Margaret Island Symposium 2023 on Particles & Plasmas

Damped-Dynamic Screening Potential of Nuclei during BBN Epoch:

The Role of Electron-Positron Plasma in the Early Universe

Chris Grayson, Cheng Tao Yang, Johann Rafelski

Publication forthcoming

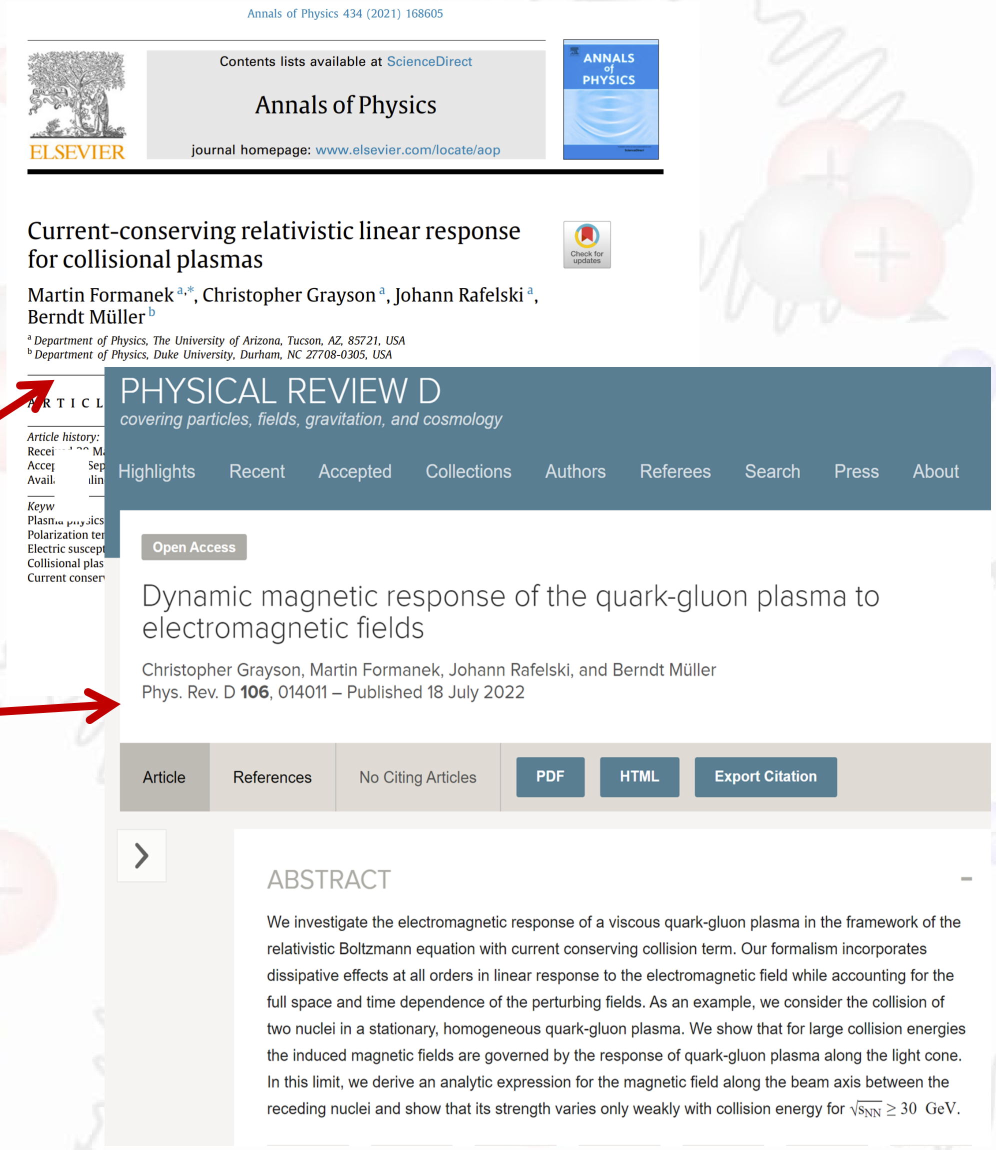
THE UNIVERSITY OF ARIZONA

Current-Conserving Relativistic Linear Response

We apply previous work to **electron-positron plasma** in the **early universe**.

Electromagnetic Fields $\leftrightarrow$ Pair Plasmas

- Our previous work develops **analytic** methods to calculate electromagnetic fields using **linear response** in infinite homogeneous plasmas with simplified collisional **damping**. (2021)
- After using these tools to study the magnetic field in QGP during heavy ion collisions (2022), we turned to study the **early universe** electron-positron plasma.
    - We find an **analytic** expression for the electric **potential** in the **damped BBN** plasma, similar to those found in dusty plasma theory (2023).


Annals of Physics 434 (2021) 168605

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Current-conserving relativistic linear response for collisional plasmas

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Dynamic magnetic response of the quark-gluon plasma to electromagnetic fields

Christopher Grayson, Martin Formanek, Johann Rafelski, and Berndt Müller

Phys. Rev. D **106**, 014011 – Published 18 July 2022

Article References No Citing Articles PDF HTML Export Citation

ABSTRACT

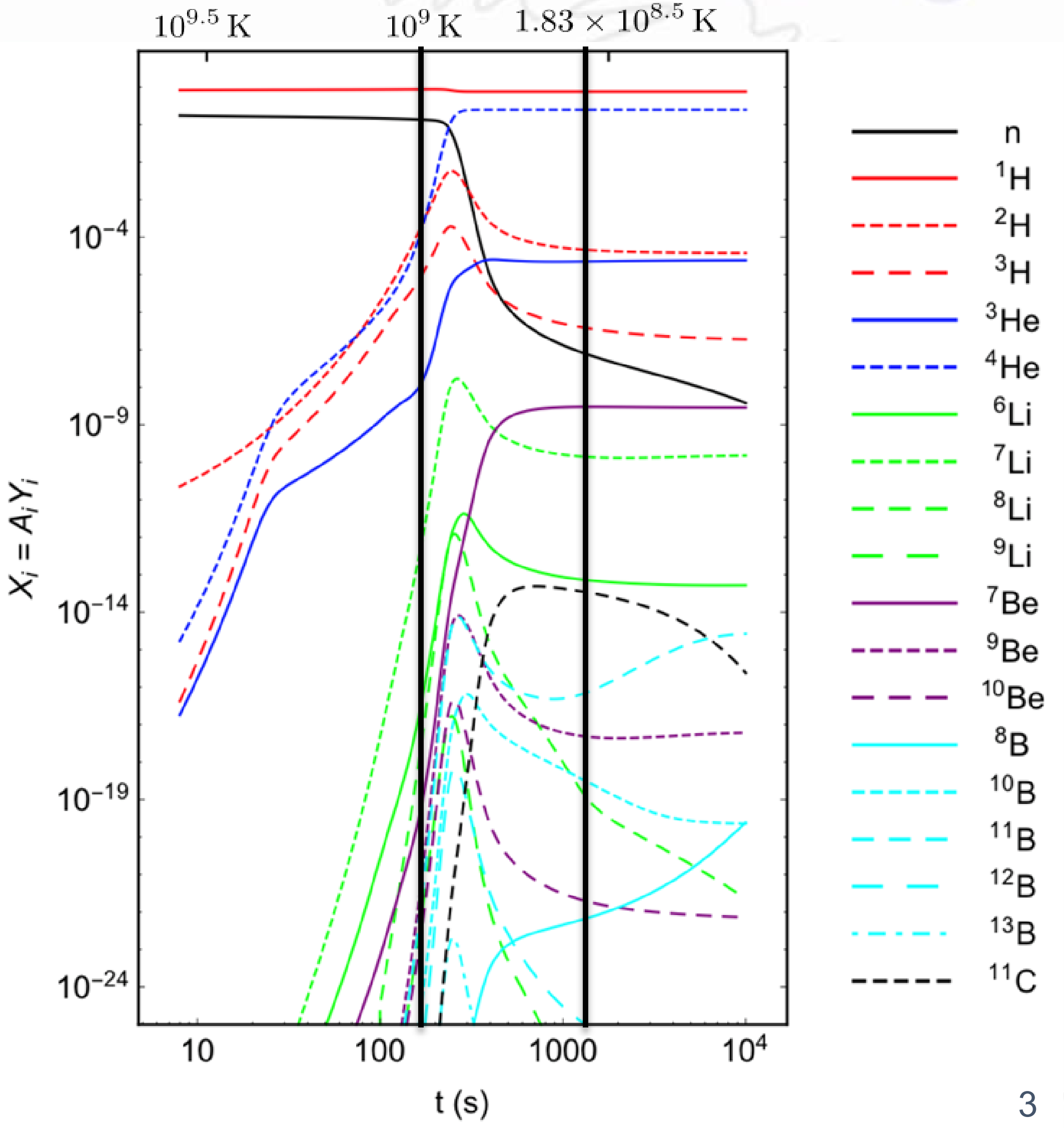
We investigate the electromagnetic response of a viscous quark-gluon plasma in the framework of the relativistic Boltzmann equation with current conserving collision term. Our formalism incorporates dissipative effects at all orders in linear response to the electromagnetic field while accounting for the full space and time dependence of the perturbing fields. As an example, we consider the collision of two nuclei in a stationary, homogeneous quark-gluon plasma. We show that for large collision energies the induced magnetic fields are governed by the response of quark-gluon plasma along the light cone. In this limit, we derive an analytic expression for the magnetic field along the beam axis between the receding nuclei and show that its strength varies only weakly with collision energy for $\sqrt{s_{NN}} \geq 30$ GeV.

Big Bang Nucleosynthesis (BBN)

$T_{BBN} = 50 - 86$ keV

Why is the Early universe of interest?

- During BBN, numerous **nuclear reactions** occur, producing primordial light **element distributions**.
- **Open questions and tensions between measurement and theory** remain about light element distributions in the universe



C.Pitrou, A.Coc, J.P.Uzan and E.Vangioni, ``Precision big bang nucleosynthesis with improved Helium-4 predictions,'' Phys. Rept. 754, 1-66 (2018)

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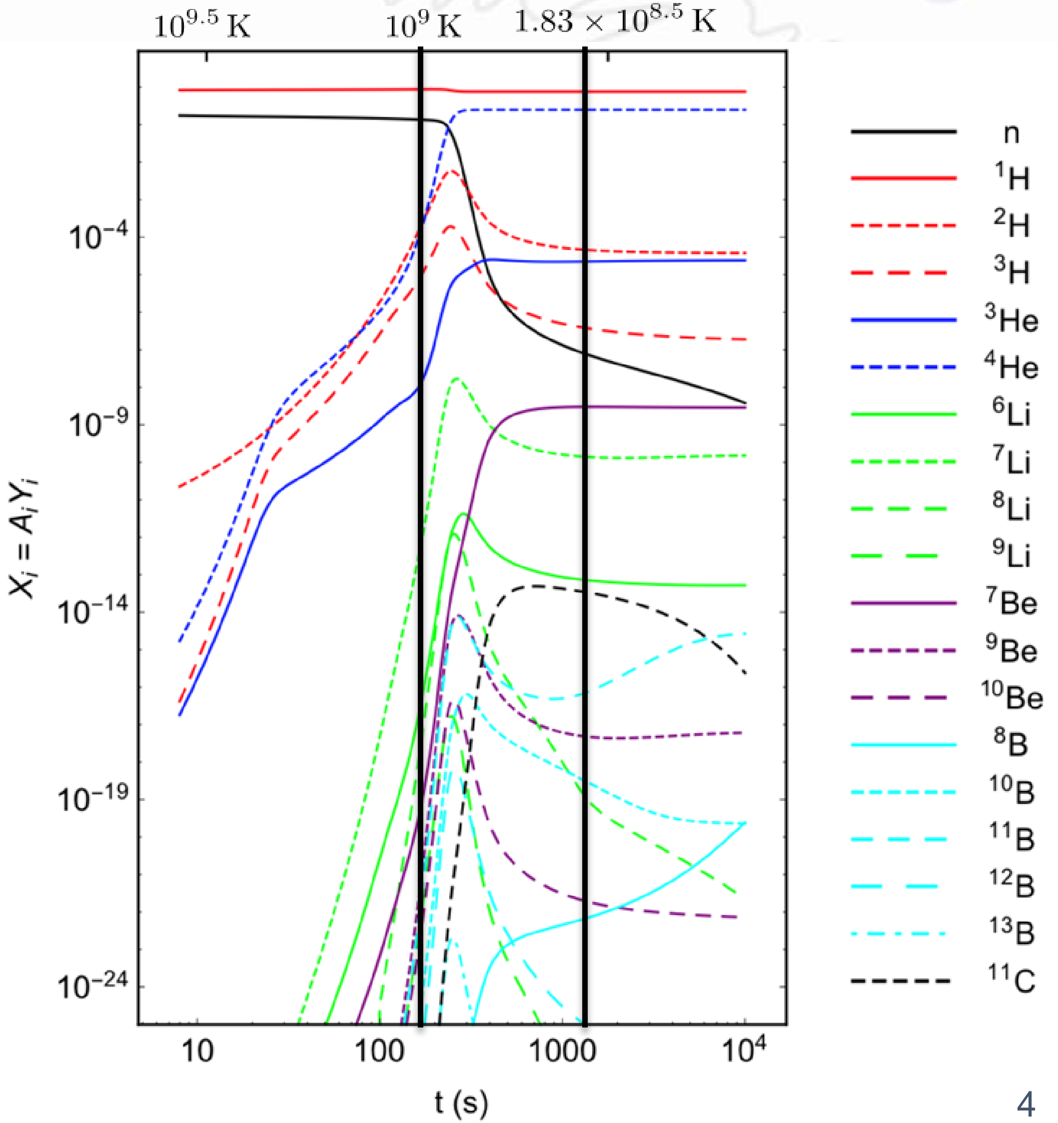
Question: How does the universe's composition affect the Big Bang Nucleosynthesis reaction network?

We acknowledge that these reactions do not occur in vacuum.



