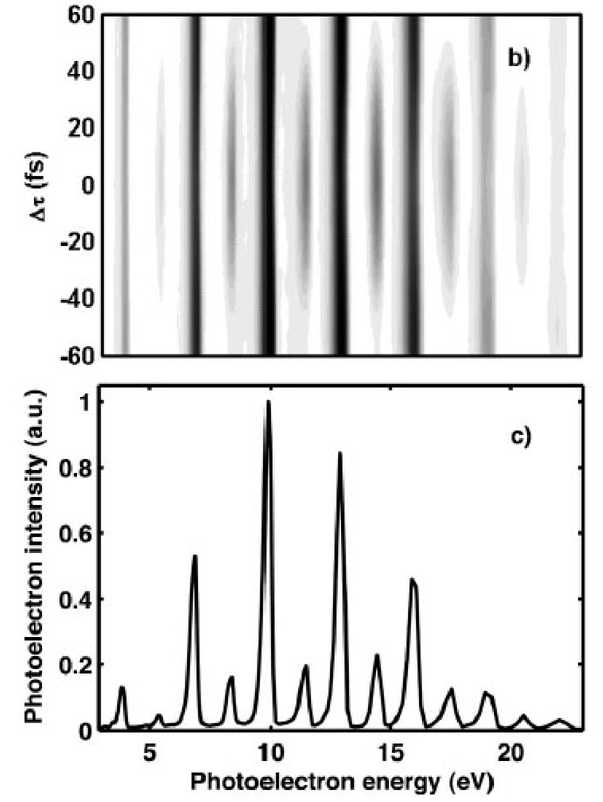


Fig. 2. The experimental setup. A beam of a titanium-sapphire laser (800 nm, 40 fs, 1 kHz) is split by a mask into an outer, annular part (3 mJ) and a small central part (30 μJ). Both parts are focused into an Ar jet, where the smaller focus of the annular part generates harmonics (XUV). The annular part is then blocked by a pinhole, and only the central part of the IR pulse and its harmonics propagates into a magnetic-



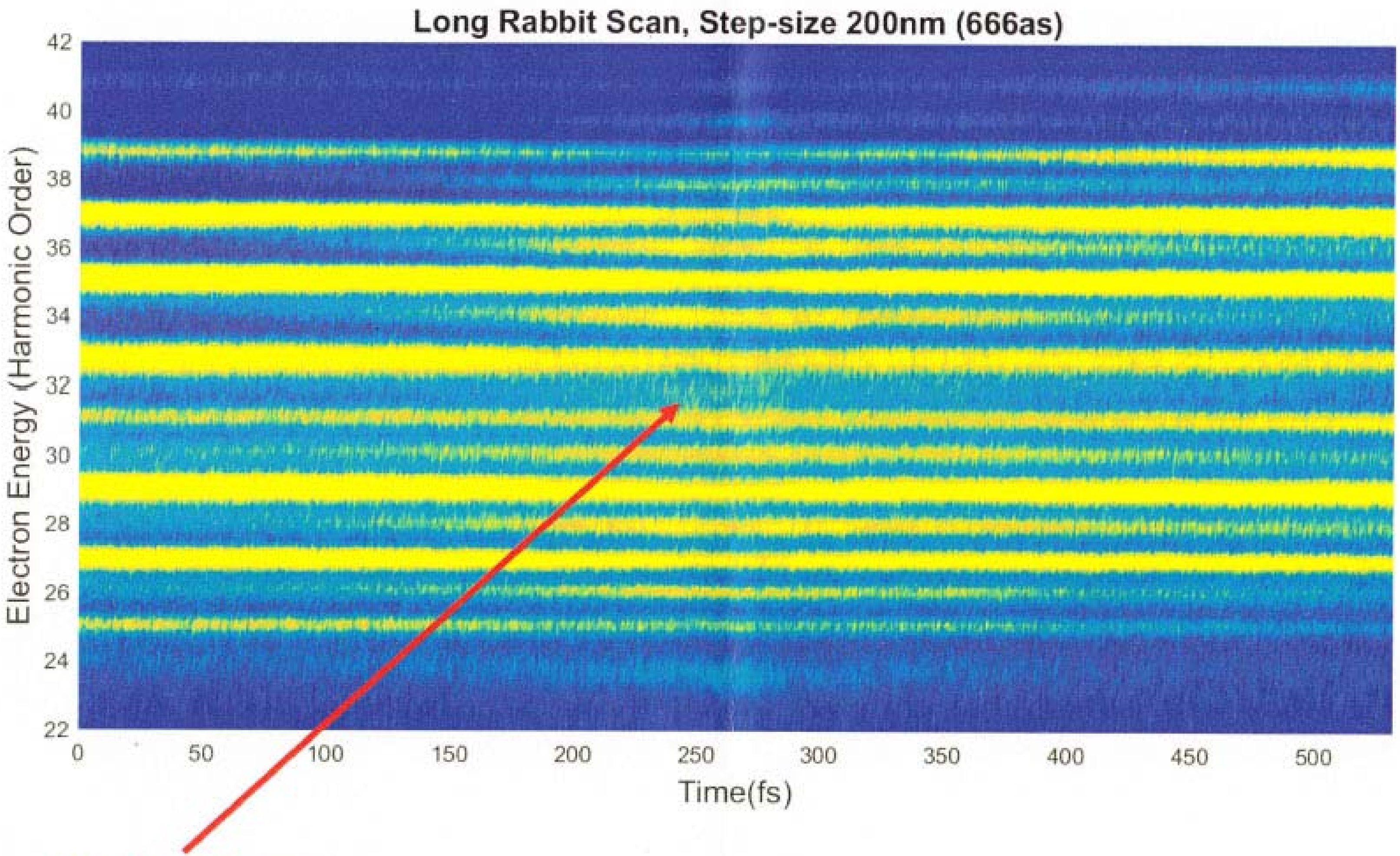
$$\cos(2\varphi_{IR} + \varphi_{2N-1} - \varphi_{2N+1} + \Delta\phi_a)$$

