Knot a Problem!

09.04.21

New Symmetries & Insights in Physics from Knots

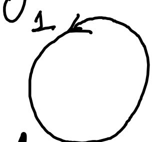
from the

University of Warsaw

talk given at

Wigner FK RMI Elméleti Szemináriuma

0_1

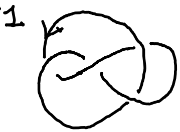


Hélder Larraguível

3_1

Jakub Jankowski

4_1



Piotr Kucharski

5_1

Dmitry Noshchenko

5_2

Piotr Sułkowski

- Do you want to compute the part. fn. for the Ising model?

- Do you want to compute the part. fn. for the Ising model?
- Do you want to build a noise-proof quantum computer?

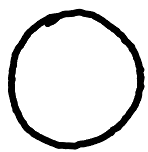
- Do you want to compute the part. fn. for the Ising model?
- Do you want to build a noise-proof quantum computer?
- Do you want to derive exact results in Quantum Field Theory?

- Do you want to compute the part. fn. for the Ising model?
- Do you want to build a noise-proof quantum computer?
- Do you want to derive exact results in Quantum Field Theory?

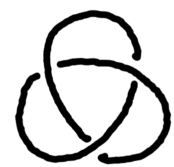
Why Knot?

I Guess Knot

Unknot

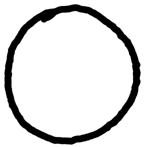


Trefoil

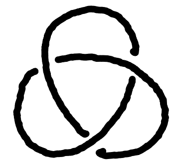


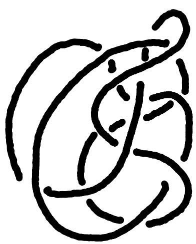
I Guess Knot

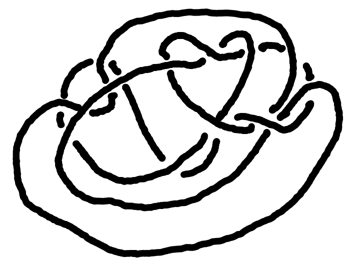
Unknot



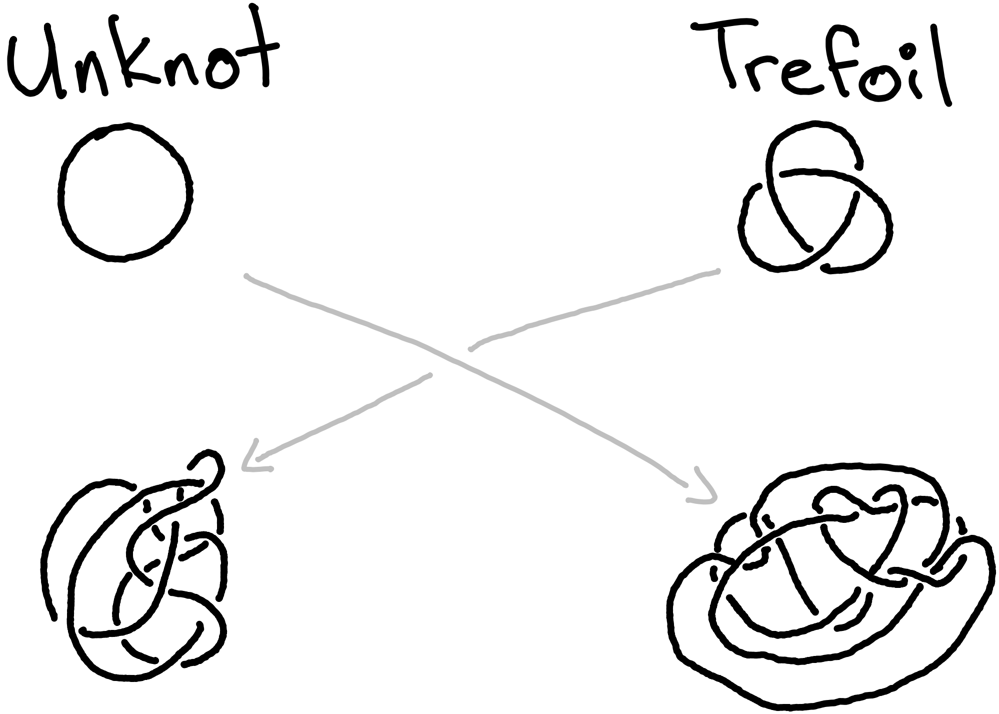
Trefoil







I Guess Knot

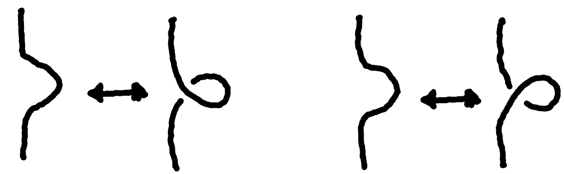


Homework, verify it.

Reidemeister Moves

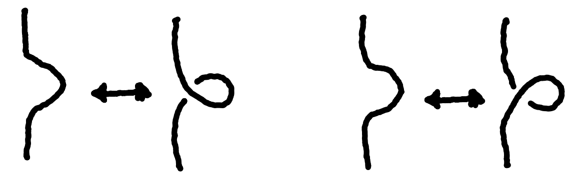
Reidemeister Moves

Type I

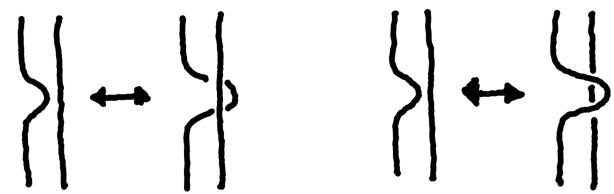


Reidemeister Moves

Type I

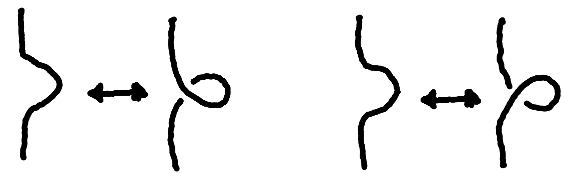


Type II

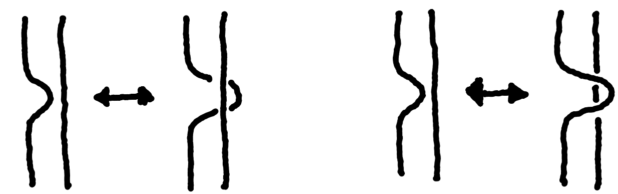


Reidemeister Moves

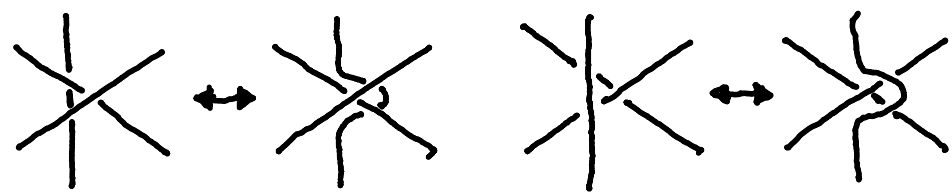
Type I



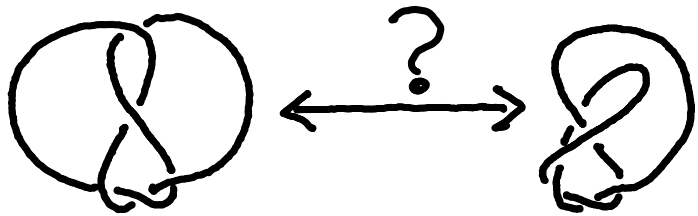
Type II



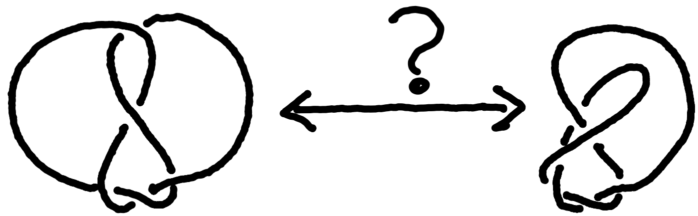
Type III

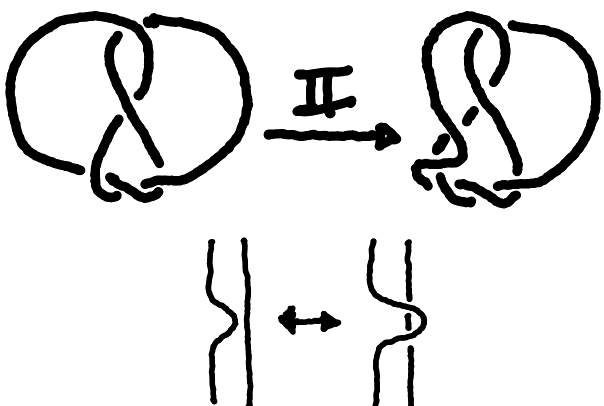


Reidemeister Moves

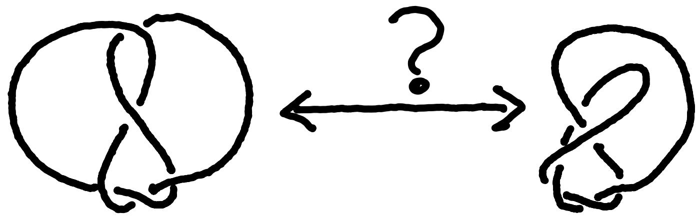


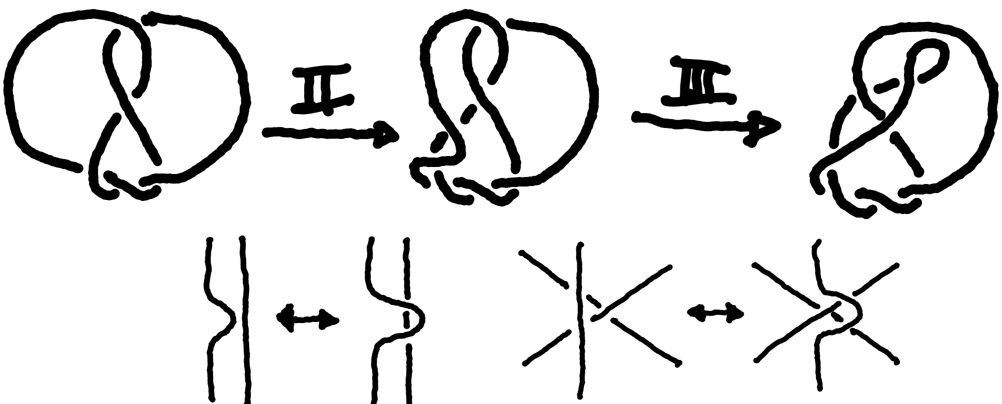
Reidemeister Moves



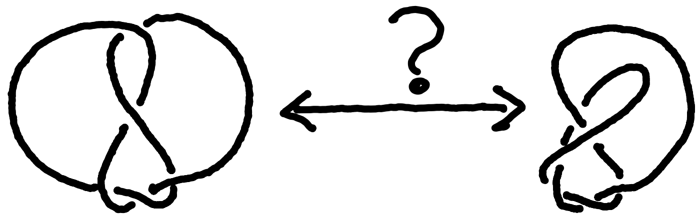


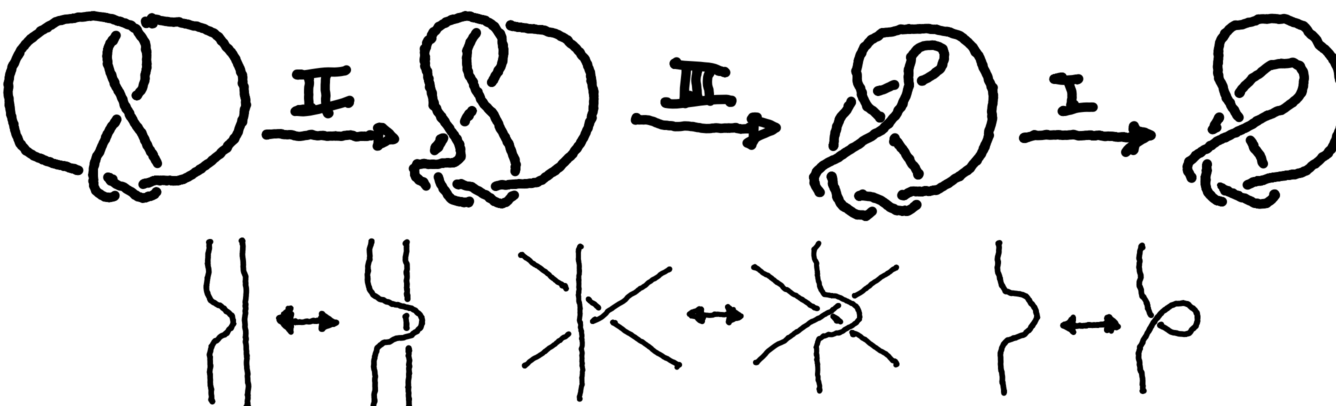
Reidemeister Moves





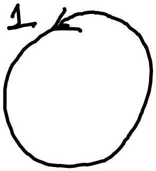
Reidemeister Moves





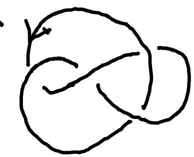
Knot Invariants

0_1

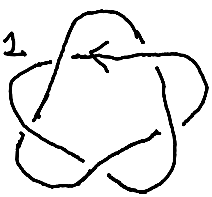


3_1

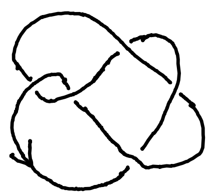
4_1



5_1



5_2



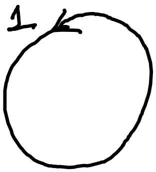
Knot Invariants

"Skein Rerations"

