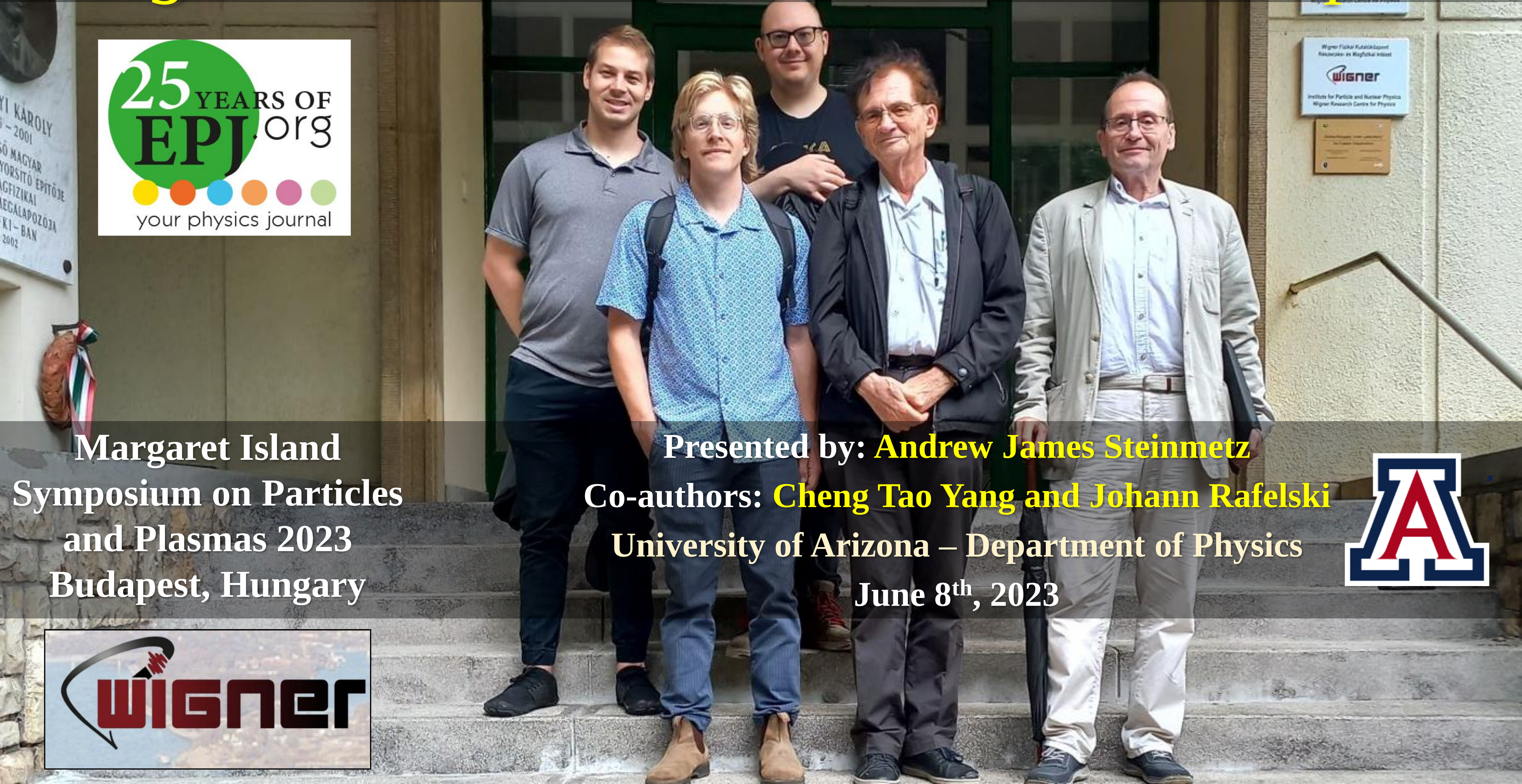
Magnetism in the Cosmic e^+e^- Plasma Epoch



Margaret Island Symposium on Particles and Plasmas 2023
Budapest, Hungary

Presented by: Andrew James Steinmetz
Co-authors: Cheng Tao Yang and Johann Rafelski
University of Arizona – Department of Physics

June 8th, 2023

Our colleagues and recent work


Cheng Tao Yang


Dr. Jeremiah Birrell
(photo at CERN cafeteria)

UA Tucson, AZ Group

Paramagentism in Primordial Universe

A Preprint

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June 5, 2023

Abstract

We explore in the temperature range 200 keV $> T >$ 20 keV the primordial Universe magnetization driven by spin paramagnetism of the ultra dense electron-positron plasma. The e^+e^- pair density was more than 10^8 greater than the baryon density. Pairs fully disappear only below $T = 20$ keV.

Keywords early universe cosmology · magnetization · electron-positron plasma · intergalactic magnetic fields

(To be submitted to arXiv in coming days!)

A Short Survey of Matter-Antimatter Evolution in the Primordial Universe

A Preprint

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May 17, 2023

Abstract

We offer a survey of the matter-antimatter evolution within the primordial Universe. While the origin of the tiny matter-antimatter asymmetry has remained one of the big questions in modern cosmology, antimatter itself has played a large role for much of the Universe's early history. In our study of the evolution of the Universe we adopt the position of the standard model Λ-CDM Universe implementing the known baryonic asymmetry. We present the composition of the Universe across its temperature history while emphasizing the epochs where antimatter content is essential to our understanding. Special topics we address include the heavy quarks in quark-gluon plasma (QGP), the creation of matter from QGP, the free-streaming of the neutrinos, the vanishing of the muons, the magnetism in the electron-positron cosmos, and a better understanding of the environment of the Big Bang Nucleosynthesis (BBN) producing the light elements. We suggest but do not explore further that the methods used in exploring the early Universe may also provide new insights in the study of exotic stellar cores, magnetars, as well as gamma-ray burst (GRB) events. We describe future investigations required in pushing known physics to its extremes in the unique laboratory of the matter-antimatter early Universe.

Keywords Particles, Plasmas and Electromagnetic Fields in Cosmology · Quarks to Cosmos

arXiv:2305.09055
(Submitted to MDPI Universe for Remo Ruffini Festschrift)

Cosmic relic magnetism I – Pre-CMB signal?

