Mirror Thermodynamic Bethe Ansatz for AdS3/CFT2

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Plan

1. Quick review of AdS3/CFT2
2. The (factorised) S matrix
3. The mirror model
4. The mirror Thermodynamic Bethe Ansatz
5. Conclusions and Outlook

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Maximally Supersymmetric AdS3 backgrounds

These backgrounds have 16 susys (**half of AdS5**):

$$AdS_3 \times S^3 \times T^4, \qquad AdS_3 \times S^3 \times K3, \qquad AdS_3 \times S^3 \times S^3 \times S^1$$

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The background has 20 moduli. [Larsen, Martinec '99]

For the closed-string spectrum, **only B-field (and tension) matter**. [Ohlsson-Sax, Stefanski '18]

Two-parameter classically integrable NLSM. [Cagnazzo, Zarembo '12]

Interpolating from zero to maximal B-field

No B field

– Most similar to AdS5

– From D1-D5 system

– Unknown dual

– Continuous tension $h > 0$

– Nondegenerate spectrum

– Spectrum through TBA

[Frolov, AS '21]

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– $sl(2) \oplus su(2)$ WZW_k model

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– Partially known dual

– Quantised tension $k \in \mathbb{N}$

– Simple, degenerate spectrum

– Can do TBA too

[Maldacena, Ooguri '00][...][Dei, AS '18]

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Intermediate case

– Hardest case

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– "Only" S-matrix so far

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This talk!

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Symmetries and lightcone-gauge fixing

The symmetries of $AdS_3 \times S^3 \times T^4$ contain

$$\mathfrak{so}(2,2) \oplus \mathfrak{so}(4) \cong \mathfrak{su}(1,1)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 2} \subset \mathfrak{psu}(1,1|2)^{\oplus 2}$$

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Irreducible representations are labeled by $(H, M, C, C^\dagger)$.

Particles (short unitary representations)

We get the conditions

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Single-particle dispersion relation ($h \geq 0$ is the tension) [Borsato, Ohlsson Sax, Stefanski, AS, Torrielli '13]

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They must be four-dimensional (B=Boson, F=Fermion)

$$\begin{tikzcd}[column sep=small] & |B\rangle \arrow[dl] \arrow[dr] & \\ |F^1\rangle \arrow[dr] & & |F^2\rangle \arrow[dl] \\ & |B'\rangle & \end{tikzcd} \qquad \text{or} \qquad \begin{tikzcd}[column sep=small] & |F\rangle \arrow[dl] \arrow[dr] & \\ |B^1\rangle \arrow[dr] & & |B^2\rangle \arrow[dl] \\ & |F'\rangle & \end{tikzcd}$$

Left, right and massless particles

For a given tension $h \geq 0$,

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  - "**chiral**" if $H(p) = +2h \sin(p/2)$, i.e. $0 < p < \pi$,
  - "**antichiral**" if $H(p) = -2h \sin(p/2)$, i.e. $-\pi < p < 0$. [Frolov, AS '21]

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For each pair of irreps we can construct an S-matrix block:

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The dressing factors are not all independent, due to unitarity, parity, and LR symmetry.

Independent S-matrix elements

Dressing factor	Particles scattered
$\sigma^{\bullet\bullet}(p_1, p_2)$	$M_1 = M_2 = \pm 1.$
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Zhukovski variables

Instead of using momenta, we introduce

$$x^{\pm}(p,|M|) = e^{\pm ip/2}\frac{|M| + \sqrt{M^2 + 4h^2\sin^2(p/2)}}{2h\sin(p/2)}\,.$$

that satisfy

$$e^{ip} = \frac{x_p^+}{x_p^-}\,, \qquad H = \frac{h}{2i}\left(x_p^+ - \frac{1}{x_p^+} - x_p^- + \frac{1}{x_p^-}\right)\,,$$

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Crossing:

$$x_p^{\pm} \to \bar{x}_p^{\pm} = \frac{1}{x_p^{\pm}}\,, \qquad x_p \to \bar{x}_p = \frac{1}{x_p}$$

Crossing equations

$$\left(\sigma^{\bullet\bullet}(x_1^{\pm},x_2^{\pm})\right)^{-2}\left(\tilde{\sigma}^{\bullet\bullet}(\bar{x}_1^{\pm},x_2^{\pm})\right)^{-2}=\left(\frac{x_2^+}{x_2^-}\right)^2\frac{(x_1^--x_2^-)(x_1^+-x_2^+)}{(x_1^--x_2^+)^2}\frac{1-\frac{1}{x_1^+x_2^-}}{1-\frac{1}{x_1^-x_2^+}}\,,$$