$$T^{(\pm)}_{2N\pm1}(n,l,m,\boldsymbol{k})$$

Paul P M, Toma E S, Breger P Mullot G, Augé F, Balcou Ph, Muller H G and Agostini P , Observation of a train of attosecond pulses from high harmonic generation. *Science* 292, 1689 (2001). Right figure taken from O'Keeffe P, López-Martens R, Mauritsson J, Johansson A, L'Huillier A, Véniard V, Taieb R, Maquet A and Meyer M, Polarization effects in two-photon nonresonant ionization of argon with extreme-ultraviolet and infrared femtosecond pulses. Phys. Rev. A 69, 051401(R) (2004).

$$\frac{1}{2}k^2 = (2N \pm 1)\omega - I_p \mp \omega$$

Laser-assisted ionization. [A puzzling recent Rabbitt scan received at ELI-ALPS.]



$$T_{2N\pm1}^{(\pm)}(n,l,m,\boldsymbol{k}) \propto J_{\pm1}(\boldsymbol{\alpha}_L \cdot \boldsymbol{k})T_1(n,l,m,\boldsymbol{k})$$

$$\boldsymbol{\alpha}_L = \boldsymbol{\varepsilon}_L \mu_0 \lambda\!\!\!^{-}{}_L$$

The behaviour of the RABBITT sidebands in a composite laser-assisting field. [In this case the assiting (original, IR) laser field is assumed to be a coherent superposition of the 'signal' and an (ASE) 'noise'.

Intensity distribution (c-number, stochastic):

$$p(w) = \frac{1}{\overline{w}_{ASE}} \exp\left(-\frac{w + w_S}{\overline{w}_{ASE}} \right) I_0\left(\frac{2\sqrt{w_S w}}{\overline{w}_{ASE}} \right)$$

Semi-classical photoelectron counting.

$$p(n) = \frac{\overline{n}_{ASE}^{\,n}}{(1+\overline{n}_{ASE})^{n+1}} \exp\left(-\frac{\overline{n}_S}{1+\overline{n}_{ASE}} \right) L_n\left(-\frac{\overline{n}_S / \overline{n}_{ASE}}{1+\overline{n}_{ASE}} \right)$$

The original coherent s-th sideband magnitudes go over into an interpolating expression, containing the (ASE) noise:

$$T_{2N\pm s}^{\pm s}(n,l,m,\boldsymbol{p}) \propto J_{\pm s}(\boldsymbol{\alpha}_L \cdot \boldsymbol{p}/\hbar) T_1(n,l,m,\boldsymbol{p})$$

The strengths of the sidebands in the composite field (coherent superposotion of signal plus noise).

$$\sum_{k=-\infty}^{\infty} J_{s-k}^2 (z_0 \sqrt{1-x}) I_k (\tfrac{1}{2} z_0^2 x) e^{-\frac{1}{2} z_0^2 x}, \qquad x = \frac{\overline{w}_{ASE}}{w_S + \overline{w}_{ASE}}$$

x = 0: Coherent limit.

$$J_n^2(z_0) \approx \frac{1}{(n!)^2} (z_0^2/4)^n$$

x = 1: Chaotic limit.

$$I_n(z_0^2/2) \approx \frac{1}{n!} (z_0^2/4)^n$$

Summary

- Introduction. ‘Pedestals’ of amplified light pulses. Amplified Spontaneous Emission (ASE).

- Bridges between strong field physics and quantum optics.

- The ‘n-factorial increase’ of multiphoton excitation probabilities. Asymmetry of absorption and emission for Bose distribution.

- Note on the RABBITT [reconstruction of attosecond beating by interference of two-photon transitions]

René Magritte, The lost jockey (1948).