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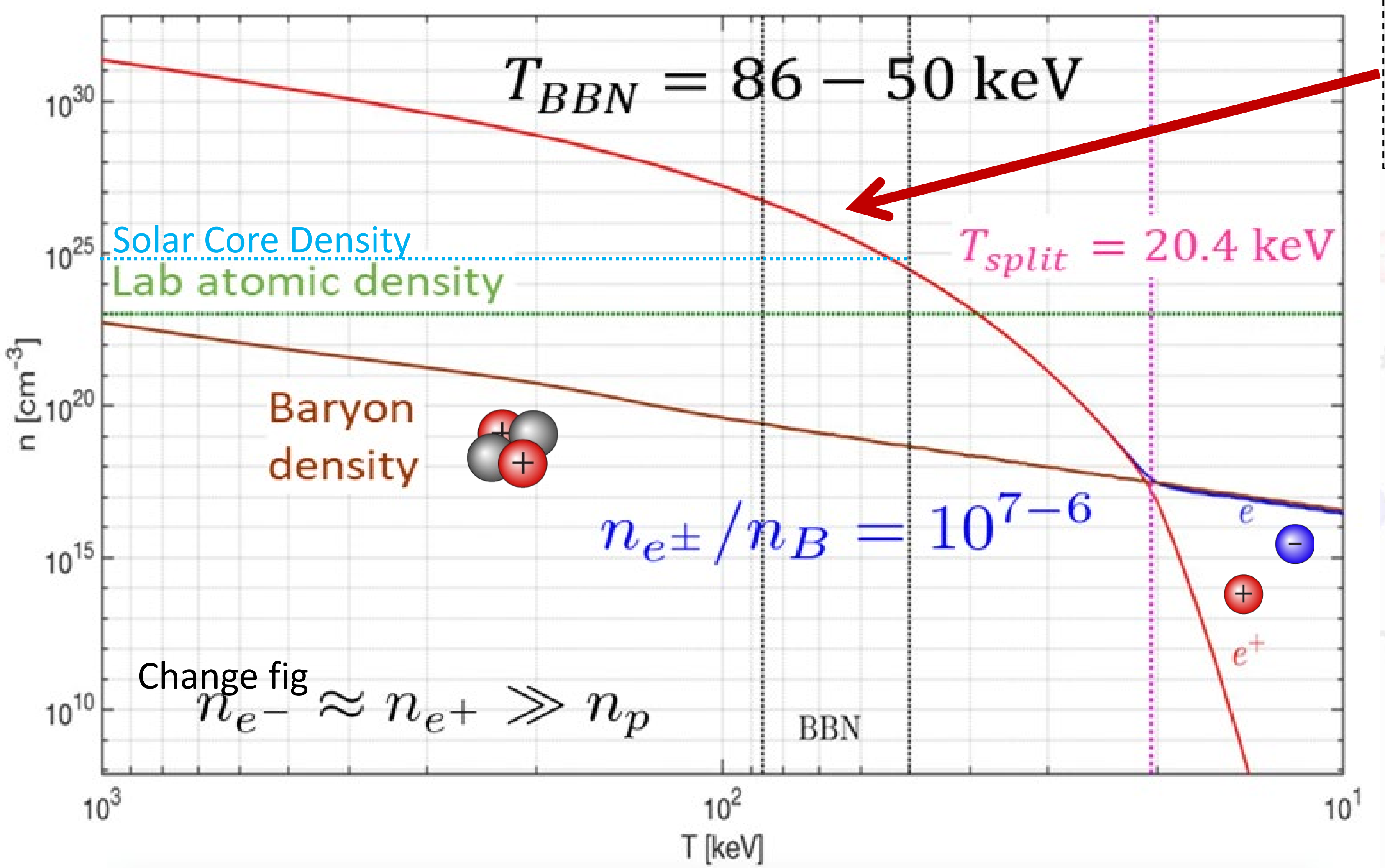
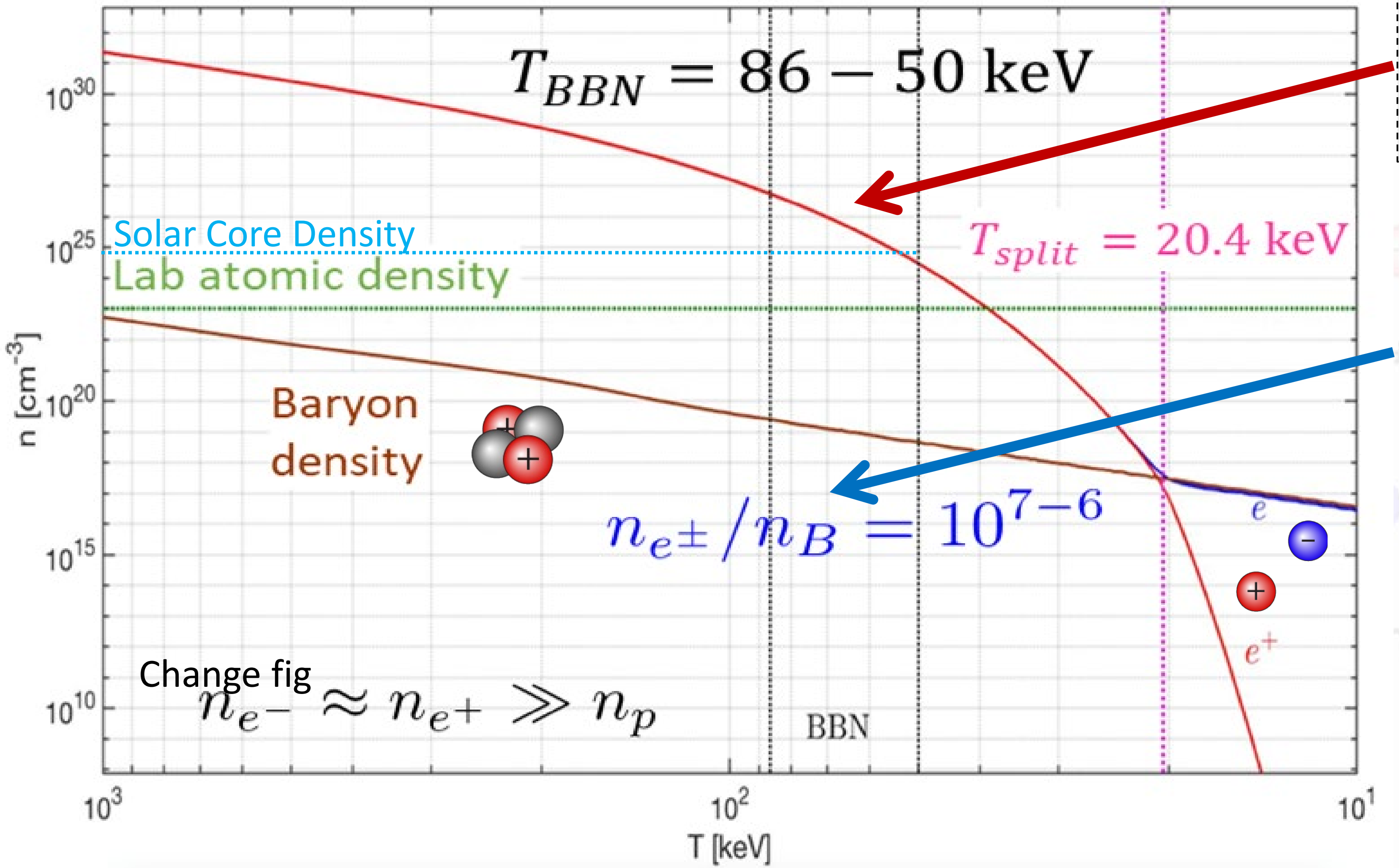
Early Universe Contents – BBN Epoch



BBN nuclear reactions occurred in the presence of a hot dense **electron-positron plasma** which must be accounted for.

Figure by Cheng Tao Yang, University of Arizona

Early Universe Contents – BBN Epoch



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Since the **baryon** density is very small, we describe them as spatially **dispersed** heavy '**impurities**' in the plasma.

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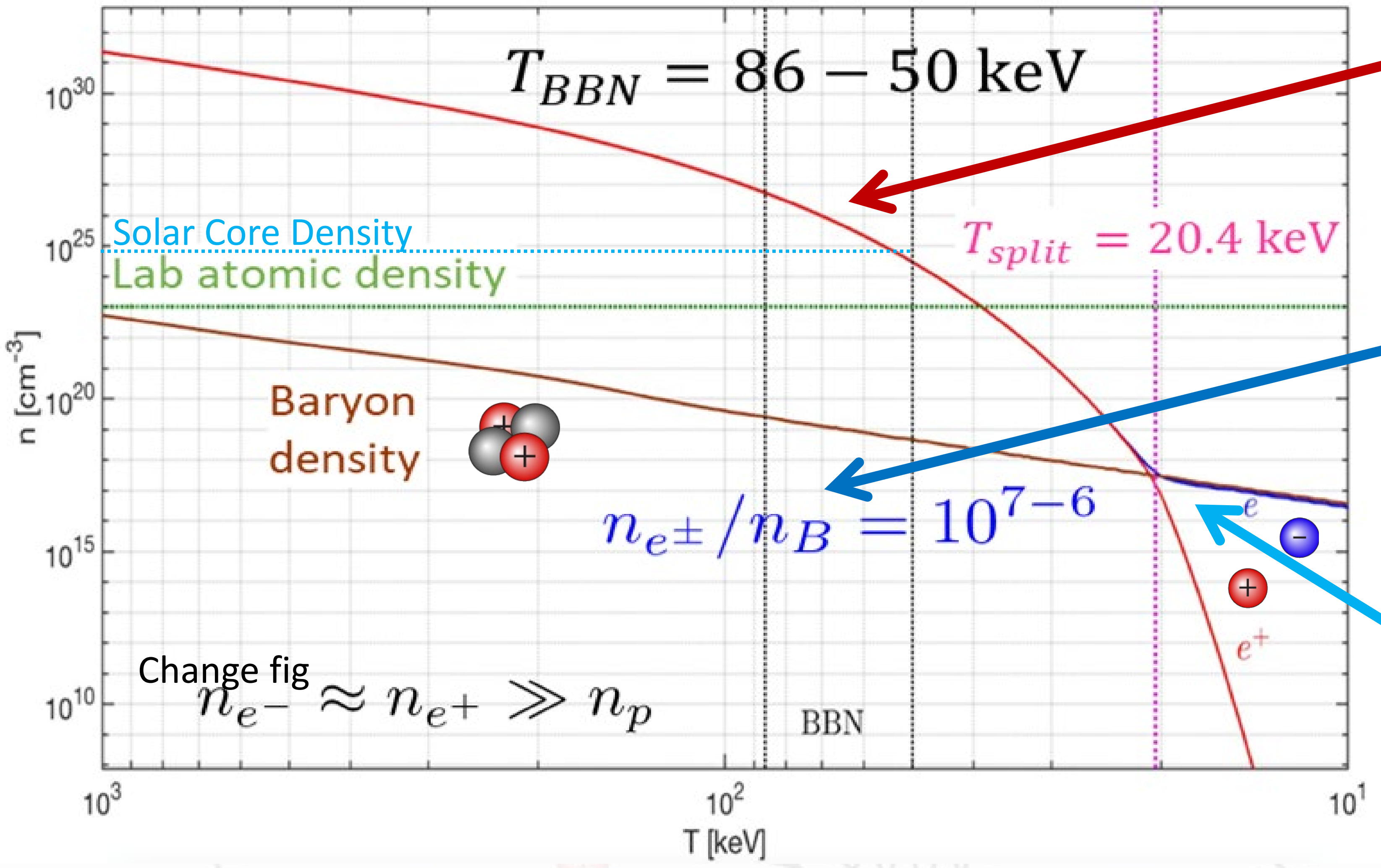

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Since the **baryon** density is very small, we describe them as spatially **dispersed** heavy '**impurities**' in the plasma.

Before T=20 keV, the **electron-positron chemical potential** is **zero**. Thus the plasma contains effectively equal parts electrons and positrons.

Figure by Cheng Tao Yang, University of Arizona

Big Bang Nucleosynthesis (BBN)

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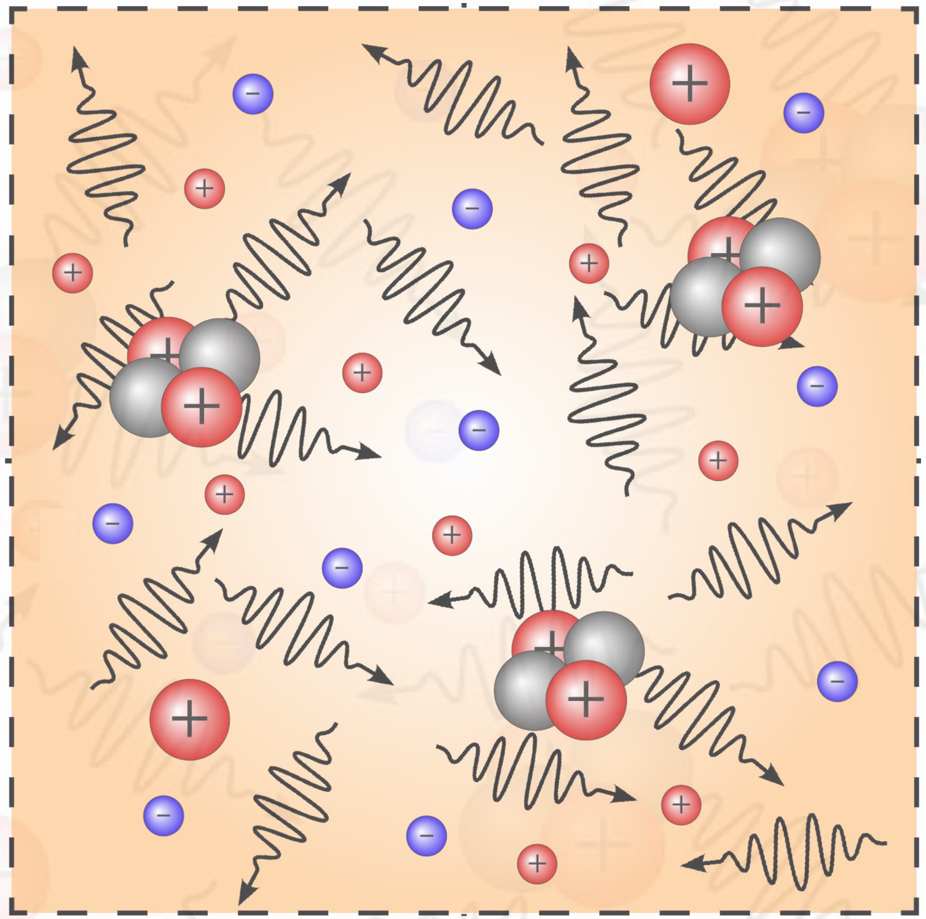
Why is the Early universe of interest?