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$$\left(\sigma^{\bullet\circ}(x_1^\pm, x_2)\right)^{-2}\left(\sigma^{\bullet\circ}(\bar{x}_1^\pm, x_2)\right)^{-2} = (x_2)^4 \frac{f(x_1^-, x_2)}{f(x_1^+, x_2)},$$

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Some (old) observations

We expect that, like in $AdS_5 \times S^5$, the **BES phase** should appear. Introduce:

$$\varsigma^{\bullet\bullet}(x_1^\pm, x_2^\pm) = \frac{\sigma^{\bullet\bullet}(x_1^\pm, x_2^\pm)}{\sigma_{\text{BES}}(x_1^\pm, x_2^\pm)}\,, \qquad \tilde{\varsigma}^{\bullet\bullet}(x_1^\pm, x_2^\pm) = \frac{\tilde{\sigma}^{\bullet\bullet}(x_1^\pm, x_2^\pm)}{\sigma_{\text{BES}}(x_1^\pm, x_2^\pm)}\,,$$

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Then we have simply

$$\Big(\varsigma^{+}(\bar{x}_1^\pm, x_2^\pm)\Big)^{-2}\Big(\varsigma^{+}(x_1^\pm, x_2^\pm)\Big)^{-2} = \frac{f(x_1^+, x_2^+)\, f(x_1^-, x_2^-)}{f(x_1^+, x_2^-)\, f(x_1^-, x_2^+)}\,,$$

$$\Big(\varsigma^{-}(\bar{x}_1^\pm, x_2^\pm)\Big)^{-2} = \Big(\text{something simple}\Big)\Big(\varsigma^{-}(x_1^\pm, x_2^\pm)\Big)^{-2}\,,$$

Another parametrisation for massless modes

It is convenient to introduce [Fontanella, Torrielli '19]

$$\gamma : \quad x = \frac{i - e^{\gamma}}{i + e^{\gamma}} , \qquad H(\gamma) = \frac{2h}{\cosh \gamma} .$$

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In these terms, massless S matrix is of **difference form**.

Moreover, the crossing equation is almost (up to a sign) the one of **Sine-Gordon**.

The “old” (incorrect) solution to crossing

Previous proposal for the phases [Borsato, Ohlsson Sax, Sfondrini, Stefanski, Torrielli ’14 –’16]

- Massive phases are give by the BES phase plus “bits and pieces” (like HL phase)
- Massless and mixed-mass are given by HL phase
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Problems:

- Only partial match with perturbation theory (maybe ok?)
- Strange analytic properties
- Match with Sine-Gordon up to an "i"
- Actually, **violates worldsheet parity invariance**.

New proposal

Intuition: in the loops, massless and massive are not separate. Treat them on the same footing.

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Assume there is a **BES factor everywhere**: not only

$$\varsigma^{\bullet\bullet}(x_1^\pm, x_2^\pm) = \frac{\sigma^{\bullet\bullet}(x_1^\pm, x_2^\pm)}{\sigma_{\text{BES}}(x_1^\pm, x_2^\pm)}\,, \qquad \tilde{\varsigma}^{\bullet\bullet}(x_1^\pm, x_2^\pm) = \frac{\tilde{\sigma}^{\bullet\bullet}(x_1^\pm, x_2^\pm)}{\sigma_{\text{BES}}(x_1^\pm, x_2^\pm)}\,,$$

but also:

$$\varsigma^{\bullet\circ}(x_1^\pm, x_2) = \frac{\sigma^{\bullet\circ}(x_1^\pm, x_2)}{\sigma_{\text{BES}}(x_1^\pm, x_2)}\,, \qquad \varsigma^{\circ\bullet}(x_1, x_2^\pm) = \frac{\sigma^{\circ\bullet}(x_1, x_2^\pm)}{\sigma_{\text{BES}}(x_1, x_2^\pm)}\,,$$

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New massive rapidities

Introduce rapidity variables for massive particles too: [Beisert, Hernandez, Lopez '06]

$$x^+ = \frac{i - e^{\gamma^+}}{i + e^{\gamma^+}}\,, \qquad x^- = \frac{i + e^{\gamma^-}}{i - e^{\gamma^-}}\,.$$

Crossing:

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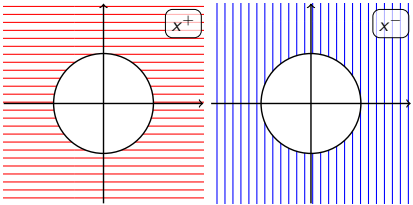

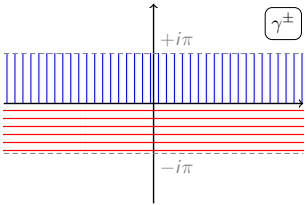

New form of the crossing equations

$$\left(\varsigma^{+}(\bar{\gamma}_1^{\pm},\gamma_2^{\pm})\right)^{-2}\left(\varsigma^{+}(\gamma_1^{\pm},\gamma_2^{\pm})\right)^{-2} = \coth\frac{\gamma_{12}^{++}}{2}\coth\frac{\gamma_{12}^{+-}}{2}\coth\frac{\gamma_{12}^{-+}}{2}\coth\frac{\gamma_{12}^{--}}{2}.$$

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Solution for the rescaled factors $\varsigma^{\pm}$

For massive modes we have

$$\left(\varsigma^{+}(x_1^{\pm}, x_2^{\pm})\right)^{-2} = -\frac{\tanh\frac{\gamma_{12}^{-+}}{2}}{\tanh\frac{\gamma_{12}^{+-}}{2}}\,\Phi(\gamma_{12}^{--})\Phi(\gamma_{12}^{++})\Phi(\gamma_{12}^{-+})\Phi(\gamma_{12}^{+-})\,,$$

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with

$$\frac{d}{d\gamma}\log\Phi(\gamma) = \frac{i\gamma}{\pi\sinh\gamma}\,, \qquad \log\Phi(0) = 0\,, \qquad \text{(Sine-Gordon)}.$$

$$\frac{d}{d\gamma}\log\widehat{\Phi}(\gamma) = \frac{\gamma\coth\gamma}{i\pi}\,, \qquad \log\widehat{\Phi}(0) = 0\,, \qquad \text{(new phase)}.$$

The remaining factors

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Plan

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2. The (factorised) S matrix
3. The mirror model
4. The mirror Thermodynamic Bethe Ansatz
5. Conclusions and Outlook

Double Wick-rotation

Just like in $AdS_5 \times S^5$, we introduce a mirror model by setting [Arutyunov, Frolov '07]

$$\tilde{H} = -i\,p\,, \qquad \tilde{p} = -i\,H\,.$$

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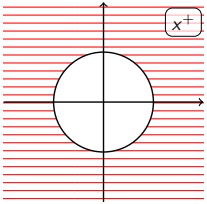
The mirror model is genuinely different: dispersion

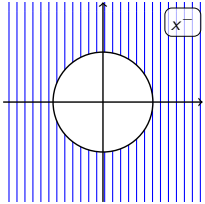
$$\tilde{H} = 2\text{arcsinh}\left(\frac{\sqrt{M^2 + \tilde{p}^2}}{2h}\right).$$

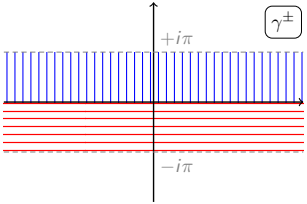
Still, it is related to the original theory by analytic continuation. [Frolov, AS '21]

Analytic continuation for massive variables

String theory







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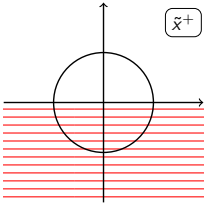
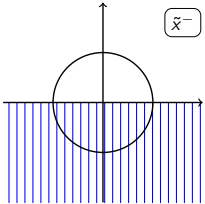
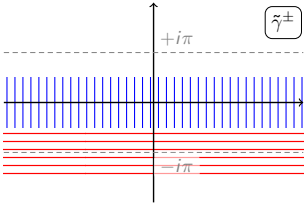
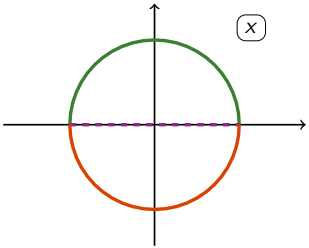


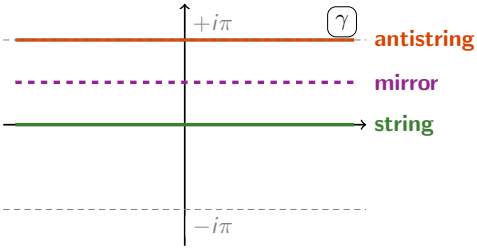




Analytic continuation for massless variables

Massless modes live on a line (no bound states allowed).





