SU(N) symmetry broken states with long range order in 2D

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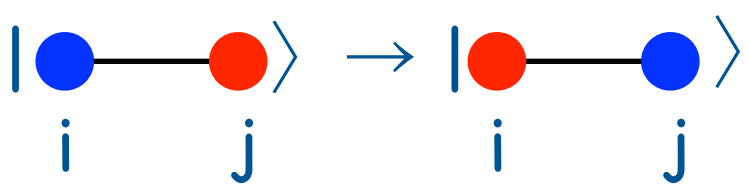
What are the SU(N) symmetric Heisenberg models that we are interested in?

$$\mathcal{H} = \sum_{i,j} \mathcal{P}_{i,j}$$

$\mathcal{P}_{i,j}$ is the transposition operator

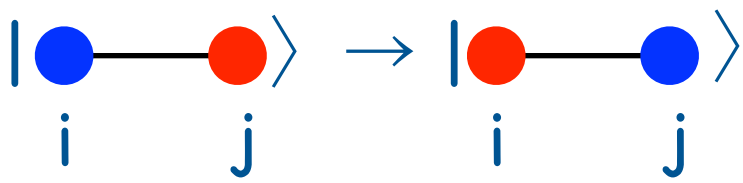
$\mathcal{P}_{i,j}$ 

N species on each site that are treated equally.

$$\mathcal{P}_{ij}|\beta_i \alpha_j\rangle = |\alpha_i \beta_j\rangle$$

simplest example:
SU(2) S=1/2 (fundamental representation)
[but not the S=1 !]

What are the SU(N) symmetric Heisenberg models that we are interested in?

$\mathcal{P}_{i,j}$ 

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = S_1^z S_2^z + \frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right)$$

$$\mathbf{S}_1 \cdot \mathbf{S}_2 |\uparrow\uparrow\rangle = \frac{1}{4}|\uparrow\uparrow\rangle \qquad \rightarrow \qquad \left(2\mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{1}{2}\right)|\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle$$

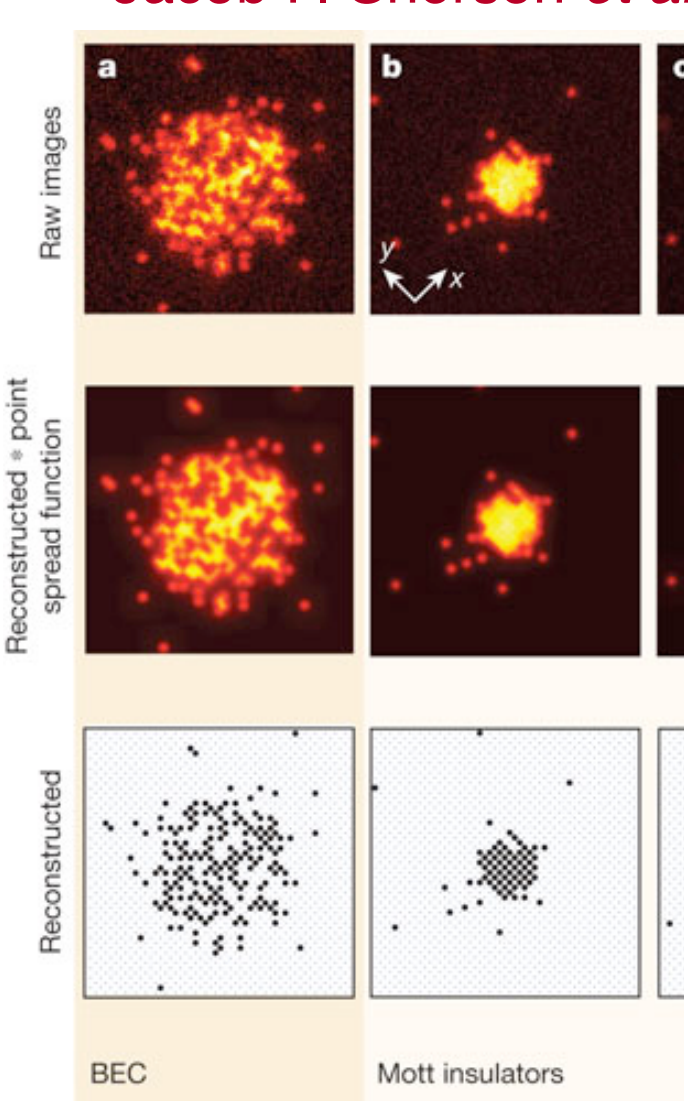
$$\mathbf{S}_1 \cdot \mathbf{S}_2 |\uparrow\downarrow\rangle = -\frac{1}{4}|\uparrow\downarrow\rangle + \frac{1}{2}|\downarrow\uparrow\rangle \qquad \rightarrow \qquad \left(2\mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{1}{2}\right)|\uparrow\downarrow\rangle = |\downarrow\uparrow\rangle$$

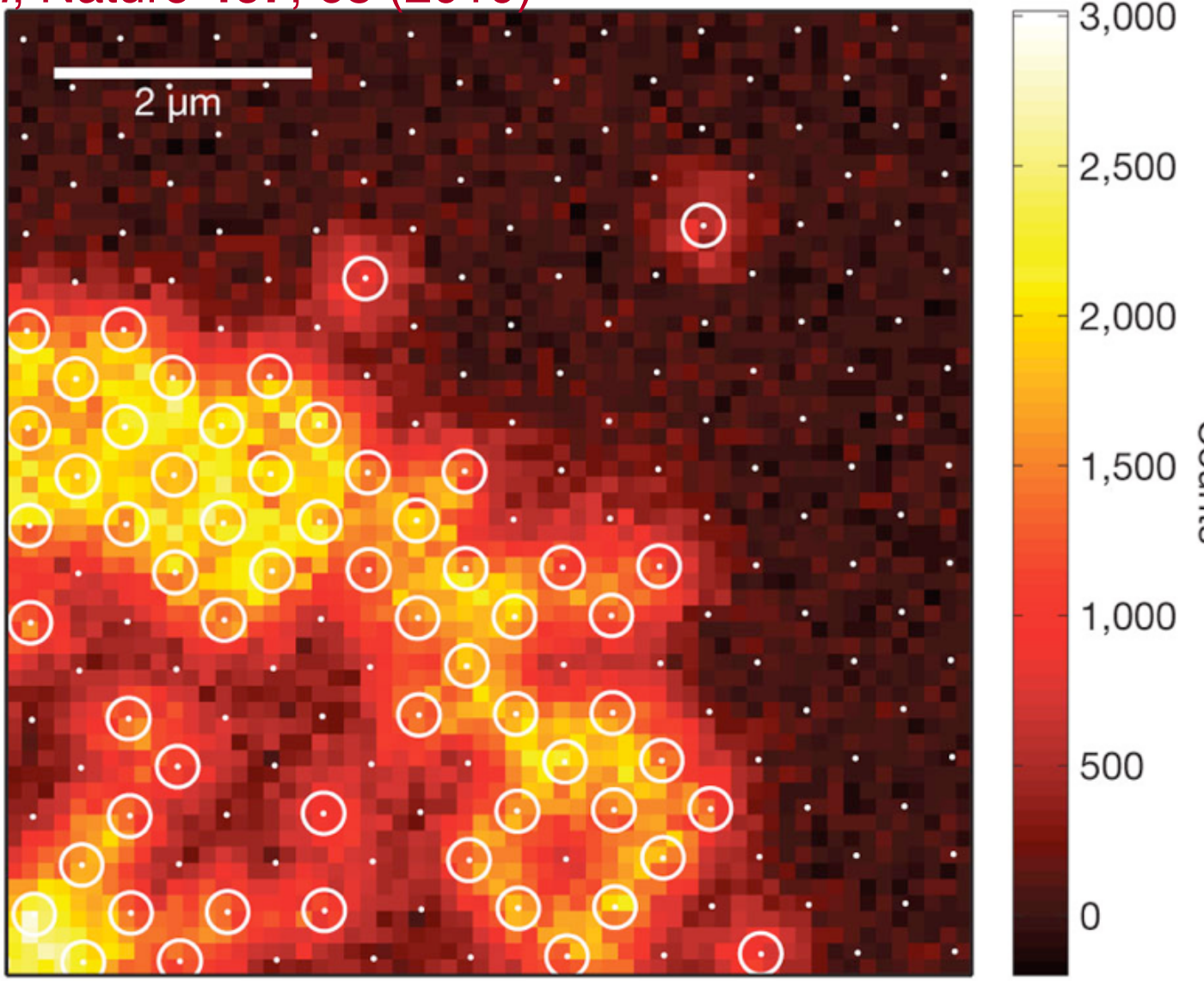
For the $S = 1/2$ fundamental representation of the SU(2):

$$\left(2\mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{1}{2}\right) = \mathcal{P}_{1,2}$$

Single-atom-resolved fluorescence imaging of an atomic Mott insulator

Jacob F. Sherson *et al.*, Nature **467**, 68 (2010)



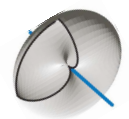


also

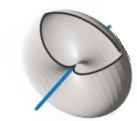
Probing the Superfluid–to–Mott Insulator Transition at the Single-Atom Level

W. S. Bakr *et al.*, Science **329**, 547 (2010)

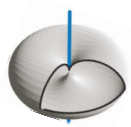
SU(3): Competing spin and quadrupolar ordering in spin models ($NiGa_2S_4$?)



$$|x\rangle = \frac{i}{\sqrt{2}}\left(|1\rangle - |\bar{1}\rangle\right)$$



$$|y\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |\bar{1}\rangle\right)$$



$$|z\rangle = -i|0\rangle$$

$$\hat{\mathbf{Q}}_i\hat{\mathbf{Q}}_j = 2\left(\hat{\mathbf{S}}_i\hat{\mathbf{S}}_j\right)^2 + \hat{\mathbf{S}}_i\hat{\mathbf{S}}_j - 8/3$$

bilinear-biquadratic Heisenberg model for S=1 spins:

$$\mathcal{H} = J\sum_{i,j}\left[\cos\vartheta\mathbf{S}_i\mathbf{S}_j + \sin\vartheta\left(\mathbf{S}_i\mathbf{S}_j\right)^2\right] - h\sum_i S_i^z$$

Increased SU(3) symmetry at $\vartheta=\pi/4$:

$$\hat{\mathbf{Q}}_i\hat{\mathbf{Q}}_j + \hat{\mathbf{S}}_i\hat{\mathbf{S}}_j = -2\mathcal{P}_{i,j} - \frac{2}{3}$$

all the 3 states of the S=1 spin become identical

What methods do we use?

(i) Variational - site factorized wave function
(ii) Flavor wave calculations
(iii) Exact diagonalization of small clusters
(iv) iPEPS: infinite project entangled pair states(variational approach based on tensor ansatz)
(v) Variational - Gutzwiller projected fermionic wave functions

Triangular lattice, S=1 and SU(3)

SU(2) vs. SU(3) – two sites

$\mathcal{H} = \mathcal{P}_{12}$

$\mathcal{P}_{12}(|\alpha\beta\rangle - |\beta\alpha\rangle) = -(|\alpha\beta\rangle - |\beta\alpha\rangle)$ E=−1 , odd wave function

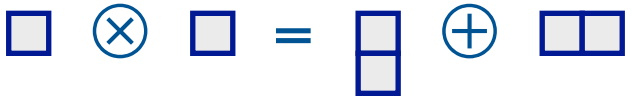
$\mathcal{P}_{12}(|\alpha\beta\rangle + |\beta\alpha\rangle) = +(|\alpha\beta\rangle + |\beta\alpha\rangle)$ E=+1, even wave function

Addition of two S=½ SU(2) spins:

½ ⊗ ½ = 0 ⊕ 1

using Young diagrams:

2 × 2 = 1 + 3



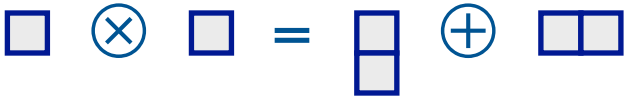
 ↑ or ↓ spin

 $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ singlet, odd (anti-symmetrical)

 $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ triplet even (symmetrical)

Addition of two SU(3) spins:

3 × 3 = 3 + 6



 $|a\rangle$, $|b\rangle$, and $|c\rangle$.

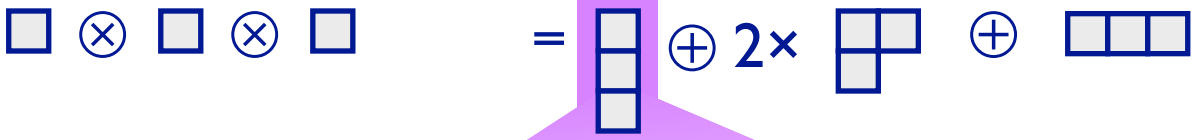
 $|ab\rangle - |ba\rangle$, $|ac\rangle - |ca\rangle$, $|bc\rangle - |cb\rangle$: odd (anti-symmetrical).

 $|aa\rangle$, $|bb\rangle$, $|cc\rangle$, $|ab\rangle + |ba\rangle$, $|ac\rangle + |ca\rangle$, and $|bc\rangle + |cb\rangle$. even (symmetrical)

SU(3) irreps on 3 sites

Addition of three SU(3) spins (27 states):

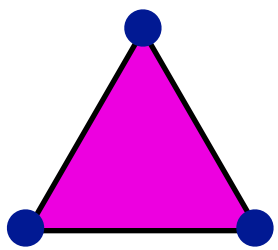
$3 \times 3 \times 3 = 1 + 2 \times 8 + 10$



SU(3) singlet

$= |ABC\rangle + |CAB\rangle + |BCA\rangle - |BAC\rangle - |ACB\rangle - |BCA\rangle$

spins fully antisymmetrized



in the SU(3) singlet the spins are fully entangled:
we cannot write it in a product form

Variational (classical) approach

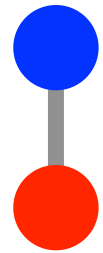
a site-product wave function for e.g. SU(3):

$$|\Psi\rangle = \prod_i |\psi_i\rangle$$

$$|\psi_i\rangle = d_{A,i}|A\rangle_i + d_{B,i}|B\rangle_i + d_{C,i}|C\rangle_i$$

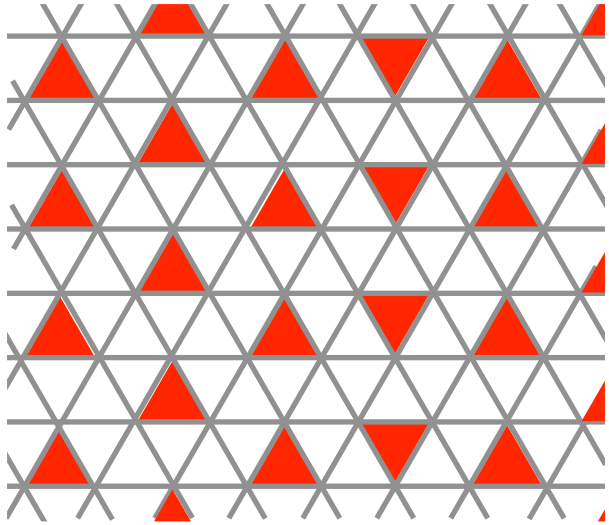
$$E_{\text{var}} = \frac{\langle\Psi|\mathcal{H}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = J\sum_{\langle i,j\rangle} |\mathbf{d}_i \cdot \bar{\mathbf{d}}_j|^2$$

minimal, when the $\mathbf{d}_i$ and $\mathbf{d}_j$ on the bond are orthogonal
different colors on a bond

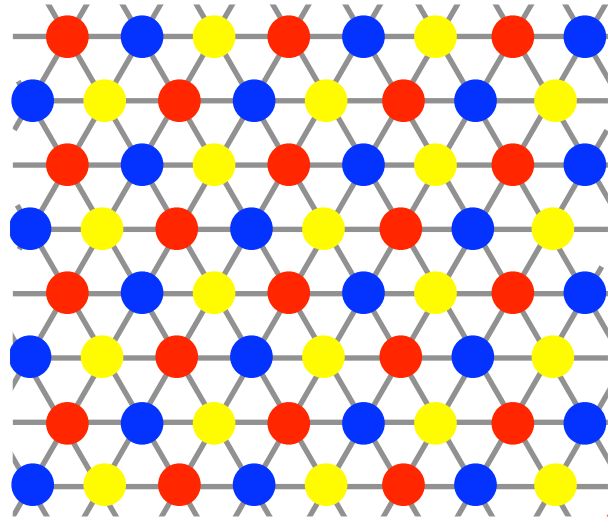


The fate of SU(3) on triangular lattice

SU(2) frustrated!



crystal of singlets?



"classical" solution?

SU(3) classical state is perfectly happy on the triangular lattice - the 3 mutually perpendicular **d**'s form a 3 sublattice structure.

H. Tsunetsugu and M. Arikawa, J. Phys. Soc. Jpn. **75**, 083701 (2006) [NiGa2S4, Nakatsuji]

A. M. Läuchli, F. Mila, and K. Penc, Phys. Rev. Lett. 97, 087205 (2006)

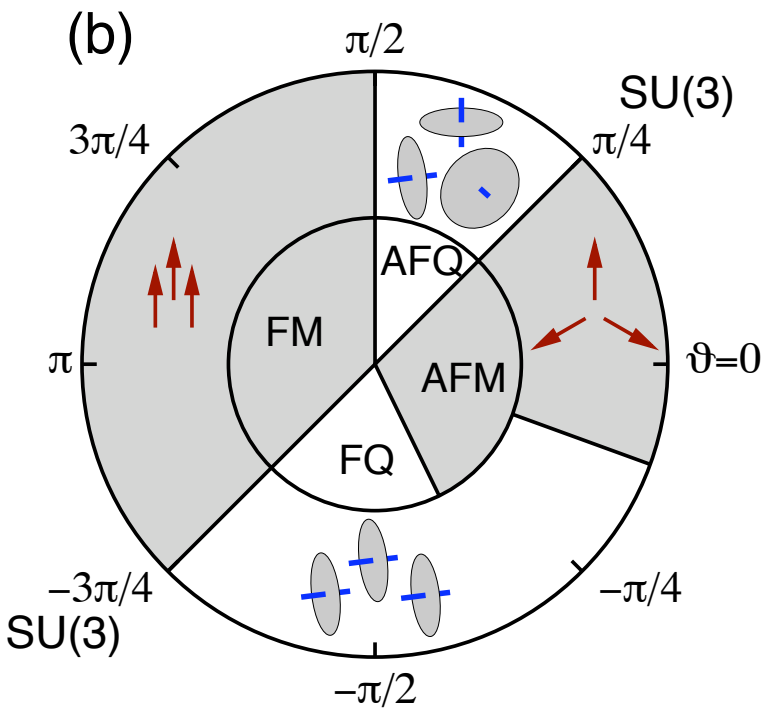
Phase diagram

mean field

$$\langle \hat{\mathbf{Q}}_i \cdot \hat{\mathbf{Q}}_j \rangle = 2(\mathbf{d}_i \cdot \mathbf{d}_j)^2 - \frac{2}{3}$$

