# Fate of the QCD critical endpoint at large $N_c$

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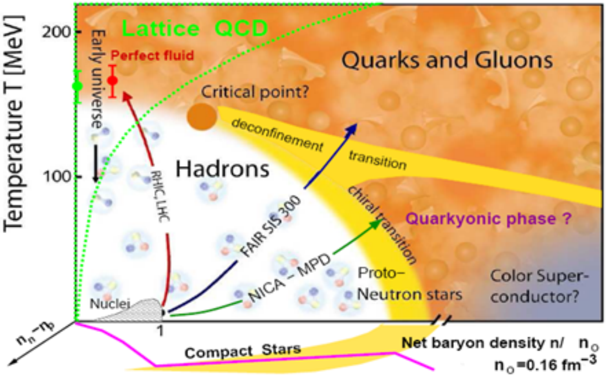
# Overview

# Envisaged phase diagram of QCD




Important details of the phase diagram is still unknown (mainly at large baryon density)

Properties of the phase diagram especially at finite baryon densities/baryochemical potential can be well investigated with the help of effective field theories of QCD $\longrightarrow$ e.g. details of the phase boundary like existence and location of the CEP, in medium dependence of meson masses, or properties of compact stars etc.

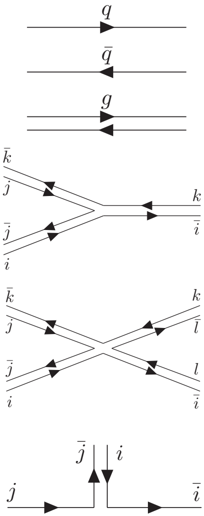
# Basics of Large $N_c$ I.

*G. 't Hooft. (1974), Nucl. Phys. B 72:461*

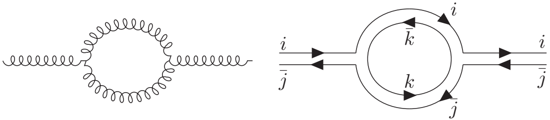
*G. 't Hooft. (1974), Nucl. Phys. B 75:461–470*

*E. Witten. (1979), Nucl. Phys. B 160:57–115*

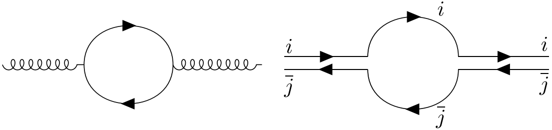
- No expansion parameter in QCD if $m_{u/d/s} \approx 0 \longrightarrow$ not so obvious expansion parameter: $N_c$
- $SU(3) \longrightarrow SU(N_c)$
- double line notation based on color structure of gluons: $A^{\mu;\,i}_{j} \sim q^i \bar{q}_j$
- 3-coupling: $A^i_{\mu;\,j} A^j_{\nu;\,k} \partial^\mu A^{\nu;\,k}_i$
- 4-coupling: $A^i_{\mu;\,j} A^j_{\nu;\,k} A^{\mu;\,k}_l A^{\nu;\,l}_i$
- quark - gluon vertex: $\bar{q}_i \gamma^\mu q^j A^i_{\mu;\,j}$



# Basics of Large $N_c$ II.




$N_c$ combinatorial factor due to closed color loop $\Longrightarrow$ $g \sim \frac{1}{\sqrt{N_c}}$




Quark loops are $1/N_c$ suppressed.

Leading diagrams are palanar diagrams with minimum number of quark loops

Investigation of $N$-point functions of quark bilinears ($J = \bar{q}q$, $\bar{q}\gamma^\mu q$) leads to the large $N_c$ properties of mesons

# Properties of mesons and baryons for Large $N_c$

- mesons are free, stable, and non-interacting
- mesons are pure $q\bar{q}$ states for large $N_c$
- meson masses $\sim N_c^0$
- meson decay amplitudes $\sim 1/\sqrt{N_c}$
- for one meson creation: $< 0|J|m > \sim \sqrt{N_c}$
- $k$ meson vertex $\sim N_c^{1-k/2}$. Specifically, the three- and four-meson vertices are $\sim 1/\sqrt{N_c}$ and $\sim 1/N_c$, respectively
- baryon masses $\sim N_c$. Consequently constituent quark masses $\sim N_c^0$

# Lagrangian of the PLeLSM model

$\mathcal{L}$ constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$
\begin{aligned}
\mathcal{L} = & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& + c_1(\det\Phi + \det\Phi^\dagger) + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\
& + \bar{\Psi}\left(i\gamma^\mu D_\mu - g_F(S - i\gamma_5 P)\right)\Psi - g_V\bar{\Psi}\left(\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\Psi,
\end{aligned}
$$

$$
\begin{aligned}
\Phi &= S + iP \equiv \sum_{a=0}^{8}(S_a\lambda_a + iP_a\lambda_a) \\
D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \left\{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\right\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \left\{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\right\}, \\
D^\mu\Psi &= \partial^\mu\Psi - iG^\mu\Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}
$$

+ Polyakov loop potential (for $T > 0$)

# Particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar ($\sim \bar{q}_i q_j$) and pseudoscalar ($\sim \bar{q}_i \gamma_5 q_j$) meson nonets

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star+} \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star-} & \bar{K}_0^{\star 0} & \sigma_S \end{pmatrix} \quad P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

multiple possible assignments
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548),\ \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$
fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

In case of compact stars, also nonzero vector condensates

# Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of $\chi^2$:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\exp}}{\delta Q_i}\right]^2,$$

$(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N) \longrightarrow$ from the model, $Q_i^{\exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

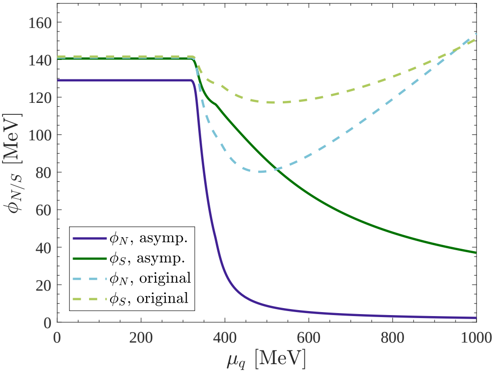
- PCAC $\rightarrow$ 2 physical quantities: $f_\pi, f_K$
- Curvature masses $\rightarrow$ 16 physical quantities: $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature $T_c$ at $\mu_B = 0$

# Importance of parameterization

- a naive parameterization → chiral symmetry would be broken at high densities
- investigating the asymptotic behavior we get an additional constraint for the parameters:

$$3h_1 + 2h_2 + 2h_3 < 0$$

- Phys.Rev.D 105 (2022) 10, 103014



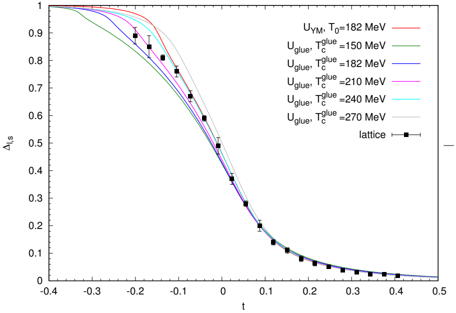
# Features of our approximation

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  – $u, d, s$ constituent quarks ($m_u = m_d$)
  – (Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\mathrm{YM}}$ or $\mathcal{U}_{\log}^{\mathrm{glue}}$ )
- no mesonic fluctuations, only fermionic ones
  $\mathcal{Z} = e^{-\beta V \Omega(T,\mu_q)} = \int_{\mathrm{PBC}} \prod_a \mathcal{D}\xi_a \int_{\mathrm{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp\left[-\int_0^\beta d\tau \int_V d^3x \left(\mathcal{L} + \mu_q \sum_f q_f^\dagger q_f\right)\right]$ approximated
  as $\Omega(T,\mu_q) = U_{\mathrm{meson}}^{\mathrm{tree}}(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$, $\tilde{\mu}_q = \mu_q - iG_4$

  $e^{-\beta V \Omega_{\bar{q}q}^{(0)}} = \int_{\mathrm{APBC}} \prod_{f,g} \mathcal{D}q_g \mathcal{D}q_f^\dagger \exp\left\{ \int_0^\beta d\tau \int_x q_f^\dagger \left[\left(i\gamma_0 \vec{\gamma} \cdot \vec{\nabla} - \frac{\partial}{\partial \tau} + \tilde{\mu}_q\right)\delta_{fg} - \gamma_0 \mathcal{M}_{fg}|_{\xi_a = 0}\right] q_g \right\}$
- tree-level (axial)vector masses
- fermionic thermal fluctuations included in the (pseudo)scalar curvature masses
- 2 (or 4) coupled $T/\mu_B$-dependent field equations for the condensates $\phi_N, \phi_S, (\Phi, \bar{\Phi})$ at $N_c = 3$
- Polyakov-loops and fermionic vacuum fluctuations

# $t$-dependence of the condensates compared to lattice results

The subtracted chiral condensate




– subtracted chiral condensate:

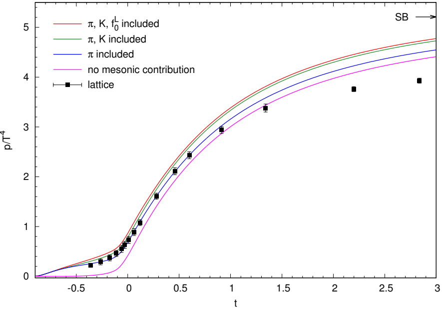
$$\Delta_{l,s} = \frac{\left(\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S\right)\Big|_T}{\left(\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S\right)\Big|_{T=0}}$$

– $U^{\text{glue}}_{\text{log}}$ with $T_c^{\text{glue}} \in (210, 240)$ MeV gives good agreement with the lattice result of

Borsányi *et al.*, JHEP 1009, 073 (2010)