Properties of the phases in string & mirror region

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- Satisfies all good properties in the mirror region (**mirror unitarity**).
- Works well with **mirror fusion**, too.

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The string hypothesis

Mirror Bethe-Yang equations: $$1 = e^{i\tilde{p}_j R} \prod_k S_{jk}$$

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In this case we have:

- Left-left bound states, or Q-particles.
- Right-right bound states, or $\overline{Q}$-particles.
- Massless particles $\times$ 2.
- Auxiliary roots $\times$ 2.

Bethe-Yang equations for the "Bethe strings"

$$1 = e^{i\tilde{p}_a R} \prod_{b\neq a}^{N_L} S_{sl}^{Q_a Q_b}(u_a, u_b) \prod_{b=1}^{N_R} \widetilde{S}_{sl}^{Q_a \overline{Q}_b}(u_a, u_b) \prod_{\dot{\alpha}=1}^{2} \prod_{j=1}^{N_0^{(\dot{\alpha})}} S^{Q_a 0}(u_a, x_j^{(\dot{\alpha})}) \prod_{\alpha=1}^{2} \prod_{b=1}^{N_y^{(\alpha)}} S^{Q_a y}(u_a, y_b^{(\alpha)})$$

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$$-1 = e^{i\tilde{p}_k R} \prod_{j\neq k}^{N_0^{(1)}} (\sigma_{kj}^{\circ\circ})^{-2} \prod_{j=1}^{N_0^{(2)}} (\sigma_{kj}^{\circ\circ})^{-2} \prod_{b=1}^{N_L} S^{0 Q_b}(x_k, u_b) \prod_{b=1}^{N_R} \overline{S}^{0 \overline{Q}_b}(x_k, u_b) \prod_{\alpha=1}^{2} \prod_{b=1}^{N_y^{(\alpha)}} S^{0y}(x_k, y_b^{(\alpha)})$$

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$$-1 = e^{i\tilde{p}_k R} \prod_{j\neq k}^{N_0^{(1)}} (\sigma_{kj}^{\circ\circ})^{-2} \prod_{j=1}^{N_0^{(2)}} (\sigma_{kj}^{\circ\circ})^{-2} \prod_{b=1}^{N_L} S^{0 Q_b}(x_k, u_b) \prod_{b=1}^{N_R} \overline{S}^{0 \overline{Q}_b}(x_k, u_b) \prod_{\alpha=1}^{2} \prod_{b=1}^{N_y^{(\alpha)}} S^{0y}(x_k, y_b^{(\alpha)})$$

$$-1 = \prod_{b=1}^{N_L} S^{y Q_b}(y_k^{(\alpha)}, u_b) \prod_{b=1}^{N_R} \overline{S}^{y \overline{Q}_b}(y_k^{(\alpha)}, u_b) \prod_{\dot{\alpha}=1}^{2} \prod_{j=1}^{N_0^{(\dot{\alpha})}} S^{y 0}(y_k^{(\alpha)}, x_j^{(\dot{\alpha})})$$

Deriving the TBA equations

From the Bethe-Yang equations, we follow a standard route [Yang, Yang '69]

- Take the thermodynamic limit $R \gg 1$, $N_* \gg 1$,
- Rewrite (log of) products as integrals for densities,
- Set a finite temperature $T = 1/L$,
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It is customary and convenient to express them in terms of **Y-functions**

$$Y_Q\,, \qquad \overline{Y}_Q\,, \qquad Y_0^{(1,2)}\,, \qquad Y_\pm^{(1,2)}\,.$$

The (simplified) mirror TBA equations

$$H(L) = -\int \frac{d\tilde{p}_Q}{2\pi} \log\Big(\big(1+Y_Q\big)\big(1+\overline{Y}_Q\big)\Big) - \int \frac{d\tilde{p}_0}{2\pi} \log\Big(\big(1+Y_0^{(1)}\big)\big(1+Y_0^{(2)}\big)\Big)$$

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$$-\log Y_Q = \log\left(1+\frac{1}{Y_{Q-1}}\right)\left(1+\frac{1}{Y_{Q+1}}\right)\star s\,, \qquad Q \geq 2$$