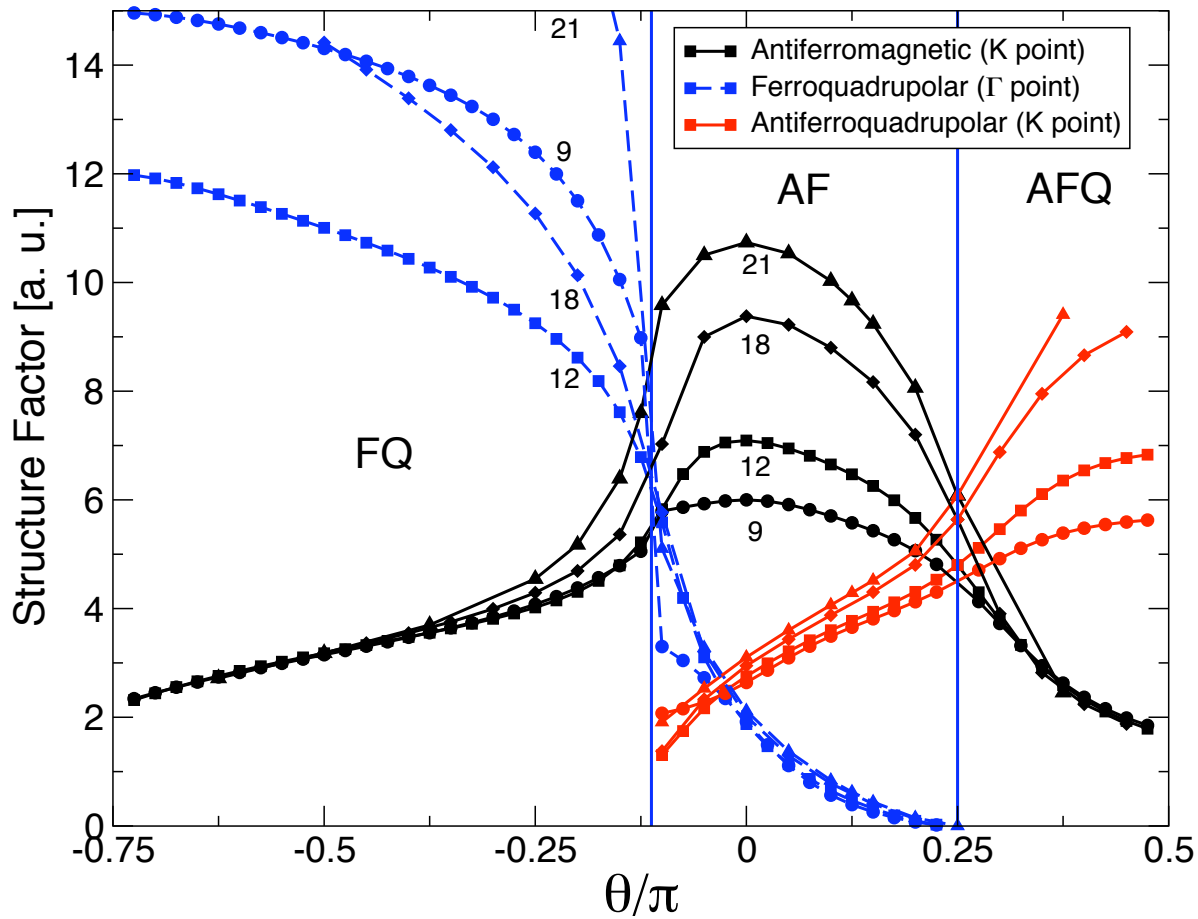
max., if $\mathbf{d}_i$ ll $\mathbf{d}_j$

min., if $\mathbf{d}_i \perp \mathbf{d}_j$



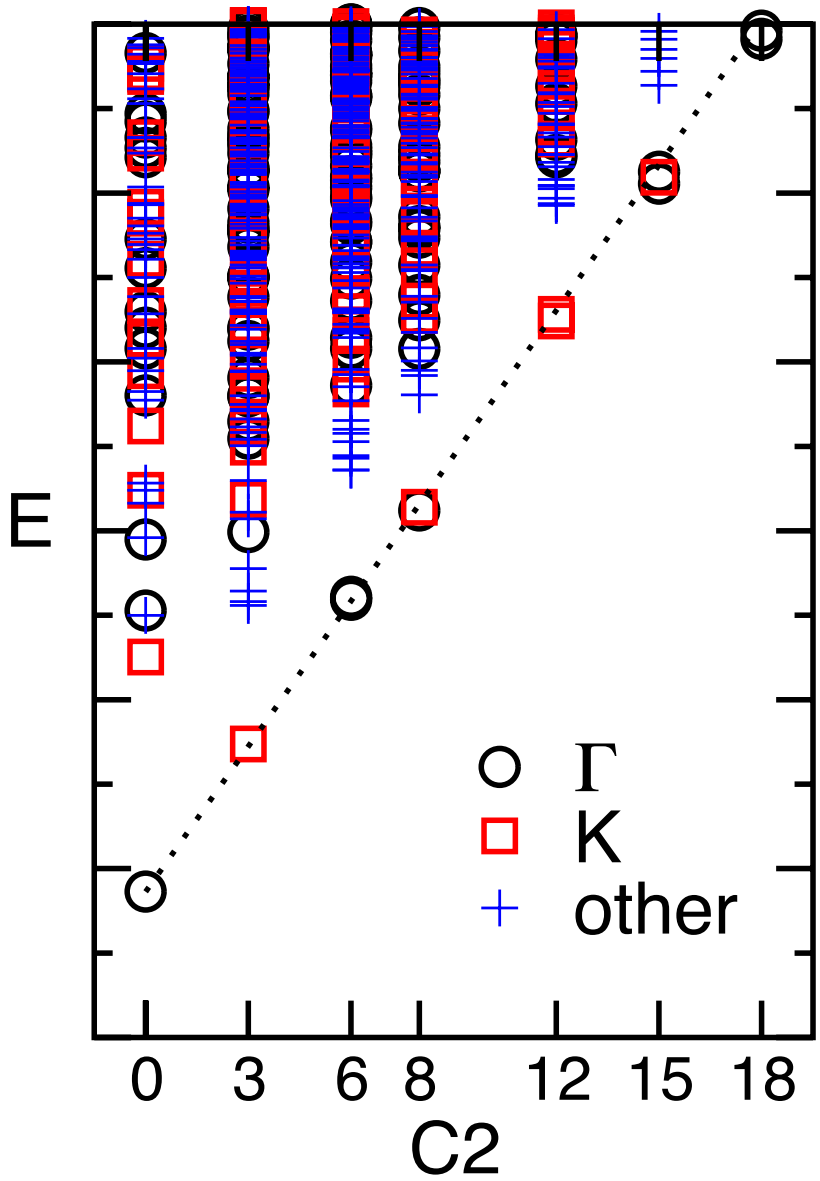
Tsunetsugu & Arikawa, JPSJ **75**, 083701 (2006)
Läuchli, Mila & Penc, PRL **97**, 087205 (2006)
Bhattacharjee, Shenoy & Senthil,PRB **74**, 092406 (2006).

exact diagonalization



Läuchli, Mila & Penc, PRL **97**, 087205 (2006)

SU(3) on triangular lattice - exact diagonalization



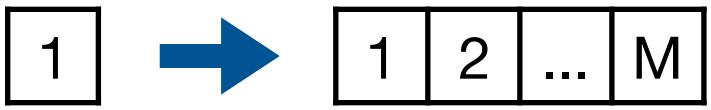
Signature of SU(3) breaking in the excitation spectrum:
Anderson tower compatible with 3 sublattice order

C2 - Casimir operator, analog of the total spin S^2

K. Penc, A. M. Läuchli, in `Introduction to Frustrated Magnetism', p. 331-362, Springer Series in Solid-State Sciences, Vol. **164**, eds. C. Lacroix, F. Mila, and P. Mendels (Springer, 2011)

SU(3) flavour-wave theory

N. Papanicolaou, Nucl. Phys. B **305**, 367 (1988)
A. Joshi *et al.* PRB **60**, 6584 (1999)



We enlarge the fundamental to the fully symmetric representation of M boxes.

States in the fully symmetric multiplet can be represented by 3 Schwinger bosons a_A,a_B and a_C

$$a^\dagger_{A,i}a_{A,i} + a^\dagger_{B,i}a_{B,i} + a^\dagger_{C,i}a_{C,i} = M$$

$$\mathcal{P}_{ij} = \sum_{\mu,\nu\in\{A,B,C\}} a^\dagger_{\mu,i}a^\dagger_{\nu,j}a_{\nu,i}a_{\mu,j}$$

The site product wave function is the “classical” solution (no quantum entanglement between sites)

$$|\Psi\rangle = \prod_i |\psi_i\rangle$$

$$|\psi_i\rangle = d_{A,i}|A\rangle_i + d_{B,i}|B\rangle_i + d_{C,i}|C\rangle_i$$

cf. SU(2) spin coherent state for SU(2)

$$|\vartheta,\varphi\rangle = \cos\frac{\vartheta}{2}|\uparrow\rangle + \sin\frac{\vartheta}{2}e^{i\varphi}|\downarrow\rangle$$

SU(3) flavour-wave theory

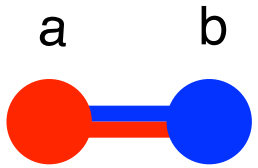
$$\mathcal{P}_{ij} = \sum_{\mu,\nu\in\{A,B,C\}} a^\dagger_{\mu,i} a^\dagger_{\nu,j} a_{\nu,i} a_{\mu,j}$$

1/M expansion:

$$\tilde{a}^\dagger_A, \tilde{a}_A \to \sqrt{M - \tilde{a}^\dagger_B \tilde{a}_B - \tilde{a}^\dagger_C \tilde{a}_C}$$

$$\to \sqrt{M} - \frac{1}{2\sqrt{M}} \left(\tilde{a}^\dagger_B \tilde{a}_B + \tilde{a}^\dagger_C \tilde{a}_C \right) + \dots$$

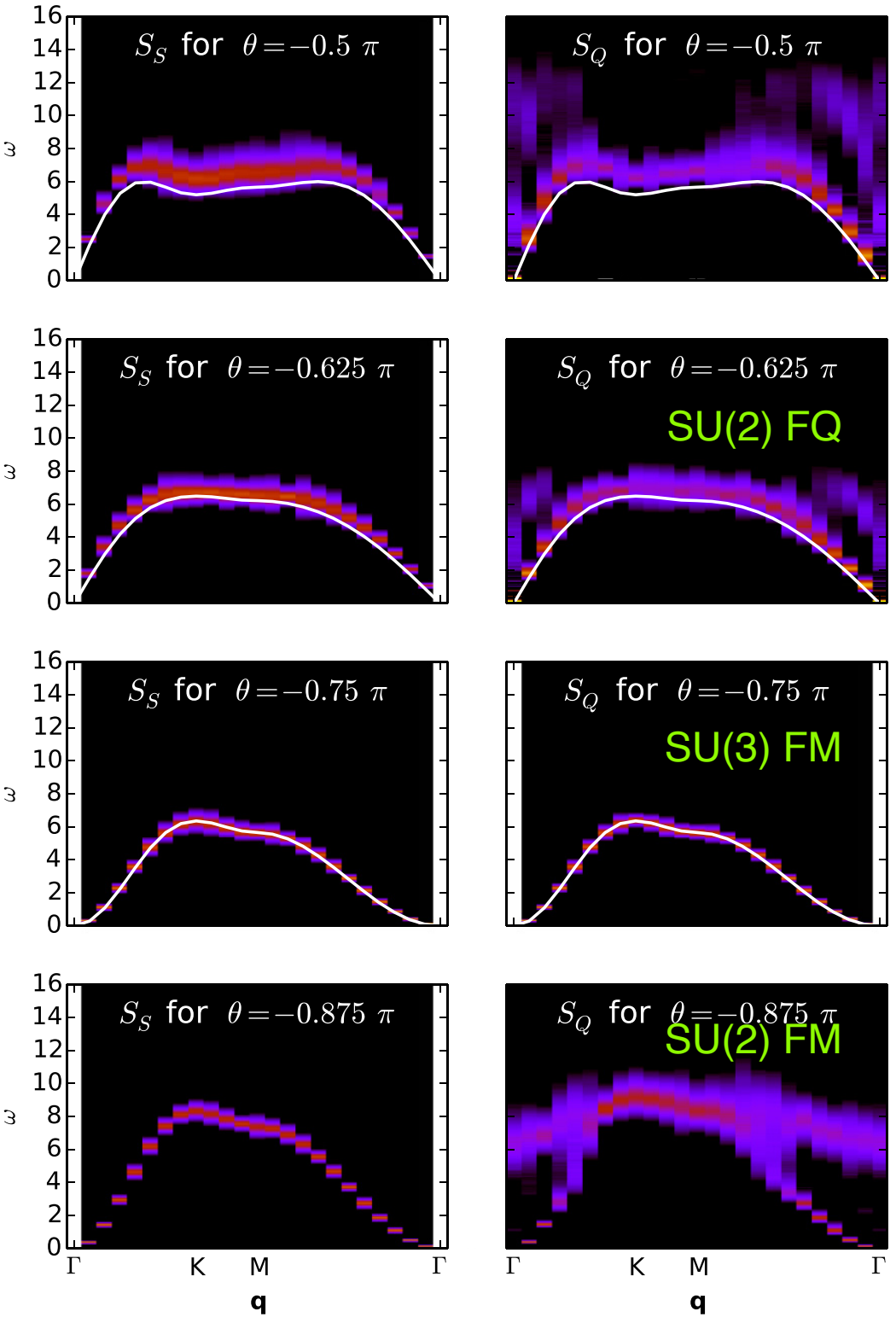
Holstein-Primakoff bosons

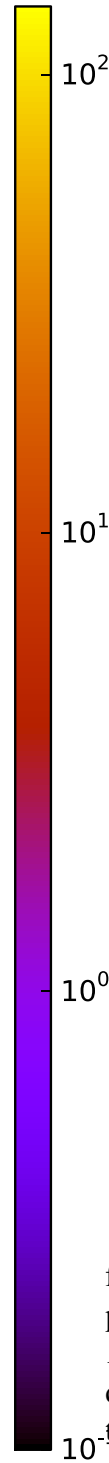


$$\mathcal{H} = (a^\dagger + b)(a + b^\dagger)$$

quadratic in operators: we know how to diagonalize it (spin wave)

$$\mathcal{H} = -MJL + M \sum_\nu \sum_{\mathbf{k}} \omega_\nu(\mathbf{k}) \left(\alpha^\dagger_\nu(\mathbf{k}) \alpha_\nu(\mathbf{k}) + \frac{1}{2} \right)$$





Structure factors in S=1 bilinear-biquadratic Heisenberg model QMC (A. Völl and S. Wessel)

Phys. Rev. B **91**, 165128 (2015)

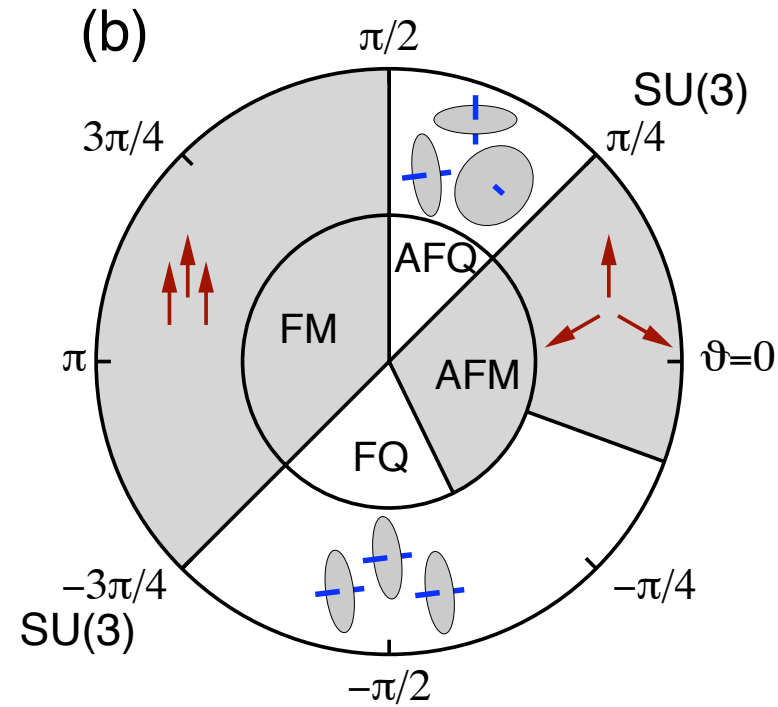
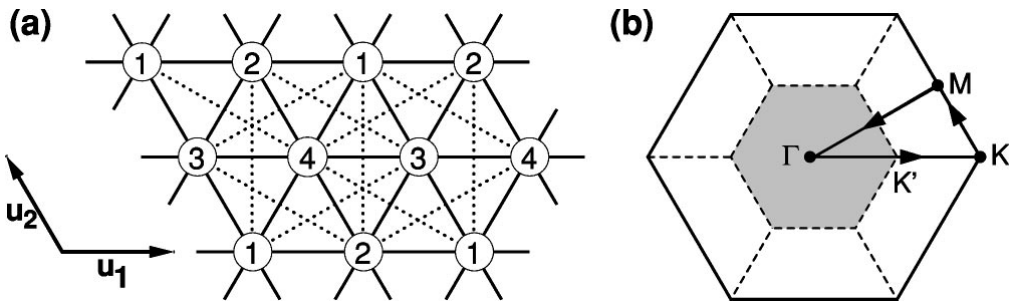


FIG. 5. (Color online) Dynamical spin and quadrupolar structure factors $S_S(\omega,\boldsymbol{q})$ and $S_Q(\omega,\boldsymbol{q})$ for different values of θ along the path $\Gamma \to K \to M \to \Gamma$ through the Brillouin zone [$\Gamma = (0,0)^\intercal$, $K = (4\pi/3,0)^\intercal$, $M = (2\pi/3,\pi/\sqrt{3})^\intercal$]. The dashed lines indicate the quadrupolar wave dispersion relations obtained within flavor-wave theory.

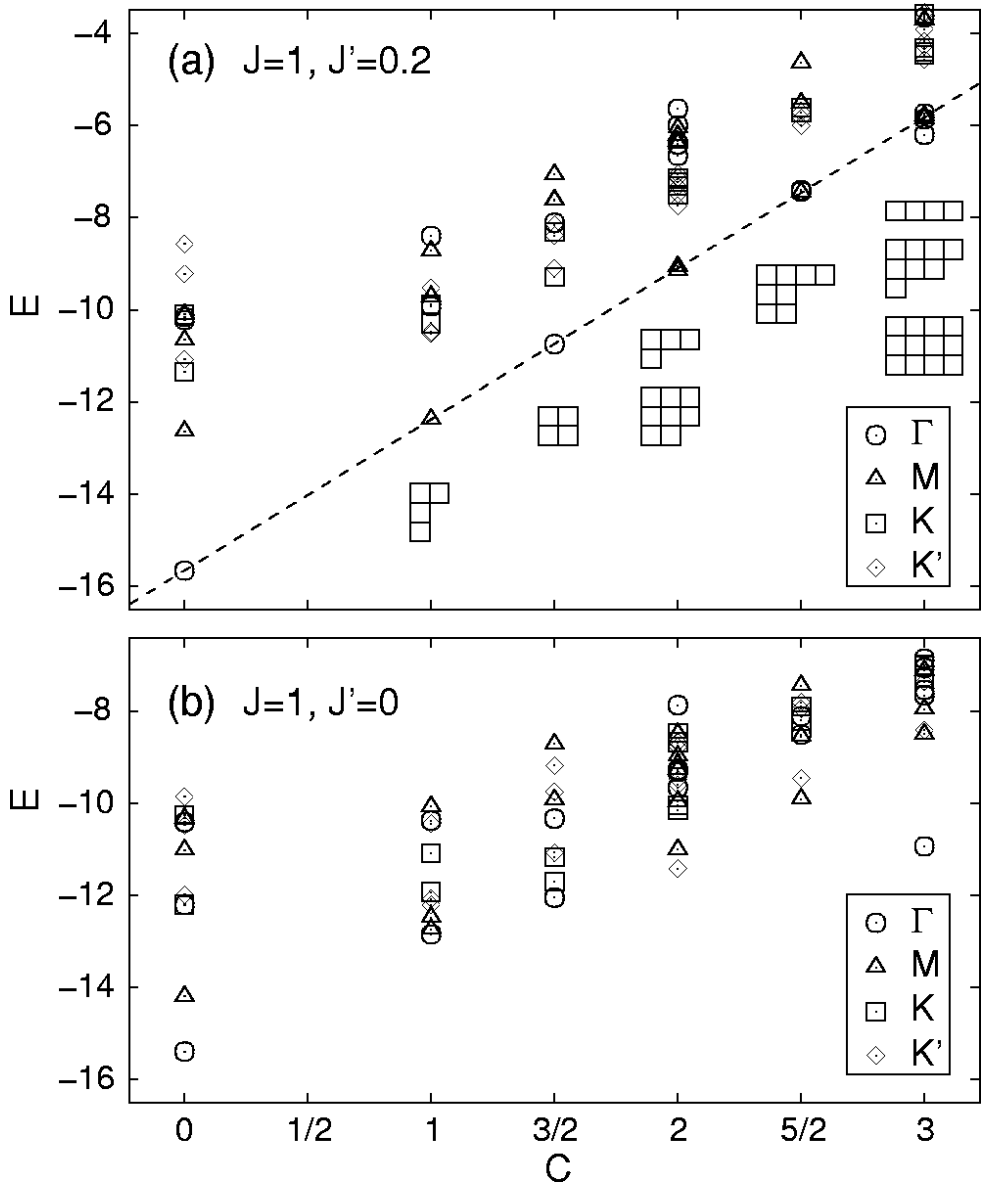
Triangular lattice, SU(4)

SU(4) on triangular lattice - exact diagonalization



Signature of SU(4) breaking in the excitation spectrum:
Anderson tower compatible with 4 sublattice order for large enough second neighbor J'

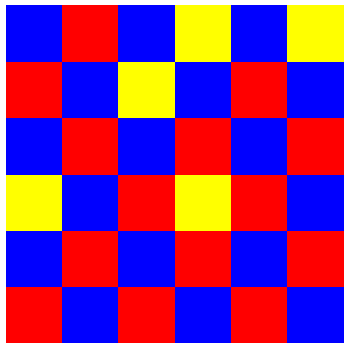
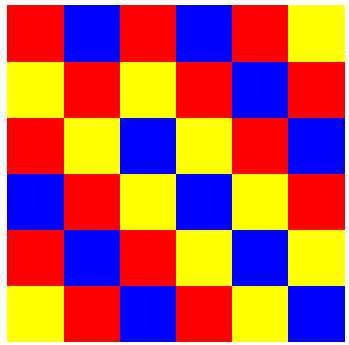
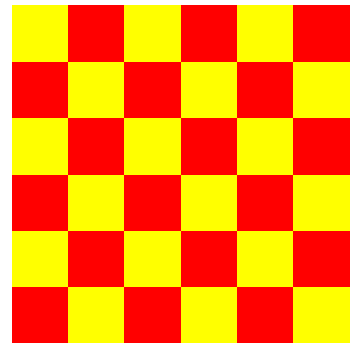
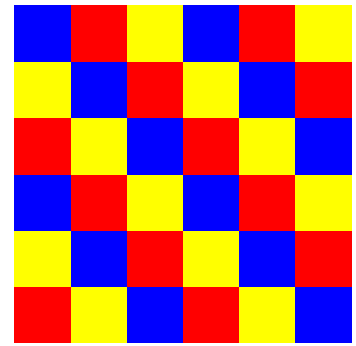
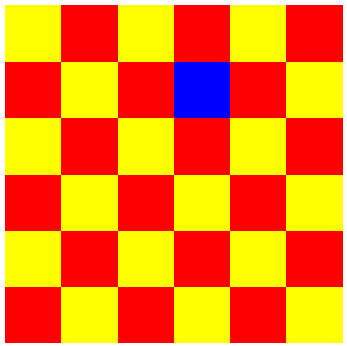
C2 - Casimir operator, analog of the total spin S^2



