Correlation functions of determinant operators in conformal fishnet theory

Edoardo Vescovi

NORDITA

MARIE CURIE ACTIONS

2110.09458 with Omar Shahpo

IGST, 25.7.2022

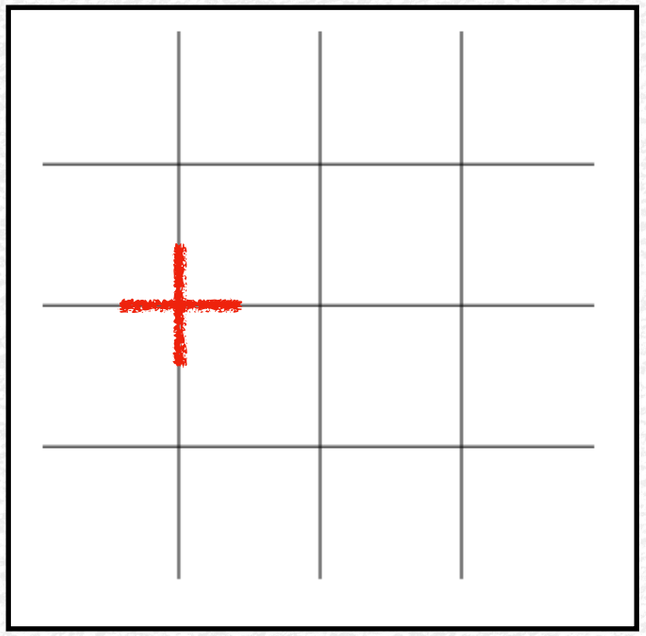
Fishnet

Fishnet vertex sets basis for:

- integrability [Zamolodchikov 80] (TBA, QSC, hexagon, …)

- CFT in 4d

…



bi-scalar fishnet

[Gürdoğan, Kazakov 15]

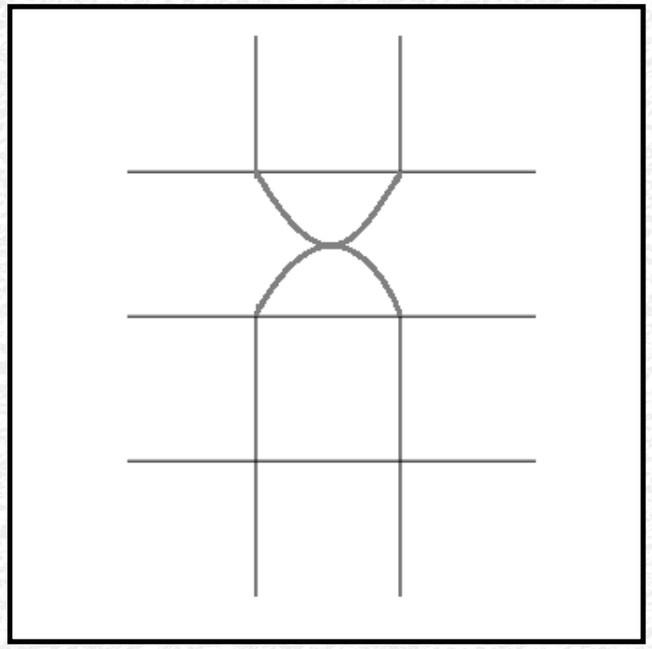
Fishnet

Fishnet vertex sets basis for:

- integrability [Zamolodchikov 80] (TBA, QSC, hexagon, …)

- CFT in 4d

…



bi-scalar fishnet

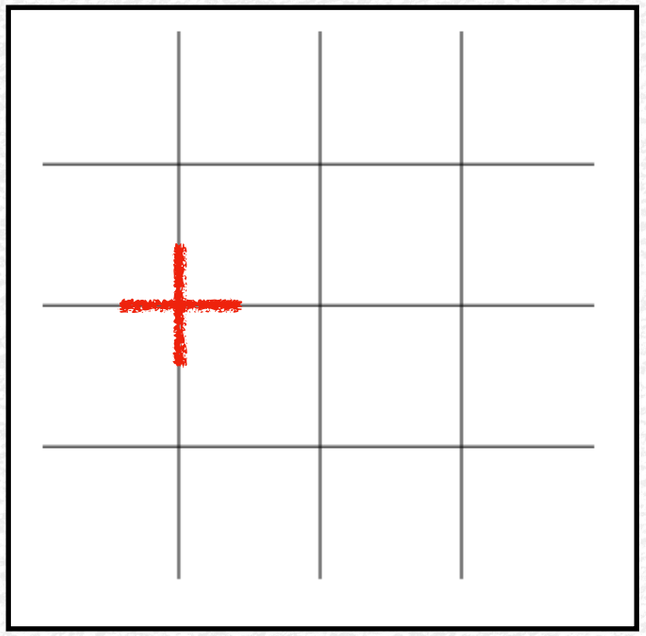
[Gürdoğan, Kazakov 15]

Fishnet

Fishnet vertex sets basis for:

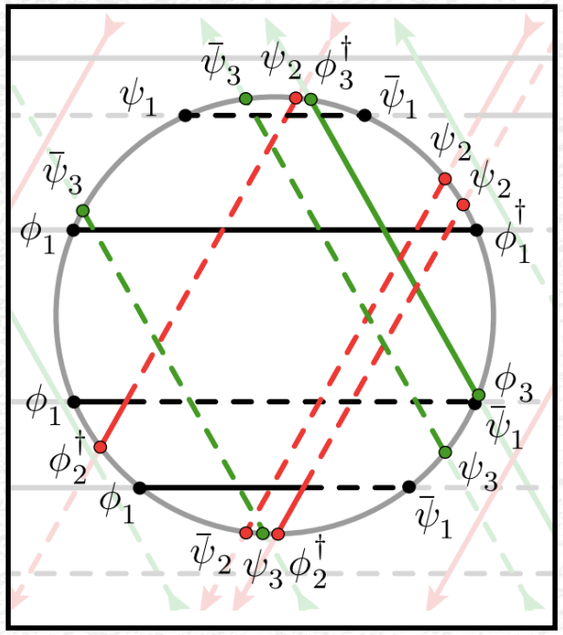
- integrability [Zamolodchikov 80] (TBA, QSC, hexagon, …)
- CFT in 4d

…



bi-scalar fishnet

[Gürdoğan, Kazakov 15]



3-coupling fishnet

= double-scaling limit of γ-deformed $\mathcal{N} = 4$ SYM

[Leigh, Strassler 95]
[Frolov 05]
[Beisert, Roiban 05]

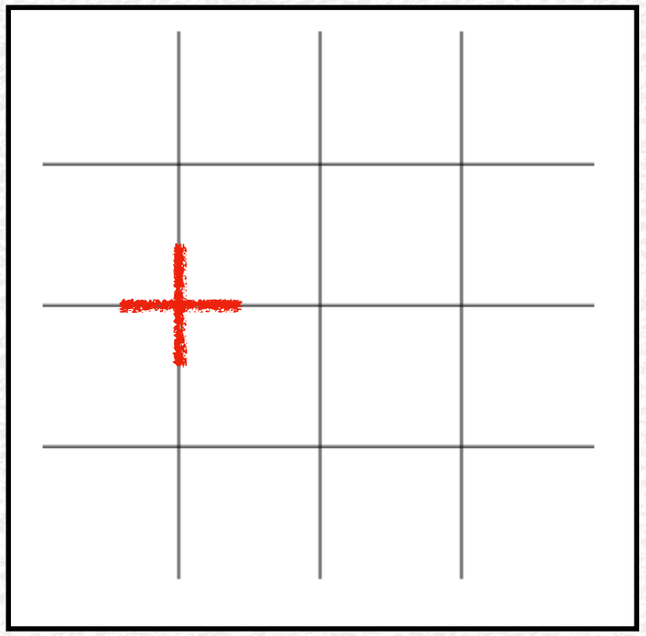
picture courtesy of [Kazakov, Olivucci, Preti 19]

