Neutrino Mixing with Discrete Flavor Symmetries and its Consequences

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Based on 2203.08185

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Outline

- **Introduction:**
  * Neutrino physics and the known unknowns.
- **Flavor Symmetry and Lepton Masses and Mixing:**
  * Flavor symmetries, why?
  * General framework
  * Family symmetry, nonzero θ_{13} and nonzero δ_{CP}
  * Flavor symmetry and CP symmetries, higher order discrete Groups, GUT etc.
- **Implications of Flavor Symmetry in Various Frontiers**
  * Dark matter
  * Baryon asymmetry of the Universe
  * Collider physics
- **New Ideas**
  * Modular symmetry
  * Flavor symmetry and neutrino experiments
- **Conclusion**

Neutrino parameters and the known unknowns

- Neutrinos are special!! It's flavor and mass eigenstates are related by : $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$.
- Pontecorvo-Maki-Nakagawa-Sakata parametrization:

$$U_{PMNS} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$.

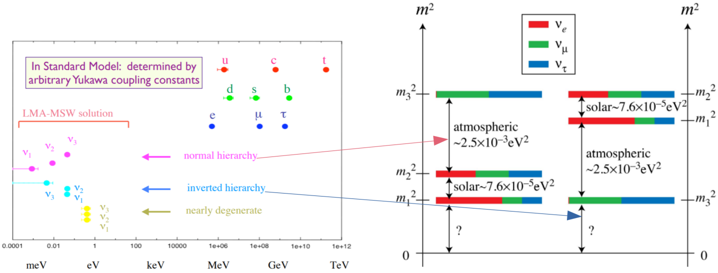
- Large Lepton Mixings

$$|U_{PMNS}| \sim \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.14 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix}$$

- Small Quark Mixings

$$|V_{CKM}| \sim \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix}$$

Neutrino parameters and the known unknowns



	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{\rm CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

Flavor symmetries, why?

$$U_{PMNS} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{pmatrix}$$

$\Downarrow$

(Prior to 2012)

$s_{23} = 1/\sqrt{2}\ (\theta_{23} = 45^\circ)$ and $\theta_{13} = 0$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$ Bimaximal Mixing
- $\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$ Tribimaximal Mixing
- $\theta_{12} = 31.7^\circ$ Golden Ratio Mixing
- $\theta_{12} = 30^\circ (s_{12} = 1/2)$ Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta,Ramond NPB03; Rodejohann et. al. EPJC10

(GR: $\tan\theta_{12} = 1/\phi$ where $\phi = (1+\sqrt{5})/2$)

Flavor symmetries, why?

- Using the diagonalization relation

$$m_\nu = U_0^\star \,\mathrm{diag}(m_1, m_2, m_3) U_0^\dagger ,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ $i.e.$, $\theta_{12} = 35.26^\circ$

General Framework

Anarchy

- Neutrino mixing anarchy is the hypothesis that the leptonic mixing matrix can be described as the result of a random draw from an unbiased distribution of unitary 3×3 matrices.
- Random analysis without imposing prior theories or symmetries on the mass and mixing matrices.
- This hypothesis does not make any correlation among the neutrino masses and mixing parameters

de Gouvea, Haba, Hall, Murayama : 9911341, 0009174, 1204.1249

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Texture

- More specific studies with imposed mass or mixing textures for which models with underlying symmetries can be sought.
- It's an intermediate approach'
- Some texture zeros of neutrino mass matrices can be eliminated.

Alejandro Ibarra, Graham Ross: Phys.Lett.B 2003

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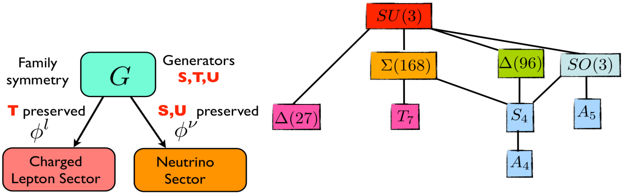
Symmetry

- Theoretical studies where some explicit symmetries at the Yukawa Lagrangian level are assumed and corresponding extended particle sector is defined.
- The symmetry based approach to explain the non-trivial mixing in the lepton sector known as family symmetry or horizontal symmetry

Reviews: Tanimoto *et.al*. 1003.3552, Altarelli, Feruglio 1002.0211, King 1301.1340

General Framework: Symmetry based approach

- Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- Incidentally, both Abelian or non-Abelian family symmetries have potential to shade light on the Yukawa couplings.
- The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a hierarchical structure of the Yukawa couplings.
- Non-Abelian symmetries are more equipped to explain the non-hierarchical structures of the observed lepton mixing as observed by the oscillation experiments.