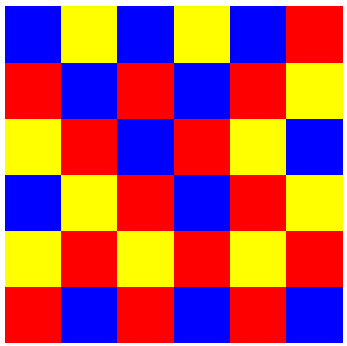
K. Penc, M. Mambrini, P. Fazekas, F. Mila, Phys. Rev. B **68**, 012408 (2003)

Square lattice, SU(3)

SU(3) square lattice, classical solutions: macroscopically degenerate

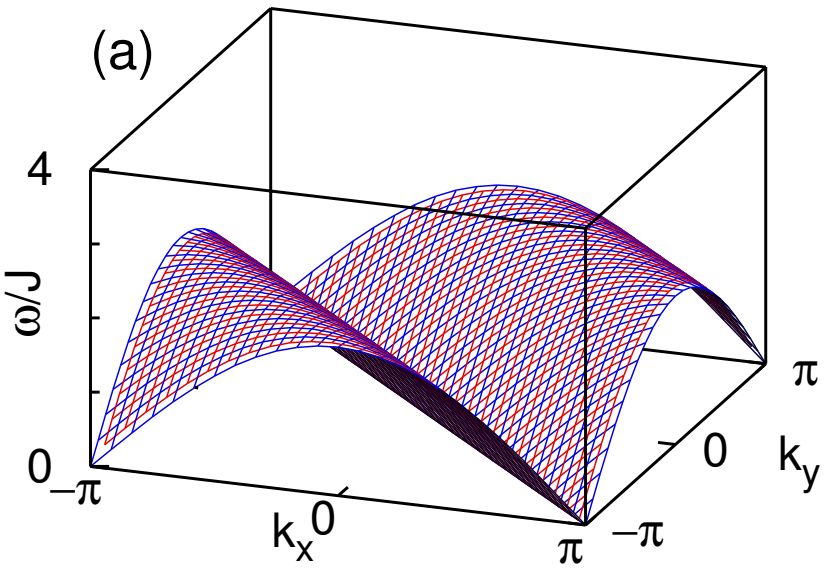
All bonds happy at the mean field level,
frustration due to abundance of choices

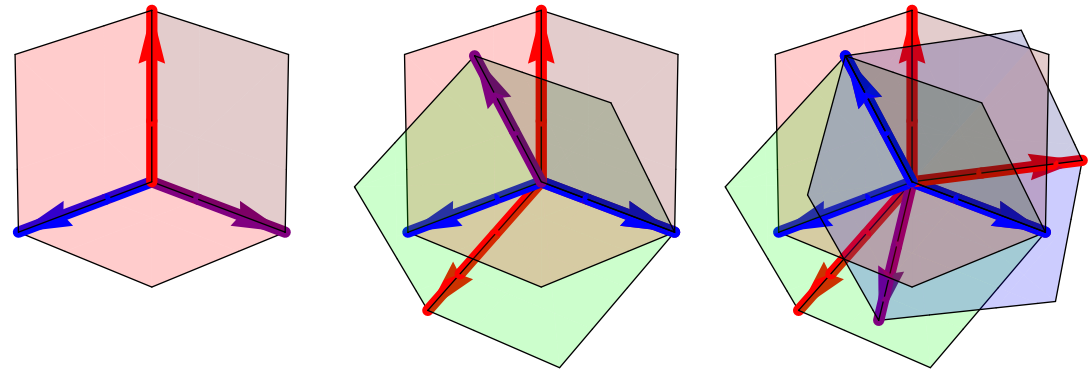


Order by disorder:
the zero point energy of the quantum fluctuations over a mean field solution selects the ground state

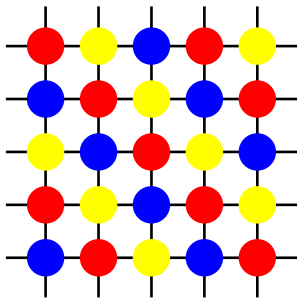
$$E_{ZP} = \frac{M}{2} \sum_{\nu} \sum_{\mathbf{k}} \omega_{\nu}(\mathbf{k})$$

SU(3) flavour-wave: helical states

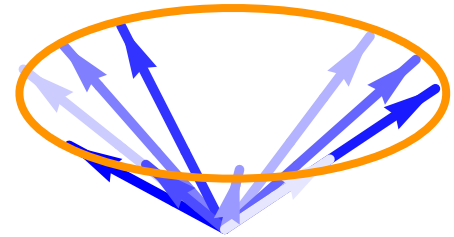




$$\mathbf{d}_{l+2} = \cos\theta \mathbf{d}_l + \sin\theta \mathbf{d}_l \times \mathbf{d}_{l+1}$$



$$\mathbf{Q} = (q, q)$$



$$|\psi_i\rangle \propto \sqrt{-\cos q}|z\rangle + \sin(\mathbf{r}_i \cdot \mathbf{Q})|x\rangle + \cos(\mathbf{r}_i \cdot \mathbf{Q})|y\rangle$$

$q = \pi$ 2-sublattice

$q = 2\pi/3$ 3-sublattice

SU(3) flavour-wave theory: 1/M expansion

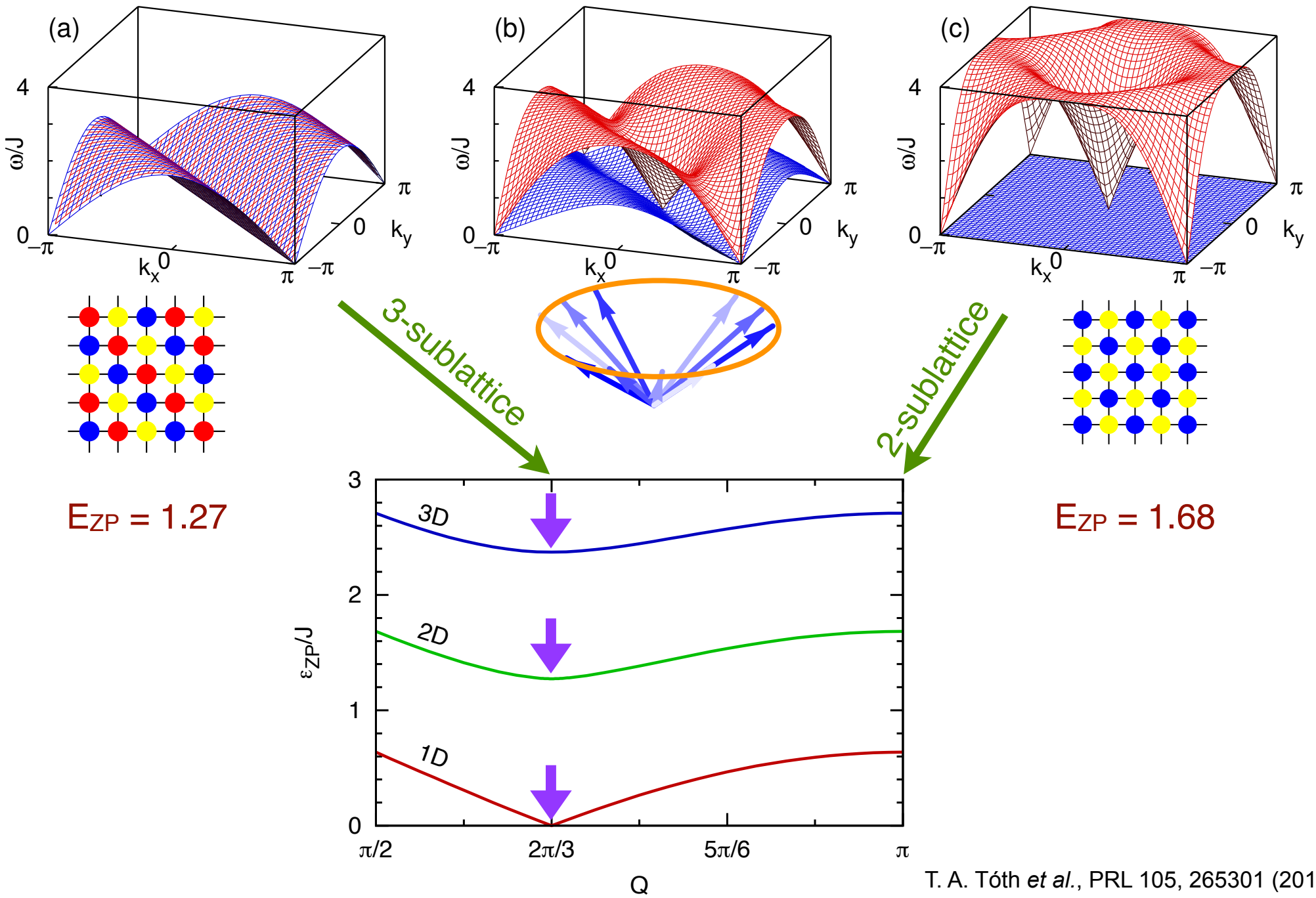
quadratic in operators → we know how to diagonalize it (spin wave)

$$\mathcal{H} = -MJL + M\sum_{\nu}\sum_{\mathbf{k}}\omega_{\nu}(\mathbf{k})\left(\alpha_{\nu}^{\dagger}(\mathbf{k})\alpha_{\nu}(\mathbf{k}) + \frac{1}{2}\right)$$

Order by disorder:
if classically degenerate,
quantum fluctuations decide the
winner

$$E_{ZP} = \frac{M}{2}\sum_{\nu}\sum_{\mathbf{k}}\omega_{\nu}(\mathbf{k})$$

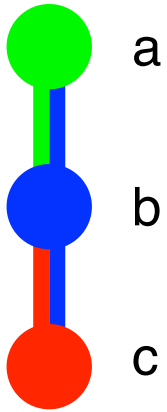
SU(3) flavour-wave: helical dispersion,zero-point energy



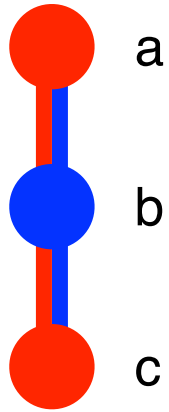
T. A. Tóth *et al.*, PRL 105, 265301 (2010).

structure of the flavor wave Hamiltonian

$$\mathcal{H} = + (a^\dagger + b)(b^\dagger + a) + (b^\dagger + c)(c^\dagger + b)$$



$$\mathcal{H} = + (a^\dagger + b)(b^\dagger + a) + (b^\dagger + c)(c^\dagger + b)$$



disconnected (commuting) terms,

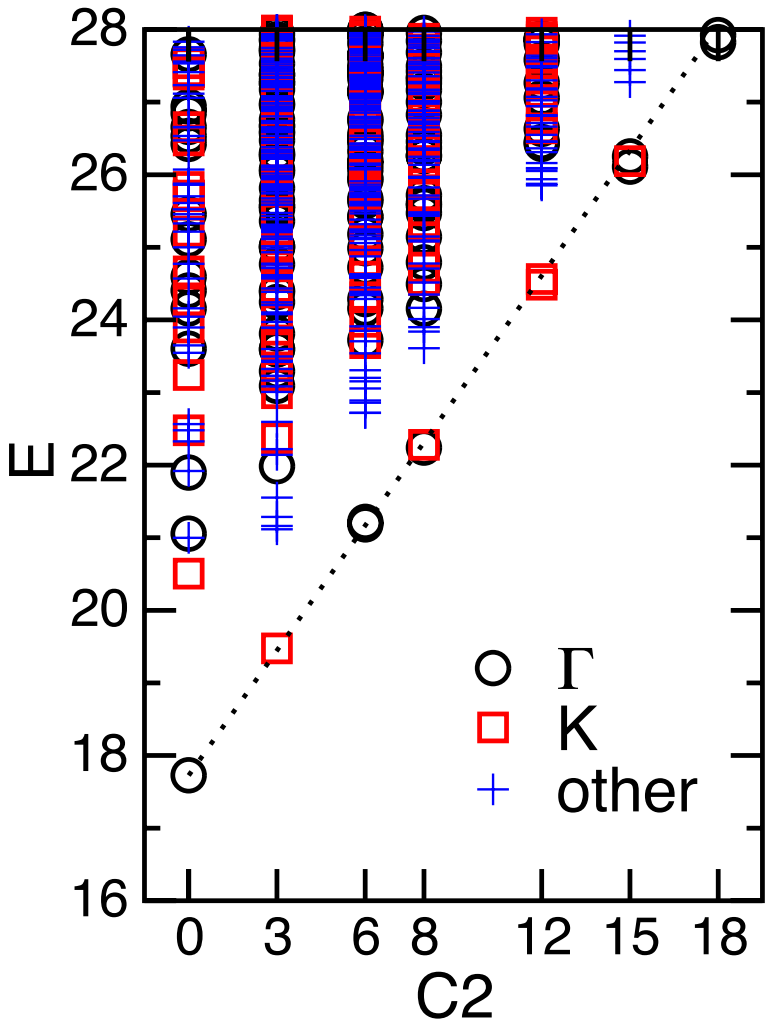
each of them gives $E_{ZP} = 0$

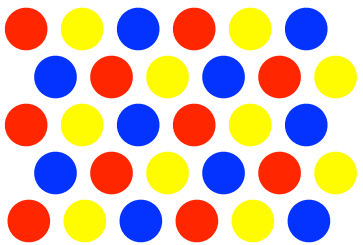
the connected term gives $E_{ZP} > 0$

fluctuation energy minimal
if next nearest neighbor spins
are also of different color

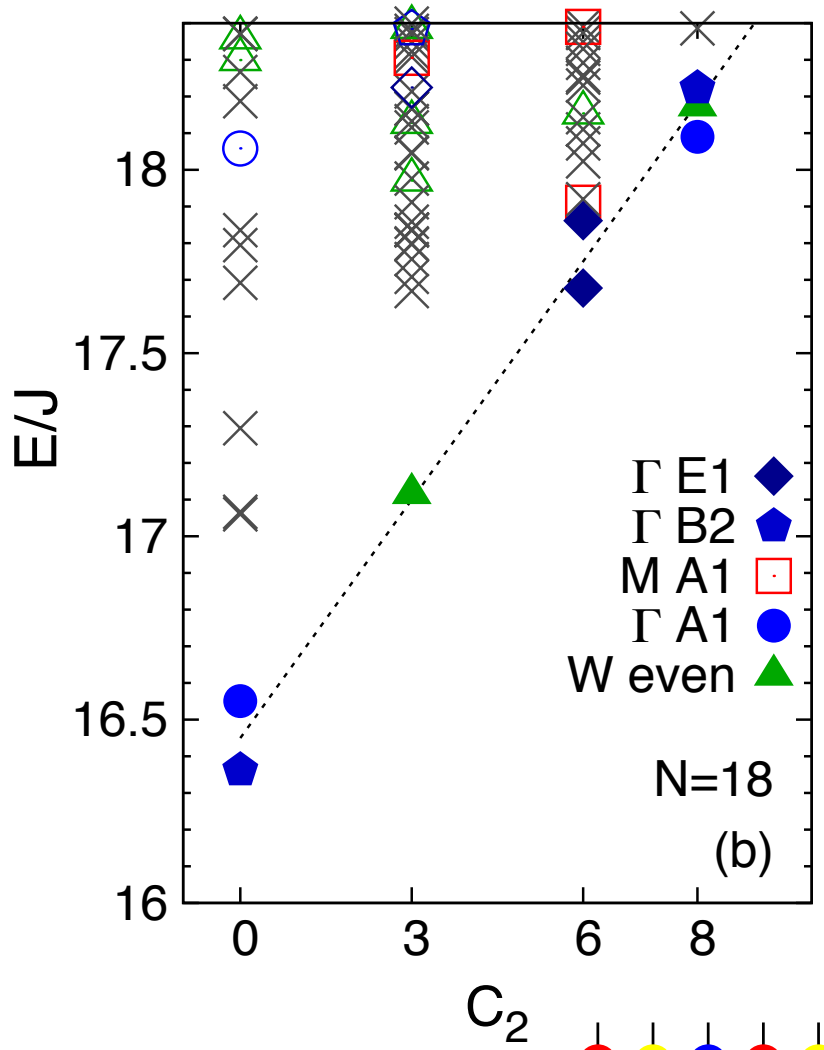
Unbiased calculation: ED, Anderson towers