Fishnet

Fishnet vertex sets basis for:

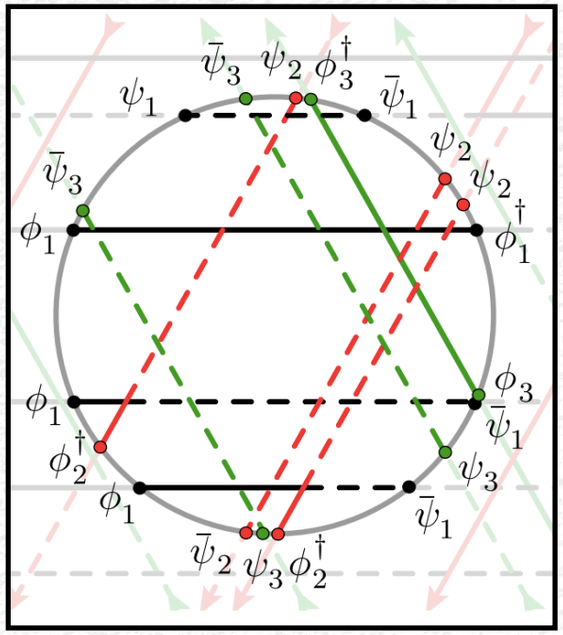
- integrability [Zamolodchikov 80] (TBA, QSC, hexagon, …)
- CFT in 4d

…



bi-scalar fishnet

[Gürdoğan, Kazakov 15]



3-coupling fishnet

= double-scaling limit of γ-deformed $\mathcal{N} = 4$ SYM

[Leigh, Strassler 95]
[Frolov 05]
[Beisert, Roiban 05]

Operators with dimension $\Delta \gg 1$?

picture courtesy of [Kazakov, Olivucci, Preti 19]

Motivation

$$\det \Phi \propto \varepsilon_{i_1 \ldots i_N} \varepsilon^{j_1 \cdots j_N} \Phi^{i_1}_{\ j_1} \Phi^{i_2}_{\ j_2} \ldots \Phi^{i_N}_{\ j_N} \qquad \Phi = y^I \phi_I$$

$$y^I y_I = 0 \quad \longrightarrow \quad \textbf{1/2-BPS}$$

Dual to maximal giant graviton D3-branes [McGreevy, Susskind, Toumbas 00]
[Balasubramanian, Berkooz, Naqvi, Strassler 01] [Corley, Jevicki, Ramgoolam 01]

Motivation

$$\det \Phi \propto \varepsilon_{i_1 \ldots i_N} \varepsilon^{j_1 \cdots j_N} \Phi^{i_1}_{\ j_1} \Phi^{i_2}_{\ j_2} \ldots \Phi^{i_N}_{\ j_N} \qquad \Phi = y^I \phi_I$$

$$y^I y_I = 0 \quad \longrightarrow \quad \textbf{1/2-BPS}$$

Dual to maximal giant graviton D3-branes [McGreevy, Susskind, Toumbas 00]
[Balasubramanian, Berkooz, Naqvi, Strassler 01] [Corley, Jevicki, Ramgoolam 01]

Tree-level SoV-type formula [Jiang, Komatsu, EV 19]

$$\langle \det \det \ \mathrm{tr}(ZZXZ\ldots) \rangle \sim \prod_k \int dx_k \frac{Q_{\mathbf{u}}(x_k) Q_{\mathbf{u}}(-x_k)}{Q^+_{\mathbf{0}}(x_k) Q^-_{\mathbf{0}}(x_k)} \qquad Q_{\mathbf{u}}(x) = \prod_k (x^2 - u_k^2)$$

cf. [Cavaglià, Gromov, Levkovich-Maslyuk 18] [Giombi, Komatsu 18] […]

SoV-observables are under full quantum control. Further lessons from fishnet?

[Cavaglià, Gromov, Levkovich-Maslyuk, Ryan, Volin, … 19-22]

Motivation

$$\varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}\,Z^{i_1}_{\ j_1}\ldots Z^{i_{N-1}}_{\ j_{N-1}}(XXZY\ldots)^{i_N}_{\ j_N}$$

Spectrum via integrability [Berenstein, Correa, Vazquez 05] [Hofman, Maldacena 07] [Bajnok, Drukker, Hegedüs, Nepomechie, Palla, Sieg, Suzuki 13] […]

Open strings on giant: match string energies/dimensions [Balasubramanian, Huang, Levi, Naqvi 02] [Berenstein, Herzog, Klebanov 02] […]

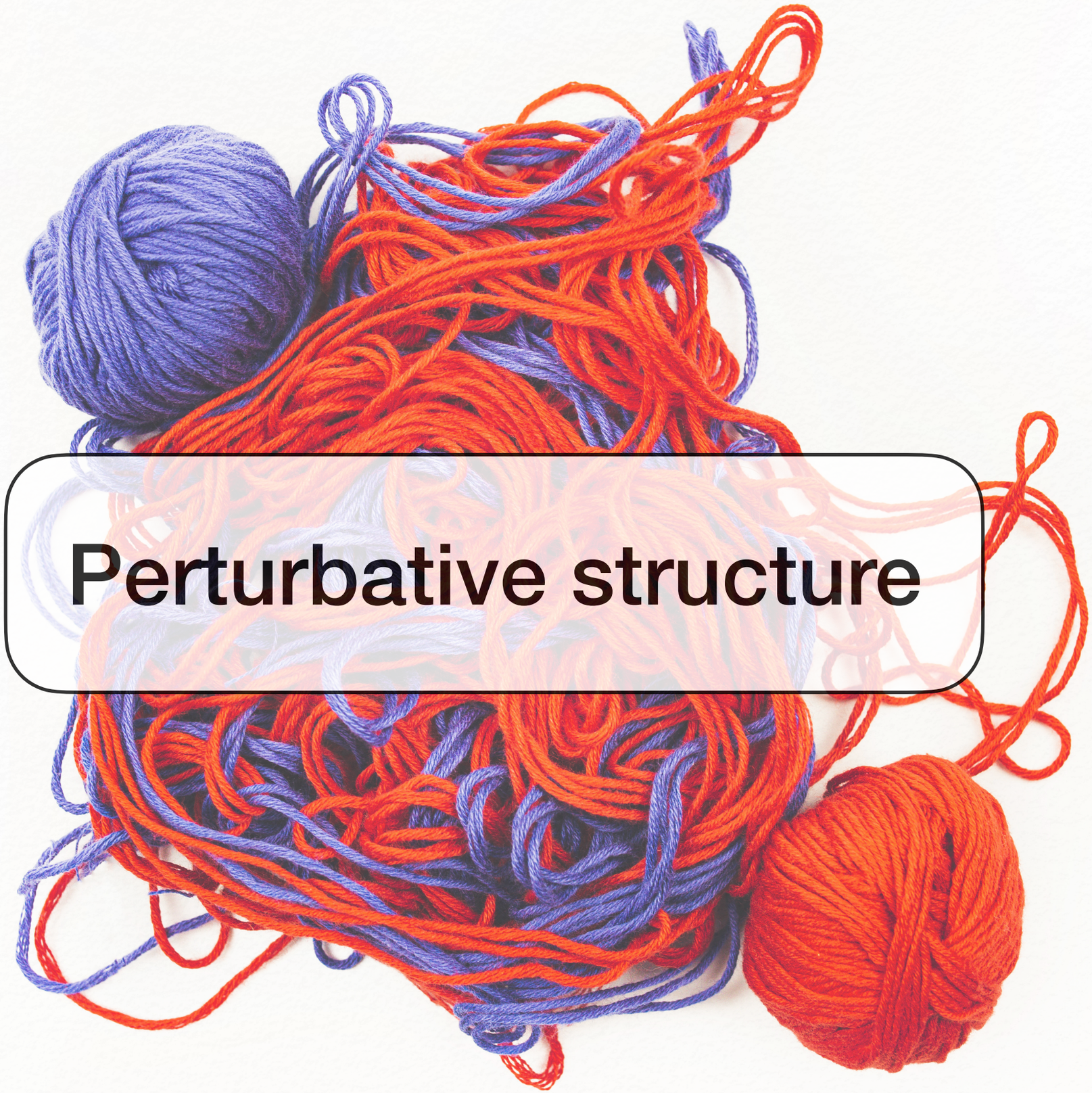
Fishnet holography for open strings + branes? [Basso, Zhong 18] [Gromov, Sever 19]

