QUANTUM SPECTRAL CURVE FOR ADS3/CFT2: A PROPOSAL

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Andrea Cavaglià, University of Turin

Based on joint work with **Nikolay Gromov, Bogdan Stefański, jr.** *and* **Alessandro Torrielli**

+ *ongoing works and discussions with* **NG, BS, AT+ Simon Ekhammar, Paul Ryan, Dmytro Volin**

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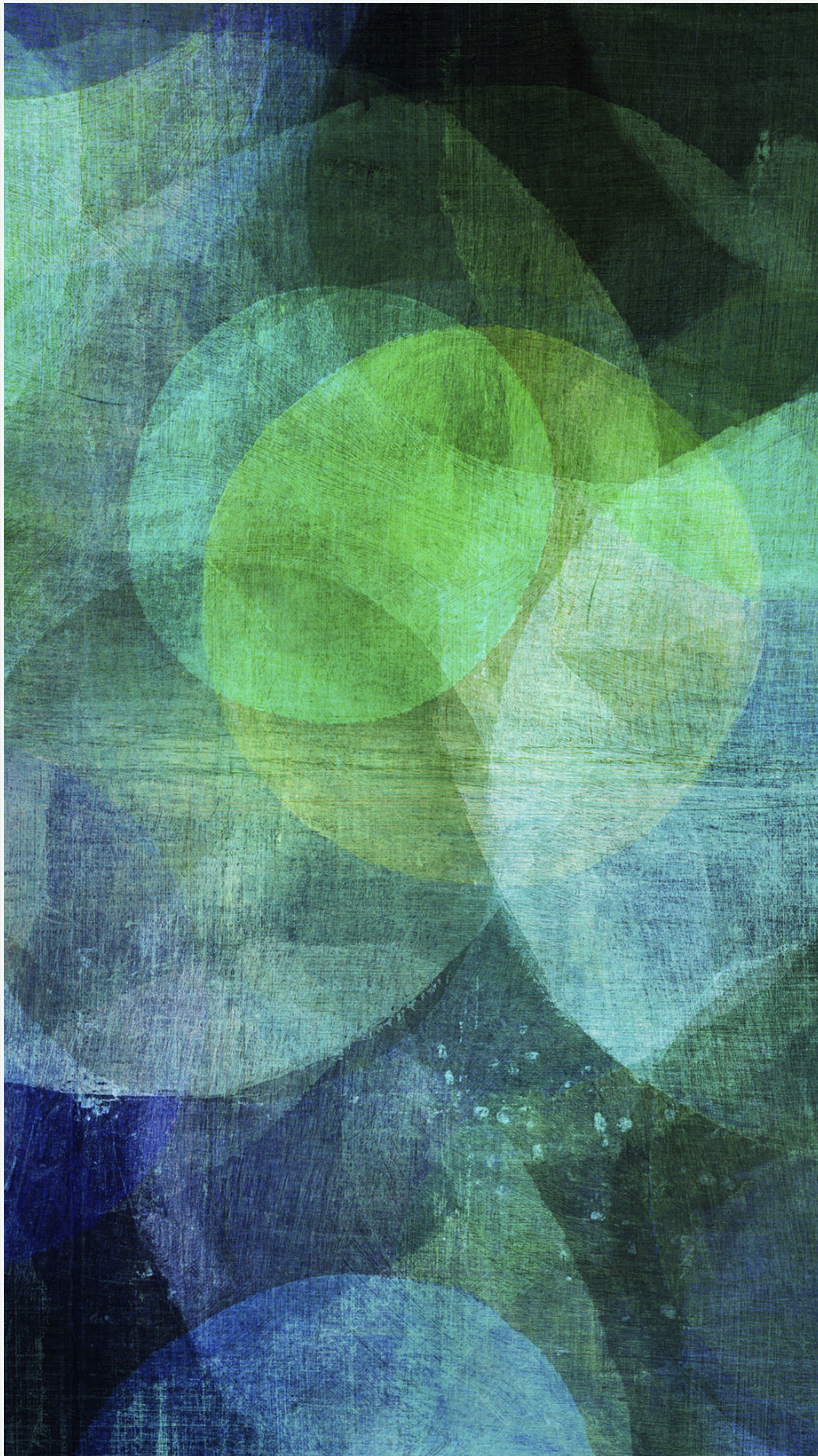


What $\mathbf{AdS_3/CFT_2}$ are we studying?

$SQCD_2$

RG

[Maldacena '97]

String theory on $AdS_3 \times S^3 \times T^4$
with **purely R-R flux**

Essentially mysterious
and complicated dual CFT:
IR limit of world volume theory
of D1-D5 system

"Polar opposite" to the NSNS case on which much progress was made
[Eberhardt, Gaberdiel, Gopakumar '18]

here the worldsheet CFT approach is much more complicated if not impossible

The theory should be integrable on the worldsheet! [Babichenko, Stefanski, Zarembo '09]
[Borsato, Ohlsson-Sax, Sfondrini, Stefanski '14] + Torrielli '13,'16]
[Frolov, Sfondrini '21]
..+Abbott, Aniceto, Beccaria, Cagnazzo, Dei, Dekel, Eden, Hoare, Levkovich-Maslyuk, le Plat, Lloyd, Macorini, Majumder, Nieto, Sebold, Tseytlin, Varga…]

… same conceptual framework as AdS5 and AdS4 with significant complications
related to massless modes in worldsheet theory

We propose the Quantum Spectral Curve for the planar spectrum at finite coupling
in the sector with zero winding/momentum on the torus

Quantum Spectral Curve

$AdS_5 \times S^5 \leftrightarrow \mathcal{N}=4$ SYM
AdS$_5$/CFT$_4$

[Gromov, Kazakov, Leurent, Volin'13]

$AdS_4 \times CP^3 \leftrightarrow$ ABJM
AdS$_4$/CFT$_3$

[AC, Gromov, Fioravanti, Tateo'14]

AdS$_3$/CFT$_2$

[AC, Gromov, Stefanski Torrielli]+[Ekhammar,Volin] '21

What is it? A complex analysis problem for "Q-functions" $Q_i(u)$

spectral parameter

't Hooft coupling = position of branch points

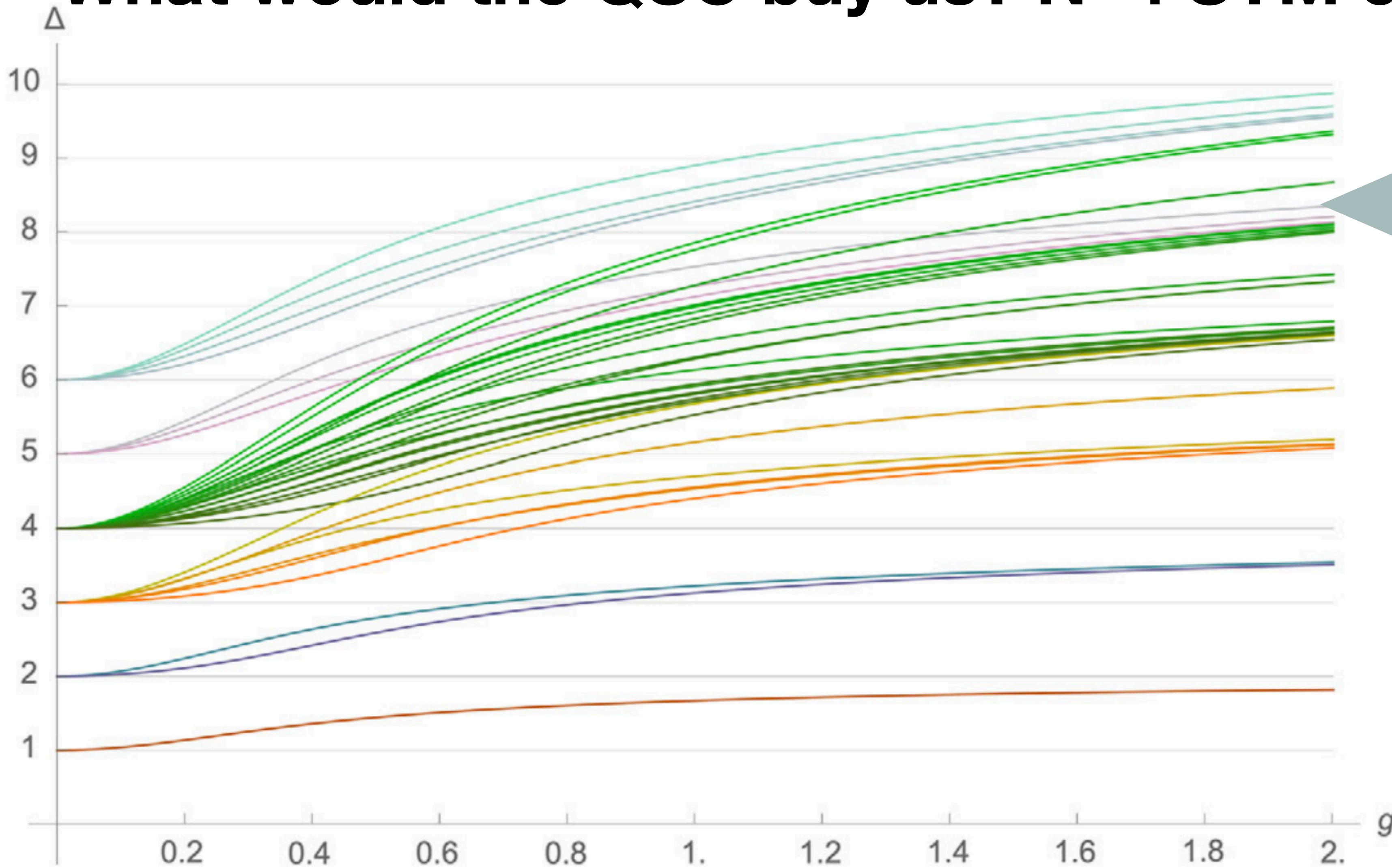
The same set of equations for the **full planar spectrum at finite coupling**