$$q\,P(\;) - q^{-1}\,P(\;) = a\,P(\;)$$

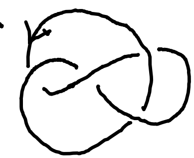
$$P(\bigcirc) = 1$$

0_1

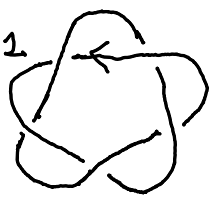


3_1

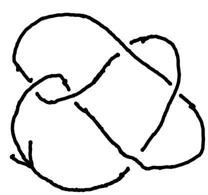
4_1



5_1



5_2



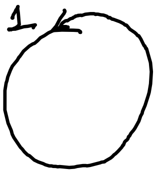
Knot Invariants

"Skein Rerations"

$$q\,P(\times) - q^{-1}\,P(\times) = a\,P(\,)(\,)$$

$$P(\bigcirc) = 1$$

0_1

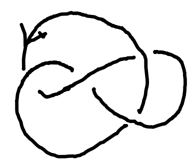


$P = 1$

3_1

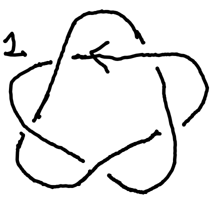
$P = aq - a^2 + aq$

4_1



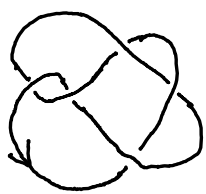
$P = 1 - q + a^{-1} + a - q$

5_1



$P = a^2 q^{-2} - a^3 q^{-1} + a^2 - a^3 q + a^2 q^2$

5_2



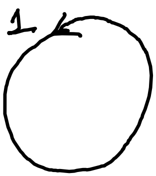
$P = a q^{-1} + a^2 q^{-1} - a - a^2 - a^3 + a^2 q + a q$

Knot Invariants

"Skein Rerations"

$$P(\bigcirc) = 1$$

0_1



$P = 1$

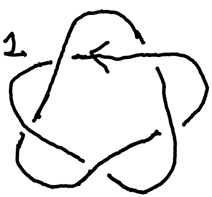
3_1

$P = aq - a^2 + aq$

4_1

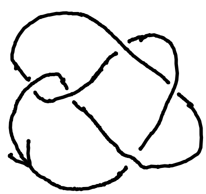
$P = 1 - q + a^{-1} + a - q$

5_1



$P = a^2 q^{-2} - a^3 q^{-1} + a^2 - a^3 q + a^2 q^2$

5_2



$P = a q^{-1} + a^2 q^{-1} - a - a^2 - a^3 + a^2 q + a q$

$P(a,q) = \text{HOMFLY-PT}$

$P(a=1, q) = \text{Jones}$

Remember, remember...

Remember, remember...

To distinguish knots we can compute their HOMFLY-PT polynomial using the skiens relations. These polynomials are invariant under Reidemeister moves.

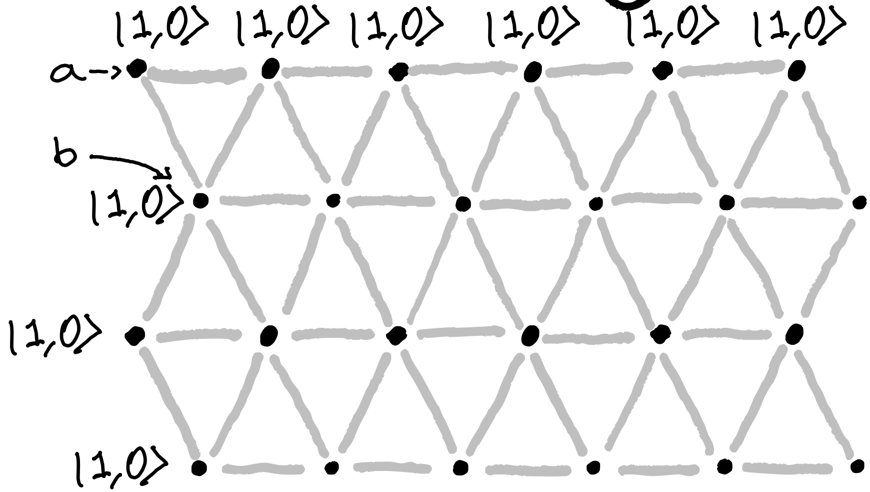
Remember, remember...

To distinguish knots we can compute their HOMFLY-PT polynomial using the skiens relations. These polynomials are invariant under Reidemeister moves.

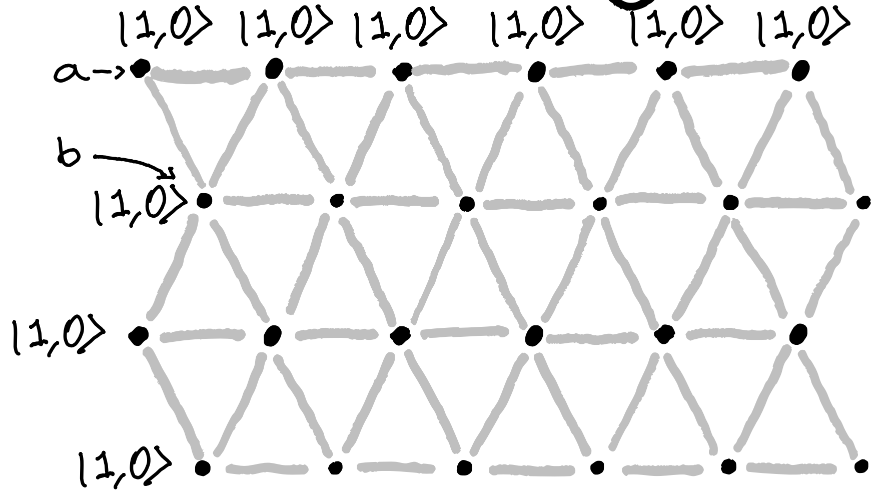
3_1

$$P = aq - a^2 + aq$$

Knots in my Potts



Knots in my Potts



Ising
$|1,0\rangle$

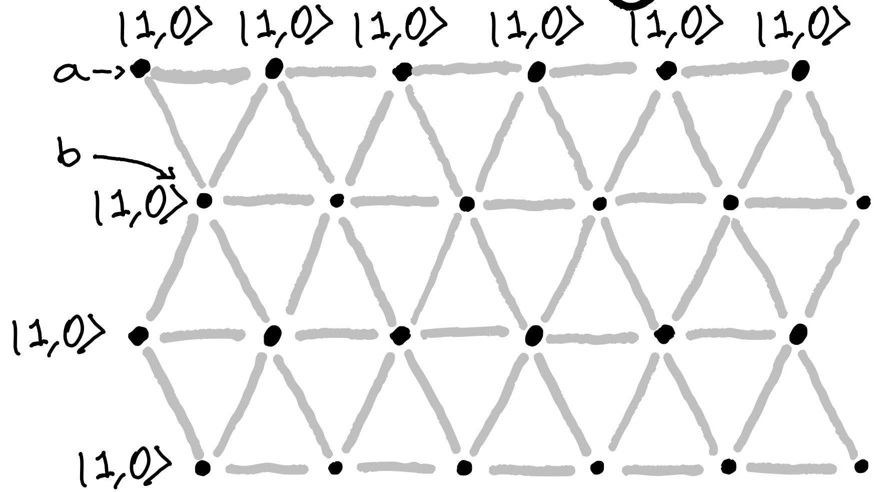
Interaction (Edges)

$$E(a,b) = \begin{cases} 1 & |a\rangle = |b\rangle \\ 0 & |a\rangle \neq |b\rangle \end{cases}$$

Partn fn.

$$Z = \sum_{\text{All states}} \exp\left[-\frac{1}{KT} \sum_{\text{All interacts.}} E \right]$$

Knots in my Potts



Ising
$|1,0\rangle$

Potts
$|0,1,\ldots,q\rangle$

Interaction (Edges)

$$E(a,b)=\begin{cases}1 & |a\rangle=|b\rangle \\ 0 & |a\rangle\neq|b\rangle\end{cases}$$

Partn fn.

$$Z=\sum_{\text{All states}} \exp\left[-\frac{1}{KT}\sum_{\text{All interacts.}} E\right]$$

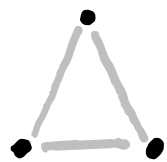
Knots in my Potts

Ising

$|1,0\rangle$

Potts

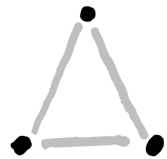
$|0,1,\ldots,q\rangle$



Partn fn.

$$Z = \sum_{\text{All states}} \exp\left[-\frac{1}{kT}\sum_{\text{All interacts.}} E\right]$$

Knots in my Potts



Ising

$|1,0\rangle$

Potts

$|0,1,\ldots,q\rangle$

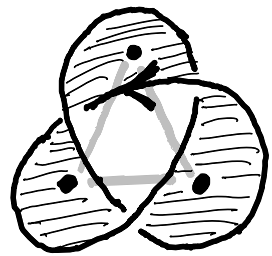
Partn fn.

"Jones"

$$Z = \sum_{\text{All states}} \exp\left[-\frac{1}{kT}\sum_{\text{All interacts.}} E\right] \propto \underbrace{P_{3_1}}_{\text{Trefoil}}(q)$$

(Suitable change of variables)

Knots in my Potts



Ising
$|1,0\rangle$

Potts
$|0,1,\ldots,q\rangle$

Partn fn.

$$Z = \sum_{\text{All states}} \exp\left[-\frac{1}{KT} \sum_{\text{All interacts.}} E\right] \propto \underset{\text{Trefoil}}{\underbrace{P_{3_1}}}(q)$$

"Jones"

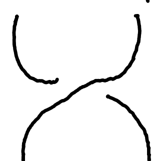
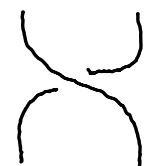
(Suitable change of variables)

Knotty Anyons

In 3+1 diminsions

Pauli's Eclusion Principle

Particles

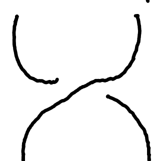
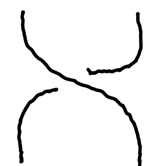
	Bosons	Fermions
	$\vert\varphi_1, \varphi_2\rangle$	$\vert\Psi_1, \Psi_2\rangle$
Interchange		
	$(+1)\vert\varphi_2, \varphi_1\rangle$	$(-1)\vert\Psi_2, \Psi_1\rangle$
Spin	$0, 1, 2, \ldots$	$1/2, 3/2, 5/2, \ldots$

Knotty Anyons

In $\cancel{3}^{2}+1$ diminsions

Pauli's Eclusion Principle

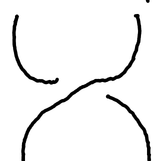
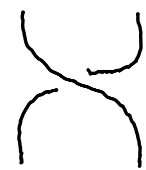
Particles

	Bosons	Fermions
	$\lvert \varphi_1, \varphi_2 \rangle$	$\lvert \psi_1, \psi_2 \rangle$
Interchange		
	$(+1)\lvert \varphi_2, \varphi_1 \rangle$	$(-1)\lvert \psi_2, \psi_1 \rangle$
Spin	$0, 1, 2, \ldots$	$1/2, 3/2, 5/2, \ldots$

Knotty Anyons

In ~~3~~ 2+1 dimensions ~~Pauli's Eclusion Principle~~

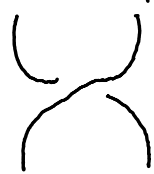
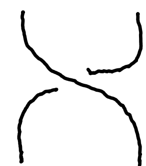
Particles

	Bosons	Fermions
	$\lvert\varphi_1, \varphi_2\rangle$	$\lvert\psi_1, \psi_2\rangle$
Interchange		
	$(+1)\lvert\varphi_2, \varphi_1\rangle$	$(-1)\lvert\psi_2, \psi_1\rangle$
Spin	$0, 1, 2, \ldots$	$1/2, 3/2, 5/2, \ldots$