Resummation techniques for large-N QFT?

$$\det\Phi \propto \varepsilon_{i_1,\ldots i_N}\varepsilon^{j_1,\ldots j_N}\,\Phi^{i_1}_{\ j_1}\ldots\Phi^{i_N}_{\ j_N}$$

baryons $\varepsilon^{i_1\cdots i_N}q_{i_1}\ldots q_{i_N}$

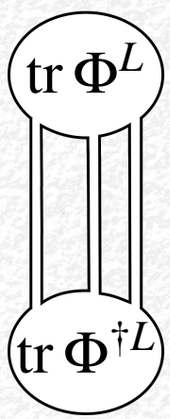
[Witten 79,83]



Perturbative structure

Conformality at large N

Dets lack standard notion of planarity



vs

Decompose $\det\Phi = \mathrm{tr}(\Phi^N) + \ldots + [\mathrm{tr}(\Phi^2)]^{N/2}$

Number $\sim N!$ of non-planar diagrams can

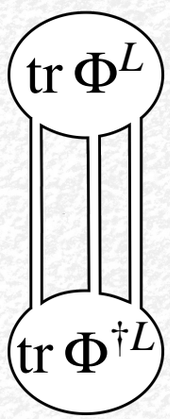
overwhelm $1/N$ suppression

correlator = (planar and non-planar diagrams) $\times N^0$ + $O(N^{-2})$

[Balasubramanian, Berkooz, Naqvi, Strassler 01] [Witten 98]

Conformality at large N

Dets lack standard notion of planarity



vs

Decompose $\det\Phi = \mathrm{tr}(\Phi^N) + \ldots + [\mathrm{tr}(\Phi^2)]^{N/2}$

Number $\sim N!$ of non-planar diagrams can

overwhelm $1/N$ suppression

correlator = (planar and non-planar diagrams) $\times N^0$ + $O(N^{-2})$

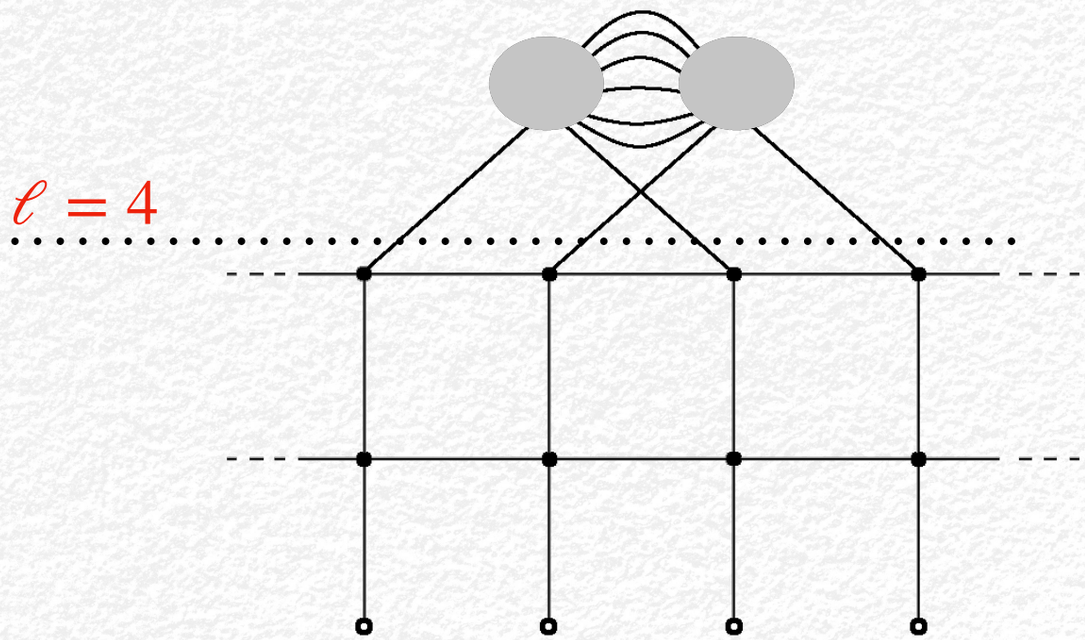
[Balasubramanian, Berkooz, Naqvi, Strassler 01] [Witten 98]

Conformality well-established only for planar fishnet

Dets correlators conformal? Non-planar counter-terms?

Conformality at large N

$$\det(\Phi)\det(\Phi^\dagger)\Big|_{\text{partial free contractions}} = \frac{1}{x^{2N}} + \frac{\mathrm{tr}(\Phi\Phi^\dagger)}{x^{2(N-1)}} + \frac{\mathrm{tr}(\Phi\Phi^\dagger\Phi\Phi^\dagger) + \mathrm{tr}(\Phi\Phi^\dagger)\mathrm{tr}(\Phi\Phi^\dagger)}{x^{2(N-2)}} + \ldots$$



