QCD vacuum as a time crystal: mirror symmetry of the ground state and vanishing cosmological constant

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Lund U.

Vacuum in Quantum Physics vs in Cosmology

Vacuum energy

"...the worst theoretical prediction in the history of physics" (Hobson 2006)

in Quantum Physics		in Cosmology
$\epsilon_{vac} \sim 10^{-2}\,\mathrm{GeV}^4$	Topological QCD vacuum unique strongly-coupled subsystem!	$\Lambda_{\rm cosm} \sim 10^{-47}\,\mathrm{GeV}^4$
$\sim 10^{8}\,\mathrm{GeV}^4$	Higgs condensate	"Old" CC problem: Why such small and positive? "New" CC problem: Why non-zeroth and exists at all?

Vacuum in Quantum Physics has incredibly wrong energy scale!

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle + \langle 0| : m_s \bar{s}s : |0\rangle\right)$$

$$\simeq -(5 \pm 1) \times 10^9\ \mathrm{MeV}^4.$$

Two possible approaches to this problem:

- **Let's forget about the "bare" vacuum (DE: "phantom", "quintessence", "ghost"… etc)**
  Zero vacuum density in the Minkowski limit, by (Casimir-like) definition, then (Zhitnitsky et al)

  $\Lambda_{\rm cosm} \equiv \epsilon_{\rm FLRW} - \epsilon_{\rm Mink}$ **simply imposing a cancellation of the "bare" vacuum by hands!!**

- **Let's look closer at the vacuum state — why/how does it become "invisible" to gravity?**

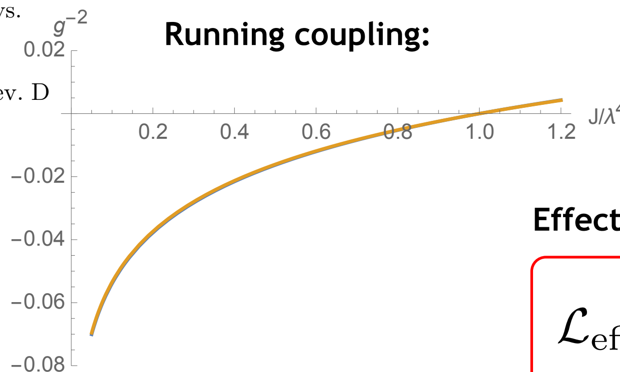
Effective YM action and Savvidy vacuum

**At least, for SU(2) gauge symmetry,
the all-loop and one-loop effective Lagrangians
are practically indistinguishable (by FRG approach)**

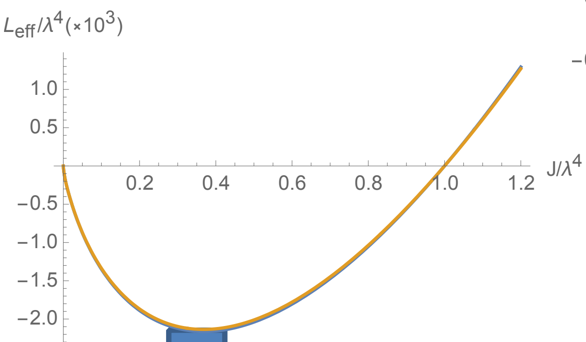
P. Dona, A. Marciano, Y. Zhang and C. Antolini, Phys. Rev. D **93** (2016) no.4, 043012.

A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D **83** (2011) 045014 [Phys. Rev. D **83** (2011) 069903].

Running coupling:



Effective Lagrangian:



gluon condensate (Savvidy vacuum)

Discovery of chromomagnetic condensate:

G. K. Savvidy, Phys. Lett. **71B**, 133 (1977)

G. Savvidy, Eur. Phys. J. C **80** (2020) 165

H. Pagels and E. Tomboulis, Nucl. Phys. B **143**, 485 (1978).

Classical YM Lagrangian:

$$\mathcal{L}_{\rm cl} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\rm YM}\, f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{A}^a_\mu \equiv g_{\rm YM} A^a_\mu$$

Effective YM Lagrangian:

$$\mathcal{F}^a_{\mu\nu} \equiv g_{\rm YM} F^a_{\mu\nu}$$

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2(\mathcal{J})}, \qquad \mathcal{J} = -\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a$$