Varró S, Multiphoton Processes in the Field of a 'Pedestal'. [Talk presented at Margaret Island Symposium 2022 on Vacuum Structure , Particles, and Plasmas, 15-18 May 2022., Budapest, Hungary].

Appendix.

Multiphoton processes in the field of a ‘pedestal’.

Sándor Varró [1,2]
1) Wigner Research Centre for Physics, Inst. Solid State Phys. and Optics, Eötvös Loránd Research Network, Budapest
2) ELI-ALPS (Attosecond Light Pulse Source) Research Institute, ELI-HU Non-profit Ltd., W. Sandner utca 3., Szeged

Abstract. The high-intensity light pulses used in mutiphoton experiments come from amplifiers, and these pulses contain quite strong (unwanted) ‘pre-pulses’, or ‘pedestals’. The main source of the pedestal is said to be the amplified spontaneous emission (ASE) of the amplifying medium. Since the photon statistics of the ASE is different from that of the main pulse, the study of multiphoton processes taking place in a pedestal may be interesting, both in theory and in experiments. Recently we have worked out a general quantum optical theory of multiphoton processes [1, 2], on the basis of which, we shall deal with the possible role of photon statistics in electron excitation, scattering and high-harmonic generation in the field of a ‘pedestal’.

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[1] Varró S, Quantum optical aspects of high-harmonic generation. Photonics 8, 269 (2021) [https://doi.org/10.3390/photonics8070269].
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The contrast problem in the experimental study of very-high-order multiphoton processes.

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Gombkötő Á, Czirják A, Varró S and Földi P, A quantum optical model for the dynamics of high harmonic generation. *Physical Review A* 94, 013853 (2016) *E-print*: arXiv: 1605.01087 [quant-ph], 2016.

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Multifoton Kramers–Heisenberg-formula;
Quantum optical treatment.

Ref.: Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269 (2021). https://doi.org/10.3390/photonics8070269 . Special Issue “Quantum Optics in Strong Laser Fields”.

A nonrelativistic QED treatment of high-harmonic generation.

$$\left[\frac{1}{2m}\left(\hat{\boldsymbol{p}}+\frac{e}{c}\hat{\boldsymbol{A}}\right)^2+V(\boldsymbol{r})+\hat{H}_{rad}\right]|\Psi\rangle = i\hbar\frac{\partial|\Psi\rangle}{\partial t}$$

$$\hat{H}_{rad} = \sum_{\boldsymbol{k},s}\hbar\omega_k(\hat{a}^+_{\boldsymbol{k},s}\hat{a}_{\boldsymbol{k},s}+\tfrac{1}{2})$$

$$\hat{\boldsymbol{A}} = \sum_{\boldsymbol{k},s}\hat{\boldsymbol{A}}_{\boldsymbol{k},s}(\boldsymbol{r}) = c\sum_{\boldsymbol{k},s}\sqrt{2\pi\hbar/\omega_k L^3}\,\boldsymbol{\varepsilon}_{\boldsymbol{k},s}(\hat{a}_{\boldsymbol{k},s}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\hat{a}^+_{\boldsymbol{k},s}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}})$$

By expanding the ‘kinetic energy’;

$$\left[\frac{\hat{\boldsymbol{p}}^2}{2m}+V(\boldsymbol{r})+\hat{K}+\hat{M}\right]|\Psi(t)\rangle = i\hbar\partial_t|\Psi(t)\rangle$$

$$\hat{K} = \sum_k \hat{K}_k \qquad M = \sum_k\sum_{k'<k}\hat{M}_{k,k'}$$

In dipole approximation:

$$\hat{K}_k = \hbar\omega_k[\sqrt{\beta_k/m\hbar\omega_k}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{\varepsilon}_k)(\hat{a}_k+\hat{a}_k^+)+\tfrac{1}{2}\beta_k(\hat{a}_k^2+\hat{a}_k^{+2})+(1+\beta_k)(\hat{a}_k^+\hat{a}_k+\tfrac{1}{2})]$$

where

$$\beta_k = \omega_p^2/2\omega_k^2 \qquad \omega_p^2 = 4\pi n_e e^2/m \qquad n_e = 1/L^3$$

ω_p is a ‘plasma frequency’ $n_e = N/L^3$

Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269 (2021). [https://doi.org/10.3390/photonics8070269]. Special Issue “Quantum Optics in Strong Laser Fields”.