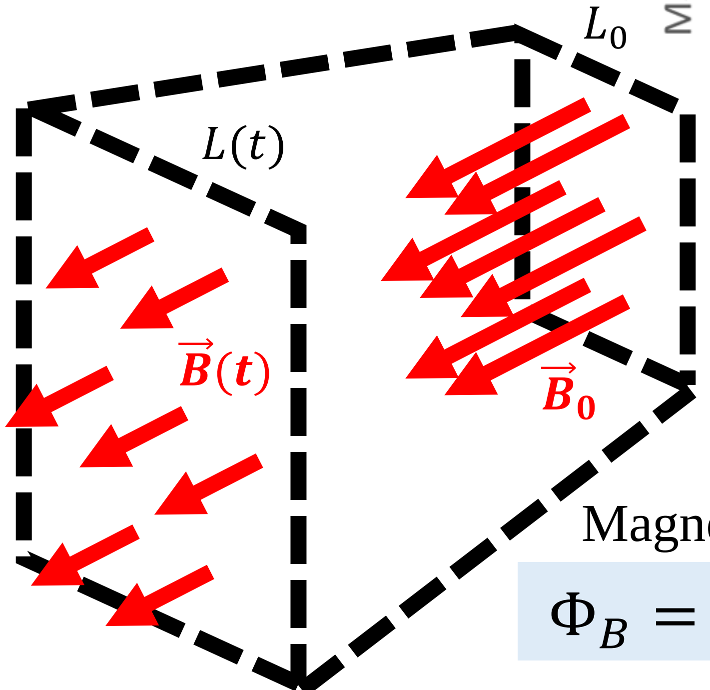
FLRW scale factor

Magnetic flux is conserved over a comoving surface

$$L(t)^2 = L_0^2\,\frac{a(t)^2}{a(t_0)^2}$$

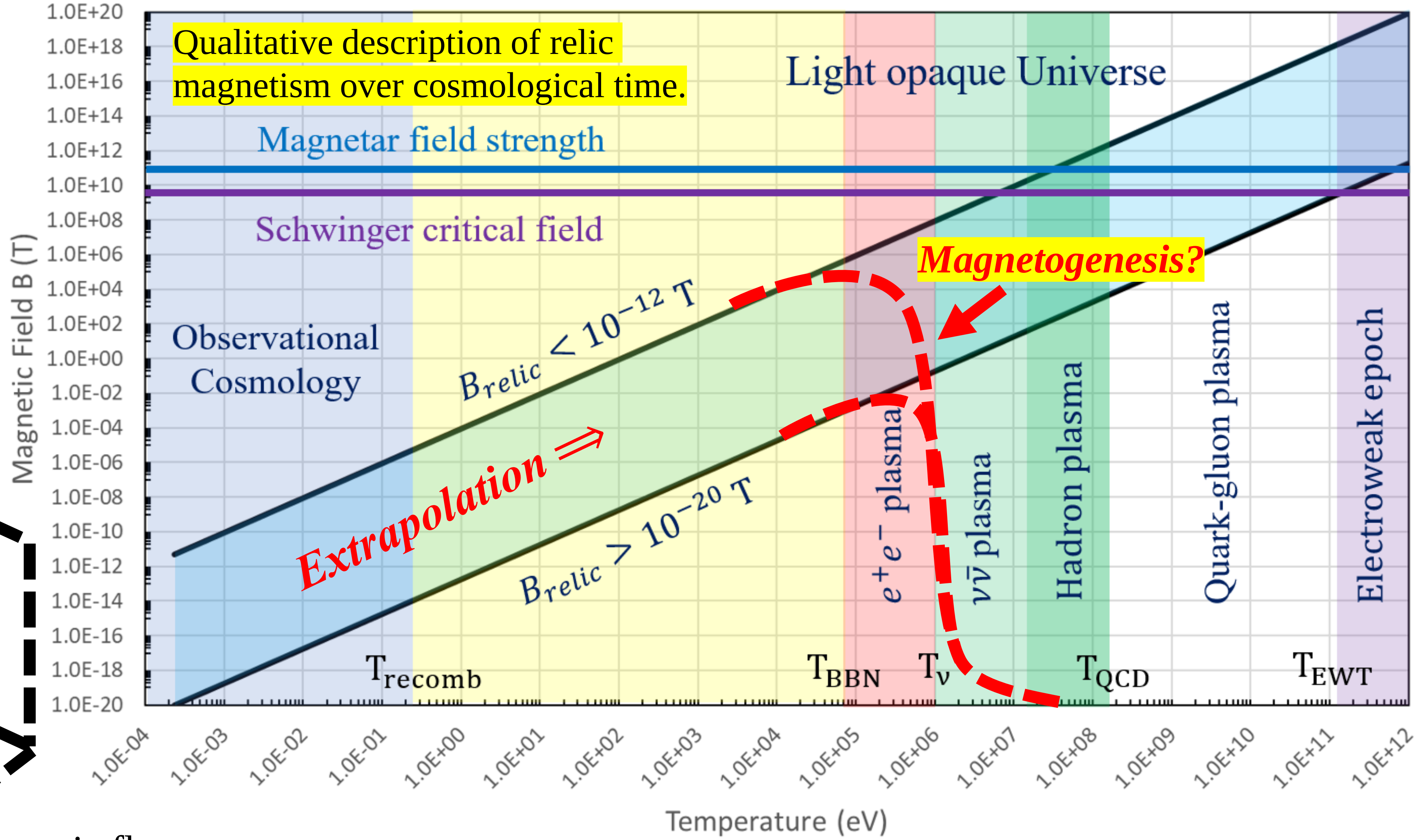
The magnetic field then dilutes over time as

$$B(t) = B_0\,\frac{a(t_0)^2}{a(t)^2}$$



Magnetic flux:

$$\Phi_B = B(t)L^2(t) = B_0 L_0^2$$



The upper and lower bounds on inter-galactic magnetic fields (IGMF) coherent over Mpc scales is shown.

Cosmic relic magnetism II – Pre-CMB signal?

FLRW scale factor

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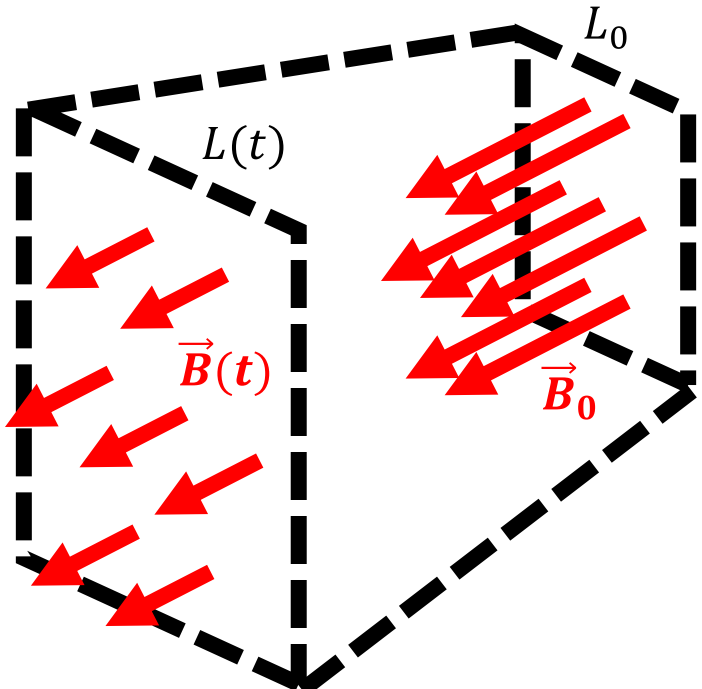


The temperature also decreases over cosmic expansion as

$$T(t) = T_0 \frac{a(t_0)^2}{a(t)^2}$$

Magnetic fields present in cosmic voids would be “uncontaminated” primordial relics.

This lets us define a conserved cosmic “magnetic scale” for charged particles

$$b_0 \equiv \frac{e\mathcal{B}(t)}{T(t)^2} = \frac{e\mathcal{B}_0}{T_0^2} = \text{const}$$

In natural units:

$$(\hbar = c = k_B = 1)$$

Contemporary B-fields

$$10^{-12}\ \text{T} > \mathcal{B}_0 > 10^{-20}\ \text{T}$$

Upper: Faraday rotation of radio AGN
Lower: Spectra of “blazar” AGN

Contemporary temperature

$$T_0 = 2.7\ \text{K}\ (2.3 \times 10^{-4}\ \text{eV})$$

As determined from the Cosmic Microwave Background (CMB)

$$10^{-3} > b_0 > 10^{-11}$$

Thus b_0 controls the strength of the magnetization of the primordial electron-positron plasma.

Note: B-field grows with temperature.

Why electron-positron epoch and cosmic magnetism? Electron-positron pair abundance

Prior to electron-positron annihilation, there was almost a 450 million e^+e^- pairs per baryon