Quite rigid mathematical structure.
Variation found describing spectrum on cusped WL [Gromov, Levkovich-Maslyuk '15]

Hugely useful in practice. In N=4 SYM and ABJM we can now answer almost any question on the planar spectrum

Let us look at some applications…

What would the QSC buy us? N=4 SYM examples for motivation



spectrum of defect CFT on a Wilson line in N=4 SYM
[AC, Julius, Gromov, Preti '21]

High precision numerics, Regge trajectories …
[Gromov Levkovich-Maslyuk, Sizov '15]+…

Solve analytically at weak coupling (and other limits)
[Marboe, Volin '14]+..

$$
\begin{aligned}
\Delta \;=\;& 4 + 12g^2 - 48g^4 + 336g^6 + g^8\left(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\right)\\
&+g^{10}\left(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2 - 8640\,\zeta_5 + 30240\,\zeta_7\right)\\
&+g^{12}\left(-7680 - 262656\,\zeta_3 - 20736\,\zeta_3^2 + 112320\,\zeta_5 + 155520\,\zeta_3\,\zeta_5 + 75600\,\zeta_7 - 489888\,\zeta_9\right)\\
&+g^{14}\big(-2135040 + 5230080\,\zeta_3 - 421632\,\zeta_3^2 + 124416\,\zeta_3^3 - 229248\,\zeta_5 + 411264\,\zeta_3\,\zeta_5\\
&\quad - 993600\,\zeta_5^2 - 1254960\,\zeta_7 - 1935360\,\zeta_3\,\zeta_7 - 835488\,\zeta_9 + 7318080\,\zeta_{11}\big)\\
&+g^{16}\Big(54408192 - 83496960\,\zeta_3 + 7934976\,\zeta_3^2 + 1990656\,\zeta_3^3 - 19678464\,\zeta_5 - 4354560\,\zeta_3\,\zeta_5\\
&\quad - 3255552\,\zeta_3^2\,\zeta_5 + 2384640\,\zeta_5^2 + 21868704\,\zeta_7 - 6229440\,\zeta_3\,\zeta_7 + 22256640\,\zeta_5\,\zeta_7\\
&\quad + 9327744\,\zeta_9 + 23224320\,\zeta_3\,\zeta_9 + \frac{65929248}{5}\,\zeta_{11} - 106007616\,\zeta_{13} - \frac{684288}{5}\,Z_{11}^{(2)}\Big)
\end{aligned}
$$

What numbers can appear?

SYM: MZV
[Marboe, Volin '14]+…

ABJM: alternating MZV
[Anselmetti Bombardelli,AC,Tateo'15].

AdS3 RR = likely more involved

Extra motivation: QSC and correlation functions

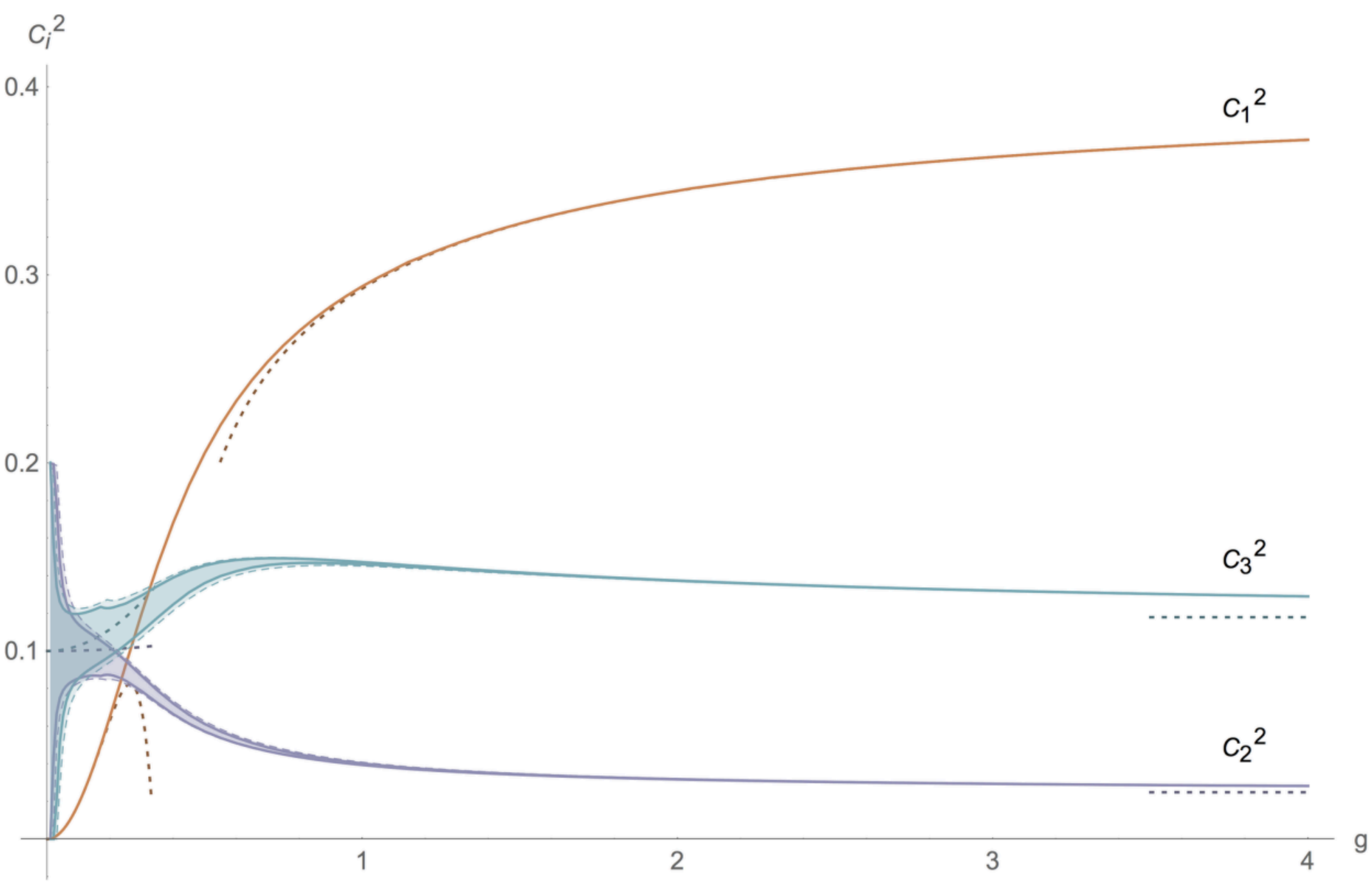
QSC/TBA + hexagons? **cf. Benjamin's talk** [Basso Georgoudis Sueiro '22]

Can Q-functions build correlators via SoV? [AC, Gromov, Levkovich-Maslyuk '18] [Komatsu, Giombi '18] [Jiang, Komatsu, Kostov, Serban '15]. [Gromov, Primi, Ryan '21]…

QSC also essential for "bootstrability"

[AC, Gromov, Julius, Preti '21, '22]
[..+N.Sokolova, in progress]
Bulk ops: [Caron-Huot, Coronado, Trinh, Zahraee '22]

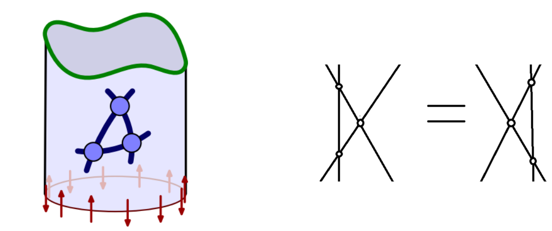
cf. poster by Julius



[Beisert, Staudacher '05]
[Bombardelli, Fioravanti, Tateo '09]
[Arutyunov, Frolov '09]
[Gromov, Kazakov, Vieira '09]
[AC, Fioravanti, Tateo '10]
[Gromov, Kazakov, Leurent, Volin '11,'13]

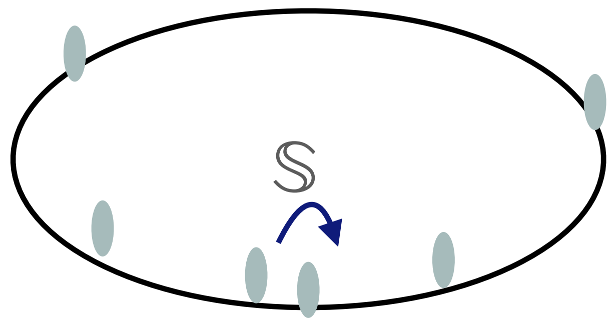
Systematic route

Large worldsheet



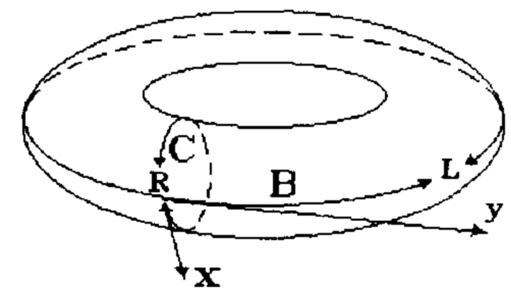
dispersion relation, worldsheet S-matrix (up to CDD)

Large but periodic



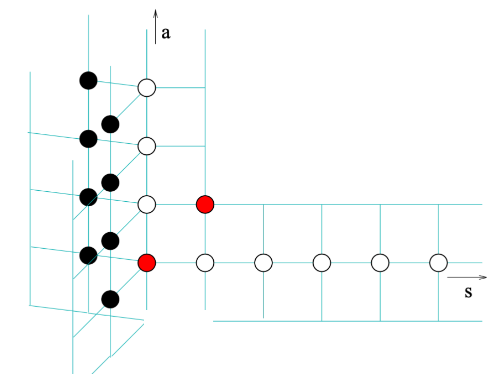
Asymptotic Bethe Ansatz

Make it finite



Thermodynamic Bethe Ansatz

Simplify (a lot)



Y- ,T- , Q-system, … QSC

[Borsato, Ohlsson-Sax, Sfondrini, Stefanski '14] + Torrielli '13,'16]

[Frolov, Sfondrini, '21]

cross-check,

QSC should fix CDD

Can we fix the QSC by self-consistency?