Diagonalization: elimination of the p.A and A² terms. The appearance of the „quantized space-translated potential”.

$$|\Psi(t)\rangle = \hat{S}\hat{D}|\Phi_{SD}(t)\rangle \qquad \hat{S} = \Pi_k \hat{S}_k(\theta_k) \qquad \hat{D} = \Pi_k \hat{D}_k(\sigma_k)$$

$$\hat{S}_k(\theta_k) = \exp[\tfrac{1}{2}\theta_k(\hat{a}_k^2 - \hat{a}_k^{+2})] \qquad \theta_k = \tfrac{1}{4}\log(1+2\beta_k)$$

(squeezing transformation)

$$\hat{D}_k(\sigma_k) = \exp[\sigma_k(\hat{a}_k^+ - \hat{a}_k)] \qquad \sigma_k = -\sqrt{\beta_k / m\hbar\omega_k}\, e^{-3\theta_k}(\hat{\boldsymbol{p}}\cdot\boldsymbol{\varepsilon}_k)$$

(displacement op.)

„quantized space-translated potential”

New equation:

$$\left[\frac{\hat{\boldsymbol{p}}^2}{2m} + V(\boldsymbol{r}+\hat{\boldsymbol{\alpha}}) + \widetilde{H}_{rad} + \hat{D}^+\hat{S}^+\hat{M}\hat{S}\hat{D}\right]|\Phi_{SD}(t)\rangle = i\hbar\partial_t|\Phi_{SD}(t)\rangle$$

where

$$\hat{\boldsymbol{\alpha}} = \sum_k \hat{\boldsymbol{\alpha}}_k \qquad \hat{\boldsymbol{\alpha}}_k = -\sqrt{\widetilde{\beta}_k / m\hbar\widetilde{\omega}_k}\,\boldsymbol{\varepsilon}_k(\hat{a}_k^+ - \hat{a}_k) = \frac{e}{mc^2}\hat{\boldsymbol{Z}}_k \qquad \widetilde{\omega}_k = \sqrt{c^2|\boldsymbol{k}|^2 + \omega_p^2}$$

