Sensitivity of finite volume effects to the vacuum contribution

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Krzystof Redlich, Wroclaw U

Size of the physical system

What are the typical sizes?

- Typical size of the fireball in heavy ion collisions is a few fm.
- Neutron stars and compact stars built up from strongly interacting matter (with extra structure) with a size $\sim$ 10 km.
- Several models with finite (different) size.
- In field theoretical calculations (LSM, NJL, DS, etc) usually the size is infinite.

Why does it matter?

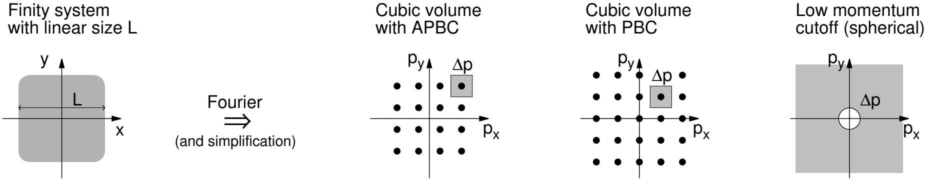
- It can be seen that the properties of the system can change significantly.
- Example: in the phase diagram of strong interaction the CEP (and the first order region) might disappear.

The vicinity of the CEP is accessible with models that are in the thermodynamic limit.

How to consider the finite size effects without losing the advantages of these models?

It is usual to modify the momentum space.

- Discretization: $\int dp \to \sum_n$
- Low momentum cutoff: $\int_0^\infty dp \to \int_\lambda^\infty dp$



Various results in (P)NJL, (P)LSM, DS, etc. calculations.
With discretization and low momentum cutoff as well.

	Discretization	Low cutoff
LSM	J. Phys. G **38**, 085101 (2011) PoS **FACESQCD**, 017 (2010)	J. Phys. G **44**, no.2, 025101 (2017) Universe **5**, no.4, 94 (2019)
NJL	arXiv:1802.00258 [hep-ph] Mod. Phys. Lett. A **33**, no.39, 1850232 (2018) Universe **8**, no.5, 264 (2022)	Phys. Rev. D **87**, no.5, 054009 (2013) Phys. Rev. D **91**, no.5, 051501 (2015) Int. J. Mod. Phys. A **32**, no.13, 1750067 (2017)
FRG	Phys. Rev. D **73**, 074010 (2006) Phys. Rev. D **90**, no.5, 054012 (2014) Phys. Rev. D **95**, no.5, 056015 (2017) Phys. Rept. **707-708**, 1-51 (2017)	
D-S	Phys. Rev. D **81**, 094005 (2010) Phys. Rev. D **102**, 114011 (2020) Phys. Rev. D **104**, no.7, 074035 (2021) Phys. Lett. B **841**, 137908 (2023)	Nucl. Phys. B **938**, 298-306 (2019)

Various results in (P)NJL, (P)LSM, DS, etc. calculations.
With discretization and low momentum cutoff as well.

Vacuum part		**Discretization**	**Low cutoff**
NO	**LSM**	J. Phys. G **38**, 085101 (2011) PoS **FACESQCD**, 017 (2010)	J. Phys. G **44**, no.2, 025101 (2017) Universe **5**, no.4, 94 (2019)
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Separated Renorm.	**FRG**	Phys. Rev. D **73**, 074010 (2006) Phys. Rev. D **90**, no.5, 054012 (2014) Phys. Rev. D **95**, no.5, 056015 (2017) Phys. Rept. **707-708**, 1-51 (2017)	
Renorm.	**D-S**	Phys. Rev. D **81**, 094005 (2010) Phys. Rev. D **102**, 114011 (2020) Phys. Rev. D **104**, no.7, 074035 (2021) Phys. Lett. B **841**, 137908 (2023)	Nucl. Phys. B **938**, 298-306 (2019)

Few examples for the phase diagram:

LSM

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QM model FRG

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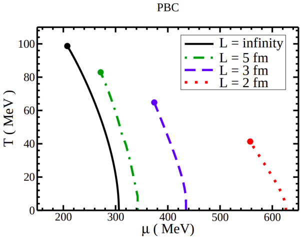
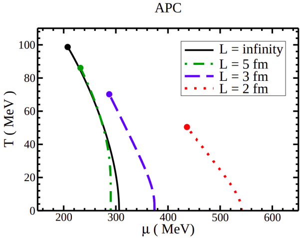