- During BBN, numerous **nuclear reactions** occur, producing primordial light **element distributions**.
- **Open questions and tensions between measurement and theory** remain about light element distributions in the universe
- It is becoming more widely recognized that the universe was filled with a **hot dense electron-positron plasma** interspersed with light nuclei during BBN.
    - Carraro et al. (1988), Famiano et al. (2016), X. Yao et al (2017), B. Wang et al. (2021)



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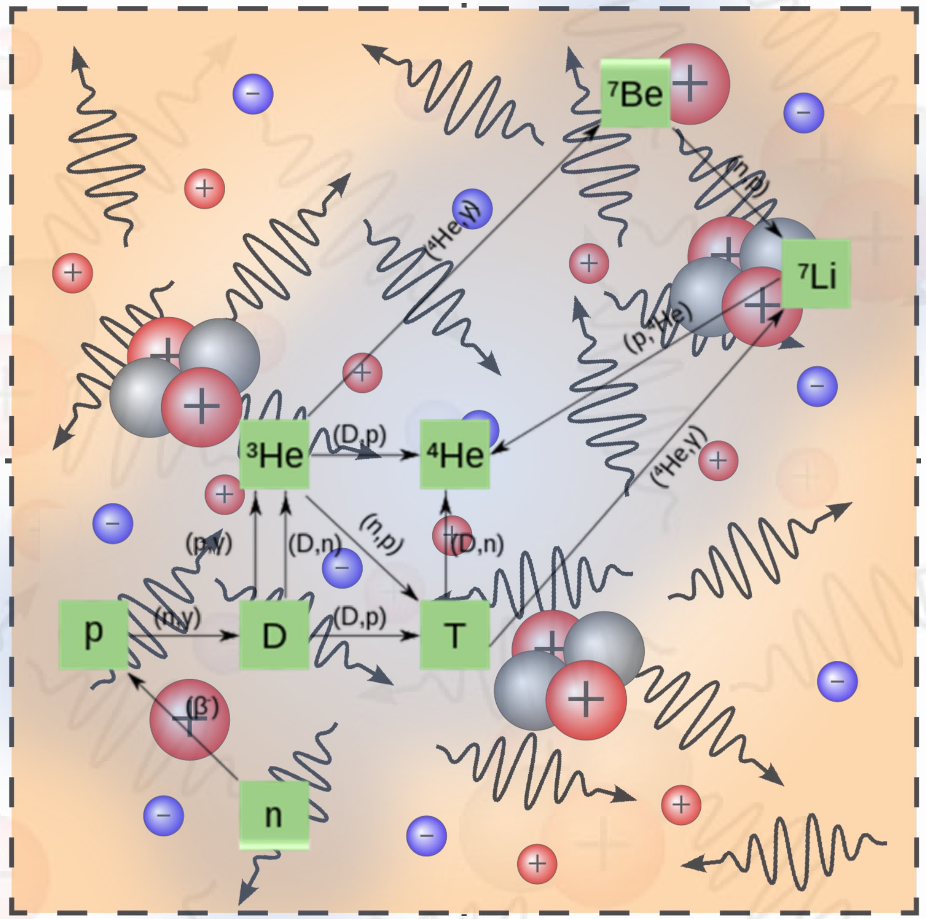
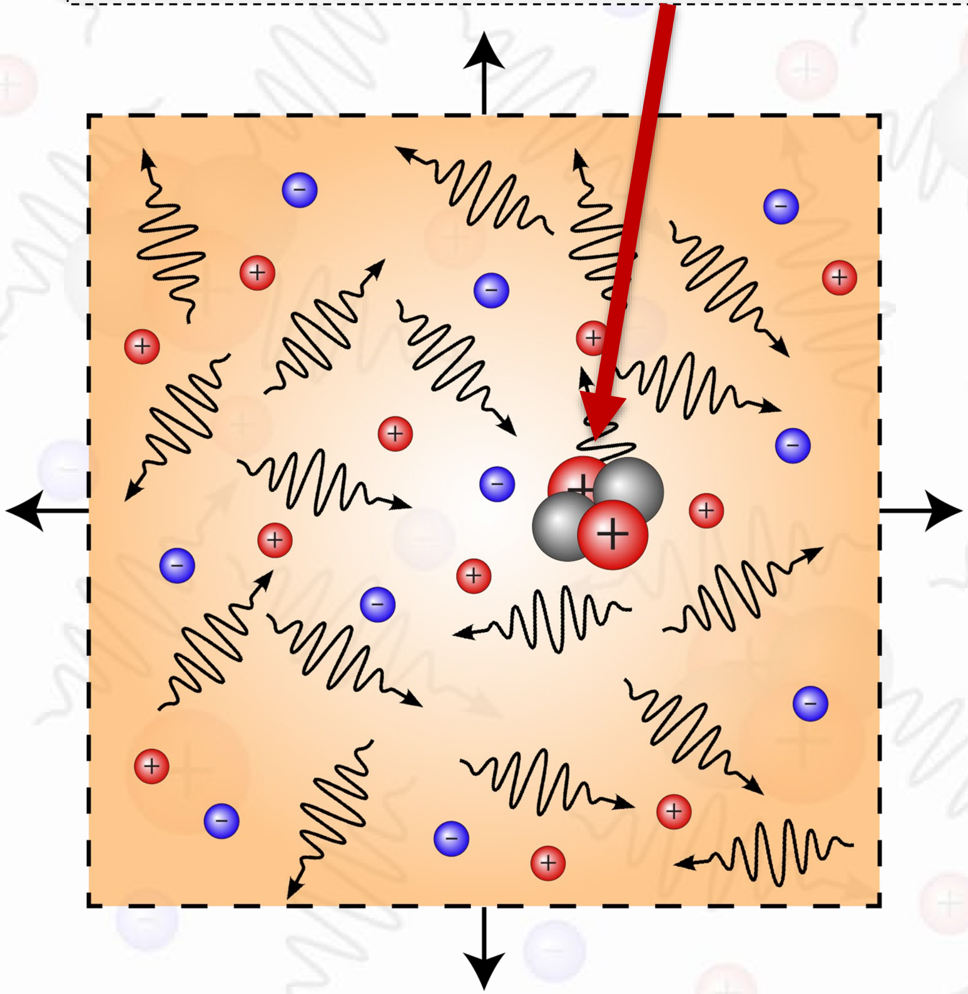


Early universe plasma suggests a review of the BBN reaction network

Overview – Screening Effect on Reaction Rates

Ion acts as an **external perturbation** in the plasma



E. E. Salpeter, "Electron screening and thermonuclear reactions," Austral. J. Phys. 7 (1954), 373-388 doi:10.1071/PH540373

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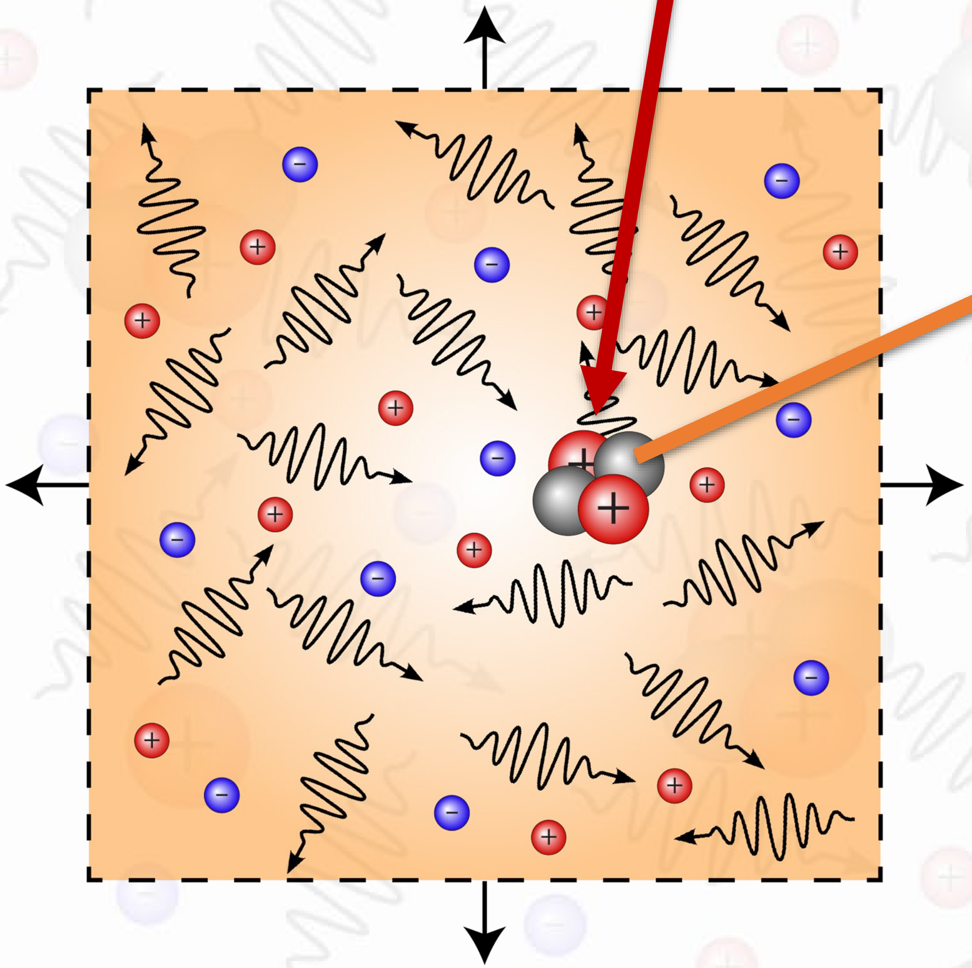
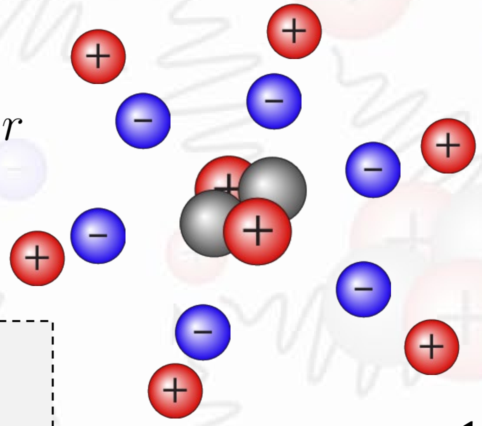


$$\phi_{\text{stat}}(r) = \frac{Ze}{4\pi\varepsilon_0 r} e^{-m_D r}$$



Polarization reduces electrostatic **repulsion**

$$m_D = {}^{1}/_{\lambda_D}$$

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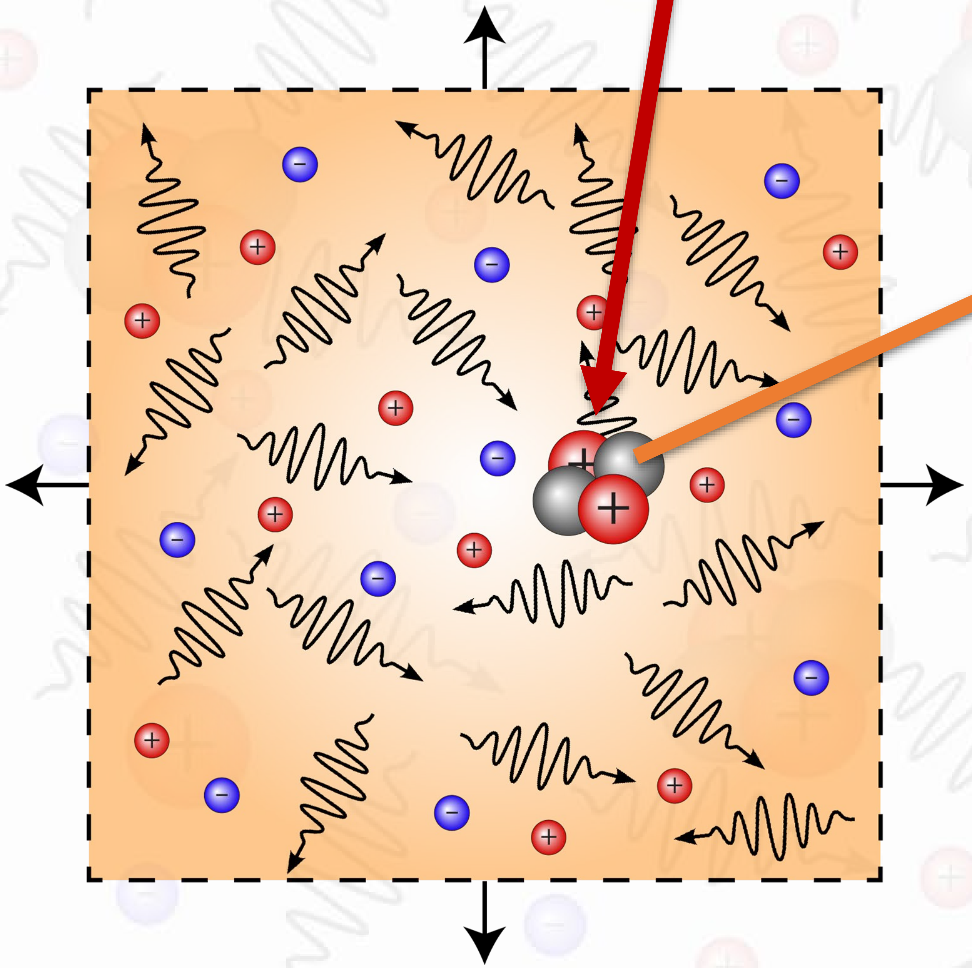

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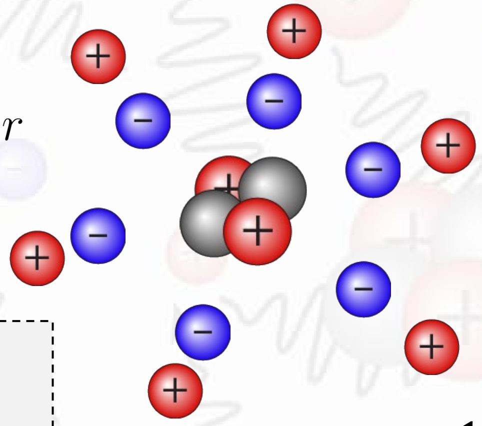

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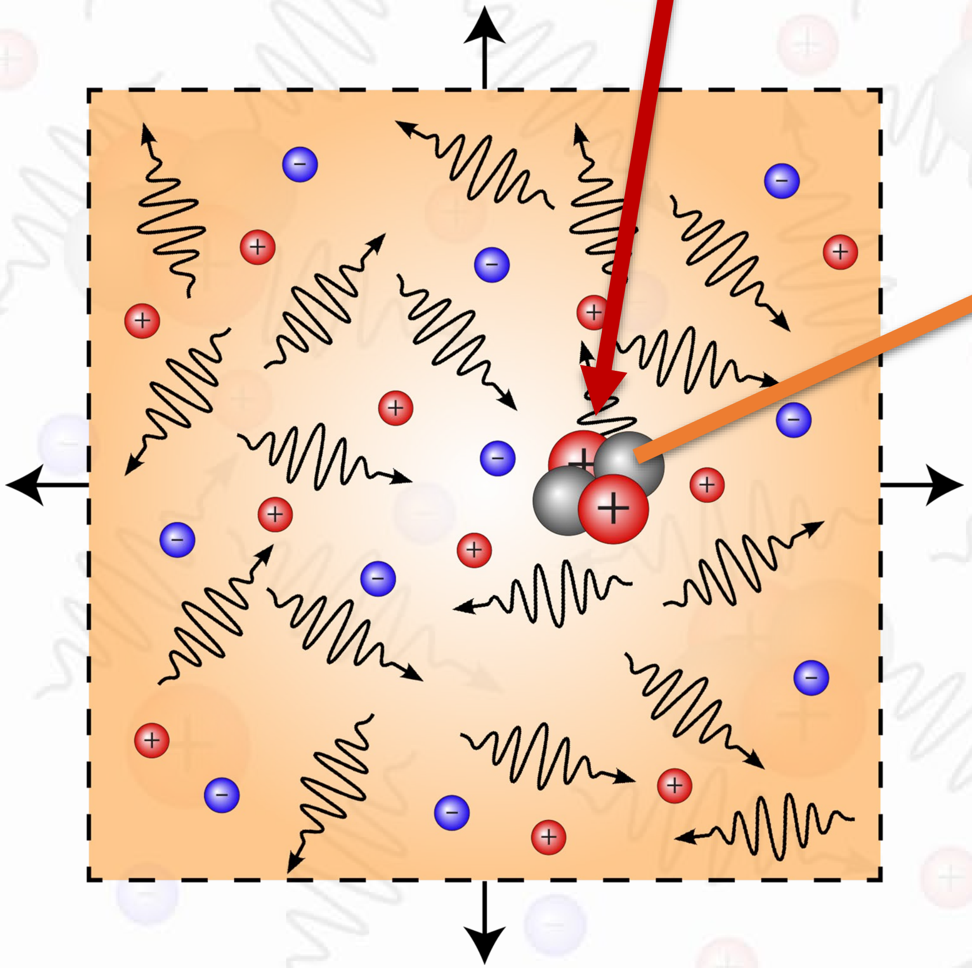