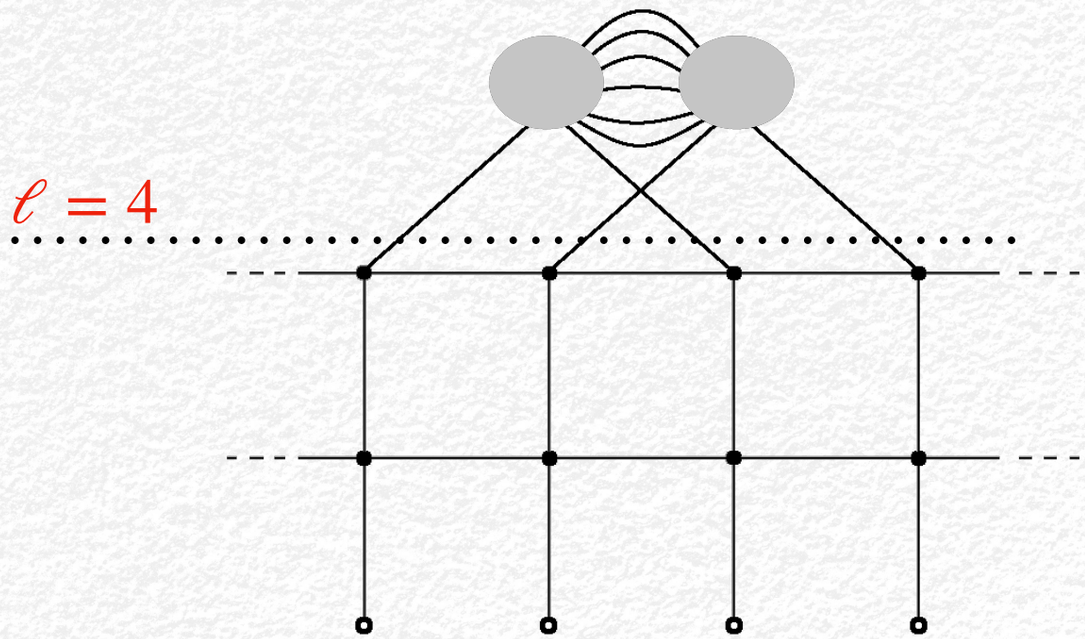
$\mathrm{tr}(\tilde{\Phi}\tilde{\Phi}^\dagger\tilde{\Phi}\tilde{\Phi}^\dagger)$

uniform N-powers

short lengths

Conformality at large N

$$\det(\Phi)\det(\Phi^\dagger)\Big|_{\text{partial free contractions}} = \frac{1}{x^{2N}} + \frac{\mathrm{tr}(\Phi\Phi^\dagger)}{x^{2(N-1)}} + \frac{\mathrm{tr}(\Phi\Phi^\dagger\Phi\Phi^\dagger) + \mathrm{tr}(\Phi\Phi^\dagger)\mathrm{tr}(\Phi\Phi^\dagger)}{x^{2(N-2)}} + \ldots$$



$\mathrm{tr}(\tilde{\Phi}\tilde{\Phi}^\dagger\tilde{\Phi}\tilde{\Phi}^\dagger)$

uniform N-powers

short lengths

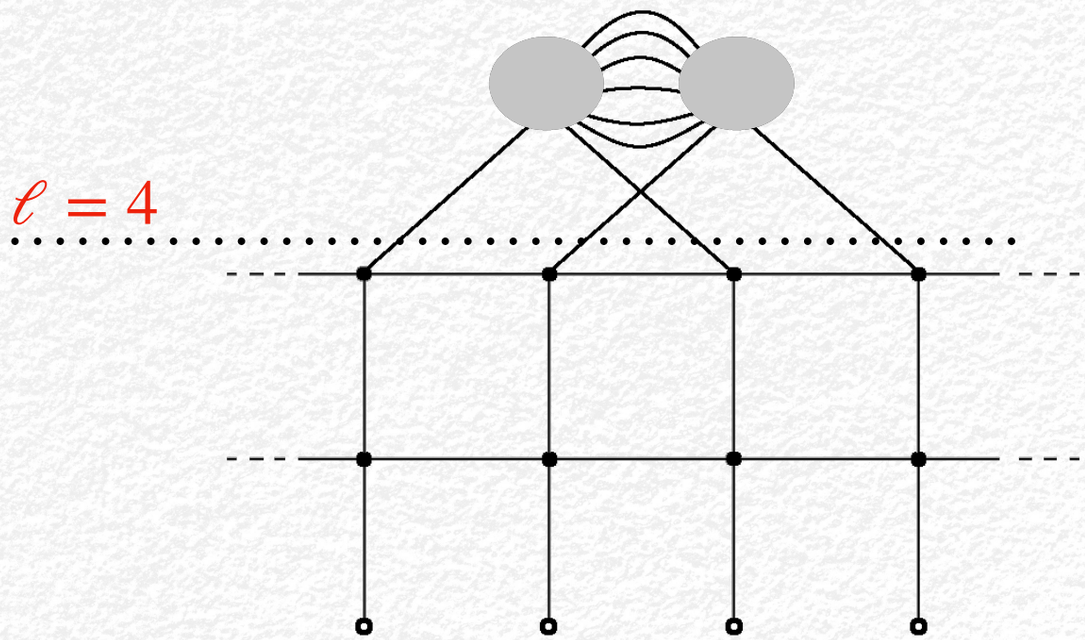
Planar fishnet (regular lattice, sprinkled with *planar* counter-terms, finite, conformal, integrable)

Conformality at large N

$$\det(\Phi)\det(\Phi^\dagger)\Big|_{\text{partial free contractions}} = \frac{1}{x^{2N}} + \frac{\mathrm{tr}(\Phi\Phi^\dagger)}{x^{2(N-1)}} + \frac{\mathrm{tr}(\Phi\Phi^\dagger\Phi\Phi^\dagger) + \mathrm{tr}(\Phi\Phi^\dagger)\mathrm{tr}(\Phi\Phi^\dagger)}{x^{2(N-2)}} + \ldots$$