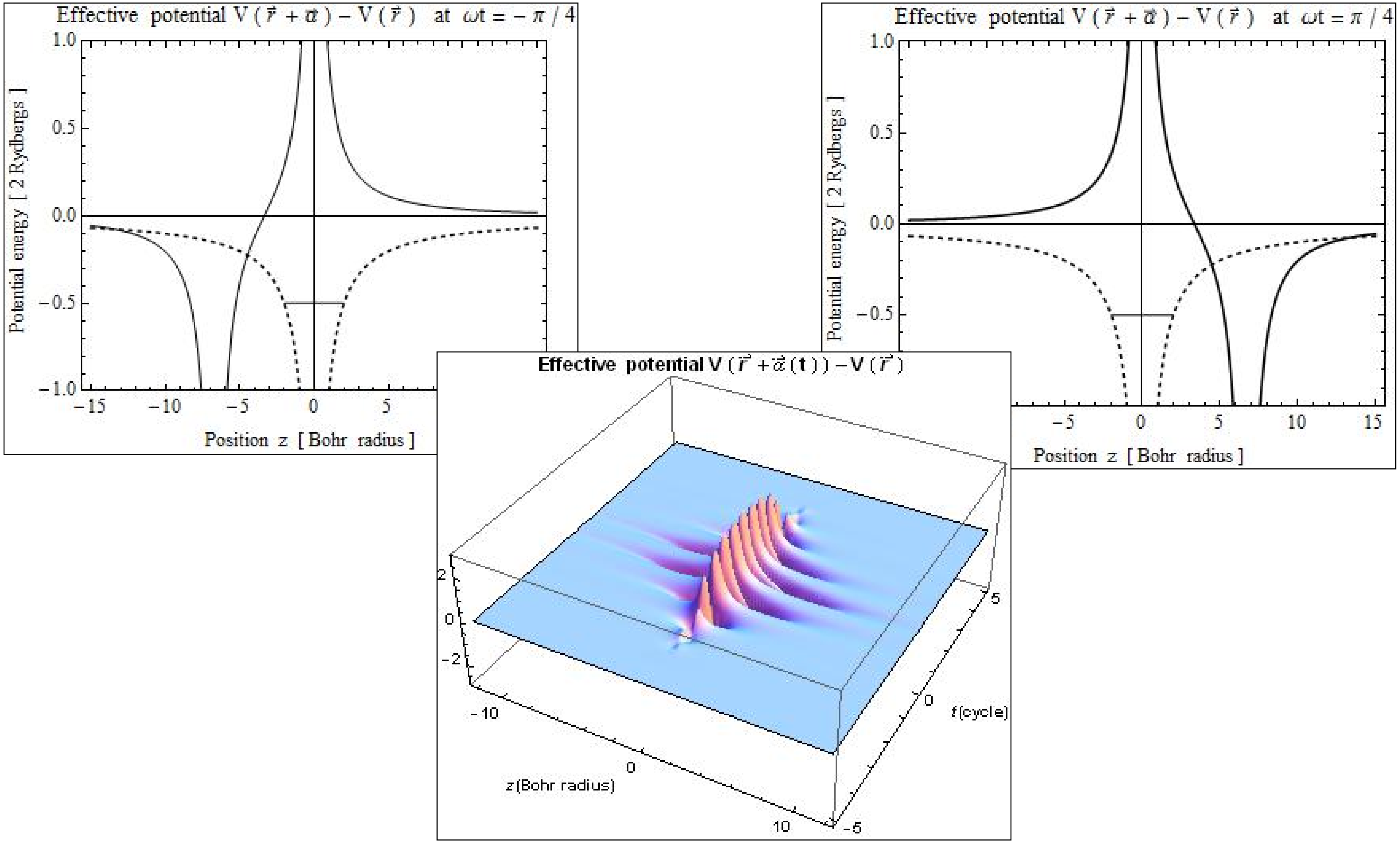
Hertz vector

plasmon dispersion, „blue-shift”, „attochirp”...

$$\widetilde{H}_{rad} = \sum_{\boldsymbol{k},s}\hbar\widetilde{\omega}_k(\hat{a}_{\boldsymbol{k},s}^+\hat{a}_{\boldsymbol{k},s} + \tfrac{1}{2})$$

Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269 (2021). [https://doi.org/10.3390/photonics8070269]. Special Issue “Quantum Optics in Strong Laser Fields”. The elimination technique is the same as in Bergou J and Varró S, J. Phys. A 14, 1469 (1981), ibid. 14, 2281 (1981) used for free Schrödinger and Dirac electrons, resp.

„Multiphoton effective potential”; V(r+α(t)) – V(r)



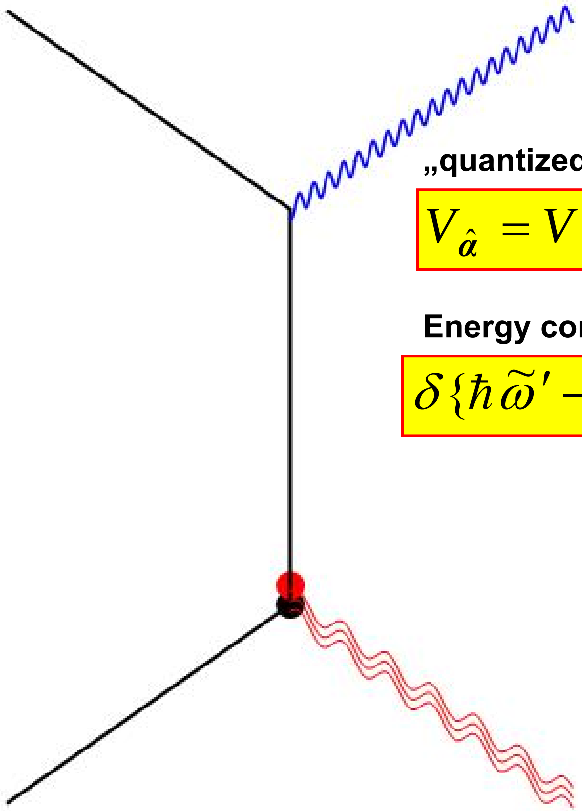
Ref. Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269 (2021). [https://doi.org/10.3390/photonics8070269]. Special Issue “Quantum Optics in Strong Laser Fields”.

Varró S, Strong-field Kramers-Heisenberg formula for high-harmonic generation in quantized radiation modes.
Talk presented at Seminar 2 of LPHYS’21. [29th International Laser Physics Workshop, 19-23 July 2021, Zoom Meeting].

Two main terms of the „Strong-field Kramers-Heisenberg formula" for high-harmonic generation in quantized radiation modes.

(optional Cooper – minimum)

$$\sum_r \left[\frac{\langle \psi_1 | (\boldsymbol{p} \cdot \boldsymbol{\varepsilon}') | \psi_{E_r} \rangle \langle \psi_{E_r} | \langle n_1 | V_{\hat{\boldsymbol{\alpha}}} | n_0 \rangle | \psi_0 \rangle}{E_r - E_0 - (n_0 - n_1)\hbar\omega} \right]$$

