Estimating Compressibility by Maximal-mass Compact Star Observations

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Extended Walecka-model ($\sigma - \omega$ model)

- **Scalar meson** $U_i(\sigma) = \gamma_3\sigma^3 + \gamma_4\sigma^4$
- Electron kinetic and mass
- Kinetic terms: $\Psi = (\Psi_p, \Psi_n)$

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m_N + g_\sigma\sigma - g_\omega\omega\!\!\!/ + g_\rho\rho\!\!\!/^2\tau_a)\Psi + \bar{\Psi}_e(i\partial\!\!\!/ - m_e)\Psi_e + \frac{1}{2}\sigma(\partial^2 + m_\sigma^2)\sigma - U_i(\sigma)$$
$$- \frac{1}{4}\omega^{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\rho^{a\mu\nu}\rho^a_{\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^a_\mu$$

- Vector mesons

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad \rho^a_{\mu\nu} = \partial_\mu\rho^a_\nu - \partial_\nu\rho^a_\mu + g_\rho\epsilon^{abc}\rho^b_\nu\rho^c_\nu \quad (1)$$

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Mean-Field Approximation

- Mean-field approximation at zero temperature and finite chemical potential
- Tree-level: $\omega = \omega_0$ and $\rho_0^3 = \rho$
- β-equilibrium: $\mu_n = \mu_p + \mu_e$
- Baryon number and electric charge conservation
- **Free-energy:**

$$
\begin{aligned}
f_T = {} & f_F(m_N - g_\sigma\sigma, \mu_p - g_\omega\omega + g_\rho\rho) + f_F(m_N - g_\sigma\sigma, \mu_n - g_\omega\omega - g_\rho\rho) \\
& + f_f(m_e, \mu_e) + \frac{1}{2}m_\sigma^2\sigma^2 + U_i(\sigma) - \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{2}m_\rho^2\rho^2
\end{aligned} \tag{2}
$$

- **Fermionic pressure:**

$$
f_F(T, m, \mu) = -2T \int \frac{d^3k}{(2\pi)^3} \ln\left[1 + e^{-\beta(E_k - \bar{\mu})}\right] \tag{3}
$$

Data Fit

The free parameters of the model are determined by using nuclear saturation data [Gle97].

The values used to fit the model

1. **Binding energy:** $B = -16.3$ **MeV**
2. Saturation density: $n_0 = 0.153 fm^{-3}$
3. Nucleon Landau mass $m_L = 0.8854 m_N$
4. Compressibility: $K = 240$ MeV
5. Asymmetry energy: $a_{sym} = 32.5$ MeV

$$K = 9n^2 \frac{\partial^2}{\partial n^2}\left(\frac{\epsilon}{n}\right) \qquad m_L = \sqrt{k_f^2 + m_{N,eff}^2} \qquad a_{sym} = \frac{1}{2}\frac{\partial^2}{\partial t^2}\left(\frac{\epsilon}{n}\right)\Big|_t \tag{4}$$

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Mass-Radius Diagrams

- Static, spherically symmetric space-time
- Tolman-Oppenheimer-Volkoff equations (TOV)

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{G\epsilon(r)m(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]^{-1}$$
$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2\epsilon(r) \qquad (5)$$

- We used core-crust model for the calculation
- Low density region: well known BPS nuclear EoS
- The calculated R in this case corresponds to the radius of the neutron star core

Maximal Mass Stars

- Determine the mass and radius of the maximal mass stars (MMS)
- Most relevant parameter: the Landau mass m_L. ($\delta m_L \overset{10x}{>} \delta K$)
- Linear dependence M_{maxM}, R_{MaxM} on the m_L
- Landau mass was optimized with fixed K, a_{sym}.

$$M_{maxM}[\mathrm{M}_\odot] = 5.418 - 0.0043\, m_L[\mathrm{MeV}]\ ,$$
$$R_{maxM}[\mathrm{km}] = 19.04 - 0.0104\, m_L[\mathrm{MeV}]\ .$$

After fixing the m_L by the MMS observations we can also obtain linear, one-parameter dependence on the parameter K.

