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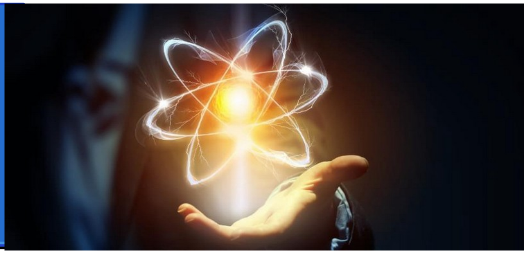
Representation learning in (artificial) intelligence

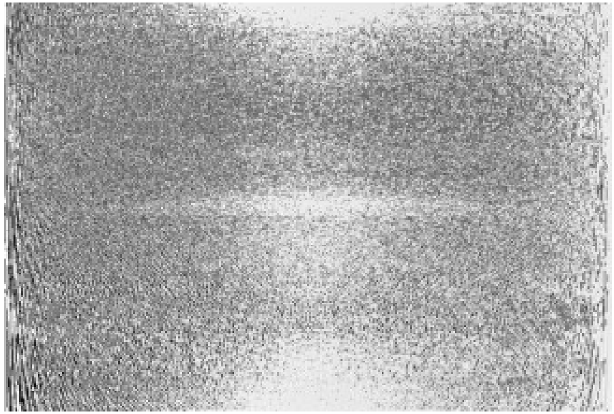
GPU day 2022, June 20-21, 2022.

Motivation: representation matters!



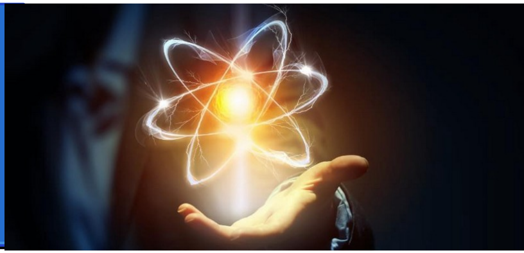
What is in the images?

First seems to be noise … although it is just a transformed variant of the second!



Human visual system uses a recognition function class that relies on the specific properties of the natural images (eg. solid bodies, forms).

Motivation: representation matters!



No “general learning machine”

- $I = \{ N \times M \text{ color images } \}$, for 1Mpx images $|I| \approx 10^{7000000}$
- a class can be any subset: number of subsets $2^{|I|} \approx 10^{10^{7000000}}$
- information in 1Pbyte $\approx 10^{10^{15}}$
- we can describe only a vast minority of all possible classes
- **for success we must exploit the specific properties of the observed class!**
  e.g. in images: important details are slowly changing, shapes, textures, translation and scale invariance
  → included in the Convolutional Neural Network (CNN) architecture

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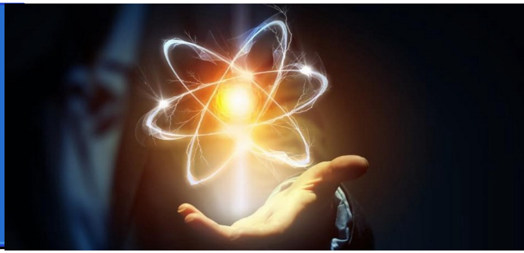


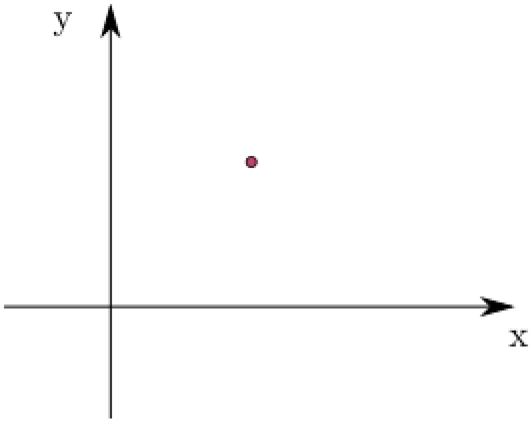
Difference between understanding and training:

- neural network $f(x, \alpha) = y$ maps input to output using a parametrizable function class
- **training**: in a given function class we refine the parametrization to fit to the external requirements (*supervision*)
- **understanding**: find the function class that best fits to the set of the inputs (*unsupervised, data-driven*)
- understanding should precede training! (*representation learning*)

Examples of data modeling

The most elementary, but generic task is to tell if an item is element of a set.

Continuous examples: single 2D data point: S={p} one element set.



We can represent it with the (x,y) coordinates.

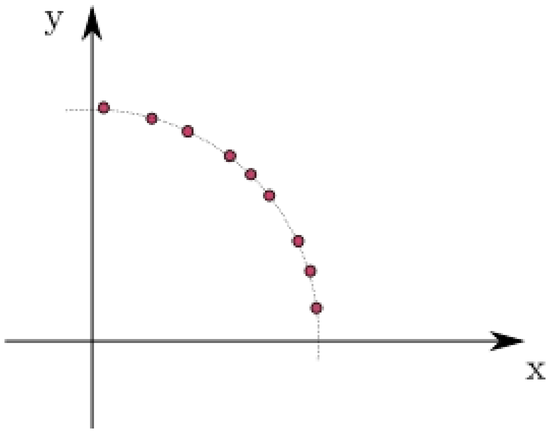
Other representations are also appropriate.

For a single data all representations are equivalent.

Examples of data modeling

The most elementary, but generic task is to tell if an item is element of a set.

Continuous examples: multiple 2D data points



In the (x,y) representation the coordinates are not independent.

In the polar coordinate system (r,φ) we find r=R for all data points! The r and φ coordinates are independent.

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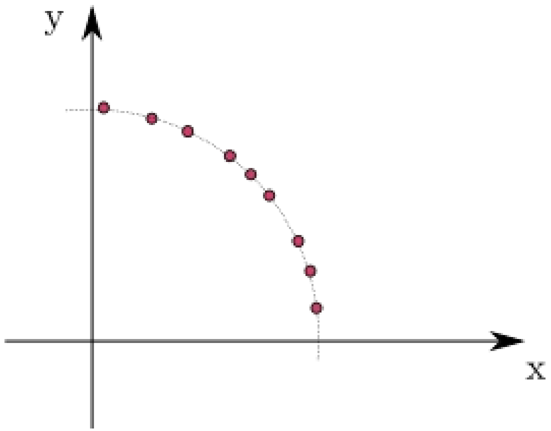


In the (x,y) representation the coordinates are not independent.

In the polar coordinate system (r,φ) we find r=R for all data points! The r and φ coordinates are independent.

In a well-chosen coordinate system the data coordinates are independent, and they are either constant (**relevant** or **selective** coordinates, or **laws**),
or variable (**irrelevant** or **descriptive** coordinates).

Coordination and understanding

If we understand a system well, elementary training is trivial!