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$$-\log Y_1 = \log\left(1+\frac{1}{Y_2}\right)\star s - \log\left(1-\frac{e^{-i\mu}}{Y_-^{(1)}}\right)\left(1-\frac{e^{i\mu}}{Y_-^{(2)}}\right)\hat{\star}\, s + F_1 \check{\star}\, s\,,$$

$$-\log \overline{Y}_1 = \log\left(1+\frac{1}{\overline{Y}_2}\right)\star s - \log\left(e^{-i\mu}-Y_+^{(1)}\right)\left(e^{i\mu}-Y_+^{(2)}\right)\hat{\star}\, s + \overline{F}_1 \check{\star}\, s$$

The mirror TBA equations /2

$$
\begin{aligned}
-\log Y_0^{(\dot\alpha)} = L\,\widetilde{H}_0 - \sum_{\dot\beta=1,2} \log\left(1+Y_0^{(\dot\beta)}\right) \,\check{\star}\, K^{00} - \log\left(1+Y_Q\right) \star K^{Q0} - \log\left(1+\overline{Y}_Q\right) \star \widetilde{K}^{Q0} \\
- \sum_{\alpha=1,2} \log\left(1-\frac{e^{i\mu_\alpha}}{Y_+^{(\alpha)}}\right) \,\hat{\star}\, K^{y0} - \sum_{\alpha=1,2} \log\left(1-\frac{e^{i\mu_\alpha}}{Y_-^{(\alpha)}}\right) \,\hat{\star}\, K^{y0},
\end{aligned}
$$

The mirror TBA equations /2

$$-\log Y_0^{(\dot\alpha)} = L\,\widetilde{H}_0 - \sum_{\dot\beta=1,2} \log\left(1+Y_0^{(\dot\beta)}\right) \check{\star}\, K^{00} - \log\left(1+Y_Q\right)\star K^{Q0} - \log\left(1+\overline{Y}_Q\right)\star \widetilde{K}^{Q0}$$

$$- \sum_{\alpha=1,2} \log\left(1-\frac{e^{i\mu_\alpha}}{Y_+^{(\alpha)}}\right) \hat{\star}\, K^{y0} - \sum_{\alpha=1,2} \log\left(1-\frac{e^{i\mu_\alpha}}{Y_-^{(\alpha)}}\right) \hat{\star}\, K^{y0},$$

$$\log Y_-^{(\alpha)} = -\log\left(1+Y_Q\right)\star K_-^{Qy} + \log\left(1+\overline{Y}_Q\right)\star K_+^{Qy} + \sum_{\dot\alpha=1,2} \log\left(1+Y_0^{(\dot\alpha)}\right) \check{\star}\, K^{0y},$$

$$\log Y_+^{(\alpha)} = -\log\left(1+Y_Q\right)\star K_+^{Qy} + \log\left(1+\overline{Y}_Q\right)\star K_-^{Qy} - \sum_{\dot\alpha=1,2} \log\left(1+Y_0^{(\dot\alpha)}\right) \check{\star}\, K^{0y}\,.$$

Plan

1. Quick review of AdS3/CFT2
2. The (factorised) S matrix
3. The mirror model
4. The mirror Thermodynamic Bethe Ansatz
5. Conclusions and Outlook

Conclusions

- We derived the **mirror TBA equations** based on minimal assumptions.
- In principle, full control on **finite-volume spectrum**. [Dorey, Tateo '96]
- In the process, we corrected the proposal for the **dressing factors**.
- Main lesson: **massless and massive modes interact** (affecting σ^{**}).
- Truncating to only massless or massive results in **different integrable models**.

Outlook

- Further simplify the TBA: Y-system, QSC, ... [see Dima's and Andrea's talks]
- Compute observables. What comparison? [Ohlsson Sax, AS, Stefanski '14]
- More ambitious observables: correlation functions! [Eden, le Plat, AS '21]
- Mixed-flux integrability [Hoare, Stepanchuk, Tseytlin '13] [Lloyd, Ohlsson Sax, AS, Stefanski '14]

$$H(p) = \sqrt{\left(M + \frac{k}{2\pi}p\right)^2 + 4h^2 \sin^2\left(\frac{p}{2}\right)}$$

- Make contact with WZW / "hybrid" formalism at

$$k \geq 1, \qquad h \to 0\,, \qquad H(p) \approx \left|M + \frac{k}{2\pi}p\right|$$

[many recent developments!]

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