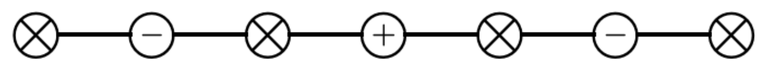
QSC = Symmetry + Analyticity

Symmetry

Q-system

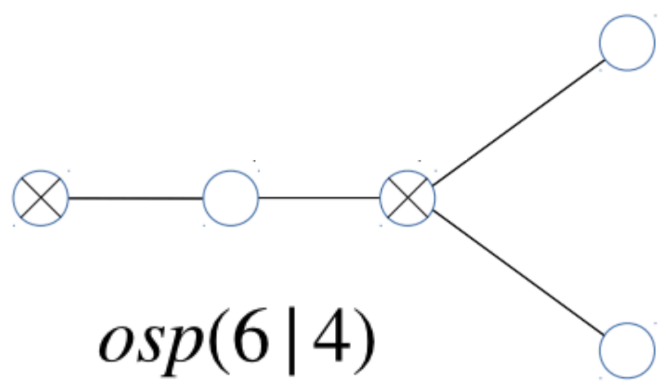
Known for all A-type superalgebras: [Tsuboi '09]

AdS5



$psu(2,2\,|\,4)$

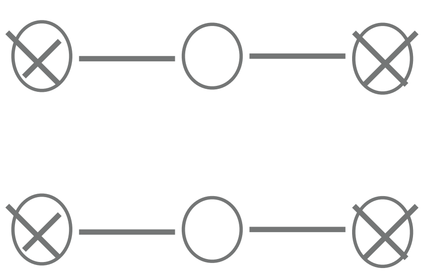
AdS4



$osp(6\,|\,4)$

QQ-relations in [Bombardelli, AC, Fioravanti, Gromov, Tateo '17]

AdS3



$$psu(1,1\,|\,2)_L \oplus psu(1,1\,|\,2)_R$$

Two copies of a small-rank version of the AdS5 case

Analyticity

Large u contains charges

The only singularities in Q-functions should be "kinematical"

We don't really know the rules a priori…

…but analytic properties were pretty similar in AdS4 and AdS5…

In RR AdS3, let's be
<u>as close as possible</u>
to other QSC's
(*based on similar dispersion relation and S-matrix*)

Note: we do not input any special information related to massless modes.

Are they an emergent feature?

Dotted **Undotted**

AdS3: $psu(1,1\,|\,2)_L \oplus psu(1,1\,|\,2)_R$ **, two copies of a known Q-system**

16 Q-functions 16 Q-functions

For each copy the structure is:

$\mathbf{Q}_a(u) \quad \mathbf{P}_a(u)$

$\mathbf{Q}_{a|b}(u),\ \ \mathcal{Q}_{12|\varnothing}(u),\ \ \mathcal{Q}_{12|\varnothing}(u)$

$\mathbf{Q}^a(u),\ \mathbf{P}^a(u) \qquad\qquad a, b \in \{1,2\}$

$$Q_{aA|I}Q_{A|Ii} = Q^+_{aA|Ii}Q^-_{A|I} - Q^-_{aA|Ii}Q^+_{A|I}\,,$$
$$Q_{12|I}Q_{\emptyset|I} = Q^+_{1|I}Q^-_{2|I} - Q^-_{1|I}Q^+_{2|I}\,,$$
$$Q_{A|12}Q_{A|\emptyset} = Q^+_{A|1}Q^-_{A|2} - Q^-_{A|1}Q^+_{A|2}\,,$$
$$A\,, I \in \{\emptyset, 1, 2, (12)\}$$

$(f^{\pm}(u) \equiv f(u \pm \frac{i}{2}))$

“as close as possible to other QSC’s”

$$\mathcal{Q}_{\varnothing|\varnothing}(u) = \mathcal{Q}_{12|12}(u) = 1$$

an “Exact Bethe Ansatz” is already contained in the Q-system

$$Q_{1|1}(u+\tfrac{i}{2}) - Q_{1|1}(u-\tfrac{i}{2}) = \mathbf{Q}_1(u)\,\mathbf{P}_1(u) \quad\longrightarrow\quad \left.\frac{Q_{1|1}^{+}}{Q_{1|1}^{-}}\right|_{u\in\{\text{zeros of } \mathbf{Q}_1\}} = 1$$

$$Q_{1|1}(u+\tfrac{i}{2})Q_{2|1}(u-\tfrac{i}{2}) - Q_{1|1}(u-\tfrac{i}{2})Q_{2|1}(u+\tfrac{i}{2}) = \mathbf{Q}_1(u)\,\mathbf{Q}^2(u) \quad\longrightarrow\quad \left.\frac{Q_{1|1}^{++}\mathbf{Q}_1^{-}\mathbf{Q}^{2-}}{Q_{1|1}^{--}\mathbf{Q}_1^{+}\mathbf{Q}^{2+}}\right|_{u\in\left\{\text{zeros of } Q_{1|1}\right\}} = -1,$$

$$\left.\frac{Q_{1|1}^{+}}{Q_{1|1}^{-}}\right|_{u\in\{\text{zeros of } \mathbf{Q}^2\}} = 1,$$

$$\left.\frac{Q_{\hat{1}|\hat{1}}^{+}}{Q_{\hat{1}|\hat{1}}^{-}}\right|_{u\in\left\{\text{zeros of } \mathbf{P}_{\hat{1}}\right\}} = 1$$

$$\left.\frac{Q_{\hat{1}|\hat{1}}^{++}\mathbf{P}_{\hat{1}}^{-}\mathbf{P}^{\dot{2}-}}{Q_{\hat{1}|\hat{1}}^{--}\mathbf{P}_{\hat{1}}^{+}\mathbf{P}^{\dot{2}+}}\right|_{u\in\left\{\text{zeros of } Q_{\hat{1}|\hat{1}}\right\}} = -1$$

$$\left.\frac{Q_{\hat{1}|\hat{1}}^{+}}{Q_{\hat{1}|\hat{1}}^{-}}\right|_{u\in\left\{\text{zeros of } \mathbf{P}^{\dot{2}}\right\}} = 1,$$