Features: independent coordinates over C, either selective or descriptive

Let ξ be the common features for $C_{1,} C_{2,} \ldots, C_a, C = \cup_i C_i$

- **classification:** $x \in C_i$ iff selective bits of $\xi(x) =$ selective bits of C_i
- **decoding:** to produce $x \in C_i$ we have to chose the relevant bits characteristic to C_i and the irrelevant bits independently, uniform randomly

$$\xi^{-1}(\sigma_{relevant} = C_{i,relevant}, \sigma_{irrelevant} = \mathrm{random}) \in C_i$$

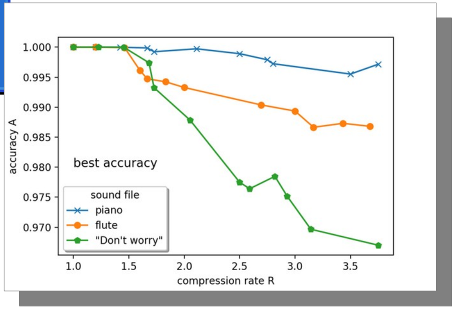
- **lossless data compression:** if we know that $x \in C_i$, the relevant bits can be built into the static part of the code, and we have to store the *irrelevant bits*.

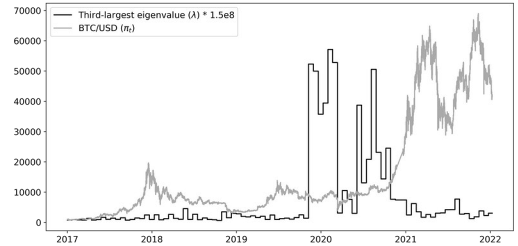
All the AI tasks can be solved by inspecting certain bits.

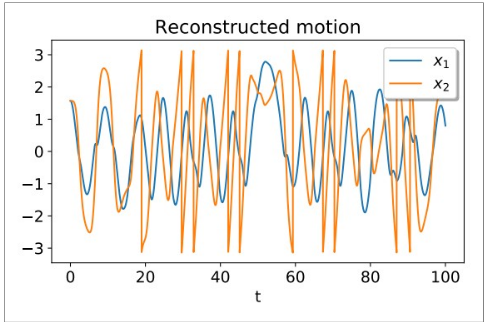
Publications in the topic

Using this technique we studied some topics:

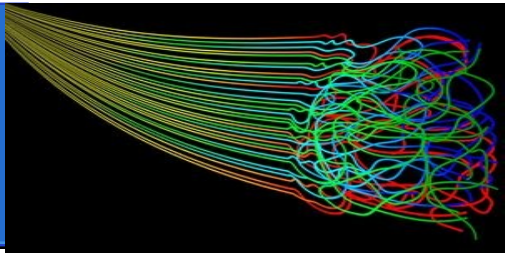
- [D.Berenyi, AJ, P. Pósfay, 2020]: paper about the theoretical basics
- [AJ, 2021]: treating linear laws, application for musical data compression
- [TS. Biró, AJ, 2022] : entropy associated to representations
- [M. Kurbucz, P. Pósfay, AJ, 2022] using linear laws we examined Bitcoin prices and identified potential external influence
- [M. Kurbucz, P. Pósfay, AJ, 2022]: reconstruction of mechanical motions using nonlinear laws
- … more in preparation







Entropy of the intelligence



Intelligence or understanding is the choice of correct representation.

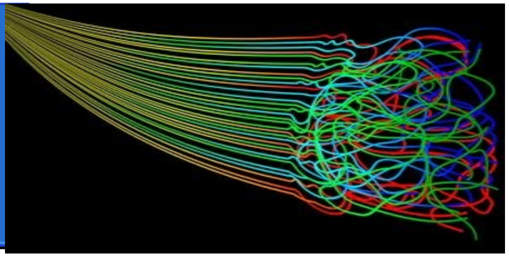
Is there a universal measure to decide, how good a given representation is?

entropy of a representation with respect to a subset

- **Shannon entropy:** $$S_{SH} = \sum_{\sigma \in B^N} p_C(\xi = \sigma) \log_2 p_C(\xi = \sigma) = \log_2 |C|$$
  - independent of the representation
  - yields the true information content of the set (i.e. the number of necessary bits)
- **representation entropy:** ξ coordination implies $p_C(\xi_i = \sigma_i)$ **bitwise distribution**

$$S_{repr} = \sum_{i=1}^{N} \left[\sum_{\sigma \in 0,1} p_C(\xi_i = \sigma_i) \log_2 p_C(\xi_i = \sigma) \right]$$

Entropy of the intelligence



Representation entropy

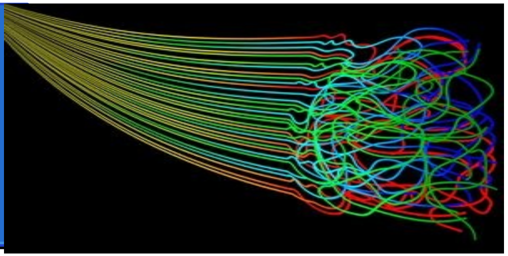
$$S_{repr} = \sum_{i=1}^{N} [\sum_{\sigma \in 0,1} p_C(\xi_i = \sigma_i) \log_2 p_C(\xi_i = \sigma)]$$

Mathematical properties

- $S_{repr} \geq S_{SH}$, equality if the coordination is independent
- minimality of S_{repr} implies independence, and the least # of descriptive coordinates
- $Loss = S_{repr} + \lambda \alpha + \mu \beta$ can be used in practice, with type one and two errors (false negative and false positive)

representation entropy is a general unsupervised loss function:
in a general learning process, by minimizing the representation entropy, we get closer to the learning of the proper representation

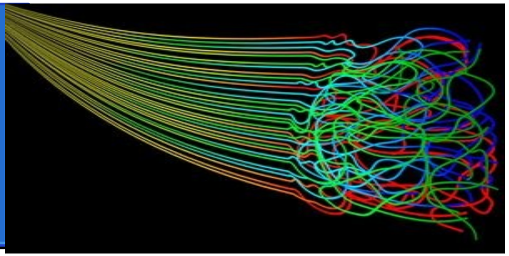
Reconstruction of mechanical motions



Task:

- observe a motion $\{x_n \in \mathbb{R}^D \mid n \in \{0,\ldots,N\}\}$
  - ‘n’ is a (discrete) time variable for $t = n\Delta$, maximal observed time $T = N\Delta t$
  - D dimensional motion
- describe/characterize the motion
- continue for t>T in a ”plausible” way

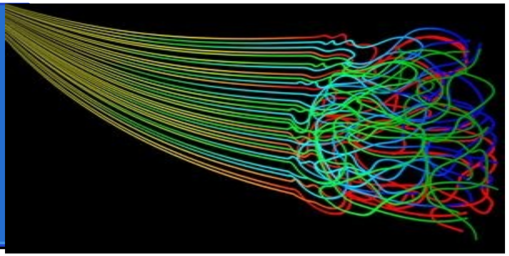
Reconstruction of mechanical motions



Method:

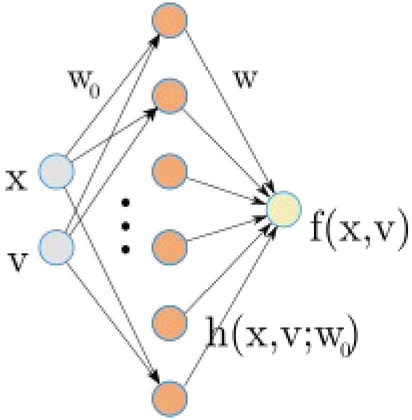
- local characterization of the motion: $v_n = \frac{x_n - x_{n-1}}{\Delta t}, \quad a_n = \frac{x_{n+1} - 2x_n + x_{n-1}}{\Delta t^2}$
- look for different level “laws”/constraints:
  - level 0, holonomic contraints: $C^{(0)}(x_n) = C^{(0)}(x_0)$
  - level 1: anholonomic constraints, conserved quantities: $C^{(1)}(x_n, v_n) = C^{(1)}(x_0, v_0)$
  - level 2: lows for acceleration / discrete Newton’s laws: $a_n = f(x_n, v_n)$
- In a consistent mechanical system Newton’s laws are compatible with lowest order constraints, but in numerical observations they are independent.
- from discrete Newton’s laws a recursion can be obtained $(x_{n-1}, x_n) \Rightarrow x_{n+1}$