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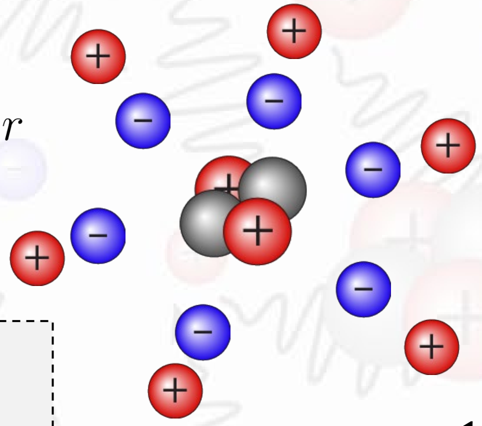

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Enhancement of reactions rates changes primordial **light element distributions.**

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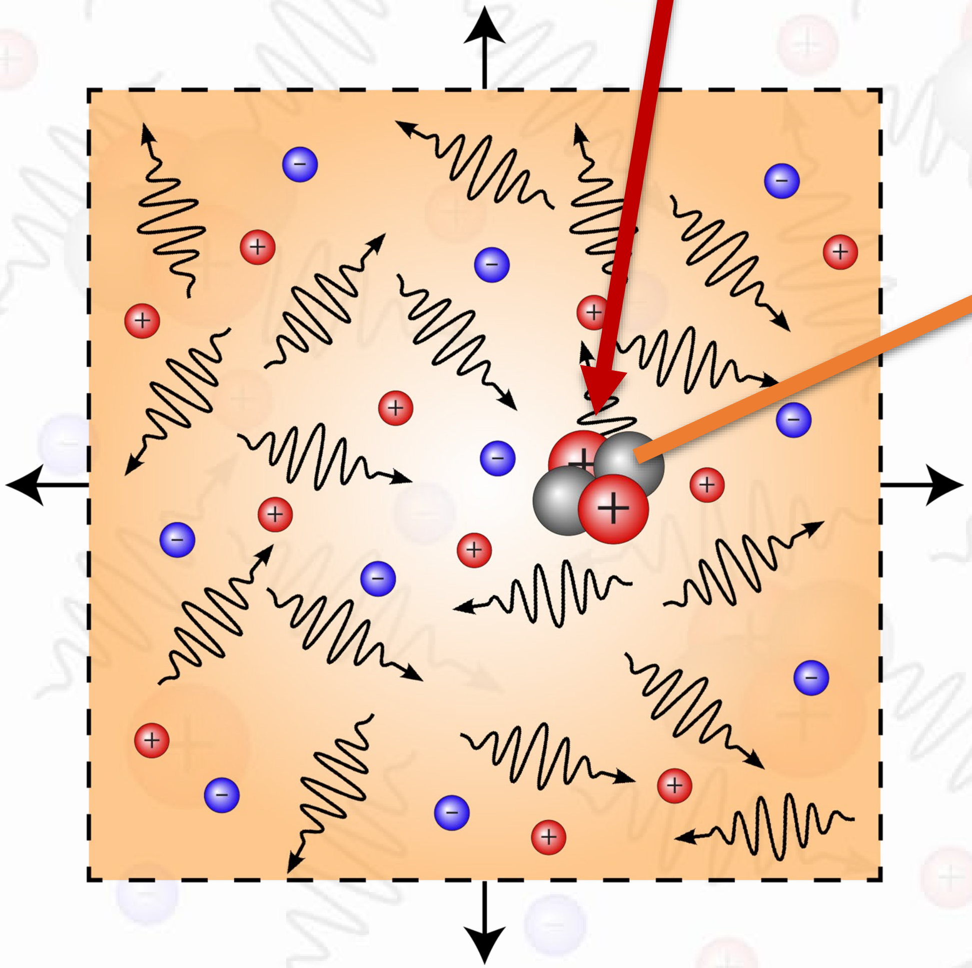

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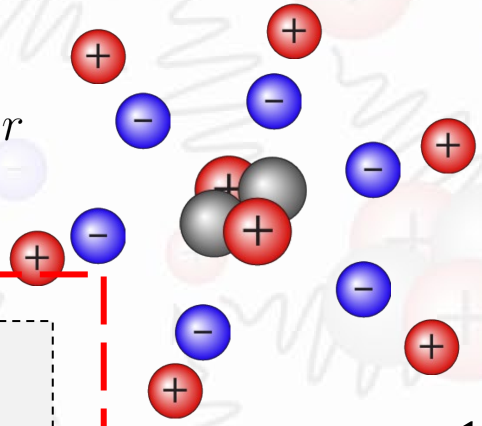

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Polarization screens the electric potential

Our focus

Reduction of the coulomb barrier **enhances** reactions rates

Enhancement of reactions rates changes primordial **light element distributions.**

E. E. Salpeter, “Electron screening and thermonuclear reactions,” Austral. J. Phys. 7 (1954), 373-388 doi:10.1071/PH540373

Plasma Screening models in BBN

All of these models assume the weak field limit $e\phi/_T \ll 1$

- **Static Screening -** nuclei sit at **rest** in the global frame of the plasma and have a standard Debye-Hückel potential.
  - E. E. Salpeter (1954)**,** Salpeter & van Horn (1969), Famiano et al. (2016)

$$\phi_{\mathrm{stat}}(r) = \frac{Ze}{4\pi\varepsilon_0 r} e^{-m_D r}$$

- **Dynamic Screening** - due to the temperature of the BBN plasma nuclei have a **nonrelativistic** thermal distribution of **velocities.**
  - Carraro et al. (1988), X. Yao et al (2017), B. Wang et al. (2021)
  - Static screening **potential** has **corrections** due to the **motion** of nuclei.

$$v_N \approx \sqrt{\frac{2T}{m_N}}$$

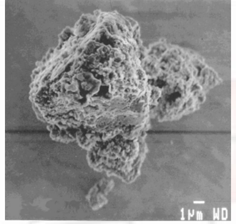
(Most probable velocity Boltzmann Distribution)

- **Damped-Dynamic Screening – collisions** between plasma particles cause **damping** in the dynamic screened potential. Our contribution
  - M. Formanek, C. Grayson, J. Rafelski and B. Müller (2021)

Analytic result matches dusty plasma theory.

We neglect primordial magnetic fields for now.

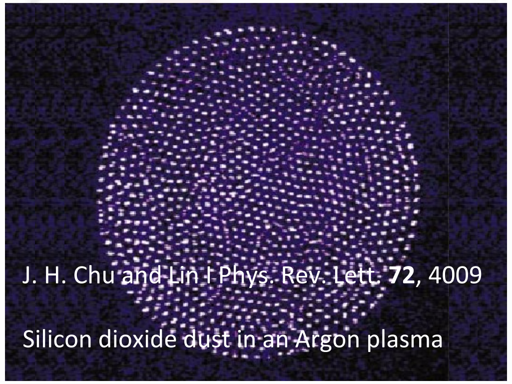
Plasma Screening Dusty (Complex) Plasmas



Current BBN screening models are **analogous** to models **previously** created to describe **dusty** (complex) **plasmas**.

- focuses on plasma **impurities** (dust) on the order of $\sim$1 μm in size.

  Areas of interest: Interplanetary space, Comets, Planetary rings, Earth's atmosphere, fusion devices.
  - Both fields study **larger**, often spatially dispersed 'impurities' or dust with Q/m different than standard plasma components.
- We expect other results from this theoretical framework may be similar or directly applicable to **BBN electron-positron plasma.**

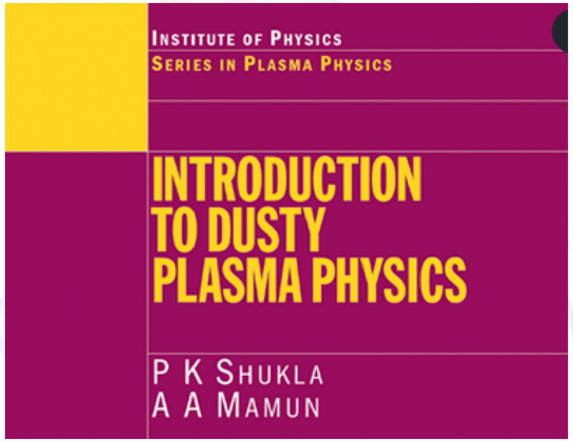

J. H. Chu and Lin I Phys. Rev. Lett. **72**, 4009

Silicon dioxide dust in an Argon plasma

L. Stenflo, M. Y. Yu, and P. K. Shukla "Shielding of a slow test charge in a collisional plasma" Phys. Fluids 16 (1973)

P. K. Shukla and A. A. Mamun "Introduction to Dusty Plasma Physics" Plasma Phys. Control. Fusion 44 395 (2002)

P. K. Shukla and N. N. Rao "Coulomb crystallization in colloidal plasmas with streaming ions and dust grains" Physics of Plasmas 3, 1770 (1996)



Self consistenet Screening Potential

The **screened potential** is found in the weak field limit ${e\phi}/{T} \ll 1$ by introducing an induced polarization current due to the BBN plasma through the **linear response relation.**

$$\widetilde{j}^{\mu}_{\mathrm{ind}}(k) = \Pi^{\mu}{}_{\nu}(k)\widetilde{A}^{\nu}(k)$$

"Linear response relation"

$$-ik_{\mu}\widetilde{F}^{\mu\nu}(k) = \widetilde{j}^{\nu}_{\mathrm{ext}}(k) + \widetilde{j}^{\nu}_{\mathrm{ind}}(k)$$

"Maxwell's equations"

One can then solve Maxwell's equations to find the usual self-consistent potential in the plasma,

$$\phi(t,\boldsymbol{x}) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{\widetilde{\rho}_{\mathrm{ext}}(\omega,\boldsymbol{k})}{\varepsilon_{\parallel}(\omega,\boldsymbol{k})(\boldsymbol{k}^2 - \omega^2)}$$

$$\varepsilon_{\parallel}(\omega,\boldsymbol{k}) = \left(\frac{\Pi_{\parallel}(\omega,\boldsymbol{k})}{\omega^2} + 1\right)$$

"Longitudinal permittivity"

D. Melrose, Quantum Plasmadynamics: Unmagnetized Plasmas, Lect. Notes Phys. 735 (Springer, New York, 2008)

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Polarization tensor describes plasma properties

$$\varepsilon_{\parallel}(\omega, \boldsymbol{k}) = \left(\frac{\Pi_{\parallel}(\omega, \boldsymbol{k})}{\omega^2} + 1\right)$$

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D. Melrose, Quantum Plasmadynamics: Unmagnetized Plasmas, Lect. Notes Phys. 735 (Springer, New York, 2008)

Electron-Positron Plasma – Linear Repsonse

The **polarization tensor** is found by calculating **the induced current** due to small perturbations from equilibrium in the **Vlasov-Boltzmann equation**

$$\tilde{j}^{\mu}_{\rm ind}(k) = 2\int \frac{d^4p}{(2\pi)^4} p^{\mu} 4\pi \delta_+(p^2 - m^2) \sum_{i=+,-} q_i \tilde{f}_i(k,p)$$

$$\widetilde{j}^{\mu}_{\rm ind}(k) = \Pi^{\mu}_{\nu}(k)\widetilde{A}^{\nu}(k)$$

The nonequilibrium distribution $f(x,p)$ is found by considering **small perturbations** δf away from **equilibrium** in the system of Vlasov-Boltzmann equations describing the plasma. $f_{\pm}(x,p) = f^{(\rm eq)}_{\pm}(p) + \delta f_{\pm}(x,p)$

$$(p\cdot\partial) f_{\pm}(x,p) + qF^{\mu\nu}p_{\nu}\frac{\partial f_{\pm}(x,p)}{\partial p^{\mu}} = C_{\pm}(x,p)$$

"Electron-Positrons"

$$(p\cdot\partial) f_{\gamma}(x,p) = C_{\gamma}(x,p)$$

"Photons" – does not couple directly to the EM field.

"Collision term"

$$C_i(x,p) = \kappa_i(p\cdot u)\left(f^{\rm eq}_i(p)\frac{n_i(x)}{n^{\rm eq}_i} - f_i(x,p)\right)$$

M. Formanek, C. Grayson, J. Rafelski and B. Müller, (2021)

Collision term - BGK term (Relaxation Term)

We assume a **collision term** in the P. L. Bhatnagar, E. P. Gross and M. Krook (**BGK) form**. This models the sum of all **scattering effects** on particles in the plasma as a dissipative medium effect which returns the system to equilibrium in time τ.

$$\frac{df(x,p)}{dt} = \frac{f_{\mathrm{eq}}(p) - f(x,p)}{\tau}$$

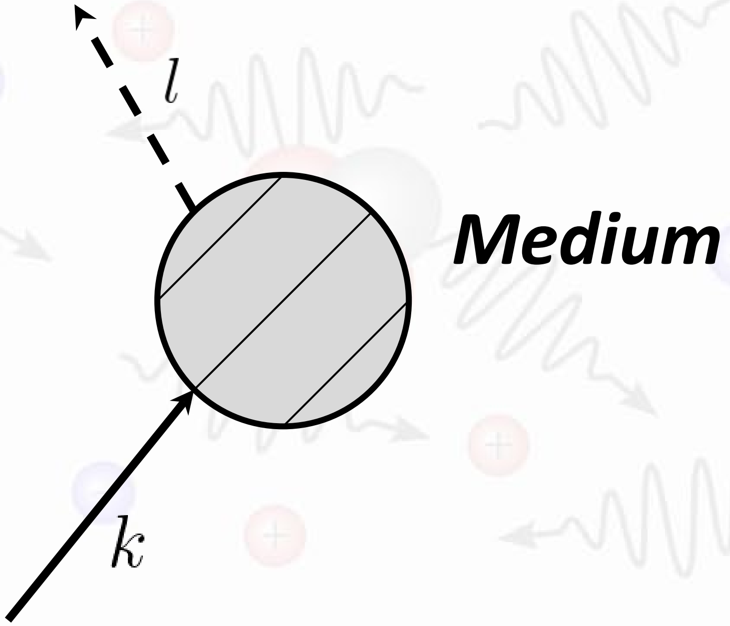
$$\kappa = 1/\tau$$

"Damping rate"

Determines the **strength** of **damping**

$$C_{kl}(x,p_k) = (p_k \cdot u)\kappa_{kl}\left(f_k^{\mathrm{eq}}(p)\frac{n_k(x)}{n_k^{\mathrm{eq}}} - f_k(x,p_k)\right)$$

Additional term **conserves particle number**. Terms can be added to ensure energy conservation. *G. S. Rocha et al. (2021)*