S. F. King 1301.1340

$\boxed{G_f \rightarrow G_e, G_\nu}$ typically, $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$.

An example:

- Let us consider $G_f = S_4$ as a guiding symmetry.
- Geometrically, it's a symmetry group of a rigid cube (group of permutation 4 objects).
- the order of the group is $4! = 24$ and the elements can be conveniently generated by the generators S, T and U satisfying the relation

$$S^2 = T^3 = U^2 = 1 \text{ and } ST^3 = (SU)^2 = (TU)^2 = 1.$$

- irreducible triplet representations:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}; T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \text{ and } U = \mp\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T^\dagger M_\ell^\dagger M_\ell T = M_\ell^\dagger M_\ell, \; S^T M_\nu S = M_\nu \text{ and } U^T M_\nu U = M_\nu$$

$$[T, M_\ell^\dagger M_\ell] = [S, M_\nu] = [U, M_\nu] = 0$$

- The non-diagonal matrices S, U can be diagonalized by

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

Tribimaximal Mixing: A_4- Ma, Rajasekaran 0106291; Altarelli, Feruglio 0504165; $\Delta(27)$-Varzielas, King, Ross-0607045; Bimaximal Mixing: Frampton, Petcov, Rodejohann 0401206; Golden Ratio Mixing: Feruglio, Paris 1101.0393; Hexagonal Mixing: Albright, Dueck, Rodejohann-1004.2798.

Non-zero θ_{13}

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Bimaximal Mixing — Tribimaximal Mixing — Golden Ratio Mixing — Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 \\ -\frac12 & \frac12 & \frac{1}{\sqrt2} \\ \frac12 & -\frac12 & \frac{1}{\sqrt2} \end{pmatrix} \begin{pmatrix} \sqrt{\frac23} & \frac{1}{\sqrt3} & 0 \\ -\frac{1}{\sqrt6} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt2} \\ -\frac{1}{\sqrt6} & \frac{1}{\sqrt3} & \frac{1}{\sqrt2} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt2} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt2} \end{pmatrix} \begin{pmatrix} \sqrt{\frac34} & \frac12 & 0 \\ -\frac{1}{2\sqrt2} & \frac{\sqrt3}{2\sqrt2} & -\frac{1}{\sqrt2} \\ -\frac{1}{2\sqrt2} & \frac{\sqrt3}{2\sqrt2} & \frac{1}{\sqrt2} \end{pmatrix}$$

Decendents of fixed pattern mixing schemes

Non-zero θ_{13}: Decendents of tribimaximal mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$|U_{\mathrm{TM}_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad |U_{\mathrm{TM}_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

- If S_4 is considered to be broken spontaneously into $Z_3 = \{1, T, T^2\}$ (for the charged lepton sector) $Z_2 = \{1, SU\}$ (for the neutrino sector) such that it satisfies : $[T, M_\ell^\dagger M_\ell] = [SU, M_\nu] = 0$

$$U_{\mathrm{TM}_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}, U_{\mathrm{TM}_2} = \begin{pmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}$$