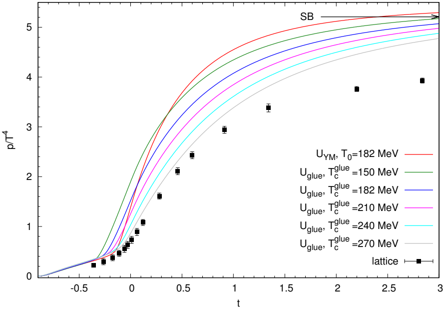
here we use the reduced temperature: $t = (T - T_c)/T_c$

# Normalized pressure and the effects of meson contributions



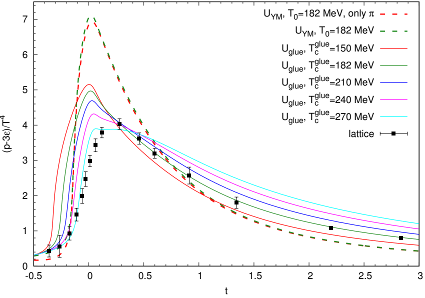

– we used $U_{\rm glue}$ with $T_c^{\rm glue} = 270$ MeV
– pions dominate the pressure at small $T$
– contribution of the kaons is important
– at high $T$ the pressure overshoots the lattice data of Borsányi *et al.*, JHEP 1011, 077 (2010)

– overshooting increases with decreasing $T_c^{\rm glue}$



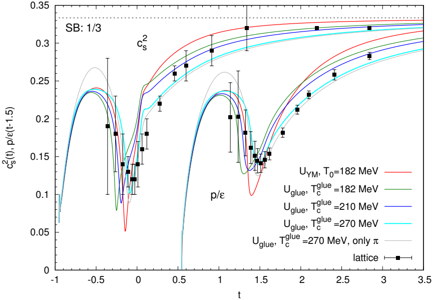
# Scaled interaction measure, speed of sound and $p/\epsilon$

## Scaled interaction measure




By properly setting the $T_c^{glue}$ parameter $\longrightarrow$ good agreement with lattice

## Speed of sound and $p/\epsilon$



# $N_c$ scaling of the Lagrange parameters

The parameters are: $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S,\ \Phi_N, \Phi_S, g_F,\ h_{N/S}$

- $m_0^2,\ m_1^2,\ \delta_s \sim N_c^0$, because terms of tree level meson masses
- $g_1,\ g_2 \sim \frac{1}{\sqrt{N_c}}$, three couplings
- $\lambda_2,\ h_2,\ h_3 \sim \frac{1}{N_c}$, four couplings
- $\lambda_1,\ h_1 \sim \frac{1}{N_c^2}$, four couplings with different trace structure
- $c_1 \sim \frac{1}{N_c^{3/2}}$ $U_A(1)$ anomaly term has extra $1/N_c$ suppression
- $h_{N/S} \sim \sqrt{N_c}$, Goldstone-theorem: $m_\pi^2 \Phi_N = Z_\pi^2 h_N$ + PCAC: $\Phi_N = Z_\pi f_\pi$
- $g_F \sim \frac{1}{\sqrt{N_c}}$, $m_{u/d} = g_F \Phi_N$

We expect $\Phi_{N/S} \sim \sqrt{N_c}$, since $\Phi_N = Z_\pi f_\pi$, $f_\pi \sim \sqrt{N_c}$, but have to check!

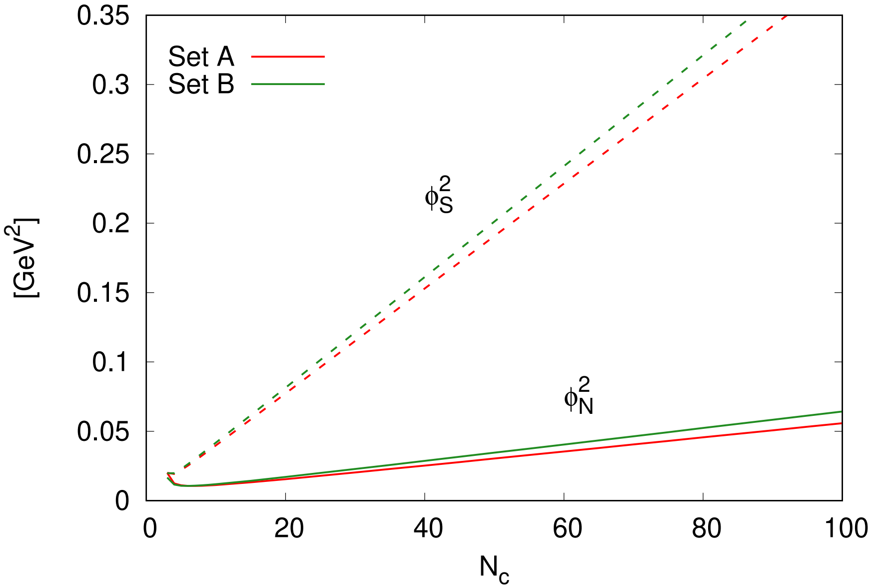
practically: $g_1 \longrightarrow g_1\sqrt{\frac{3}{N_c}}$, $\Phi_{N/S} \longrightarrow \Phi_{N/S}\sqrt{\frac{N_c}{3}} \ldots$ *etc.*

