Numerical Simulation of Mirages above Water Bodies

Anna Horváth, Gergely Gábor Barnaföldi, Balázs Bámer
Wigner Research Center for Physics,
29-33 Konkoly-Thege Miklós Str., H-1121 Budapest, Hungary

Introduction

When light travels through a medium with a **changing refractive index**, it gets bent towards its higher values. This can produce upside down **'mirror' images** of a scenery in deserts, above over-heated roads, surfaces, or bodies of water. The so called *mirage* or *fata morgana* appears in stable weather conditions, if the **temperature difference** between the two layers is high-enough to create a large refraction index gradient. We implemented a computer program, which simulates mirages above water using the method of **ray tracing**.

Inferior mirage over Lake Balaton (top) and the scenery of the same area without the mirage (bottom) taken from a height of approximately 50m, so that the surface level would not get cut off by Earth's curvature. Latter was used as a base for simulations.

Strategy & Model:

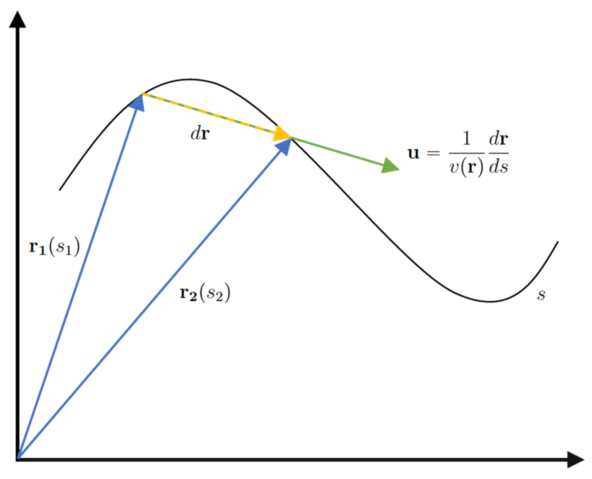
- Developing a **model for** the **temperature profile** above water bodies based on measurement data
- Using a model **calculation of** the **refractive index** profile from the temperature values
- Implementation of a **computer program** that simulates the mirage:
  - Solving the **eikonal equation** with Runge-Kutta methods – **ray tracing**
  - **Pinhole camera** setup
  - Taking into account the **curvature of Earth**
- **Simulation** of mirages
  - Examining the path of **individual rays**
  - Simulating views of distant objects with **different temperature** and **distance** values
  - **Comparing photos** of mirages with our simulations

Numerical Solution by Ray Tracing

Ray paths were calculated using an **eikonal equation** of the form:

$$|\nabla t|^2 = \frac{1}{v^2}$$

where t is travel tin[...]ont and v = c/n(***r***) is speed of light in the [...]**equations of motion** are:

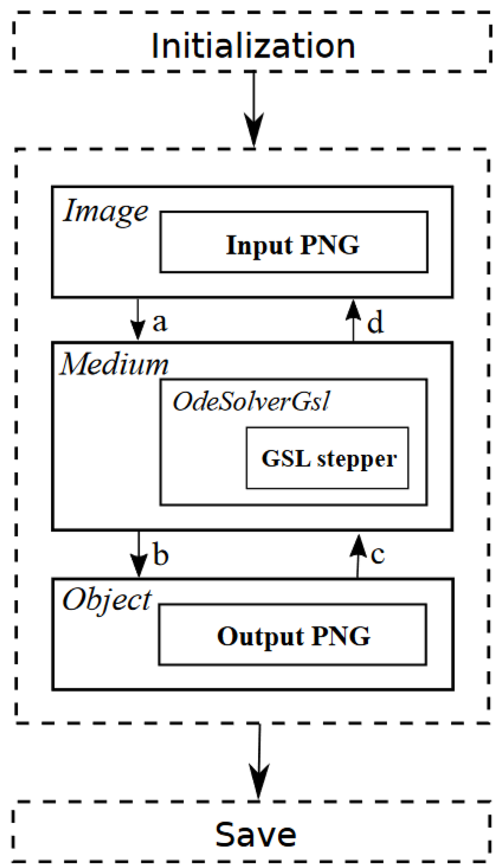


$$\frac{d\mathbf{r}}{ds} = v(\mathbf{r})\mathbf{u}(\mathbf{r})$$
$$\frac{d\mathbf{u}}{ds} = -\frac{1}{v(\mathbf{r})^2}\nabla v(\mathbf{r})$$

where ***u*** is the slowness. The integration of the equations was performed using the **second order adaptive Runge-Kutta** method of the GNU Scientific Library [3].