Knotty Anyons

In ~~3~~ 2+1 diminsions ~~Pauli's Eclusion Principle~~

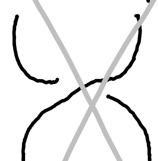
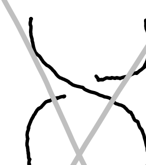
Quasi-Particles

	Bosons	Fermions
	$\lvert\varphi_1, \varphi_2\rangle$	$\lvert\psi_1, \psi_2\rangle$
Interchange		
	$(+1)\lvert\varphi_2, \varphi_1\rangle$	$(-1)\lvert\psi_2, \psi_1\rangle$
Spin	$0, 1, 2, \ldots$	$1/2, 3/2, 5/2, \ldots$

Knotty Anyons

In $\cancel{3}^{2}$+1 dimensions ~~Pauli's Eclusion Principle~~

Quasi-Particles

	~~Bosons~~	~~Fermions~~
	~~$\lvert\varphi_1, \varphi_2\rangle$~~	~~$\lvert\Psi_1, \Psi_2\rangle$~~
Interchange		
	~~$(+1)\lvert\varphi_2, \varphi_1\rangle$~~	~~$(-1)\lvert\Psi_2, \Psi_1\rangle$~~
Spin	~~$0, 1, 2, \ldots$~~	~~$1/2, 3/2, 5/2, \ldots$~~

Knotty Anyons

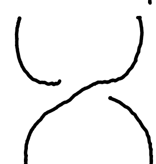
In $\cancel{3}^{2}+1$ diminsions ~~Pauli's Eclusion Principle~~

Quasi-Particles

Any-ons (Fractons)

$$|\varphi_1, \varphi_2\rangle$$

Interchange



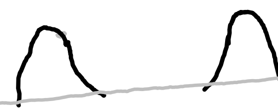
$$(e^{i\theta})|\varphi_2, \varphi_1\rangle$$

Spin Q^+

Knotty Anyons

Two Virtual pairs are created

$$|\varphi_1, \varphi_2, \varphi_3, \varphi_4\rangle$$



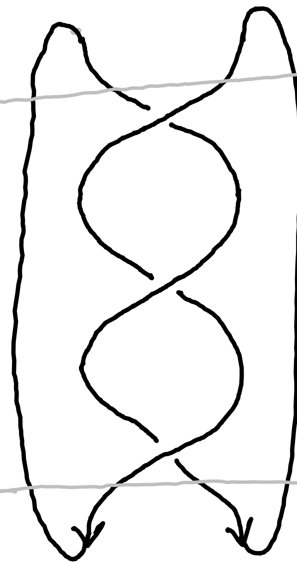
Knotty Anyons

Two Virtual pairs are created

$|\varphi_1, \varphi_2, \varphi_3, \varphi_4\rangle$

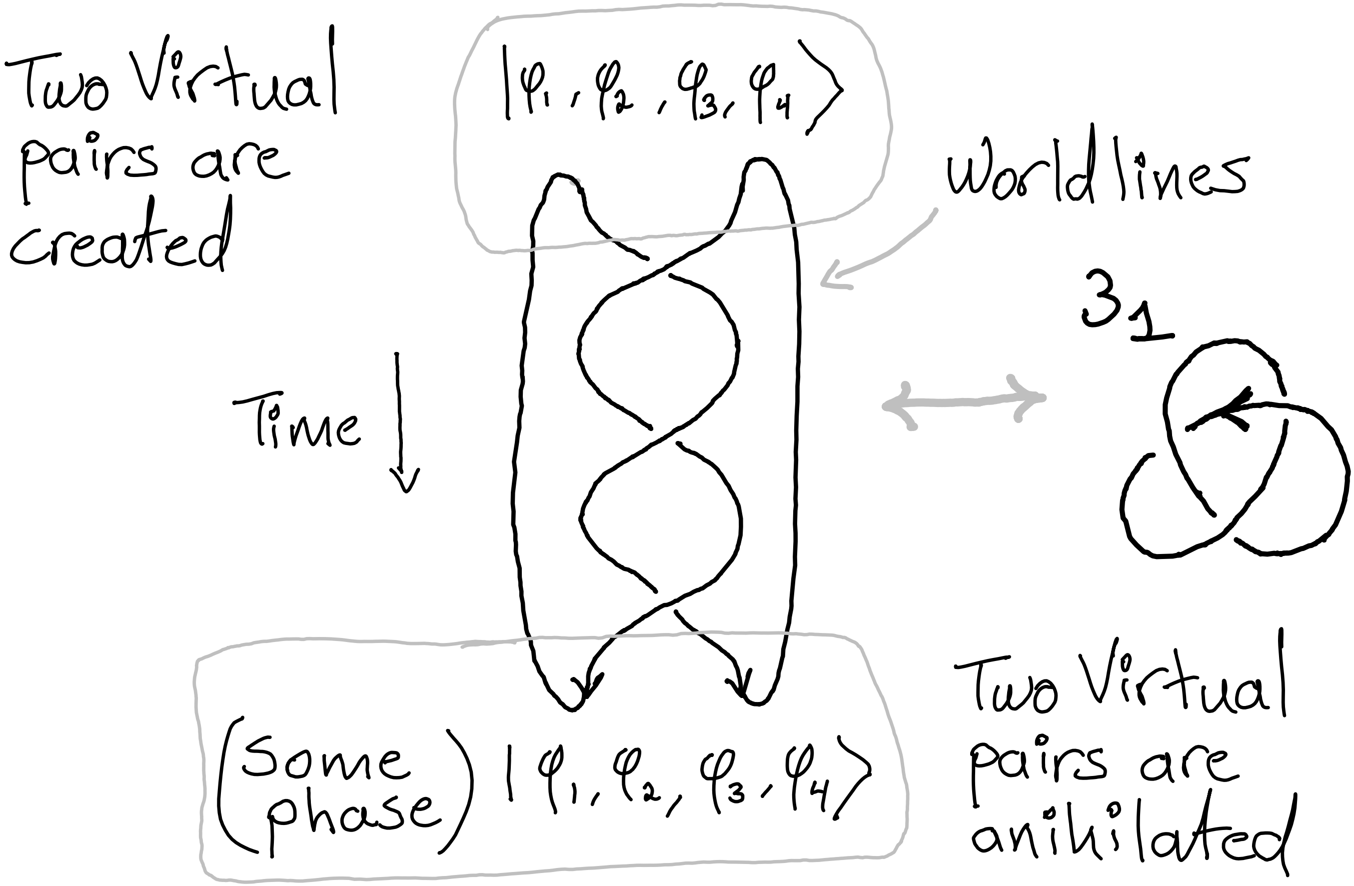
Worldlines

Time



(Some phase) $|\varphi_1, \varphi_2, \varphi_3, \varphi_4\rangle$

Knotty Anyons



Chern-Simons Thr.

$$S_{CS} = \frac{k}{4\pi}\int_{S^3} d^3x \, \varepsilon^{\mu\nu\rho} \, \mathrm{Tr}\left[A_\mu \partial_\nu A_\rho + i\frac{2}{3} A_\mu A_\nu A_\rho\right]$$

$$A_\mu = A_\mu^a(x) T^a, \quad T^a \in su(N) \qquad N = 1, 2, 3, \ldots$$

Color

Chern-Simons Thr.

$$S_{cs} = \frac{k}{4\pi}\int_{S^3} d^3x\, \varepsilon^{\mu\nu\rho}\, \mathrm{Tr}\left[A_\mu \partial_\nu A_\rho + i\frac{2}{3} A_\mu A_\nu A_\rho\right]$$

$A_\mu = A^a_\mu(x) T^a$, $T^a \in su(N)$ $\quad N = 1, 2, 3, \ldots$

Color

Wilson line operators

$$\langle W_\rho(K)\rangle = \mathrm{Tr}\, \vec{P} \exp\left(i\oint_K A^a_\mu(x)\, T^a_{(\rho)}\, dx^\mu\right)$$

Knot

Fundamental represent.

Chern-Simons Thr.

$$S_{CS} = \frac{k}{4\pi}\int_{S^3} d^3x\, \varepsilon^{\mu\nu\rho}\,\mathrm{Tr}\left[A_\mu\partial_\nu A_\rho + i\frac{2}{3}A_\mu A_\nu A_\rho\right]$$

$A_\mu = A^a_\mu(x)T^a$, $T^a \in su(N)$ $\quad N=1,2,3,\dots$

Color

Wilson line operators

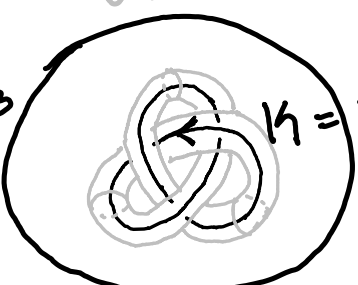
$$\langle W_\rho(K)\rangle = \mathrm{Tr}\,\vec{P}\exp\left(i\oint_K A^a_\mu(x)\,T^a_{(\rho)}\,dx^\mu\right)$$

Knot

Fundamental represent.

$q = e^{-\frac{2\pi i}{k+N}}$

S_3

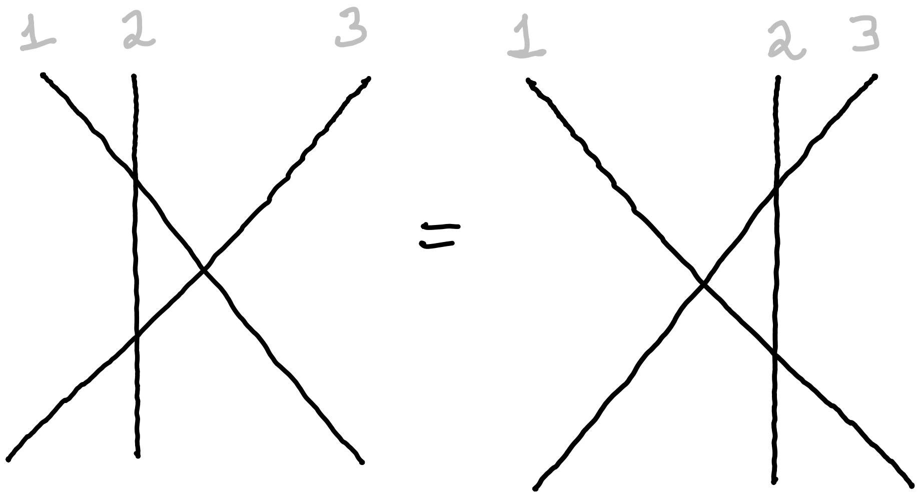


$K = 3_1$ $\quad = P_N(q)$ "Colored Jones"