# Parameter sets at $N_c = 3$

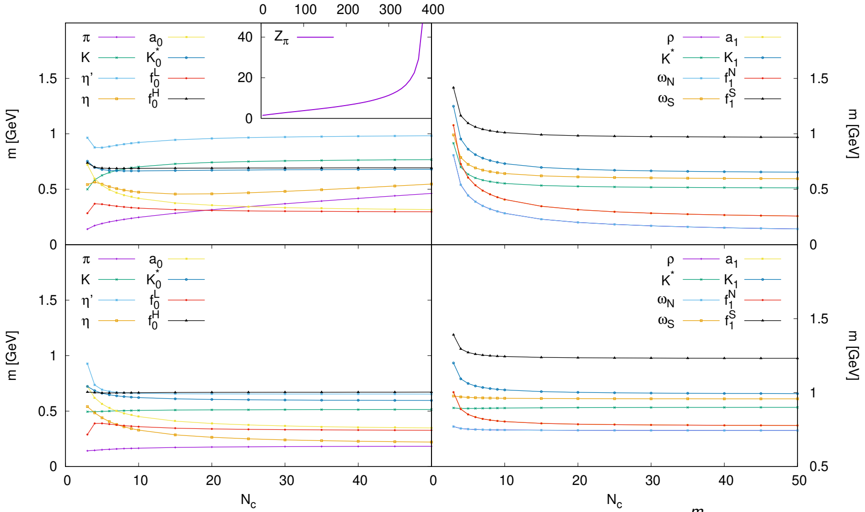
| Parameter | Set A | Set B |
|---|---|---|
| $\phi_N$ [GeV] | 0.1411 | 0.1290 |
| $\phi_S$ [GeV] | 0.1416 | 0.1406 |
| $m_0^2$ [GeV$^2$] | 2.3925$_{E-4}$ | −1.2370$_{E-2}$ |
| $m_1^2$ [GeV$^2$] | 6.3298$_{E-8}$ | 0.5600 |
| $\lambda_1$ | −1.6738 | −1.0096 |
| $\lambda_2$ | 23.5078 | 25.7328 |
| $c_1$ [GeV] | 1.3086 | 1.4700 |
| $\delta_S$ [GeV$^2$] | 0.1133 | 0.2305 |
| $g_1$ | 5.6156 | 5.3295 |
| $g_2$ | 3.0467 | −1.0579 |
| $h_1$ | 37.4617 | 5.8467 |
| $h_2$ | 4.2281 | −12.3456 |
| $h_3$ | 2.9839 | 3.5755 |
| $g_F$ | 4.5708 | 4.9571 |
| $M_0$ [GeV] | 0.3511 | 0.3935 |

- Set A: $m_\sigma = 290$ MeV from Phys.Rev.D 93 (2016) 11, 114014
- Set B: similar $m_\sigma$ mass and additional constraint, $3h_1 + 2h_2 + 2h_3 < 0$ from Phys.Rev.D 105 (2022) 10, 103014

# $N_c$ scaling of the $\phi_{N/S}$ condensates



# $N_c$ scaling of the tree level masses




Top figures using Set A, bottom figures using Set B;

$$Z_\pi = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2 \phi_N^2}}$$

for every meson we see: $m_{meson} \sim N_c^0$

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# Introduction to Polyakov-loops/Polyakov-loop variables

Definition of Polyakov-loop

$$L(\vec{x}) = \mathcal{P} \exp\left\{ i \int_0^\beta A_4 dt \right\}$$

Polyakov-loop variables:

$$\Phi(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}_c L(\vec{x}), \text{ and } \bar{\Phi}(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}_c L(\vec{x})^\dagger, \text{(non center } (C_n) \text{ symmetric)}$$

If $\Delta F_{q/\bar{q}}$ is a change in the free energy, when an infinitely heavy quark (or antiquark) is added to the system, than

$$\langle \Phi(\vec{x}) \rangle_\beta = e^{-\beta \Delta F_q(\vec{x})} \quad , \quad \langle \bar{\Phi}(\vec{x}) \rangle_\beta = e^{-\beta \Delta F_{\bar{q}}(\vec{x})}$$ *Phys. Rev. D* 24 (1981) 450

- $C_n$ symm. phase $\rightarrow \langle \Phi(\vec{x}) \rangle_\beta = 0 \rightarrow \Delta F_{q/\bar{q}} = \infty \rightarrow$ confinement
- $C_n$ NON symm. phase $\rightarrow \langle \Phi(\vec{x}) \rangle_\beta \neq 0 \rightarrow \Delta F_{q/\bar{q}} < \infty \rightarrow$ deconfinement

Thus $\Phi(\vec{x})$ and $\bar{\Phi}(\vec{x})$ can be used as order parameters for confinement

# Polyakov-loop variables at large $N_c$ (I.)

Polyakov gauge $\rightarrow$ $A_4$ is diagonal and time independent
$\rightarrow$ further simplification: homogeneous gluon filed

$$L = e^{i\beta A_4} = \mathrm{diag}\left(e^{iq_1}, \ldots, e^{iq_{N_c}}\right), \quad q_j \in \mathbb{R}, \quad \sum_j q_j = 0$$