Only zero mode of vacuum part



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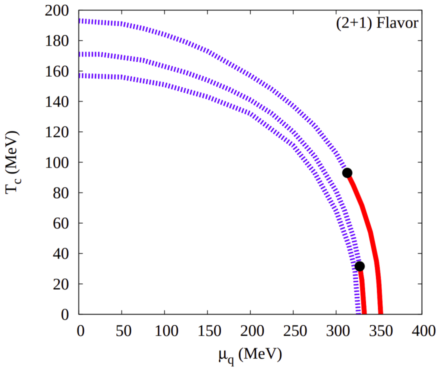

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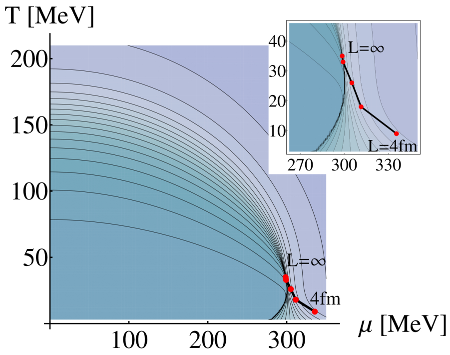

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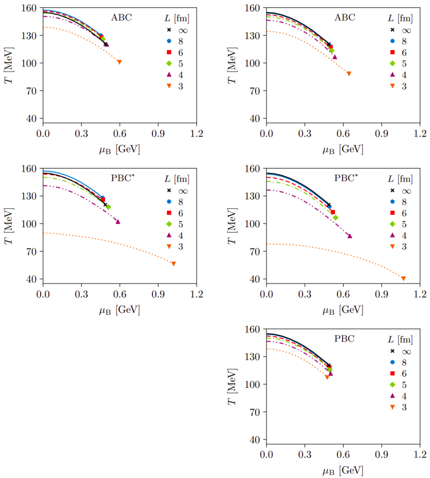

Vector and axial vector meson **E**xtended **P**olyakov **Q**uark-**M**eson model. (ePQM or ELSM)
Effective model to study the phase diagram of strongly interacting matter at finite T and μ.
Phys. Rev. D 93, no. 11, 114014 (2016)

- **Q**uark-**M**eson model: "simple" linear sigma model with quarks and mesons
- **E**xtended: Vector and Axial vector nonets (besides to Scalar and Pseudoscalar)
  Isospin symmetric case: 16 mesonic degrees of freedom.
- **P**olyakov: Polyakov loop variables give 2 order parameters Φ, $\bar{\Phi}$.
- The mesonic Lagrangian $\mathcal{L}_m$ with chiral symmetry

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \to SU(2)_I \times U(1)_V$$

broken explicitly (and spontaneously) and with the axial anomaly taken into account

- $\mathcal{L}_m$ contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
  - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
  - Each meson-meson terms upto 4th order that are allowed by the chiral symmetry.
- Constituent quarks ($N_f = 2+1$) in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi \tag{1}$$

  In the 2016 version $g_V = 0$ was used. **Phys. Rev. D 104, 056013 (2021)**

- SSB with nonzero vev. for scalar-isoscalar sector $\phi_N,\ \phi_S$.
  $\Rightarrow m_{u,d} = \frac{g_F}{2}\phi_N,\ m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.
- Mean field level effective potential $\rightarrow$ the meson masses and the thermodynamics are calculated from this.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \mathrm{diag}(m_u, m_d, m_s)\right)\psi$$

Functional integration over the fermionic fields.
The momentum integrals are renormalized.

- Polyakov loop potential.

$$\Omega(T, \mu_q) = U_{Cl} + \mathrm{tr}\int_K \log\left(iS_0^{-1}\right) + U(\Phi, \bar{\Phi}) \tag{2}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{3}$$

Curvature meson masses:

$$M_{ab}^2 = \left.\frac{\partial^2\Omega}{\partial\varphi_a\partial\varphi_b}\right|_{\{\varphi_i\}=0} \tag{4}$$

Thermodynamics: **Mean field level** effective potential:

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$$\bar{\psi}\,(i\gamma^\mu\partial_\mu - \mathrm{diag}(m_u, m_d, m_s))\,\psi$$

Functional integration over the fermionic fields.
The momentum integrals are renormalized.