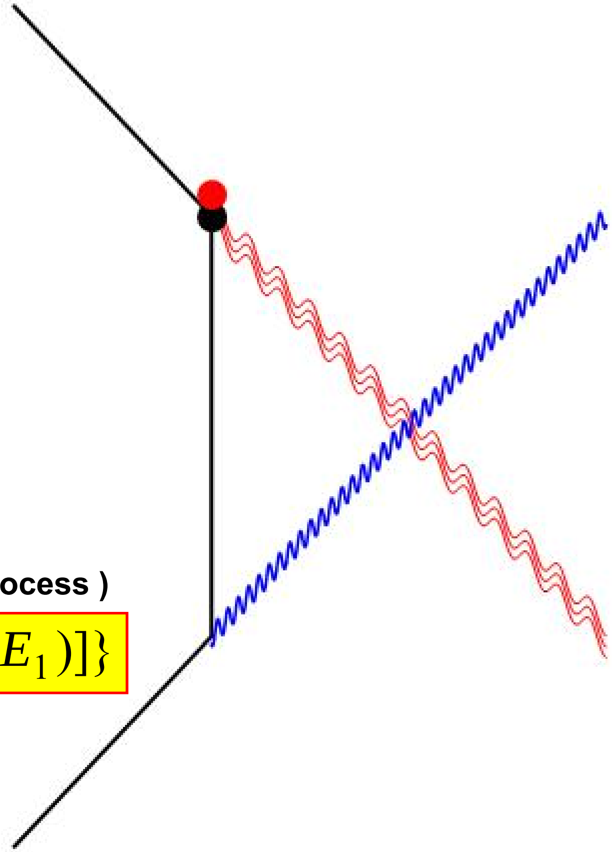


„quantized multiphoton potential"

$$V_{\hat{\boldsymbol{\alpha}}} = V(\boldsymbol{r} + \hat{\boldsymbol{\alpha}}) - V(\boldsymbol{r})$$

Energy conservation (optional Raman – process)

$$\delta\{\hbar\tilde{\omega}' - [(n_0 - n_1)\hbar\tilde{\omega} - (E_0 - E_1)]\}$$



$$\sum_r \left[\frac{\langle \psi_1 | \langle n_1 | V_{\hat{\boldsymbol{\alpha}}} | n_0 \rangle | \psi_{E_r} \rangle \langle \psi_{E_r} | (\boldsymbol{p} \cdot \boldsymbol{\varepsilon}') | \psi_0 \rangle}{E_r - E_0 + \hbar\omega'} \right]$$

Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269.
Varró S, Strong-field Kramers-Heisenberg formula for high-harmonic generation in quantized radiation modes. Seminar 2 of LPHYS'21. [29th International Laser Physics Workshop, 19-23 July 2021, Zoom Meeting].

Generalized semi-classical limit of transition amplitudes. “Imaginary satellite trajectory” can be connected to the depletion.

$$V(\boldsymbol{r}+\hat{\boldsymbol{\alpha}}) = \int d^3q V(\boldsymbol{q}) e^{\frac{i}{\hbar}\boldsymbol{q}\cdot\boldsymbol{r}} \hat{D}[\gamma(\boldsymbol{q})]$$

$$\hat{D}[\gamma(\boldsymbol{q})] = \exp[\gamma(\boldsymbol{q})(\hat{a}^+ - \hat{a})]$$

Generalized coherent superposition of transition amplitudes:

$$T_m^{(cl)} = \sum_{n=k}^{\infty} \frac{(\beta^*)^{n-m}}{\sqrt{(n-m)!}} e^{-\frac{1}{2}|\beta|^2} \langle n-m|\hat{D}(\gamma)|n\rangle \frac{(\alpha)^n}{\sqrt{(n)!}} e^{-\frac{1}{2}|\alpha|^2}$$

$$T_m^{(cl)} = \mathrm{e}^{-\frac{1}{2}|\gamma|^2} \mathrm{e}^{-\frac{1}{2}|\beta|^2 - \frac{1}{2}|\alpha|^2 + \beta^*\alpha} (s^m J_m(z))$$

$$s = \mathrm{e}^{i\chi}\sqrt{\beta^*/\alpha}$$

$$z = 2|\gamma|\sqrt{\beta^*\alpha}$$

$$s^m J_m(z) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \mathrm{e}^{-im\varphi} \exp(\gamma\beta^*\mathrm{e}^{i\varphi} - \gamma^*\alpha\mathrm{e}^{-i\varphi})$$

„imaginary satellite trajectory” **„Volkov”**

$$\mathrm{e}^{-|\gamma|(|\alpha|-|\beta|)\cos(\varphi+\chi-\phi)} \times \mathrm{e}^{i|\gamma|(|\alpha|+|\beta|)\sin(\varphi+\chi-\phi)}$$