Non-zero θ_{13}: Decendents of tribimaximal mixing

- TM_1 vs TM_2

	TM_1	TM_2
$\lvert U_{e2}\rvert$	$\left\lvert\frac{\cos\theta}{\sqrt{3}}\right\rvert$	$\frac{1}{\sqrt{3}}$
$\lvert U_{e3}\rvert$	$\left\lvert\frac{\sin\theta}{\sqrt{3}}e^{-i\gamma}\right\rvert$	$\left\lvert\frac{2\sin\theta}{\sqrt{6}}e^{-i\gamma}\right\rvert$
$\lvert U_{\mu 3}\rvert$	$\left\lvert\frac{\cos\theta}{\sqrt{2}}+\frac{\sin\theta}{\sqrt{3}}e^{-i\gamma}\right\rvert$	$\left\lvert-\frac{\cos\theta}{\sqrt{2}}-\frac{\sin\theta}{\sqrt{6}}e^{-i\gamma}\right\rvert$
$\sin^2\theta_{12}$	$1-\frac{2}{3-\sin^2\theta}$	$\frac{1}{3-2\sin^2\theta}$
$\sin^2\theta_{13}$	$\frac{1}{3}\sin^2\theta$	$\frac{2}{3}\sin^2\theta$
$\sin^2\theta_{12}$	$\frac{1}{2}\left(1-\frac{\sqrt{6}\sin 2\theta\cos\gamma}{3-\sin^2\theta}\right)$	$\frac{1}{2}\left(1+\frac{\sqrt{3}\sin 2\theta\cos\gamma}{3-\sin^2\theta}\right)$
J_{CP}	$-\frac{1}{6\sqrt{6}}\sin 2\theta\sin\gamma$	$-\frac{1}{6\sqrt{3}}\sin 2\theta\sin\gamma$
$\sin\delta_{CP}$	$-\frac{(5+\cos 2\theta)\sin\gamma}{\sqrt{(5+\cos 2\theta)^2-24\sin^2 2\theta\cos^2\gamma}}$	$-\frac{(2+\cos 2\theta)\sin\gamma}{\sqrt{(2+\cos 2\theta)^2-3\sin^2 2\theta\cos^2\gamma}}$

Non-zero θ_{13}: Explicit Model for TM_2

	e^c	μ^c	τ^c	L	N^c	H_u	H_d	ϕ_S	ϕ_T	ξ	ξ'
SU(2)	1	1	1	2	1	2	2	1	1	1	1
A_4	1	$1''$	$1'$	3	3	1	1	3	3	1	$1'$
Z_3	ω	ω	ω	ω	ω^2	1	ω	ω^2	1	ω^2	ω^2

BK, A.Sil PRD16

- Neutrino Sector: $y(LN^c)H_u + x_A\xi(N^cN^c) + x_B\phi_S(N^cN^c) + x_N\xi'(N^cN^c)$

$$
\begin{aligned}
U_\nu &\propto U_{TB}\, U_1\, diag(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) \\
&\propto U_{TB}\begin{bmatrix} \cos\theta & 0 & -\sin\theta e^{-i\lambda} \\ 0 & 1 & 0 \\ \sin\theta e^{i\lambda} & 0 & \cos\theta \end{bmatrix} diag(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) \\
&\propto \begin{bmatrix} \sqrt{\frac{2}{3}}\cos\theta & 1/\sqrt{3} & -\sqrt{\frac{2}{3}}\sin\theta e^{-i\lambda} \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{\sin\theta}{\sqrt{2}}e^{i\lambda} & 1/\sqrt{3} & \frac{\cos\theta}{\sqrt{2}} + \frac{\sin\theta}{\sqrt{6}}e^{-i\lambda} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{\sin\theta}{\sqrt{2}}e^{i\lambda} & 1/\sqrt{3} & -\frac{\cos\theta}{\sqrt{2}} + \frac{\sin\theta}{\sqrt{6}}e^{-i\lambda} \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix},
\end{aligned}
$$

- Sum Rules $\Longrightarrow$ $\boxed{\frac{1}{m_1} - \frac{2\mathrm{K}}{m_2(1+\alpha_1)}e^{i\alpha_{21}} = \frac{e^{i\alpha_{31}}}{m_3}}$

- Other examples : Shimizu, Tanimoto, Watanabe 1105.2929; King, Luhn 1107.5332; Varzielas, Lavoura 1212.3247; Chakraborty, Krishnan, Ghosal 2003.00506; Ding, Lu, Valle 2009.04750 etc