$$\frac{n_{e^\pm}}{n_B} = 4.47 \times 10^8$$

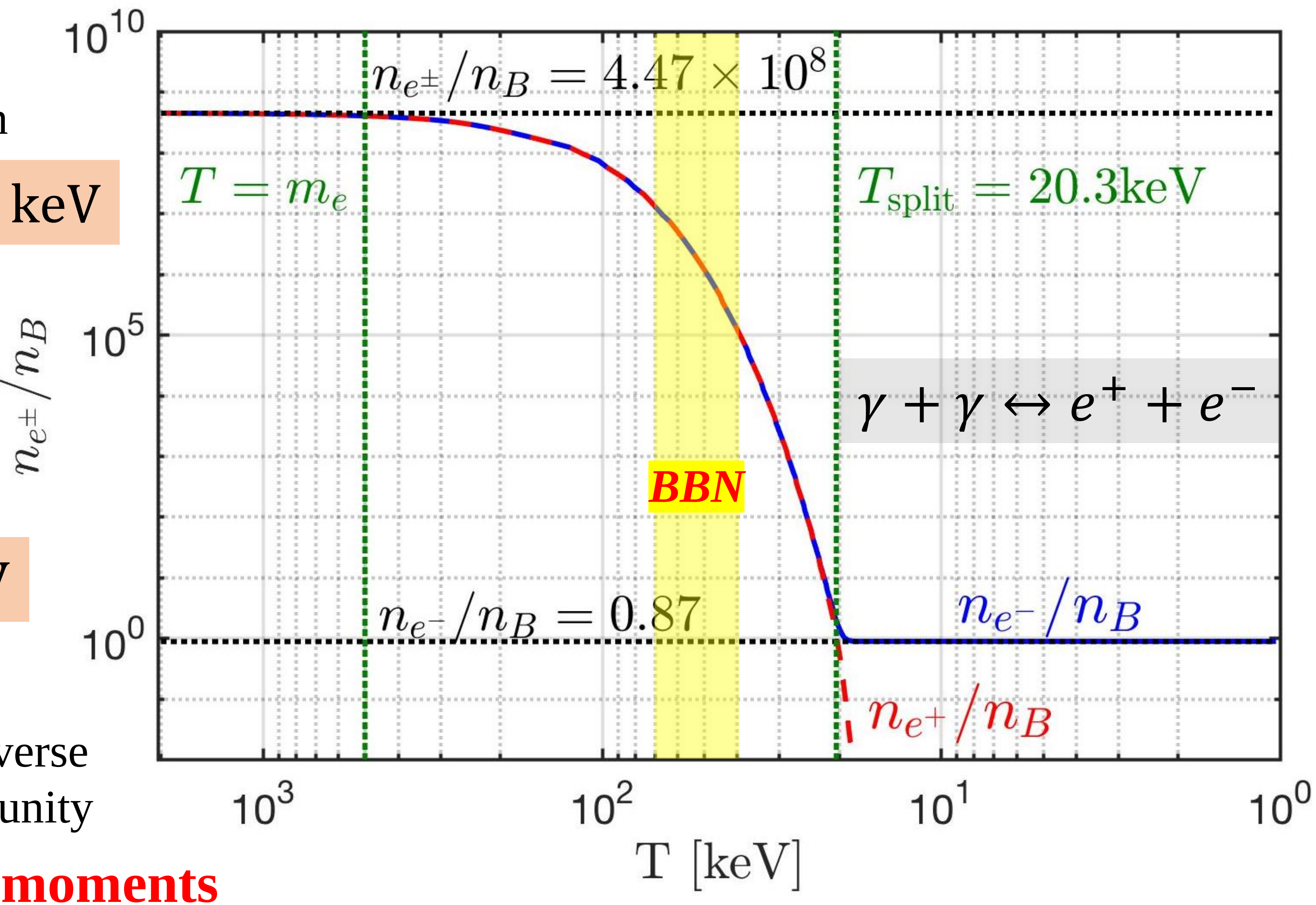
$T > 511$ keV

This was the density prior to Big Bang nucleosynthesis (BBN) (occurring in range $T = 70 - 40$ keV). After annihilation this, the universe was left with

$$\frac{n_{e^-}}{n_B} = 0.87$$

$T < 20.3$ keV

determined by the **charge neutrality** of the universe and **baryon asymmetry**. Slight deviation from unity due to bound neutrons.



g-factor

Ratio of magnetic moments

$$\frac{|\vec{\mu}_{e^\pm}|}{|\vec{\mu}_p|} = \frac{g_e}{g_p}\frac{m_p}{m_e} \approx 633$$

We propose that this dense matter-antimatter plasma was highly magnetized (and may be responsible for cosmic magnetism).

First principles derivation of high temperature ratio – I

Using the **charge neutrality** of the universe, we can write the baryon density as a function of the **charged lepton asymmetry**.

$$(n_{e^-} - n_{e^+}) = n_p \rightarrow \left(\frac{n_p}{n_B}\right) n_B \qquad X_p \equiv \left(\frac{n_p}{n_B}\right)$$

The charged lepton asymmetry is written as

$$\ln \mathcal{Z} = \sum_{states} \ln\left[1 + \lambda \exp -\frac{E}{T}\right]$$

Partition Function

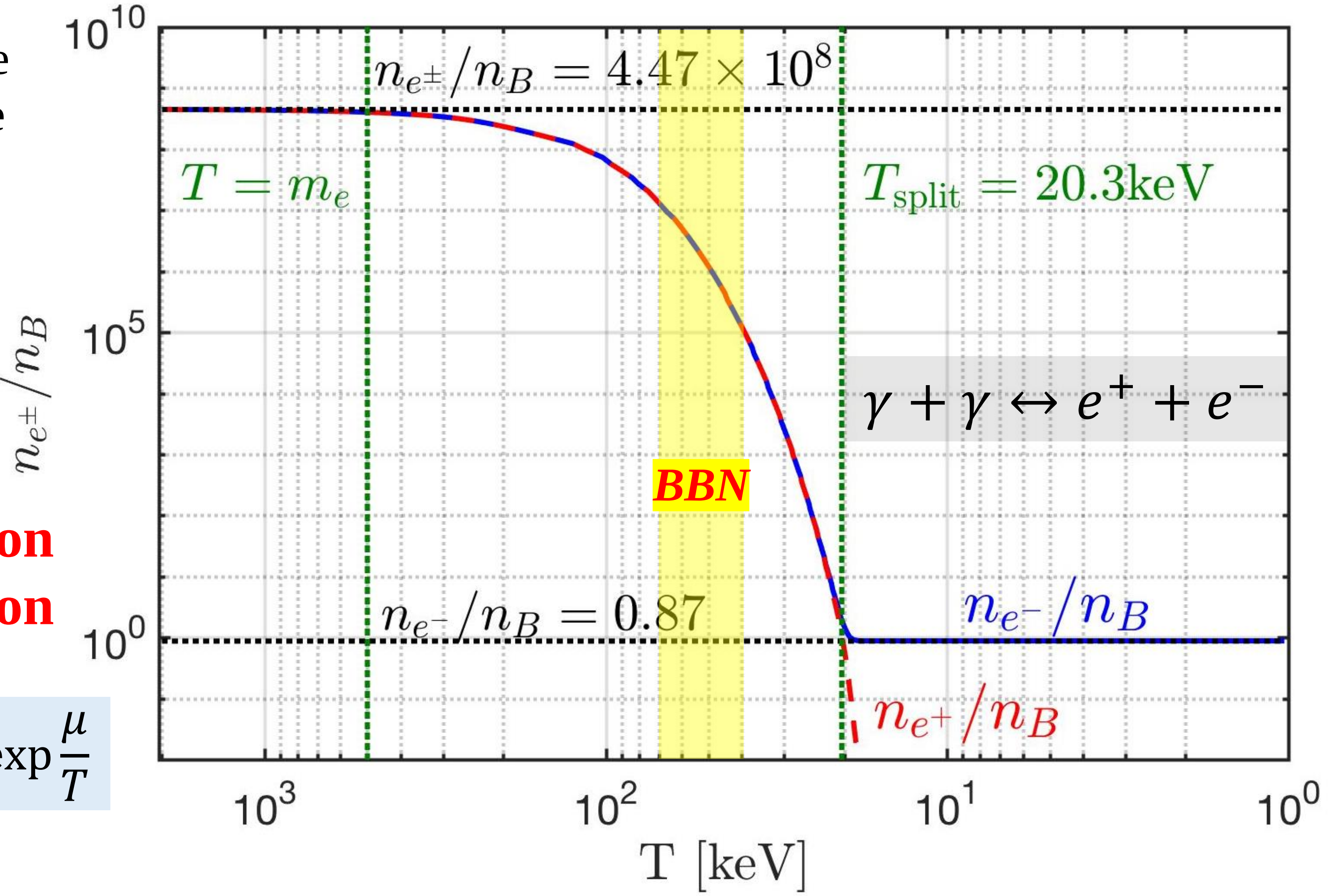
$$(n_{e^-} - n_{e^+}) = \frac{1}{V}\lambda \frac{\partial \ln \mathcal{Z}_{e^+e^-}}{\partial \lambda}$$

Fugacity

$$\lambda = \exp\frac{\mu}{T}$$

The working equation for lepton-to-baryon ratio is then

$$\frac{n_{e^\pm}}{n_B} = X_p \frac{n_{e^\pm}}{(n_{e^-} - n_{e^+})}$$



Contrary to the traditional case, magnetization will increase at higher temperatures. How? – Stay tuned for next slides!