triangular lattice

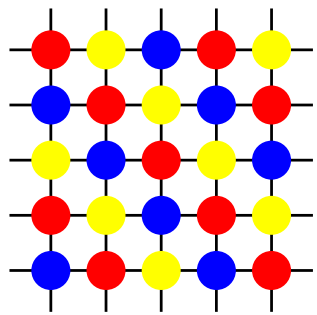




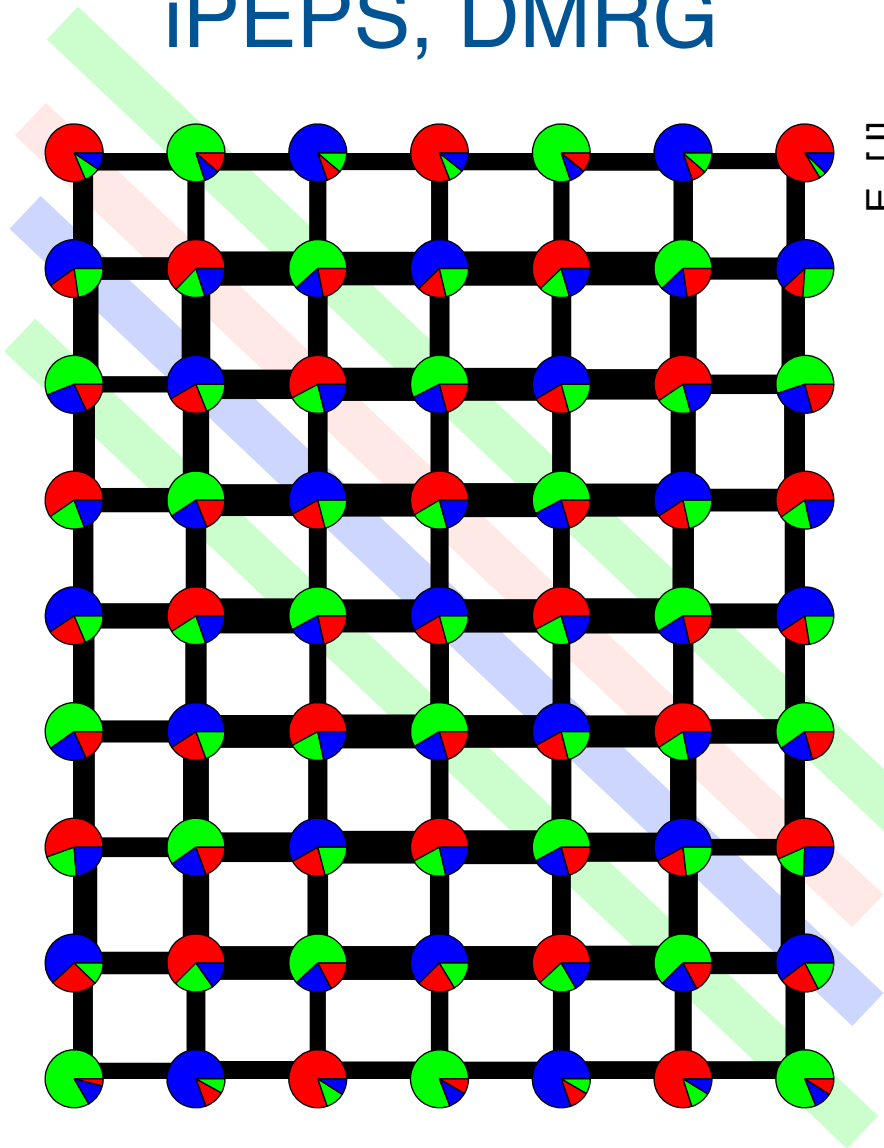
square lattice

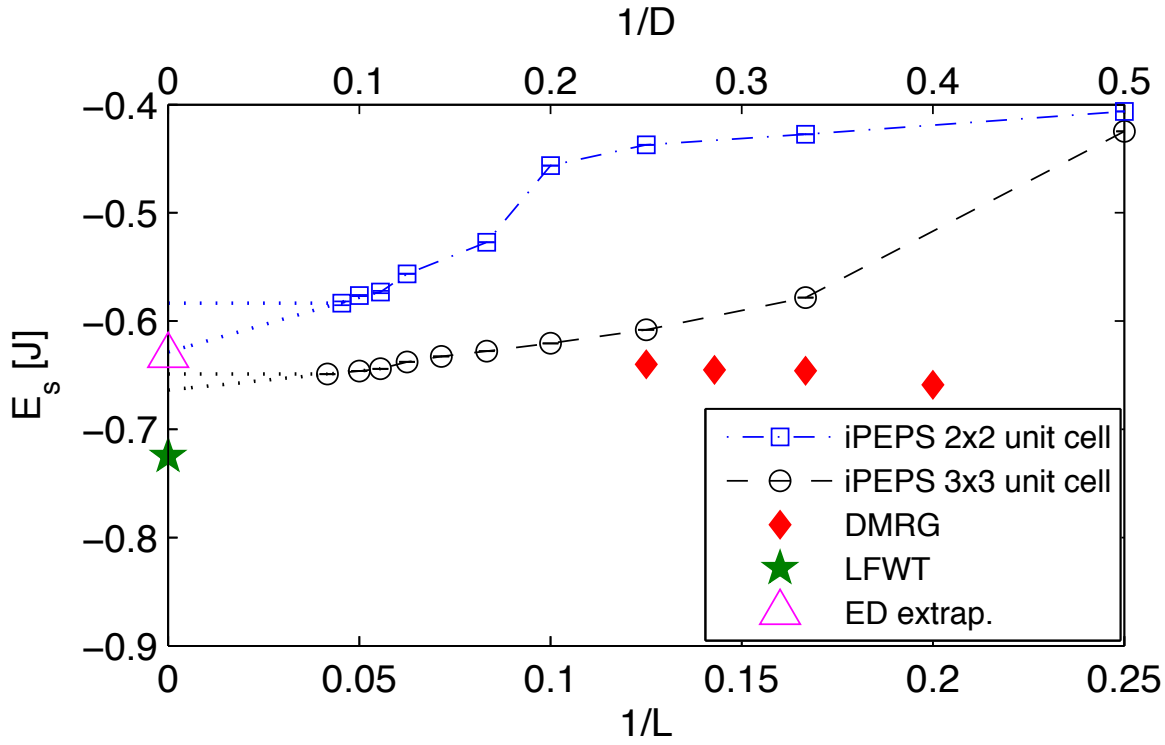


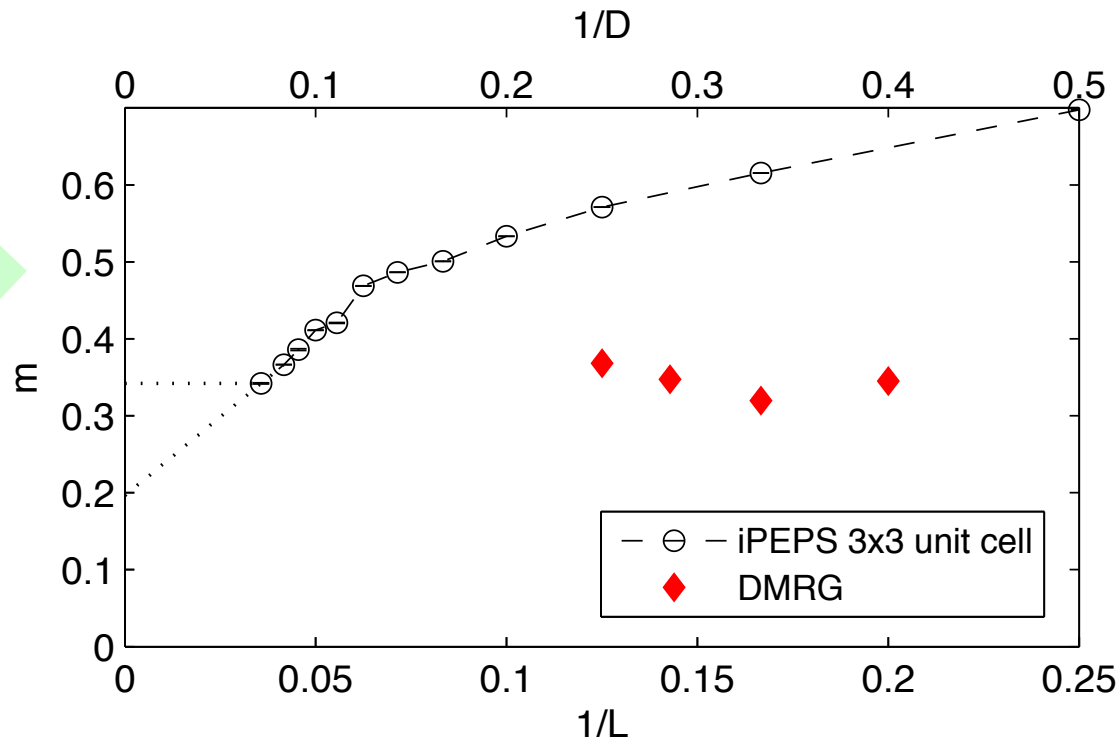
additional Z2 symmetry



Unbiased calculation: iPEPS, DMRG

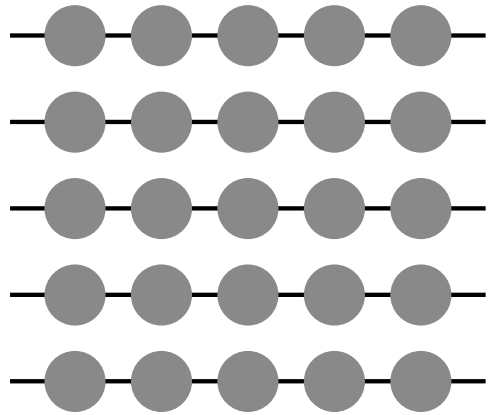




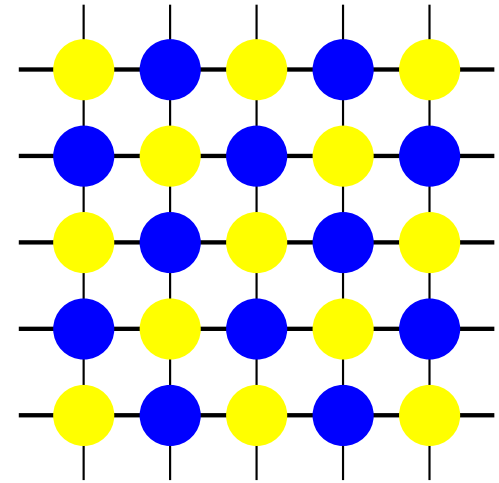


B. Bauer, P. Corboz, A. M. Läuchli, L. Messio, K. Penc, M. Troyer, F. Mila: Phys. Rev. B **85**, 125116/1-11 (2012)

SU(2) coupled chains

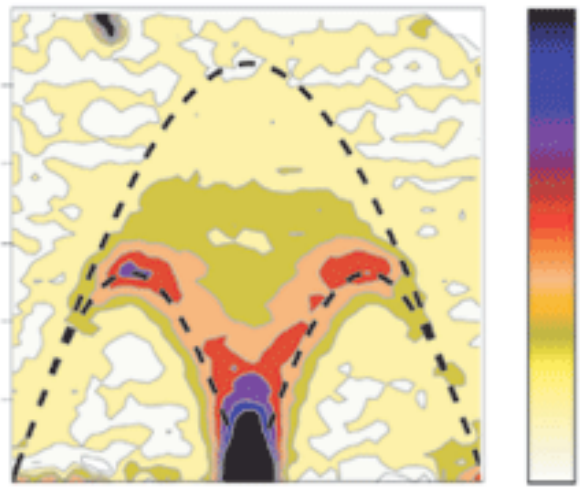


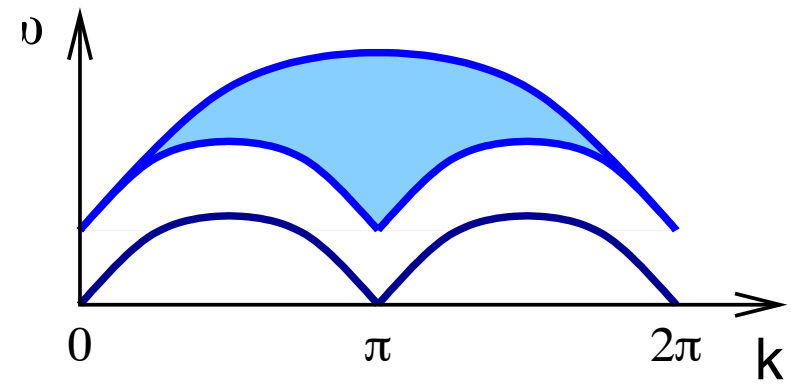
decoupled SU(2) spin chain:
Luttinger liquid with a
spinon continuum



coupled SU(2) spin chain:
antiferromagnetic LRO, with magnons as
bound states developing from the lower
edge of the continuum

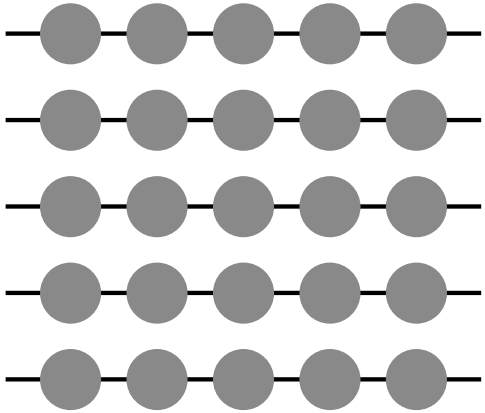
KCuF3

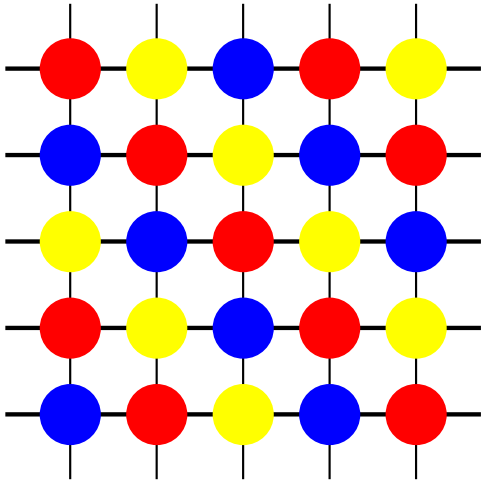


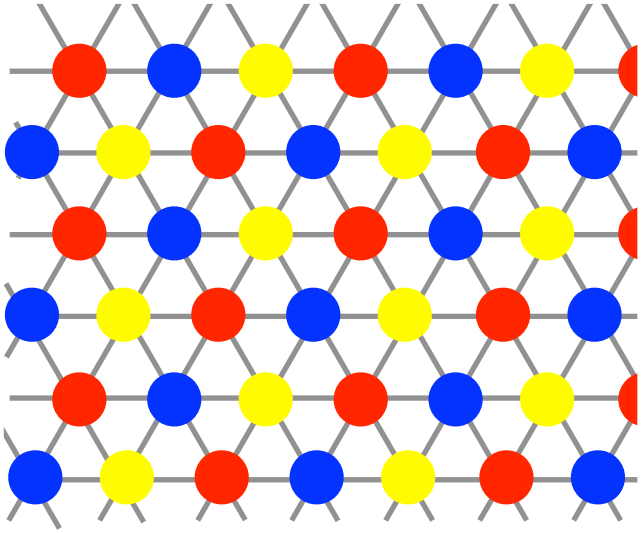


ladder in field

SU(3) coupled chains

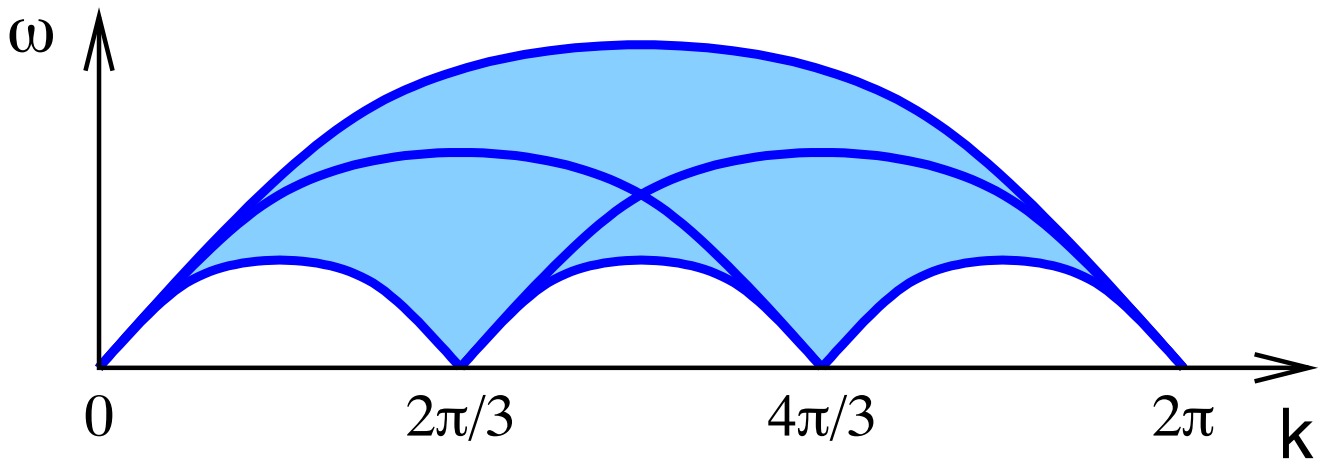






decoupled SU(3) spin chain:
Luttinger liquid,
excitations form a continuum

softening at 2Pi/3 leads to
3 sublattice LRO
for the coupled SU(3) chains?



Square lattice, SU(4)

SU(2) vs. SU(4) - two sites

$\mathcal{H} = \mathcal{P}_{12}$

$\mathcal{P}_{12}(|\alpha\beta\rangle - |\beta\alpha\rangle) = -(|\alpha\beta\rangle - |\beta\alpha\rangle)$ E=−1 , odd wave function

$\mathcal{P}_{12}(|\alpha\beta\rangle + |\beta\alpha\rangle) = +(|\alpha\beta\rangle + |\beta\alpha\rangle)$ E=+1, even wave function

Addition of two S=½ SU(2) spins:

½ ⊗ ½ = 0 ⊕ 1

using Young diagrams:

2 × 2 = 1 + 3

□ ⊗ □ =  ⊕ □□

□ ↑ or ↓ spin

 $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ singlet, odd (anti-symmetrical) comb.

□□ $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ triplet even (symmetrical) comb.

Addition of two SU(4) spins:

4 × 4 = 6 + 10

□ ⊗ □ =  ⊕ □□

□ $|\uparrow\rangle$, $|\downarrow\rangle$, $|\uparrow\rangle$, and $|\downarrow\rangle$
$|a\rangle$, $|b\rangle$, $|c\rangle$, and $|d\rangle$.

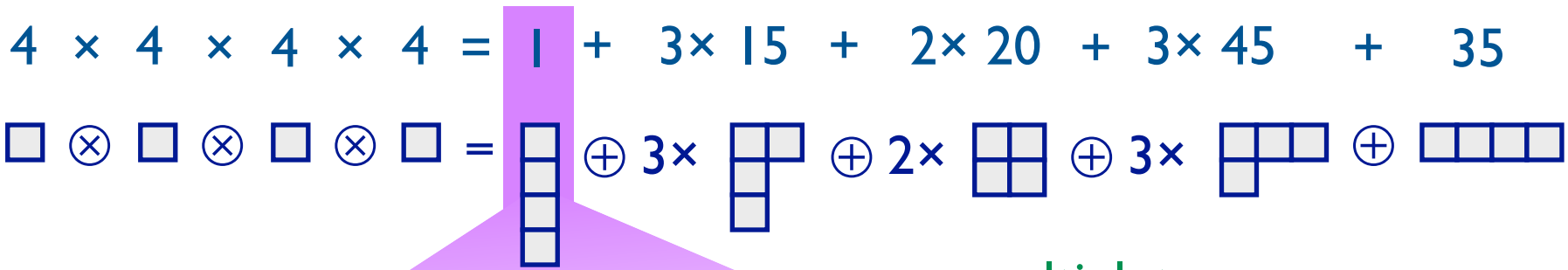
 $|ab\rangle - |ba\rangle$, $|ac\rangle - |ca\rangle$, etc. odd (anti-symmetrical).

□□ $|aa\rangle$, $|bb\rangle$, $|cc\rangle$, $|dd\rangle$, $|ab\rangle + |ba\rangle$, $|ac\rangle + |ca\rangle$, etc. even (symmetrical).

SU(4) irreps on 4 sites

Addition of four SU(4) spins (256 states):

$4 \times 4 \times 4 \times 4 = 1 + 3\times 15 + 2\times 20 + 3\times 45 + 35$

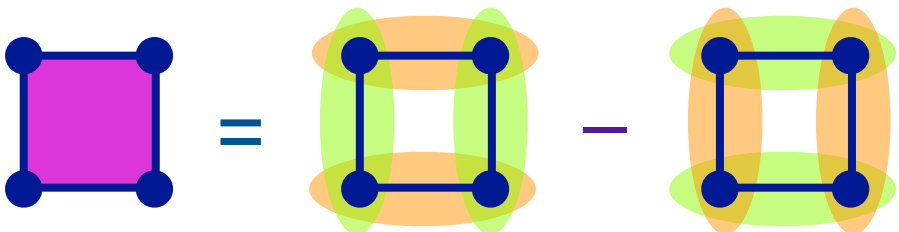


multiplets

SU(4) singlet

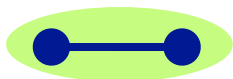
$= |abcd\rangle - |bacd\rangle + |badc\rangle - |bdac\rangle - \ldots$

spins fully antisymmetrized



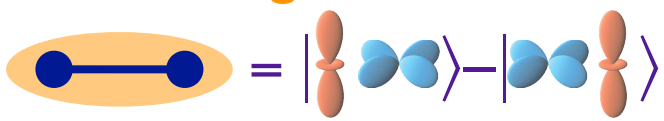
SU(4) singlet plaquette
entangled spins and orbitals

spin singlet bond:



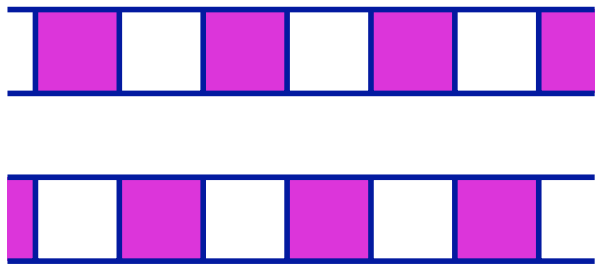
$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

orbital singlet bond:

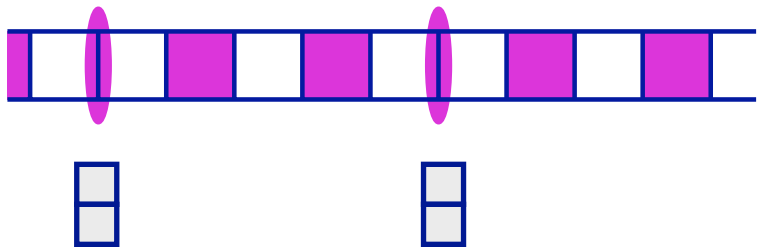


SU(4) ladder

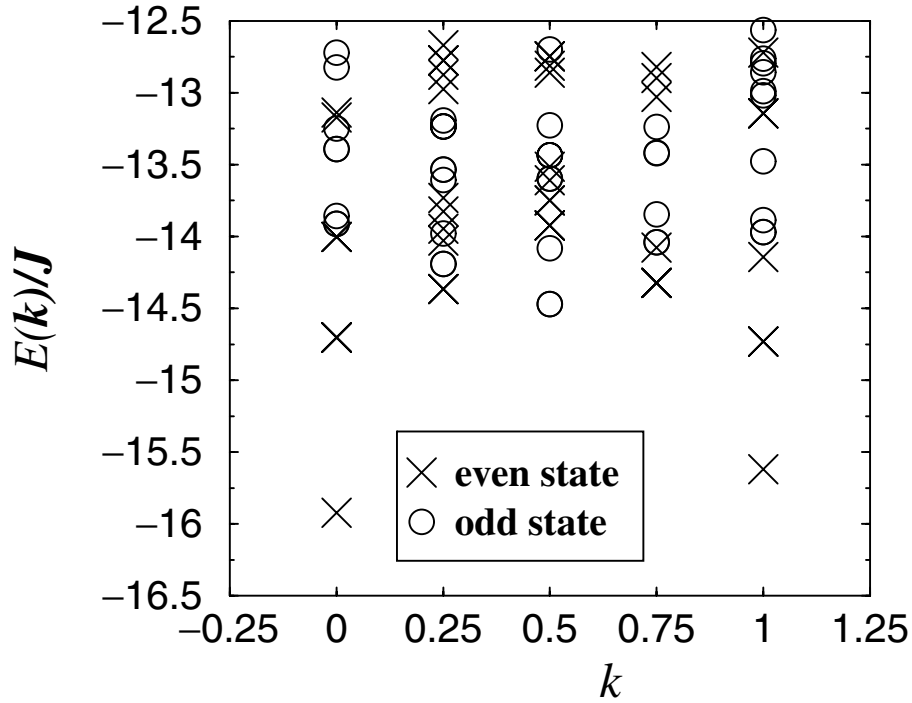
GS twofold degenerate
(translational invariance broken):



gapped excitation spectrum,
situation similar to
J_1-J_2 SU(2) Heisenberg chain:

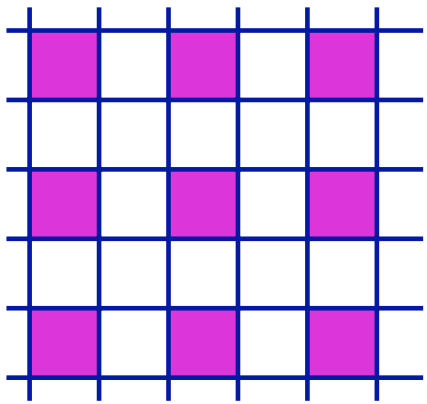


M. van den Bossche et al.,
Phys. Rev. Lett. **86**, 4124 (2001).

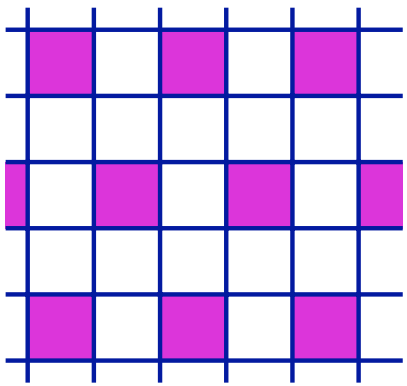


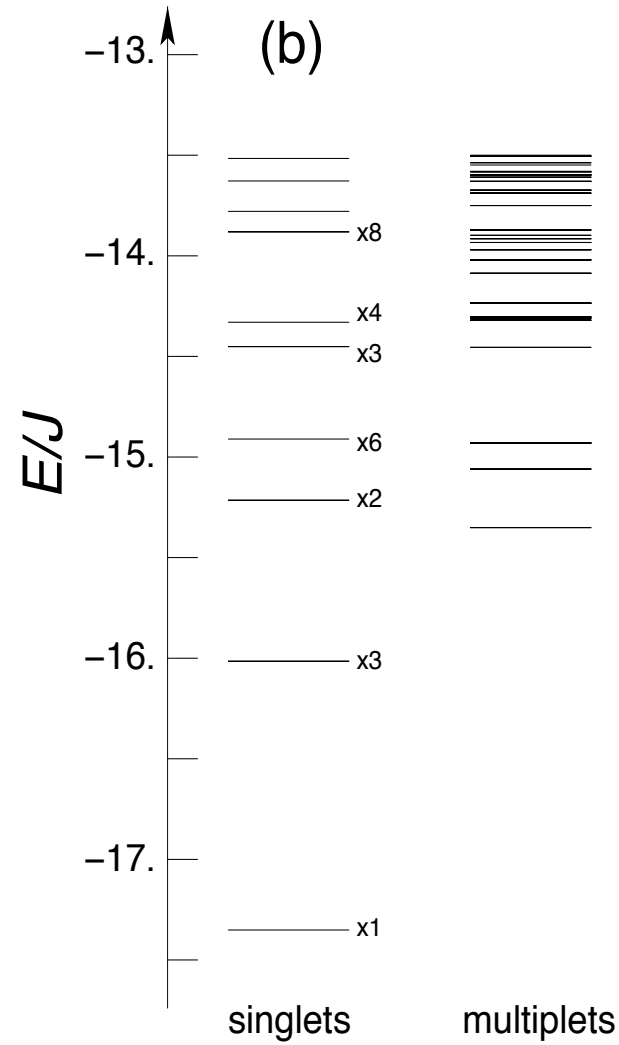
SU(4) on 2D-square lattice: is it a crystal of singlets ?