Non-zero θ_{13}:Cobimaximal Mixing

- $\mu - \tau$ permutation symmetry : $\nu_e \rightarrow \nu_e, \; \nu_\mu \rightarrow \nu_\tau, \; \nu_\tau \rightarrow \nu_\mu$
- $\mu - \tau$ symmetry + CP : $\nu_e \rightarrow \nu_e^c, \; \nu_\mu \rightarrow \nu_\tau^c, \; \nu_\tau \rightarrow \nu_\mu^c$
- The mixing matrix satisfy the condition :

$$|U_{\mu i}| = |U_{\tau i}| \;\; \text{with } i = 1, 2, 3.$$

- Predicts specific values for the atmospheric mixing angle $\theta_{23} = 45^\circ$ and Dirac CP phase $\delta = -90^\circ$.
- The neutrino mixing matrix can be parametrized as

$$U_0 = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1^* & v_2^* & v_3^* \end{pmatrix},$$

where the entries in the first row, u_i's are real (and non-negative) with trivial values of the Majorana phases.

- The mass matrix leading to the above mixing matrix can be written as

$$m_\nu = \begin{pmatrix} a & b & b^\star \\ b & c & d \\ b^\star & d & c^\star \end{pmatrix},$$

where b and c are in general complex while c and d remain real.

Fukuura, Miura, Takasugi, Yoshimura PRD 99; Miura, Takasugi, Yoshimura PRD01; Harrison, Scott PLB02; Grimus, Lavoura PLB04; Babu, Ma, Valle, PLB03

Cobimaximal Mixing: A flavor model

	$\ell_{e,\mu,\tau}$	e_R, μ_R, τ_R	H	N_R	$\phi_{1,2,3}$	ξ	ϕ_S
A_4	$1, 1', 1''$	$1, 1'', 1'$	1	3	3	1	3
Z_3	1	1	1	ω^2	ω	ω^2	ω^2
Z_4	-i,-1,i	i,-1,- i	1	1	i,-1,- i	1	1

BK, appearing shortly on arXiv

- **Neutrinos:**

$$-\mathcal{L}_\nu = \frac{y_1}{\Lambda}(\bar{\ell_e})_1 \tilde{H}(N_R\phi_1)_1 + \frac{y_2}{\Lambda}(\bar{\ell_\mu})_{1'} \tilde{H}(N_R\phi_2)_{1''} + \frac{y_3}{\Lambda}(\bar{\ell_\tau})_{1''} \tilde{H}(N_R\phi_3)_{1'}$$
$$+(y_x\xi + y_\phi\phi_S)\overline{N_R^c}N_R + h.c.$$

- **Light neutrino mass via type-I seesaw:**

$$m_\nu \sim -m_D M^{-1} m_D^T$$
$$\sim\lambda \begin{pmatrix} 1-\kappa_1^2 & (\kappa_1\kappa_2-\kappa_2)\omega & (\kappa_1\kappa_2-\kappa_2)\omega^2 \\ (\kappa_1\kappa_2-\kappa_2)\omega & (1-\kappa_2^2)\omega^2 & \kappa_2^2-\kappa_1 \\ (\kappa_1\kappa_2-\kappa_2)\omega^2 & \kappa_2^2-\kappa_1 & (1-\kappa_2^2)\omega \end{pmatrix};$$

$$m_\nu = U^\star \mathrm{diag}(m_1, m_2, m_3)U^\dagger$$

$$U == \begin{pmatrix} \cos\vartheta_{12}\cos\vartheta_{13} & -\sin\vartheta_{12}\cos\vartheta_{13} & -\sin\vartheta_{13} \\ \frac{\sin\vartheta_{12} - i\cos\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & \frac{\cos\vartheta_{12} + i\sin\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & -\frac{i\cos\vartheta_{13}}{\sqrt{2}} \\ \frac{\sin\vartheta_{12} + i\cos\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & \frac{\cos\vartheta_{12} - i\sin\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & \frac{i\cos\vartheta_{13}}{\sqrt{2}} \end{pmatrix}.$$

$$\delta = \arcsin\left[\frac{\mathrm{Im}[U_{23}U_{13}^\star U_{12}U_{22}^\star]}{\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}}\right] = -\pi/2; \quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2} = \frac{1}{2}$$

Flavor symmetry with CP invariance:

- $\mu - \tau$ symmetry + CP : $\nu_e \to \nu_e^c, \ \nu_\mu \to \nu_\tau^c, \ \nu_\tau \to \nu_\mu^c$.
- Residual symmetries with CP transformations may lead to new invariance conditions on the mass matrices.
- The cobimaximal matrix

$$M_0 = \begin{pmatrix} a & b & b^\star \\ b & c & d \\ b^\star & d & c^\star \end{pmatrix}$$

is invariant under

$$\mathcal{S}^T M_0 \mathcal{S} = M_0^*,$$

where the transformation matrix is given by

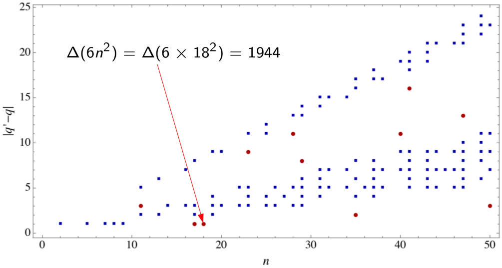
$$\mathcal{S} \quad = \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and such transformations are usually referred to as **generalized CP symmetry transformation**.

- The existence of both discrete flavor and generalized CP symmetries determines the possible structure of the generalized CP symmetry matrices and predictions involving Dirac and Majorana CP phases are made.
- For further readings: Feruglio, Hagedorn 1211.5560; Nishi 1306.0877; Li, Ding 1408.0785; Ding, King 1510.03188; Penedo Petcov, Titov 1803.11009; Iura, López-Ibáñez Meloni 1811.09662
- **Flavor symmetry and GUT** S. F. King, Unified Models of Neutrinos, Flavour and CP Violation, 1701.04413

Flavor symmetry and Higher Order Discrete Groups:

- Fixed mixing schemes such as BM, TBM, GR, HG are dead after measurement of non-zero θ_{13}
- Mixing schemes such as TM_1 ,TM_2, CBM are still consistent with observations.
- Smaller discrete groups such as S_3, A_4, S_4, A_5, $\Delta(27)$ etc. can be used to reproduce TM_1 ,TM_2, CBM or to generate appropriate "**clever/ugly**" modifications to BM, TBM, GR, HG mixings.
- Lepton mixing with larger groups : $G_f \to G_e, G_\nu$, G_f any higher order group.
- Example : $G_e = Z_3$ $G_\nu = Z_2$



Holthausen, Lim, Lindner 1212.2411; Joshipura, Patel 1610.07903

- The values of $n \leq 50$ and $|q' - q|$ $(q, q' = 0, 1, \ldots, n-1)$ leading to the viable columns of leptonic mixing matrix. The blue squares (red dots) indicate that the corresponding prediction is consistent with the first (third) column of U_{PMNS} matrix within 3σ. Each point represents a unique solution obtained by the smallest possible values of n and $|q' - q|$.

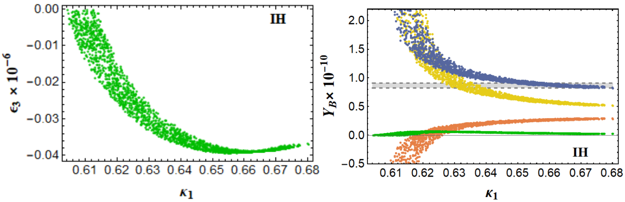
Flavor Symmetries in Various Frontiers: Leptogenesis

- The origin of tiny neutrino mass is often best explained by various seesaw mechanisms.
- New heavy fermions and scalar are introduced to justify lightness of the active neutrinos.
- Out-of-equilibrium decay of these heavy particles can generate observed matter anti-matter asymmetry
- Type-I seesaw, heavy right-handed neutrinos are introduced.
- The CP-violating out-of-equilibrium decay of RH neutrinos into lepton and Higgs doublets in the early universe produces a net lepton asymmetry Fukugita, Yanagida, 1986; Covi, Roulet, Vissani 9605319
- The CP asymmetry parameter :

$$\epsilon_i^\alpha = \frac{\Gamma(N_i \to \ell_\alpha H) - \Gamma(N_i \to \bar{\ell}_\alpha \bar{H})}{\Gamma(N_i \to \ell_\alpha H) + \Gamma(N_i \to \bar{\ell}_\alpha \bar{H})} = \frac{1}{8\pi} \sum_{j \neq i} \frac{\mathrm{Im}\left[\left((\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij}\right)^2\right]}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii}} f\left(\frac{m_i^2}{m_j^2}\right),$$

$$f(x) = \sqrt{x}\left[\frac{2-x}{1-x} - (1-x)\ln\left(1+\frac{1}{x}\right)\right] \text{ with } x = m_i^2/m_j^2$$