(a) Density grows far faster than temperature.

(b) B-field grows with temperature.

First principles derivation of high temperature ratio – II

The **fugacity** can be determined by **entropy conservation** and **baryon asymmetry**

Yang, Cheng Tao, and Johann Rafelski. "Cosmological strangeness abundance." *Physics Letters B* 827 (2022): 136944.

$$\boldsymbol{n_B} = \left(\frac{n_B}{s_{\gamma,\nu,e^+e^-}}\right) \boldsymbol{s_{\gamma,\nu,e^+e^-}}$$

$$\left(\frac{n_B}{s_{\gamma,\nu,e^+e^-}}\right) = \left(\frac{n_B}{s}\right)_{t_0} = (0.856 \pm 0.008) \times 10^{-10}$$

$$\boldsymbol{s_{\gamma,\nu,e^+e^-}} = \frac{2\pi^2}{45} T_\gamma^3 \left(g_\gamma + g_\nu \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^3\right) + \frac{(\rho_{e^+e^-} + P_{e^+e^-})}{T_\gamma} - \frac{\mu}{T_\gamma}(n_{e^-} - n_{e^+})$$

γ, ν entropy **e^+e^- entropy**

Putting it all together, our working equation is…

$$\frac{n_{e^\pm}}{n_B} = X_p \left[\frac{\partial \ln \mathcal{Z}_{e^\pm}}{\partial \boldsymbol{\lambda}}\right] \left[\frac{\partial \ln \mathcal{Z}_{e^+e^-}}{\partial \boldsymbol{\lambda}}\right]^{-1}$$

Degrees of freedom:

$$g_\gamma = 2 \qquad g_\nu = 2 \times 3 = 6$$

$$\boldsymbol{\lambda} = \exp \frac{\boldsymbol{\mu}}{\boldsymbol{T}}$$

The **fugacity** and thus chemical potential is numerically evaluated.

Thanks to Cheng Tao teaching me this derivation!

$$\frac{1}{V}\boldsymbol{\lambda}\frac{\partial \ln \mathcal{Z}_{e^+e^-}}{\partial \boldsymbol{\lambda}} = X_p \left(\frac{n_B}{s}\right)_{t_0} \left(\frac{2\pi^2}{45}(T_\gamma)^3 \left(g_\gamma + g_\nu \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^3\right) + \frac{(\rho_{e^+e^-} + P_{e^+e^-})}{T_\gamma} - \frac{\boldsymbol{\mu}}{\boldsymbol{T_\gamma}}\frac{1}{V}\boldsymbol{\lambda}\frac{\partial \ln \mathcal{Z}_{e^+e^-}}{\partial \boldsymbol{\lambda}}\right)$$

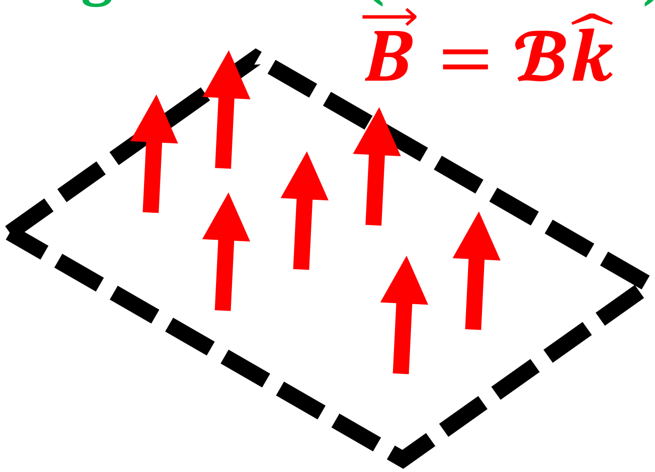
Statistical properties of the electron-positron gas – I

Let us look at the magnetized fermion partition function to describe the e^+e^- gas.

$$\ln \mathcal{Z}_{e^+e^-} = \frac{2e\mathcal{B}V}{(2\pi)^2}\sum_{\sigma}^{\pm}\sum_{s}^{\pm}\sum_{n=0}^{\infty}\int_0^{\infty} dp_z \left[\ln\left[1+\lambda_\sigma \xi_s \exp\left(-\frac{E_n^s}{T}\right)\right]\right]$$

Quantum energy eigenvalues (next slide)

Fugacity **"Spin" Fugacity**



We sum over particles and antiparticles (σ), spin polarizations (s), and Landau orbital levels (n). In principle we could include other particles if needed: baryons, neutrinos, etc…

We also introduce the following two kinds of fugacity

a. **Chemical Fugacity:** $\lambda_\sigma = \exp\frac{\mu_\sigma}{T}$ → $\mu \equiv \mu_{e^-} = -\mu_{e^+}$

b. **Spin Fugacity:** $\xi_s = \exp\frac{\eta_s}{T}$ → $\xi \equiv \xi_+ = -\xi_-$

Generalized Fugacity → $\Upsilon_\sigma^s = \lambda_\sigma \xi_s = \exp\frac{\lambda_\sigma + \xi_s}{T}$

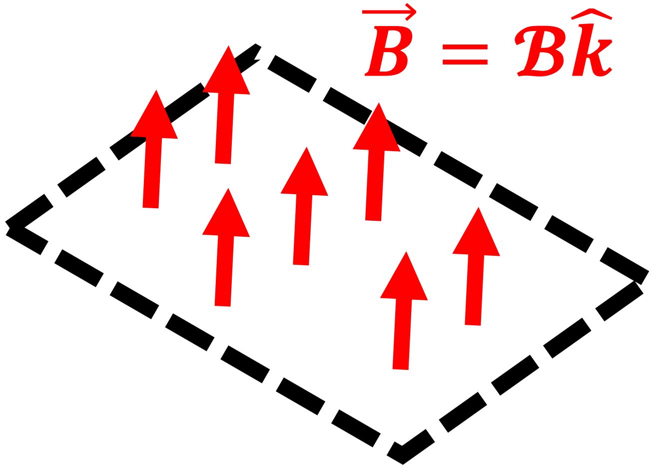
The "spin" fugacity represents an **imbalance of spins** within the gas and is constrained by conservation of angular momentum.

A value of $\xi \neq 1$ indicates angular momentum in other species, orbital motion, or a locally polarized domain.

We will return to ξ_s at the end!

Statistical properties of the electron-positron gas – II

$$\ln \mathcal{Z}_{e^+e^-} = \frac{2q\mathcal{B}V}{(2\pi)^2} \sum_{\sigma}^{\pm} \sum_{s}^{\pm} \sum_{n=0}^{\infty} \int_0^{\infty} dp_z \left[\ln \left[1 + \lambda_\sigma \exp\left(-\frac{E_n^s}{T} \right) \right] \right]$$



The Klein-Gordon-Pauli (KGP) **energy eigenvalues** of the magnetized fermion are given by

$$E_n^{\pm}(p_z, \mathcal{B}) = \sqrt{m_e^2 + p_z^2 + e\mathcal{B}\left(2n + 1 \mp \frac{g}{2}\right)}$$

We can rearrange into a more convenient form

$$E_n^{\pm}(p_z, \mathcal{B}) = m_{\pm}^2 \sqrt{1 + \frac{p_z^2}{m_{\pm}^2} + \frac{2e\mathcal{B}}{m_{\pm}^2}}$$

$$m_{\pm}^2 = m_e^2 + e\mathcal{B}\left(1 \mp \frac{g}{2}\right)$$

This effective "polarized mass" bundles the spin and the Landau ground state which is **ultimately responsible for the magnetization**.