$N_c - 1$ independent diagonal $SU(N_c)$ matrix $\rightarrow$ $\Phi$ and $\bar{\Phi}$ above are not enough

$$\Phi_n = \frac{1}{N_c}\mathrm{Tr}_c L^n, \quad \bar{\Phi}_n = \frac{1}{N_c}\mathrm{Tr}_c L^{\dagger n}, \quad n \in \left(1, \ldots, \lfloor\frac{N_c}{2}\rfloor\right),$$ Phys. Rev. D 86, 105017 (2012)

our approx. $\rightarrow$ quarks prop. on a const. gluon background $\rightarrow$ color dep. chem. pot.

$$\Omega^{(0)}_{\bar{q}q}(T,\mu_q) = \Omega^{(0)\mathrm{v}}_{\bar{q}q} + \Omega^{(0)\mathrm{T}}_{\bar{q}q}(T,\mu_q),$$

$$\Omega^{(0)\mathrm{T}}_{\bar{q}q}(T,\mu_q) = -2T\sum_f \int \frac{d^3p}{(2\pi)^3}\left[\ln g_f^+(p) + \ln g_f^-(p)\right]$$

$$\ln g_f^+(p) \equiv \mathrm{Tr}_c \ln\left[\mathbb{1} + L^\dagger e^{-\beta(E_f(p)-\mu_q)}\right] = \ln \mathrm{Det}_c\left[\mathbb{1} + L^\dagger e^{-\beta(E_f(p)-\mu_q)}\right]$$

$$\ln g_f^-(p) \equiv \mathrm{Tr}_c \ln\left[\mathbb{1} + L e^{-\beta(E_f(p)+\mu_q)}\right] = \ln \mathrm{Det}_c\left[\mathbb{1} + L e^{-\beta(E_f(p)+\mu_q)}\right]$$

# Polyakov-loop variables at large $N_c$ (*II.*)

$$g_f^+ = 1 + e^{-N_c \beta E_f^+} + N_c \left[ \bar{\Phi}_1 e^{-\beta E_f^+} + \Phi_1 e^{-(N_c-1)\beta E_f^+} \right] \\ + [\text{terms with } 2 \text{ to } N_c - 2 \text{ phases}]$$

Only first line in case of $N_c = 3$. For $N_c > 3$ more and more $\Phi_k$s appear:

$$\begin{aligned} g_f^+ = & 1 + e^{-N_c \beta E_f^+} \\ & + N_c \left[ \bar{\Phi} e^{-\beta E_f^+} + \Phi e^{-(N_c-1)\beta E_f^+} \right] \\ & + \frac{1}{2} \left( N_c^2 \bar{\Phi}^2 - N_c \bar{\Phi}_2 \right) e^{-2\beta E_f^+} \\ & + \frac{1}{2} \left( N_c^2 \Phi^2 - N_c \Phi_2 \right) e^{-(N_c-2)\beta E_f^+} \\ & + [\text{terms with } 3 \text{ to } N_c - 3 \text{ phases}] \end{aligned}$$

All the $\Phi_k$s are unknown $\rightarrow$ At $N_c$ there would be $N_c + 1$ field equations $\rightarrow$ approximation: Uniform eigenvalue ansatz

# Uniform eigenvalue ansatz

We express the $q_j$ angles with the $N_c - 1$ group angles of the Cartan subgroup of $SU(N_c)$: Phys. Rev. D 103, 074026 (2021)

$$\vec{q} \equiv (q_1, \dots, q_{N_c}) = \sum_{j=1}^{N_c-1} \gamma_j \vec{v}_j, \quad \{\vec{v}_j\}_{j=1}^{N_c-1} \text{ (set of basis vectors)}$$

Ansatz (UEA): $\gamma_1 \neq 0$, $\gamma_i = 0, i \neq 1$, a direction is fixed in the subalgebra,

$$\vec{v}_1 = \left(-1, -\left(1 - \frac{2}{N_c - 1}\right), \dots, -\left(1 - (j-1)\frac{2}{N_c - 1}\right), \dots, 1\right), \; j = 1, \dots, N_c$$

$$L = \text{diag}\left(e^{-i\gamma}, e^{-i\left(1-\frac{2}{N_c-1}\right)\gamma}, e^{-i\left(1-2\frac{2}{N_c-1}\right)\gamma}, \dots, (e^0), \dots, e^{i\left(1-2\frac{2}{N_c-1}\right)\gamma}, e^{i\left(1-\frac{2}{N_c-1}\right)\gamma}, e^{i\gamma}\right)$$

Instead of $N_c - 1$ unknown variables, only one: $\gamma_1 \equiv \gamma$ and $\Phi_n = \bar{\Phi}_n \in \mathbb{R}$

$$\Phi_n = \frac{1}{N_c}\left(2\sum_{j=1}^{\lfloor \frac{N_c}{2} \rfloor} \cos\left[\left(1 - 2\frac{j-1}{N_c - 1}\right) n\gamma\right] + \alpha\right)$$