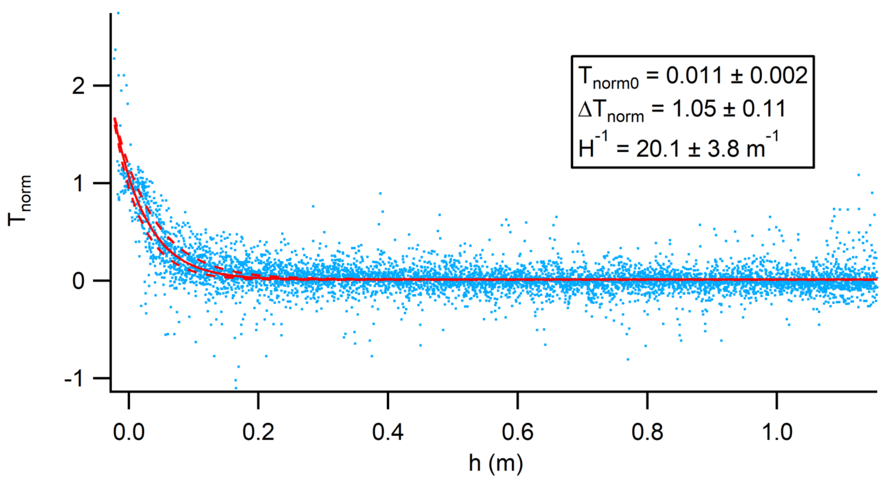
Structure of the Program:

- Rays **start** from a pixel on the **optical screen** of a pinhole camera
- **Initial angle** of the rays is determined by the relative **position of** each **pixel and** the **hole**
- Rays **travel through** the **medium** with a predetermined temperature distribution and pressure **(a)**
- Traced until **reaches** the water **surface or** plane of the target (**object**) **(b)**
- If it **hits the object**, it will be **assigned a colour** characteristic of the target at that point; if it does **not hit** anything, it will be assigned **black (c)**
- This **colour** will be **given to** the **pixel, where** the ray was **initialized**, thus forming the picture that the camera sees **(d)**
- **Foreground** is **added** to the pictures, which is not part of the simulation



Temperature and Refractive Index Profile Above Water

We used **measurement data** [1] taken at **Lake Geneva** to build our model. Altogether twelve measurements were taken at different times of the day **at the surface**. For some of the cases the ambient temperature was lower, for others, it was higher than that of water. We concentrated on inferior mirages, however, from this data superior mirages could also be simulated (ones appearing above the real images). The temperature of water in the data was about 10°C, and the ambient temperature varied approximately between 8-15°C.



We **fitted** an **exponential** function on **normalized** measurement data (temperature as a function of height) of the form:

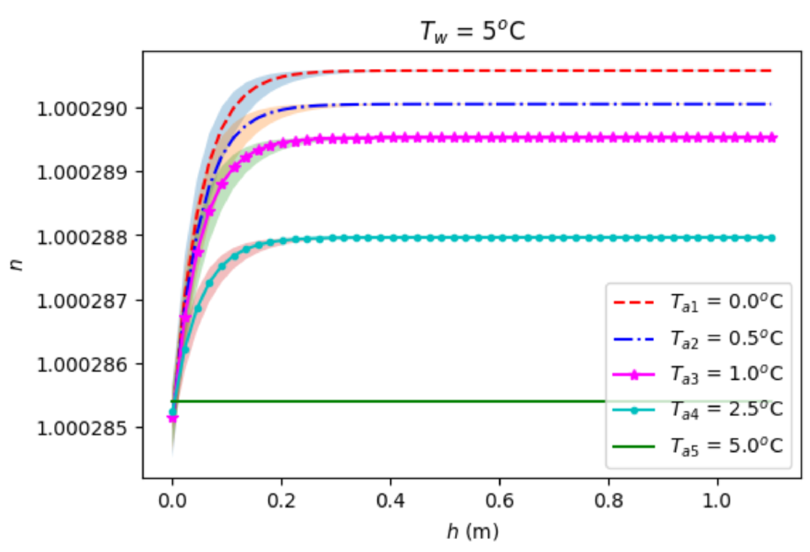
$$T(h) = T_a + \Delta T e^{-h/H}$$

where T_a is ambient temperature (sufficiently far from the surface), H is characteristic length and $\Delta T = T_w - T_a$ is the temperature difference between the water and ambient air.