Ground state 4-fold degenerate?



Excitations?



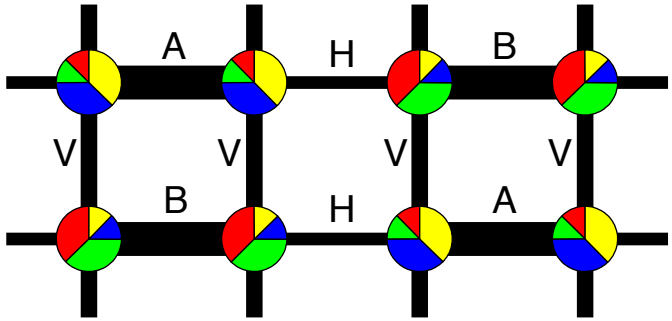


Exact diagonalization, 16 sites

M. van den Bossche et al.,
Eur. Phys. J. B **17**, 367 (2000).

SU(4) on 2D-square lattice: iPEPS

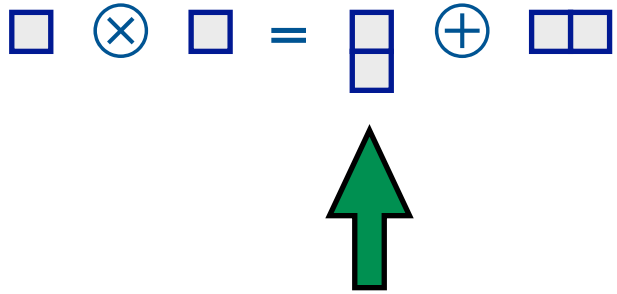
iPEPS: infinite project entangled pair states



D = 12 and a unit cell 4 × 2

dimerization and Neel-like state: both spatial and the SU(4) symmetry is broken

$4 \times 4 = 6 + 10$



the 6 dimensional irreducible representation is realized on the dimers, can Neel order

2-step scenario:

(i) Dimerization: 6-dimensional irreps are formed

(ii) the 6-imensional irreps can possibly Néel order

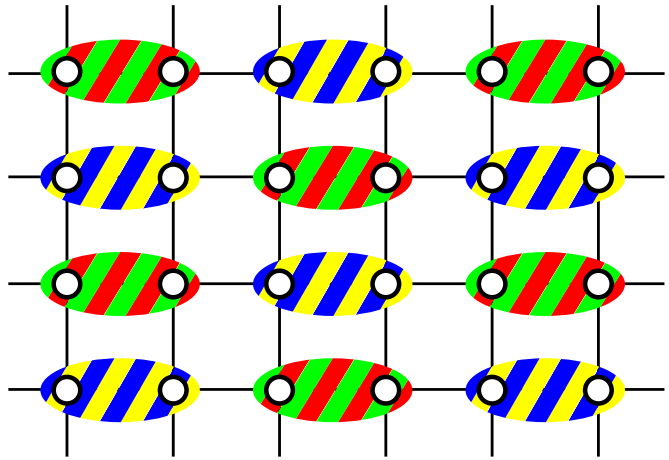
P. Corboz, A. M. Läuchli, K. Penc, M. Troyer, F. Mila, PRL **107**, 215301 (2011).

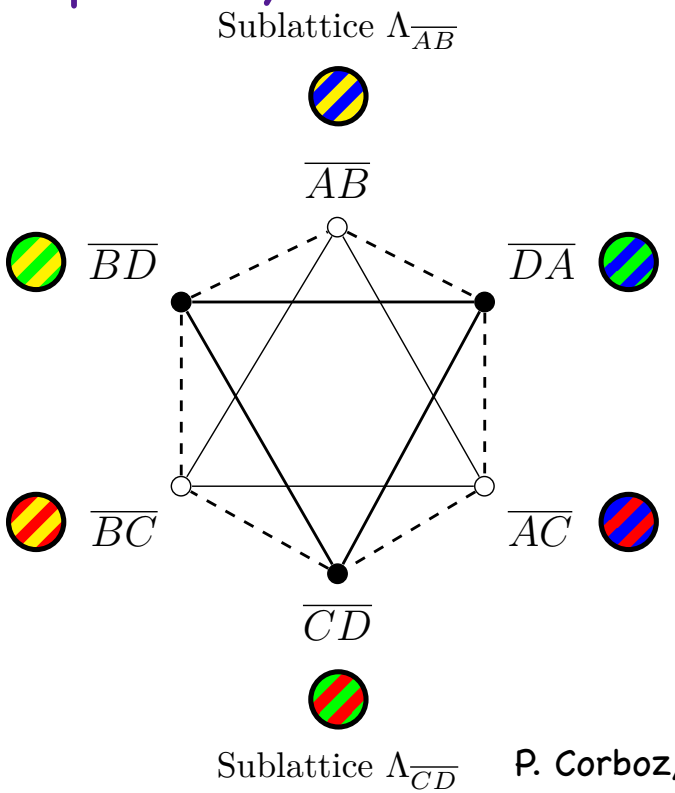
SU(4) on square lattice: 2 step scenario

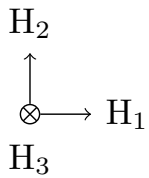
2-step scenario:

(i) Dimerization: **6**-dimensional irreps are formed

(ii) the antipodal pairs in the 6-imensional irreps can possibly Néel order





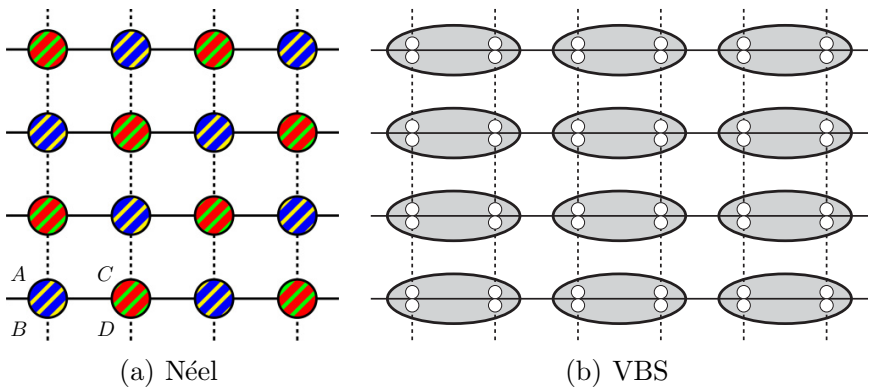


Weight diagram of the **6**-dimensional irrep.

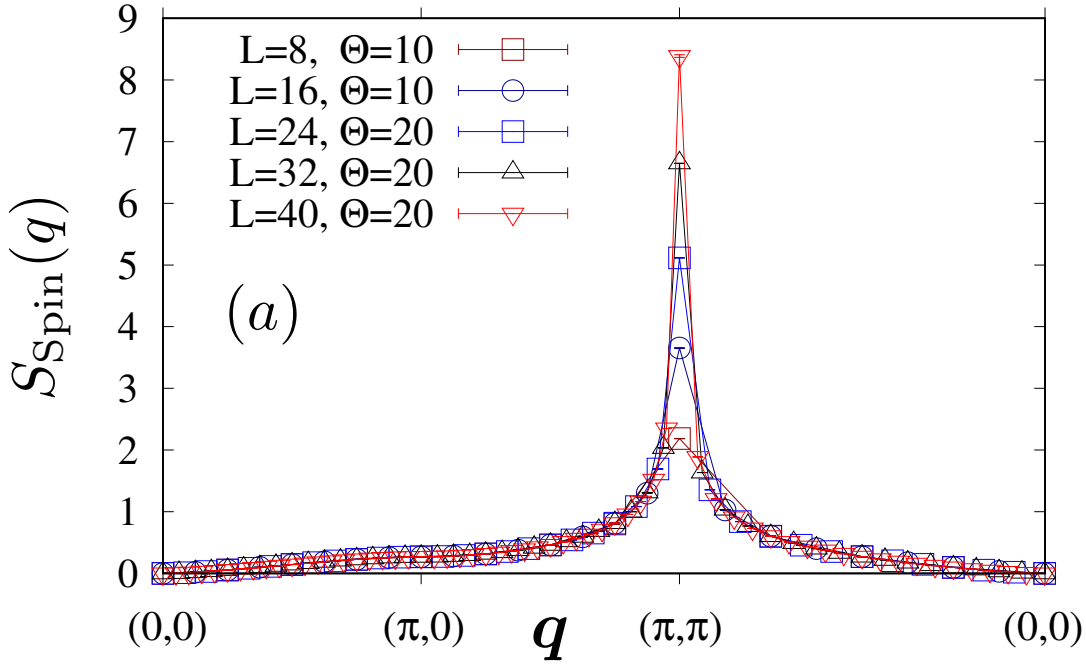
P. Corboz, A. M. Läuchli, K. Penc, M. Troyer, F. Mila, PRL **107**, 215301 (2011).

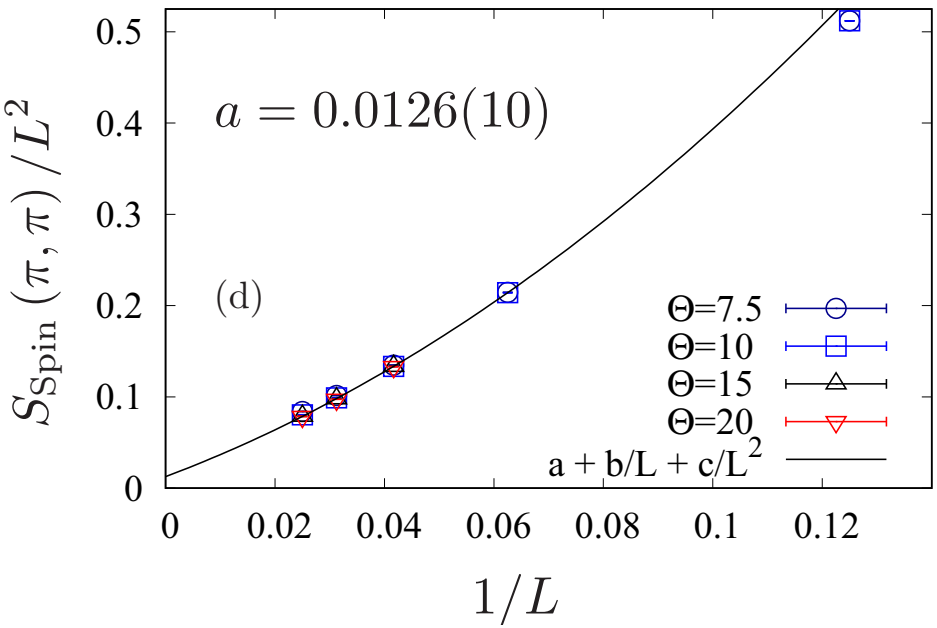
SU(4) 6-dim irrep on square lattice

2 competing phases



sign problem free QMC possible,
structure factor is peaked at (π,π)

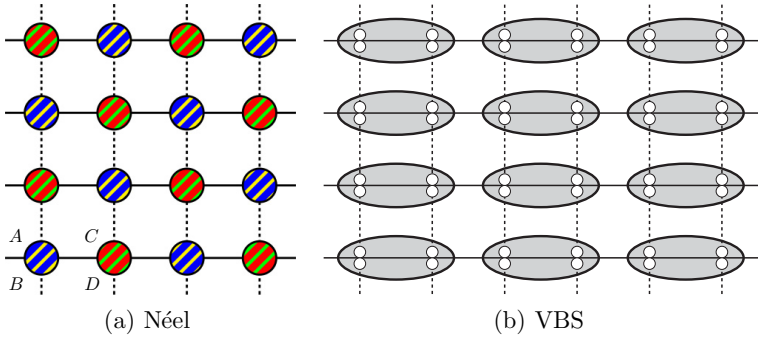




QMC finite size scaling
suggests spin LRO

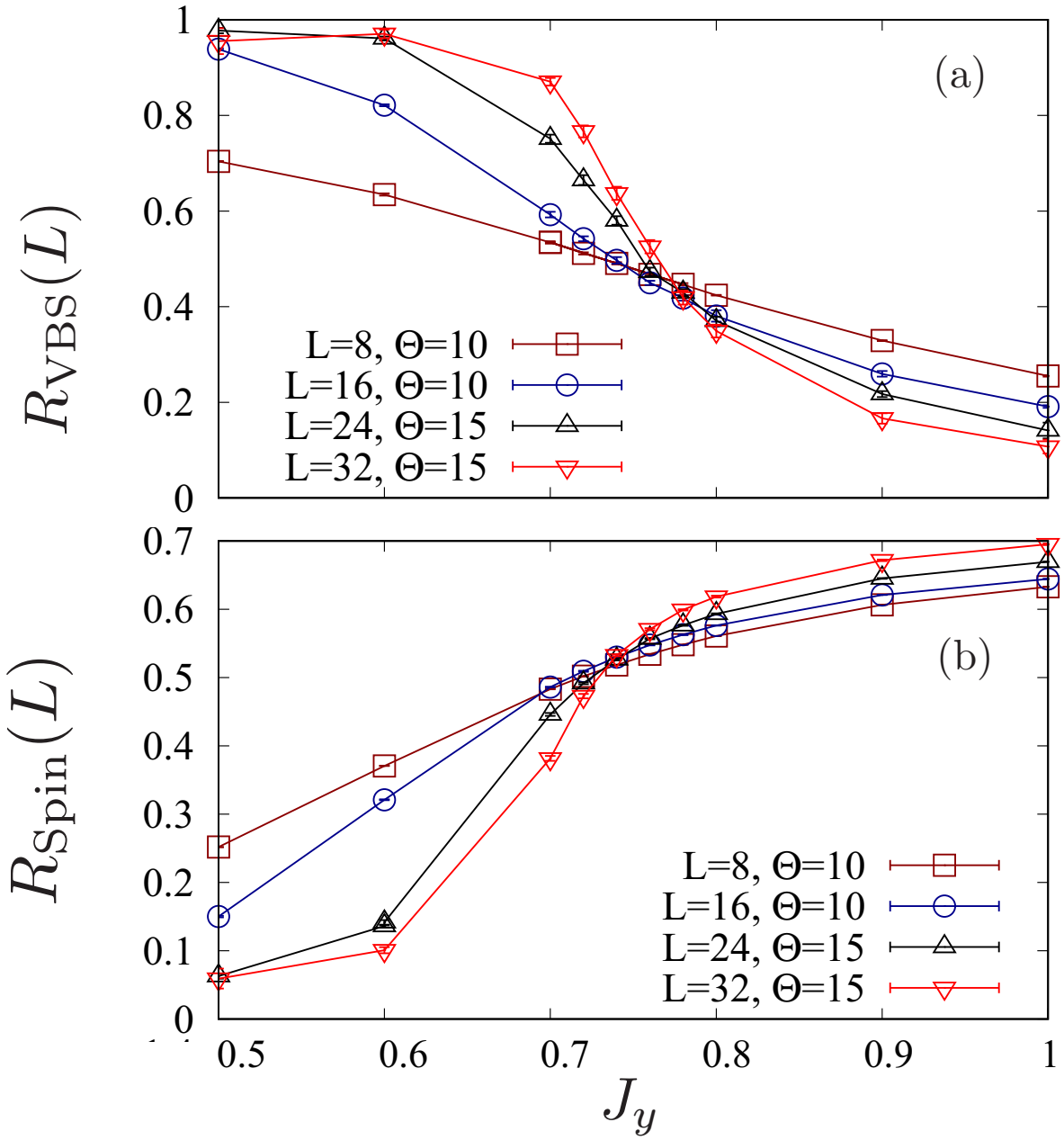
F. H. Kim, F. F. Assaad, K. Penc, F. Mila,
Phys. Rev. B **100**, 085103/1-8 (2019)

SU(4) 6-dim irrep on square lattice



$$R_{\rm Spin}(L) = 1 - \frac{S_{\rm Spin}(\mathbf{Q} - (0, 2\pi/L))}{S_{\rm Spin}(\mathbf{Q})}.$$

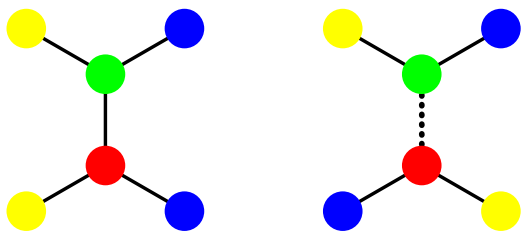
changing the anisotropy (decreasing the coupling between the chains), there is a phase transition between the VBS and ordered state — **deconfined quantum critical point** ?



Honeycomb, SU(4)

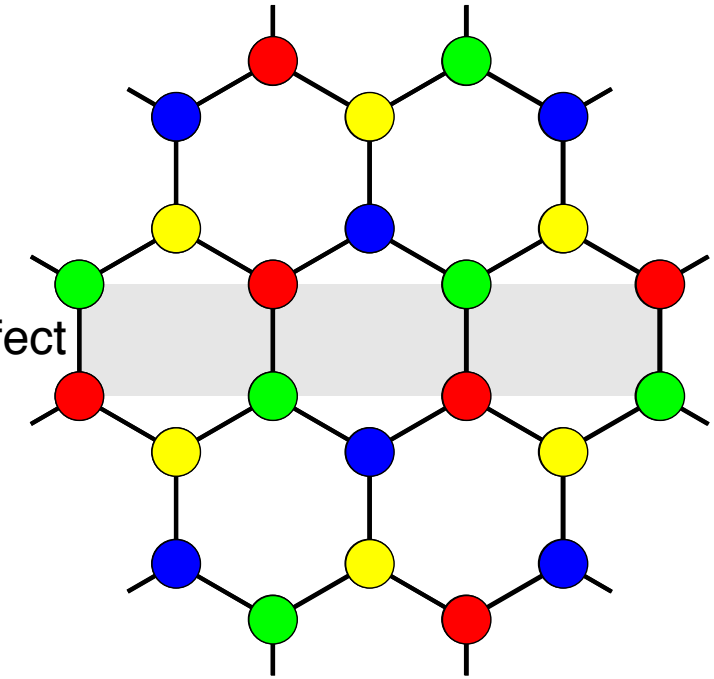
Lifting of the degeneracy in flavor wave theory

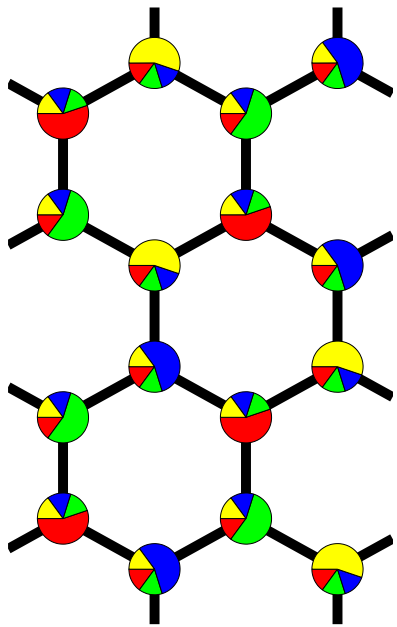
Basic building blocks:
nearest (mean field)
and
next nearest (fluctuations) neighbor
colors different.



Order by disorder does not work!