# Polyakov-loop potential at large $N_c$

$$U_{\mathrm{Pol}} = U_{\mathrm{conf}} + U_{\mathrm{glue}},$$ Phys. Rev. D 103, 074026 (2021)

$$U_{\mathrm{conf}} = -\frac{b}{2} T \ln H = -\frac{b}{2} T \ln g'_A, \quad g'_A = \mathrm{Det}'\left(\mathbb{1}_A - L_A e^{-\beta E_A(p)}\right)$$

$$U_{\mathrm{glue}} = n_{\mathrm{glue}} T \int \frac{d^3p}{(2\pi)^3} \ln g_A, \qquad g_A = \mathrm{Det}\left(\mathbb{1}_A - L_A e^{-\beta E_A(p)}\right)$$

$$L_A = \mathrm{diag}(e^{iQ_1}, \ldots, e^{iQ_{N_c^2-1}}),$$

$$\vec{Q} = (\underbrace{0, \ldots, 0}_{N_c-1}, \underbrace{q_1 - q_2, \ldots, (q_j - q_k)|_{j\neq k}, \ldots q_{N_c-1} - q_{N_c}}_{N_c(N_c-1)}).$$

$L_A$ Polyakov-loop op. in the adjoint repr.; $Q_j$ adjoint angles
$b = (0.1745\ \mathrm{GeV})^3$, $n_{\mathrm{glue}} = 2$, and $m_A = 0.756$ GeV

# Field equation in the UEA

$$0 = \frac{dU_{\mathrm{Pol}}}{d\gamma} - 2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(h_f^+(p) + h_f^-(p)\right),$$

$$0 = m_0^2\phi_N + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - \frac{c}{\sqrt{2}}\phi_N\phi_S - h_{0N} + \frac{g_F}{2}\sum_{l=u,d}\langle\bar{q}_l q_l\rangle_T,$$

$$0 = m_0^2\phi_S + (\lambda_1 + \lambda_2)\phi_S^3 + \lambda_1\phi_N^2\phi_S - \frac{\sqrt{2}c}{4}\phi_N^2 - h_{0S} + \frac{g_F}{\sqrt{2}}\langle\bar{q}_s q_s\rangle_T,$$

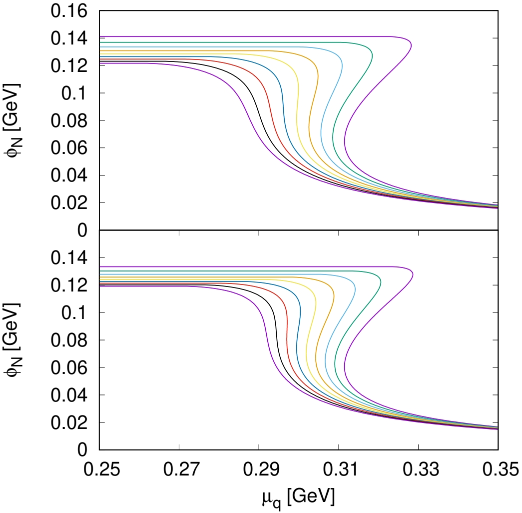
with

$$h_f^\pm = \frac{1}{g_f^\pm}\frac{\partial g_f^\pm}{\partial\gamma},$$

$$\langle\bar{q}_f q_f\rangle_T = -4N_c m_f\left[\frac{m_f^2}{16\pi^2}\left(\frac{1}{2} + \ln\frac{m_f^2}{M_0^2}\right) + \mathcal{T}_f^{\mathrm{matt}}\right]$$

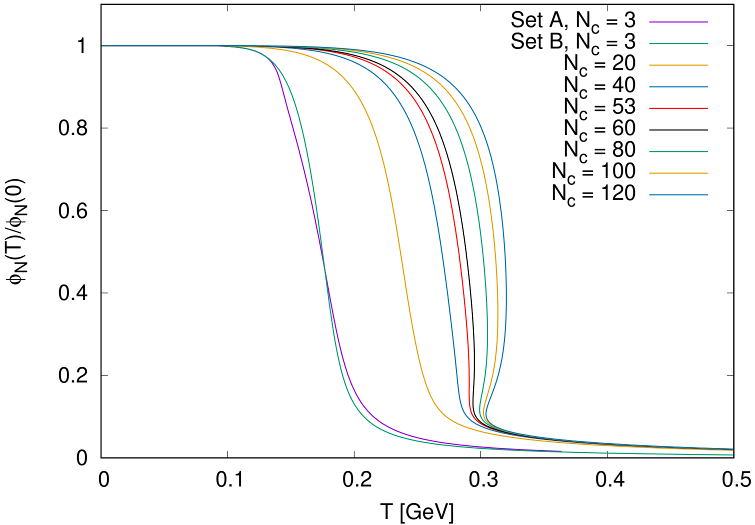
$$\mathcal{T}_f^{\mathrm{matt}} = \frac{T}{N_c}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2m_f}\left(\frac{1}{g_f^+}\frac{\partial g_f^+}{\partial m_f} + \frac{1}{g_f^-}\frac{\partial g_f^-}{\partial m_f}\right)$$