From the temperature profile (for which the **input** is **ambient** temperature **and water temperature**) we calculated the **refractive index** profile as a function of height using [2]:

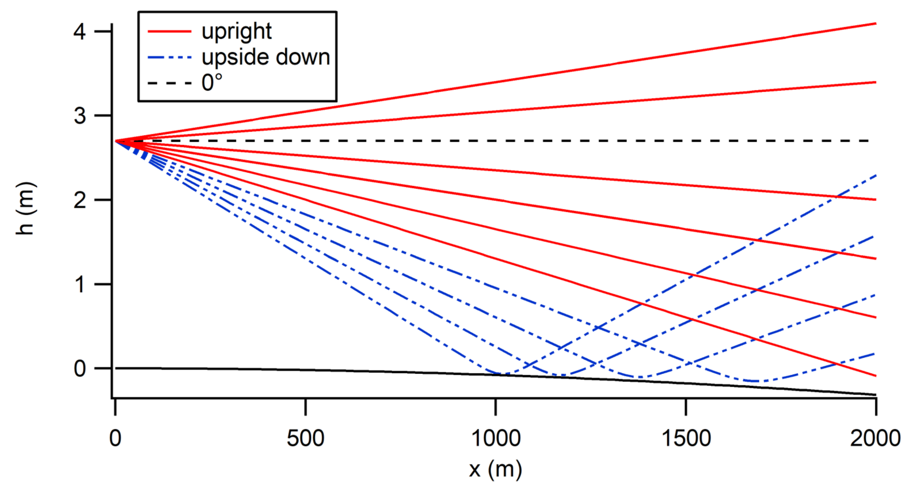
$$n = 1 + \frac{7.86 \cdot 10^{-4} p}{273.15 + T}$$

where p is pressure in kPa. In our simulations it was set to 101kPa.

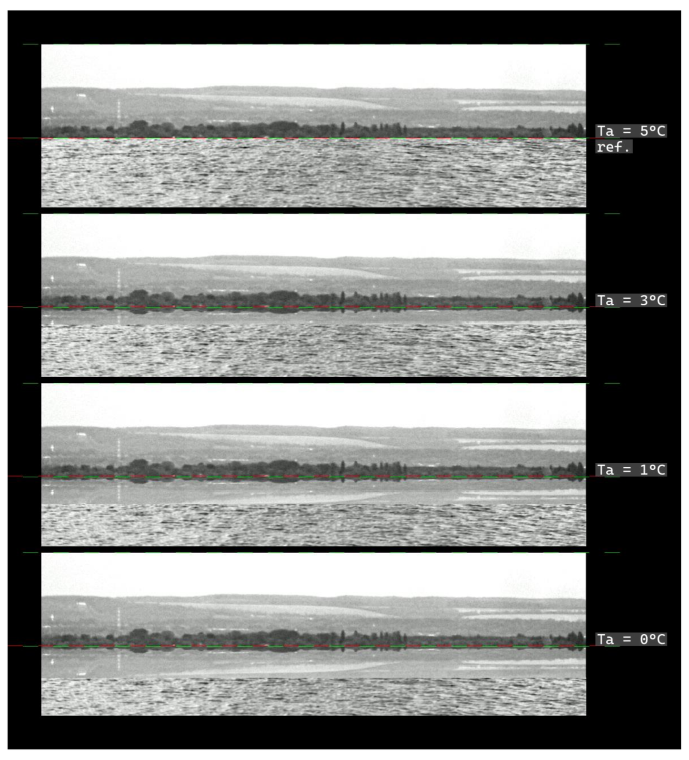


Results

Bending of rays happens primarily on the **first 10 centimetres** from the surface, since the temperature gradient is the greatest here. The **curvature of Earth** makes a huge difference already on the scale of a few kilometers. **Rays that just pass** over the surface by bending upwards (shown in **blue**) produce an **upside down image**, which is what we call an inferior mirage..