P. L. Bhatnagar, E. P. Gross and M. Krook, Phys. Rev. 94, 511 (1954)

Electron-Positron Plasma – Linear Repsonse

$$m_D = \frac{1}{\lambda_D}$$

After solving for the **small perturbation** δf in frequency space we calculate the induced current, assuming the equilibrium distribution is a Boltzmann distribution.

$$\tilde{j}^{\mu}_{\text{ind}}(k) = 4e \int \frac{d^3p}{(2\pi)^3 p^0} p^{\mu} \delta \tilde{f}(k,p) \quad \text{with} \quad f^{(\text{eq})}_{\pm}(p) \approx \exp\left(-\frac{m}{T}\left(1 + \frac{|\boldsymbol{p}|^2}{2m^2}\right)\right)$$

Then keeping up to 2nd order in $|\boldsymbol{p}|/m$ one finds the polarization functions

M. Formanek, C. Grayson, J. Rafelski and B. Müller, (2021)

"Non-relativistic Boltzmann Distribution"
$T_{BBN} \ll m_e$

$T_{BBN} = 86 - 50\ \text{keV}$

$$\Pi_{\parallel}(\omega, \boldsymbol{k}) = -\omega_p^2 \frac{\omega^2}{(\omega + i\kappa)^2} \frac{1}{1 - \frac{i\kappa}{\omega + i\kappa}\left(1 + \frac{T|\boldsymbol{k}|^2}{m(\omega + i\kappa)^2}\right)}$$

The longitudinal polarization function corresponds to the movement of the charge

$$\Pi_{\perp}(\omega) = -\omega_p^2 \frac{\omega}{\omega + i\kappa}$$

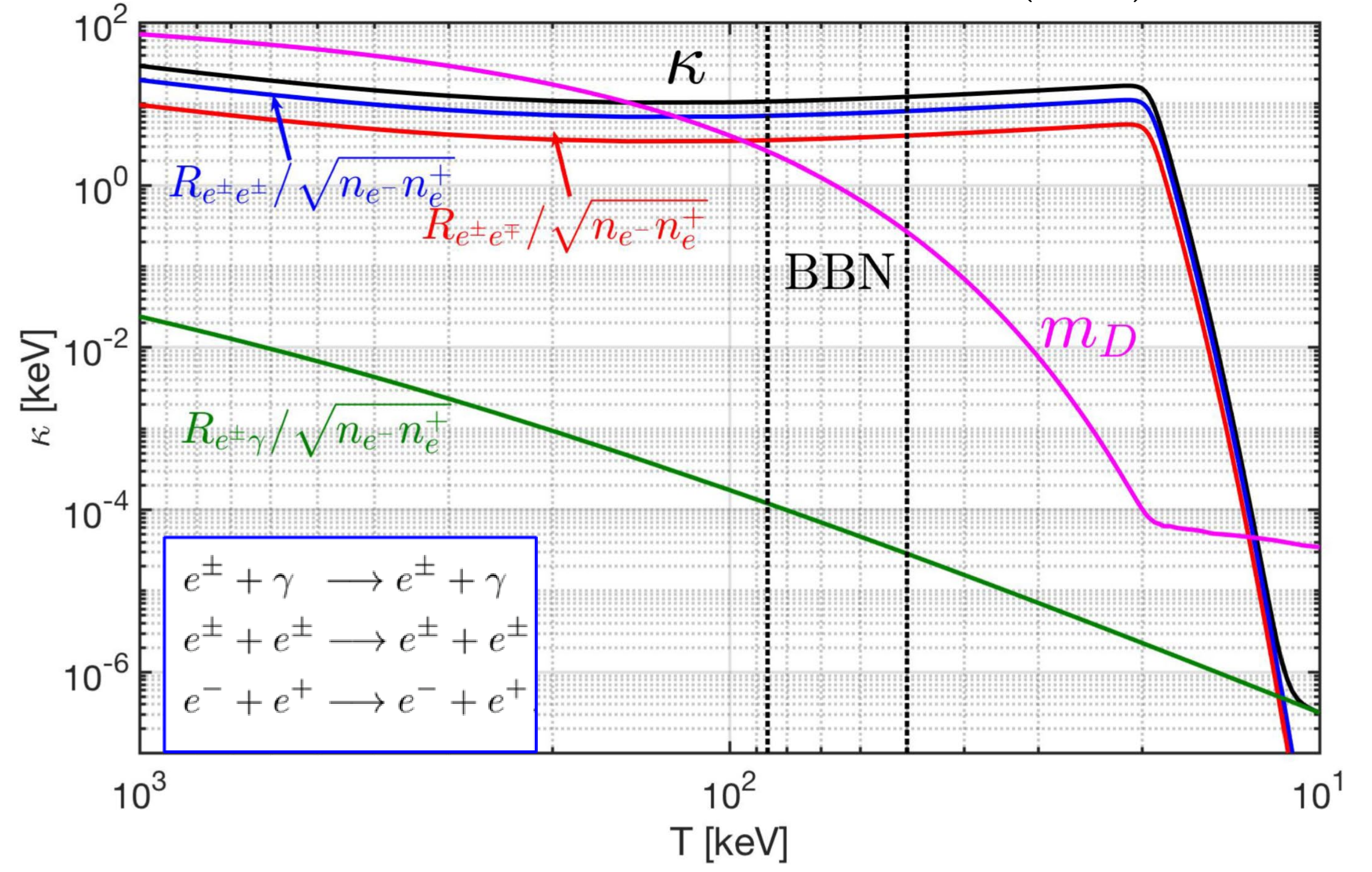
The transverse polarization function corresponds to the dispersion of electromagnetic waves.

$$\omega_p^2 = m_D^2 \frac{T}{m}$$

Electron Positron Plasma Damping Rate

$$m_D^2 = 4\pi\alpha \left(\frac{2mT}{\pi}\right)^{3/2} \frac{e^{-m/T}}{2T}$$

$$\kappa = \frac{R_{e^\pm e^\pm} + R_{e^\pm e^\mp} + R_{e^\pm \gamma}}{\sqrt{n_{e^-} n_{e^+}}}$$



The **damping rate** κ is found by **summing** the most relevant reaction rates for 2 ↔ 2 scatterings.

Figure by Cheng Tao Yang, University of Arizona

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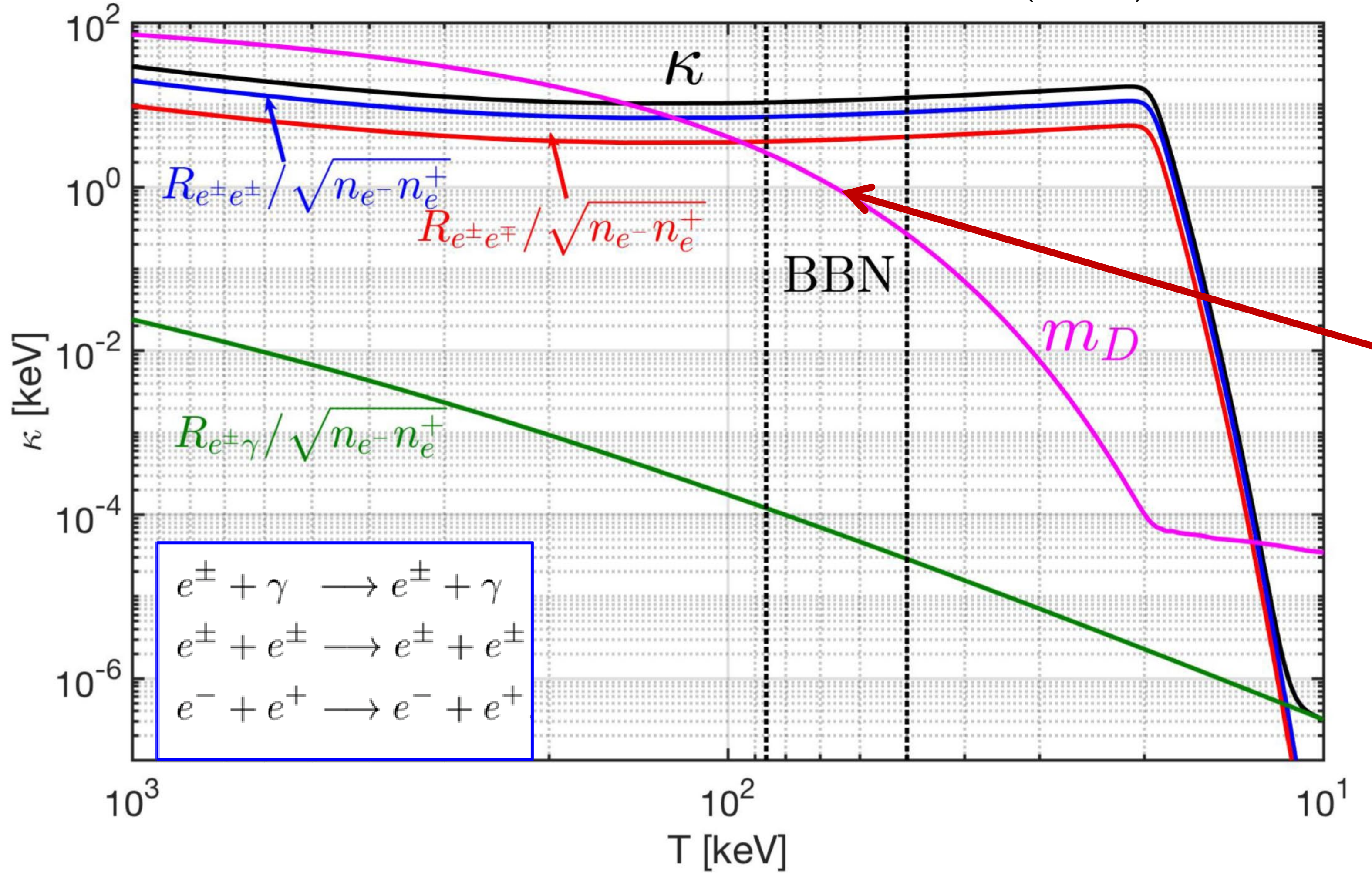

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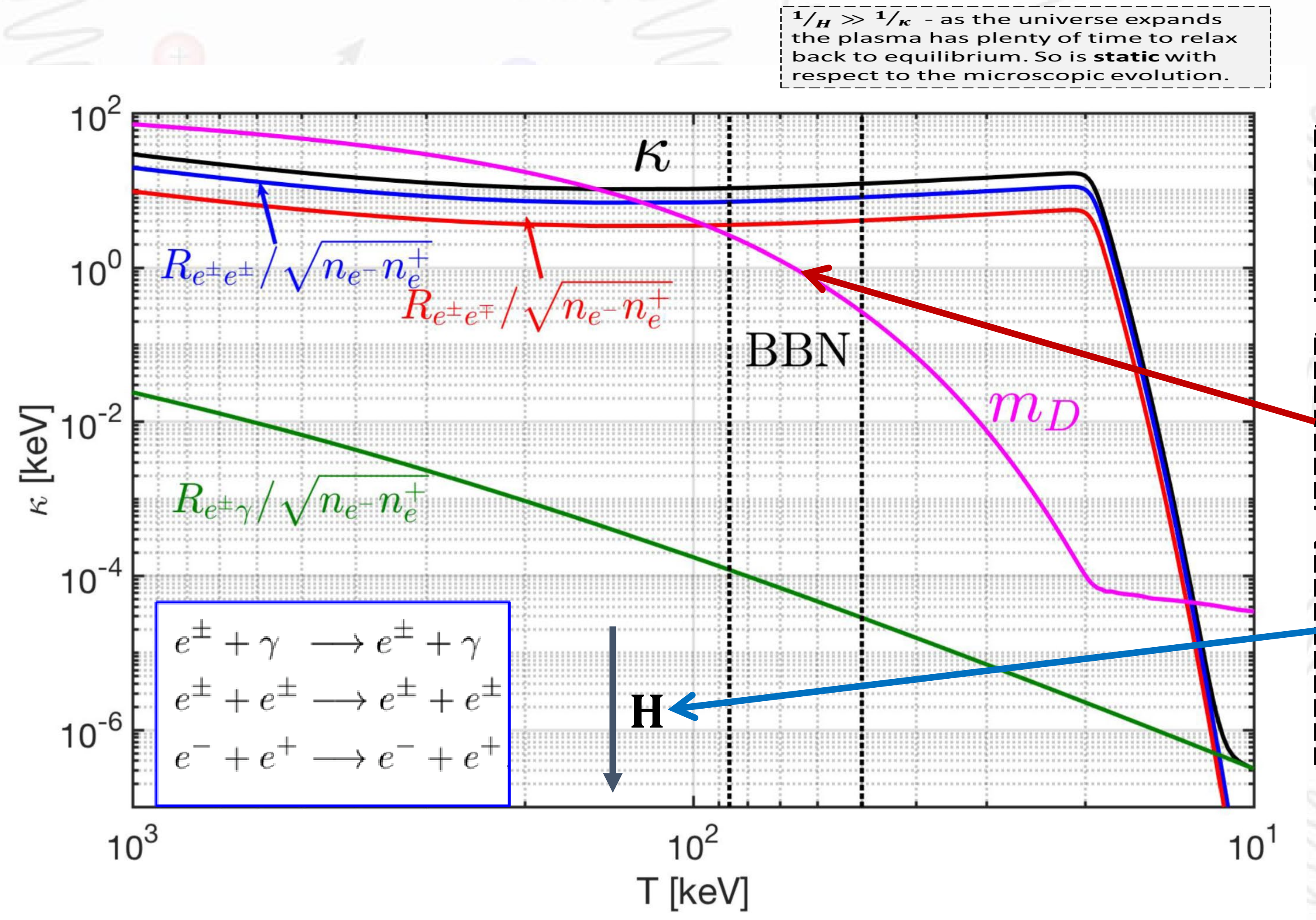
$\boldsymbol{\kappa \gg m_D}$ - during BBN indicating electromagnetic perturbations of the plasma will be **over-damped**.