It should match the ABA in appropriate limit

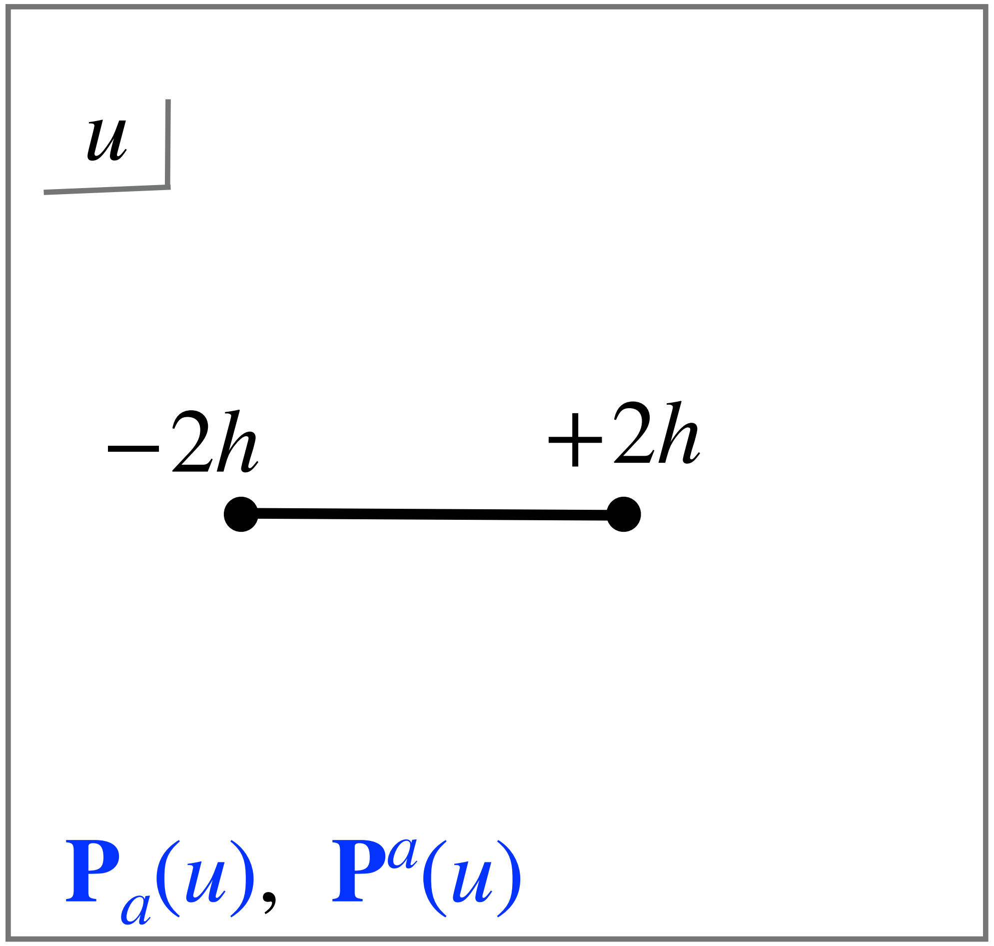
Analyticity

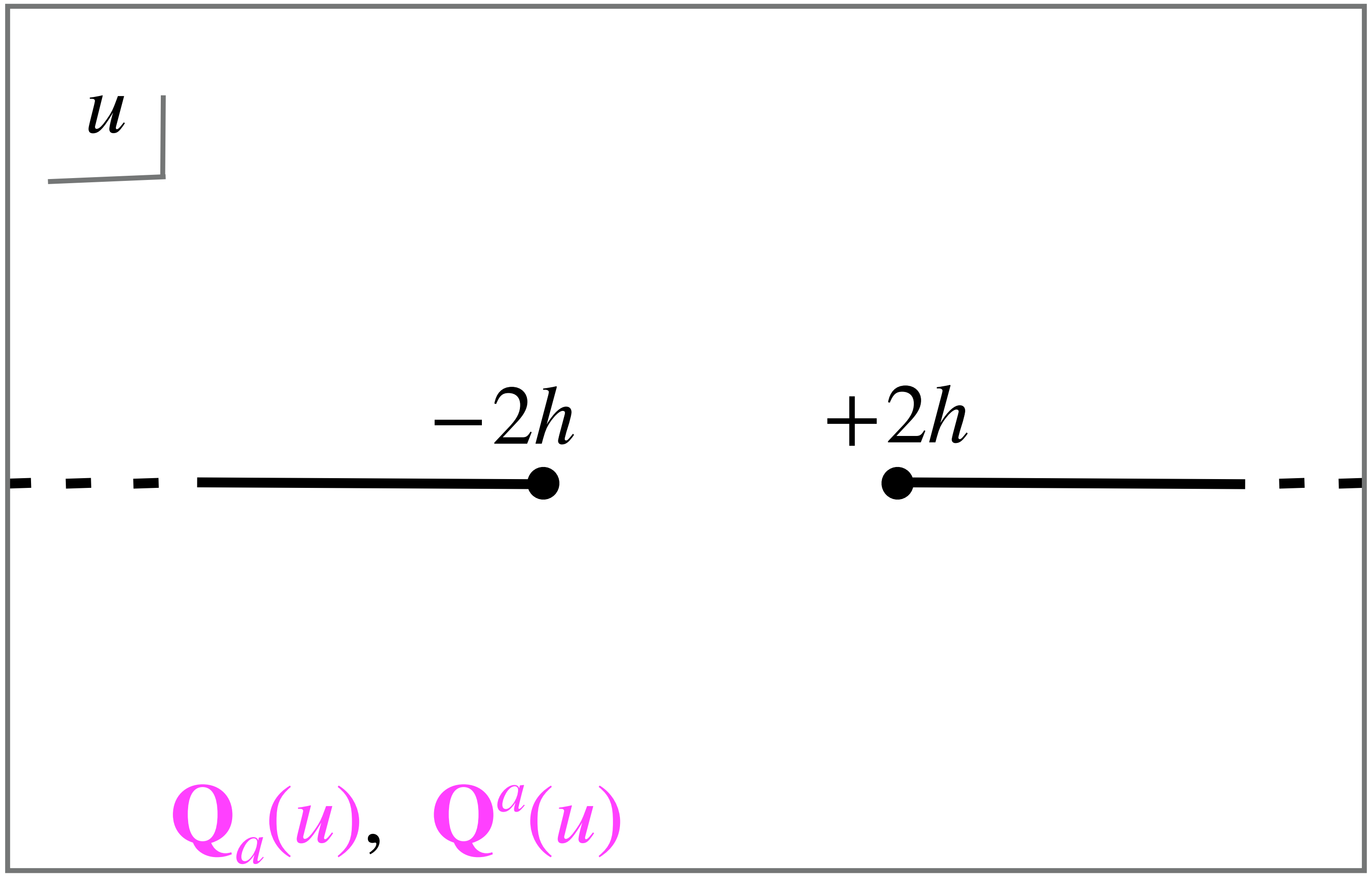
In the classical limit, $Q_a(u) \sim e^{\int^u q_a(z)dz}$

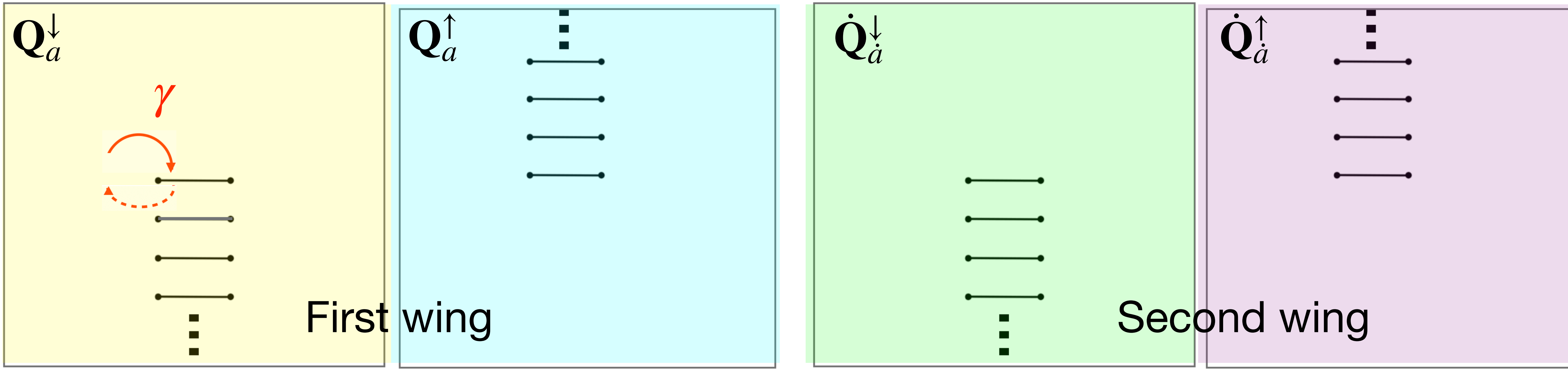
$$(\mathbf{Q}_1(u), \mathbf{Q}_2(u) \,|\, \mathbf{P}_1(u), \mathbf{P}_2(u)) \sim \left(u^{\frac{\Delta}{2}+\cdots},\, u^{-\frac{\Delta}{2}+\cdots} \,|\, u^{-\frac{J}{2}+\cdots},\, u^{\frac{J}{2}+\cdots} \right), \quad u \to \infty$$

$(\mathbf{Q}_1(u), \mathbf{Q}_2(u) \,|\, \mathbf{P}_1(u), \mathbf{P}_2(u))$ should parametrise motion in $\mathbf{AdS_3}$ or $\mathbf{S^3}$

Basic "kinematical singularities" on the simplest Riemann sheets:



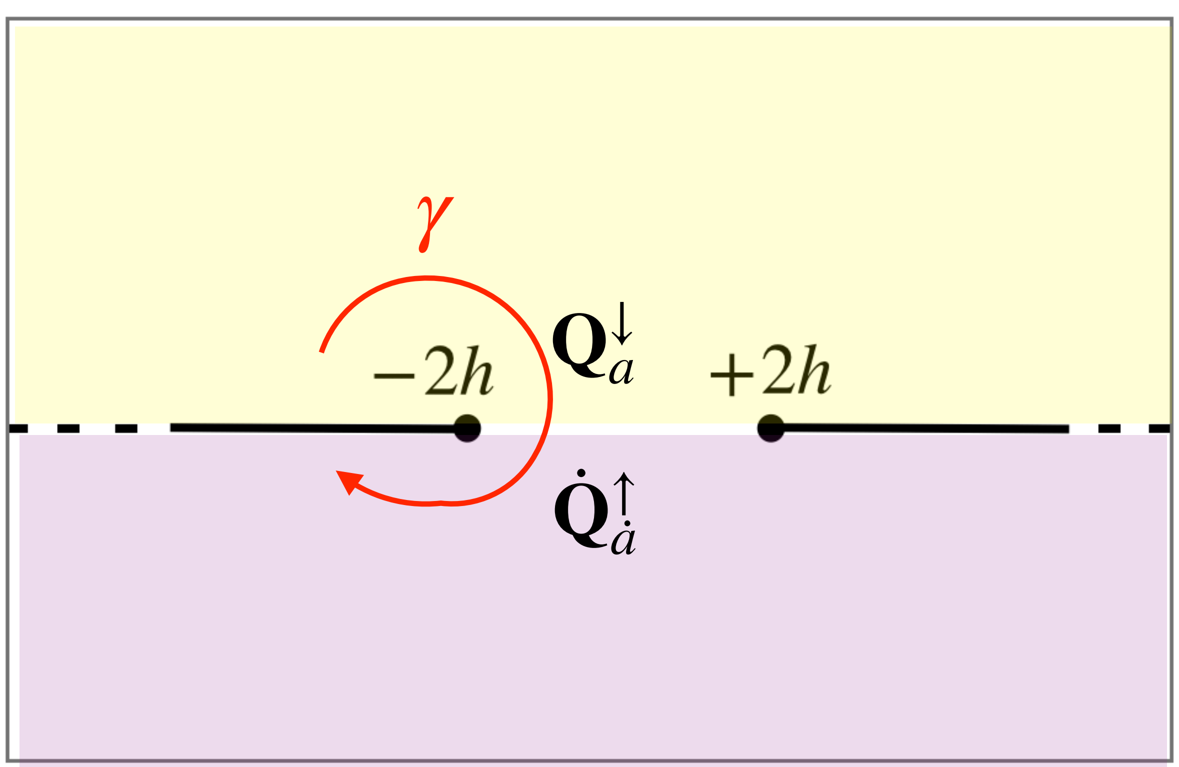


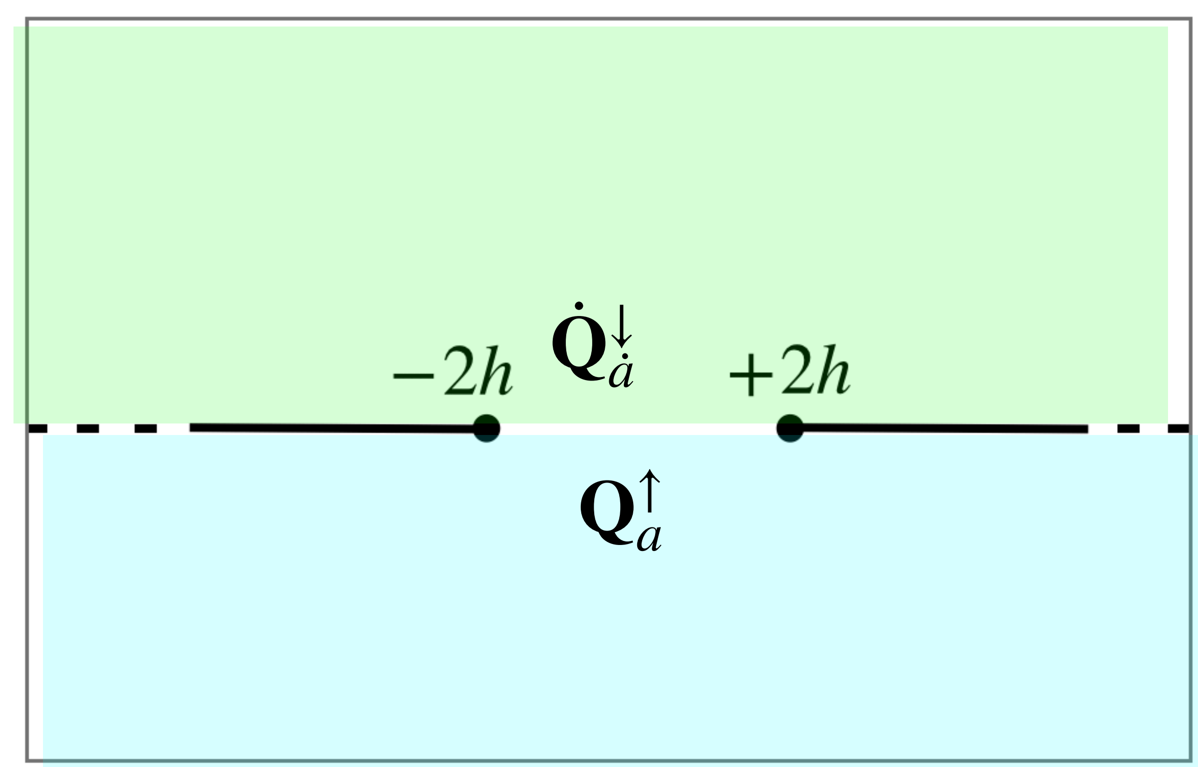


We couple them in a minimal way:

$$\mathbf{Q}_a^{\downarrow}(u^{\gamma}) = G_a^{\ \dot b}\,\dot{\mathbf{Q}}_{\dot b}^{\uparrow}(u) \qquad \dot{\mathbf{Q}}_{\dot a}^{\downarrow}(u^{\gamma}) = G_{\dot a}^{\ b}\,\mathbf{Q}_b^{\uparrow}(u)$$

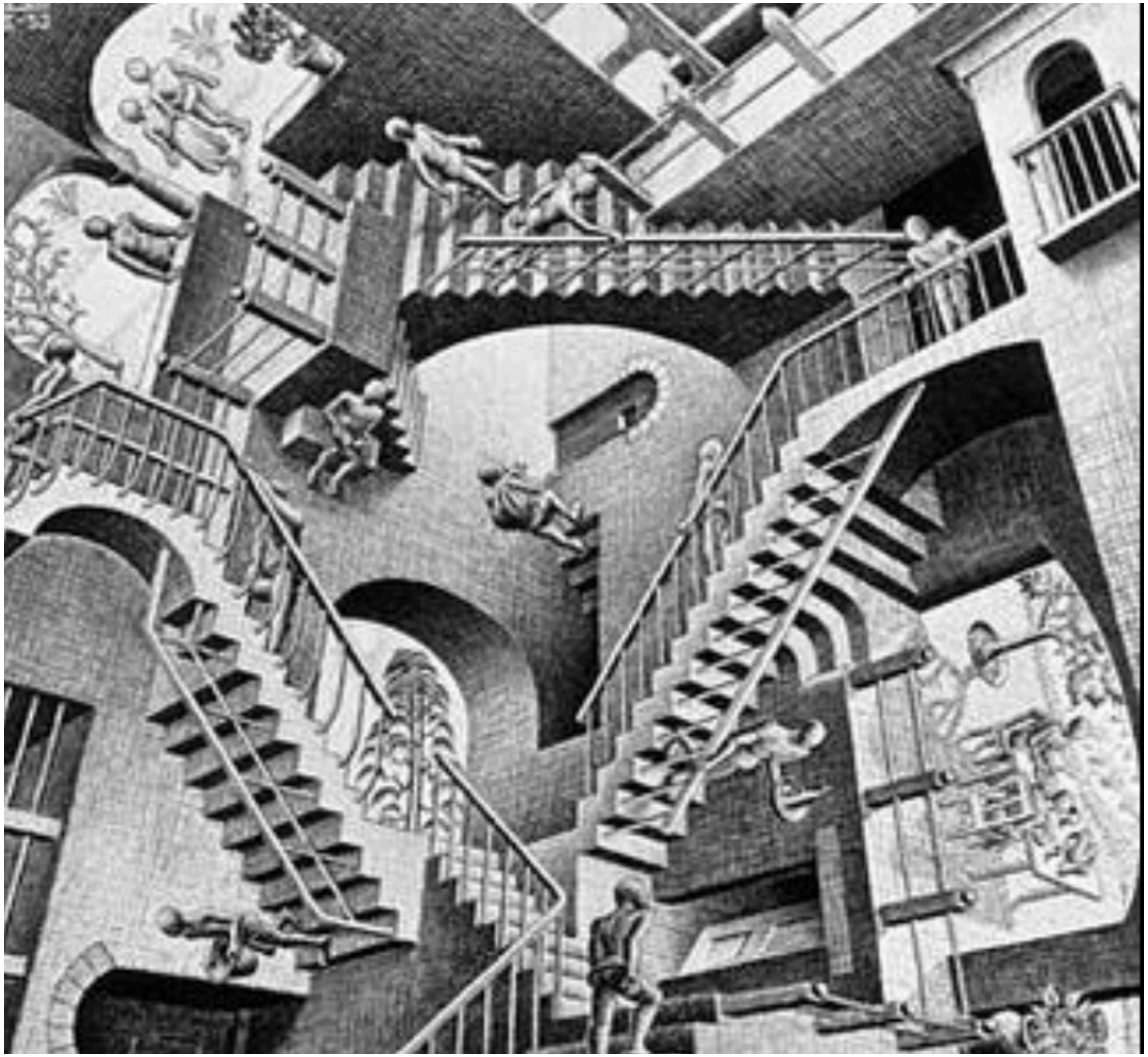
constant gluing matrices





The Riemann surface is complicated (as in previous cases):

infinitely many sheets, infinitely many branch points,…



More on analytic continuation

$$\mathbf{Q}_a^{\downarrow}(u^{\gamma}) = G_a^{\ \dot{b}}\, \dot{\mathbf{Q}}_{\dot{b}}^{\uparrow}(u) \longrightarrow \mathbf{Q}_a^{\downarrow}(u^{\gamma}) = \omega_a^{\ \dot{b}}(u)\, \dot{\mathbf{Q}}_{\dot{b}}^{\downarrow}(u)$$

$$\omega(u) = \omega(u+i)$$

gluing Q's with same type of cuts

$$(\omega_{\dot{k}}^{\ \ l})^{\bar{\gamma}} - \omega_{\dot{k}}^{\ \ l} = \mathbf{Q}_{\dot{k}}\mathbf{Q}^{l\,\bar{\gamma}} - \mathbf{Q}_{\dot{k}}^{\gamma}\mathbf{Q}^{l}$$

$\longrightarrow$ $U_a^{\ \dot{b}} \equiv \omega_a^{\ \dot{b}} \cdot (\delta_{\dot{b}}^{\dot{c}} - \dot{\mathbf{Q}}_{\dot{b}}\dot{\mathbf{Q}}^{\dot{c}})$ has no cut on the real axis

same for $\dot{U}_{\dot{a}}^{\ b} \equiv \dot{\omega}_{\dot{a}}^{\ b} \cdot (\delta_b^c - \mathbf{Q}_b\mathbf{Q}^c)$

$$\mathbf{Q}_a(u^{\gamma}) = U_a^{\ \dot{b}}(u)\, \dot{\mathbf{Q}}_{\dot{b}}(u)$$

$$\mathbf{Q}_a(u^{\gamma^2}) = U_a^{\ \dot{c}}(u)\, \dot{U}_{\dot{c}}^{\ b}(u)\, \mathbf{Q}_b(u)$$

No reason to expect $U \cdot \dot{U} = 1$ ...