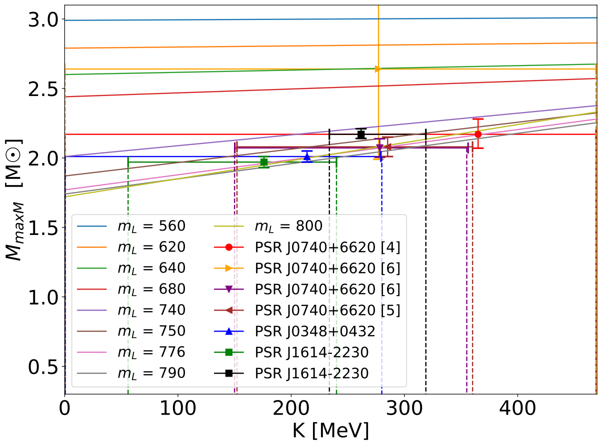
$$M_{maxM}[\mathrm{M}_\odot] = 1.940 + 0.000880\, K\, [\mathrm{MeV}]\ ,$$
$$R_{maxM}[\mathrm{km}] = 9.248 + 0.00718\, K\, [\mathrm{MeV}]\ .$$

Observation Data

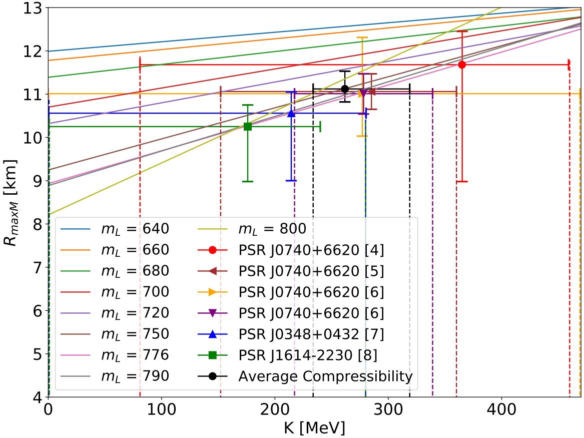
Ref.	Pulsar	M_{maxM}[$M_\odot$]	m_L[MeV]	K[MeV]	R_{maxM}[km]
[**J0740**]	PSR J0740+6620	$2.17^{+0.11}_{-0.10}$ *	$748.39^{+63.3}_{-57.2}$	$351.8^{+115}_{-84.5}$	$11.25^{+1.06}_{-1.04}$
[**J0348**]	PSR J0348+0432	$2.01^{+0.04}_{-0.04}$ *	$785.25^{+20.0}_{-20.3}$	$206.4^{+42.7}_{-20.5}$	$10.87^{+0.82}_{-0.80}$
[**J1614**]	PSR J1614-2230	$1.97^{+0.04}_{-0.04}$ *	$794.47^{+20.1}_{-20.4}$	$170.0^{+15.5}_{-20.9}$	$10.77^{+0.82}_{-0.80}$
[**NICER**]	PSR J0740+6620	$2.07^{+0.07}_{-0.07}$ *	$778.14^{+15.3}_{-15.5}$	$278.2^{+60.9}_{-60.8}$	$11.01^{+0.46}_{-0.47}$
[**NICER**]	PSR J0740+6620	$2.64^{+1.98}_{-0.65}$	639.42^{+159}_{-125}	277.3^{+257}_{-178}	$12.39^{+1.30}_{-0.98}$ *
[**XMNS**]	PSR J0740+6620	$2.08^{+0.07}_{-0.01}$ *	$769.12^{+16.9}_{-16.9}$	$285.1^{+54.8}_{-54.8}$	$11.06^{+0.41}_{-0.41}$

Table 1: The Landau mass, m_L and compressiblity, K values calculated via eq. (**??**)- from measured pulsar mass data, assuming that these are maximal-mass neutron stars. Maximal radius of the maximal-mass neutron star is also calculated form eq. (**??**). [Ant+13; al20; Dem10]

K dependence of the mass of MMS



K dependence of the radius of MMS



Landau mass values:

Remark I.

The range of 550 MeV$< m_L <$ 800 MeV is compatible with effective nucleon mass given by the various equation of state at saturation [Klä+06].

Remark II.

- Extended Walecka Model: $m_L = 776.0^{+38.5}_{-84.9}$ MeV
- Bayesian Analysis I. [Alv+20a]: (PSR J0030+0451 + PSR 0740+6620 + GW170817)
  - $m_L = 750^{+15}_{-15}$ MeV
- Bayesian Analysis II. [Alv+20b]: (PSR J0740+6620 + PSR J0348+0432 + GW170817 + PSR J0030+0451)
  - $m_L = 750^{+15}_{-15}$ MeV

Summary

Input:

- Extended Walecka Model
- Mean-Field Approximation

Method:

- Tollman-Oppenheimer-Volkoff equation
- Using the linear relations between the microscopic and macroscopic parameters
- Observation data

Results:

- Landau mass: $m_L = 776.6^{+40.1}_{-84.2}$ MeV
- Compressibility: $K = 287.3^{59.1}_{-29.2}$ MeV

References

Thank you for your time. Questions?

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