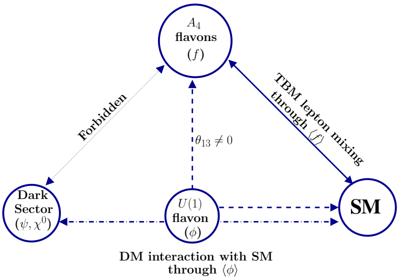
- Flavor symmetry dictates the structure of Y_ν and M_R, hence leaves its imprint on leptogenesis
- Leptogenesis with cobimaximal mixing (**BK, appearing soon**):

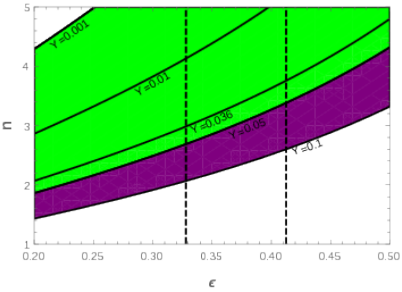


Flavor Symmetries in Various Frontiers: Dark Matter

- Can we extend flavor symmetry to the dark sector as well?
- Can discrete symmetry play any role to ensure the stability of dark matter?
- Example :

$$\mathcal{L}_{int} = \left(\frac{\phi}{\Lambda}\right)^n \bar{\psi}\tilde{H}\chi^0 + \frac{(HL^T LH)\phi\eta}{\Lambda^3} \text{ with } Y = \left(\frac{\phi}{\Lambda}\right)^n = \epsilon^n$$



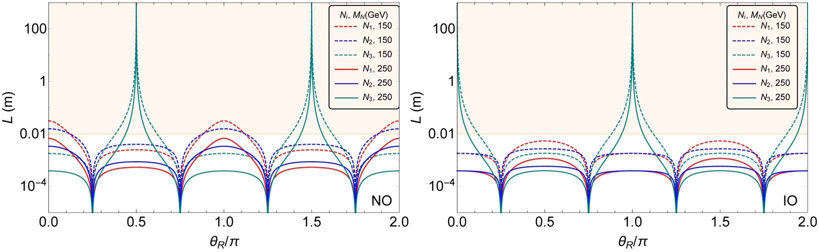


- A schematic representation of dark matter (ψ, χ^0) interaction with SM to generate non-zero θ_{13} in the presence of the $U(1)$ flavor symmetry. The A_4 flavons help in generating base TBM mixing.

S. Bhattacharya , B.K., N. Sahu, A. Sil 1603.04776

Flavor Symmetries in Various Frontiers: Collider Physics

- The high-energy CP phases present in Y_D that are responsible for leptogenesis are in general unrelated to the low-energy CP phases in U_{PMNS}.
- Since the experiments are only sensitive to the low-energy CP phases
- As discussed earlier, incorporating residual flavor and CP symmetries the high- and low-energy CP phases can be related.
- Since in this case the PMNS mixing matrix depends on a single free parameter, this turns out to be highly constraining and predictive for both low- and high-energy CP phases as well as the lepton mixing angles
- Example : $\Delta(6n^2) \times CP$



G. Chauhan, P. S. Bhupal Dev 2112.09710

New Ideas: Modular Symmetry

Criticism for conventional model building with flavor symmetry :

- Traditionally discrete flavor symmetry groups are very useful to explain correct neutrino masses and mixing due to its high predictability.
- The spectrum of the models here is so large that it is difficult to obtain clear clue of the underlying flavour symmetry.
- Often introduces many parameters and auxiliary symmetries $\rightarrow$ non-minimal.

Reason for non-minimality:

Introduce flavons (gauge singlet scalars) to discuss dynamics of flavours. Write down an effective Lagrangian including flavons. Flavour symmetry is broken spontaneously by VEV of flavons.