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THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Magnetic dipole moment in relativistic quantum mechanics

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Euler-Maclaurin integration of the partition function – I

for $|x| < 1$

$$\ln \mathcal{Z}_{e^+e^-} = \frac{2q\mathcal{B}V}{(2\pi)^2} \sum_{\sigma}^{\pm} \sum_{s}^{\pm} \sum_{n=0}^{\infty} \int_0^{\infty} dp_z \left[\ln \left[1 + \lambda_\sigma \exp\left(-\frac{E_n^s}{T} \right) \right] \right]$$

$$\ln(1+x) = \sum_{k}^{\infty} (-1)^{k+1} \frac{x^k}{k}$$

Log replacement

We can replace the sum over Landau orbitals with an integral using the Euler-Maclaurin formula

Remainder Error

$$\sum_{n=a}^{b} f(n) = \int_a^b f(n)\mathrm{dn} + \frac{f(b)+f(a)}{2} + \sum_{j=1}^{\infty} \frac{B_{2j}}{(2j)!} \left[f^{(2j-1)}(b) - f^{(2j-1)}(a) \right] + R[a,b,j]$$

Note that B_{2j} are Bernoulli numbers. In general, $R[a,b,j]$ is nonzero. We do not return to its size here.
We obtain a partition function ($j = 1$) with the form

$$\ln \mathcal{Z}_{e^+e^-} = \ln \mathcal{Z}_{free} + \ln \mathcal{Z}_B + \ln \mathcal{Z}_R$$

Euler-Maclaurin integration of the partition function – II

More explicitly, our partition function (with $\ln \mathcal{Z}_R$ truncated) has the structure

Bessel functions of the second kind

$$\ln \mathcal{Z}_{free} = \frac{T^3 V}{2\pi^2} \sum_s^{\pm} \sum_k^{\infty} \frac{(-1)^{k+1}}{k^4} \left[2\cosh\frac{k\mu}{T}\right] k^2 x_s^2 K_2(x_s)$$

$$\ln \mathcal{Z}_B = \frac{T^3 V}{2\pi^2} \sum_s^{\pm} \sum_k^{\infty} \frac{(-1)^{k+1}}{k^2} \left[2\cosh\frac{k\mu}{T}\right] \left[\frac{k x_s \boldsymbol{b_0}}{2} K_1(k x_s) + \frac{k^2 \boldsymbol{b_0^2}}{12} K_0(k x_s)\right]$$

The first portion is a “free” Fermi gas partition function, however as it depends on $x_{\pm}$, the spin magnetic response still manifests. It turns out this will be the dominant term in most phenomenon.

$$x_{\pm} \equiv \frac{m_{\pm}}{T} = \sqrt{\frac{m_e^2}{T^2} + 2\boldsymbol{b_0}\left(1 \mp \frac{g}{2}\right)}$$

We write a combined form in Boltzmann approximation ($k = 1; T > m_e$)

$$\ln \mathcal{Z}_{e^+e^-} \simeq \frac{T^3 V}{2\pi^2} \sum_s^{\pm} \left[2\cosh\frac{\mu}{T}\right] \left[x_s^2 K_2(x_s) + \frac{\boldsymbol{b_0}}{2} x_s K_1(x_s) + \frac{\boldsymbol{b_0^2}}{12} K_0(x_s)\right]$$

Evaluation of the chemical potential

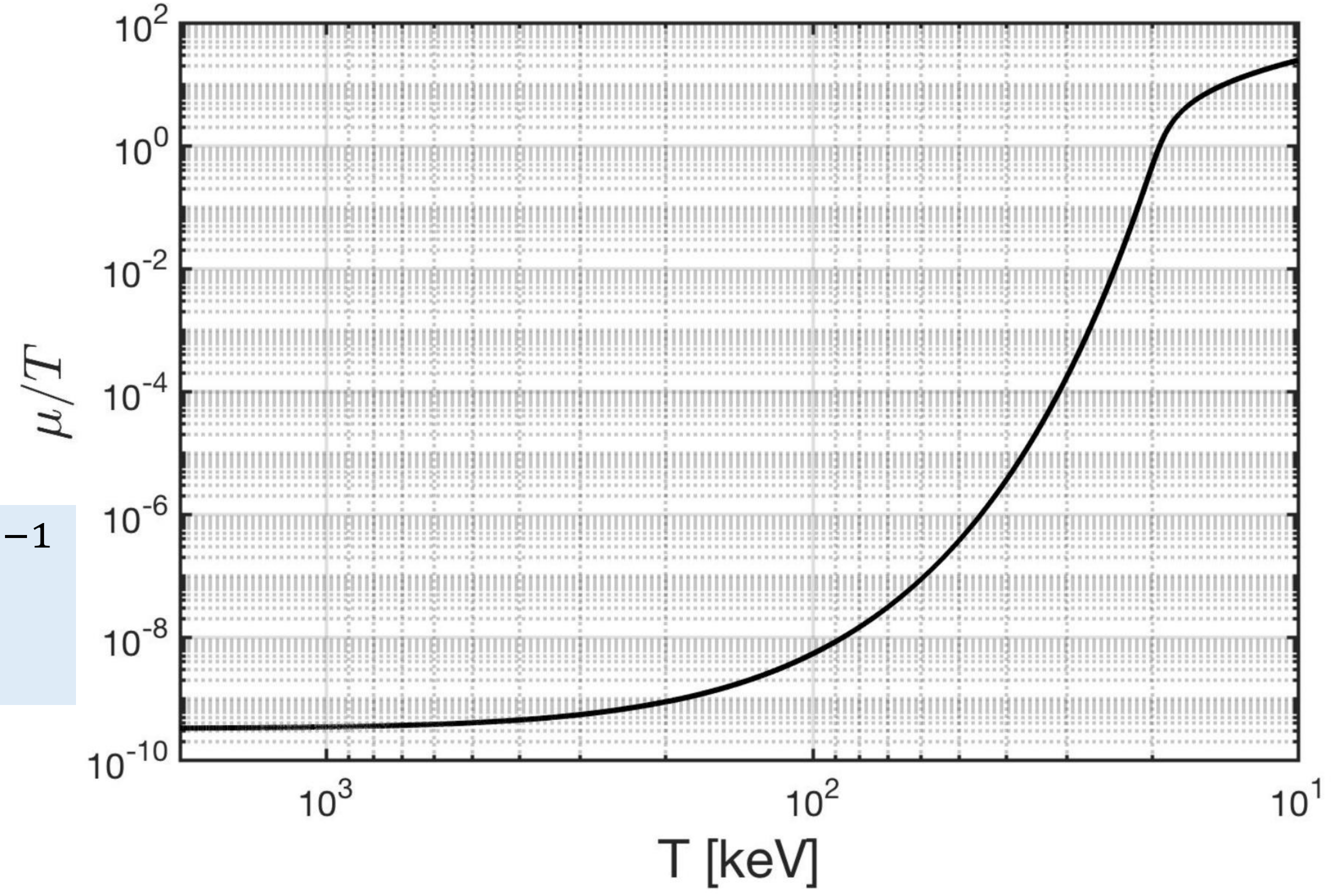
The chemical potential of the partition function is controlled by the charge neutrality condition which connects the number density of excess electrons in the universe to the proton density.

$$n_p = n_{e^-} - n_{e^+} = \frac{1}{V}\lambda \frac{\partial \ln \mathcal{Z}_{e^+e^-}}{\partial \lambda}$$

$$\sinh\frac{\mu}{T} = \frac{\pi^2 n_p}{T^3}\left[x_s^2 K_2(x_s) + \frac{\boldsymbol{b_0}}{2} x_s K_1(x_s) + \frac{\boldsymbol{b_0^2}}{12} K_0(x_s)\right]^{-1}$$

Dominant for the ranges of b_0 considered

$$10^{-3} > b_0 > 10^{-11}$$



The chemical potential resembles the free Fermi case as the magnetic response only becomes significant at unrealistically large external field strengths.