The energy-momentum tensor:

$$T^\nu_\mu = \frac{1}{\bar{g}^2}\left[\frac{\beta(\bar{g}^2)}{2} - 1\right]\left(\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{J}\right) - \delta^\nu_\mu \frac{\beta(\bar{g}^2)}{8\bar{g}^2}\mathcal{J}$$

Equations of motion:

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2}\left(1 - \frac{\beta(\bar{g}^2)}{2}\right)\right] = 0,$$

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \equiv \left(\delta^{ab}\overrightarrow{\partial}_\nu - f^{abc}\mathcal{A}^c_\nu\right),$$

trace anomaly:

$$T^\mu_\mu = -\frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{J}$$

NOTE: the RG equation

$$\frac{d\ln|\bar{g}^2|}{d\ln|\mathcal{J}|/\mu_0^4} = \frac{\beta(\bar{g}^2)}{2}$$

appears to be invariant under

$$\mathcal{J} \longleftrightarrow -\mathcal{J}$$

$$\bar{g}^2 = \bar{g}^2(|\mathcal{J}|)$$

Real-time evolution of the gluon condensate

$$\mathcal{J} = \frac{2}{\sqrt{-g}} \sum_a (\mathbf{E}_a \cdot \mathbf{E}_a - \mathbf{B}_a \cdot \mathbf{B}_a) \equiv \frac{2}{\sqrt{-g}} (\mathbf{E}^2 - \mathbf{B}^2)$$

FLRW metric in conformal time:

$$g \equiv \det(g_{\mu\nu}),\; g_{\mu\nu} = a(\eta)^2 \mathrm{diag}(1,\, -1,\, -1,\, -1)$$

$$\sqrt{-g} = a^4(\eta)\,, \qquad t = \int a(\eta) d\eta$$

- **Basic qualitative features on the non-perturbative YM action are noticed already at one loop**

Einstein-YM equations of motion for the effective YM theory:

$$\frac{1}{\varkappa}\left(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R\right) = \bar{\epsilon}\delta_\mu^\nu + \frac{b}{32\pi^2}\frac{1}{\sqrt{-g}}\left[\left(-\mathcal{F}^a_{\mu\lambda}\mathcal{F}_a^{\nu\lambda} + \frac{1}{4}\delta_\mu^\nu \mathcal{F}^a_{\sigma\lambda}\mathcal{F}_a^{\sigma\lambda}\right)\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}_a^{\alpha\beta}|}{\sqrt{-g}\,\lambda^4} - \frac{1}{4}\delta_\mu^\nu\, \mathcal{F}^a_{\sigma\lambda}\mathcal{F}_a^{\sigma\lambda}\right],$$

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\overrightarrow{\partial}_\nu\sqrt{-g} - f^{abc}\mathcal{A}_\nu^c\right)\left(\frac{\mathcal{F}_b^{\mu\nu}}{\sqrt{-g}}\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}_a^{\alpha\beta}|}{\sqrt{-g}\,\lambda^4}\right) = 0$$

temporal (Hamilton) gauge $\quad A_0^a = 0 \qquad e_i^a A_k^a \equiv A_{ik} \qquad e_i^a e_k^a = \delta_{ik} \qquad e_i^a e_i^b = \delta_{ab}$

due to local SU(2) ~ SO(3) isomorphism $\quad A_{ik}(t,\vec{x}) = \delta_{ik}U(t) + \widetilde{A}_{ik}(t,\vec{x})$

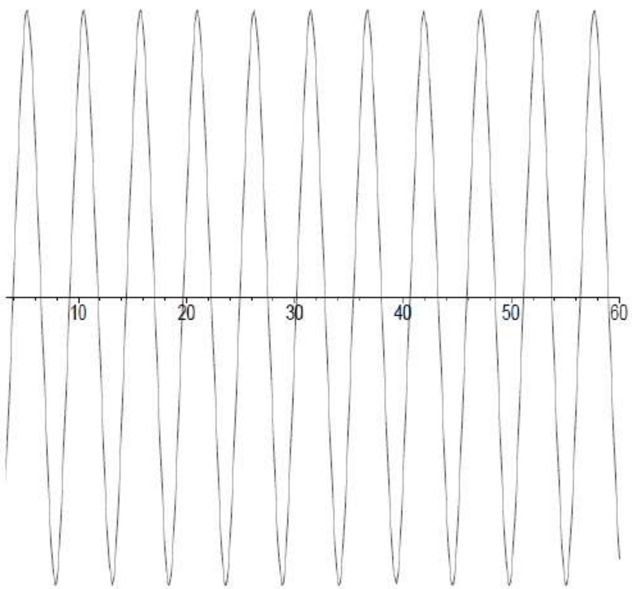
The resulting equations:

$$\frac{6}{\varkappa}\frac{a''}{a^3} = 4\bar{\epsilon} + T_\mu^{\mu,\mathrm{U}}\,, \qquad T_\mu^{\mu,\mathrm{U}} = \frac{3b}{16\pi^2 a^4}\left[(U')^2 - \frac{1}{4}U^4\right],$$

$$\frac{\partial}{\partial\eta}\left(U'\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4\lambda^4}\right) + \frac{1}{2}U^3\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4\lambda^4} = 0$$

Gluon condensate on non-stationary (FLRW) background

Classical YM condensate



"Radiation" medium

$$\epsilon_{\rm YM} \propto 1/a^4$$

Unstable solution!

$$Q \equiv \frac{32}{11}\pi^2 e(\xi\Lambda_{\rm QCD})^{-4} T^\mu_\mu[U]$$
$$= 6e\left[(U')^2 - \frac{1}{4}U^4\right] a^{-4}(\xi\Lambda_{\rm QCD})^{-4}$$

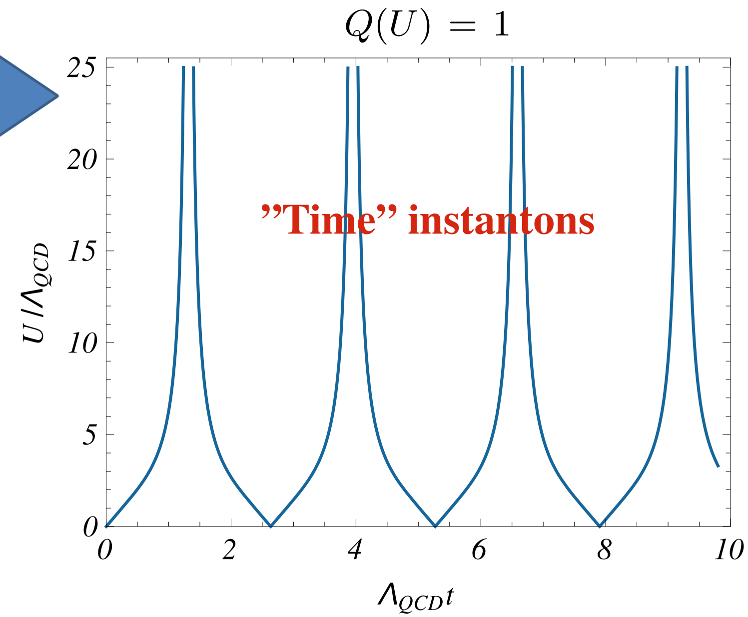
Exact partial solution:

$$|Q| = 1$$

Quantum corrections

QCD vacuum: a ferromagnetic undergoing spontaneous magnetisation (Pagels&Tomboulis)

Quantum YM vacuum



Asymptotic (attractor) solution

$$\epsilon_{\rm CE} \to +{\rm const} \quad t \to \infty$$

Stable solution!

- **In fact, both chromoelectric and chromomagnetic condensates are stable on non-stationary (FLRW) background of expanding Universe**

"Mirror" symmetry of the ground state

In a vicinity of the ground state, the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2} \qquad \mathcal{J} \simeq \mathcal{J}^*$$

is invariant under

$$\mathbb{Z}_2: \qquad \mathcal{J}^* \longleftrightarrow -\mathcal{J}^*, \quad \bar{g}^2(\mathcal{J}^*) \longleftrightarrow -\bar{g}^2(\mathcal{J}^*), \quad \beta(\bar{g}_*^2) \longleftrightarrow -\beta(\bar{g}_*^2)$$

For pure gluodynamics at one-loop: $\beta_{(1)} = -\frac{bN}{48\pi^2}\,\bar{g}^2_{(1)} \qquad b = 11$

$$\alpha_s = \frac{\bar{g}^2}{4\pi} \qquad \alpha_{\rm s}(\mu^2) = \frac{\alpha_{\rm s}(\mu_0^2)}{1 + \beta_0\,\alpha_{\rm s}(\mu_0^2)\ln(\mu^2/\mu_0^2)} \qquad \mu^2 \equiv \sqrt{|\mathcal{J}|}$$

Choosing the ground state value of the condensate $\mu_0^2 \equiv \sqrt{|\mathcal{J}^*|}$ **as the physical scale**

we observe that the mirror symmetry, indeed, holds provided

$$\mathcal{J} \simeq \mathcal{J}^* \qquad \alpha_s(\mu_0^2) \longleftrightarrow -\alpha_s(\mu_0^2)$$

i.e. in the ground state only!