- Possible Origin $\longrightarrow$ Unknown

New Ideas: Modular Symmetry

Possible Origin:

Superstring theory on certain compactifications may lead to Modular groups. In fact, torus compactification leads to Modular symmetery, which includes S_3, A_4, S_4, A_5 as its congruence subgroup.

Use of Modular Symmetry:

- Very recently, it has been showed that neutrino mass might be of modular form (F. Feruglio, [arXiv:1706.08749 [hep-ph]]), introducing modular invariance approach to the lepton sector.
- The primary advantage is that the flavon fields might not be needed and the Yukawa couplings are written as modular forms, functions of only one complex parameter.
- T. Kobayashi, K. Tanaka, T. H. Tatsuishi 1803.10391, J. T. Penedo, S. T. Petcov 1806.11040,F. J. de Anda, S. F. King, E. Perdomo 1812.05620, Wang, Zhou 2102.04358

Rich phenomenology : Yet to be explored

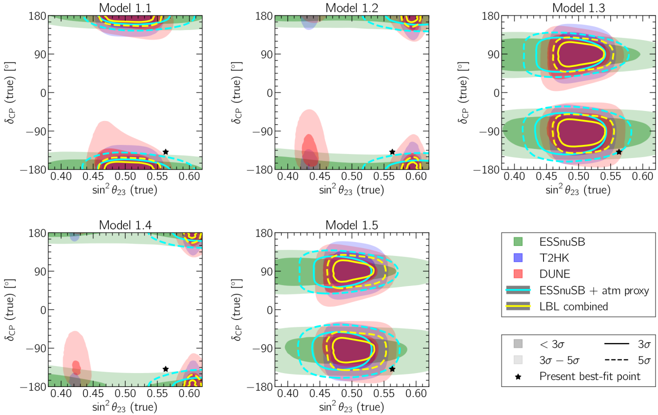
Flavor Symmetry and Oscillation Experiments:

- We need to test the existence underlying flavor symmetry G_f, if any.
- We look for the possibilities of testing its predictions at the current and future neutrino experiments.
- Such studies crucially depend on the breaking pattern of G_f into its residual subgroups for charged lepton sector G_e and neutrino sector G_ν.
- Example : $G_e = Z_k, k > 2$ or $Z_m \times Z_n,\ m, n \geq 2$ and $G_\nu = Z_2 \times CP$
- Correlations among $\theta_{23}, \theta_{12}, \theta_{12}$ and δ_{CP} are obtained and studied in the context of various experiments.

Flavor Symmetry and Oscillation Experiments:

Model	Case [Ref.]	Group	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ_{CP}	χ^2_{min}
1.1	VII-b [25]	$A_5 \rtimes CP$	0.331	0.523	180°	5.37
1.2	III [25]	$A_5 \rtimes CP$	0.283	0.593	180°	5.97
1.3	IV [24]	$S_4 \rtimes CP$	0.318	1/2	±90°	7.28
1.4	II [24]	$S_4 \rtimes CP$	0.341	0.606	180°	8.91
1.5	IV [25]	$A_5 \rtimes CP$	0.283	1/2	±90°	11.3

M. Blennow, M. Ghosh, T. Ohlsson, A. Titov 2005.12277



Conclusion

- Is there any guiding principle behind observed pattern of lepton mixing ?
- (Discrete) flavor symmetry is one such potential candidate.
- What is the origin of such symmetries?
- What additional role they can play?
- How to falsify these plethora of models?
- If flavor symmetry is not the guiding principle, what else?

Thank you for your attention!!