Ref. : Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269.
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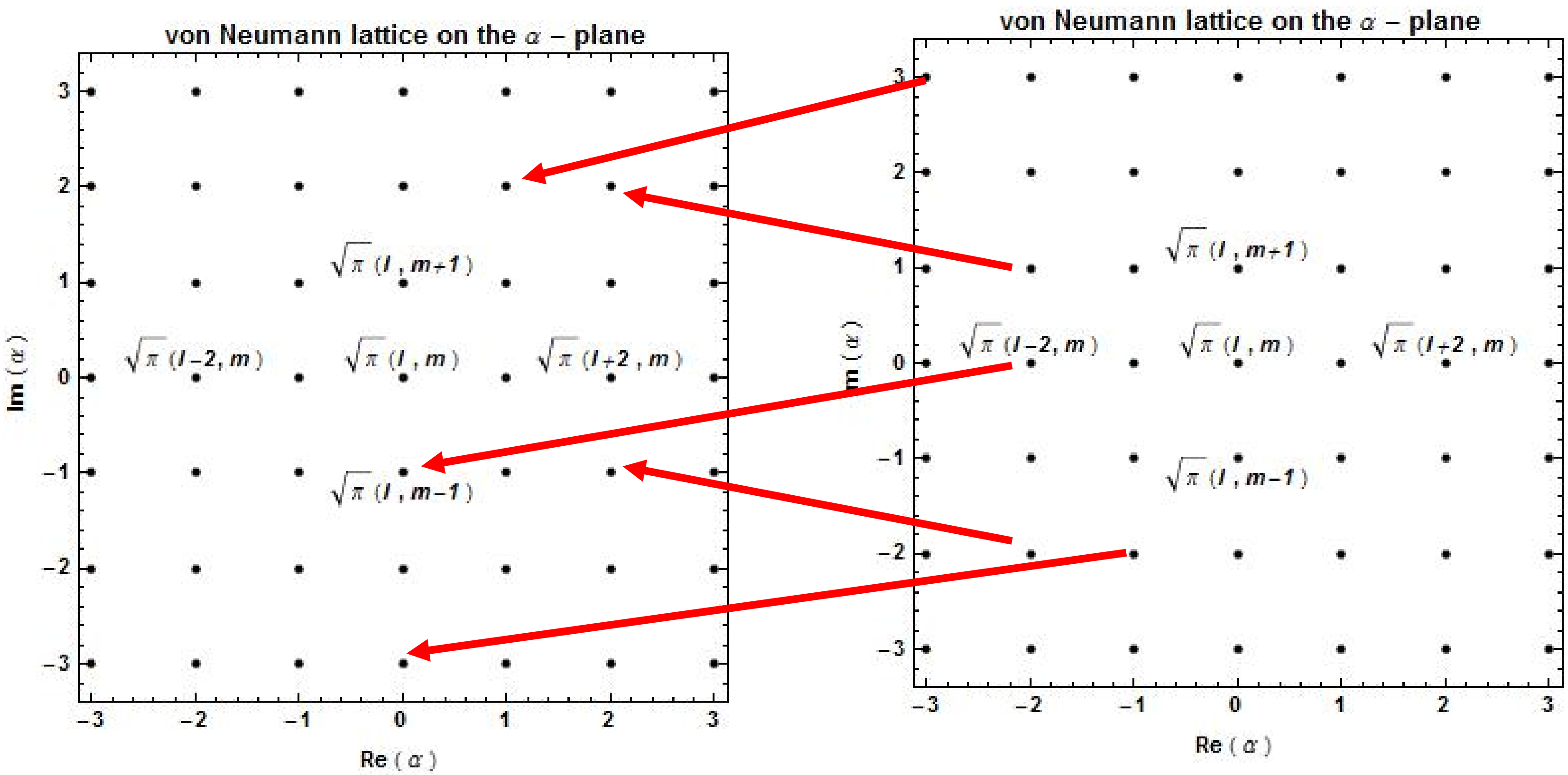
Illustration; Transitions between Neumann lattice coherent states.



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New class of exact solutions of the Dirac and Klein-Gordon equation in a strong laser field [2013].

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LETTER

New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium

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Abstract

Exact solutions are presented of the Dirac equation of a charged particle moving in a classical monochromatic electromagnetic plane wave in a medium of index of refraction $n_m < 1$. The found solutions are expressed in terms of new complex trigonometric polynomials, which form a doubly infinite set labelled by two integer quantum numbers. These quantum numbers represent quantized spectra of the energy–momentum components of the charged particle along the polarization vector and along the propagation direction of the applied electromagnetic plane wave field (which is considered as a laser field of arbitrarily high intensity propagating in an underdense plasma). The found solutions may serve as a basis for the description of possible quantum features of mechanisms for the acceleration of electrons by high-intensity laser fields.

(Some figures may appear in colour only in the online journal)

New class of exact solutions of the Dirac and Klein-Gordon equation in a strong laser field [2013].

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Laser Phys. Lett. **11** (2014) 016001 (14pp) doi:10.1088/1612-2011/11/1/016001

Letter

A new class of exact solutions of the Klein–Gordon equation of a charged particle interacting with an electromagnetic plane wave in a medium

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Abstract

Exact solutions are presented of the Klein–Gordon equation of a charged particle moving in a transverse monochromatic plasmon wave of arbitrary high amplitude, which propagates in an underdense plasma. These solutions are expressed in terms of Ince polynomials, forming a doubly infinite set, parametrized by discrete momentum components of the charged particle's de Broglie wave along the polarization vector and along the propagation direction of the plasmon radiation. The envelope of the exact wavefunctions describes a high-contrast periodic structure of the particle density on the plasma length scale, which may have relevance in novel particle acceleration mechanisms.