Reconstruction of mechanical motions



Numerical implementation:

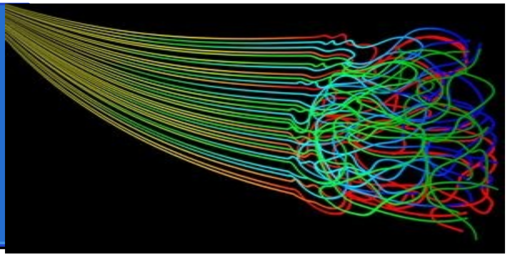
- input: (x_n, v_n) 2D dimensional
- output: $C^{(1)}(x_n, v_n)$ conserved quantity or $f(x_n, v_n)$ force function
- network: Extreme Learning Machine, 1 hidden layer, only last weights are trained, nonlinear activation function ensures smoothness of output



Issues:

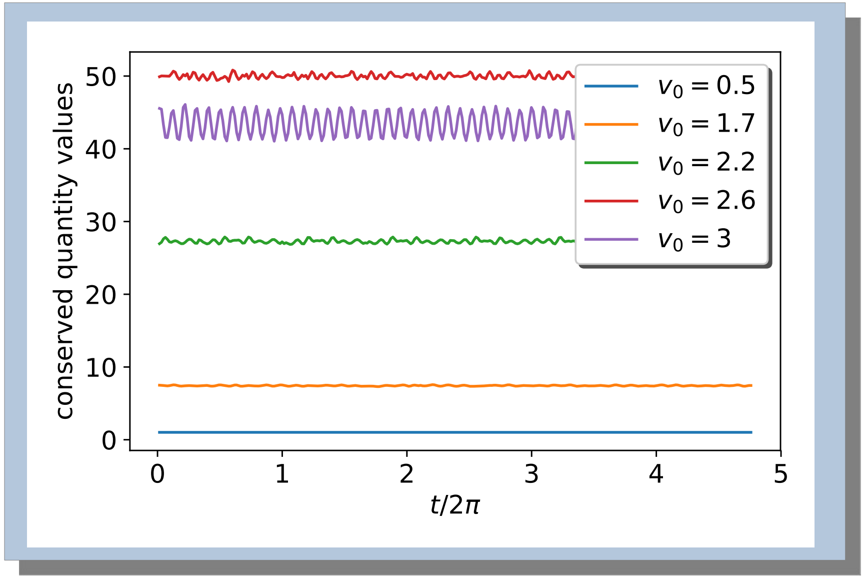
- chaoticity: if nearby motions diverge fast, even “exact” methods give different results. Comparison: force and qualitative features
- renormalization: recursion can be determined for different Δt, multi-step algorithms are possible.

Reconstruction of mechanical motions



Results:

- gravity pendulum: integrable motion $\ddot{x} = -\sin x$



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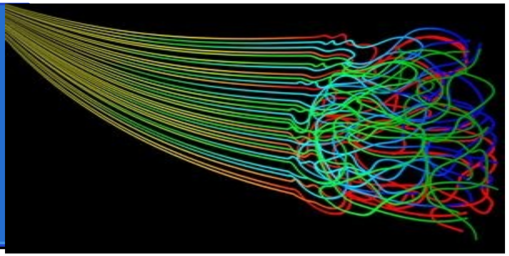

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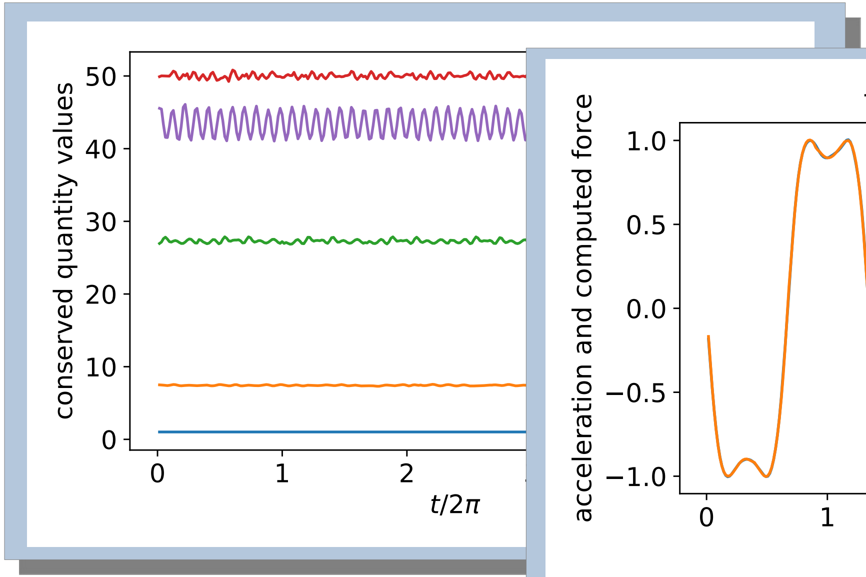

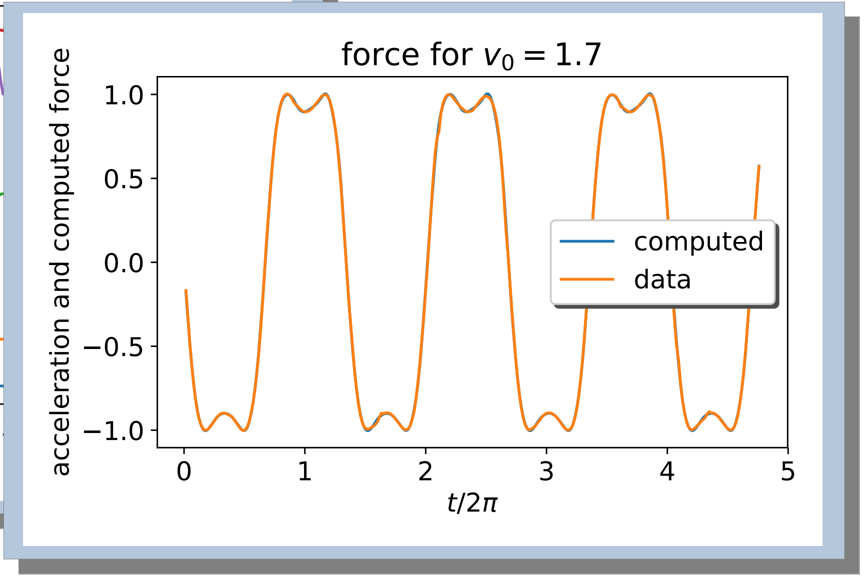

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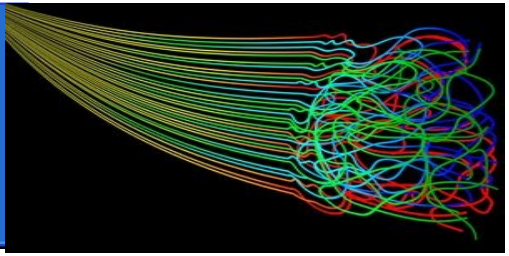