Figure by Cheng Tao Yang, University of Arizona

Electron Positron Plasma Damping Rate



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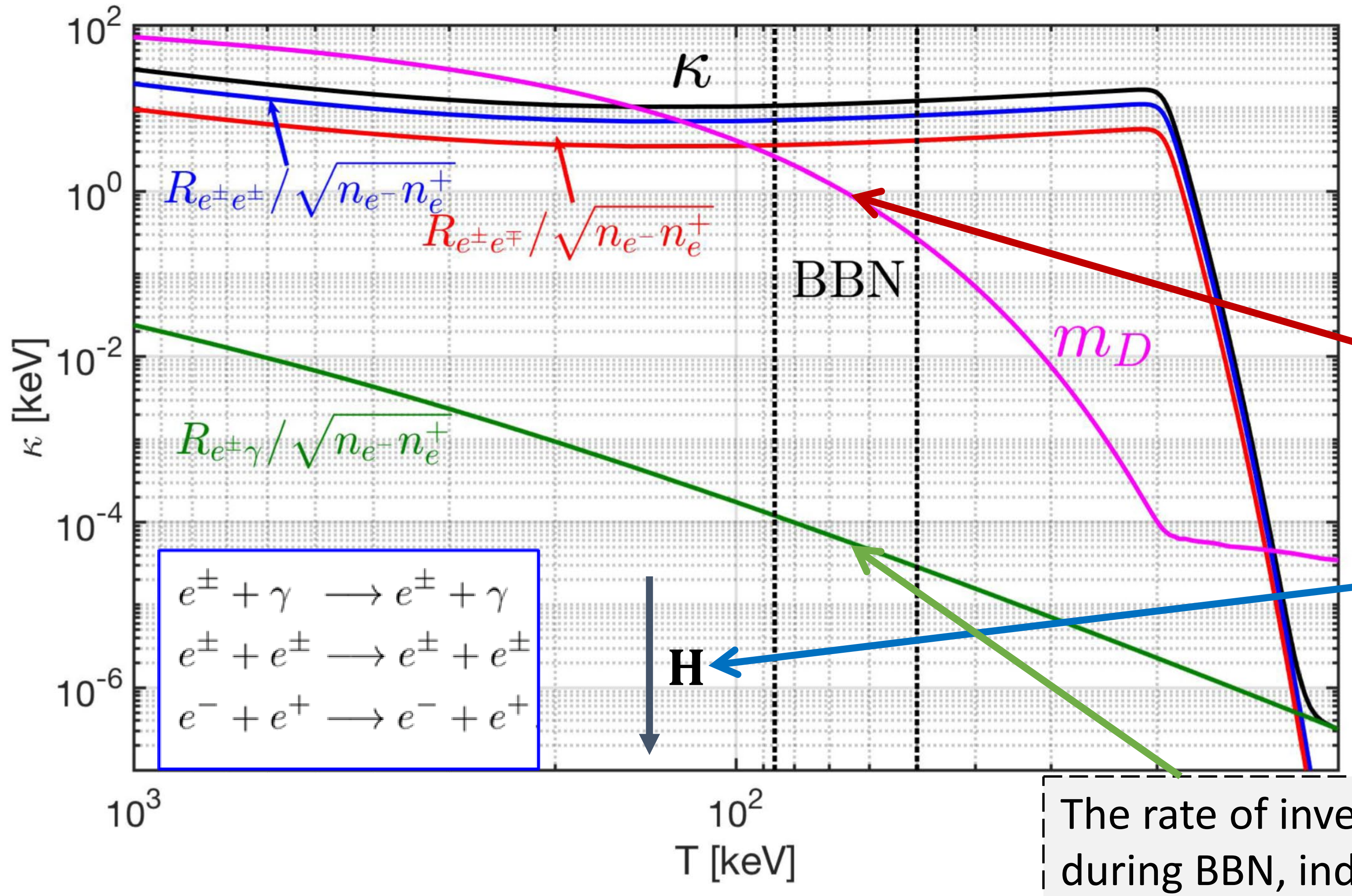

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The rate of inverse Compton scattering is very **small** during BBN, indicating that photons will not influence the electron-positron distribution.

Electron-Positron Plasma – Calculating the potential

The **self-consistent potential** is found by Fourier transforming its momentum space relation

$$\phi(\boldsymbol{x},t) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{\widetilde{\rho}_{\text{ext}}(\omega,\boldsymbol{k})}{\varepsilon_{\|}(\omega,\boldsymbol{k})(\boldsymbol{k}^2-\omega^2)}$$

where $\varepsilon_{\|}(\omega,\boldsymbol{k}) = \frac{\Pi_{\|}(\omega,\boldsymbol{k})}{\varepsilon_0\omega^2} + 1$

$$\Pi_{\|}(\omega,\boldsymbol{k}) = -\omega_p^2 \frac{\omega^2}{(\omega+i\kappa)^2} \frac{1}{1-\frac{i\kappa}{\omega+i\kappa}\left(1+\frac{T|\boldsymbol{k}|^2}{m(\omega+i\kappa)^2}\right)}$$

For $\widetilde{\rho}_{\text{ext}}(\omega,\boldsymbol{k})$ we prescribe a **moving** gaussian

$$\widetilde{\rho}_{\text{ext}}(\omega,\boldsymbol{k}) = 2\pi Ze\, e^{-\boldsymbol{k}^2\frac{R^2}{4}}\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{\beta}_N)$$

$$\beta_N = {}^{v_N}/_{c} \approx \sqrt{\frac{2T}{m_N}}$$

We perform the frequency integral using the delta function

$$\phi(t,\boldsymbol{x}) = Ze\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{\beta}_{\text{N}}t)} \frac{e^{-\boldsymbol{k}^2\frac{R^2}{4}}}{\boldsymbol{k}^2\varepsilon_{\|}(-\boldsymbol{\beta}_{\text{N}}\cdot\boldsymbol{k},\boldsymbol{k})}$$

We use contention $-\boldsymbol{\beta}_N\cdot\boldsymbol{k}$ from dusty plasma to get the correct causal behavior of the field.

Electron-Positron Plasma – Calculating the potential

We then expand the potential in the limit of **large damping** and **small velocity** of the source, due to the comparatively large mass of nuclei.

$$(\boldsymbol{k}\cdot\boldsymbol{\beta}_{\mathrm{N}})^2 \ll \boldsymbol{k}^2\frac{T}{m} \ll \kappa^2$$

$$\phi(t,\boldsymbol{x}) = \phi_{\mathrm{stat}}(t,\boldsymbol{x}) - Ze\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{\beta}_{\mathrm{N}}t)}\frac{i\boldsymbol{k}\cdot\boldsymbol{\beta}_{\mathrm{N}}m_D^2(\frac{\boldsymbol{k}^2}{\kappa}-\frac{m}{T}\kappa)}{\boldsymbol{k}^2(\boldsymbol{k}^2+m_D^2)^2}e^{-\boldsymbol{k}^2\frac{R^2}{4}}$$

Zeroth order term is simply **static** screening

Linear order contribution due to **damped-dynamic screening**

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Zeroth order term is simply **static** screening

contribution due to **damped-dynamic screening**

This can then be Fourier transformed back into position space for a **point charge ($\boldsymbol{R}=\boldsymbol{0}$)**

$$\phi(t,\boldsymbol{x}) = \phi_{\mathrm{stat}}(t,\boldsymbol{x}) + \frac{Ze\beta_N\cos(\psi)}{4\pi\varepsilon_0}\left[\left(\frac{\kappa\frac{m}{T}}{m_D^2 r(t)^2}+\frac{\kappa\frac{m}{T}}{m_D r(t)}+\frac{(m_D^2+\frac{m}{T}\kappa^2)}{2\kappa}\right)e^{-m_D r(t)} - \frac{\kappa\frac{m}{T}}{m_D^2 r(t)^2}\right]$$

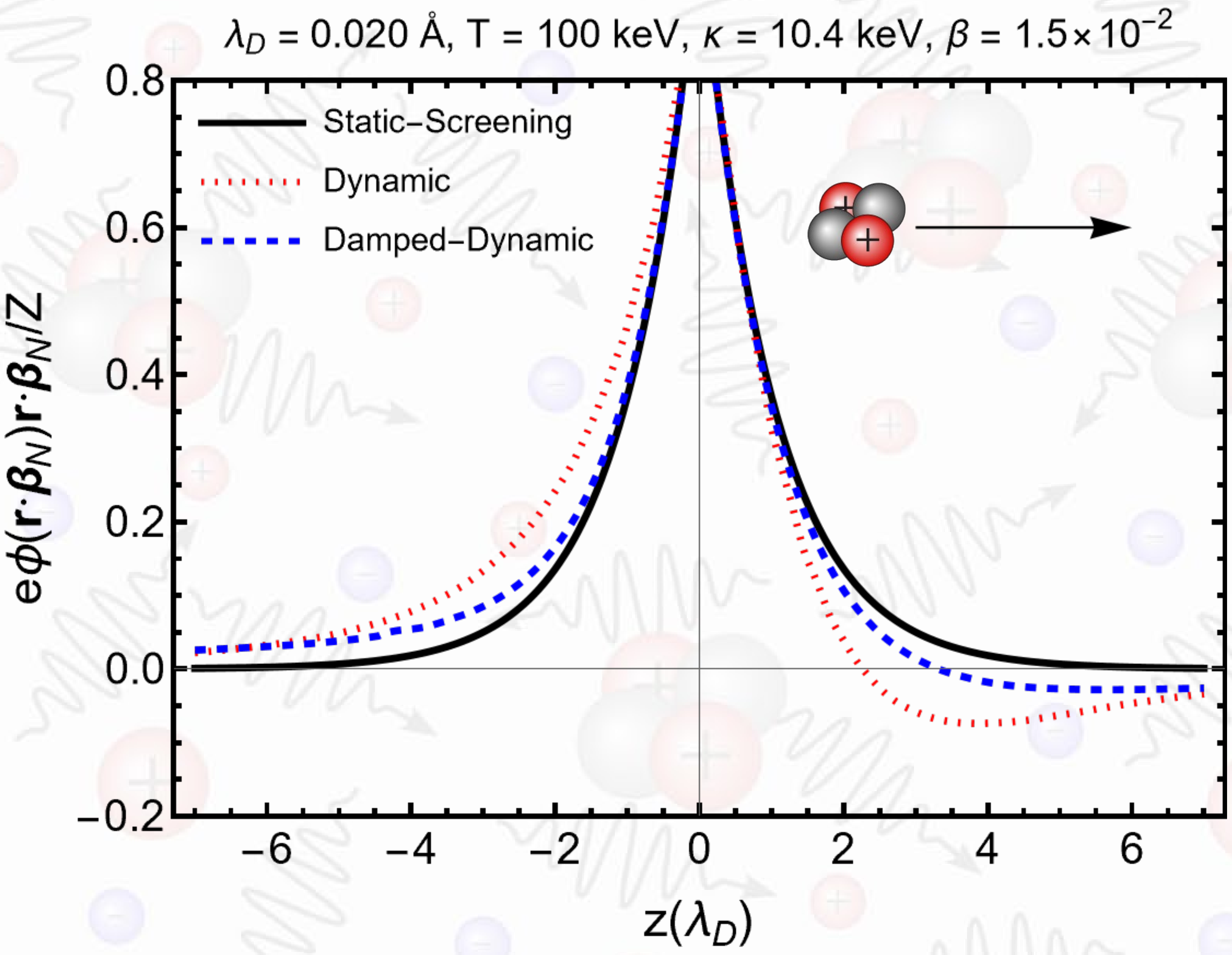
“Small distance behavior”

$r(t) = (\boldsymbol{x}-\boldsymbol{\beta}_{\mathrm{N}}t)$

ψ is the angle between $(\boldsymbol{x}-\boldsymbol{\beta}_N t)$ and $\boldsymbol{\beta}_N$

L. Stenflo, M. Y. Yu, and P. K. Shukla ”Shielding of a slow test charge in a collisional plasma” Phys. Fluids 16 (1973)

Screening Model Comparisons



Dynamic - numerical calculation predicts **asymmetry** in the potential leading to negative polarization charge in front and positive charge behind the moving nucleus.

Damped–Dynamic - **numerical** calculation predicts a screening effect that is **similar** but smaller because of damping.

Dynamic screening result adapted from E. Hwang et al. "Dynamical screening effects on big bang nucleosynthesis," JCAP 11 (2021)

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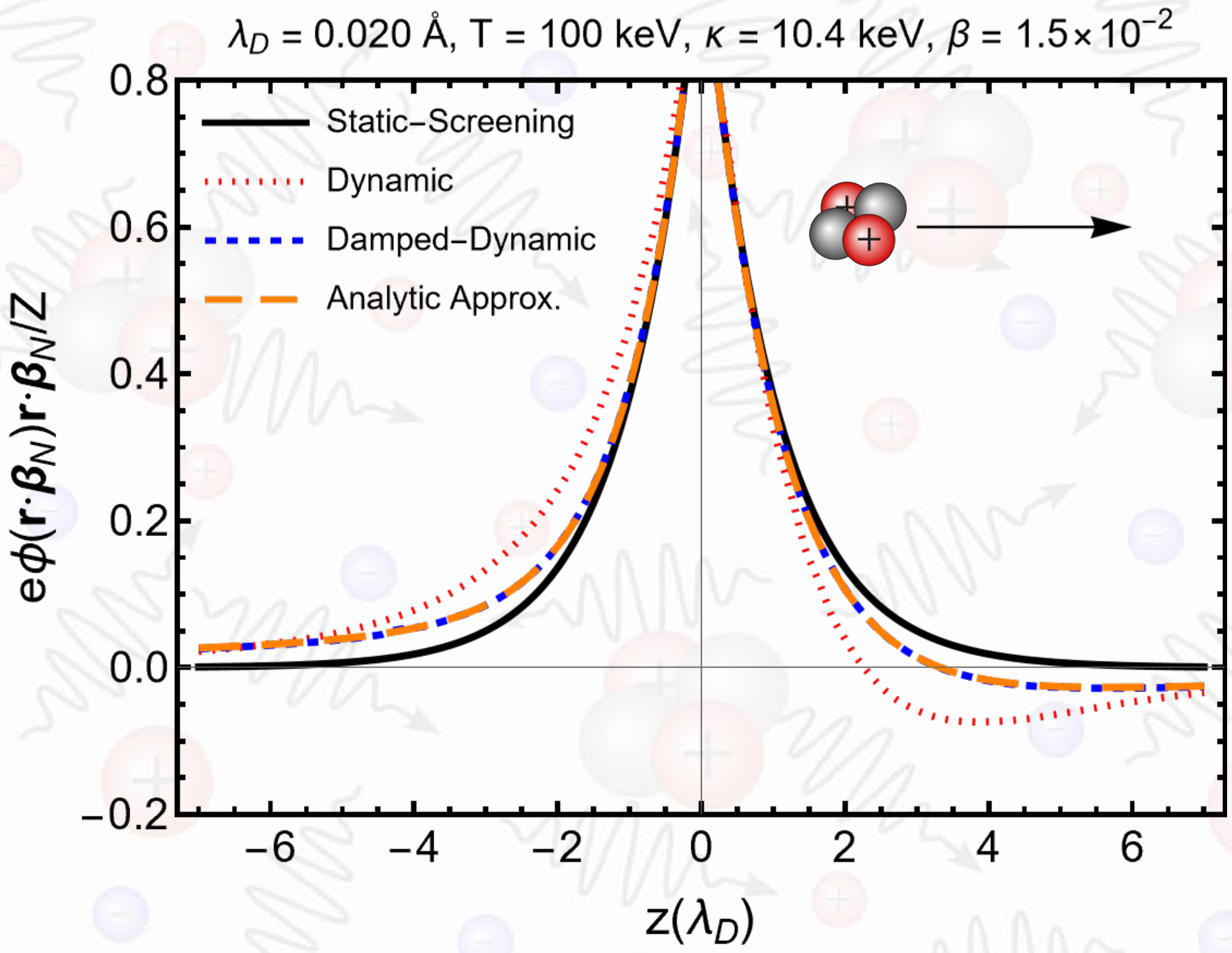