Color

Integrability

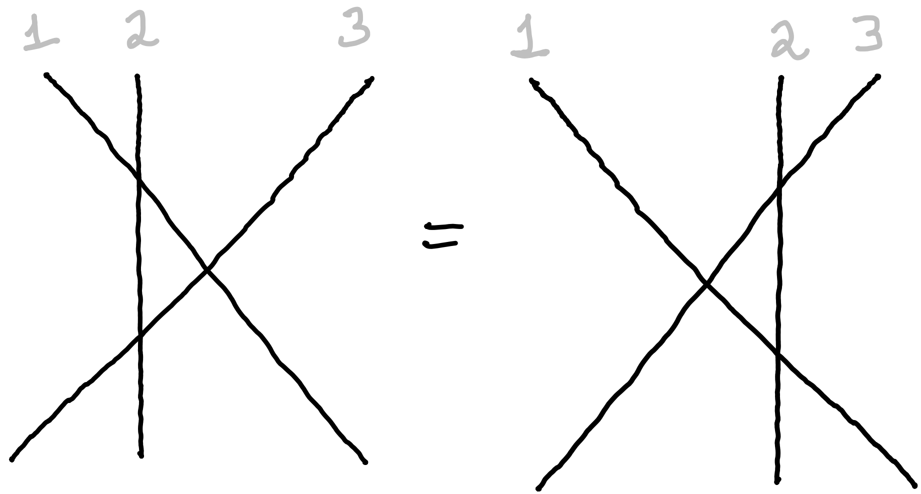
Yang-Baxter



$$R_{12} R_{23} R_{12} = R_{23} R_{12} R_{23}$$

Integrability

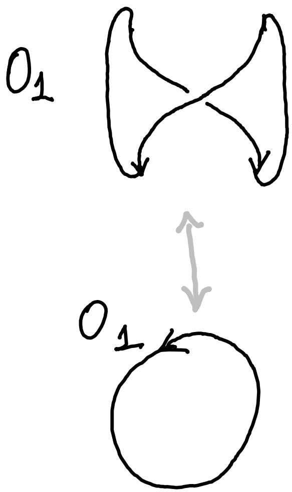
Yang-Baxter = Reidemeister move III

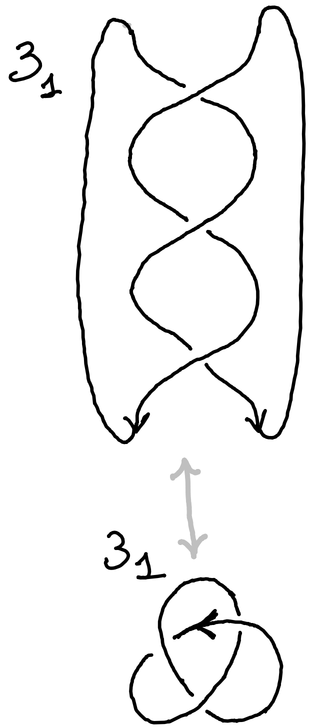


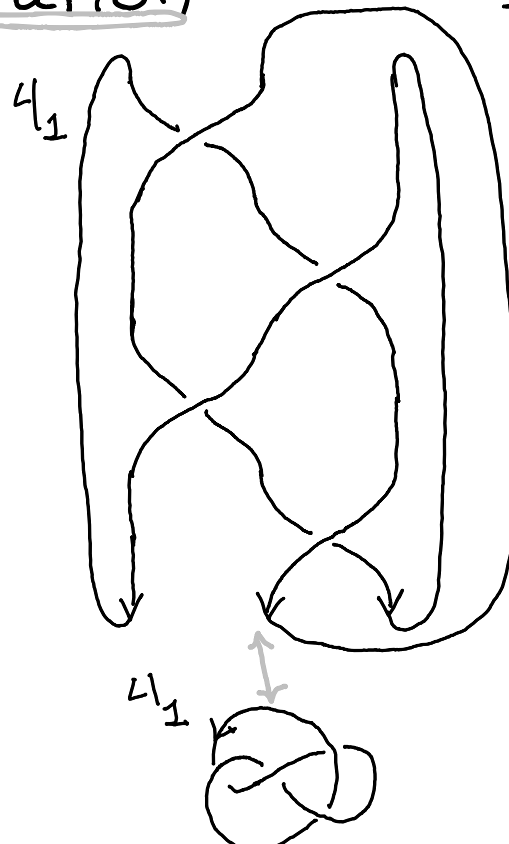
$$R_{12} R_{23} R_{12} = R_{23} R_{12} R_{23}$$

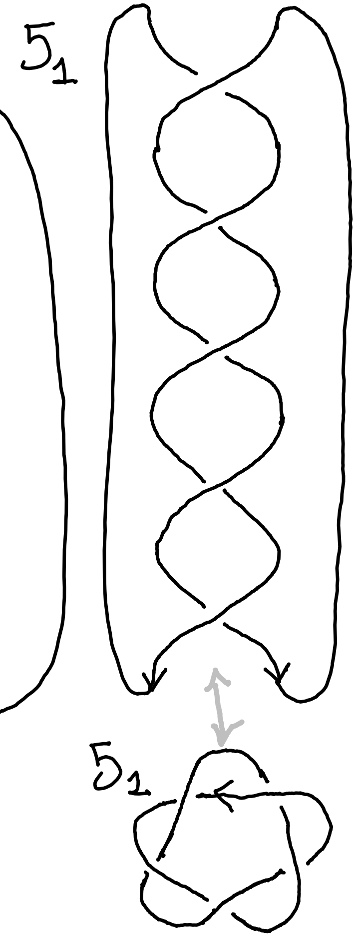
Integrability

Braiding Representation





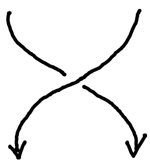




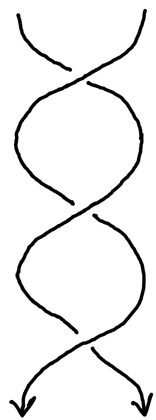
Integrability

Braiding Representation

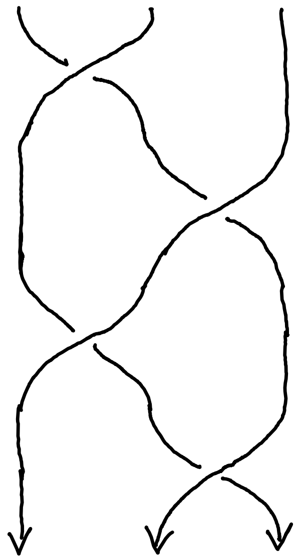
0_1



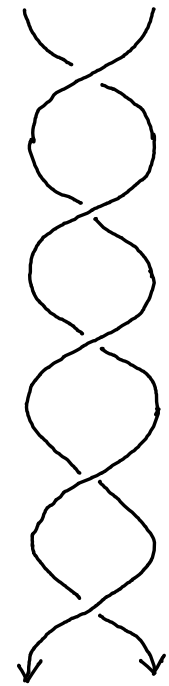
3_1



4_1



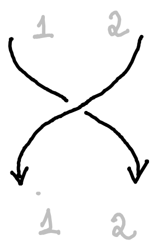
5_1



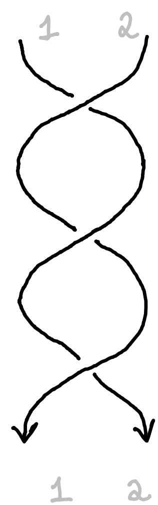
Integrability

Braiding Representation

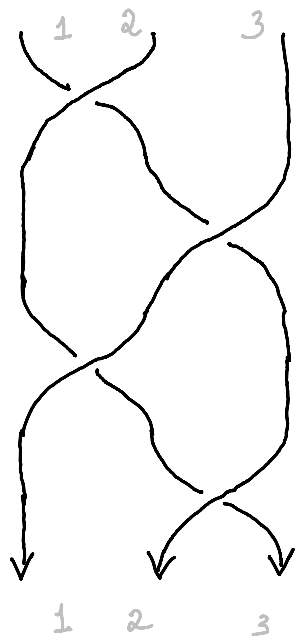
0_1



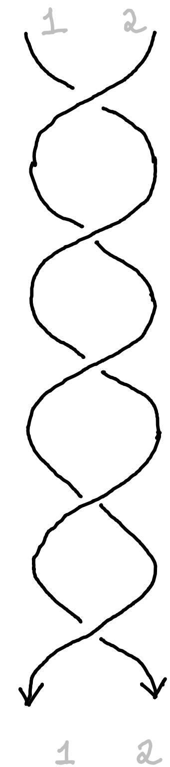
3_1



4_1



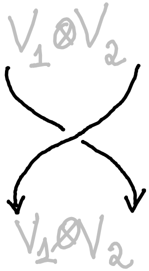
5_1



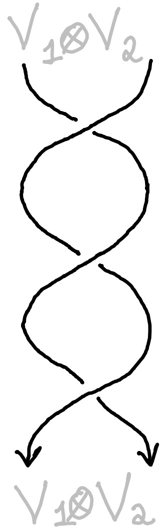
Integrability

Braiding Representation

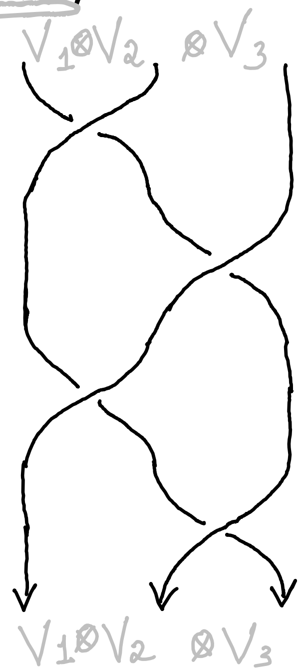
0_1



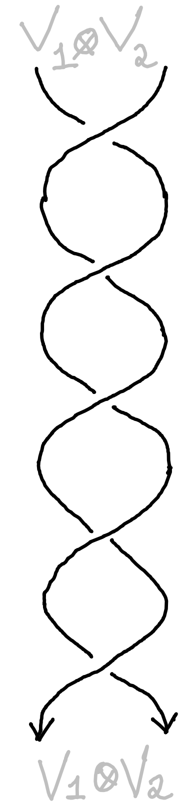
3_1



4_1



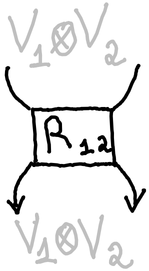
5_1



Integrability

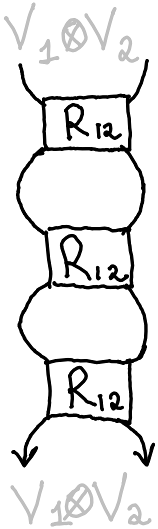
Braiding Representation

0_1



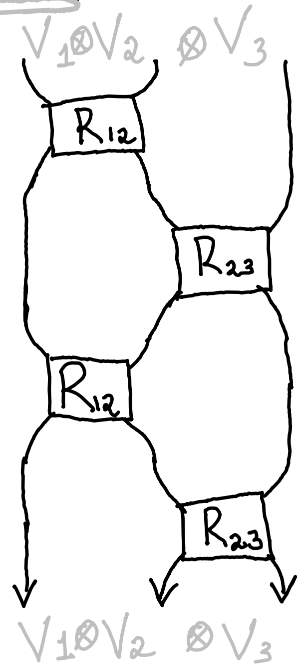
R_{12}

3_1



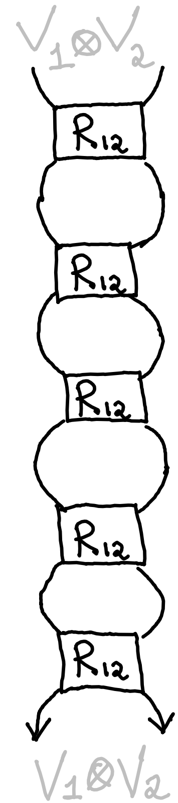
R_{12}^3

4_1



$R_{12} R_{23} R_{12} R_{23}$

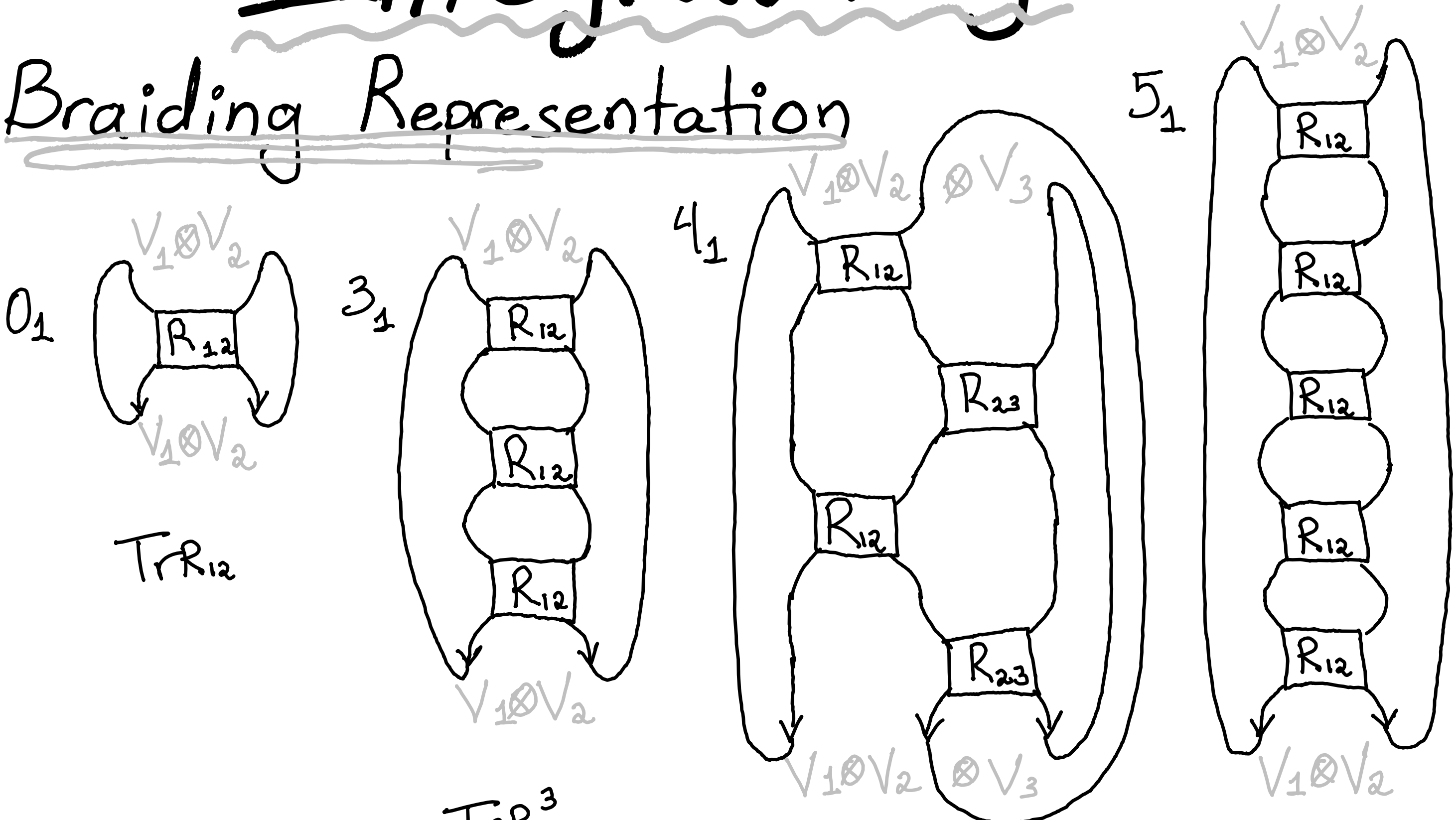
5_1



R_{12}^5

Integrability

Braiding Representation



$\mathrm{Tr}\, R_{12}$

$\mathrm{Tr}\, R_{12}^{3}$

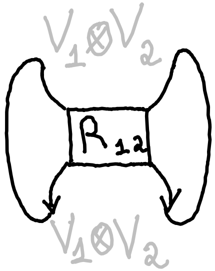
$\mathrm{Tr}\, R_{12} R_{23} R_{12} R_{23}$