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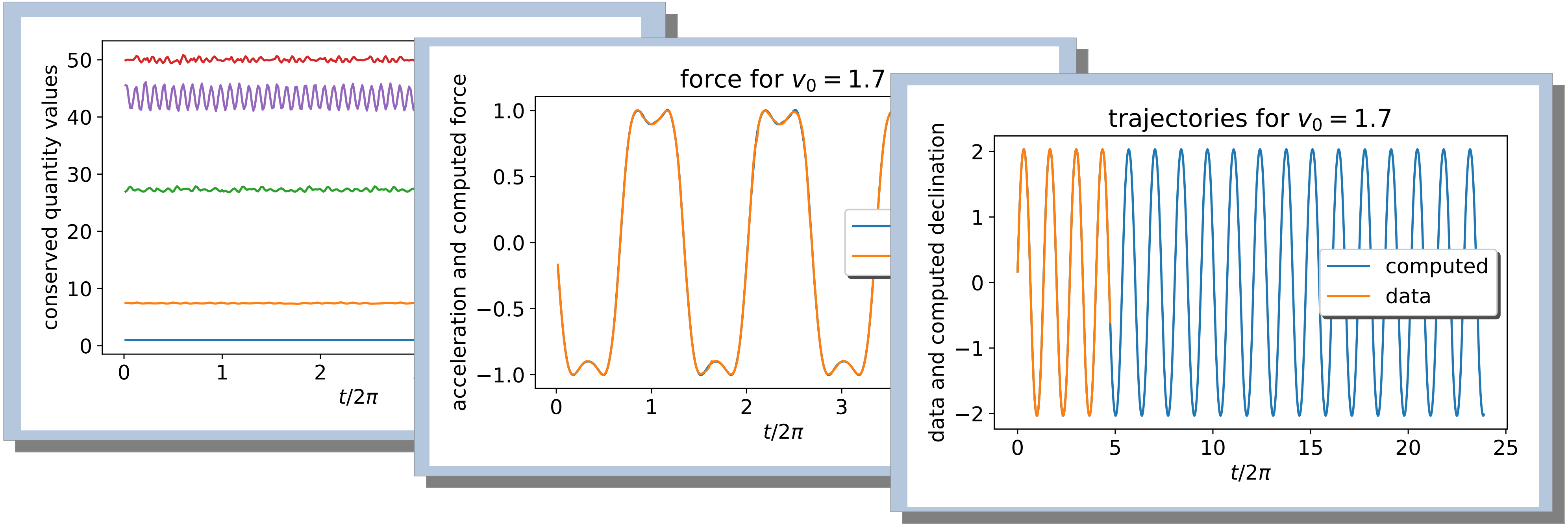

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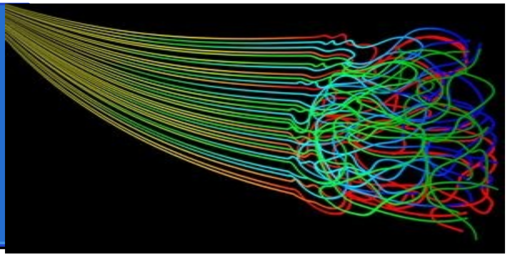
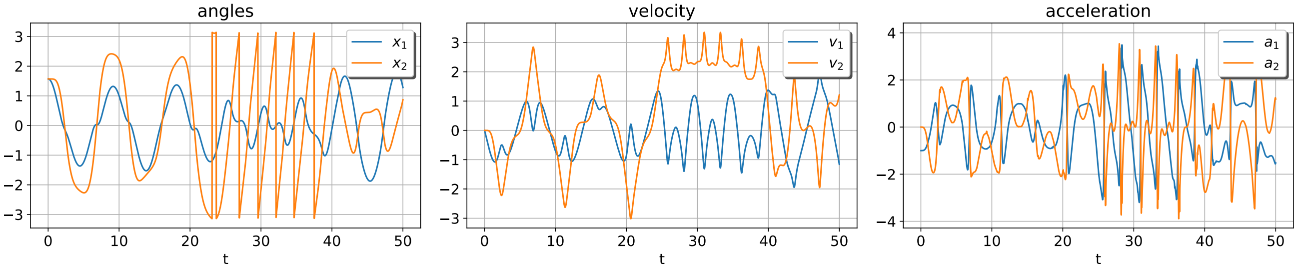
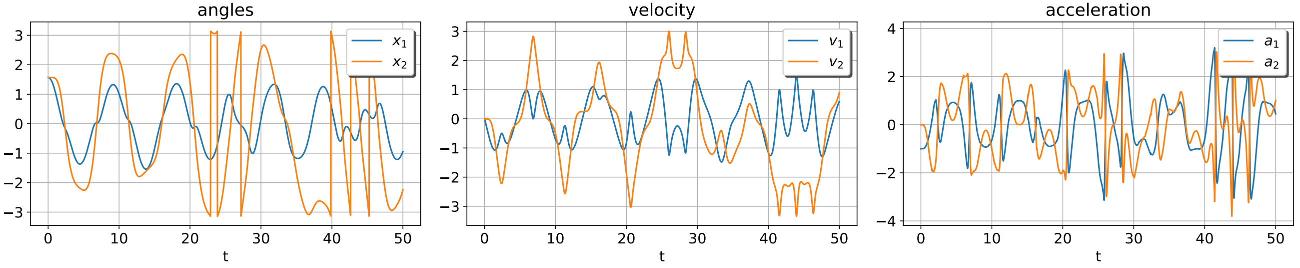


Results:

- double pendulum: 2D chaotic motion
- “exact solution” can not be found
  - Python scipy DOP853



  - Python scipy RK45



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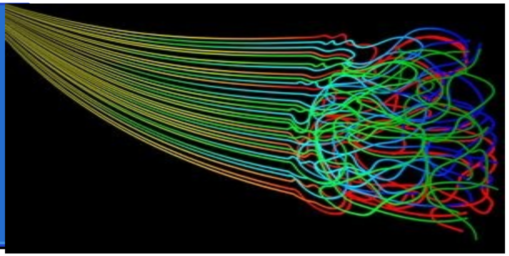

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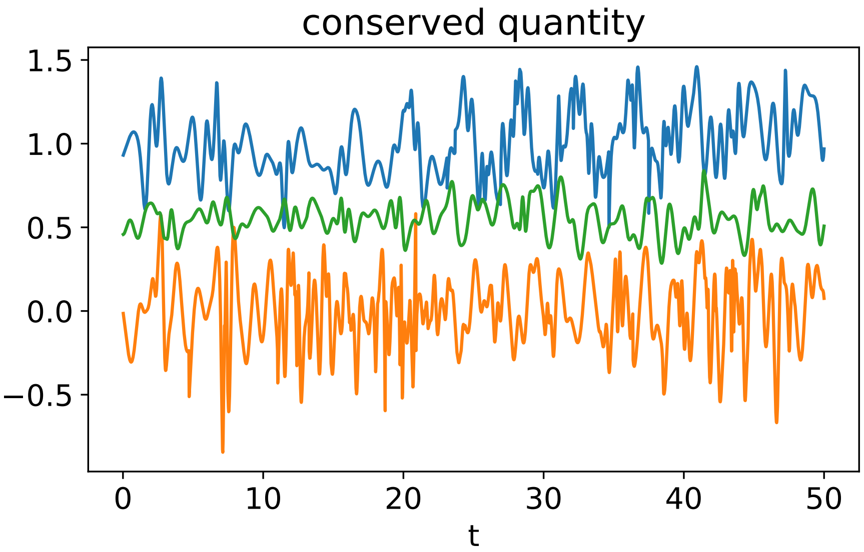