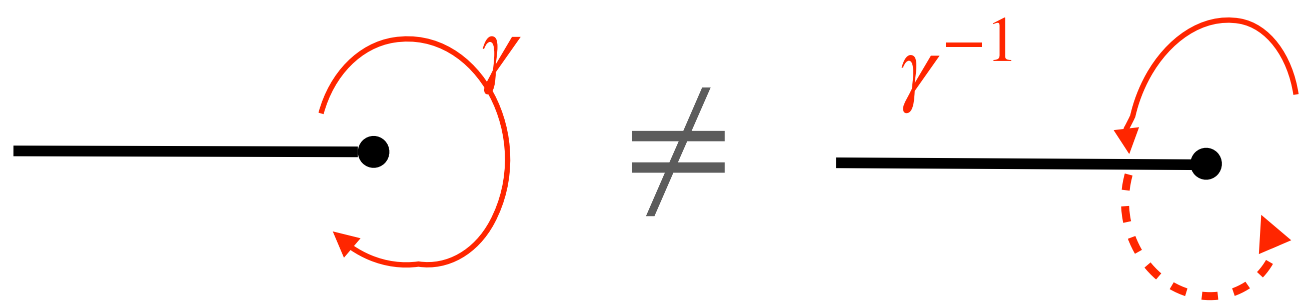
$\vdots$

$$\mathbf{Q}_a(u^{\gamma^{2n}}) = (\underbrace{U(u) \cdot \dot{U}(u) \cdot \ldots \cdot \dot{U}(u)}_{2\,n})_a^{\ b}\, \mathbf{Q}_b(u)$$

In fact, forcing quadratic branch points collapses us to the Hubbard case ([AC, Cornagliotto, Mattelliano, Tateo '15])

[Volin Ekhammar '21]

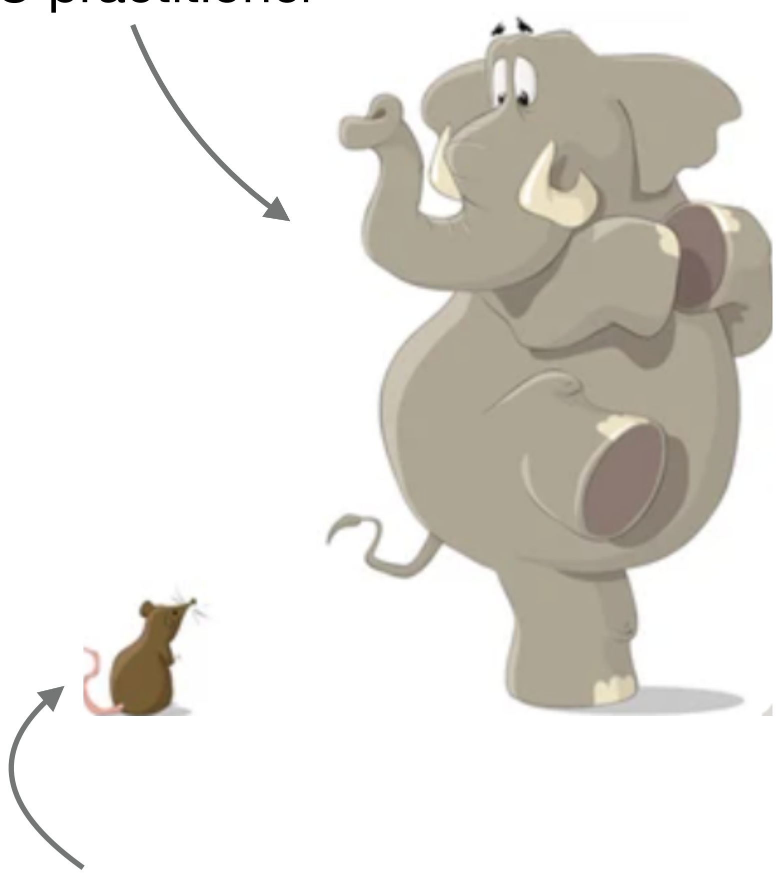
Therefore branch points now have infinite order!



$$\gamma \neq \gamma^{-1}, \quad \gamma^n \neq 1$$

We did not input (or expect) this...

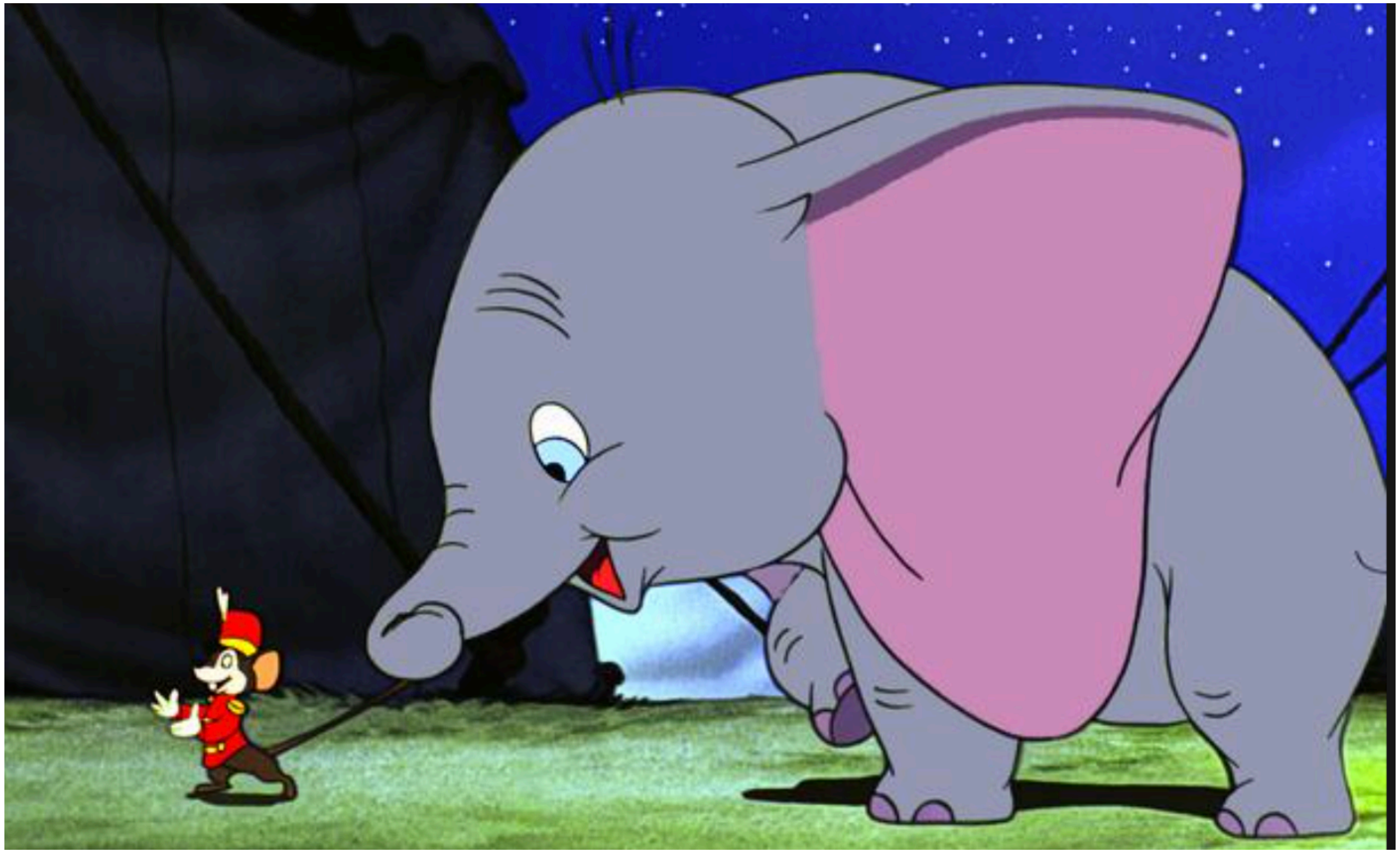
QSC practitioner



non-quadratic branch point

… but we think it is a
signature of massless modes.
We have to make friends with it!

Let's try to follow the branch points...



1. Large volume solution
2. Preliminary numerics

[AC, Ekhammar, Gromov, Ryan in progress]

Word of caution on the "ABA limit"

$J \to \infty \qquad \Delta \sim J + O(1) \qquad$, fixed h

The ABA should be valid only in a tiny region around this limit
because of massless modes

Massless virtual particles $\sim O(1/J)$ wrapping corrections to ABA
True also in the "massive" sector (*virtual* massless particles are still there)

[Abbott, Aniceto '15, '20]

Still, we can take formally the limit of the QSC and the full ABA with massive roots emerges (with a unique choice of the massive-massive dressing phases).

Asymptotic solutions with massless Bethe roots are a bit more singular, I will not discuss them today.

$J \to \infty \qquad\qquad \Delta \sim J + O(1)$

$$(\mathbf{Q}_1(u), \mathbf{Q}_2(u) \,|\, \mathbf{P}_1(u), \mathbf{P}_2(u)) \sim \left(u^{\frac{\Delta}{2}+\cdots}, u^{-\frac{\Delta}{2}+\cdots} \,|\, u^{-\frac{J}{2}+\cdots}, u^{\frac{J}{2}+\cdots}\right), \quad u \to \infty$$

$$\sim \left(\epsilon^{-1}, \epsilon \,|\, \epsilon, \epsilon^{-1}\right)$$

some Q's are large/small

The QSC simplifies, we find some Q-functions explicitly
S-matrix elements appear as building blocks

$$\mathbf{Q}_1(u) \sim$$

Exact Bethe equations → ABA

We find some quantities in terms of “Bethe roots”.

zeros of Q:
will become Bethe roots

$$\frac{Q_{1|1}(u-\frac{i}{2})}{Q_{1|1}(u+\frac{i}{2})}=\prod_i\frac{(u-u_i-\frac{i}{2})}{(u-u_i+\frac{i}{2})}\,\frac{(\frac{1}{x(u)}-x^-(u_i))^2}{(\frac{1}{x(u)}-x^+(u_i))^2}$$

$$x(u)=\frac{u+\sqrt{u-2h}\sqrt{u+2h}}{2h}$$

$$Q_{1|1}(u)\propto\prod_{i=1}^{K}\left[(u-u_i)\,(f(u+\tfrac{i}{2},u_i))^2\right]\qquad Q_{\dot{1}|\dot{1}}(u)\propto\prod_{i=K+1}^{K+\dot{K}}\left[(u-u_i)\,(f(u+\tfrac{i}{2},u_i))^2\right]$$

$$\log f(u,v)\equiv-\oint\frac{dz}{2\pi i}\log\left(\frac{\frac{1}{x(u)}-x^+(v)}{\frac{1}{x(u)}-x^-(v)}\right)\,\partial_z\log\Gamma(iz-iu)$$