$\mathrm{Tr}\, R_{12}^{5}$

Integrability

Braiding Representation

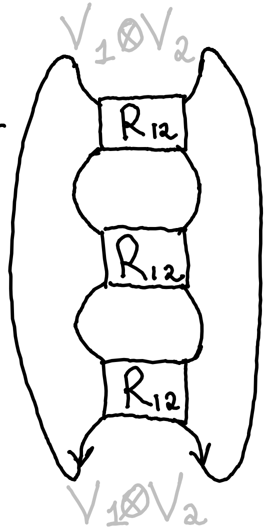
0_1



$P = \mathrm{Tr}\, R_{12}(2)$

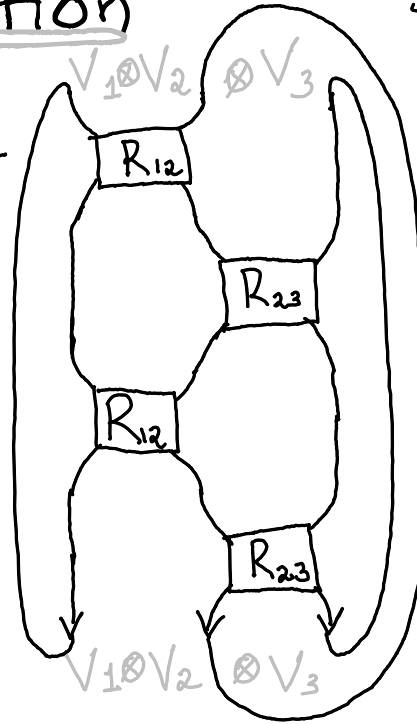
$\dim V_{1,2} = 2$

3_1



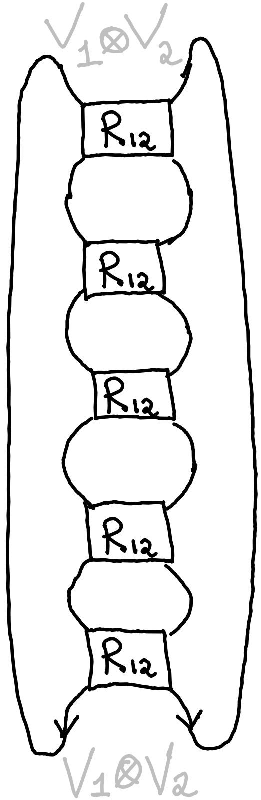
$P = \mathrm{Tr}\, R_{12}^3(2)$

4_1



$P = \mathrm{Tr}\, R_{12} R_{23} R_{12} R_{23}$

5_1



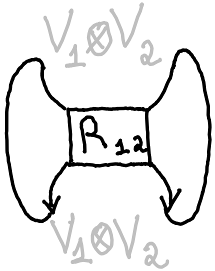
$P = \mathrm{Tr}\, R_{12}^5(2)$

HOMFLY-PT

Integrability

Braiding Representation

0_1



$P_{N=1} = 1$

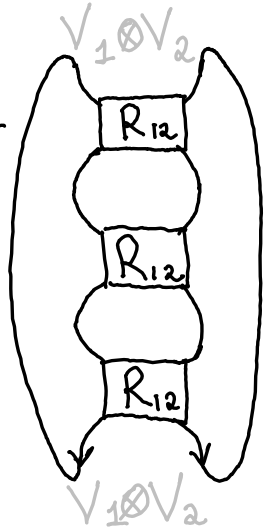
$P_{N=2} = \mathrm{Tr}\, R_{12}(2)$

$\vdots$

$\dim V_{1,2} =$

Colored HOMFLY-PT

3_1

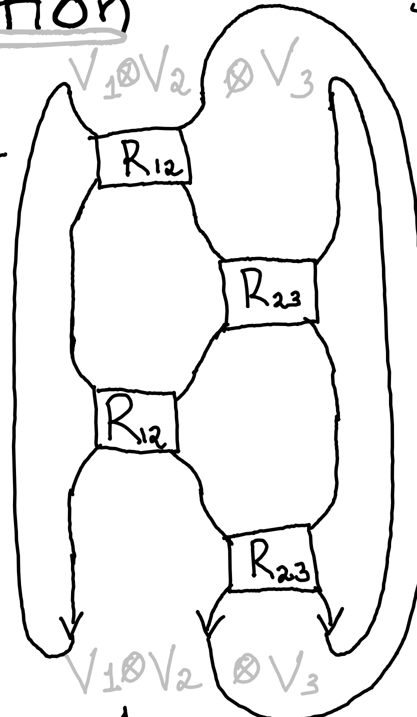


$P_{=1} = 1$

$P_{=2} = \mathrm{Tr}\, R_{12}^{3}(2)$

$\vdots$

4_1

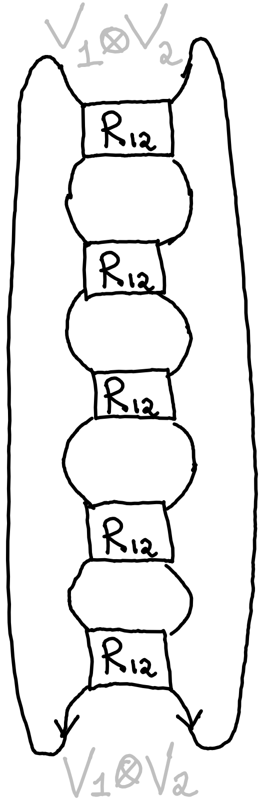


$P_{=1} = 1$

$P_{=2} = \mathrm{Tr}\, R_{12} R_{23} R_{12} R_{23}$

$\vdots$

5_1



$P_{=1} = 1$

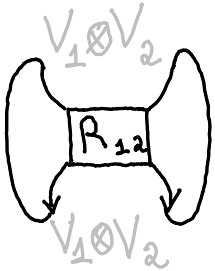
$P_{=2} = \mathrm{Tr}\, R_{12}^{5}(2)$

$\vdots$

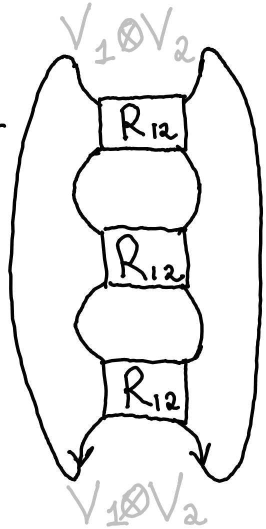
Integrability

Braiding Representation

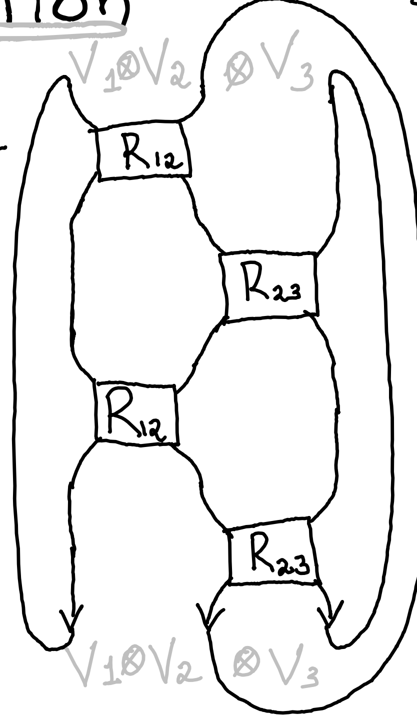
0_1



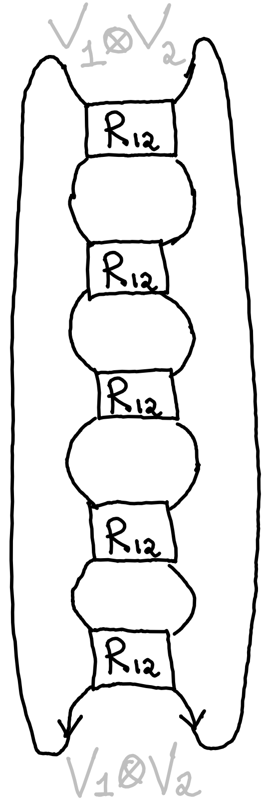
3_1



4_1



5_1



$$P_{0_1}(x) = \sum_{n=0}^{\infty} P_{n+1} x^n$$

$\dim V_{1,2} = n$

Generat. Fnt. of Colored HOMFLY-PT

$$P_{3_1}(x) = \sum_{n=0}^{\infty} P_{n+1} x^n$$

$$P_{4_1}(x) = \sum_{n=0}^{\infty} P_{n+1} x^n$$

$$P_{5_1}(x) = \sum_{n=0}^{\infty} P_{n+1} x^n$$

Remember, remember...

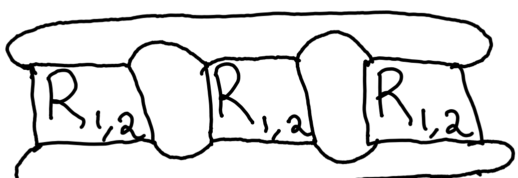
Remember, remember...

Knots appear naturaly in various physical systems when the interactions are topological. Knot invariants like colored HOMFL-PT polynomials charectarise those systems.

Remember, remember...

Knots appear naturaly in various physical systems when the interactions are topological. Knot invariants like colored HOMFL-PT polynomials charectarise those systems.

3_1



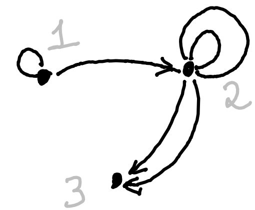
$$P_N(a,q) = \mathrm{Tr}\left(R_{1,2}^{(N)} R_{1,2}^{(N)} R_{1,2}^{(N)}\right)$$

Quiver

- A quiver Q is a directed graph

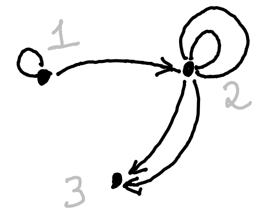
Quiver

- A quiver Q is a directed graph



Quiver

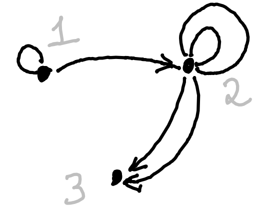
- A quiver Q is a directed graph



- A quiver matrix $\boxed{C}$ is the incidence matrix of the quiver

Quiver

- A quiver Q is a directed graph

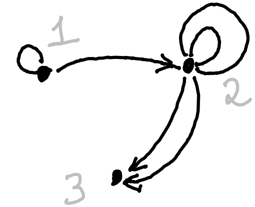


- A quiver matrix $\boxed{C}$ is the incidence matrix of the quiver

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Quiver

- A quiver Q is a directed graph



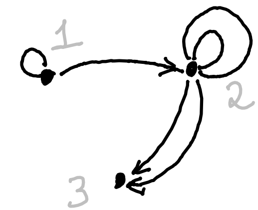
- A quiver matrix $\boxed{C}$ is the incidence matrix of the quiver

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 3 & 0 & 0 & 0 \end{array}$$

- A symmetric quiver Q is, for every arrow of node $i \longrightarrow j$ there is another arrow frome node $j \longrightarrow i$

Quiver

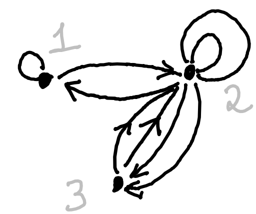
- A quiver Q is a directed graph



- A quiver matrix $\boxed{C}$ is the incidence matrix of the quiver

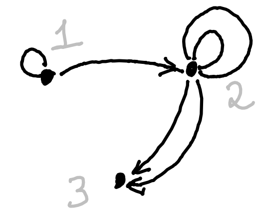
$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 3 & 0 & 0 & 0 \end{array} \quad = \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