Heterogenous quantum ground state: two-scale vacuum

The running coupling at one-loop $\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1 + \frac{bN}{96\pi^2}\bar{g}_1^2(\mu_0^4)\ln(|\mathcal{J}|/\mu_0^4)} = \frac{96\pi^2}{bN\ln(|\mathcal{J}|/\lambda_\pm^4)}$

$$\mathcal{L}_{\rm eff}^{(1)} = \frac{bN}{384\pi^2}\mathcal{J}\ln\left(\frac{|\mathcal{J}|}{\lambda_\pm^4}\right)$$

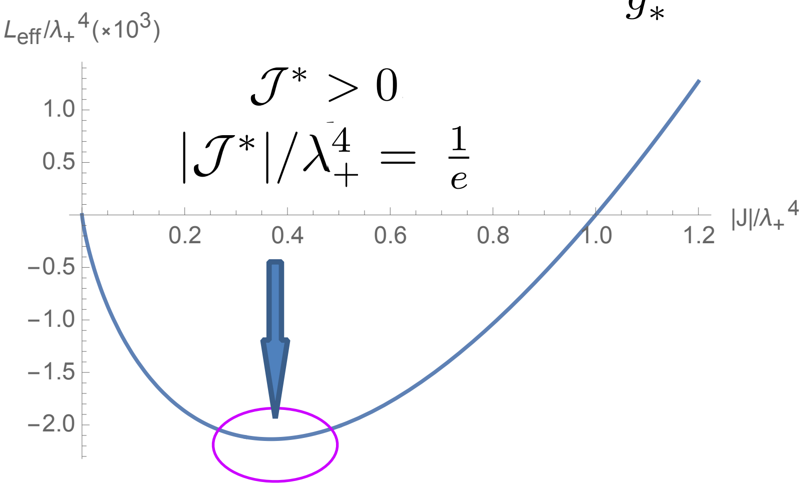
with two energy scales

$$\lambda_\pm^4 \equiv |\mathcal{J}^*|\exp\left[\mp\frac{96\pi^2}{bN|\bar{g}_1^2(\mathcal{J}^*)|}\right] \qquad |\mathcal{J}^*| = \lambda_+^2\lambda_-^2$$

CE vacuum: $\beta(\bar{g}_*^2) = 2$

e.o.m. is automatically satisfied!

Trace anomaly: $T^\mu_{\mu,\rm CE} = -\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



Cosmological CE attractor

Mirror symmetry

One-loop:

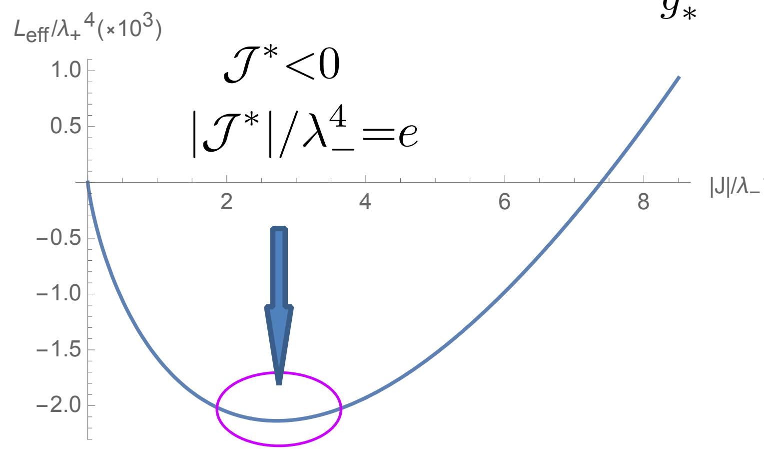
$$\lambda_+^2/\lambda_-^2 = e$$

CM vacuum: $\beta(\bar{g}_*^2) = -2$

Reduces to the standard YM e.o.m. discussed in e.g. in instanton theory

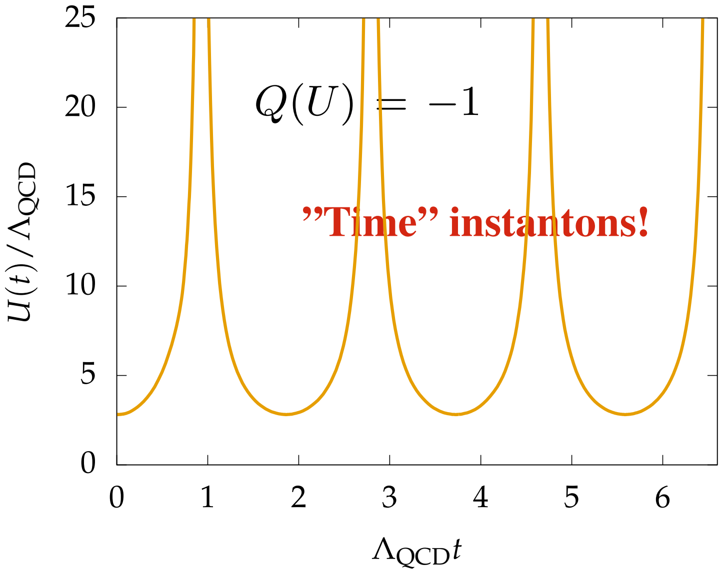
$$\overrightarrow{\mathcal{D}}_\nu^{ab}\left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2}\right] = 0, \quad \bar{g}^2 \simeq \bar{g}_*^2$$

Trace anomaly: $T^\mu_{\mu,\rm CM} = +\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



Cosmological CM attractor

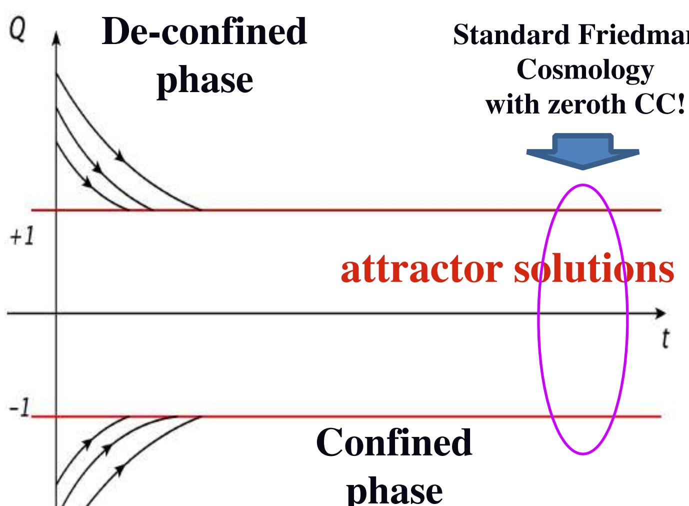
Infrared restoration of conformal invariance



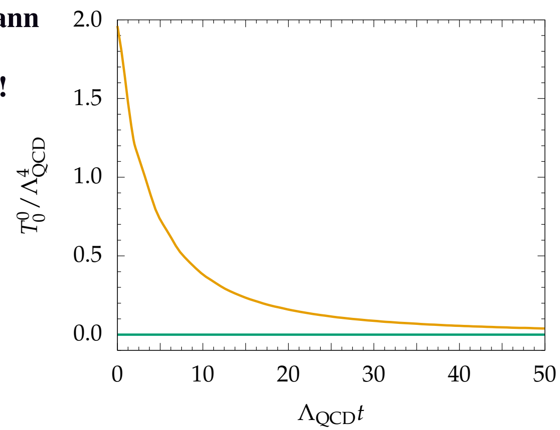
$$\epsilon_{\rm vac} \equiv \frac{1}{4}\langle T^{\mu}_{\mu}\rangle_{\rm vac} = \mp\mathcal{L}_{\rm eff}(\mathcal{J}^*)$$

$$\epsilon^{\rm CE}_{\rm vac}\Big|_{\mathcal{J}^*>0} + \epsilon^{\rm CM}_{\rm vac}\Big|_{\mathcal{J}^*<0} \equiv 0$$