uniform N-powers

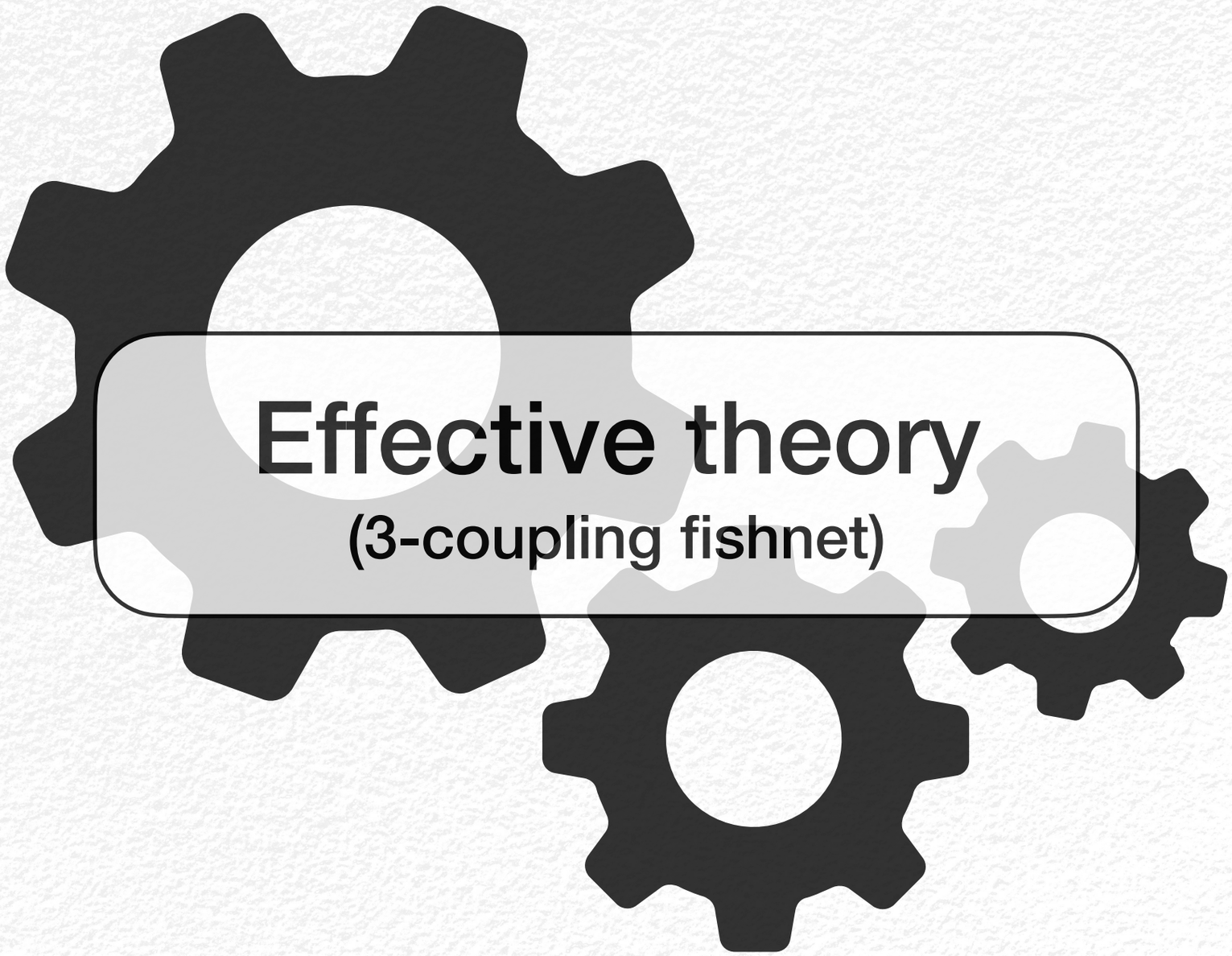
short lengths



$\mathrm{tr}(\tilde{\Phi}\tilde{\Phi}^\dagger\tilde{\Phi}\tilde{\Phi}^\dagger)$

Projection on traces is cumbersome:

- ℓ is unfixed $\Rightarrow$ find relevant intermediate states
- $\ell = 2$ generally contributes $\Rightarrow$ no direct way to exclude double-trace vertices
- 3-coupling fishnet $\Rightarrow$ diagram proliferation



Effective theory

(3-coupling fishnet)

Mathematica code

github.com/edoardo-vescovi/research-papers/tree/main/wick-contractions-fishnet

Effective theory

Rewrite as matrix integral; idea from [Jiang, Komatsu, EV 19]

$$\left\langle \det(y_1 \cdot \Phi(x_1)) \ldots \det(y_m \cdot \Phi(x_m))\, \mathcal{O}(\phi, \psi)(x_{m+1}, \ldots) \right\rangle = \int d\rho \, \left\langle \mathcal{O}^{(S)} \right\rangle_\chi e^{-NS[\rho]}$$

ρ_{kl} = $m \times m$ matrix

$\hat{\rho}_{kl}$ = $\sqrt{y_k \cdot y_l} \left| x_k - x_l \right|^{-1} \rho_{kl}$

$\chi_k^a, \bar{\chi}_{k,a}$ = aux fermions

$k, l = 1,\ldots,m$

$a = 1,\ldots,N = \square, \overline{\square}$ of $SU(N)$

Effective theory

Rewrite as matrix integral; idea from [Jiang, Komatsu, EV 19]

$$\left\langle \det(y_1 \cdot \Phi(x_1)) \ldots \det(y_m \cdot \Phi(x_m))\, \mathcal{O}(\phi,\psi)(x_{m+1},\ldots) \right\rangle = \int d\rho \, \left\langle \mathcal{O}^{(S)} \right\rangle_\chi e^{-NS[\rho]}$$

ρ_{kl} = $m \times m$ matrix

$\hat{\rho}_{kl}$ = $\sqrt{y_k \cdot y_l}\, \left| x_k - x_l \right|^{-1} \rho_{kl}$ $\qquad k,l = 1,\ldots,m$

$\chi_k^a, \bar{\chi}_{k,a}$ = aux fermions $\qquad a = 1,\ldots,N = \square, \overline{\square}$ of $SU(N)$

Action $\quad NS[\rho] = N(\mathrm{tr}\left(\rho^2\right) - \log\det\left(-2\, i\hat{\rho}\right)) \quad \rightarrow$ saddle-pt approx

Integrand $\quad \mathcal{O}^{(S)} = \int D\phi\, D\psi \;\; \mathcal{O}(\phi + S, \psi) \;\; e^{S_{\text{free}}(\phi,\psi) + S_{\text{int}}(\phi+S,\psi)} \quad \rightarrow$ expand perturbatively

Finite-coupling generalisation of free theory [Jiang, Komatsu, EV 19] and 1 loop [EV 21]

Effective theory

Rewrite as matrix integral; idea from [Jiang, Komatsu, EV 19]

$$\left\langle \det(y_1 \cdot \Phi(x_1)) \ldots \det(y_m \cdot \Phi(x_m))\, \mathcal{O}(\phi,\psi)(x_{m+1},\ldots) \right\rangle = \int d\rho \, \left\langle \mathcal{O}^{(S)} \right\rangle_\chi e^{-NS[\rho]}$$

ρ_{kl} = $m \times m$ matrix

$\hat{\rho}_{kl}$ = $\sqrt{y_k \cdot y_l}\, \left| x_k - x_l \right|^{-1} \rho_{kl}$ $\qquad k,l = 1,\ldots,m$

$\chi_k^a, \bar{\chi}_{k,a}$ = aux fermions $\qquad a = 1,\ldots,N = \square, \overline{\square}$ of $SU(N)$

Action $\quad NS[\rho] = N(\mathrm{tr}\left(\rho^2\right) - \log\det\left(-2\, i\hat{\rho}\right)) \quad \rightarrow$ saddle-pt approx

Integrand $\quad \mathcal{O}^{(S)} = \int D\phi\, D\psi \;\; \mathcal{O}(\phi + S, \psi) \;\; e^{S_{\text{free}}(\phi,\psi) + S_{\text{int}}(\phi+S,\psi)} \quad \rightarrow$ expand perturbatively

Finite-coupling generalisation of free theory [Jiang, Komatsu, EV 19] and 1 loop [EV 21]

$$S^{j,a}{}_b = \sum_{k=1}^{m} (y_k^{2j-1} + i y_k^{2j}) \left| x - x_k \right|^{-1} \chi_k^a\, \bar{\chi}_{k,b}$$

$\langle \ldots \rangle_\chi$ via Feynman rule $\bar{\chi}_{k,a}\, \chi_l^b \rightarrow \delta_a^b \left(\hat{\rho}^{-1}\right)_{kl}$

$$\left\langle \mathcal{O}^{(S)} \right\rangle_\chi = \sum_{I_1,\ldots I_L} \mathrm{Tr}\left[M^{I_1} \ldots M^{I_L}\right] \left\langle \Phi^{I_1} \ldots \Phi^{I_L} \middle| \mathcal{O} \right\rangle$$

matrix product state, free SYM [Jiang, Komatsu, EV 19]

Output

Deform inserting 1-letter words

cf. [Basso, Coronado, Komatsu, Lam, Vieira, Zhong 17]

$$\frac{d}{da_1}\det(\phi^2+a_1\phi^1)(x_1)\Big|_{a_1=0} \propto \varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}\left(\phi^1\right)^{i_1}_{\ j_1}\left(\phi^2\right)^{i_2}_{\ j_2}\ldots\left(\phi^2\right)^{i_N}_{\ j_N}$$

$$\frac{d}{da_2}\det(\phi_2^\dagger+a_2\phi^1)(x_2)\Big|_{a_2=0} \propto \varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}\left(\phi^1\right)^{i_1}_{\ j_1}\left(\phi_2^\dagger\right)^{i_2}_{\ j_2}\ldots\left(\phi_2^\dagger\right)^{i_N}_{\ j_N}$$