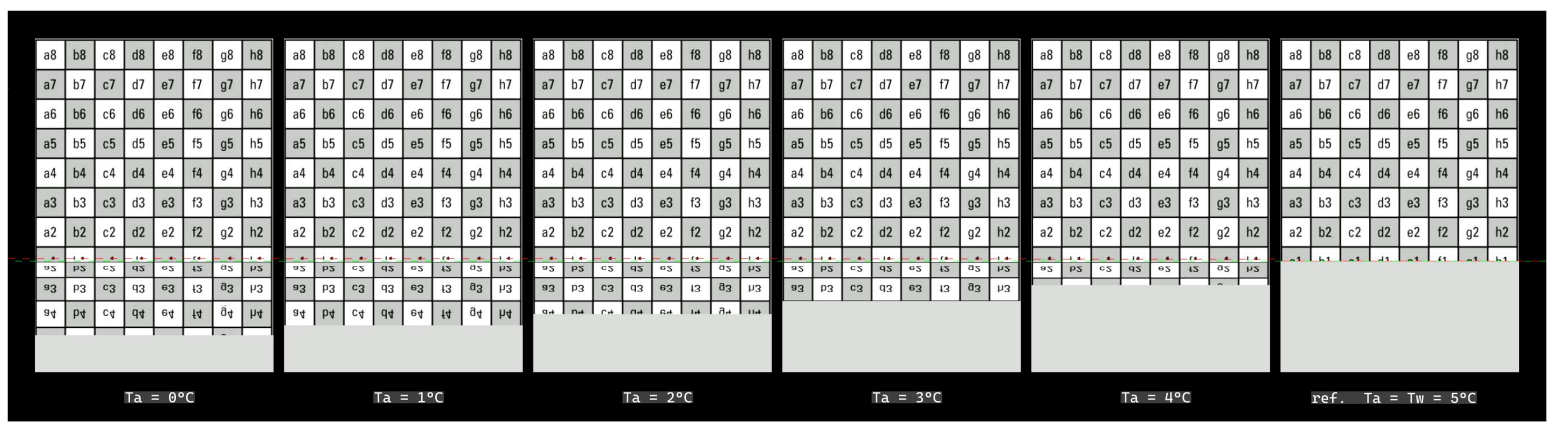


Bending of light rays in air over a water surface with ambient temperature 1°C and water temperature 5°C, on a distance of 2000m. Each ray starts from a height of 2.7m in different angles (-0.16-0.04°) from horizontal in 0.02° steps. The surface of Earth is indicated with a black curve.



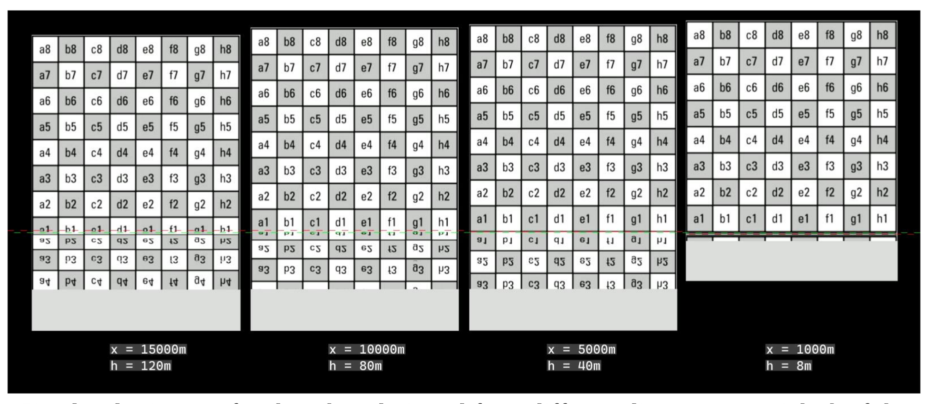
When the **temperature difference** between the water and ambient air is **greater**, the **upside down image** we can see is **taller**. The picture gets distorted (more precisely **compressed**), especially close to the mirror axis (red). Changing temperature does not seem to effect the amount of compression. **Bottom** of the images get **cropped** both **due to Earth's curvature** and the **mirage** itself. Edges of the former (the reference image, when $T_a = T_w$) is shown by green. When there is a temperature gradient, the **mirror axis** (red) and the **horizon** (green) are **not the same**: bottom of distant objects get obscured by the mirage.

Simulated mirages over Lake Balaton with T_w = 5°C and different ambient temperatures. Water shown underneath is not part of the simulation, it is located between the object and the observer. Green dashed line indicates the reference, red the mirror axis.



Simulated mirages of a chess board [4] with T_w = 5°C and different ambient temperatures. The uniformly gray area shown underneath is not part of the simulation, it is located between the object and the observer. Green dashed line indicates the reference, red the mirror axis.

For greater distances the mirror axis is either the same as the horizon (when $T_a = T_w$), or is higher up. However, on the scale of about **1 kilometer** the **mirror axis** is **below** the **horizon**. This suggests that between the upright image and the mirrored one we would see objects from the foreground of the scenery.



Simulated mirages of a chess board viewed from different distances x. Height h of the base image was adjusted to the distance. The uniformly gray area shown underneath is not part of the simulation, it is located between the object and the observer. Green dashed line (aligned for all cases) indicates the reference (where $T_a = T_w$, not shown here), red the mirror axis. T_w = 5°C, T_a = 1°C.

Acknowledgements

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References

[1] Selker, J. S. et al., "Distributed fiber-optic temperature sensing for hydrologic systems" (2006)
[2] Stone, J. and Zimmerman, J., "Index of Refraction of Air" (2001)
[3] Free Software Foundation, Inc. "GNU Scientific Library: Ordinary Differential Equations" (last visited: 22nd April 2022)
[4] Source of base image (last visited: 17th May 2022)

Scan to watch our simulations as videos!