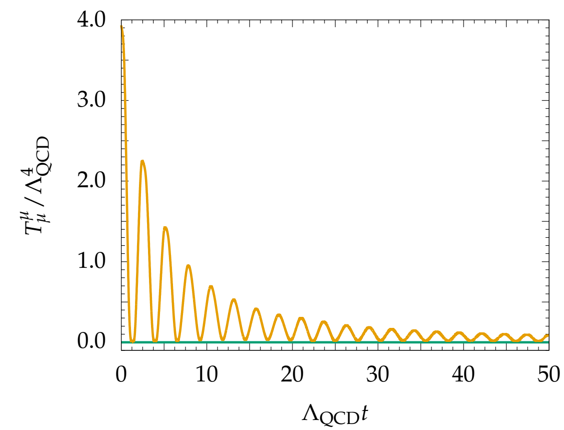
Exact compensation of CM and CE vacua as soon as the cosmological attractor is achieved!



CE energy density



CE EMT trace

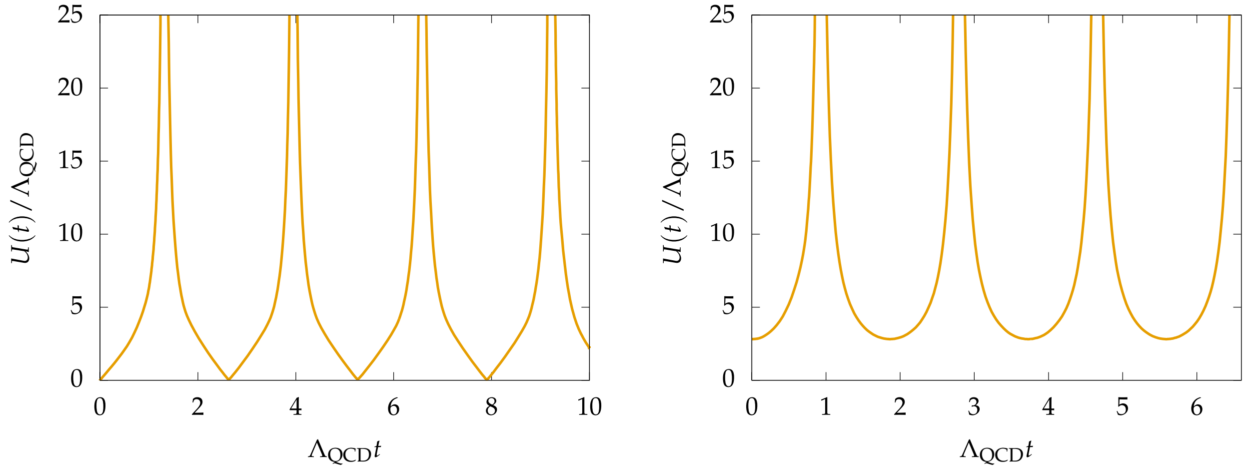


System with very unusual dynamical properties!

Addazi, A.; Marcianò, A.; Pasechnik, R.; Prokhorov, G. Mirror Symmetry of quantum Yang-Mills vacua and cosmological implications. *Eur. Phys. J. C* **2019**, *79*, 251, [arXiv:hep-th/1804.09826].

QCD "time crystal"

- **The emergence of spikes localised in time at a characteristic QCD time lapse** $\Delta t \simeq \Lambda_{\rm QCD}^{-1}$ **and extended in 3-space dimensions reveals the presence of an order state of space-like soliton/domain wall solutions (chronons)**



- **A time-ordered classical solution spontaneously breaking time translational invariance down to a discrete time shift symmetry** $T_n : t \to t + n\Lambda_{\rm QCD}^{-1}$ **is known as the "time crystal" first discovered by Wilczek in the context of superconductors and superfluids in** F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012)

- **The kink (anti-kink) profile localised in time corresponds to a space-like domain wall**

$$U(\eta) \simeq \frac{v}{\sqrt{2}} \tanh^{-1}\left[\frac{v}{\sqrt{2}}(\eta - \eta_0)\right] \qquad v \simeq \Lambda_{\rm QCD}$$

- **As the T-invariance is broken, a massless moduli field** $\eta_0(x, y, z)$ **localised on the domain wall world sheet** x, y, z **arises and corresponds to a Nambu-Goldstone boson**