Magnetization of the electron-positron gas

Warning: Because we're using the Boltzmann approximation, the high temperature behavior is uncertain. A more complete analysis is required. Magnetization as a derivative is sensitive to corrections.

$$\mathcal{M} \equiv \frac{T}{V}\frac{\partial}{\partial \mathcal{B}}\ln \mathcal{Z}_{e^+e^-} = \frac{T}{V}\left(\frac{\partial b_0}{\partial \mathcal{B}}\right)\frac{\partial}{\partial b_0}\ln \mathcal{Z}_{e^+e^-}$$

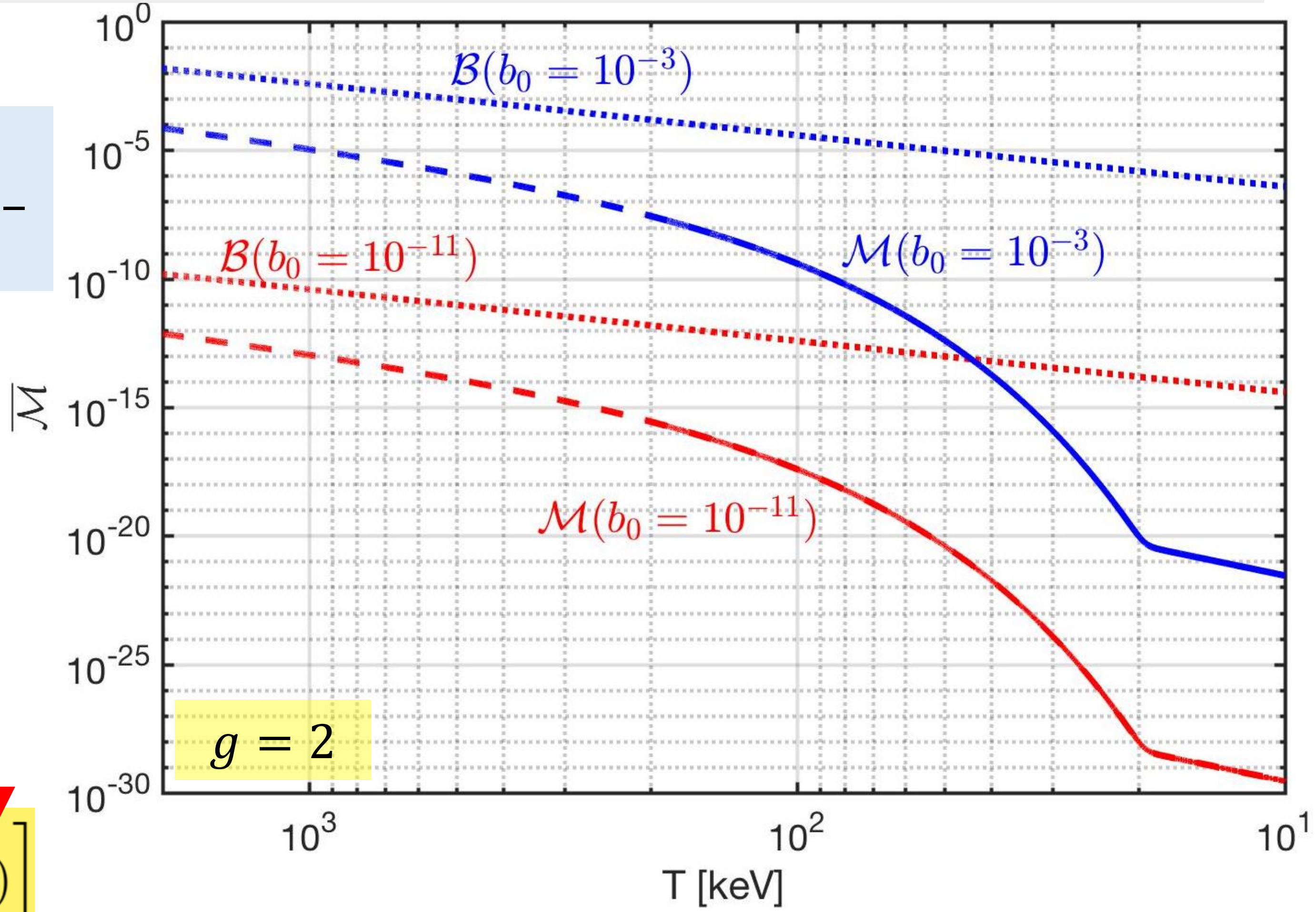
The magnetization can be written as a derivative of the cosmic magnetic scale. We define dimensionless magnetization based on critical field strength.

$$\overline{\mathcal{M}} \equiv \frac{\mathcal{M}}{\mathcal{B}_C} \qquad \mathcal{B}_C = \frac{m_e^2}{e} = 4.41 \times 10^9\ \mathrm{T}$$

Finally, what we were after!

$$\overline{\mathcal{M}}_+ = \frac{q^2}{\pi^2}\frac{T^2}{m_e^2}\xi \cosh\frac{\mu}{T}\left[\frac{1}{2}x_+K_1(x_+) + \frac{b_0}{6}K_0(x_+)\right]$$

$$-\overline{\mathcal{M}}_- = \frac{q^2}{\pi^2}\frac{T^2}{m_e^2}\xi^{-1} \cosh\frac{\mu}{T}\left[\left(\frac{1}{2} + \frac{b_0^2}{12x_-^2}\right)x_-K_1(x_-) + \frac{b_0}{3}K_0(x_-)\right]$$