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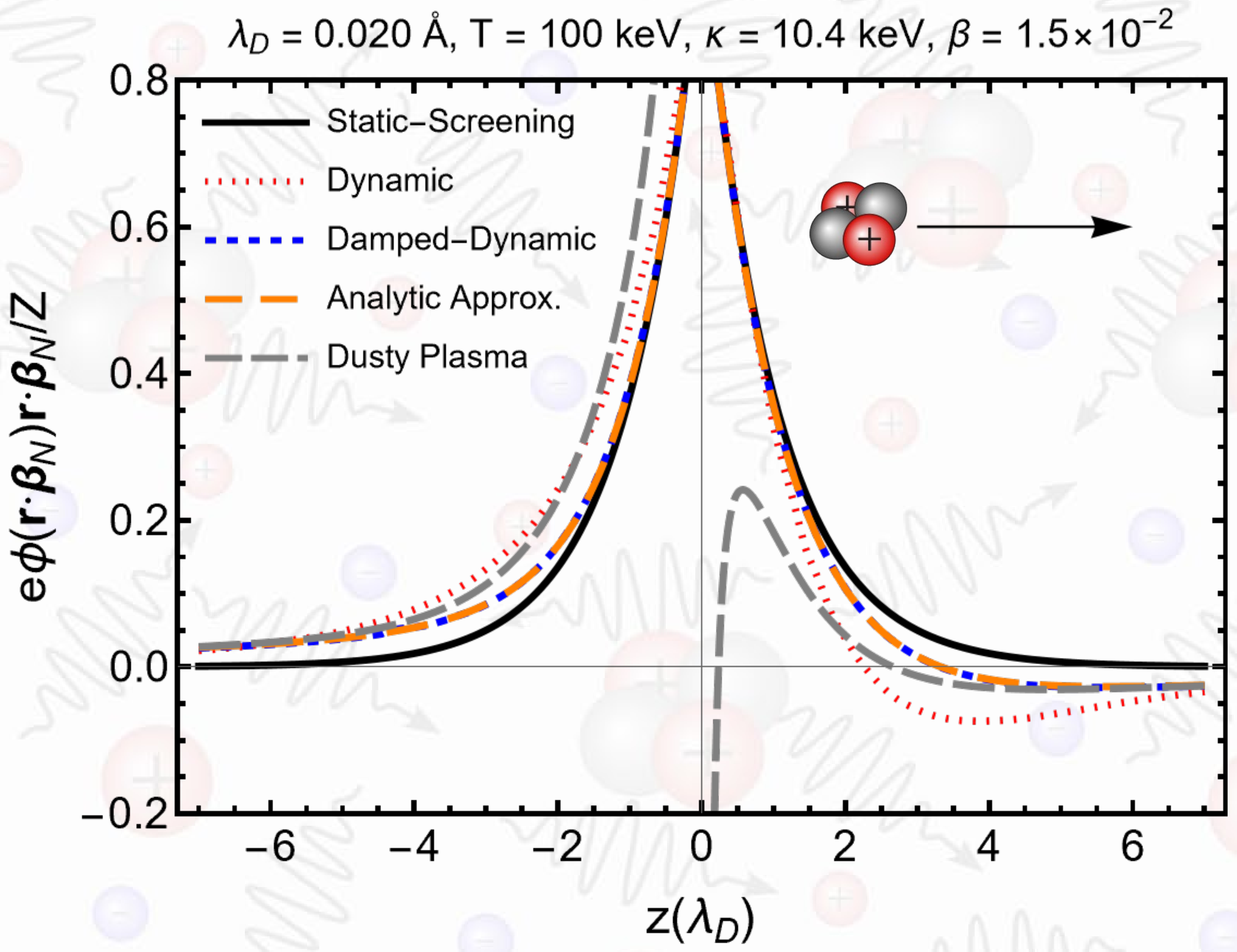

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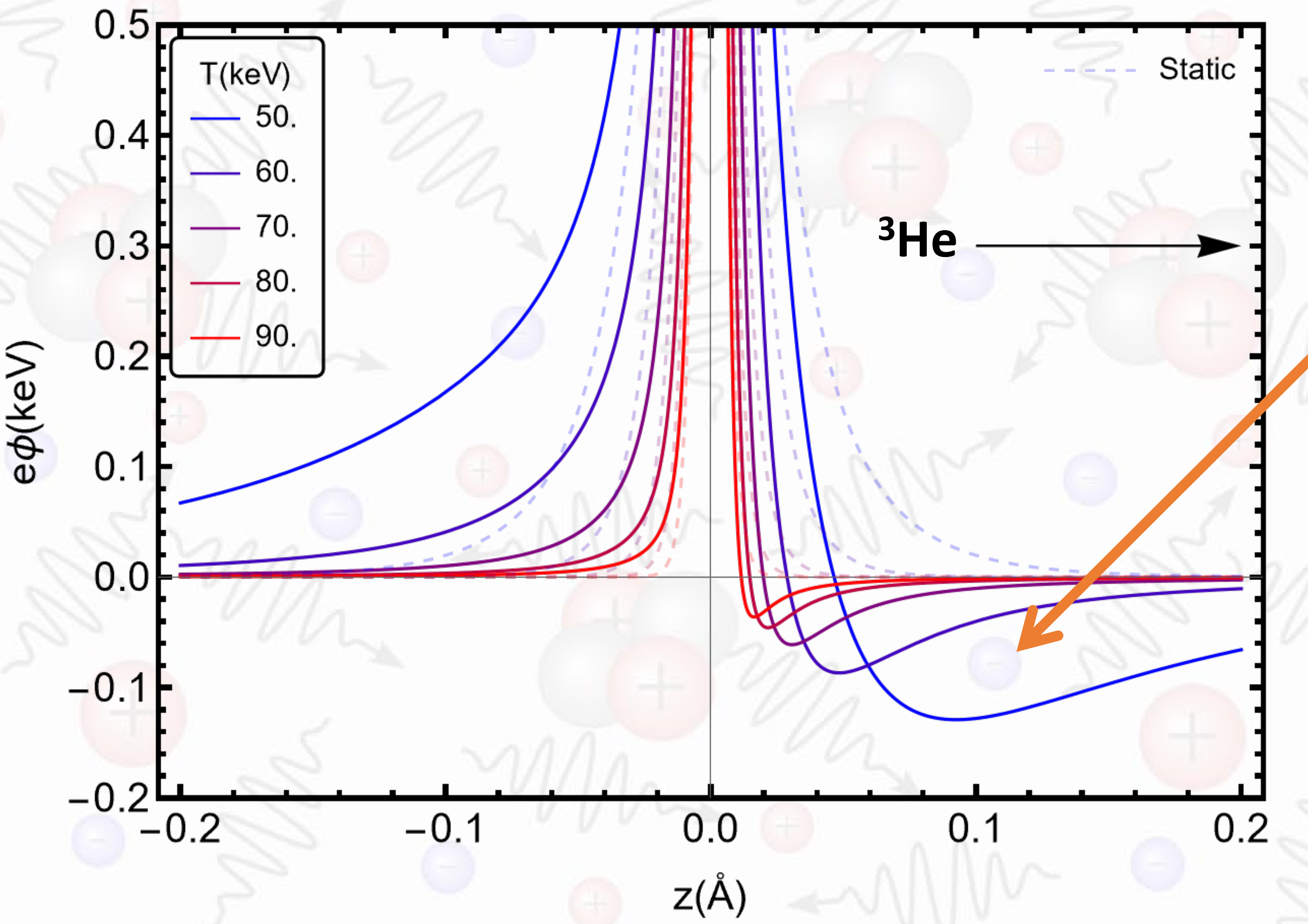
Analytic Approx. - our analytic approximation **matches** the numerical result for T=100 keV. (temperature used for comparison).

Dusty Plasma - $^1/_{r^2}$ analytic result from Stenflo et al. (1973) matches the **large** distance behavior.

Dynamic screening result adapted from E. Hwang et al. "Dynamical screening effects on big bang nucleosynthesis," JCAP 11 (2021)

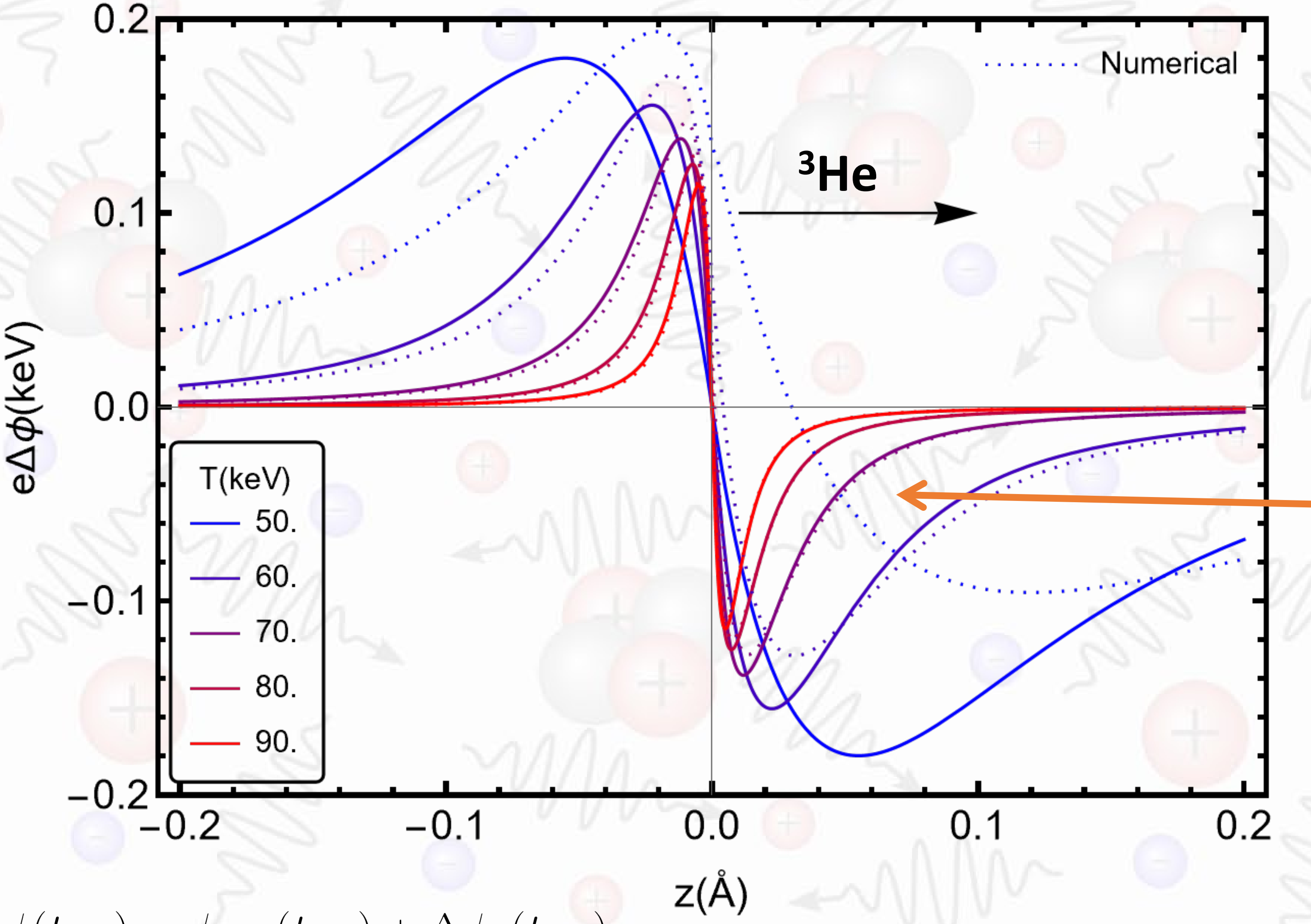
Damped-Dyanmic Screening – Temperature Variation



The size of screening polarization **increases** with decreasing temperature during BBN

Small **attractive** portion of the potential

Damped-Dyanmic Screening – Temperature Variation



The size of screening polarization **increases** with decreasing temperature during BBN

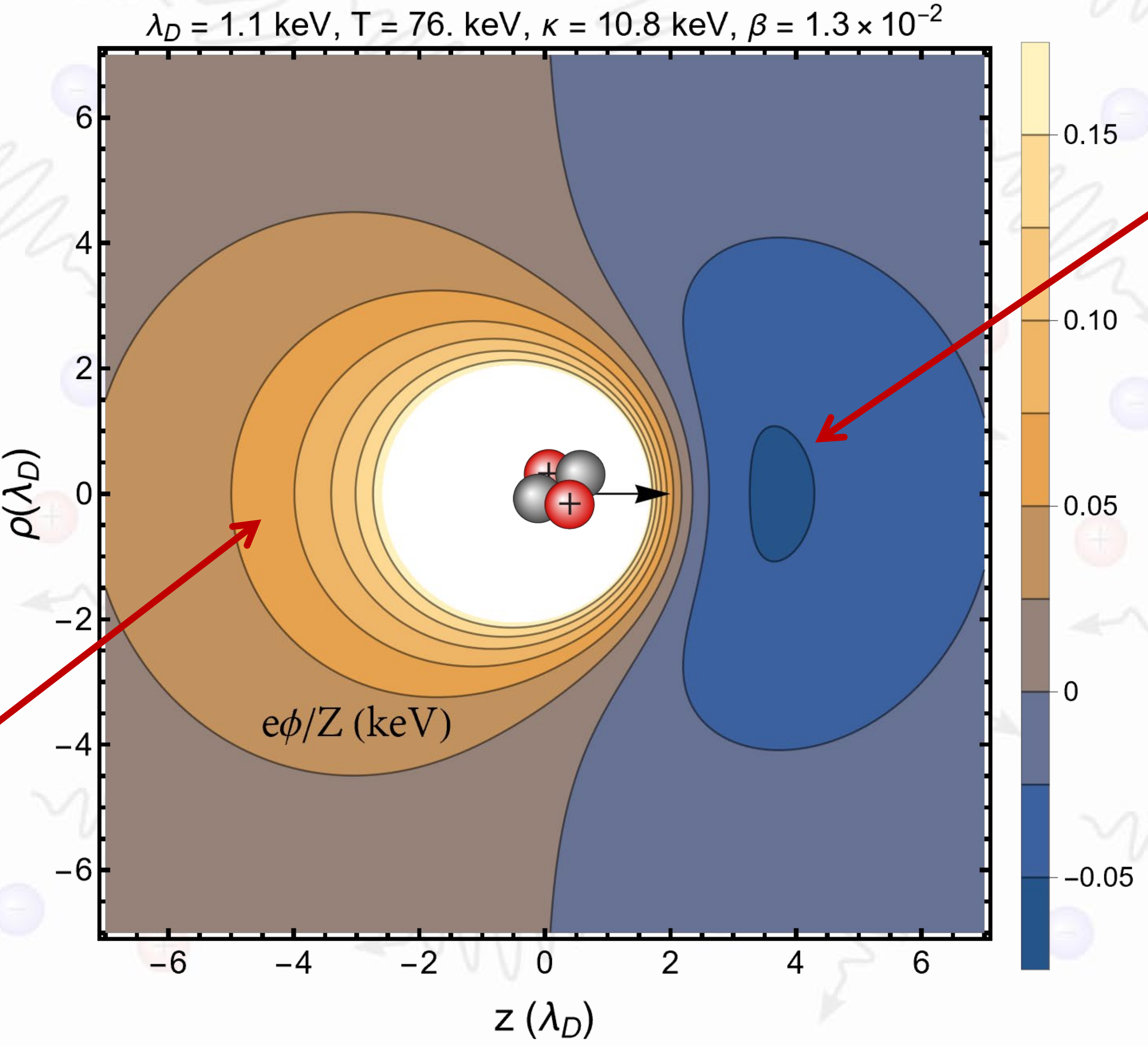
Small **attractive** portion of the potential

Analytic expression is **valid** late in BBN $T > 70$ keV.