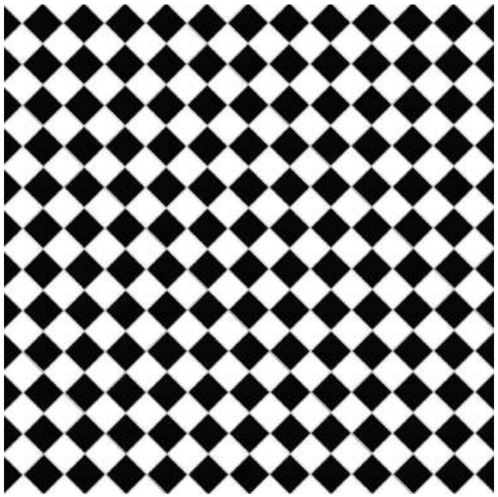
Breaking of Mirror symmetry and Cosmological Constant

Exact mirror symmetry of the YM ground state

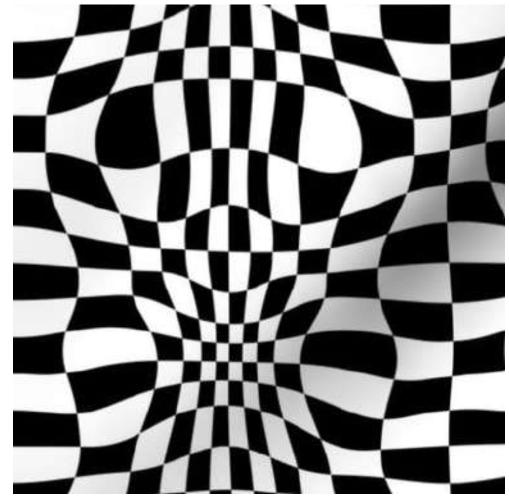
Exact conformal invariance at macroscopic scales

Quantum Gravity in the quasi classical approximation

Mirror symmetry and conformal invariance breakdown at cosmological scales



Gravity



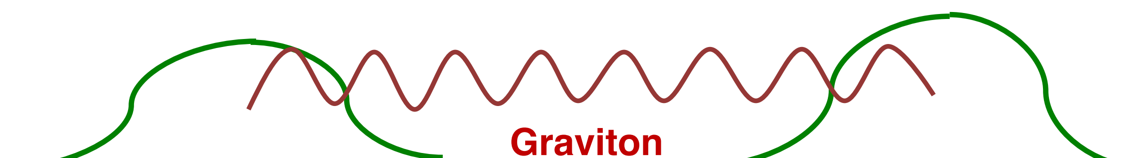
Pasechnik, R.; Beylin, V.; Vereshkov, G. Dark Energy from graviton-mediated interactions in the QCD vacuum. *JCAP* **2013**, *06*, 011, [arXiv:gr-qc/1302.6456].

Ya. Zeldovich (1967):

$$\Lambda \sim G m^6$$

A. Sakharov (1967):

extra terms describing an effect of graviton exchanges between *identical particles (bosons occupying the same quantum state) should appear in the right hand side of Einstein equations (averaged over quantum ensemble)*



$$\varepsilon_\Lambda \sim G\Lambda^6_{\mathrm{QCD}}$$

Concluding remarks

- Local loss of continuous time-translational invariance leads to "time crystal"-type configurations in the QCD vacuum

- Nielsen-Olsen proof of instability of CE condensate on a rigid Minkowski in NOT in contradiction with our picture: we consider YM evolution on a dynamical (FLRW) spacetime while equilibrium is achieved only asymptotically.

- A possible decay of CE condensate into an anisotropic vacuum after a cosmological relaxation time would be exponentially suppressed and is practically never realised

- Even starting from an initial non-zero energy-density, the evolution of localised 3-space "pockets" of the CE and CM condensates trigger a mutual screening, flowing towards a zero-energy density attractor and accompanying by a formation of the domain walls corresponding to an asymptotic restoration of the Z_2 (Mirror) symmetry and effectively protecting the "false" CE vacua pockets from further decay

- The vacua cancellation mechanism seems to naturally marry the existing confinement pictures related to a formation of a network of t'Hooft monopoles or chromovortices. In this approach, the scalar kink profile may correspond the J-invariant whose change may be related to the presence of monopole or vortex solutions localise inside the space-time domain walls. This implies the existence of space-time solitonic objects of a new type.