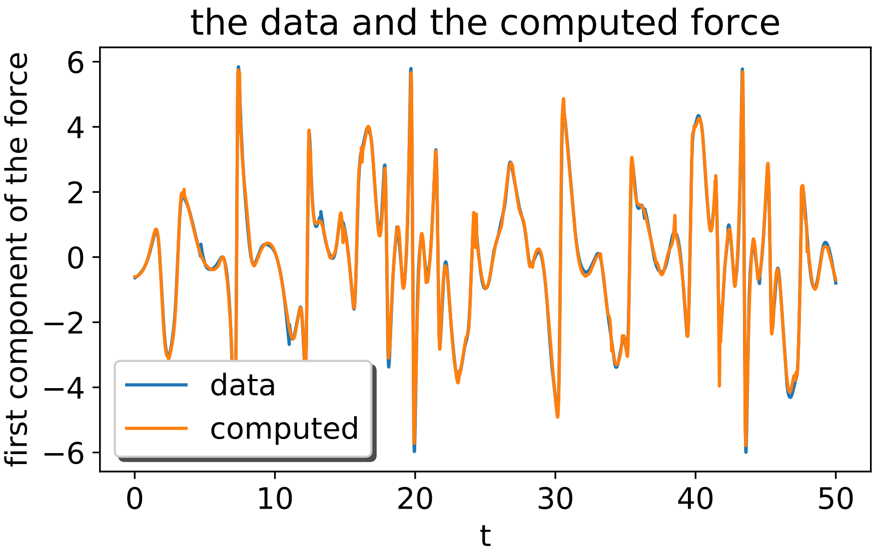

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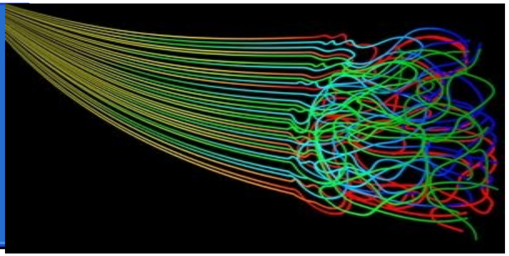
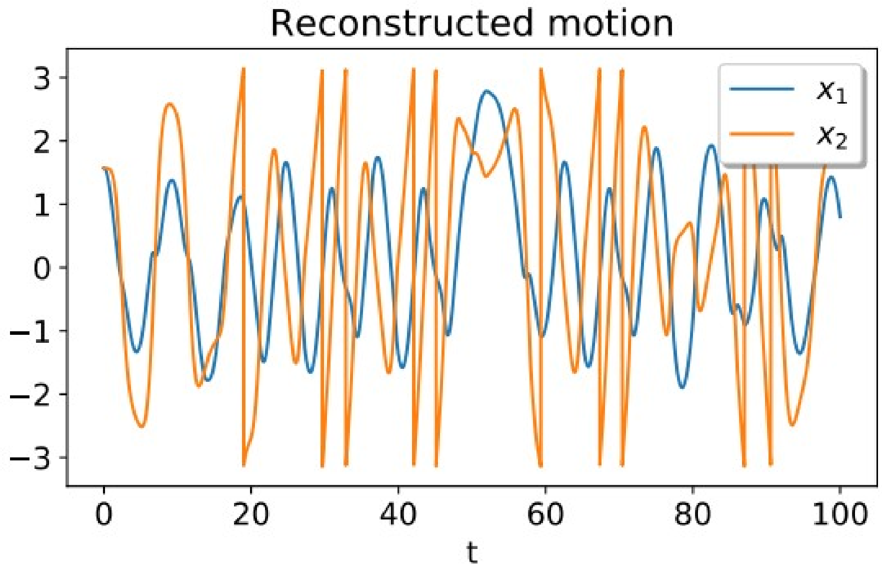


Results:

- double pendulum: 2D chaotic motion reconstructed force with 93% accuracy
- motion qualitatively correct, no runaway solutions