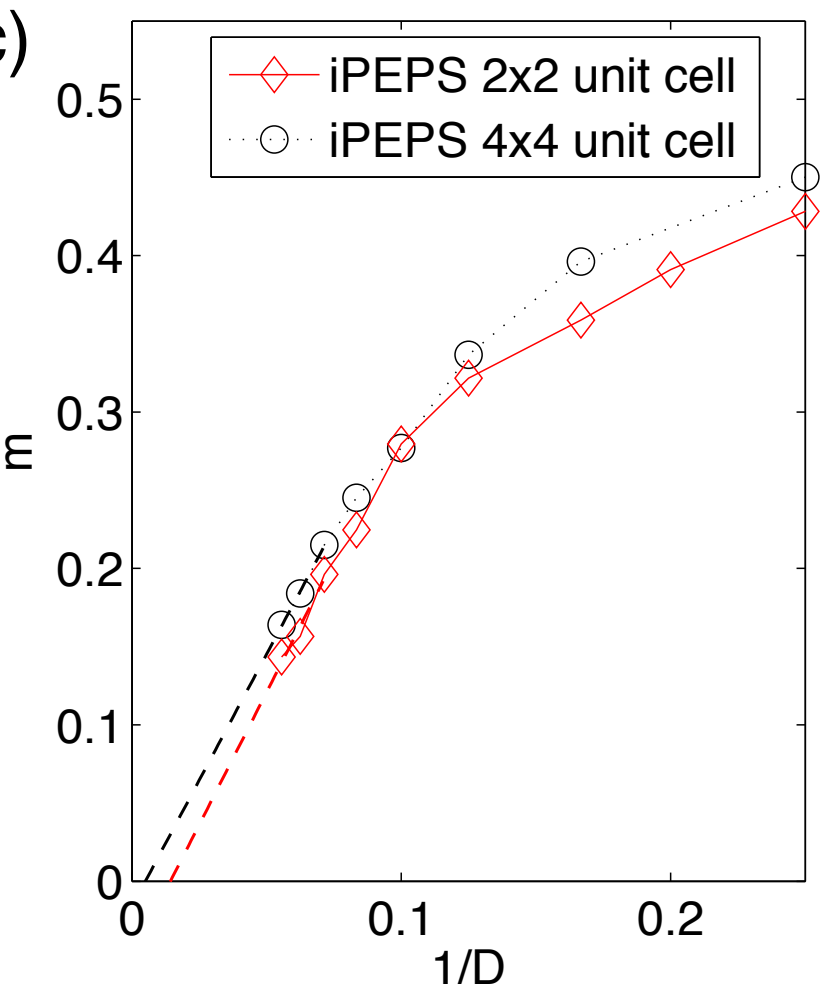
A linear defect

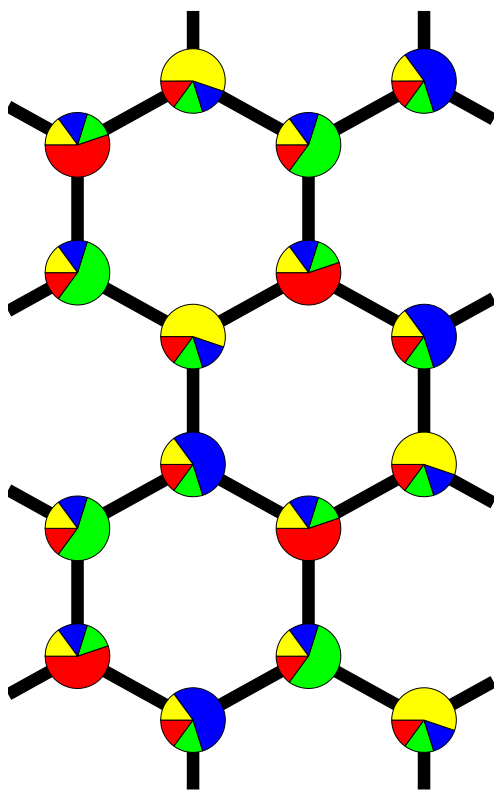




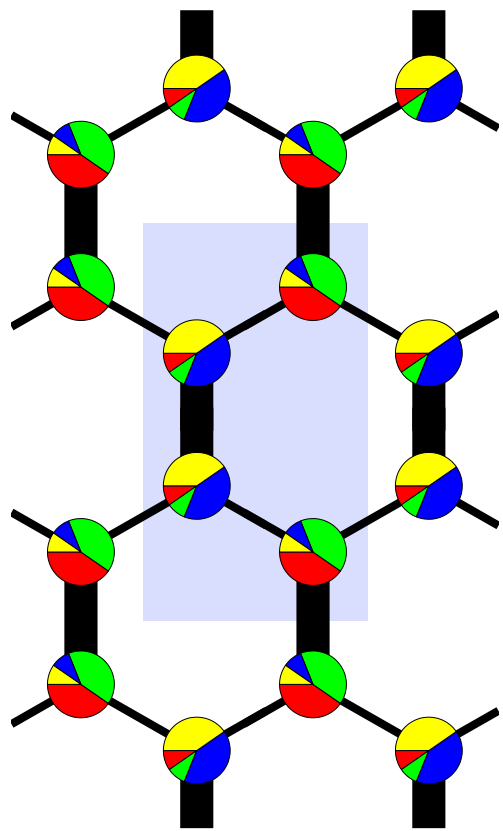
iPEPS,
D=6

iPEPS - local magnetization vanishes





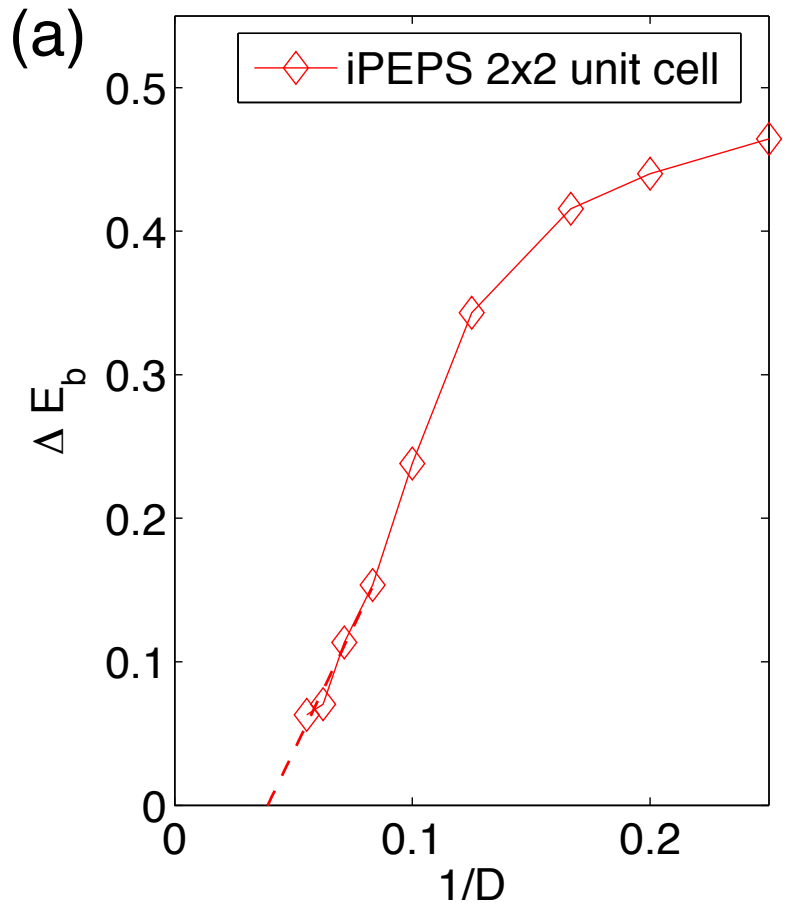
4x4 unit cell,
D=6

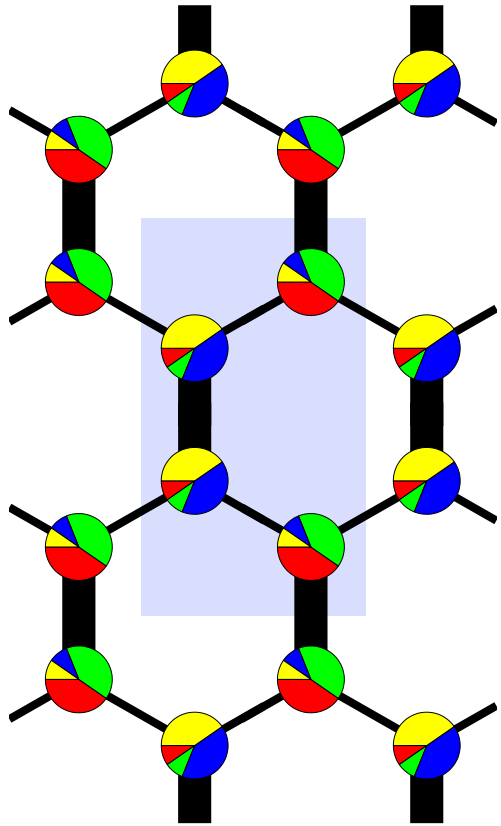


2x2 unit cell,
D=6

P. Corboz, M. Lajkó, A. M. Läuchli, K. Penc, F. Mila:
Phys. Rev. X **2**, 041013/1-11 (2012).

iPEPS - dimerization vanishes





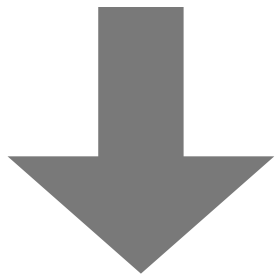
2x2 unit cell,
D=6

P. Corboz, M. Lajkó, A. M. Läuchli, K. Penc, F. Mila:
Phys. Rev. X **2**, 041013/1-11 (2012).

Summary of iPEPS results

- dimerization vanishes
  (actually no point group symmetry breaking)

- local magnetization vanishes
  (no SU(4) symmetry breaking)



spin-orbital liquid

How to characterize it?

Fermionic mean-field

we use the fermionic representation:

$$\mathcal{P}_{ij} = \sum_{\mu,\nu \in \text{colors}} f^\dagger_{\alpha,i} f_{\beta,i} f^\dagger_{\beta,j} f_{\alpha,j}$$

$$\mathcal{P}^{\text{MF}}_{ij} = \sum_{\alpha,\beta \in \text{colors}} \langle f_{\beta,i} f^\dagger_{\beta,j} \rangle f^\dagger_{\alpha,i} f_{\alpha,j}$$

$$= -\sum_{\alpha \in \text{colors}} t^\alpha_{ij} f^\dagger_{\alpha,i} f_{\alpha,j}$$

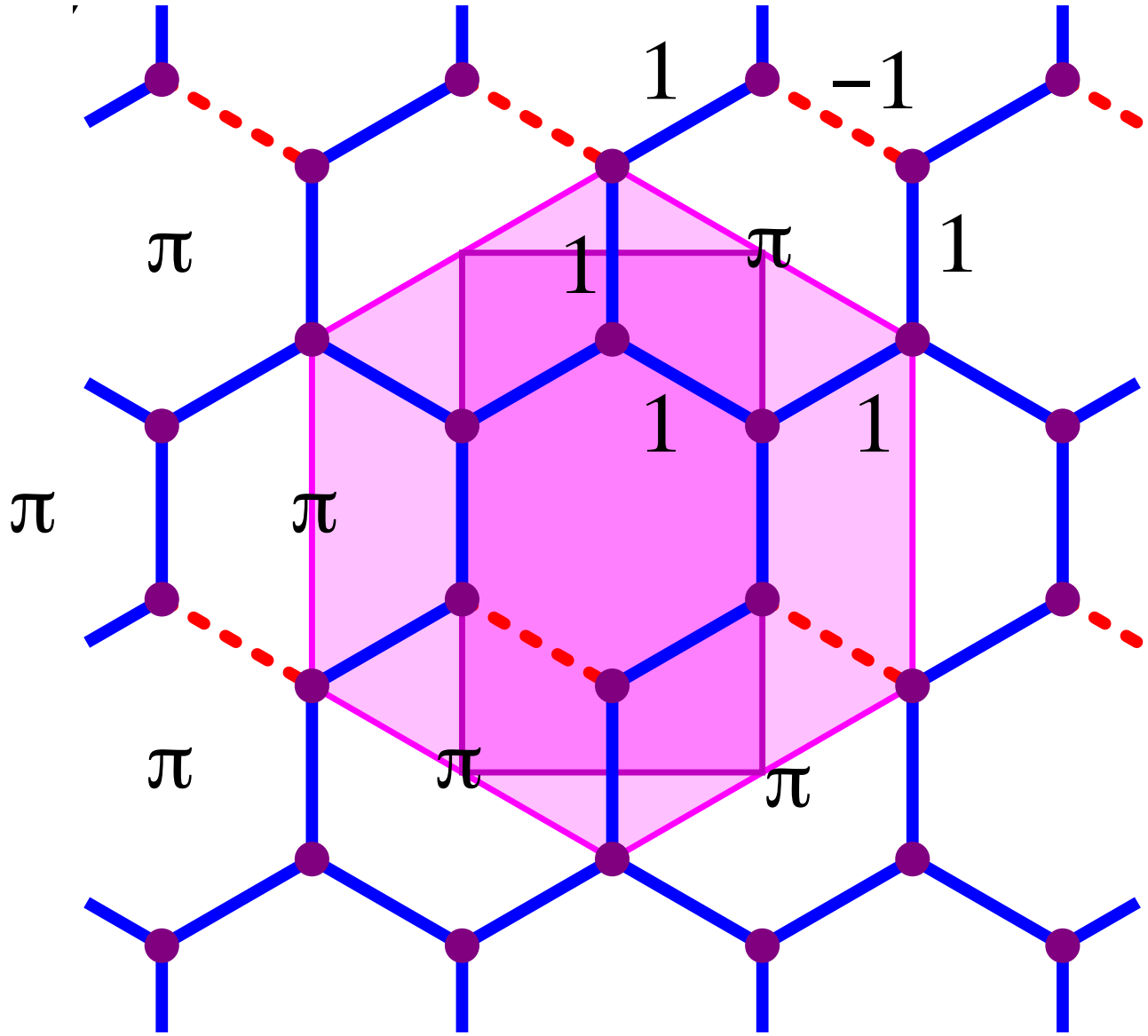
Mean-field decoupling of the fermionic Hamiltonian gives a hopping Hamiltonian and a variational wave function

$$|\Psi_{\text{vari}}\rangle = P_{\text{Gutzwiller}} |\Psi_{\text{FS}}\rangle$$

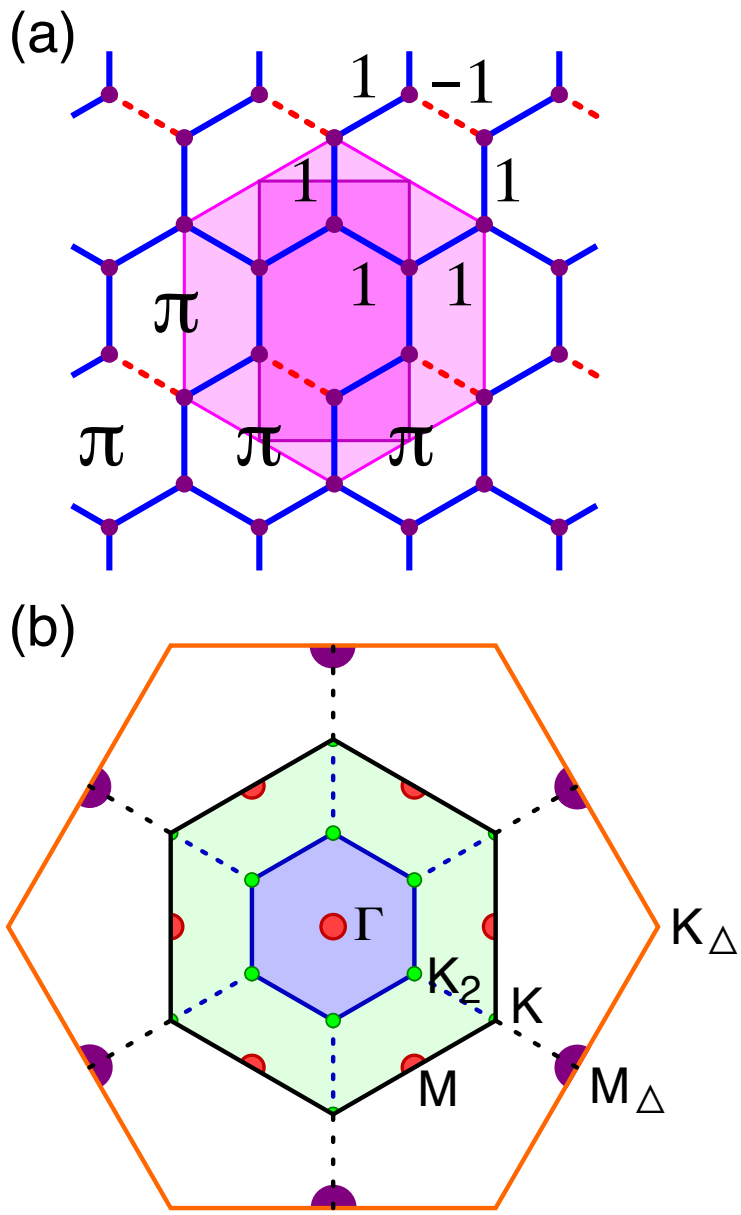
Using different Ansätze for the hoppings, we evaluate the expectation value of the Hamiltonian

$$E_{\text{vari}} = \frac{\langle \Psi_{\text{vari}} | \mathcal{H} | \Psi_{\text{vari}} \rangle}{\langle \Psi_{\text{vari}} | \Psi_{\text{vari}} \rangle}$$

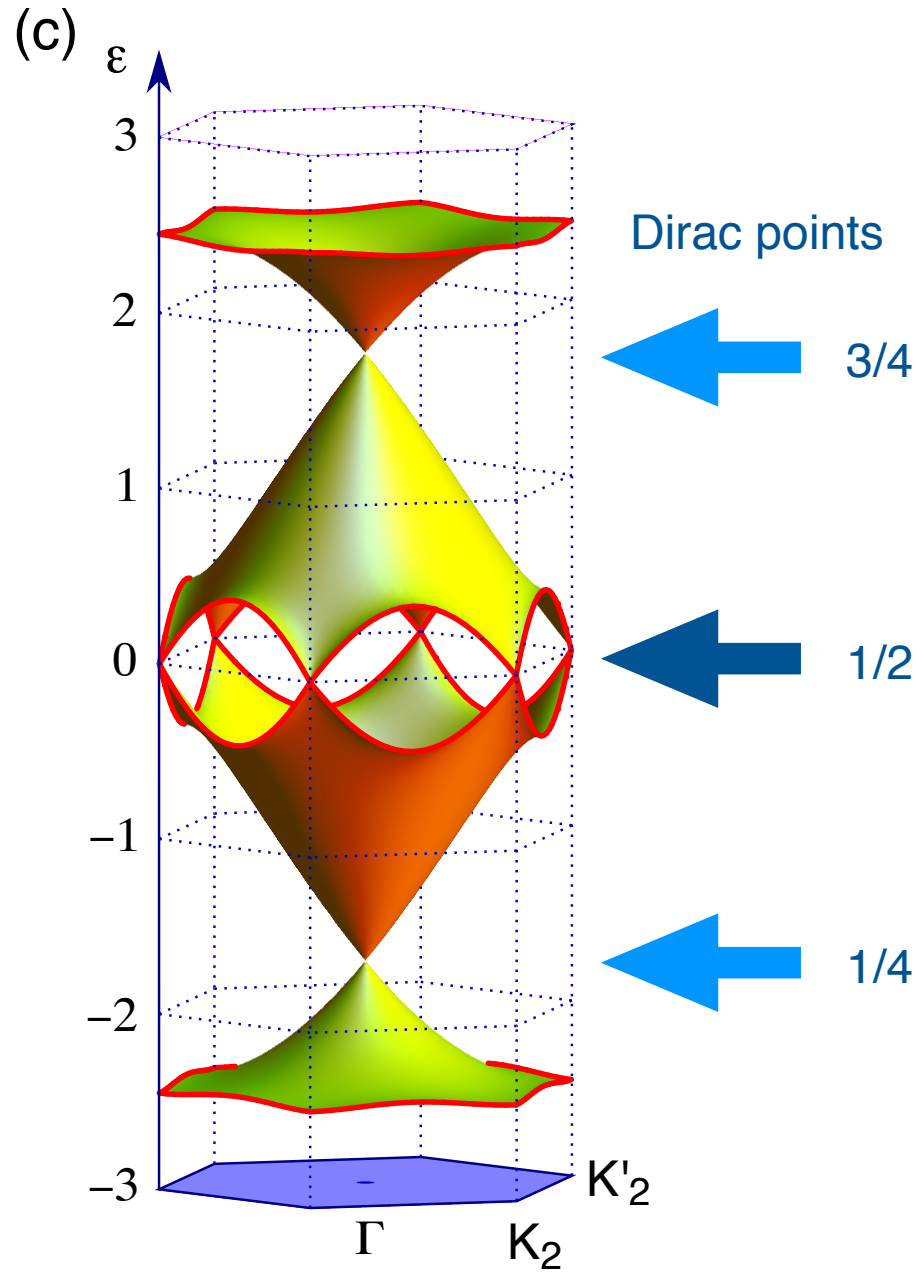
The fermionic wave function of the pi-flux state