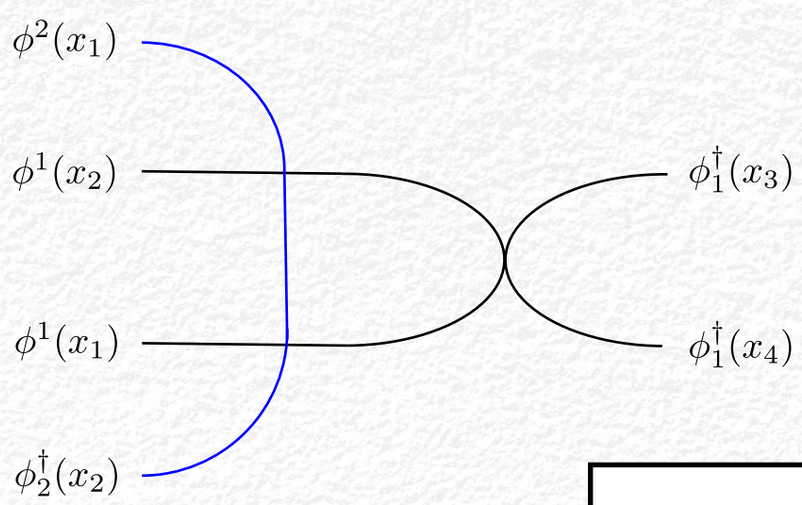
Output

Deform inserting 1-letter words

cf. [Basso, Coronado, Komatsu, Lam, Vieira, Zhong 17]

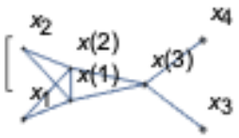
$$\frac{d}{da_1}\det(\phi^2+a_1\phi^1)(x_1)\Big|_{a_1=0} \propto \varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}\left(\phi^1\right)^{i_1}_{\ j_1}\left(\phi^2\right)^{i_2}_{\ j_2}\ldots\left(\phi^2\right)^{i_N}_{\ j_N}$$

$$\frac{d}{da_2}\det(\phi_2^\dagger+a_2\phi^1)(x_2)\Big|_{a_2=0} \propto \varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}\left(\phi^1\right)^{i_1}_{\ j_1}\left(\phi_2^\dagger\right)^{i_2}_{\ j_2}\ldots\left(\phi_2^\dagger\right)^{i_N}_{\ j_N}$$

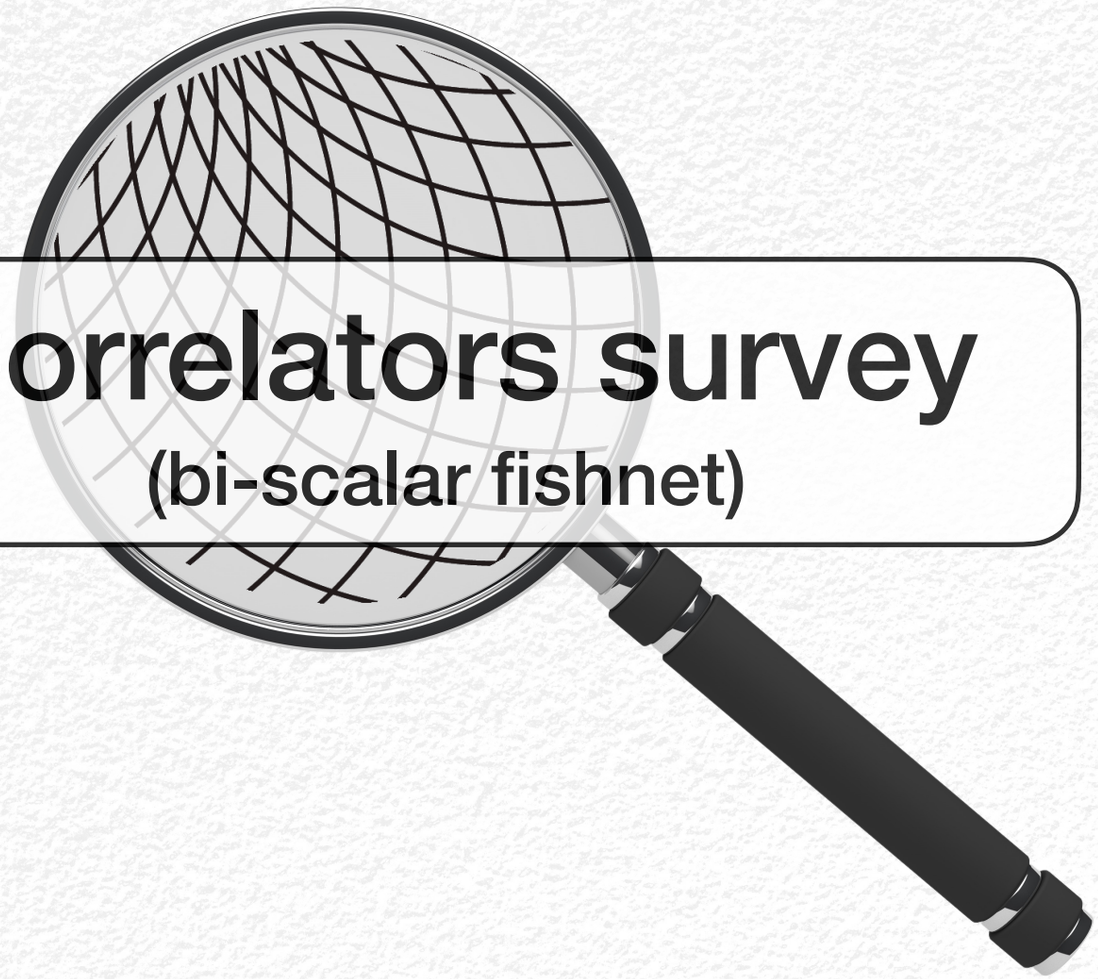


unintegrated diagram

$$\frac{\alpha_1^2\xi^4}{x_{12}^{2N-4}}\int\frac{d^4x_1d^4x_2d^4x_3}{x_{11'}^2x_{21'}^2x_{12'}^2x_{22'}^2x_{1'2'}^2x_{1'3'}^2x_{2'3'}^2x_{33'}^2x_{43'}^2}$$

```
16 384 N^(-2-N) π^6 α1^2 ξ^4 N ! graphSpacetime[  ] II[{x1, x2}]^(-2+N)

II[{x1, x[1]}] × II[{x1, x[2]}] × II[{x2, x[1]}] × II[{x2, x[2]}] × II[{x3, x[3]}] ×
II[{x4, x[3]}] × II[{x[1], x[2]}] × II[{x[1], x[3]}] × II[{x[2], x[3]}]
```



Correlators survey
(bi-scalar fishnet)

Challenges

$\mathcal{N}=4$ SYM	bi-scalar fishnet
R-symmetry $SO(6)$	$U(1)\times U(1)$
	selection of scalars, e.g. keep $y^I y_I = 0$
SUSY	no SUSY
dets are 1/2-BPS, $\Delta = N$	anomalous dimensions? cf. BMN “vacuum” $\mathrm{tr}(\phi_1^L)$
2-,3-pt functions have no quantum corrections	non-trivial spectral problem

Challenges

$\mathcal{N}=4$ SYM	bi-scalar fishnet
R-symmetry $SO(6)$	$U(1)\times U(1)$
	selection of scalars, e.g. keep $y^I y_I = 0$
SUSY	no SUSY
dets are 1/2-BPS, $\Delta = N$	anomalous dimensions? cf. BMN "vacuum" $\mathrm{tr}(\phi_1^L)$
2-,3-pt functions have no quantum corrections	non-trivial spectral problem

Dets can mix with other operators (scalar, same bare dimension and R-charges)
Large number! Effective theory quantifies transitions $\det \leftrightarrow \det$ only!