- A symmetric quiver Q is, for every arrow of node $i \longrightarrow j$ there is another arrow frome node $j \longrightarrow i$



Quiver

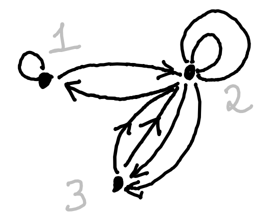
- A quiver Q is a directed graph



- A quiver matrix $\boxed{C}$ is the incidence matrix of the quiver

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \multicolumn{3}{c}{\multirow{3}{*}{$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$}} \\ 2 & & & \\ 3 & & & \end{array}$$

- A symmetric quiver Q is, for every arrow of node $i \longrightarrow j$ there is another arrow frome node $j \longrightarrow i$

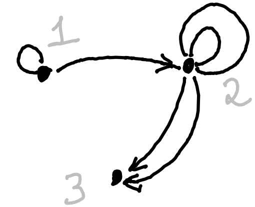


- A symmetric quiver matrix $\boxed{C}$

homework

Quiver

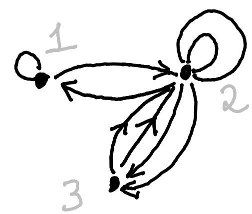
- A quiver Q is a directed graph



- A quiver matrix $\boxed{C}$ is the incidence matrix of the quiver

$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

- A symmetric quiver Q is, for every arrow of node $i \longrightarrow j$ there is another arrow frome node $j \longrightarrow i$



- A symmetric quiver matrix $\boxed{C}$

$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 2 \\ 0 & 2 & 0 \end{bmatrix} \end{array}$$

<u>homework</u>

Dualities

$$Z_C(x_1, x_2, \dots, x_m) = \sum_{d_1, \dots, d_m = 0}^{\infty} q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

Dualities

$$Z_C(x_1, x_2, \dots, x_m) = \sum_{d_1,\dots,d_m=0}^{\infty} q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

$$\updownarrow$$

$$\int_{-\infty}^{\infty} dy_1 \dots \int_{-\infty}^{\infty} dy_m$$

Dualities

$$Z_C(x_1, x_2, \ldots, x_m) = \sum_{d_1,\ldots,d_m=0}^{\infty} q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

$$\int_{-\infty}^{\infty} dy_1 \ldots \int_{-\infty}^{\infty} dy_m \, e^{\sum_{i,j=1}^{m} \sigma^{-1}_{ij} y_i y_j}$$

Dualities

$$Z_C(x_1, x_2, \dots, x_m) = \sum_{d_1,\dots,d_m=0}^{\infty} q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

$$\int_{-\infty}^{\infty} dy_1 \dots \int_{-\infty}^{\infty} dy_m \, e^{\sum_{i,j=1}^{m} \sigma_{ij}^{-1} y_i y_j + \sum_{k=1}^{m} \mu_k y_k}$$

Dualities

$$Z_C(x_1, x_2, \ldots, x_m) = \sum_{d_1,\ldots,d_m=0}^{\infty} q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

Secret ingredient

$$\int_{-\infty}^{\infty} dy_1 \ldots \int_{-\infty}^{\infty} dy_m \, e^{\sum_{i,j=1}^{m} \sigma^{-1}_{ij} y_i y_j + \sum_{k=1}^{m} \mu_k y_k}$$

Dualities

$$Z_C(x_1, x_2, \dots, x_m) = \sum_{d_1, \dots, d_m = 0}^{\infty} \underbrace{q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j}}_{\text{Gaussian-like factor}} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

q-Pochhammer symbol $\prod_{n=1}^{d_k} (1 - q^{2n})$

Dualities

2d CFTs
Partn. Fn. or Nahm Sums

(For particular matrices C_{ij})

$$Z_C(x_1, x_2, \ldots, x_m) = \sum_{d_1,\ldots,d_m=0}^{\infty} \underbrace{q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j}}_{\text{Gaussian-like factor}} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

q-Pochhammer symbol

$$\prod_{n=1}^{d_k} (1 - q^{2n})$$

Dualities

2d CFTs	3d $\mathcal{N}=2$
Partn. Fn. or Nahm Sums	Vortex Partn. Fn.

(For any $C_{i,j} \in \mathbb{Z}$)

$$Z_C(x_1, x_2, \ldots, x_m) = \sum_{d_1,\ldots,d_m=0}^{\infty} \underbrace{q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j}}_{\text{Gaussian-like factor}} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

q-Pochhammer symbol

$$\prod_{n=1}^{d_k} (1 - q^{2n})$$

Dualities

2d CFTs	3d $\mathcal{N}=2$
Partn. Fn. or Nahm Sums	Vortex Partn. Fn.

$$Z_C(x_1, x_2, \ldots, x_m) = \sum_{d_1,\ldots,d_m=0}^{\infty} \underbrace{q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j}}_{\text{Gaussian-like factor}} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

q-Pochhammer symbol $\prod_{n=1}^{d_k} (1-q^{2n})$

Knot Thr.
Genr. Fn. of HOMFLY-PT

(For certain $C_{i,j} \in \mathbb{Z}$)

Dualities

2d CFTs
Partn. Fn. or Nahm Sums

3d $\mathcal{N}=2$
Vortex Partn. Fn.

$$Z_C(x_1, x_2, \ldots, x_m) = \sum_{d_1,\ldots,d_m=0}^{\infty} q^{\sum_{i,j=1}^{m} C_{ij} d_i d_j} \prod_{k=1}^{m} \frac{x_k^{d_k}}{(q)_{d_k}}$$

Knot Thr.
Genr. Fn. of HOMFLY-PT

$$Z_C\left(a^{a_1} q^{l_1} (-1)^{C_{11}} x, a^{a_2} q^{l_2} (-1)^{C_{22}} x, \ldots, a^{a_m} q^{l_m} (-1)^{C_{mm}} x\right)$$

$$= P_K(a, q, x)$$

Remember, remember…

Remember, remember...

There is a class of 3D SUSY thrs. which are characterised by a quiver diagram. The partn. fn. of certain of those theories coincides with the generating series of colored HOMFLY-PT from knots.

Remember, remember...

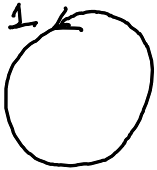
There is a class of 3D SUSY thrs. which are characterised by a quiver diagram. The partn. fn. of certain of those theories coincides with the generating series of colored HOMFLY-PT from knots.

$$\boxed{\begin{aligned} &Z_C\left(a^{a_1}q^{l_1}(-1)^{C_{11}}X, a^{a_2}q^{l_2}(-1)^{C_{22}}X, \dots, a^{a_m}q^{l_m}(-1)^{C_{mm}}X\right) \\ &\qquad = P_K(a,q,X) \end{aligned}}$$

Knots-Quivers Correspondence

Knots

0_1



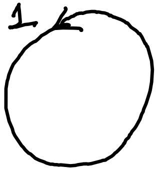
Quivers Matrices

$$[0]_{1\times 1}$$

Knots-Quivers Correspondence

Knots

0_1



3_1

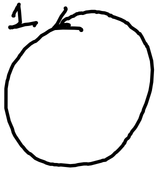
Quivers Matrices

$$\begin{bmatrix} 0 \end{bmatrix}_{1\times 1} \qquad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}_{3\times 3}$$

Knots-Quivers Correspondence

Knots

0_1



3_1

4_1

Quivers Matrices

$$\begin{bmatrix} 0 \end{bmatrix}_{1\times 1}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 1 & -1 \\ -1 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 1 & -1 \\ -1 & -1 & -2 & -1 & -2 \end{bmatrix}_{5\times 5}$$

Knots-Quivers Correspondence

Torus Knots $T_{\underbrace{2,2p+1}_{\#\text{ of crossings}}}$ $p = 1, 2, 3, \ldots$

3_1

$p=1$

5_1

$p=2$

Quivers Matrices

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}_{3\times 3} \qquad \begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5}$$

Knots-Quivers Correspondence

Torus Knots

$T_{\underbrace{2,2p+1}_{\#\text{ of crossings}}}$ $\quad p = 1, 2, 3, \ldots$

$p=1$ $\quad 3_1$

$p=2$ $\quad 5_1$

Quivers Matrices

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5}$$

3×3

Knots-Quivers Correspondence

Torus Knots $\underbrace{T_{2,2p+1}}_{\#\text{ of crossings}}$ $p = 1, 2, 3, \ldots$

3_1

$p=1$

$p=2$

5_1

Quivers Matrices

$$\begin{array}{ccc}
 & \begin{matrix} F_0 & \quad F_1 \quad \end{matrix} & \\
\begin{matrix} \\ F_1^T \\ \\ \end{matrix} & \left[\begin{array}{c|cc} 0 & 1 & 1 \\ \hline 1 & 2 & 2 \\ 1 & 2 & 3 \end{array}\right]_{3\times 3} & \begin{matrix} \\ D_1 \\ \\ \end{matrix} \\
 & \begin{matrix} \quad\quad & D_1 \end{matrix} &
\end{array}$$

$$\begin{array}{ccc}
 & \begin{matrix} F_0 & \quad F_1 \quad & \quad F_2 \quad \end{matrix} & \\
\begin{matrix} \\ F_1^T \\ \\ F_2^T \\ \\ \end{matrix} & \left[\begin{array}{c|cc|cc} 0 & 1 & 1 & 3 & 3 \\ \hline 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ \hline 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{array}\right]_{5\times 5} & \begin{matrix} \\ U_2 \\ \\ D_2 \\ \\ \end{matrix} \\
 & \begin{matrix} \quad & U_2^T & \quad D_2 \end{matrix} &
\end{array}$$

Knots-Quivers Correspondence

Torus Knots $\underbrace{T_{2,2p+1}}_{\#\text{ of crossings}}$ $p = 1, 2, 3, \ldots$

$p=1$ 3_1

$p=2$ 5_1

Quivers Matrices

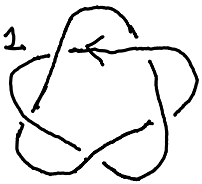
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}_{3\times3} = \begin{bmatrix} F_0 & F_1 \\ F_1^T & D_1 \end{bmatrix}_{3\times3}, \quad \begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times5} = \begin{bmatrix} F_0 & F_1 & F_2 \\ F_1^T & D_1 & U_2 \\ F_2^T & U_2^T & D_2 \end{bmatrix}_{5\times5}$$

Knots-Quivers Correspondence

Torus Knots $\underbrace{T_{2,2p+1}}_{\#\text{ of crossings}}$ $p = 1, 2, 3, \ldots$

$p=1, 3_1$

$p=2, 5_1$



$\ldots$ $p=p$

Quivers Matrices

$$F_0 = [0], \quad F_k = [2k-1, \; 2k-1]$$

$$D_k = \begin{bmatrix} 2k & 2k \\ 2k & 2k+1 \end{bmatrix}, \quad U_k = \begin{bmatrix} 2k-1 & 2k-1 \\ 2k & 2k \end{bmatrix}$$

$p=1$

$$\left[\begin{array}{c|c} F_0 & F_1 \\ \hline F_1^T & D_1 \end{array}\right]_{3\times 3}$$

$p=2$

$$\left[\begin{array}{c|c|c} F_0 & F_1 & F_2 \\ \hline F_1^T & D_1 & U_2 \\ \hline F_2^T & U_2^T & D_2 \end{array}\right]_{5\times 5} \ldots$$

$$\left[\begin{array}{c|c|c|c|c} F_0 & F_1 & F_2 & \ldots & F_p \\ \hline F_1^T & D_1 & U_2 & \ldots & U_p \\ \hline F_2^T & U_2^T & D_2 & \ldots & U_p \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline F_p^T & U_p^T & U_p^T & \ldots & D_p \end{array}\right]_{(2p+1)\times(2p+1)}$$

Remember, remember...

Remember, remember...

We can associate a quiver (3d SUSY) with a knot. We then can construct colored HOMFY-PT polyns. using simple quiver matrices as building blocks.

Remember, remember...

We can associate a quiver (3d SUSY) with a knot. We then can construct colored HOMFY-PT polyns. using simple quiver matrices as building blocks.

$$\begin{bmatrix} F_0 & F_1 & F_2 & \cdots & F_p \\ F_1^T & D_1 & U_2 & \cdots & U_p \\ F_2^T & U_2^T & D_2 & \cdots & U_p \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_p^T & U_p^T & U_p^T & \cdots & D_p \end{bmatrix}_{(2p+1)\times(2p+1)}$$

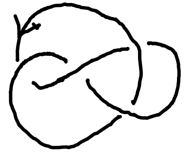
$F_0 = [0]$, $\quad F_k = [2k-1, 2k-1]$

$D_k = \begin{bmatrix} 2k & 2k \\ 2k & 2k+1 \end{bmatrix}$, $\quad U_k = \begin{bmatrix} 2k-1 & 2k-1 \\ 2k & 2k \end{bmatrix}$

But, wait...!