# $\mu_q$ dependence of the $\phi_N$ condensates at diff. $N_c$s



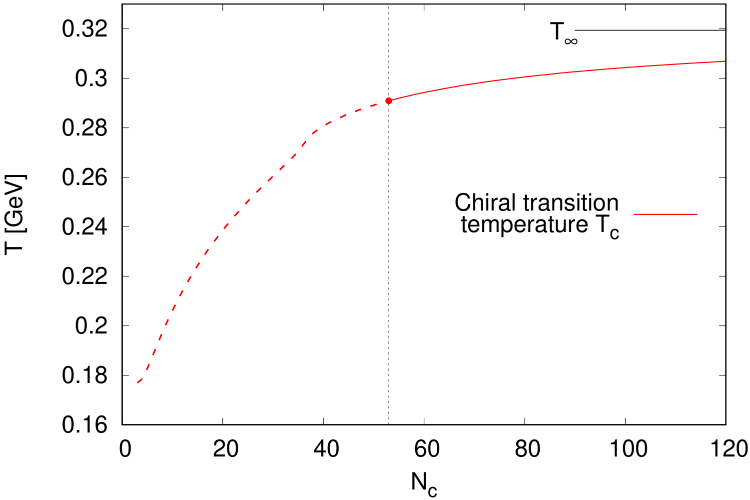

- Upper fig.: set A
- Lower fig.: set B
- Crossover already for $N_c = 3.45$

# $T$ dependence of the $\phi_N$ condensates at diff. $N_c$s




$N_c = 3$: $T_c = 178.6$ MeV and $T_c = 176.9$ MeV, for set A and set B

# Saturation of the pseudocritical temp. $T_c$




fit:

$T_c(N_c) = \alpha/(N_c + \beta) + T_\infty;\ T_\infty = 0.3192$ MeV

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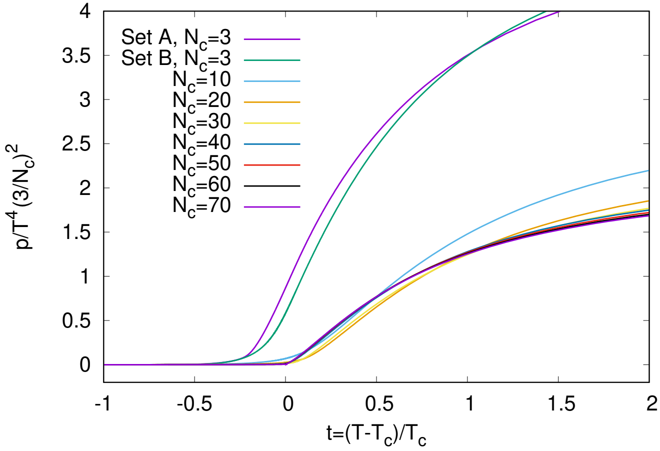
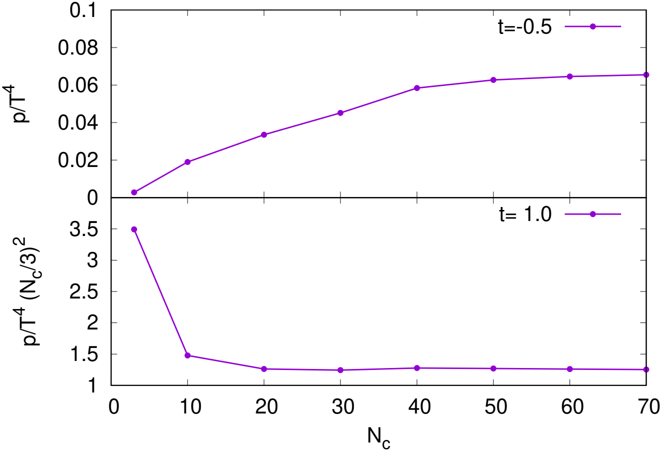
# Pressure for different $N_c$ values at $\mu_q = 0$

$$p(T,\mu_q) = -\left(\Omega(T,\mu_q,\phi_{N/S}(T,\mu_q),\gamma(T,\mu_q)) - \Omega(T,\mu_q,\phi_{N/S}(0,0),\gamma(0,0))\right)$$