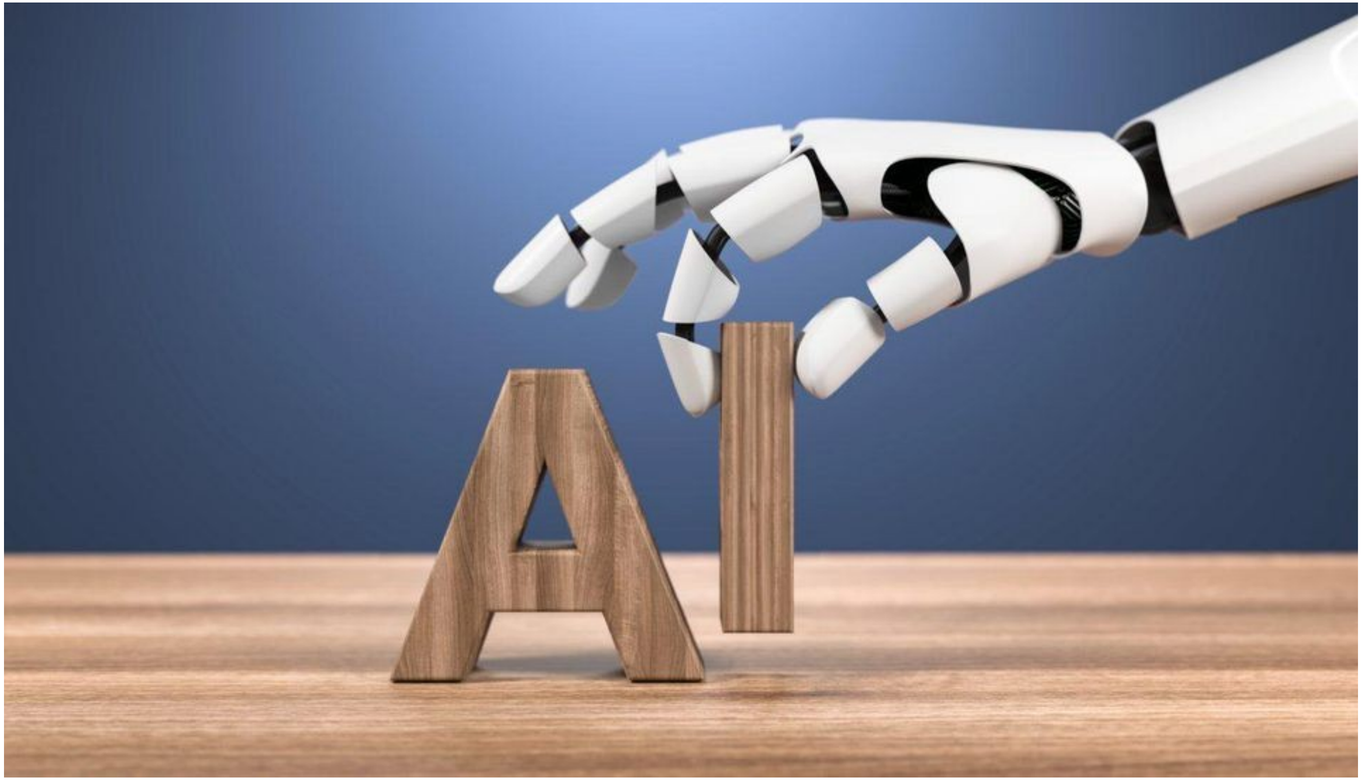


Conclusions

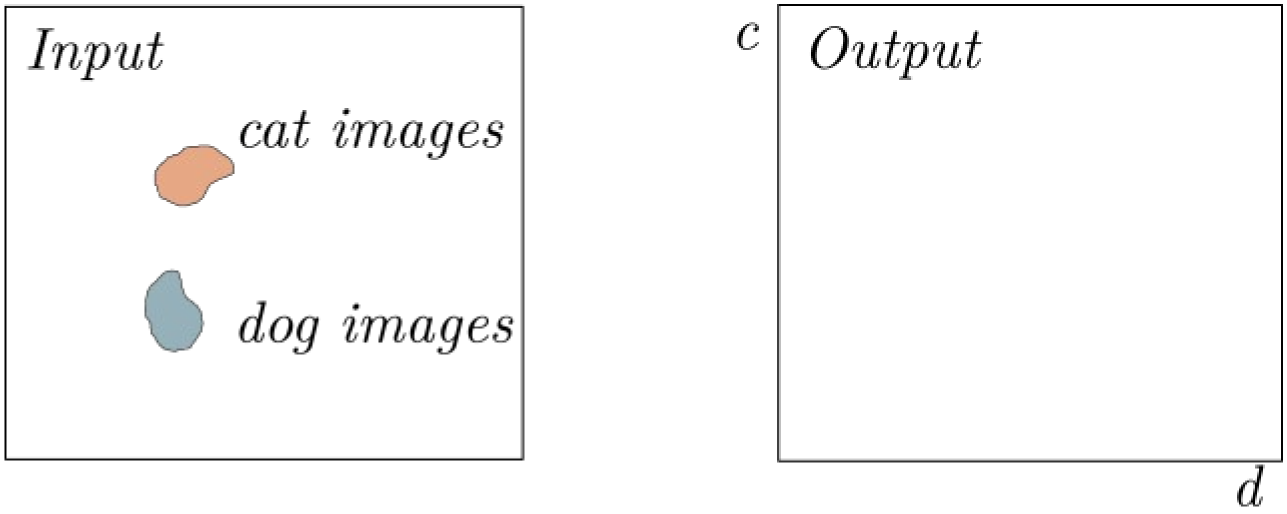
understanding ≡ best representation of data

- independent features (coordinates) over a set C: either selective or descriptive
- selective/relevant features: constant over C, good for classification
- descriptive/irrelevant features: variable over C, good for compression
- representation entropy: universal unsupervised loss function, by minimizing it we improve understanding
- in mechanical systems laws ≡ conserved quantities & Newton’s law
  - good reconstruction for integrable systems
  - qualitatively correct reconstruction for chaotic motions

The end



Interpretation of AI



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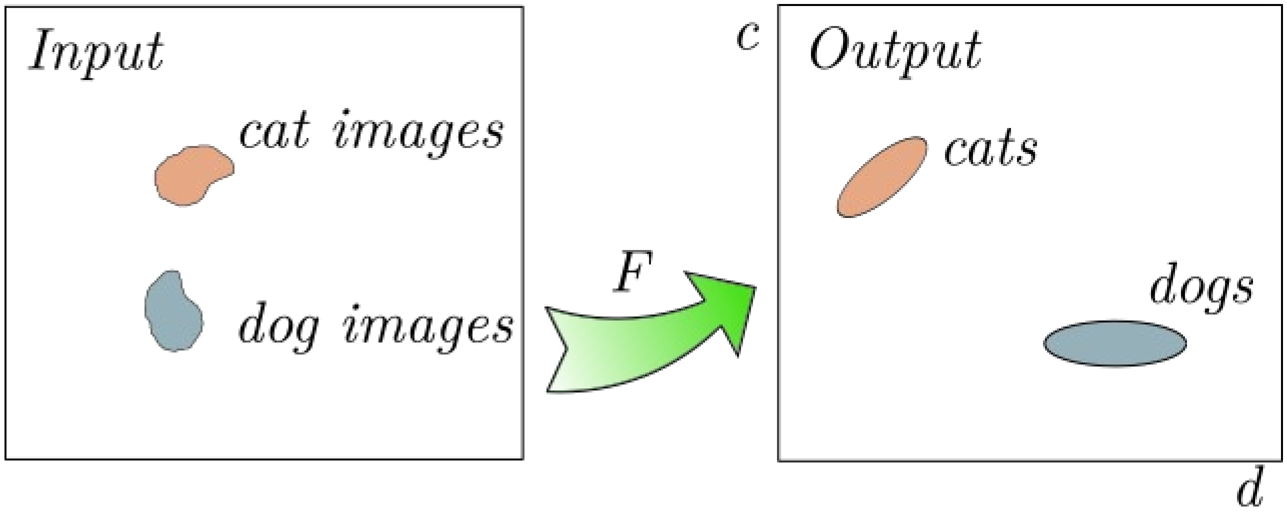

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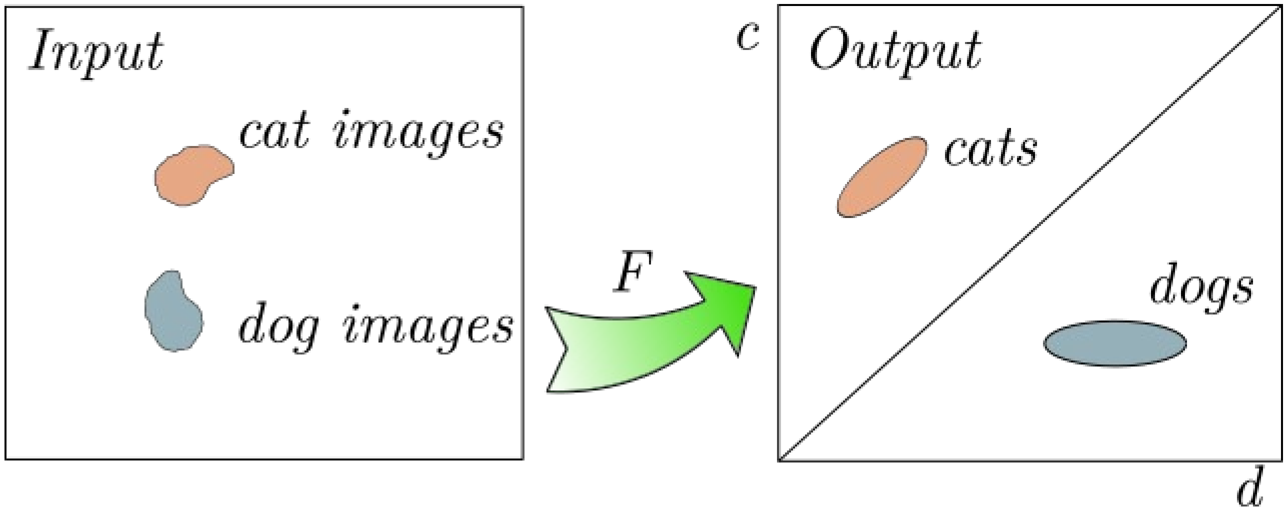