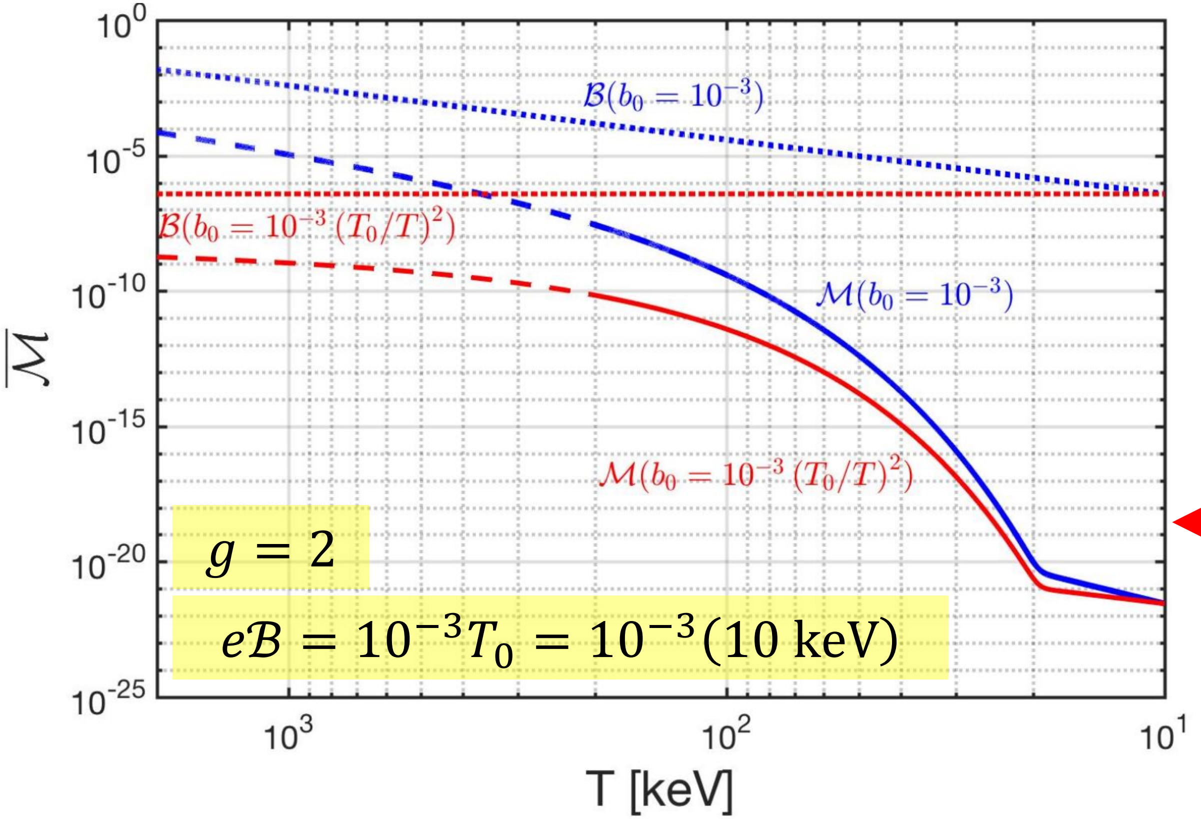


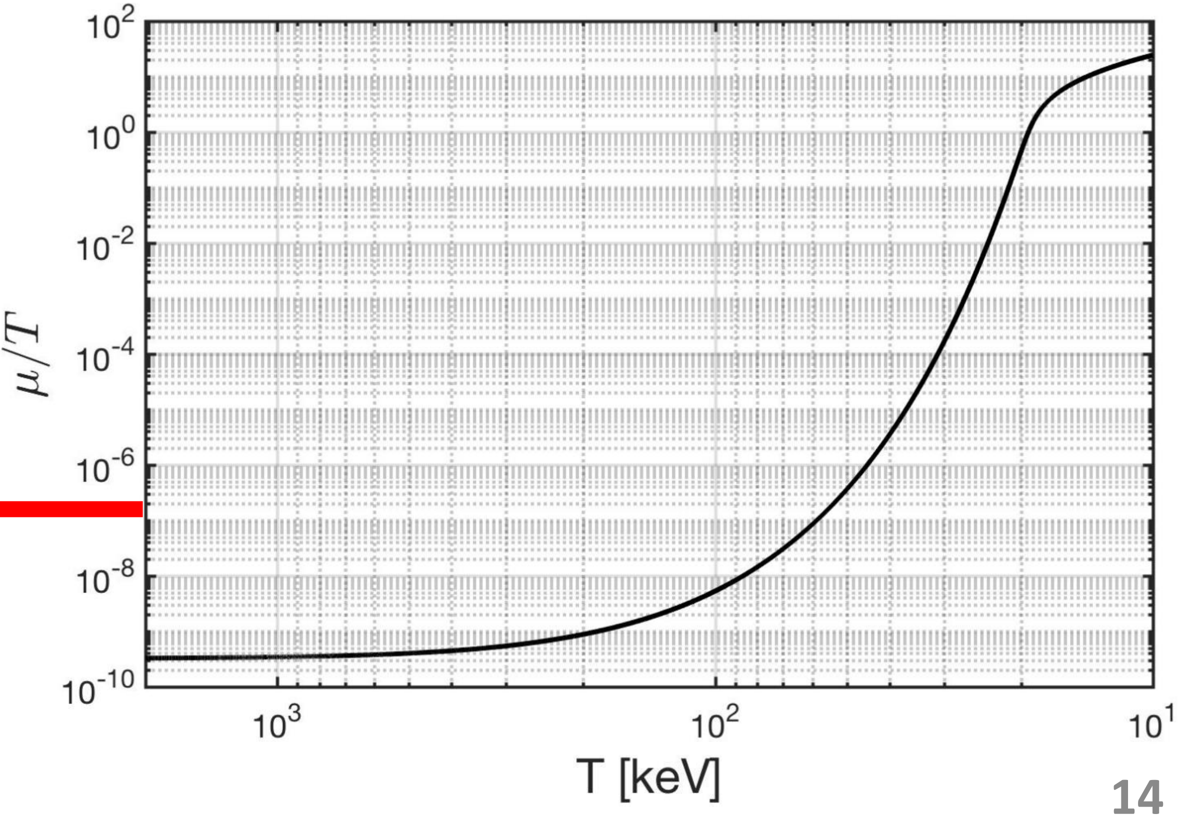
$$x_\pm \equiv \frac{m_\pm}{T} = \sqrt{\frac{m_e^2}{T^2} + 2b_0\left(1 \mp \frac{g}{2}\right)}$$

Hot magnetization: (a) Density rise overwhelms temperature

Warning: Because we're using the Boltzmann approximation, the high temperature behavior is uncertain. A more complete analysis is required. Magnetization as a derivative is sensitive to corrections.

As promised, we can demonstrate why the magnetization increases with temperature by fixing the magnetic field to a constant value. Without the increasing magnetic field, the magnetization rise with temperature is shown to be dominated by the huge increase in pair density.





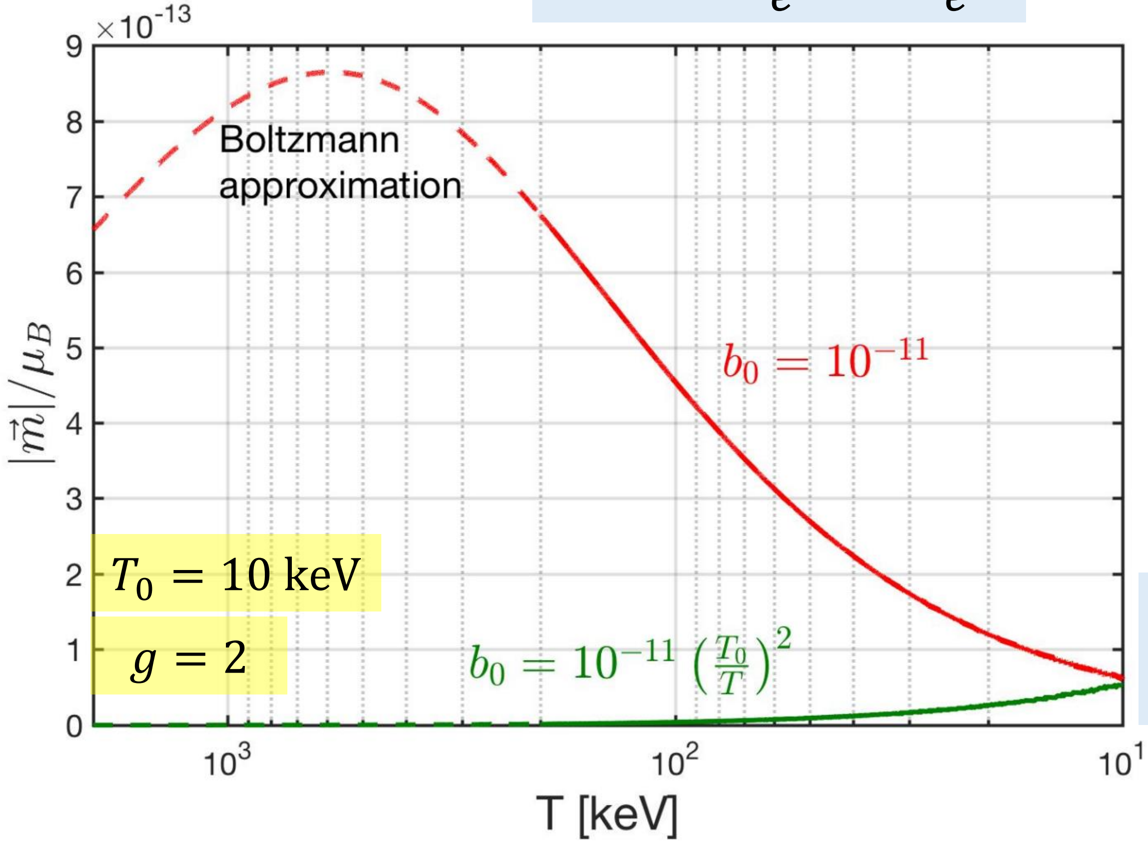
Hot magnetization: (b) Average moment-per-lepton

Warning: Because we're using the Boltzmann approximation, the high temperature behavior is uncertain. A more complete analysis is required. Magnetization as a derivative is sensitive to corrections.