Assemble 3-,4-pt functions ahead

Higher-point functions ($L = 2$)

$$\left.\begin{array}{l}\varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_2)^{i_1}{}_{j_1}(\phi_2)^{i_2}{}_{j_2}\ldots(\phi_2)^{i_N}{}_{j_N}(x_1)\\[2ex] \varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_2)^{i_1}{}_{j_1}(\phi_2^\dagger)^{i_2}{}_{j_2}\ldots(\phi_2^\dagger)^{i_N}{}_{j_N}(x_2)\end{array}\right\}\sim\frac{1}{x_{12}^{2N-2}}\mathrm{tr}(\phi_2(x_1)\phi_2(x_2))$$

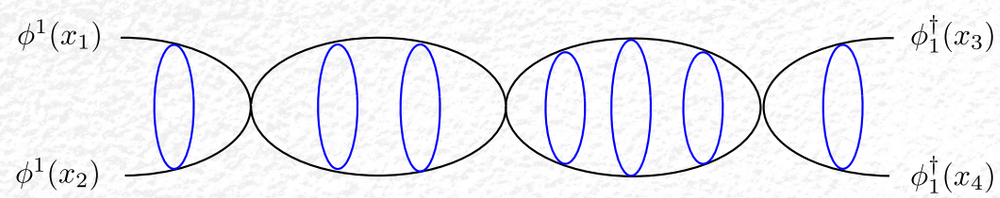
$$\mathrm{tr}(\phi_2^\dagger(x_3)\,\phi_2^\dagger(x_4))$$

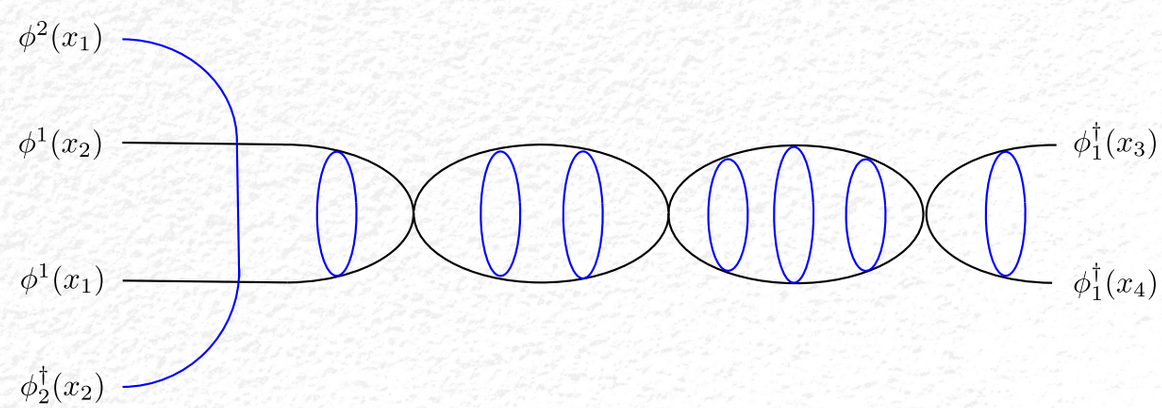
Exact Bethe-Salpeter resummation
[Grabner, Gromov, Kazakov, Korchemsky 17] [Gromov, Kazakov, Korchemsky 18]

$$\varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_1)^{i_1}{}_{j_1}(\phi_2)^{i_2}{}_{j_2}\ldots(\phi_2)^{i_N}{}_{j_N}(x_1)$$

$$\varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_1)^{i_1}{}_{j_1}(\phi_2^\dagger)^{i_2}{}_{j_2}\ldots(\phi_2^\dagger)^{i_N}{}_{j_N}(x_2)$$

$$\mathrm{tr}(\phi_1^\dagger(x_3)\,\phi_1^\dagger(x_4))$$





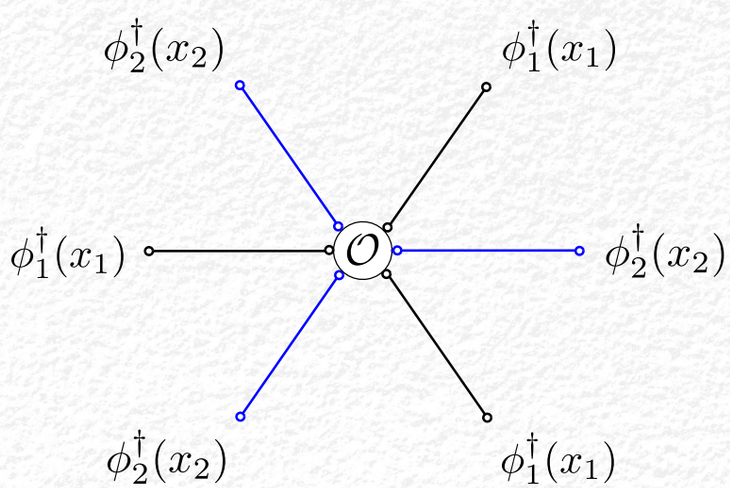
Higher-point functions ($L > 2$)

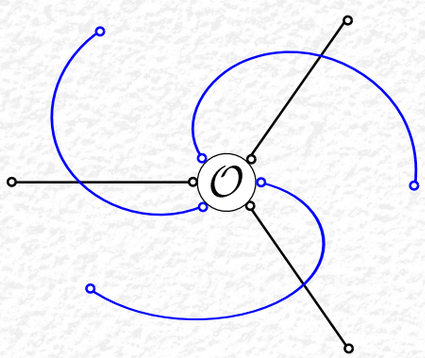
$$\varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi^2+\phi_1^\dagger)^{i_1}{}_{j_1}\ldots(\phi^2+\phi_1^\dagger)^{i_{L/2}}{}_{j_{L/2}}(\phi^2)^{i_{L/2+1}}{}_{j_{L/2+1}}\ldots(\phi^2)^{i_N}{}_{j_N}(x_1)$$

$$\varepsilon_{i_1\ldots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_2^\dagger)^{i_1}{}_{j_1}(\phi_2^\dagger)^{i_2}{}_{j_2}\ldots(\phi_2^\dagger)^{i_N}{}_{j_N}(x_2)$$

Multi-magnon state $\quad \mathrm{tr}(\phi^1(x_3)\phi^2(x_3))^{L/2}$

[Gürdoğan, Kazakov 15] [Caetano, Gürdoğan, Kazakov 16]





Different from spectral problem:

- alternating boundary $x_1x_2x_1x_2\ldots$
- observable is renormalised structure constant

