Backreaction of mesonic fluctuations on the axial anomaly at finite temperature

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Outline

- $U_A(1)$ symmetry:
  Anomalous breakdown and
  characteristic signal-quantities
  Thermal evolution of signal-quantities
  T-dependence of Kobayashi-Maskawa-'t Hooft coupling

- Thermal behavior of the anomaly in an extended linear meson model *
  Interpretation of non-monotonic behavior of KMT coupling

- Conclusion and Outlook

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$U_A(1)$ anomaly and signatures of its restoration

- The $U_A(1)$ anomaly

$$\partial_\mu j_{5\mu}(x) = 2im_f\rho_{5f}(x) - 2N_f g^2 q(x),$$
$$j_{5\mu} = \sum_f \bar{\psi}_f\gamma_\mu\gamma_5\psi_f, \quad \rho_5 = \bar{\psi}_f\gamma_5\psi_f, \quad q(x) = \frac{1}{32\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}.$$

- Topological susceptibility

$$\chi = \int d^3x\langle q^*(x)q(0)\rangle$$

- Topological mass splitting (chiral limit, $m_{Goldstone} = 0$)

$$\chi = \frac{f_\pi^2 m_{\eta_0}^2}{2N_f} + \mathcal{O}(1/N_c)$$

$U_A(1)$ anomaly and signatures of its restoration

- Instanton-induced $2N_f$-quark interaction

$$V_{KMT} = -K\left[\det\bar{\psi}P_+\psi + \det\bar{\psi}P_-\psi\right]$$

At high temperature:
exponential suppression of KMT-coupling due to suppression of instanton density

$$K_T = K_0 \exp\left(-\lambda T^2\right), \qquad \lambda \sim \frac{8}{3}(\pi R_{size})^2$$

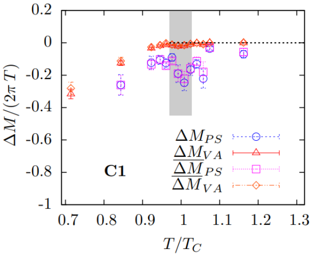
$U_A(1)$ anomaly and signatures of its restoration

- Characteristic patterns of mass degeneracy

$$\begin{tikzcd}
\pi:\ \bar\psi\gamma_5\frac{\tau^i}{2}\psi = P^i \arrow[rr, leftrightarrow, "U_A(1)"] \arrow[d, leftrightarrow, "SU_A(2)"'] & & S^i = \bar\psi\frac{\tau^i}{2}\psi\ :\ a_0 \arrow[d, leftrightarrow, "SU_A(2)"] \\
\sigma:\ \bar\psi\psi = S^0 \arrow[rr, leftrightarrow, "U_A(1)"'] & & P^0 = \bar\psi\gamma_5\psi\ :\ \eta'
\end{tikzcd}$$

$$\Delta M_{VA} = m_\pi - m_\sigma, \qquad \Delta M_{PS} = m_\pi - m_{a_0}$$

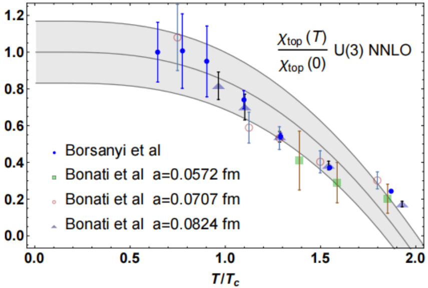
- Brandt *et al* (2018)



- Similar pattern in meson susceptibilities: $\chi_\Phi = \int_x \langle \Phi(x)\Phi(0)\rangle$

$U_A(1)$ anomaly and signatures of its restoration

- Monotonic decrease of χ with increasing temperature



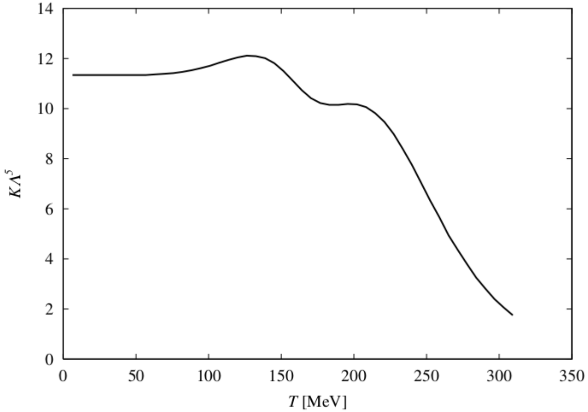
Combined effect of chiral symmetry restoration (f_π) and suppression of anomalous singlet mass ($m^2_{\eta_0}$)