• Multiplication Rules:
It has four irreducible representations: three one-dimensional and one three dimensional which are denoted by $1, 1', 1''$ and 3 respectively. The multiplication rules of the irreducible representations are given by

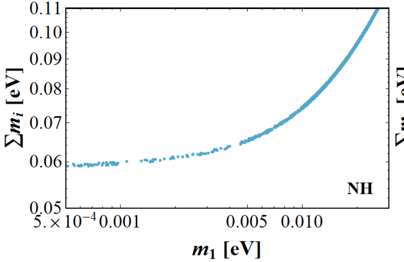
$$1 \otimes 1 = 1, 1' \otimes 1' = 1'', 1' \otimes 1'' = 1, 1'' \otimes 1'' = 1', 3 \otimes 3 = 1 + 1' + 1'' + 3_a + 3_s \tag{2}$$

where a and s in the subscript corresponds to anti-symmetric and symmetric parts respectively. Now, if we have two triplets as $A = (a_1, a_2, a_3)^T$ and $B = (b_1, b_2, b_3)^T$ respectively, their direct product can be decomposed into the direct sum mentioned above. The product rule for this two triplets in the S diagonal basis[1] can be written as

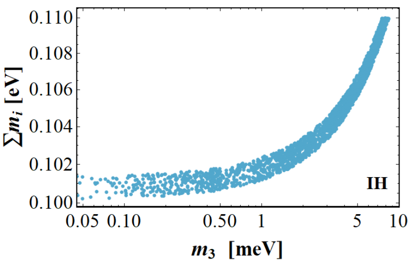
$$(A \times B)_1 \backsim a_1 b_1 + a_2 b_2 + a_3 b_3, \tag{3}$$
$$(A \times B)_{1'} \backsim a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \tag{4}$$
$$(A \times B)_{1''} \backsim a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \tag{5}$$
$$(A \times B)_{3_s} \backsim (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1), \tag{6}$$
$$(A \times B)_{3_a} \backsim (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1), \tag{7}$$

here ω $(= e^{2i\pi/3})$ is the cube root of unity

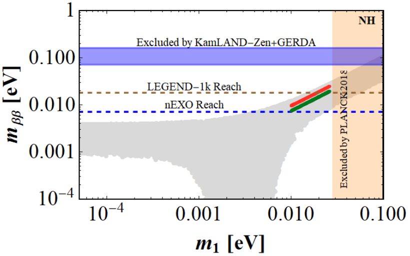
[1]Here S is a 3×3 diagonal generator of A_4.



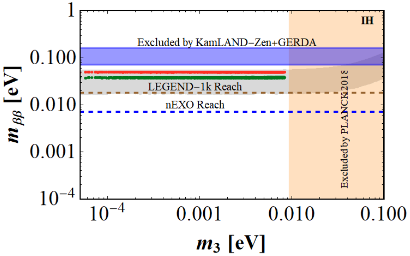
$\sum m_i$ [eV]
0.11
0.10
0.09
0.08
0.07
0.06
0.05
$5.\times10^{-4}$ 0.001 0.005 0.010
m_1 [eV]
NH



$\sum m_i$ [eV]
0.110
0.108
0.106
0.104
0.102
0.100
0.05 0.10 0.50 1 5 10
m_3 [meV]
IH



$m_{\beta\beta}$ [eV]
1
0.100
0.010
0.001
10^{-4}
10^{-4} 0.001 0.010 0.100
m_1 [eV]
NH
Excluded by KamLAND–Zen+GERDA
LEGEND–1k Reach
nEXO Reach
Excluded by PLANCK2018



$m_{\beta\beta}$ [eV]
1
0.100
0.010
0.001
10^{-4}
10^{-4} 0.001 0.010 0.100
m_3 [eV]
IH
Excluded by KamLAND–Zen+GERDA
LEGEND–1k Reach
nEXO Reach
Excluded by PLANCK2018

$$Y_B \approx \sum Y_{Bi} \tag{8}$$

where

$$Y_{Bi} \simeq -1.48 \times 10^{-3} \epsilon_i \eta_{ii}. \tag{9}$$

Y_{Bi}'s are coming from decay of each RH neutrinos and η_{ii} stands for efficiency factor [hep-ph/0310123] when $M_i < 10^{14}$ GeV,

$$\frac{1}{\eta_{ii}} \approx \frac{3.3 \times 10^{-3}\ \mathrm{eV}}{\tilde{m}_i} + \left(\frac{\tilde{m}_i}{0.55 \times 10^{-3}\ \mathrm{eV}}\right)^{1.16}, \tag{10}$$

with washout mass parameter, $\tilde{m}_i = \frac{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii} v_u^2}{M_i}$.