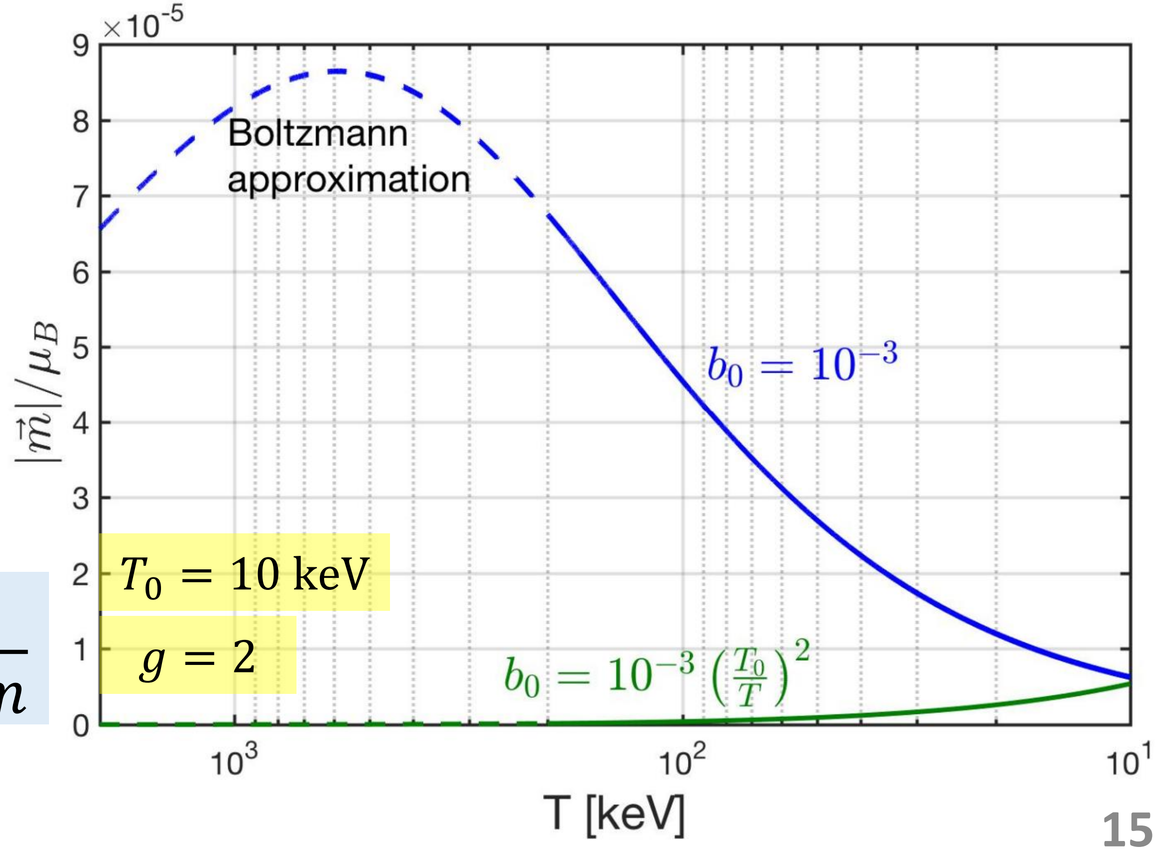
The second reason for "hot magnetization" is that the magnetic scale $\boldsymbol{b_0}$ ensures the magnetic strength $\boldsymbol{\mathcal{B}}$ rises in the past with the temperature. Show this more clearly, we define the average **magnetization-per-lepton** with or without the rising field:

$$|\vec{m}| \equiv \frac{\mathcal{M}}{n_{e^+} + n_{e^-}}$$

Without the rising field, the magnetization is suppressed at higher temperatures, as traditionally expected.



$$\mu_B = \frac{e}{2m}$$



Spin potential (fugacity) and possible cosmic ferromagnetism

Up to this point, we've neglected the spin potential we originally introduced. As an interesting aside, let us determined the magnetization of the electron-positron gas with zero external fields $\boldsymbol{b_0} = \mathbf{0}$ and $\boldsymbol{\xi} \neq \mathbf{1}$.

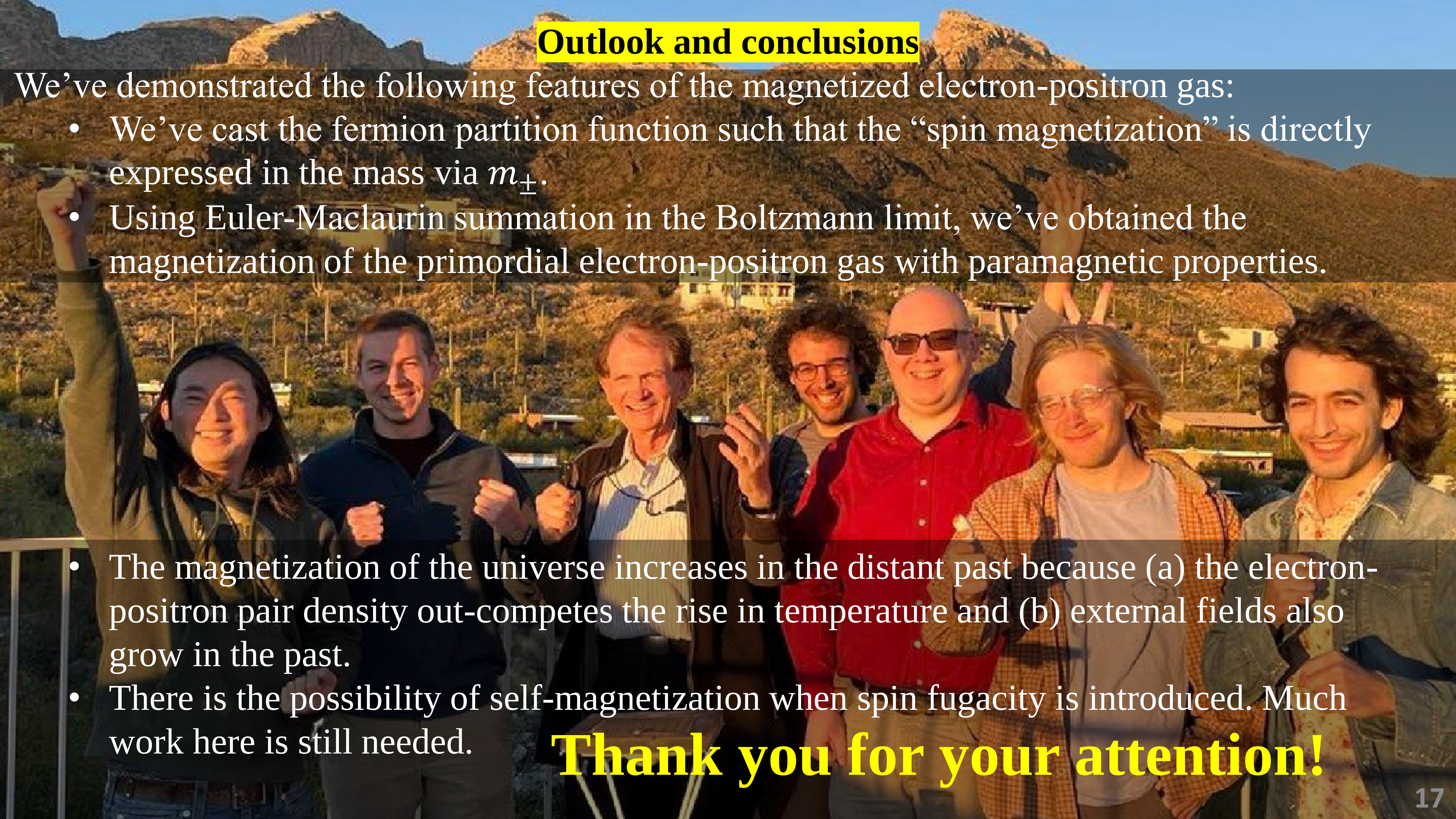
$$\lim_{b_0 \to 0} \overline{\mathcal{M}} = \frac{q^2}{\pi^2} \frac{T^2}{m_e^2} \sinh\frac{\eta}{T} \cosh\frac{\mu}{T} \left[\frac{m_e}{T} K_1\left(\frac{m_e}{T}\right)\right], \qquad 2\sinh\frac{\eta}{T} = \xi - \xi^{-1}$$

This has a "ferromagnetic" character as the magnetization in non-vanishing in zero external fields. As hyperbolic-sine is odd, the sign of the spin potential $\boldsymbol{\eta}$ controls the directionality of magnetization along a preferred axis.

Still in progress tidbit you'll have to wait for the publication to read: Certain domains of self-magnetization controlled by $\boldsymbol{\eta}$ are suspiciously near the upper bound of the cosmic magnetic field strength.

$$10^{-3} > b_0 > 10^{-11}$$

$$10^{-12}\ \mathrm{T} > \mathcal{B} > 10^{-20}\ \mathrm{T}$$



Outlook and conclusions

We've demonstrated the following features of the magnetized electron-positron gas:

- We've cast the fermion partition function such that the "spin magnetization" is directly expressed in the mass via $m_{\pm}$.
- Using Euler-Maclaurin summation in the Boltzmann limit, we've obtained the magnetization of the primordial electron-positron gas with paramagnetic properties.
- The magnetization of the universe increases in the distant past because (a) the electron-positron pair density out-competes the rise in temperature and (b) external fields also grow in the past.
- There is the possibility of self-magnetization when spin fugacity is introduced. Much work here is still needed.

Thank you for your attention!