New class of exact solutions of the Dirac and Klein-Gordon equation in a strong laser field [2013].

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New exact solutions of the Dirac and Klein–Gordon equations of a charged particle propagating in a strong laser field in an underdense plasma

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A R T I C L E I N F O

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Dirac equation
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Laser acceleration of particles

A B S T R A C T

Exact solutions are presented of the Dirac and Klein–Gordon equations of a charged particle propagating in a classical monochromatic electromagnetic plane wave in a medium of index of refraction $n_m < 1$. In the Dirac case the solutions are expressed in terms of new complex polynomials, and in the Klein–Gordon case the found solutions are expressed in terms of Ince polynomials. In each case they form a doubly infinite set, labeled by two integer quantum numbers. These integer numbers represent quantized momentum components of the charged particle along the polarization vector and along the propagation direction of the electromagnetic radiation. Since this radiation may represent a plasmon wave of arbitrary high amplitude, propagating in an underdense plasma, the solutions obtained may have relevance in describing possible quantum features of novel acceleration mechanisms.

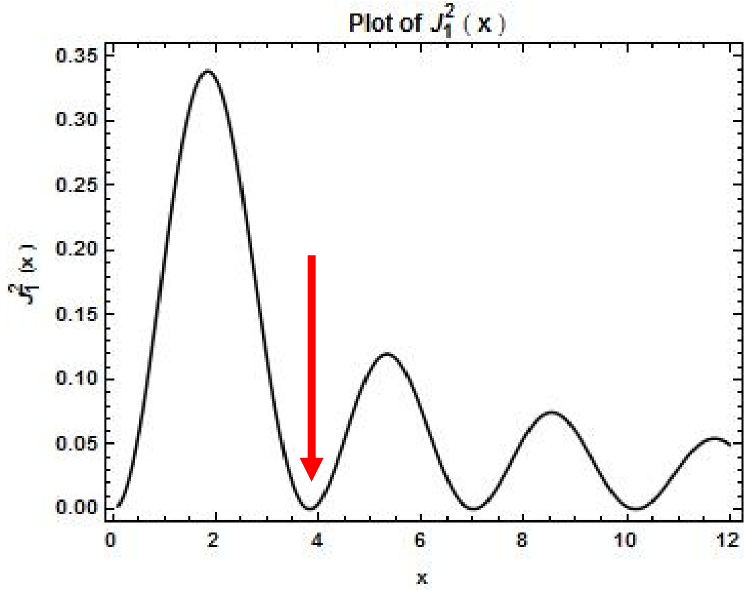
RABBITT as a ‘laser-assisted (two-colour) ionization’.

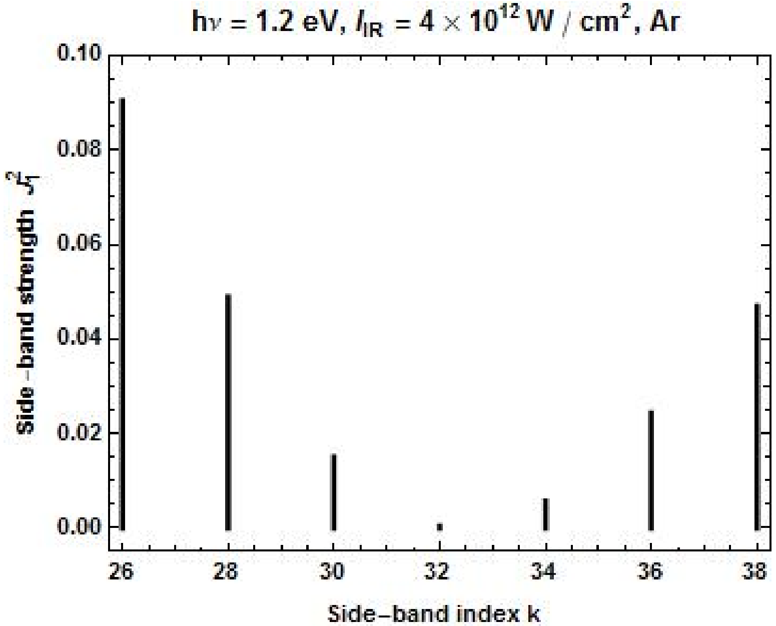
The side-band amplitudes are proportional with the Bessel functions J_{+1}, J_{-1}:

$$T_{2N\pm1}^{(\pm)}(n,l,m,\boldsymbol{k}) \propto J_{\pm1}(\boldsymbol{\alpha}_L \cdot \boldsymbol{k}) T_1(n,l,m,\boldsymbol{k})$$

$$\boldsymbol{\alpha}_L = \boldsymbol{\varepsilon}_L \mu_0 \bar{\lambda}_L$$

$$\tfrac{1}{2}k^2 = (2N \pm 1)\omega - I_p \mp \omega$$





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