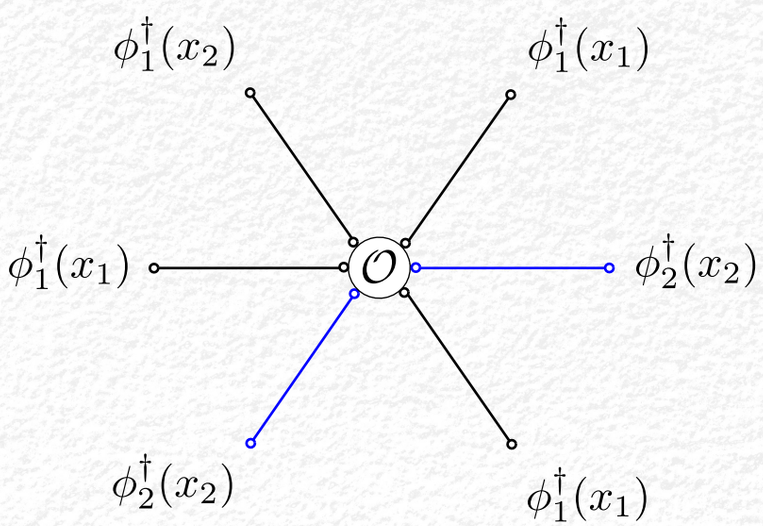
Higher-point functions ($L > 2$)

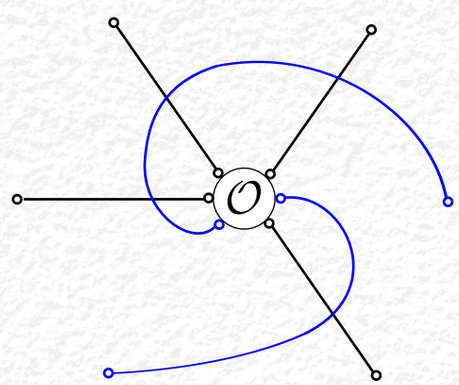
$$\varepsilon_{i_1\dots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_1^\dagger)^{i_1}_{\ j_1}\dots(\phi_1^\dagger)^{i_{L/2-M}}_{\ j_{L/2-M}}(\phi^2)^{i_{L/2-M+1}}_{\ j_{L/2-M+1}}\dots(\phi^2)^{i_N}_{\ j_N}(x_1)$$

$$\varepsilon_{i_1\dots i_N}\,\varepsilon^{j_1\cdots j_N}(\phi_1^\dagger)^{i_1}_{\ j_1}\dots(\phi_1^\dagger)^{i_{L/2}}_{\ j_{L/2}}(\phi_2^\dagger)^{i_{L/2+1}}_{\ j_{L/2+1}}\dots(\phi_2^\dagger)^{i_N}_{\ j_N}(x_2)$$

Multi-magnon state $\quad \mathcal{O} = \mathrm{tr}(\phi^1\dots\phi^1\phi^2\,\underbrace{\phi^1\dots\phi^1}_{\text{odd}}\,\phi^2\phi^1\dots\phi^1)(x_3)$

[Gürdoğan, Kazakov 15] [Caetano, Gürdoğan, Kazakov 16]





Different from spectral problem:

- alternating boundary $x_1x_2x_1x_2\dots$
- observable is renormalised structure constant

Conclusion

First study of operators of R-charge $\sim N$ in planar γ-deformed $\mathcal{N} = 4$ SYM in fishnet limit

- solve spectral problem $\sim$ measure baryons' mass
- exhaust survey, access conformal data
- Bethe-Salpeter graph-building operators for length-L traces $\sim$ diagonalise L-site spin chain for Basso-Dixon integral

[+ Derkachov, Ferrando, Kazakov, Olivucci, … 18-21]

Conclusion

First study of operators of R-charge $\sim N$ in planar γ-deformed $\mathcal{N} = 4$ SYM in fishnet limit

- solve spectral problem $\sim$ measure baryons' mass
- exhaust survey, access conformal data
- Bethe-Salpeter graph-building operators for length-L traces $\sim$ diagonalise L-site spin chain for Basso-Dixon integral

[+ Derkachov, Ferrando, Kazakov, Olivucci, … 18-21]

Search for integrable boundary states

- only giants seem integrable [Chen, de Mello Koch, Kim, Van Zyl 19]
- open strings on maximal giants [Berenstein, Vazquez 05] [Hofman, Maldacena 07]
- open strings on non-maximal giants? [Berenstein, Correa, Vazquez, 06] [Ciavarella 10]

Conclusion

First study of operators of R-charge $\sim N$ in planar γ-deformed $\mathcal{N}=4$ SYM in fishnet limit

- solve spectral problem $\sim$ measure baryons' mass
- exhaust survey, access conformal data
- Bethe-Salpeter graph-building operators for length-L traces $\sim$ diagonalise L-site spin chain for Basso-Dixon integral
  [+ Derkachov, Ferrando, Kazakov, Olivucci, … 18-21]

Search for integrable boundary states

- only giants seem integrable [Chen, de Mello Koch, Kim, Van Zyl 19]
- open strings on maximal giants [Berenstein, Vazquez 05] [Hofman, Maldacena 07]
- open strings on non-maximal giants? [Berenstein, Correa, Vazquez, 06] [Ciavarella 10]

Other fishnets

- $\mathcal{N}=2$ SCFTs: integrable sector [Pittelli, Preti 19]
- ABJM: (sub-)determinants seem integrable [Chen, De Mello Koch, Kim, Van Zyl 19] [Yang, Jiang, Komatsu, Wu 21]

Conclusion

First study of operators of R-charge $\sim N$ in planar γ-deformed $\mathcal{N}=4$ SYM in fishnet limit

- solve spectral problem $\sim$ measure baryons' mass
- exhaust survey, access conformal data
- Bethe-Salpeter graph-building operators for length-L traces $\sim$ diagonalise L-site spin chain for Basso-Dixon integral [+ Derkachov, Ferrando, Kazakov, Olivucci, … 18-21]

Search for integrable boundary states

- only giants seem integrable [Chen, de Mello Koch, Kim, Van Zyl 19]
- open strings on maximal giants [Berenstein, Vazquez 05] [Hofman, Maldacena 07]
- open strings on non-maximal giants? [Berenstein, Correa, Vazquez, 06] [Ciavarella 10]

Other fishnets

- $\mathcal{N}=2$ SCFTs: integrable sector [Pittelli, Preti 19]
- ABJM: (sub-)determinants seem integrable [Chen, De Mello Koch, Kim, Van Zyl 19] [Yang, Jiang, Komatsu, Wu 21]

Holography

- 4-pt functions scale semi-classically at large coupling?
- open strings from quantisation of fluctuations around large R-charge/momentum states [Balasubramanian, Huang, Levi, Naqvi 02]

Extra slides

Derivation

Integrate in $\chi, \bar{\chi}$

$$\left\langle \det(y_1 \cdot \Phi) \ldots \det(y_m \cdot \Phi)\, \mathcal{O} \right\rangle \sim \int D\phi\, D\psi\, d\chi\, d\bar{\chi}\;\; \mathcal{O}\;\; e^{S_{\text{free}} + S_{\text{int}} + \sum_{k=1}^{m} \bar{\chi}(y_k \cdot \Phi)\chi}$$

Shift $\phi \to \phi + S$ and integrate out ϕ, ψ

$$\int d\chi\, d\bar{\chi} \underbrace{\int D\phi\, D\psi\;\; \mathcal{O}(\phi + S, \psi)\;\; e^{S_{\text{free}}(\phi,\psi) + S_{\text{int}}(\phi + S, \psi)}\;\; e^{S^\dagger \Box S}}_{\mathcal{O}^{(S)}}$$

Integrate in ρ

$$\int d\rho\, d\chi\, d\bar{\chi}\;\; \mathcal{O}^{(S)}\;\; e^{\rho \bar{\chi} \chi}$$

Integrate out $\chi, \bar{\chi}$

$$\int d\rho\;\; \left\langle \mathcal{O}^{(S)} \right\rangle_{\chi}\;\; e^{-NS[\rho]}$$