The fermionic wave function of the pi-flux state



Brillouin zone

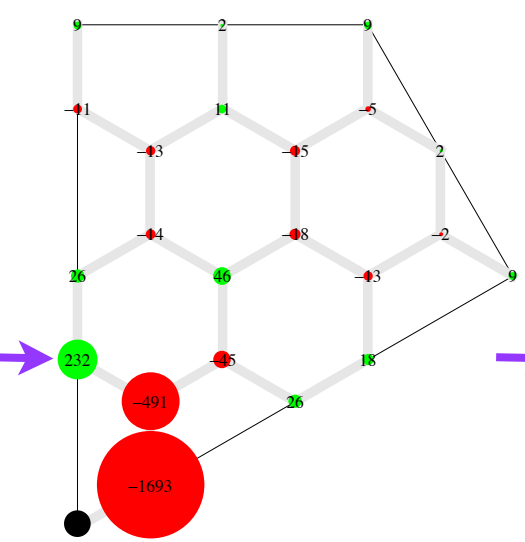


two-fold degenerate bands

96-site cluster - real space correlations from Gutzwiller projected wavefunction

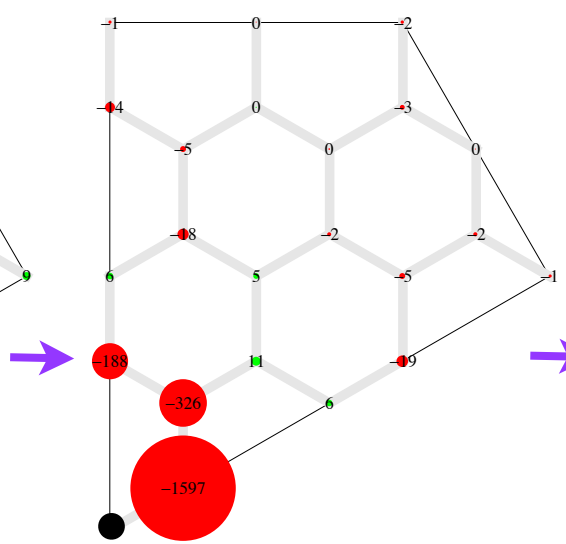
$$\langle S_\bullet S_\delta \rangle \propto \langle P_{\bullet,\delta} \rangle - \frac{1}{4}$$

red- AF, green - F



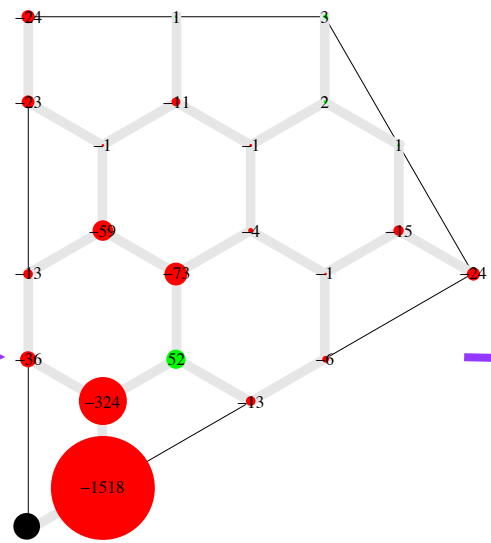
Pi-flux

E/N = –0.89466



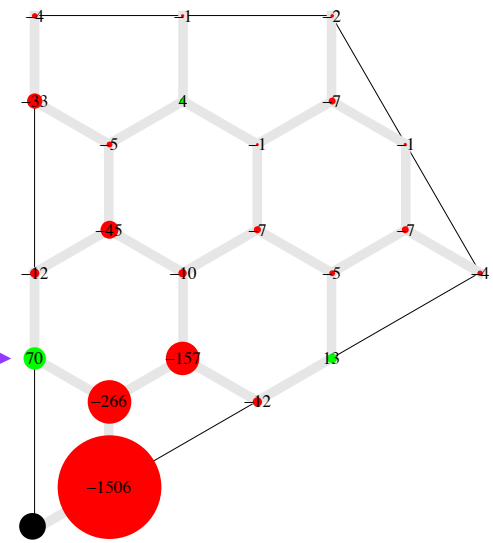
Majorana 0-flux

E/N = –0.822479



0-flux

E/N = –0.763304

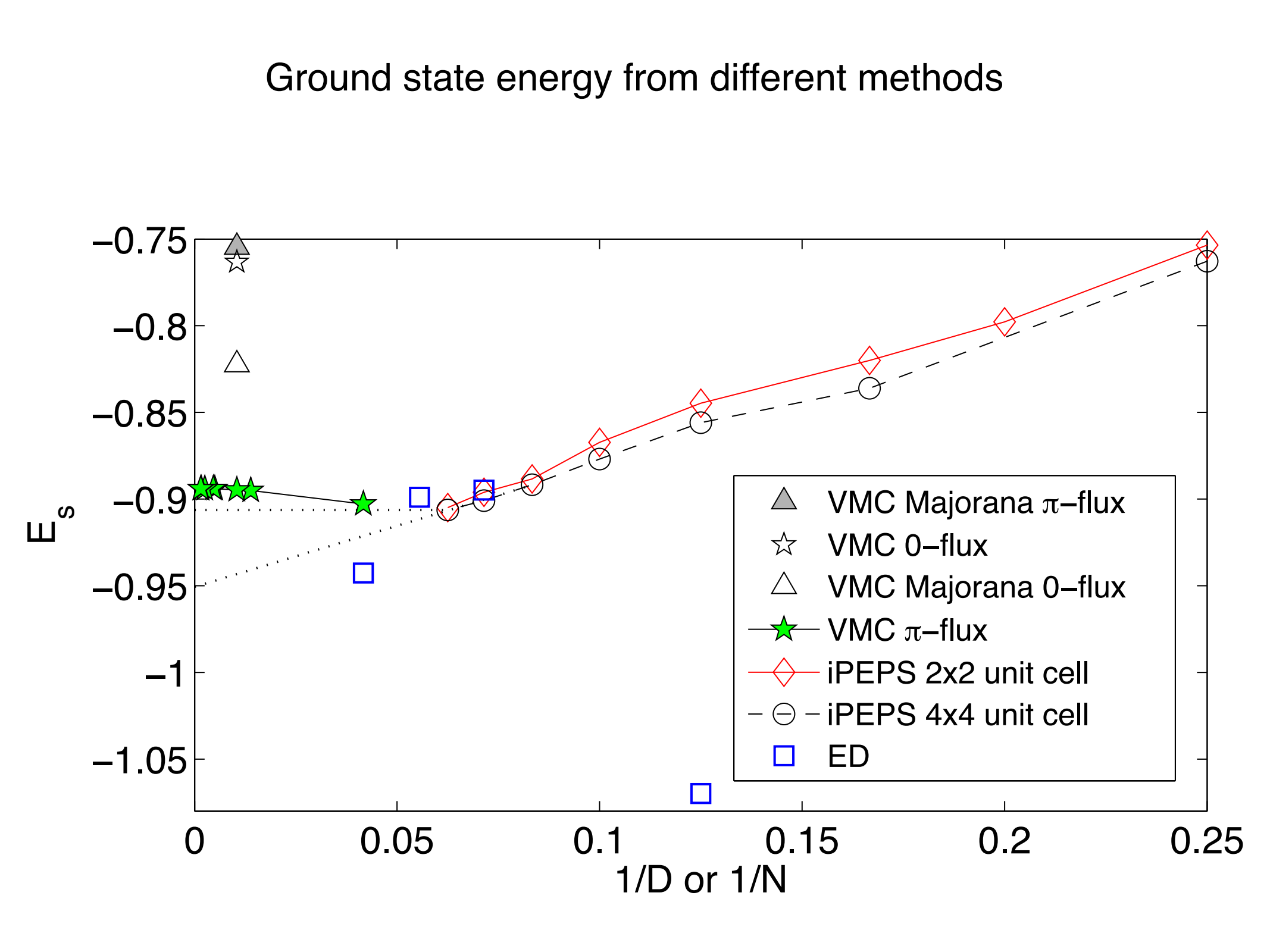


Majorana Pi-flux

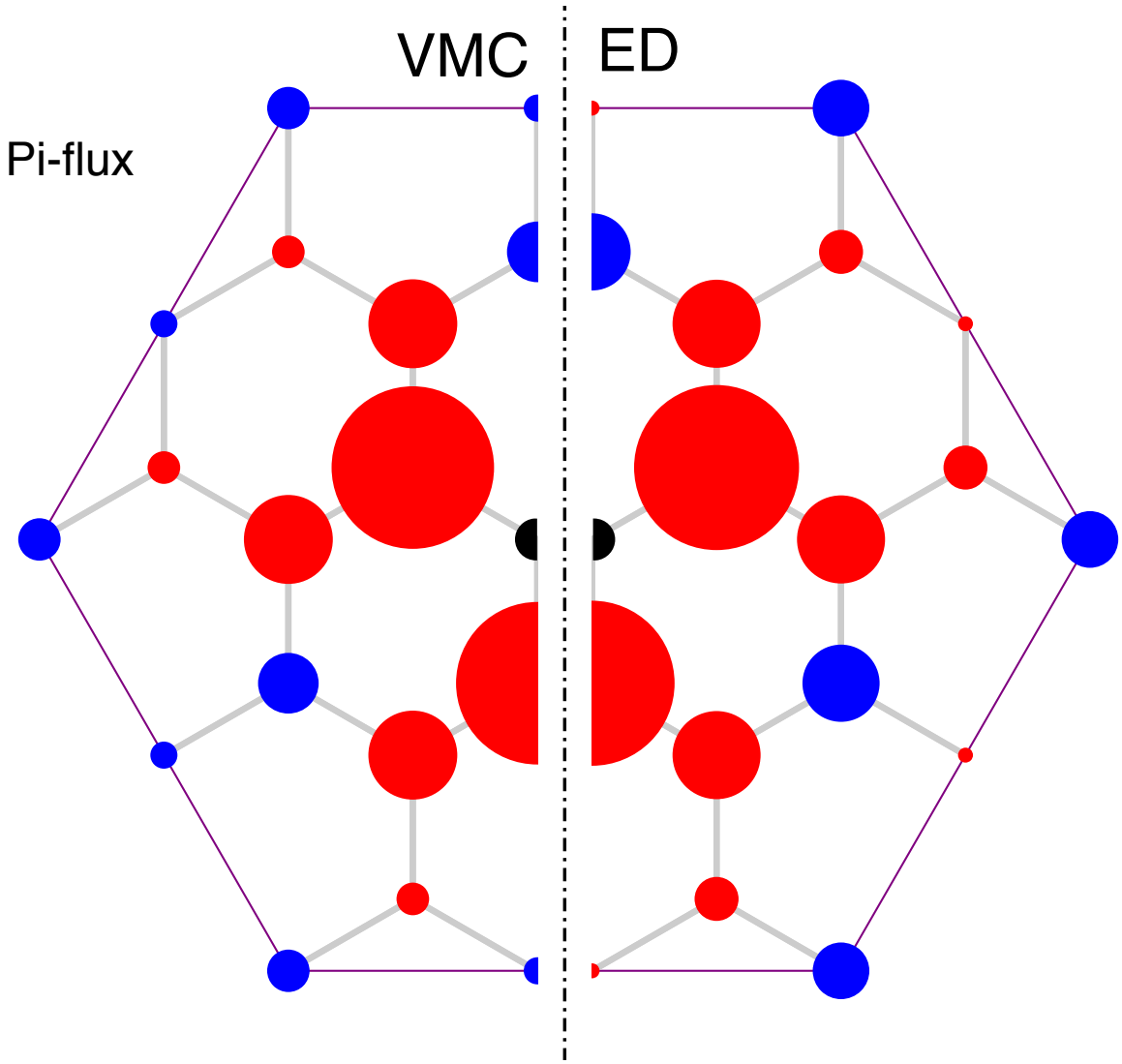
E/N = –0.754662

marked differences in
3rd neighbor correlations

Majoranna fermions for square lattice: F. Wang and A. Vishwanath, Phys. Rev. B **80**, 064413 (2009).


Ground state energy from different methods
−0.75
−0.8
−0.85
−0.9
−0.95
−1
−1.05
E_s
0
0.05
0.1
0.15
0.2
0.25
1/D or 1/N
VMC Majorana π−flux
VMC 0−flux
VMC Majorana 0−flux
VMC π−flux
iPEPS 2x2 unit cell
iPEPS 4x4 unit cell
ED

24-site cluster - real space correlations

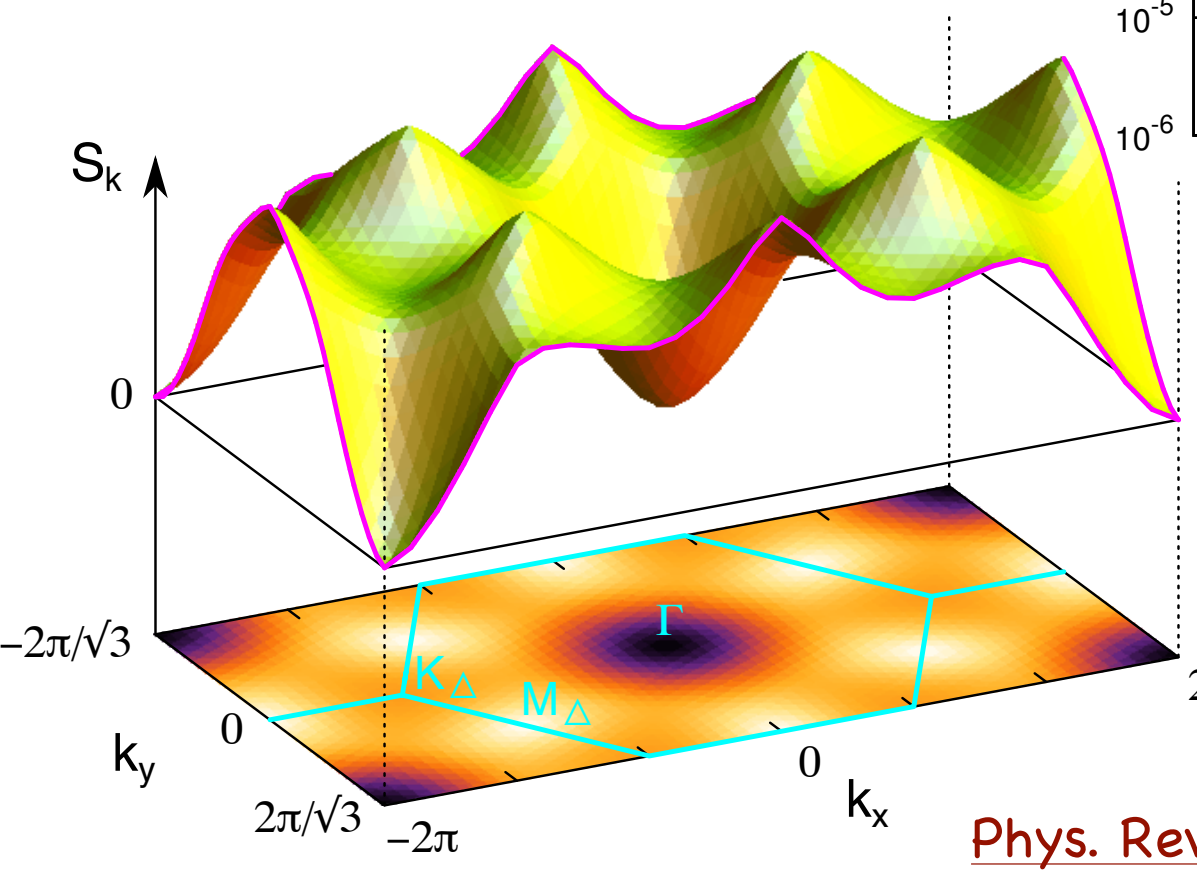


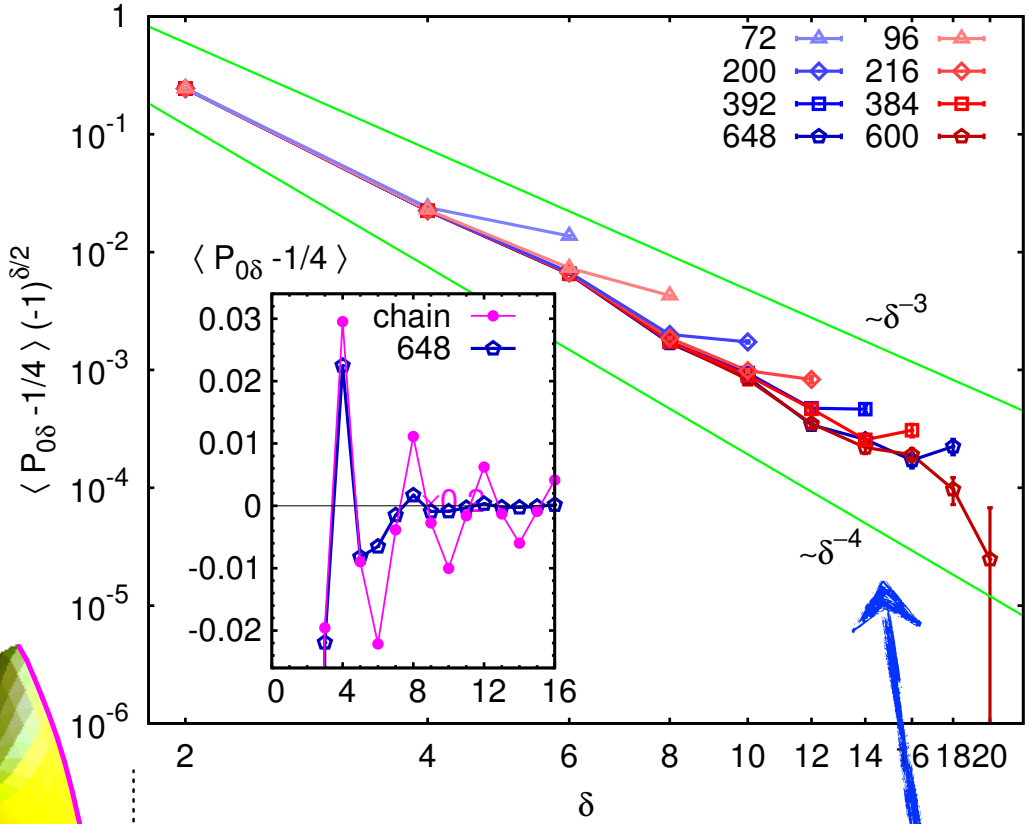
Dimension of the Hilbert space is $24!/(6!)^4 =$ 2 308 743 493 056
using symmetries makes it tractable

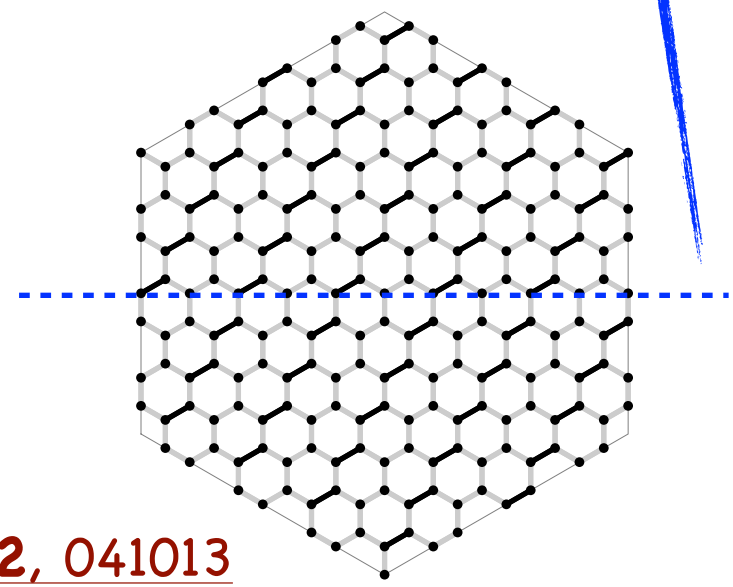
algebraic correlations
and structure factor
-typical for 1D critical system,
but not for the 2D ones

$$|\langle S_0 S_\delta \rangle| \propto \left| \langle P_{0,\delta} \rangle - \frac{1}{4} \right| \propto \delta^\eta$$

$$\eta \approx -3.5$$







Phys. Rev. X **2**, 041013

Suggestion

Model for the metal-insulator transition in graphene superlattices and beyond

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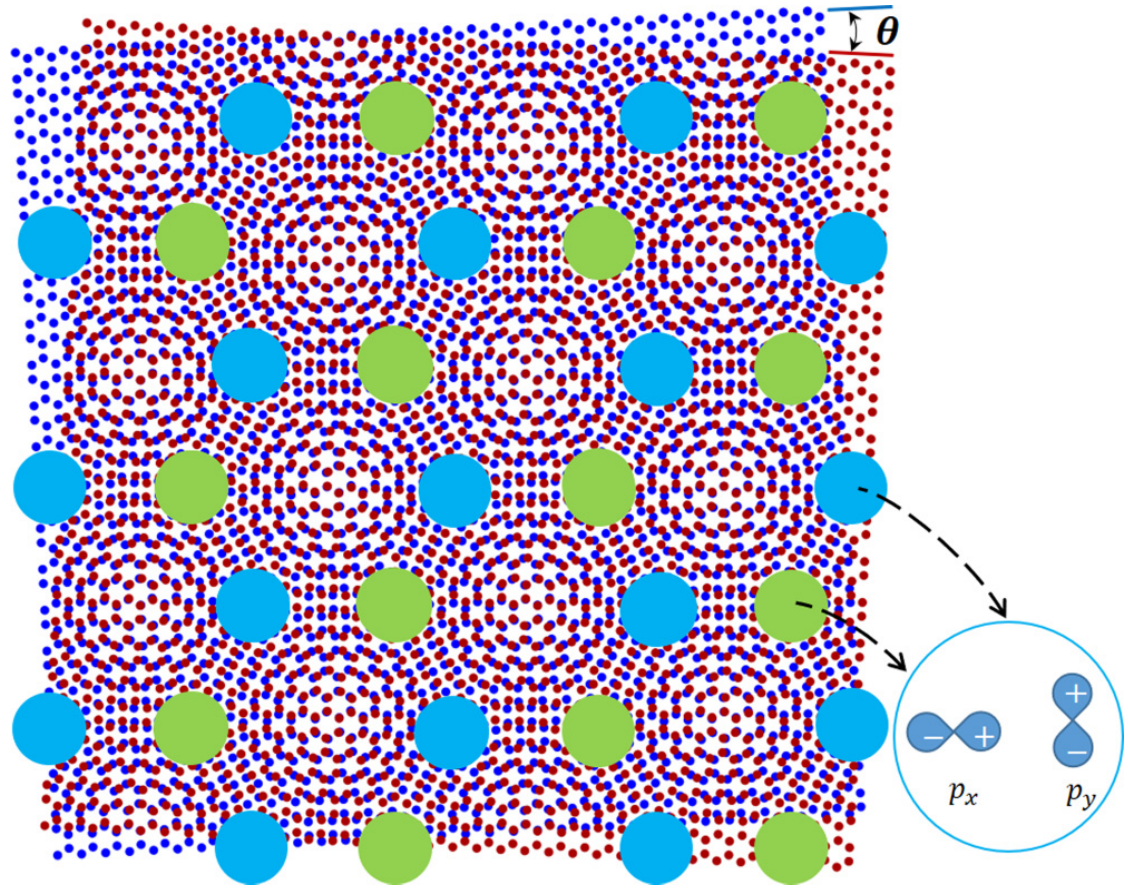
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

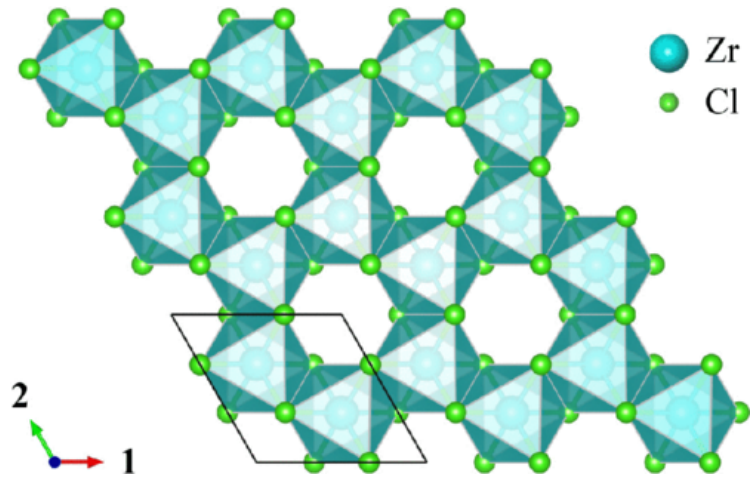
(Received 27 March 2018; published 3 July 2018)

We propose a two-orbital Hubbard model on an emergent honeycomb lattice to describe the low-energy physics of twisted bilayer graphene. Our model provides a theoretical basis for studying metal-insulator transition, Landau level degeneracy lifting, and unconventional superconductivity that are recently observed.