Closely related to Beisert-Eden-Staudacher’s AdS5 dressing phase

Until this point, everything was similar to AdS5…

Here: simplest case with no auxiliary roots

Gluing in ABA limit:

$$(\mathbf{P}_1)^\gamma = Q^+_{1|l}\,\omega^l{}_{\dot{k}}\,\mathbf{Q}^{\dot{k}} \sim Q^+_{1|1}\,\omega^1{}_{\dot{2}}\,\mathbf{Q}^{\dot{2}},$$

known

We make a generic ansatz

$$\mathbf{P}_1(u) \propto x(u)^{-\frac{L}{2}} \prod_{i=1}^{K} \left(\frac{1}{x(u)} - x^+(u_i)\right)^{\frac{1}{2}} \left(\frac{1}{x(u)} - x^-(u_i)\right)^{\frac{1}{2}} a(u)$$

$$\mathbf{Q}^{\dot{2}}(u) \propto x(u)^{\frac{L}{2}} \prod_{i=1}^{\dot{K}} \left[\frac{f(u,\dot{u}_i)}{(\frac{1}{x(u)} - x^-(\dot{u}_i))}\right] \left[\prod_{j=1}^{K} \frac{(\frac{1}{x(u)} - x^+(u_j))^{\frac{1}{2}} f(u,u_j)}{(\frac{1}{x(u)} - x^-(u_j))^{\frac{1}{2}}}\right] \frac{1}{\dot{a}(u)}$$

$a(u)$, $\dot{a}(u)$: **generic functions**

(with no zeros, asymptotically constant, and with a single short cut)

They also parametrise $\mathbf{P}_i$, $\mathbf{Q}^2$ due to Q-system

gluing $\Longleftrightarrow$

$$(a(u))^{\gamma}\ \dot{a}(u) \propto \mathscr{F}(u)\ \mathscr{G}^{\frac{1}{2}}(u)$$

$$(\dot{a}(u))^{\gamma}\ a(u) \propto \mathscr{F}(u)\ \dot{\mathscr{G}}^{\frac{1}{2}}(u)$$

$$\mathscr{F}(u) \equiv \prod_{i}^{K} \bar{f}^{--}(u,u_i) f^{++}(u,u_i) \prod_{i}^{\dot{K}} \bar{f}^{--}(u,\dot{u}_i) f^{++}(u,\dot{u}_i)$$

$$\mathscr{G}(u) \equiv \prod_{i=1}^{K} \frac{x(u)-x^{-}(u_i)}{x(u)-x^{+}(u_i)} \prod_{i=1}^{\dot{K}} \frac{\frac{1}{x(u)}-x^{+}(\dot{u}_i)}{\frac{1}{x(u)}-x^{-}(\dot{u}_i)}$$

$$\dot{\mathscr{G}}(u) \neq \mathscr{G}^{\gamma}(u)$$

cut for $a(u)$, $\dot{a}(u)$ is not quadratic

gluing $\longleftrightarrow$

$$(a(u))^\gamma\ \dot a(u) \propto \mathscr{F}(u)\ \mathscr{G}^{\frac{1}{2}}(u)$$
$$(\dot a(u))^\gamma\ a(u) \propto \mathscr{F}(u)\ \dot{\mathscr{G}}^{\frac{1}{2}}(u)$$

$$\mathscr{F}(u) \equiv \prod_i^K \bar f^{--}(u,u_i) f^{++}(u,u_i) \prod_i^{\dot K} \bar f^{--}(u,\dot u_i) f^{++}(u,\dot u_i)$$

$$\mathscr{G}(u) \equiv \prod_{i=1}^K \frac{x(u)-x^-(u_i)}{x(u)-x^+(u_i)} \prod_{i=1}^{\dot K} \frac{\frac{1}{x(u)}-x^+(\dot u_i)}{\frac{1}{x(u)}-x^-(\dot u_i)}$$

Assumption: proportionality constants take the forms
$\left(\prod_i^K C_1(u_i) \prod_k^{\dot K} C_2(\dot u_k)\right),\ \left(\prod_i^K C_2(u_i) \prod_k^{\dot K} C_1(\dot u_k)\right)$

Solution: $$\frac{a(u)}{\prod_i \sigma^{1,BES}(u,u_i)} = \prod_i^K \sigma^1(u,u_i) \prod_i^{\dot K} \tilde\sigma^1(u,\dot u_i) \qquad \frac{\dot a(u)}{\prod_i \sigma^{1,BES}(u,u_i)} = \prod_i^K \tilde\sigma^1(u,u_i) \prod_i^{\dot K} \sigma^1(u,\dot u_i)$$

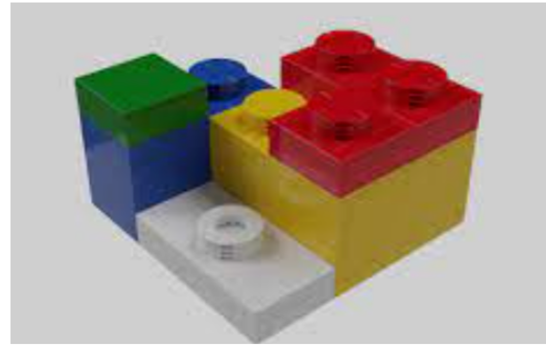
$$\left(\sigma^1(u^\gamma,v)\tilde\sigma^1(u,v)\right)^2 = C_1(v)\,\frac{x(u)-x^-(v)}{x(u)-x^+(v)} \qquad \left(\tilde\sigma^1(u^\gamma,v)\sigma^1(u,v)\right)^2 = C_2(v)\,\frac{1/x(u)-x^+(v)}{1/x(u)-x^-(v)}$$

These relations imply crossing in the u variable for the "dressing phases"

$$\frac{\sigma^1(u+\frac{i}{2},v)}{\sigma^1(u-\frac{i}{2},v)} \equiv \sigma^{\bullet\bullet}_{dress}(u,v) \qquad \frac{\tilde\sigma^1(u+\frac{i}{2},v)}{\tilde\sigma^1(u-\frac{i}{2},v)} \equiv \tilde\sigma^{\bullet\bullet}_{dress}(u,v)$$

Summary

$\mathbf{Q}_1(u) \sim$



Parametrised by roots
and functions $\sigma^1(u,v)$, $\tilde{\sigma}^1(u,v)$

$$\left.\frac{Q_{1|1}^{++}\mathbf{Q}_1^{-}\mathbf{Q}^{2-}}{Q_{1|1}^{--}\mathbf{Q}_1^{+}\mathbf{Q}^{2+}}\right|_{u\in\{\text{zeros of } Q_{1|1}\}} = -1,$$

ABA

The role of dressing phase is played by $\dfrac{\sigma^1(u+\frac{i}{2},v)}{\sigma^1(u-\frac{i}{2},v)} \equiv \sigma^{\bullet\bullet}_{dress}(u,v)$

Which solution of crossing does the QSC choose?

Let's look e.g. at the ratio of the two dressing phases.

$$\rho(u,v) \equiv \left(\frac{\sigma^1(u,v)}{\tilde{\sigma}^1(u,v)} \right)^2 \qquad \frac{\rho(u^\gamma, v)}{\rho(u,v)} = K(v)\, \frac{u - v^+}{u - v^-}$$

$$K(v) = C_1(v)/C_2(v)$$

No singularities except one cut, constant asymptotics.
This is solved by Cauchy kernel

$$\log \rho(u,v) = \frac{1}{2\pi i} \int_{-2h}^{2h} \frac{dz}{z-u} \ \log \frac{v^+ - z}{v^- - z} + \frac{1}{2\pi i} \log K(v) \ \log \frac{u - 2h}{u + 2h}$$

$$\log \frac{\sigma^{\bullet\bullet}(u,v)}{\tilde{\sigma}^{\bullet\bullet}(u,v)} = \frac{1}{4\pi i}\int_{-2h}^{2h} dz \, \log \frac{v^+ - z}{v^- - z} \, \partial_z \log \frac{u^+ - z}{u^- - z} \; + \frac{1}{4\pi i} \log K(v) \; \log \frac{(u^+ - 2h)(u^- + 2h)}{(u^+ + 2h)(u^- - 2h)}$$

$K(v)$ is tricky to fix in our present calculation

But there is a unique possibility that guarantees **braiding unitarity**

$$\mathbb{S}^{\bullet\bullet}(u,v)\mathbb{S}^{\bullet\bullet}(v,u) = 1$$

… which leads to the prediction for the dressing phases:

$$\log \frac{\sigma^{\bullet\bullet}(u,v)}{\tilde{\sigma}^{\bullet\bullet}(u,v)} = \frac{1}{4\pi i}\int_{-2h}^{2h} dz \, \log \frac{v^+ - z}{v^- - z} \, \partial_z \log \frac{u^+ - z}{u^- - z}$$

$$+ \frac{1}{8\pi i} \log \frac{(v^+ - 2h)(v^+ + 2h)}{(v^- - 2h)(v^- + 2h)} \; \log \frac{(u^+ + 2h)(u^- - 2h)}{(u^+ - 2h)(u^- + 2h)}$$

matching the recent proposal of [Frolov Sfondrini ’21]

Can we solve it at finite quantum numbers?