Knots-Quivers Correspondence

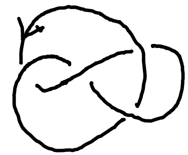
4_1



$$C = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 1 & -1 \\ -1 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 1 & -1 \\ -1 & -1 & -2 & -1 & -2 \end{bmatrix}_{5\times 5} \end{array}$$

Knots-Quivers Correspondence

4_1



$$C = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 1 & \boxed{-1} \\ -1 & 0 & -1 & \boxed{0} & -2 \\ 0 & 1 & \boxed{0} & 1 & -1 \\ -1 & \boxed{-1} & -2 & -1 & -2 \end{bmatrix}_{5\times 5} \end{array}$$

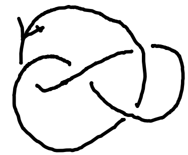
$$C_{2,5} \longleftrightarrow C_{3,4}$$

$$\longleftrightarrow$$

$$\tilde{C} = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 1 & \boxed{0} \\ -1 & 0 & -1 & \boxed{-1} & -2 \\ 0 & 1 & \boxed{-1} & 1 & -1 \\ -1 & \boxed{0} & -2 & -1 & -2 \end{bmatrix}_{5\times 5} \end{array}$$

Knots-Quivers Correspondence

4_1



$$C = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & -1 & 0 & -1 \\ 2 & 0 & 2 & 0 & 1 & \boxed{-1} \\ 3 & -1 & 0 & -1 & \boxed{0} & -2 \\ 4 & 0 & 1 & \boxed{0} & 1 & -1 \\ 5 & -1 & \boxed{-1} & -2 & -1 & -2 \end{array}_{5\times 5}$$

$$C_{2,5} \longleftrightarrow C_{3,4}$$

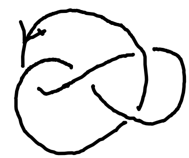
$$\longleftrightarrow$$

$$\tilde{C} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & -1 & 0 & -1 \\ 2 & 0 & 2 & 0 & 1 & \boxed{0} \\ 3 & -1 & 0 & -1 & \boxed{-1} & -2 \\ 4 & 0 & 1 & \boxed{-1} & 1 & -1 \\ 5 & -1 & \boxed{0} & -2 & -1 & -2 \end{array}_{5\times 5}$$

$$Z_C(x_1, x_2, x_3, x_4, x_5) = Z_{\tilde{C}}(x_1, x_2, x_3, x_4, x_5)$$

Knots-Quivers Correspondence

4_1



$$\underset{\|}{C} \quad \bullet\!\!\frac{\qquad\qquad\qquad\qquad}{\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}}\!\!\bullet \quad \underset{\|}{\tilde{C}}$$

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 1 & \boxed{-1} \\ -1 & 0 & -1 & \boxed{0} & -2 \\ 0 & 1 & \boxed{0} & 1 & -1 \\ -1 & \boxed{-1} & -2 & -1 & -2 \end{bmatrix}_{5\times 5} \end{array} \quad \begin{array}{c} C_{2,5} \longleftrightarrow C_{3,4} \\ \longleftrightarrow \end{array} \quad \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 1 & \boxed{0} \\ -1 & 0 & -1 & \boxed{-1} & -2 \\ 0 & 1 & \boxed{-1} & 1 & -1 \\ -1 & \boxed{0} & -2 & -1 & -2 \end{bmatrix}_{5\times 5} \end{array}$$

$$Z_C(x_1, x_2, x_3, x_4, x_5) = Z_{\tilde{C}}(x_1, x_2, x_3, x_4, x_5)$$

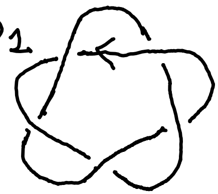
Colorful Knot Symmetries

5_1

$$\begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5}$$

Colorful Knot Symmetries

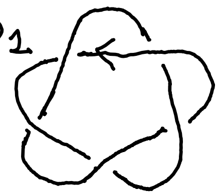
5_1



$$\begin{bmatrix} 0 & 1 & 1 & 3 & \boxed{2} \\ 1 & 2 & \boxed{3} & 3 & 3 \\ 1 & \boxed{3} & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ \boxed{2} & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5} \quad \begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5} \quad \begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & \boxed{4} \\ 1 & 2 & 3 & \boxed{3} & 4 \\ 3 & 3 & \boxed{3} & 4 & 4 \\ 3 & \boxed{4} & 4 & 4 & 5 \end{bmatrix}_{5\times 5}$$

Colorful Knot Symmetries

5_1

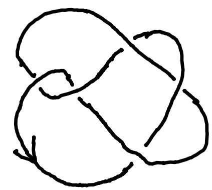


$\begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix}$ $\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$

$$\begin{bmatrix} 0 & 1 & 1 & 3 & \boxed{2} \\ 1 & 2 & \boxed{3} & 3 & 3 \\ 1 & \boxed{3} & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ \boxed{2} & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5} \quad \begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 & 4 \\ 3 & 3 & 4 & 4 & 5 \end{bmatrix}_{5\times 5} \quad \begin{bmatrix} 0 & 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 & \boxed{4} \\ 1 & 2 & 3 & \boxed{3} & 4 \\ 3 & 3 & \boxed{3} & 4 & 4 \\ 3 & \boxed{4} & 4 & 4 & 5 \end{bmatrix}_{5\times 5}$$

Colorful Knot Symmetries

5_2

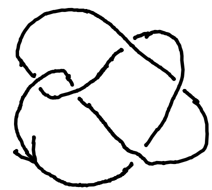


$$\begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 & 3 & 2 & 3 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 4 & 3 & 4 \\ 1 & 0 & 2 & 1 & 3 & 2 & 3 \\ 2 & 1 & 3 & 2 & 4 & 3 & 5 \end{bmatrix}_{7\times 7}$$

Colorful Knot Symmetries

- Which are the equivalent matrices?

5_2

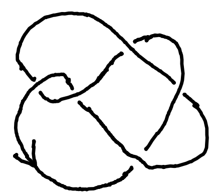


$$\begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 & 3 & 2 & 3 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 4 & 3 & 4 \\ 1 & 0 & 2 & 1 & 3 & 2 & 3 \\ 2 & 1 & 3 & 2 & 4 & 3 & 5 \end{bmatrix}_{7\times 7}$$

Colorful Knot Symmetries

- Which are the equivalent matrices?

5_2



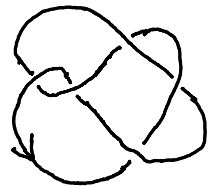
- As Fermat would say, "the margin is too small."

$$\begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 & 3 & 2 & 3 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 4 & 3 & 4 \\ 1 & 0 & 2 & 1 & 3 & 2 & 3 \\ 2 & 1 & 3 & 2 & 4 & 3 & 5 \end{bmatrix}_{7\times 7}$$

Colorful Knot Symmetries

- Which are the equivalent matrices?

5_2



- As Fermat would say, "the margin is too small."

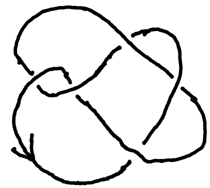
- What then?

$$\begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 & 3 & 2 & 3 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 4 & 3 & 4 \\ 1 & 0 & 2 & 1 & 3 & 2 & 3 \\ 2 & 1 & 3 & 2 & 4 & 3 & 5 \end{bmatrix}_{7\times 7}$$

Colorful Knot Symmetries

- Which are the equivalent matrices?

5_2



- As Fermat would say, "the margin is too small."

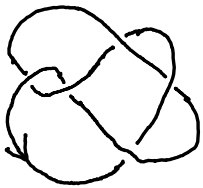
$$\begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 & 3 & 2 & 3 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 4 & 3 & 4 \\ 1 & 0 & 2 & 1 & 3 & 2 & 3 \\ 2 & 1 & 3 & 2 & 4 & 3 & 5 \end{bmatrix}_{7 \times 7}$$

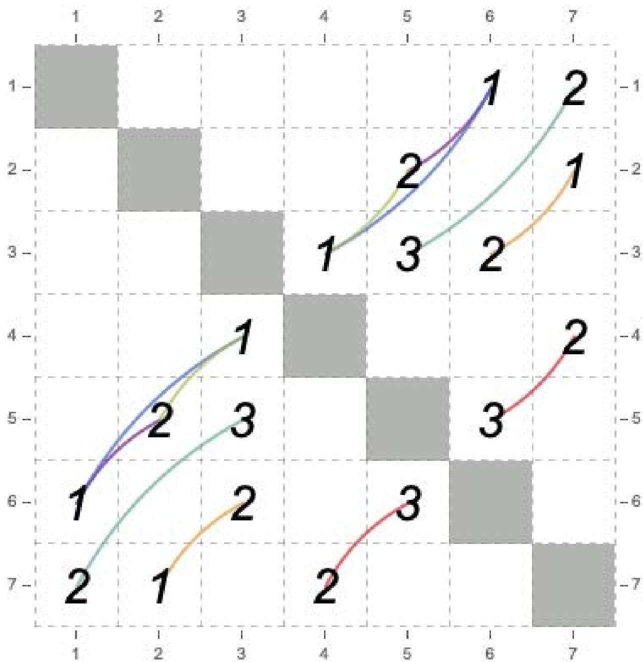
- What then?

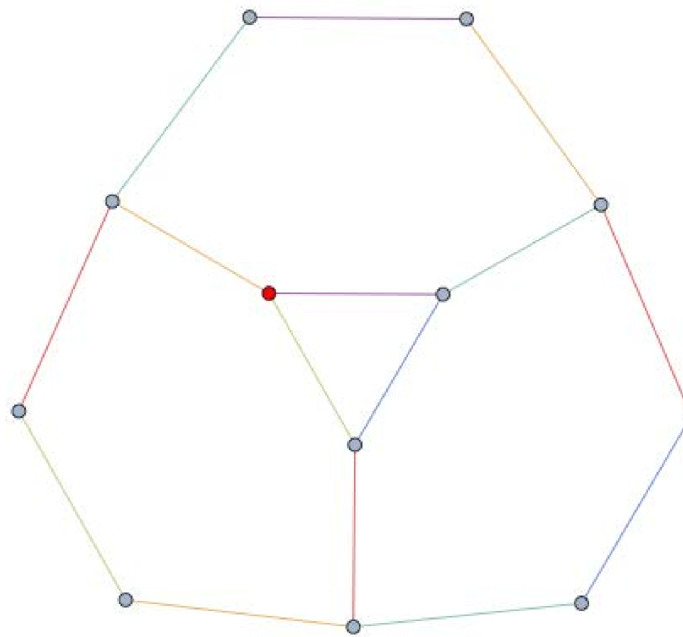
- I can show you the graph.

Colorful Knot Symmetries

5_2



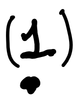




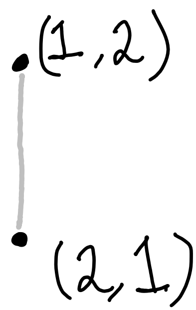
Permutahedron (Hidden Gems)

$\vec{v} = (1, 2, \ldots, n)$

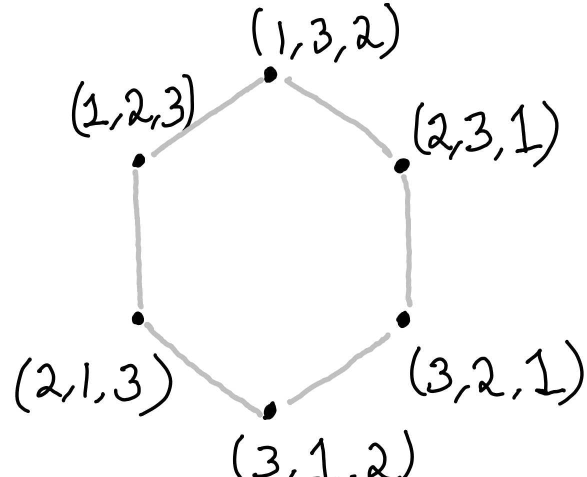
Transposition, a permutation of two elements.



