Altered Probability States

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Probability necesse est - that was the preliminary subject

- A characteristic of a system is said to be random when it is not known or cannot be predicted with complete certainty.
- For complete certainty a perfect precision and accuracy is necessary.
- There are no perfect experiments as collections of data are always statistical samples.
- Measurements involve a large successio of macroscopic and microscopic processes that randomly alter their outcome.
- To decipher meaning of statistical samples it's necessary to decipher underlying probabilistic distributions.
- To understand the meaning of probabilistic distributions an understanding of the probability is necessary.

Probabilities of deconfinement of finite size systems - general approach

On the level of Quantum Mechanics. Restriction to the finite volume V - equivalent to the presence of the external potential $U(\vec{r})$:

$$U(\vec{r}) = \infty \qquad \text{if} \qquad \vec{r}\,\bar{\epsilon} V$$

If $U(\vec{r})$ ceases at some time, then the system is subjected to the deconfinement process.

$$\hat{H} = \begin{cases} -\frac{\hbar^2}{2m}\Delta + U(\vec{r}), & \text{for } t \leq 0\,, \\ -\frac{\hbar^2}{2m}\Delta, & \text{for } t > 0\,. \end{cases}$$

General case

Let ψ be any solution of the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + U(\vec{r})\psi\,.$$

A general form of the free Schrödinger equation is a wave packet

$$\int d^3p\, g(\vec{p})\, e^{-i\frac{p^2}{2m\hbar}t}\, e^{\frac{i}{\hbar}\vec{p}\cdot\vec{r}}$$

A function

$$\Psi(\vec{r},t) = \begin{cases} \psi(\vec{r},t), & \text{for } t \leq 0\,, \\ \int d^3p\, g(\vec{p})\, e^{-i\frac{p^2}{2m\hbar}t}\, e^{\frac{i}{\hbar}\vec{p}\cdot\vec{r}}, & \text{for } t > 0 \end{cases}$$

is a solution of the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi\,,$$

The wave function $\Psi(\vec{r},t)$ is continuous at $t = 0$.

General case

$$\psi(\vec{r},0) = \int d^3p\, g(\vec{p})\, e^{\frac{i}{\hbar}\vec{p}\cdot\vec{r}}\,;$$

If a function ψ is a stationary solution then

$$\psi_E(\vec{r},t) = \phi_E(\vec{r})\, e^{-\frac{iE}{\hbar}t}\,;$$

with ϕ_E satisfying a stationary Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\phi_E + U(\vec{r})\phi_E = E\phi_E\,;$$

$$\phi_E(\vec{r}) = \int d^3p\, g(\vec{p})\, e^{\frac{i}{\hbar}\vec{p}\cdot\vec{r}}\,;$$

Subjected to deconfinement

A general interpretation of the Fourier transform of a wave function

The momentum distribution of a particle which was influenced by a potential and at time $t = 0$ was suddenly freed.

There is no question about the energy conservation

because of the time-dependency of the hamiltonian

More about this subject, including some related mathematical problems: L. Turko, *"Finite size universe or perfect squash problem"*, J. Math. Phys. **45** (2004), 3659-3675, doi:10.1063/1.1782671, [arXiv:quant-ph/0310128 [quant-ph]]. Be careful, the paper is unnecessary lengthy :-)

The simplest example - an infinite square wall

A one-dimensional potential

$$U(x) = \begin{cases} 0\,, & \text{for } 0 \le x \le a\,, \\ \infty\,, & \text{for } x \text{ everywhere else}\,, \end{cases}$$

with boundary conditions

$$\psi(0) = \psi(a) = 0\,,$$

The solutions

$$\psi_N(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{N\pi}{a} x, & \text{for } 0 \le x \le a\,, \\ 0, & \text{for } x \text{ everywhere else}\,, \end{cases}$$

Energy levels:

$$E_N = \frac{\pi^2 \hbar^2}{2ma^2} N^2 \,.$$

The Fourier integral of the wave function

$$\tilde{\psi}_N(k) = \frac{1}{\sqrt{2\pi}} \int_0^a dx\, \psi_N(x)\, e^{-ikx} \,.$$

$$\tilde{\psi}_N(k) = -\sqrt{\pi a}\, \frac{2N}{a^2k^2 - N^2\pi^2}\, e^{-iak/2} \begin{cases} i \sin \frac{ak}{2} \,, & \text{for } N \text{ even} \,, \\ \cos \frac{ak}{2} \,, & \text{for } N \text{ odd} \,. \end{cases}$$

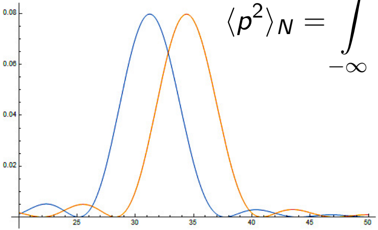
The simplest example - an infinite square wall

Momentum probability distribution

$$\mathcal{P}_N(p) = \frac{4\pi a\hbar^3 N^2}{(a^2p^2 - \hbar^2 N^2\pi^2)^2}\begin{cases}\sin^2\frac{ap}{2\hbar}\,, & \text{for } N \text{ even}\,,\\ \cos^2\frac{ap}{2\hbar}\,, & \text{for } N \text{ odd}\,.\end{cases}$$

This gives an average value of the momentum equal to zero, and an average value of the squared momentum

$$\langle p^2\rangle_N = \int_{-\infty}^{+\infty} dp\, p^2\mathcal{P}_N(p) = \frac{N^2\pi^2\hbar^2}{a^2}\,.$$



David fights Goliath

David's sling can be considered as a two-dimensional quantum rotator with a potential

$$U(\vec{r}) = \frac{1}{2} m\omega^2 (x^2 + y^2) \, .$$

Stationary solutions

$$E_{n_1,n_2} = \hbar\omega(n_1 + n_2 + 1) \, ,$$

$$\phi_{n_1,n_2}(x, y) = C_{n_1,n_2} e^{-\frac{m\omega}{2\hbar}(x^2+y^2)} H_{n_1}\left(x\sqrt{\frac{m\omega}{\hbar}}\right) H_{n_2}\left(y\sqrt{\frac{m\omega}{\hbar}}\right) \, .$$

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David fights Goliath

If Goliath were hit directly by a stone still on a cord then he would absorb an impact energy E_{n_1,n_2} , maybe not enough for him. But related momentum distribution is

$$e^{-\frac{{p_x}^2+{p_y}^2}{m\omega\hbar}} H_{n_1}^2\left(\frac{p_x}{\sqrt{m\omega\hbar}}\right) H_{n_2}^2\left(\frac{p_y}{\sqrt{m\omega\hbar}}\right) .$$

and David defeated Goliath - taking into account the power of his protector.

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