Lattice: Bonati *et al.* (2016), Borsányi *et al.* (2016)

$U(3)$ ChPT: Gómez Nicola *et al.* (2019)

$U_A(1)$ anomaly and signatures of its restoration

- Non-monotonic T-dependence of KMT coupling
  Leading large N_c expression of χ
  in Nambu–Jona-Lasinio model (Fukushima *et al.* (2001))
  Fixing K_T by requiring accurate agreement with lattice data.



Physical interpretation?

't Hooft coupling with condensate backreaction

Effective S+P Meson Model:

$$\Gamma = \int_x \Big(\mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] + V[M] \Big), \qquad M = (s_a + i\pi_a)T^a.$$

$U_L(3) \times U_R(3)$ symmetry $\longrightarrow V = V_{sym}[M]$
depends on group invariants

$$\rho = \mathrm{Tr}\,(M^\dagger M), \quad \tau = \mathrm{Tr}\,\big(M^\dagger M - \mathrm{Tr}\,(M^\dagger M)/3\big)^2,$$

$$\rho_3 = \mathrm{Tr}\,\big(M^\dagger M - \mathrm{Tr}\,(M^\dagger M)/3\big)^3.$$

Anomaly represented by KMT-determinant: $\Delta = \det M^\dagger + \det M$
Ansatz for the full effective potential (with linear explicit breaking fields $H = h_0 T^0 + h_8 T^8$ and backreaction on couplings through ρ):
$V_{sym} = U(\rho) + C(\rho)\tau + D(\rho)\rho_3,$
$V(\rho, \tau, \rho_3, \Delta; H) = V_{sym} + A(\rho)\Delta - \mathrm{Tr}\,\big(H(M + M^\dagger)\big),$

't Hooft coupling with condensate backreaction

Momentum scale (k) dependent couplings due to meson-fluctuations below hadronisation scale $\Lambda \sim 1$GeV

$$\partial_k V_k = \partial_k U_k(\rho) + \partial_k C_k(\rho)\tau + \partial_k A_k(\rho)\Delta + \partial_k D(\rho)\rho_3$$

Set of RGE derived for U_k, C_k, A_k, D_k and integrated $k \in (\Lambda \to 0)$
Initial functional (in principle determined by QCD dynamics)

$$U_\Lambda(\rho) = m^2\rho + g_1\rho^2, \quad C_\Lambda(\rho) = g_2, \quad D_\Lambda(\rho) = 0$$

Anomaly suppression at very high T:

$$A_\Lambda(\rho) = a\left[\Theta(T_0 - T) + e^{-\frac{8}{3}(\pi R_{size})^2(T^2 - T_0^2)}\Theta(T - T_0)\right]$$

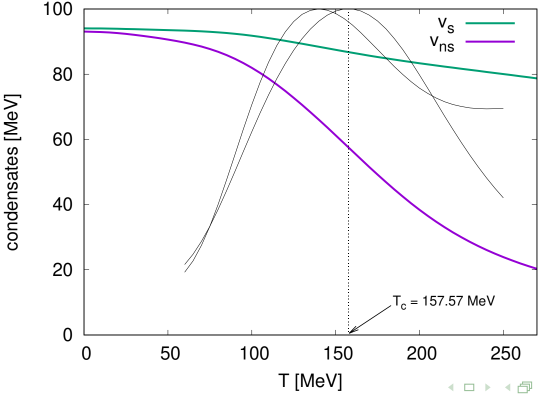
m^2, g_1, g_2, a determined from observed $T = 0$ pseudoscalar spectra $(m_\pi^2, m_K^2, m_\eta^2, m_{\eta'}^2)$; PCAC determines h_0, h_8 through condensates

$$h_0 = \frac{1}{\sqrt{6}} m_\pi^2 v_{n-s} + \sqrt{\frac{2}{3}} m_K^2 v_s, \quad h_8 = \frac{2}{\sqrt{3}} m_\pi^2 v_{n-s} - \frac{2}{\sqrt{3}} m_K^2 v_s,$$