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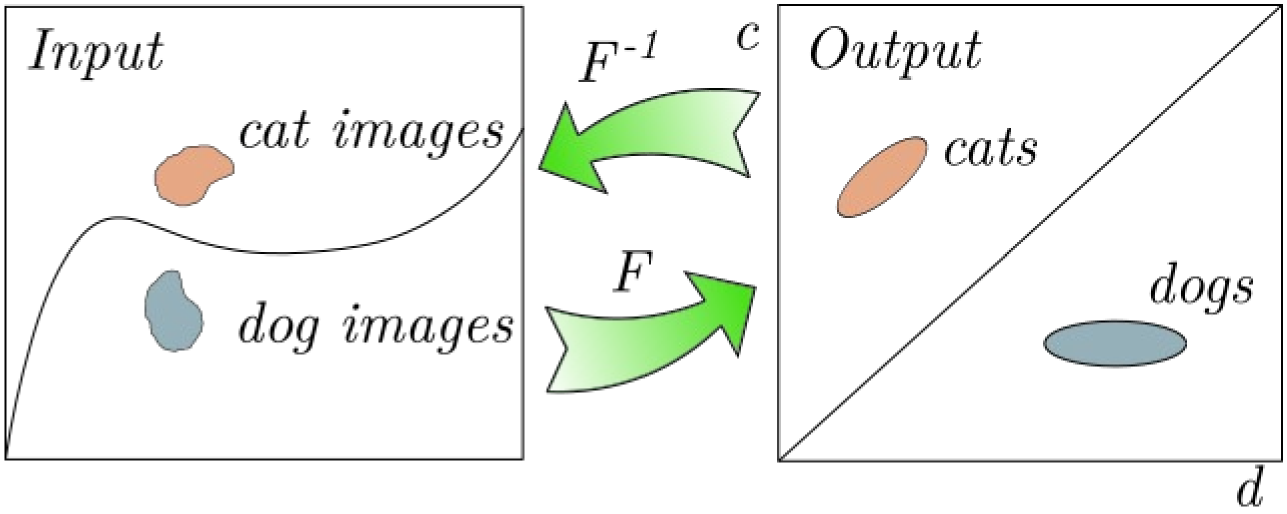

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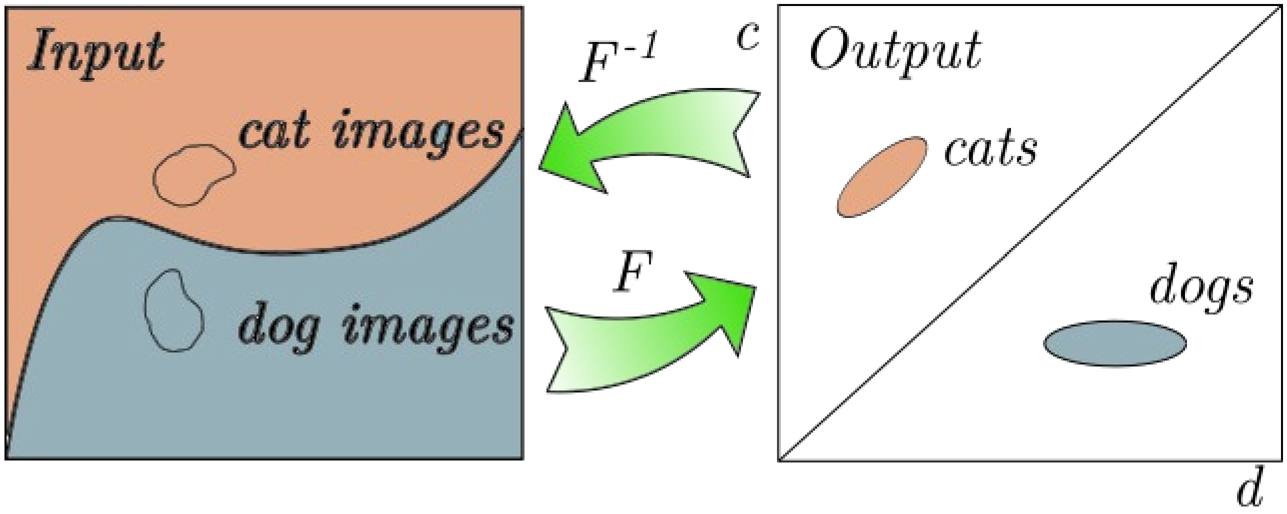

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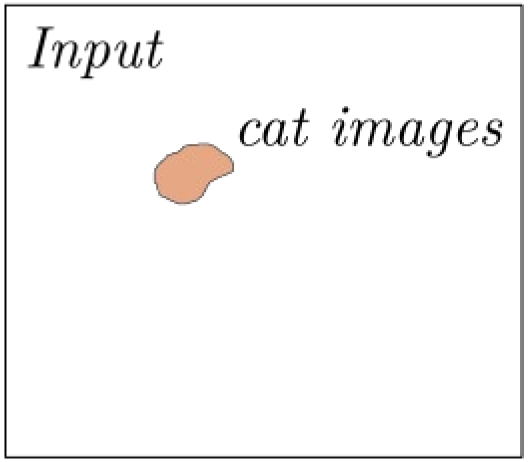