non-trivial subtraction, however field eq. doesn't change

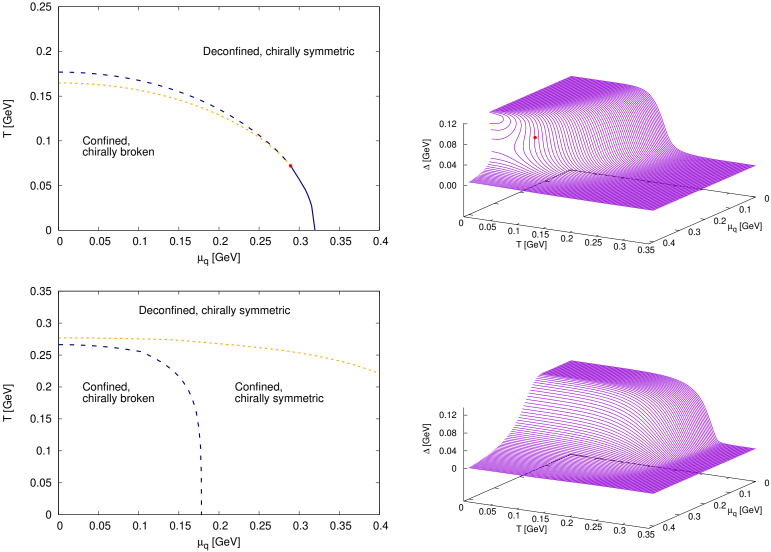


# Scaling of the pressure at $\mu_q = 0$



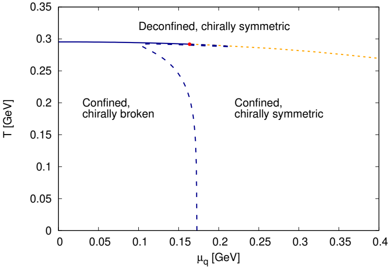

$t = -0.5$ (top: below ph. t. $\sim N_c^0$) and $t = 1$ (bottom: above ph. t. $\sim N_c^2$)

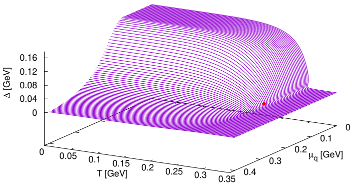
# Phase boundary for different $N_c$s *I.*




top: $N_c = 3$, CEP exist, crossover for small $T$; *bottom* : $N_c = 33$ crossover everywhere

# Phase boundary for different $N_c$s *II.*

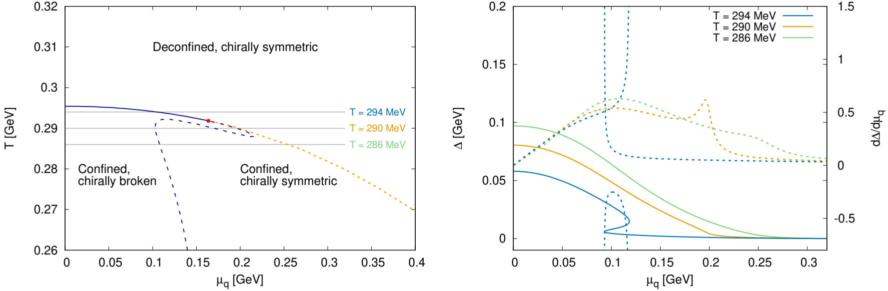



$N_c = 63$ CEP exist again, crossover for large $T$

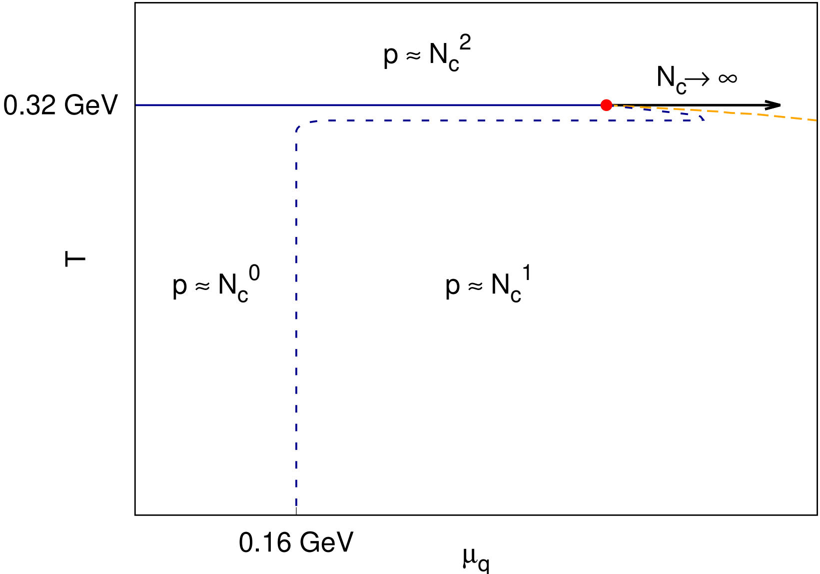
$$\Delta(T,\mu_q^{\rm fix}) = \frac{(\phi_N - \frac{h_N}{h_S}\phi_S)|_{T,\mu_q^{\rm fix}}}{(\phi_N - \frac{h_N}{h_S}\phi_S)|_{T=0,\mu_q^{\rm fix}}} \quad (1)$$

# A close look on the phase structure




$N_c = 63$: The subtracted condensate $\Delta(T_0, \mu_q)$ (solid lines) and its $\mu_q$ derivatives (dashed lines) calculated along the horizontal–$\mu_q$–directions (right)

# Schematic phase structure for large $N_c$



# Conclusion

- PLeLSM model can be used to investigate large $N_c$
- Expected $N_c$ scaling of the condensates and masses are fulfilled
- Pressure scales as $N_c^0$ in confined and $N_c^2$ in the deconfined region
- Existence of 'quarkyonic' phase is confirmed
- More detail in: Phys. Rev. D **106**, 116016

Köszönöm a figyelmet!