Results: A) T-independent $A_\Lambda = a$

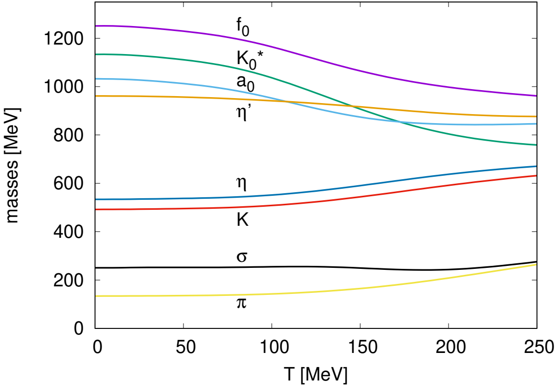
T_c: crossover temperature from inflection of

- $v_{non-strange} = \frac{1}{\sqrt{3}}\left(\sqrt{2}\langle s_0\rangle + \langle s_8\rangle\right) \to 158\mathrm{MeV}$,
- $v_{strange} = \frac{1}{\sqrt{3}}\left(\langle s_0\rangle - \sqrt{2}\langle s_8\rangle\right) \to 148\mathrm{MeV}$,



't Hooft coupling with condensate backreaction

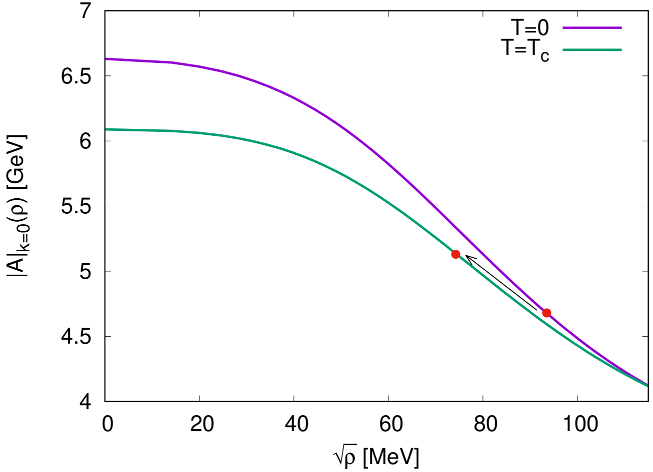
- $|m_\pi - m_\sigma| \to 167\mathrm{MeV}$



- $$|m_\pi - m_{a_0}| = \sqrt{2}A_{k=0}(\rho_{min})v_{strange} + C_{k=0}(\rho_{min})v^2_{non-strange}$$
$$\approx \sqrt{2}A_{k=0}(\rho_{min})v_{strange}, \qquad T \geq 1.5T_c$$