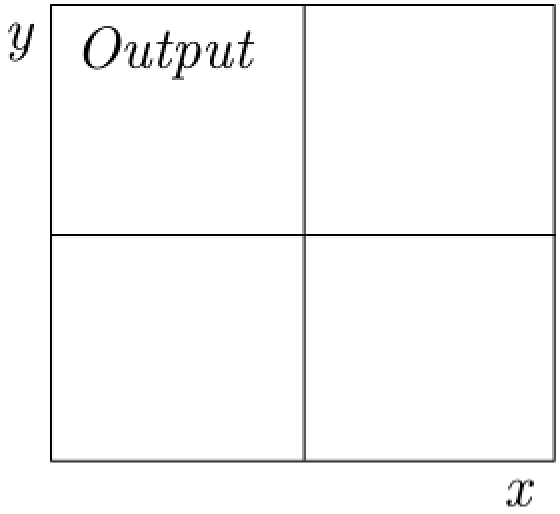

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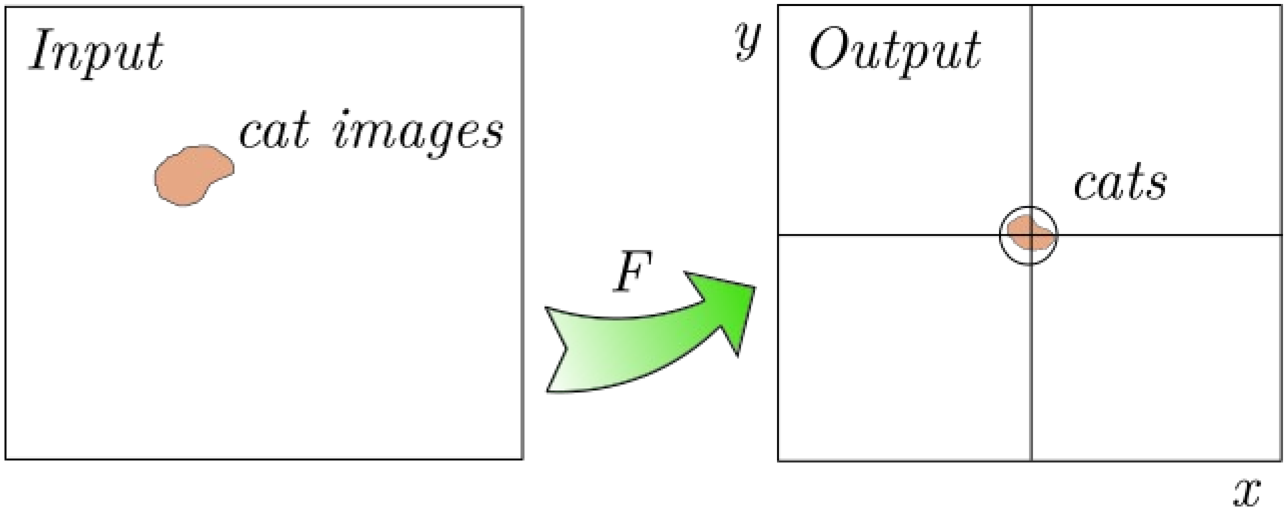

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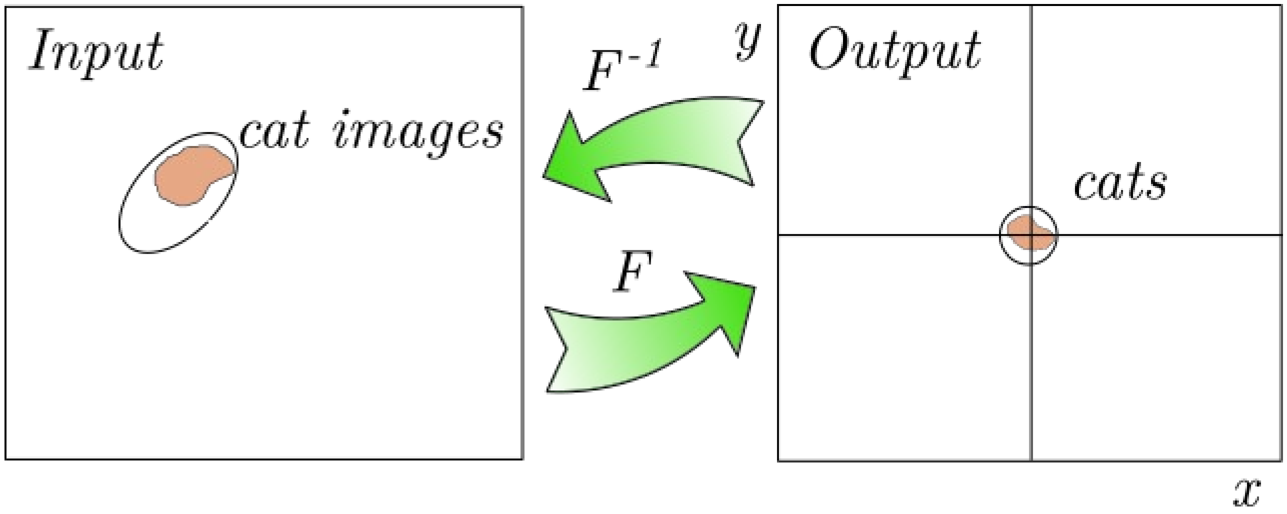

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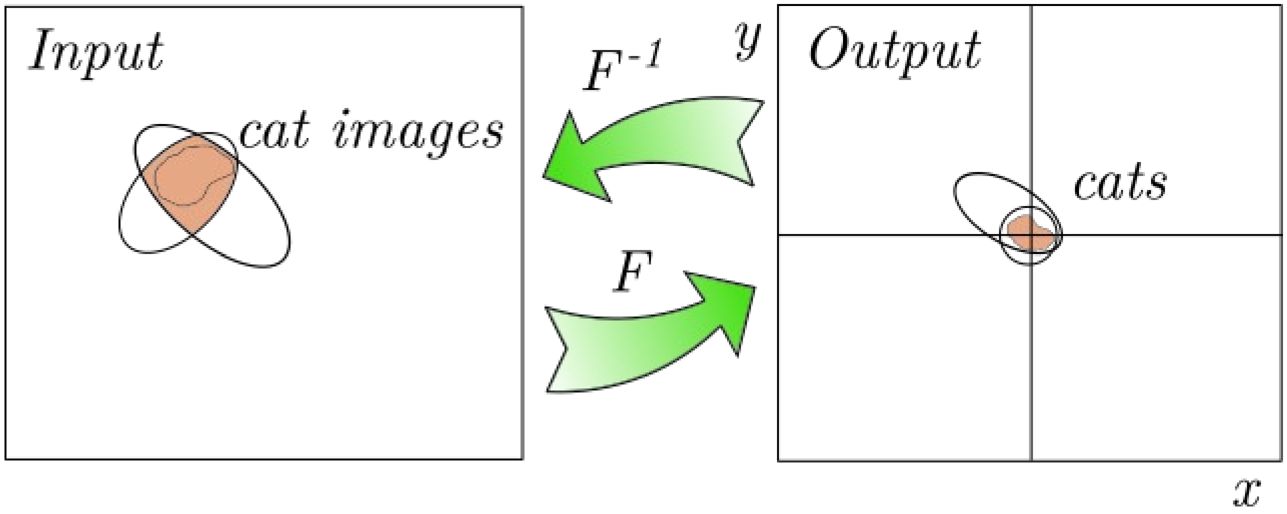
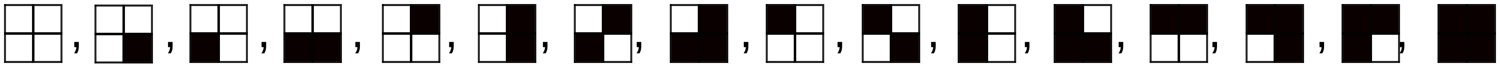


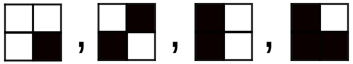
Examples of data modeling

The most elementary, but generic task is to tell if an item is element of a set.

Discrete examples: consider *2x2 bitmap "images"*, and choose a subset. Can we find the proper representation of the set where the identification of the subset is easy?

We can list all images:

X = {  }

choose an arbitrary subset, our abstract "cat images": C={  }

- the pixel-wise coordination C={0001,0110,1010,1011} : no regularity
- the pixels are not independent in C:

$$P(\xi_1=0, \xi_2=0)=1/4 \neq P(\xi_1=0)P(\xi_2=0)=1/2*3/4$$

Examples of data modeling

Find a coordination that fits the best to the problem!

X = {→0100, →0000, →0101, →0110, →0111, →1000, →0001, →1001,
→1010, →1011, →0010, →0011, →1100, →1101, →1110, →1111}

This is *not the original bit coordinates,* but it fits well to our chosen C subset!

In the new coordinates: C = {0000,0001,0010,0011}

- first two bits are 0 for elements of C: these are the relevant (selective) coordinates:
  $x \in C \Leftrightarrow x_0 = x_1 = 0$:appropriate to select the elements of C
- last two bits are variable: these are the irrelevant (descriptive) coordinates:
  to tell apart elements of C (compression) we need to consider only these coordinates