P_1



P_2

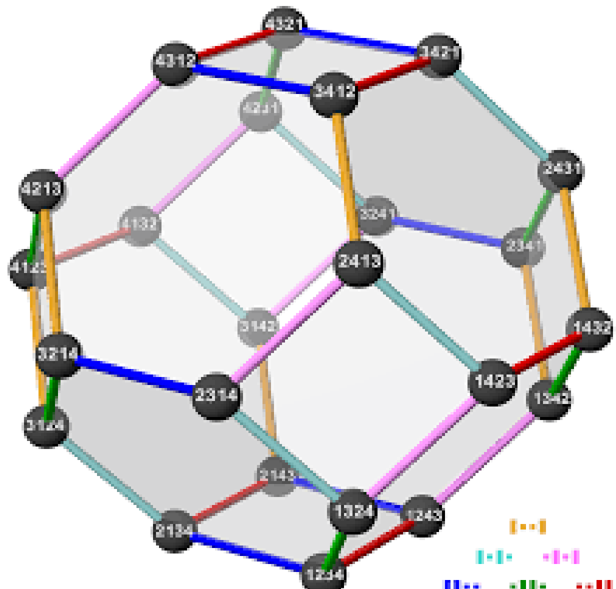


P_3

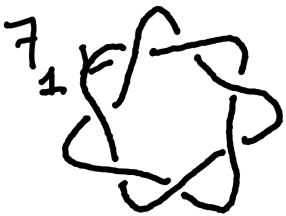
Permutahedron (Hidden Gems)

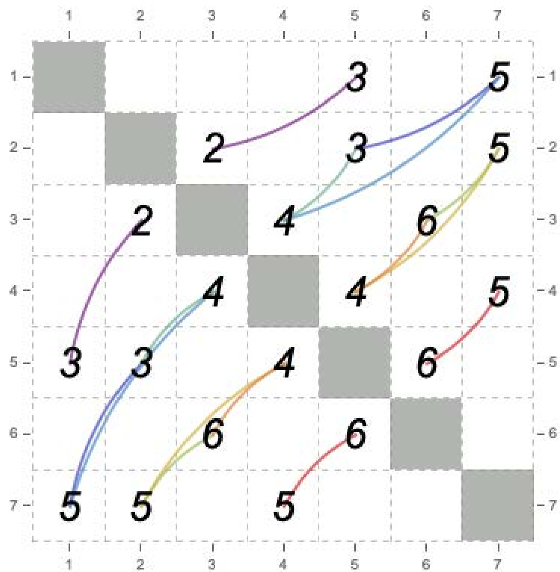
$\vec{v} = (1, 2, \ldots, n)$

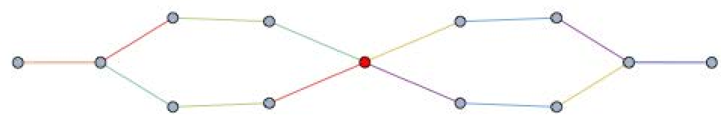
Transposition, a permutation of two elements.



Colorful Knot Symmetries

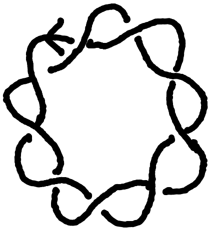


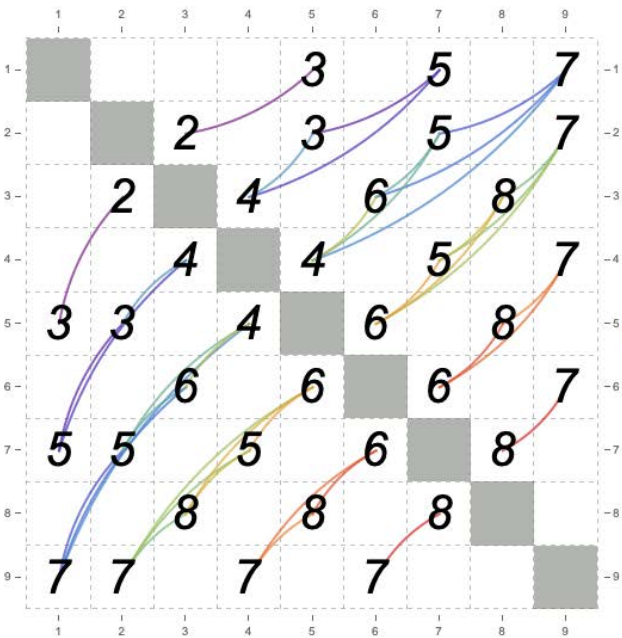


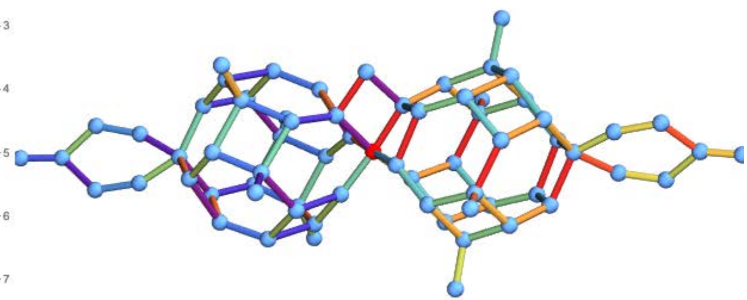


Colorful Knot Symmetries

9_1

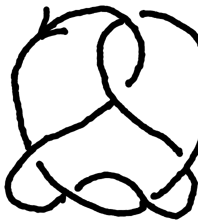


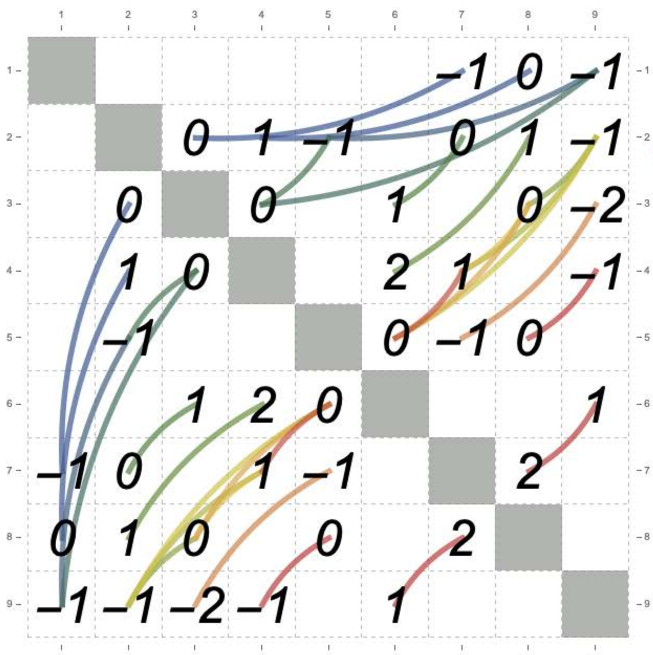


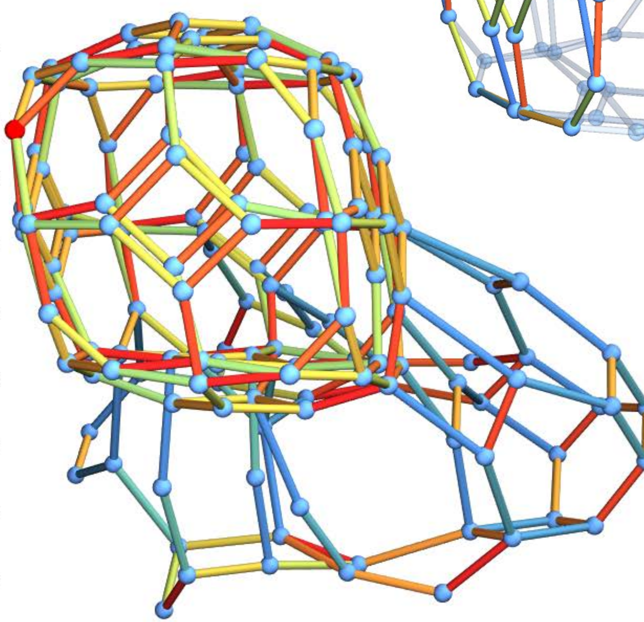


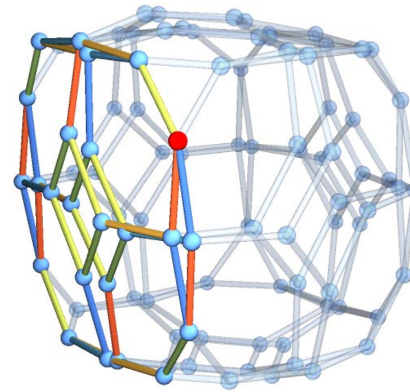
Colorful Knot Symmetries

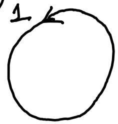
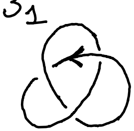
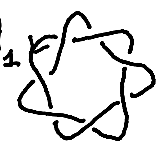
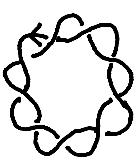
6_1







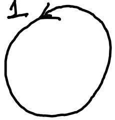


Unknot	N_s	N_e
0_1 	0	1
Torus Knots		
3_1 	0	1
5_1 	2	3
7_1 	8	13
9_1 	20	68

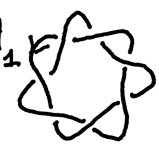
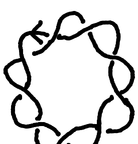
N_s: # of symms.

N_Q: # of equivalent quivers.

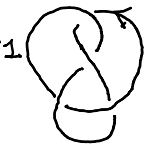
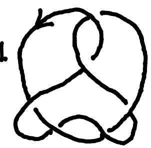
Unknot

Knot		N_s	N_e
0_1		0	1

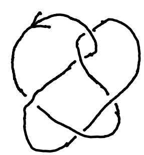
Torus Knots

Knot		N_s	N_e
3_1		0	1
5_1		2	3
7_1		8	13
9_1		20	68

Twist Knots I

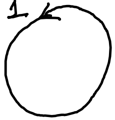
Knot		N_s	N_e
4_1		1	2
6_1		16	141
8_1		61	36,555

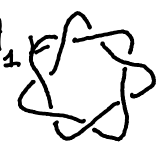
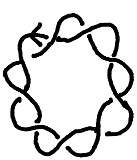
Twist Knots II

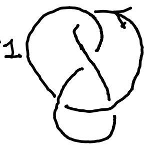
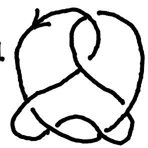
Knot		N_s	N_e
5_2		6	12
7_2		34	1,983

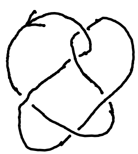
N_s: # of symms.

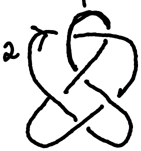
N_Q: # of equivalent quivers.

Unknot	N_s	N_e
0_1 	0	1

Torus knots	N_s	N_e
3_1 	0	1
5_1 	2	3
7_1 	8	13
9_1 	20	68

Twist knots I	N_s	N_e
4_1 	1	2
6_1 	16	141
8_1 	61	36,555

Twist knots II	N_s	N_e
5_2 	6	12
7_2 	34	1,983

Separate examples	N_s	N_e
6_2 	36	3,534
6_3 	72	142,368
7_3 	67	109,636

N_s: # of symms.

N_Q: # of equivalent quivers.

Conclusions & Outlook

1) Knots-Quivers (3D-3D) correspondence is many-to-one.

Conclusions & Outlook

1) Knots-Quivers (3D-3D) correspondence is many-to-one.

2) Physically the multiplicity implies dualities between 3D $N=2$ thrs.

Conclusions & Outlook

1) Knots-Quivers (3D-3D) correspondence is many-to-one.

2) Physically the multiplicity implies dualities between 3D $N=2$ thrs.

3) All equivalent 3D thrs are related by permutat. of ceirtain elements of the quiver matrix.

Conclusions & Outlook

1) Knots-Quivers (3D-3D) correspondence is many-to-one.

2) Physically the multiplicity implies dualities between 3D $\mathcal{N}=2$ thrs.

3) All equivalent 3D thrs are related by permutat. of ceirtain elements of the quiver matrix.

4) Graph with all possible quivers for a given Knot is a complicated gluing of permutahedrons.

Conclusions & Outlook

1) Is there a formula for the # of dual 3D thrs. for a given knot?

Conclusions & Outlook

1) Is there a formula for the # of dual 3D thrs. for a given knot?

2) Besides dualities of 3D thrs. are there other physical implications of such symms.?

Conclusions & Outlook

1) Is there a formula for the # of dual 3D thrs. for a given knot?

2) Besides dualities of 3D thrs. are there other physical implications of such symms.?

3) What are the interpretations of these symms. in other systems (Potts, Anyons and Integrability)?

Conclusions & Outlook

1) Is there a formula for the # of dual 3D thrs. for a given knot?

2) Besides dualities of 3D thrs. are there other physical implications of such symms.?

3) What are the interpretations of these symms. in other systems (Potts, Anyons and Integrability)?

4) Is there a simple way to construct the symm. graphs for a general knot?

Conclusions & Outlook

1) Is there a formula for the # of dual 3D thrs. for a given knot?

2) Besides dualities of 3D thrs. are there other physical implications of such symms.?

3) What are the interpretations of these symms. in other systems (Potts, Anyons and Integrability)?

4) Is there a simple way to construct the symm. graphs for a general knot?

Thank You :)