As temperature **decreases** the dynamic contribution goes to zero.

$\phi(t, \boldsymbol{x}) = \phi_{\text{stat}}(t, \boldsymbol{x}) + \Delta\phi, (t, \boldsymbol{x})$

Transverse Potential dependence



Positive wake field

Negative charge build up in front

Outlook

- In summary, we derived an **analytic** expression for the **damped-dynamic** potential in the **BBN epoch**, a result which has previously been only calculated **numerically**.
- Our **damped-dynamic** potential predicts **similar** changes to **reaction rates** as found in B. Wang et al. (2021), But we **reserve** a full calculation of the change to reaction rates until we study the **strong field limit** of screening.
- We expect other results from **dusty plasma** physics will have direct application to the **BBN plasma**.
- Other applications and extensions
  - Screened Heavy quark potential in QGP.
  - Stellar and Laboratory fusion
  - Effect of primordial magentic field.

Thank You

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Dispersion Relation of the Plasma

Solve for the poles of the propagator in fourier space

$$\frac{1}{(k\cdot u)^2}((k\cdot u)^2 + \mu_0 \Pi_L(k))(k^2 + \mu_0 \Pi_T(k))^2 = 0$$

Focusing just on the longitudinal modes (related to potential screening)

$$\frac{1}{(k\cdot u)^2}((k\cdot u)^2 + \mu_0 \Pi_L(k)) = 1 + \frac{\Pi_L(\omega, \boldsymbol{k})}{\omega^2} = 0$$

$$1 - \frac{m_D^2}{(\omega + i\kappa)^2} \frac{1}{1 - \frac{i\kappa}{\omega + i\kappa}\left(1 + \frac{T|\boldsymbol{k}|^2}{m(\omega+i\kappa)^2}\right)} = 0$$

$$\omega' = \omega + i\kappa$$

$$\frac{1}{\omega'^3 - i\kappa\omega'^2 + \frac{i\kappa T|\boldsymbol{k}|^2}{m}}\left(\omega'^3 - i\kappa\omega'^2 - m_D^2\omega' + \frac{i\kappa T|\boldsymbol{k}|^2}{m}\right) = 0$$

Dispersion Relation of the Plasma

Solve for the poles of the propagator in fourier space

$$\frac{1}{\omega'^3 - i\kappa\omega'^2 + \frac{i\kappa T|\boldsymbol{k}|^2}{m}}\left(\omega'^3 - i\kappa\omega'^2 - m_D^2\omega' + \frac{i\kappa T|\boldsymbol{k}|^2}{m}\right) = 0 \qquad \omega' = \omega + i\kappa$$

General solutions to a cubic equation are given by

$$\omega_n(|\mathbf{k}|) = \frac{1}{3}\left(i\kappa - \xi^n C - \frac{\Delta_0}{\xi^n C}\right), \qquad n \in \{0, 1, 2\}$$

$$\xi = \frac{i\sqrt{3} - 1}{2} \qquad \Delta_1 = 2i\kappa^3 - 9i\kappa m_D^2 + 27\frac{i\kappa T|\boldsymbol{k}|^2}{m}$$

$$C = \sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}} \qquad \Delta_0 = -\kappa^2 + 3m_D^2$$

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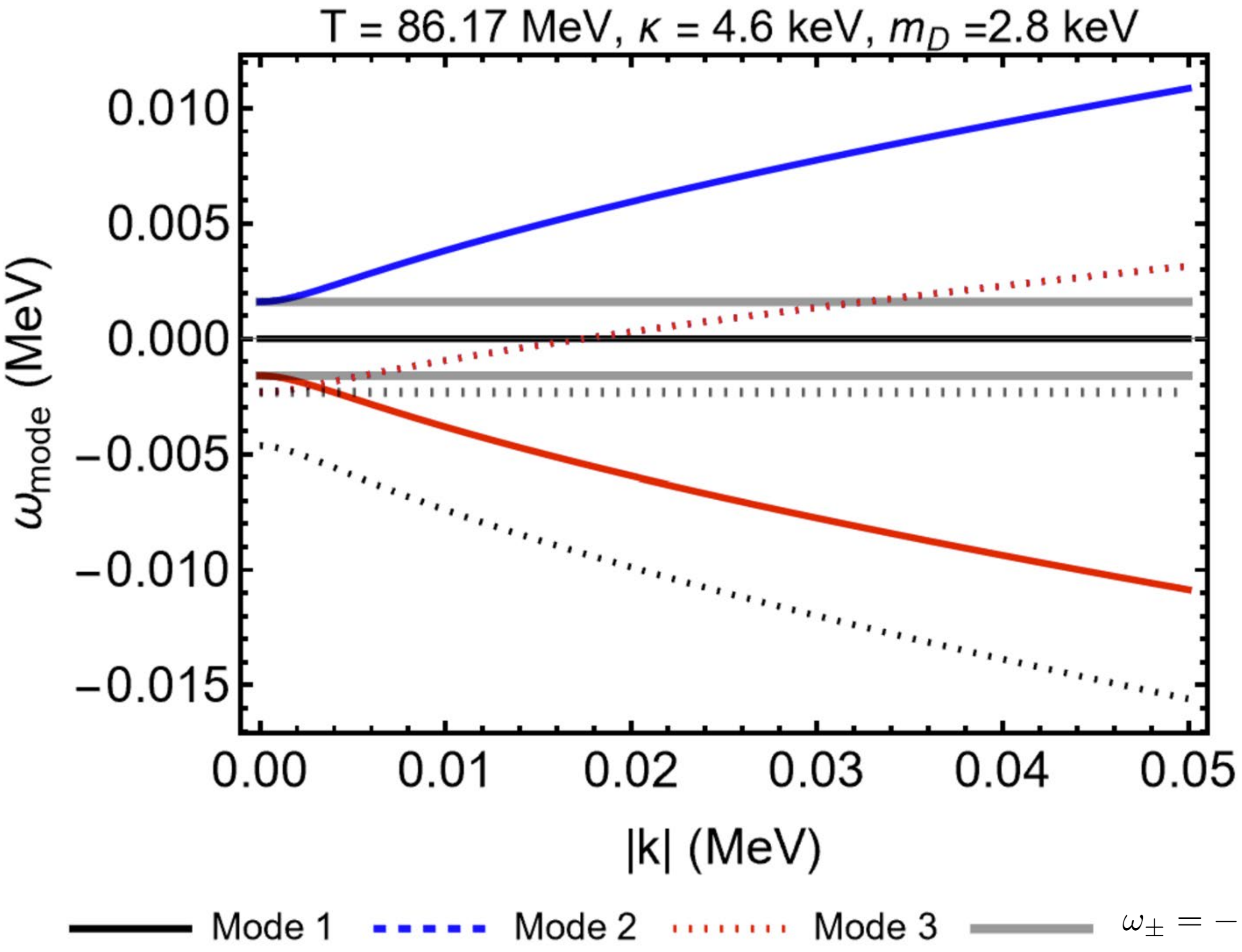
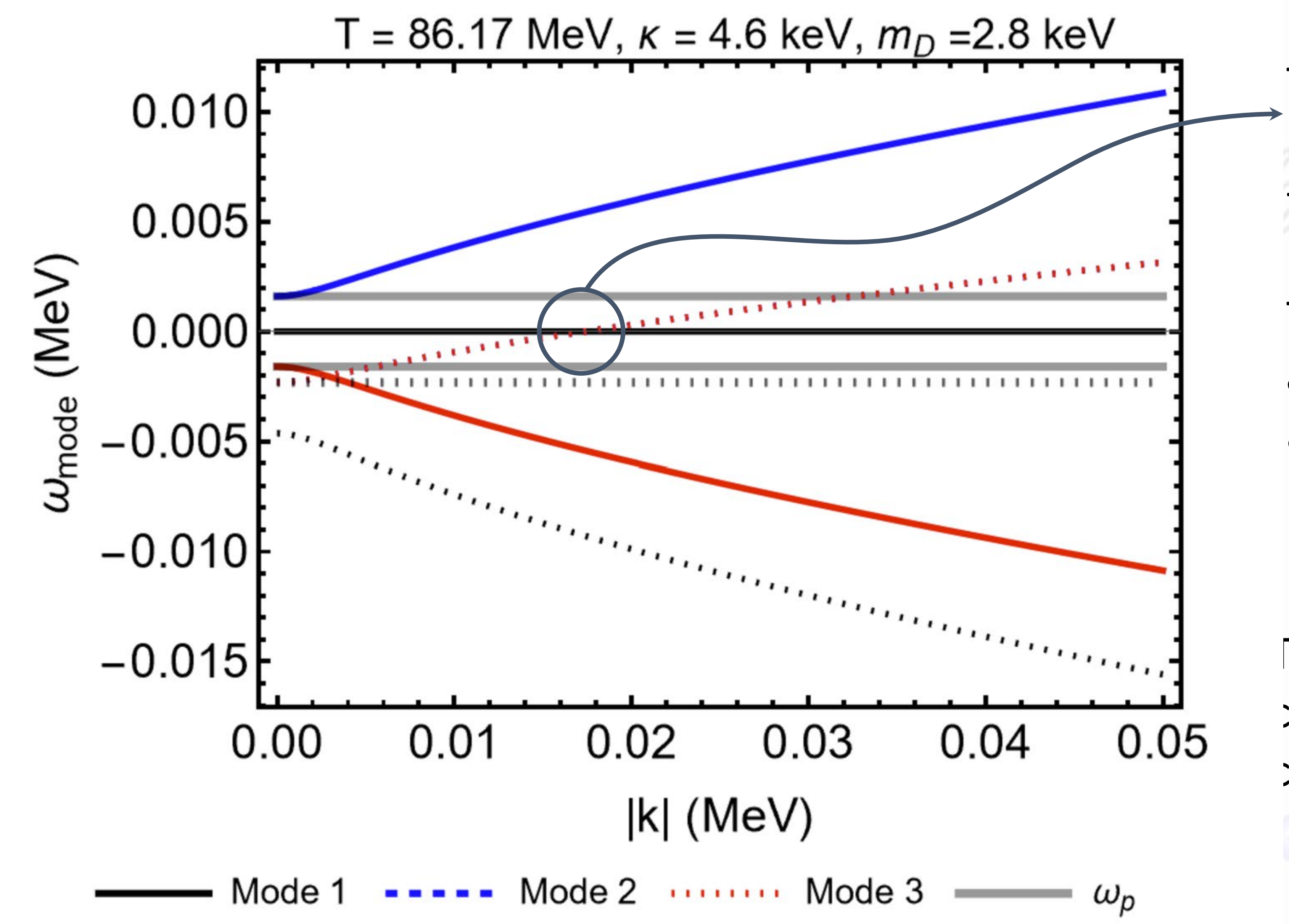
Dotted lines are imaginary parts and solid lines are the real parts



$$\omega_\pm = -\frac{i\kappa}{2} \pm \sqrt{\omega_p^2 - \frac{\kappa^2}{4}}$$

Dispersion Relation of the Plasma

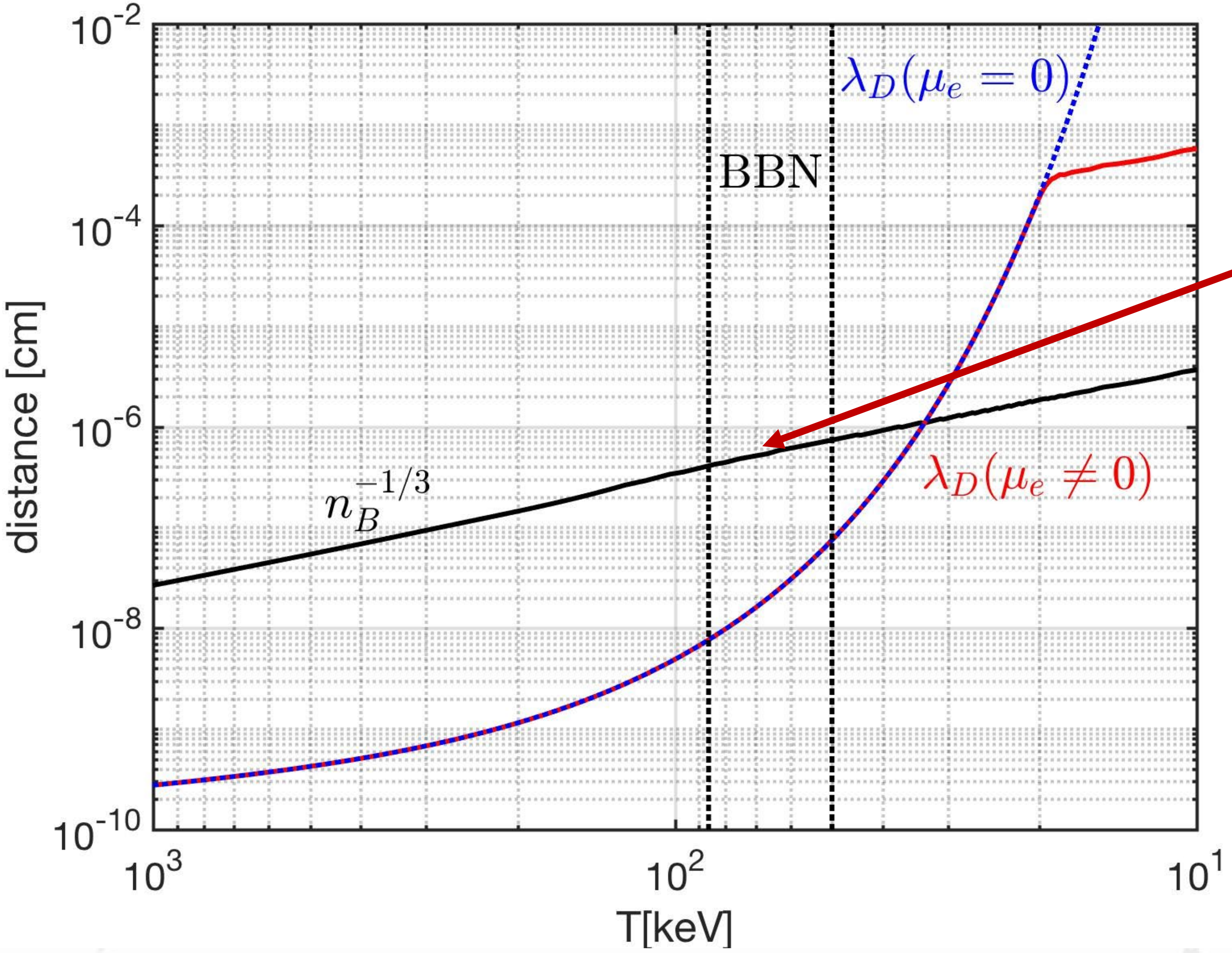


The red mode gains a position imaginary part at finite **k**

This either indicates an **acausality** in the system or an **instability** in the plasma

The value of **k** here corresponds to waves of characteristic size

Baryon Separation vs Debye screening



Baryons are to dispersed in comparison to the size of the Debye sphere to play a role in screening