Emergent SU(4) Symmetry in α-$ZrCl_3$ and Crystalline Spin-Orbital Liquids

Masahiko G. Yamada,[1,*] Masaki Oshikawa,[1] and George Jackeli[2,3,†]



PHYSICAL REVIEW B **100**, 205131 (2019)

SU(4) Heisenberg model on the honeycomb lattice with exchange-frustrated perturbations: Implications for twistronics and Mott insulators

W. M. H. Natori,[1,2] R. Nutakki,[3] R. G. Pereira,[4] and E. C. Andrade[1]

Triangular with ring exchange, SU(N)

The model

Hubbard model with artificial flux:

$$H = -t\sum_{\langle i,j\rangle}\sum_{\alpha=1}^{N}(e^{\phi_{ij}}c^{\dagger}_{i,\alpha}c_{j,\alpha} + \text{H.c.}) + U\sum_{i,\alpha<\beta} n_{i,\alpha}n_{i_\beta}.$$

the effective low energy model includes ring exchange:

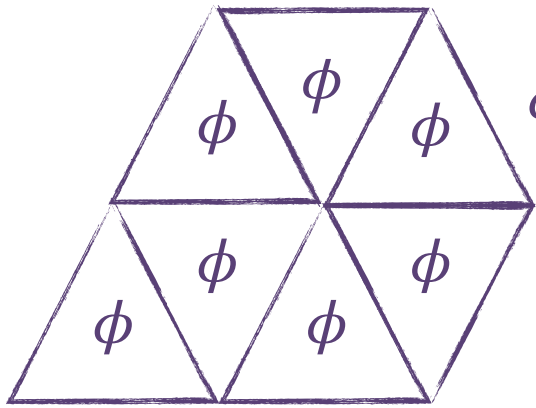
$$H = J\sum_{\langle i,j\rangle} P_{ij} + K_3\sum_{(i,j,k)}(e^{i\Phi}P_{ijk} + \text{H.c.}),$$

$$J = 2t^2/U, \quad K_3 = 6t^3/U^2$$

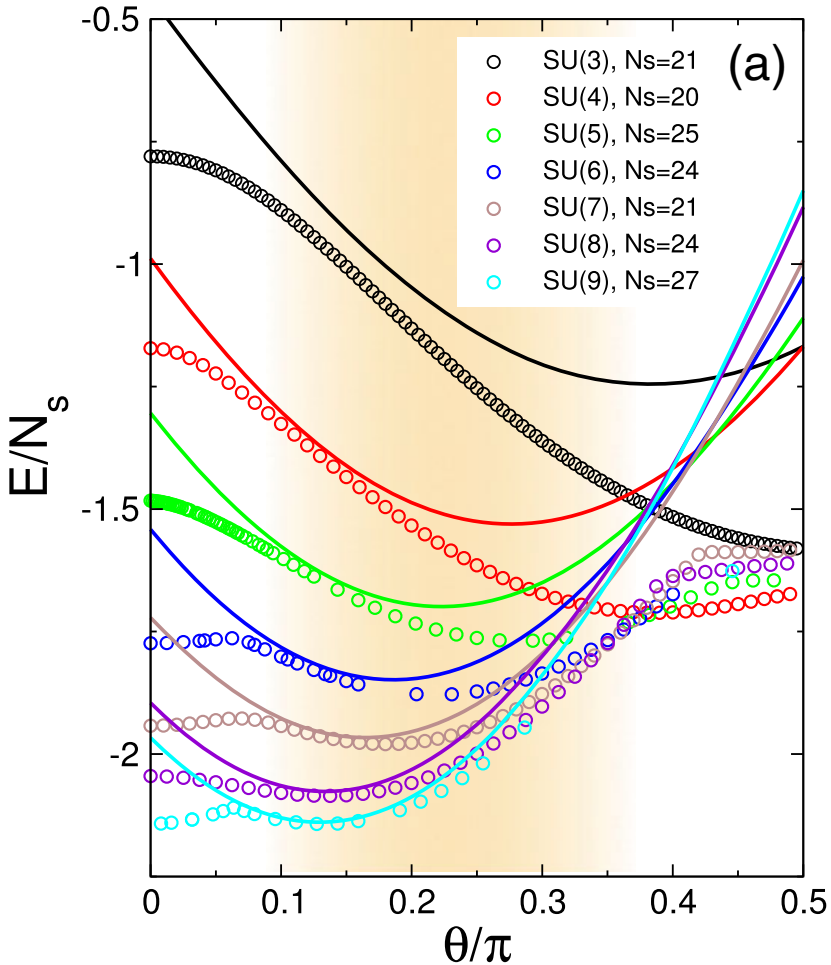
The ground state is chiral: a gaped featureless liquid which breaks time reversal invariance

P. Nataf, M. Lajkó, A. Wietek, K. Penc, F. Mila, A. M. Läuchli:
Chiral spin liquids in triangular lattice SU(N) fermionic Mott insulators with artificial gauge fields
Phys. Rev. Lett. **117**, 167202/1-6 (2016)

Exact diagonalization vs. variational Monte Carlo



$\phi=\pi/N$



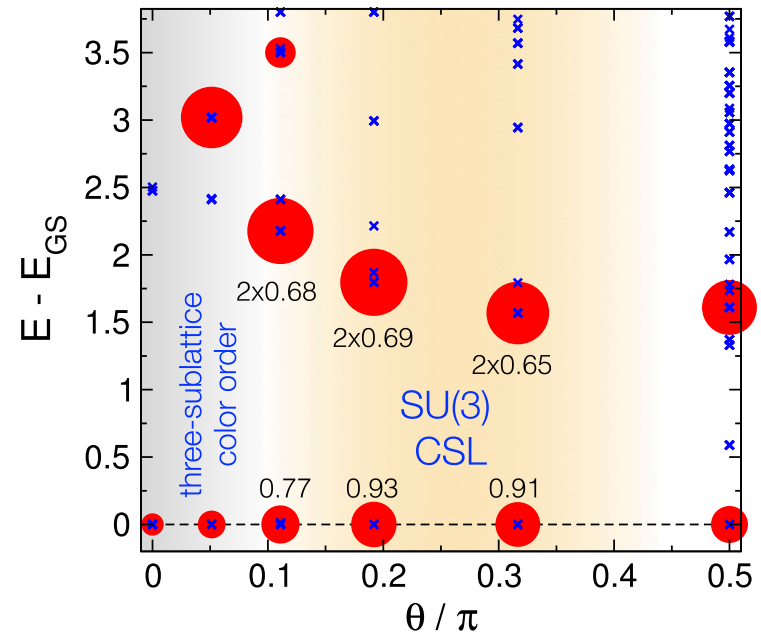


FIG. 3. Summed squared overlaps of the VMC model wave functions with ED eigenstates for $N = 3$ and $N_s = 12$. The blue crosses denote ED eigenstates, while the area of the filled red circles denotes the total squared overlap on those eigenstates. Around $\theta/\pi \approx 0.25$, the summed overlaps on the lowest three ED eigenstates (degeneracy $1 + 2$) account for over 75% of the total weight, while the ground state alone is at 90%.

P. Nataf, et al., Phys. Rev. Lett. 117, 167202 (2016)

N-fold ground state degeneracy

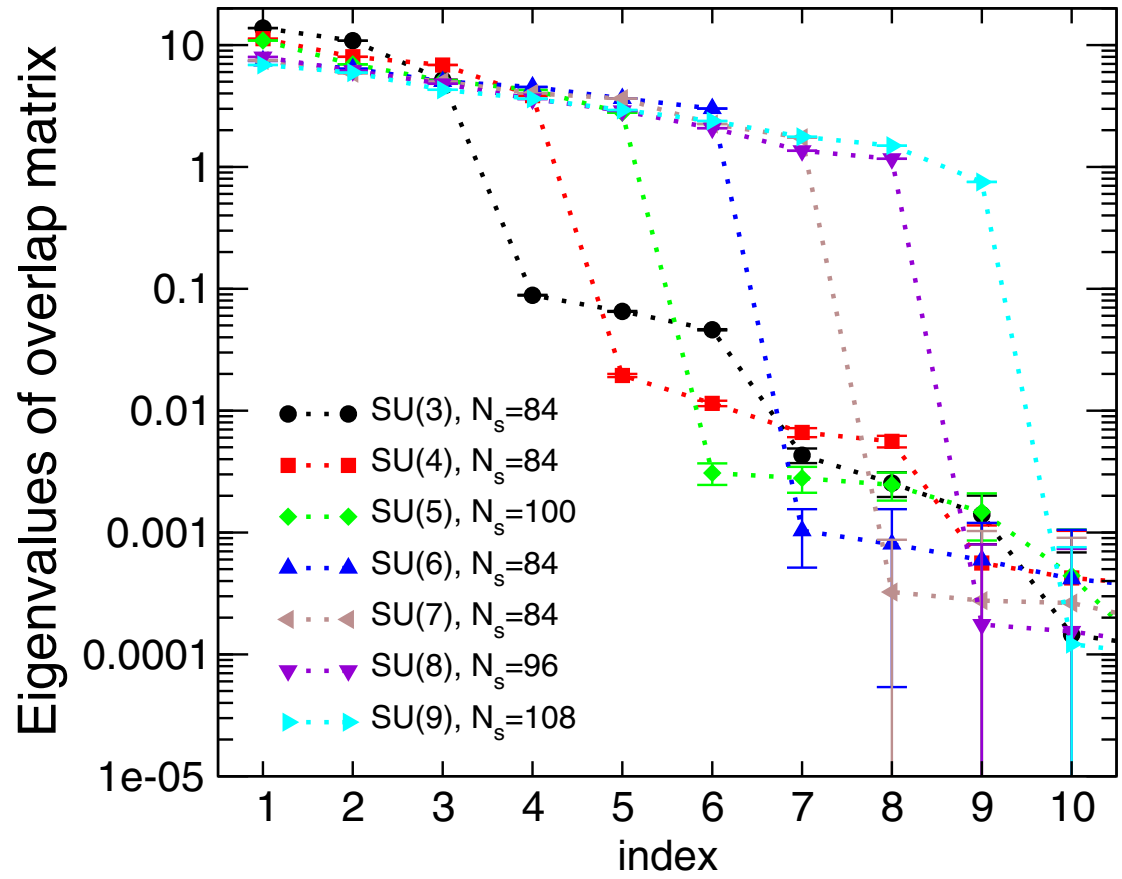


FIG. 2. VMC ground space degeneracy: ordered sequence of eigenvalues of the overlap matrix of Gutzwiller projected wave functions with 30 random values of threaded flux. The overlap matrix has precisely N large eigenvalues for an SU(N) CSL.

P. Nataf, et al., Phys. Rev. Lett. 117, 167202 (2016)

spectrum of the edge modes

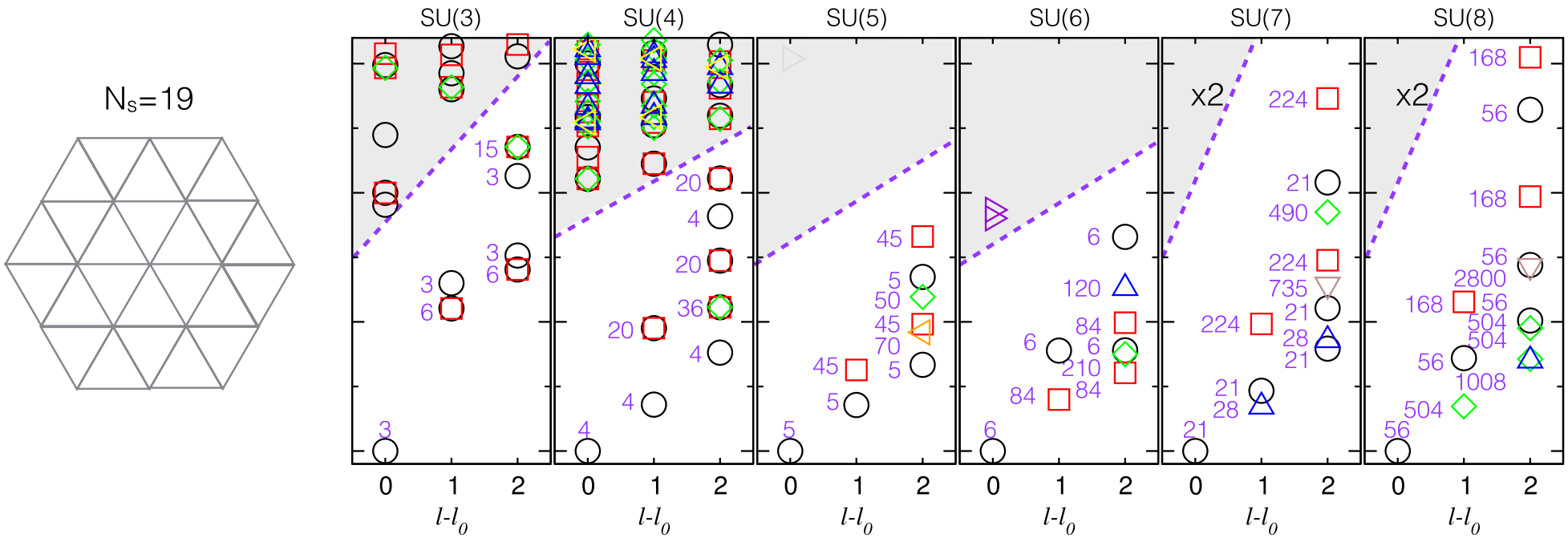


FIG. 4. Edge states in SU(N) CSLs: the leftmost panel displays the $N_s = 19$ site triangular cluster with open boundary conditions used. In the various other panels we exhibit the low energy spectrum as a function of the angular momentum around the central site (l_0 denotes the ground state angular momentum). The chiral edge states are clearly visible, with a characteristic SU(N) multiplet structure, which corresponds to a particular sector of a chiral $SU(N)_1$ Wess-Zumino-Novikov-Witten conformal field theory. The analytical predictions are indicated by the dimensions of the SU(N) multiplets and can be found in Table I of the Supplemental Material [37].

P. Nataf, et al., Phys. Rev. Lett. 117, 167202 (2016)

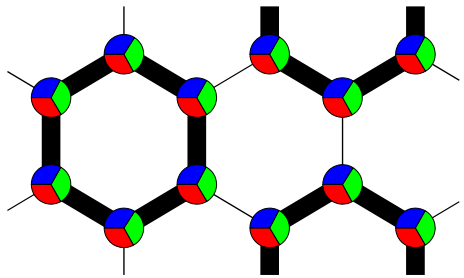
SU(2)

SU(3)

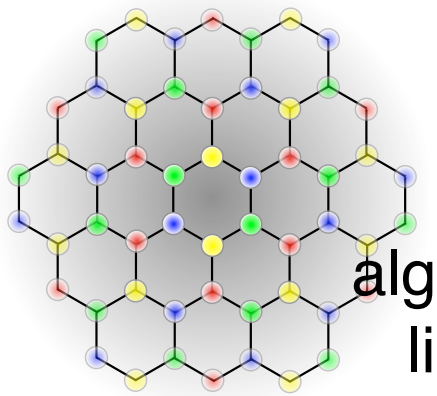
SU(4)

honeycomb

Neel

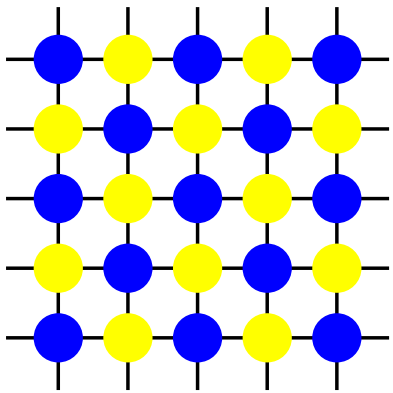


plaquette

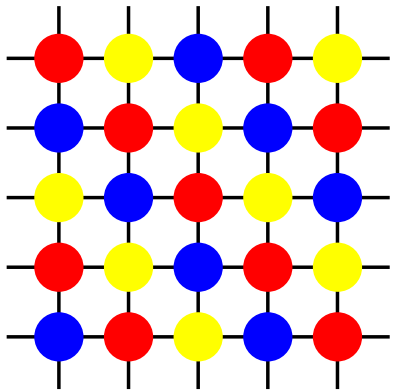


algebraic liquid

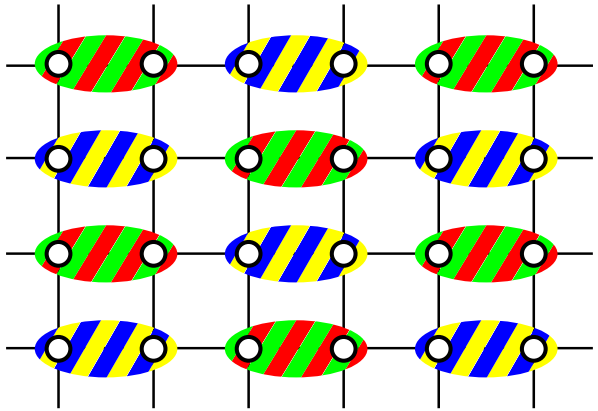
square



happy

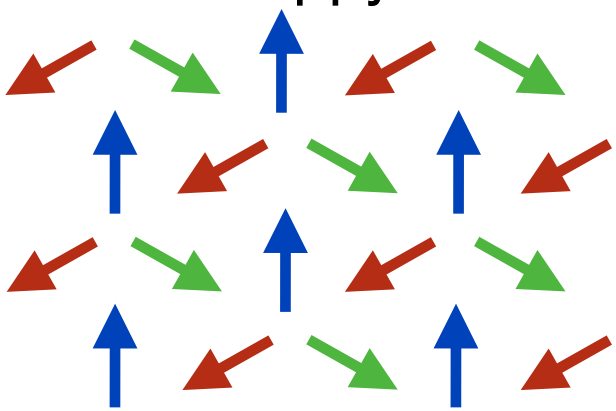


fluctuation stabilized

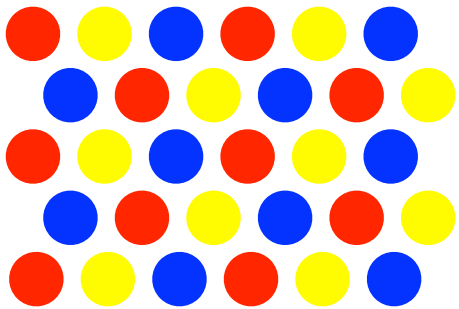


dimerization+Neel

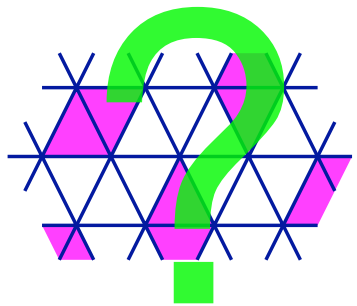
triangular



frustrated



happy



resonating liquid

Thank you for your attention!