't Hooft coupling with condensate backreaction

$|A_{k=0}(\rho, T)|$ and the backreaction of condensate evaporation

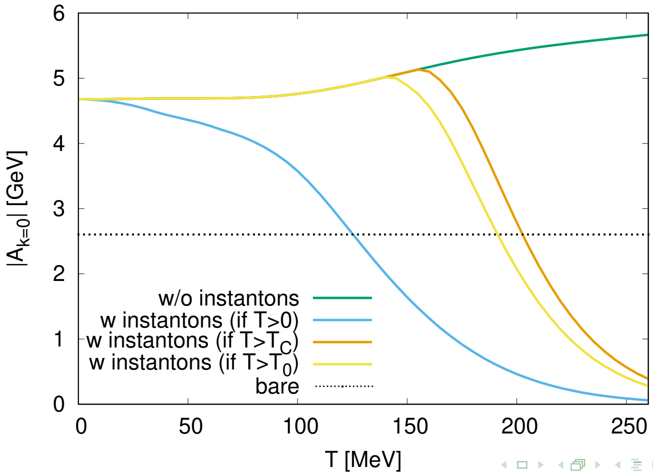


't Hooft coupling with condensate backreaction

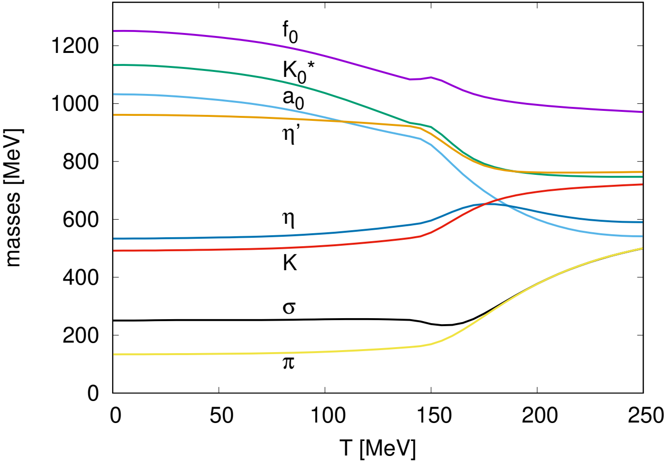
Results: B) T-dependent A_Λ

Instanton motivated T-dependence sets in for $T \geq T_0$

$T_0 = 143\mathrm{MeV}$ (leaves T_c from $v_{non-strange}$ unchanged)



't Hooft coupling with condensate backreaction



$U_A(1)$ related mass differences vary non-monotonically

Conclusion and Outlook

Conclusion

Phenomenological non-monotonic temperature dependence of the KMT coupling first established by Fukushima *et al.* (2001) is reconstructed with FRG method in a linear meson model with extended effective potential to result from

- non-perturbative backreaction of the chiral condensate on the RG-evolution of the field dependent coefficient of the 't Hooft determinant;
- instanton motivated explicit T-dependence introduced into this coefficent at the cut-off scale.

Results confirm results of earlier less complete investigations of Fejős and Hosaka (2016)

Conclusion and Outlook

Outlook

- Linearizing the effective potential only in τ and allowing general dependence on the 't Hooft-determinant

$$V(\rho, \tau, \Delta) = U(\rho, \Delta) + C(\rho)\tau$$

  brings in effects of higher charged topological configurations (Pisarski, Rennecke, 2019);

- Introducing θ-term into the effective action of the linear meson model provides access also to the topological susceptibility;

- Wave function renormalisation effects are expected to have modest influence.

Technology

FRG equations of A_k and U_k

$$\partial_k U_k(\rho) = \frac{\Omega}{2} T \sum\nolimits_n \tilde{\partial}_k (8 \log D_8 + \log D_0),$$

$$\begin{aligned}\partial_k A_k(\rho) = \Omega T \sum\nolimits_n \tilde{\partial}_k \Big[&\tfrac{8}{D_8} \Big(A'_k(\omega_n^2 + k^2 + U'_k) + \tfrac{2}{3}\rho C_k A'_k + A_k C_k \Big) \\ &+ \tfrac{1}{D_0} \Big((4A'_k + \rho A''_k)(\omega_n^2 + k^2 + U'_k) + U''_k(\rho A'_k - 3A_k) \Big) \Big]\end{aligned}$$

with

$$D_8 = (\omega_n^2 + k^2 + U'_k)(\omega_n^2 + k^2 + U'_k + \tfrac{4}{3}\rho C_k) - \tfrac{1}{3}\rho A_k^2,$$

$$D_0 = (\omega_n^2 + k^2 + U'_k)(\omega_n^2 + k^2 + U'_k + 2\rho U''_k) - \tfrac{4}{3}\rho(A_k + \rho A'_k)^2.$$