[AC, Ekhammar, Gromov, Ryan in progress]

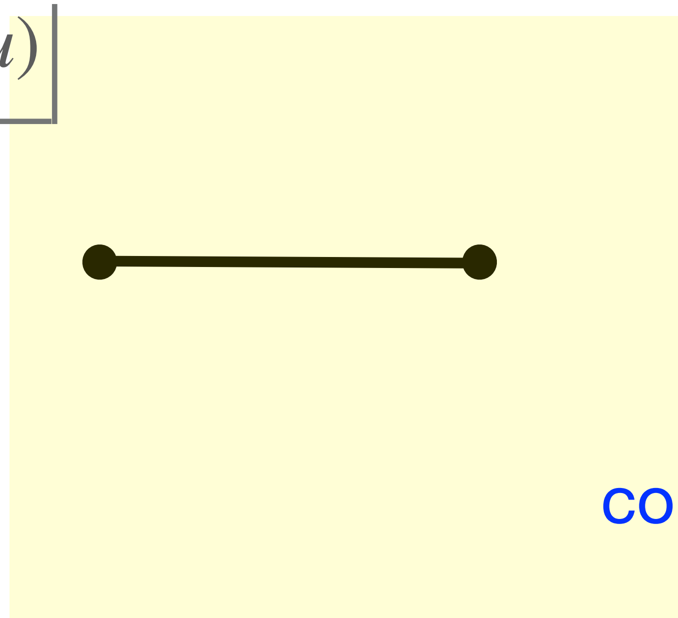
AdS5 method perturbative [Marboe, Volin '14] numerical [Gromov, Levkovich-Maslyuk, Sizov '15]

1. Parametrise Q-system
2. Impose gluing on the cut

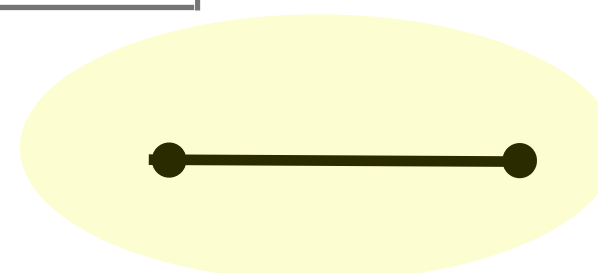
parameters

$$\mathbf{P}_a(u) = x^{M_a}(u) \sum_{n=0}^{\infty} \frac{c_{a,n}}{x^n(u)}$$

$\mathbf{P}_a(u)$



$\mathbf{P}_a(u^\gamma)$

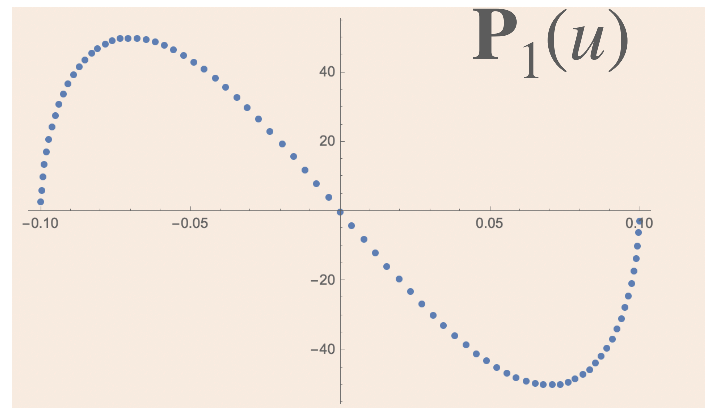


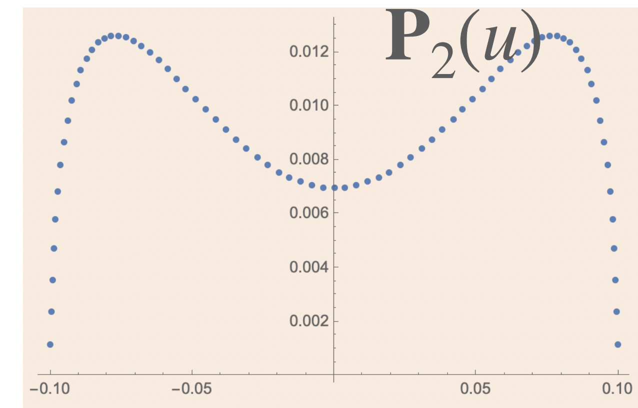
convergence region in previous cases

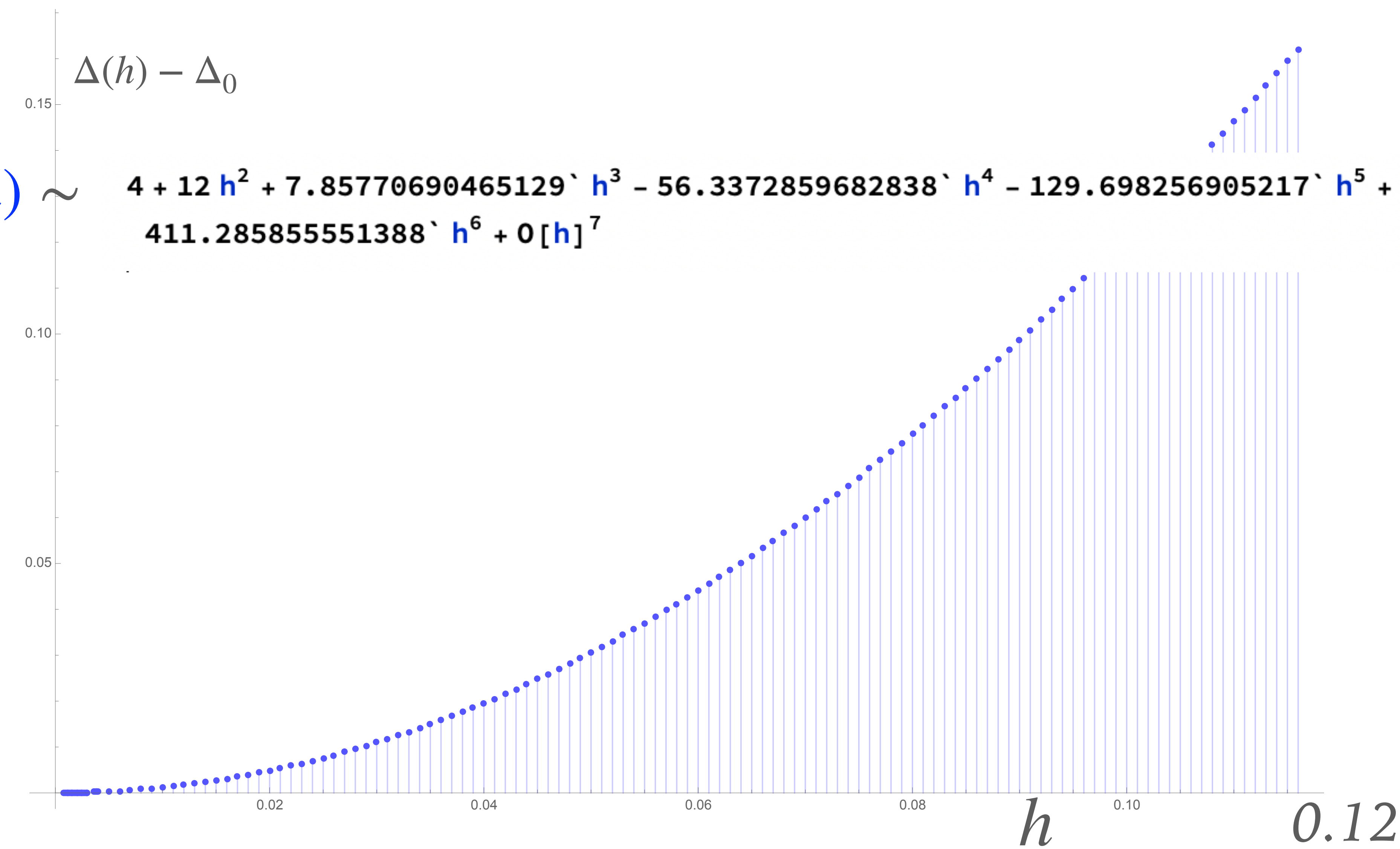
In AdS3, x-series does not converge uniformly on the cut.

We need a new method

We are trying new methods. Evidence of numerical convergence on discrete solutions! It's not an empty mathematical object







$$\Delta(h) \sim 4 + 12\,h^2 + 7.85770690465129\grave{}\ h^3 - 56.3372859682838\grave{}\ h^4 - 129.698256905217\grave{}\ h^5 + 411.285855551388\grave{}\ h^6 + O[h]^7$$

(same quantum numbers as)

state with ABA

representation:

$$x_1 = x_2$$

$$\left(\frac{x_1^+}{x_1^-}\right)^{L=2} = \mathbb{S}^{\bullet\bullet}(x_1, x_2)$$

$$\Delta_{ABA}(h) = 4 + 12h^2 + O(h^4)$$

Directions for the future

We have a concrete proposal for the QSC for AdS3 with Ramond-Ramond flux.
We propose it also contains the massless sector. Very important point to verify.
We should of course test it much more!
A derivation from TBA would be useful.

Preliminary numerical method, but there are still challenges.

Let's make it work! Then we can:

Try to check conjectures for the dual numerically

Develop integrability together with conformal bootstrap for AdS3 and study correlation functions

Other models:

Can we cover the full AdS3/CFT2 landscape
(in general mixed RR+NSNS flux)?

More exotic things should exist: non zero winding/momentum on the torus, moduli, other compact manifolds…

What modifications of the analytic properties will be needed? **What are the rules?**

What is the "integrability coupling"? cf. in ABJM $h(\lambda)$ [Gromov, Sizov '14]

What survives of the QSC at the NSNS point?

AdS_2 ? [Zarembo '10] [Hoare Pittelli Torrielli '14]

Thank you for listening