FRG equation of C_k

$$\partial_k C_k = \Omega T \sum_n \tilde{\partial}_k \Bigg\{ \frac{7}{2D_8} \left(2C_k'(\omega_n^2 + k^2 + U_k') + \frac{4}{3}\rho C_k C_k' + 2C_k^2\right)$$

$$+\frac{2}{D_8}\left(\frac{3}{2}C_k'(\omega_n^2 + k^2 + U_k') + \frac{1}{3}\rho C_k C_k' - \frac{1}{4}A_k A_k'\right)$$

$$-\frac{2}{3D_8^2}\Big(A_k^2 + \frac{4}{3}\rho C_k^2 + 4C_k(\omega_n^2 + k^2 + U_k')\Big)^2$$

$$+\frac{1}{D_0}\Big((3C_k' + \rho C_k'')(\omega_n^2 + k^2 + U_k') + \frac{3}{2}A_k'(A_k + \rho A_k') + \rho C_k' U_k''\Big)$$

$$-\frac{4}{3D_8^2}\Bigg(\frac{1}{16}A_k^4 + \frac{7}{12}\rho A_k^2 C_k^2 + \frac{2}{9}\rho^2 C_k^4(\omega_n^2 + k^2 + U_k')\Big(A_k^2 + \frac{1}{3}\rho C_k^2\Big)C_k$$

$$+\frac{5}{4}(\omega_n^2 + k^2 + U_k')^2 C_k^2\Bigg) -$$

FRG equation of C_k contd.(1)

$$
\begin{aligned}
&-\frac{8}{D_0D_8}\Bigg((\omega_n^2+k^2+U_k')^2\Big(\frac{5}{12}C_k^2+\frac{3}{16}(U_k''+\frac{4}{3}\rho C_k')^2+\frac{1}{2}C_k(U_k''+\frac{4}{3}\rho C_k')\Big)\\
&\qquad+(\omega_n^2+k^2+U_k')\Big(\frac{1}{6}\rho C_k^2(U_k''+\frac{2}{3}C_k')+\frac{1}{16}(U_k''+\frac{4}{3}\rho C_k')(A_k^2\\
&\qquad\qquad-4\rho A_kA_k'-4\rho^2A_k'^2)+\frac{C_k}{24}(3A_k^2-4\rho^2A_k'^2)\Big)\\
&\qquad+\frac{2}{9}\rho^2U_k''C_k^3-\frac{1}{9}\rho C_k^2(A_k^2-\rho A_kA_k'-2\rho^2A_k'^2)-\frac{1}{4}\rho C_kA_k^2U_k''\\
&\qquad-\frac{2}{9}\rho^2C_kC_k'A_k(A_k+\rho A_k')-\frac{A_k}{48}(A_k+\rho A_k')(A_k^2-4\rho^2A_k'^2)\Bigg)\\
&+\frac{A_k^2}{6D_0D_8}\Big(4C_k(\omega_n^2+k^2+U_k')-A_k^2\Big)\\
&+\frac{(\omega_n^2+k^2+U_k')^2A_k^2}{4D_0D_8^2}\Big(A_k^2+\frac{8}{3}\rho A_kA_k'+\frac{4}{3}\rho^2A_k'^2-\frac{8}{3}\rho C_k(U_k''-\frac{2}{3}C_k)\Big)
\end{aligned}
$$

FRG equation of C_k contd.(2)

$$
\begin{aligned}
&+\tfrac{(\omega_n^2+k^2+U_k')}{4D_0D_8}\Bigg(6(\omega_n^2+k^2+U_k')(U_k''-\tfrac{2}{3}C_k)^2 \\
&\qquad -\tfrac{2A_k^2}{D_8}(\omega_n^2+k^2+U_k')^2(U_k''-\tfrac{2}{3}C_k) \\
&\qquad +(A_k^2+\tfrac{8}{3}\rho A_kA_k'+\tfrac{4}{3}\rho^2A_k'^2)\Big(\tfrac{4\rho C_kA_k^2}{3D_8}-3(U_k''-\tfrac{2}{3}C_k)\Big)\Bigg) \\
&+\tfrac{1}{D_0}\Bigg(A_kA_k'+\rho A_k'^2\left(\tfrac{1}{2}-\frac{(\omega_n^2+k^2+U_k')^2}{D_8}\right) \\
&\qquad -\tfrac{\omega_n^2+k^2+U_k'}{4D_8}\Big(A_k^2(2C_k+U_k'')-4\rho A_kA_k'(U_k''-\tfrac{2}{3}C_k) \\
&\qquad\qquad +4\rho^2A_k'^2(U_k''+\tfrac{2}{3}C_k)\Big)\Bigg)\Bigg\},
\end{aligned}
$$