Only this term is modified

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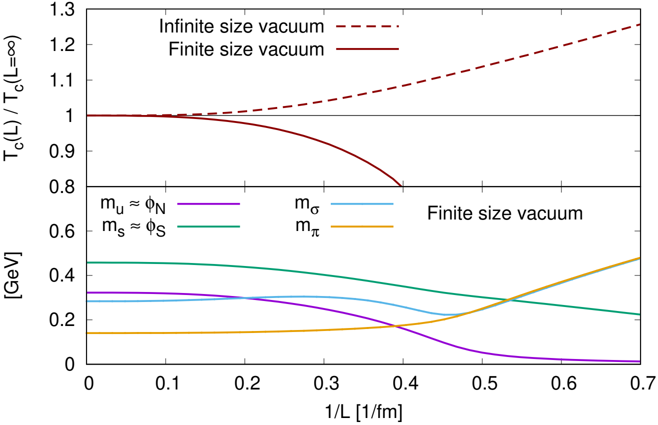
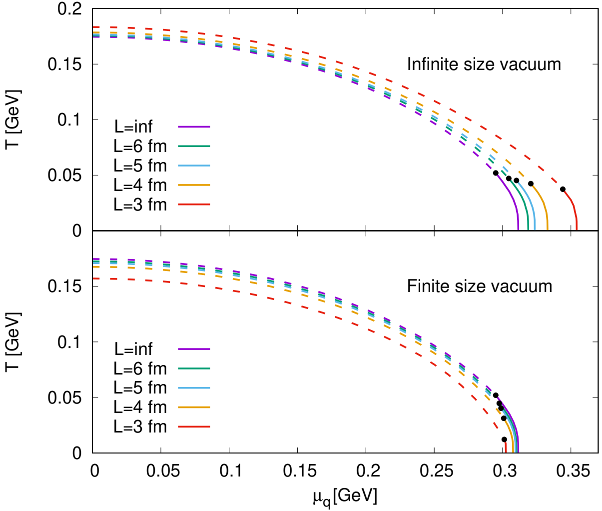
- Fermionic vacuum and matter contribution:

$$\Omega_{\bar{q}q}(T,\mu_q) = \Omega^{\mathrm{vac}}_{\bar{q}q} + \Omega^{\mathrm{mat}}_{\bar{q}q}(T,\mu_q)$$

- The size dependence of $\Omega^{\mathrm{vac}}_{\bar{q}q}$ pushes the system towards chiral restoration

- At $T = 0$ and $\mu_q = 0$ the physical quantities also show the restoration

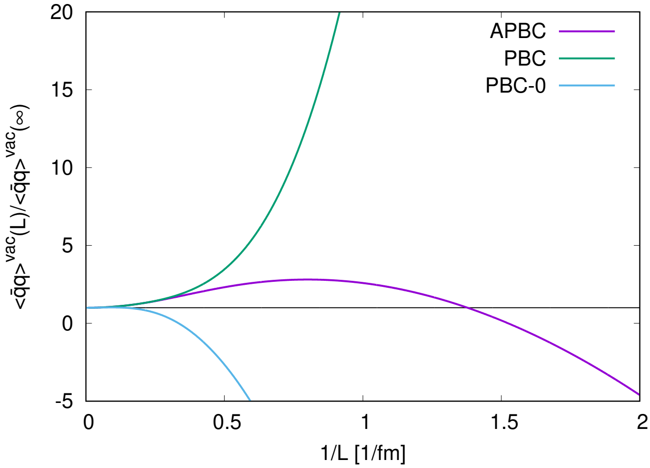




- With $L = \infty$ vacuum part the chirally broken phase expands and the CEP is present even for very small sizes
- With L dependent vacuum part the chirally broken phase will be reduced.

  The CEP disappears at $L = 2.5$ fm, as well as the broken phase at $L = 2$ fm.

- APBC and PBC is considered
- With size-dependent vacuum contribution the condensates will increase for APBC (if $L > 1$ fm) and for PBC
- The common solution of the field equation is lost around 4.5 fm.



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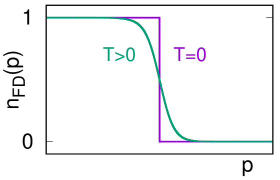
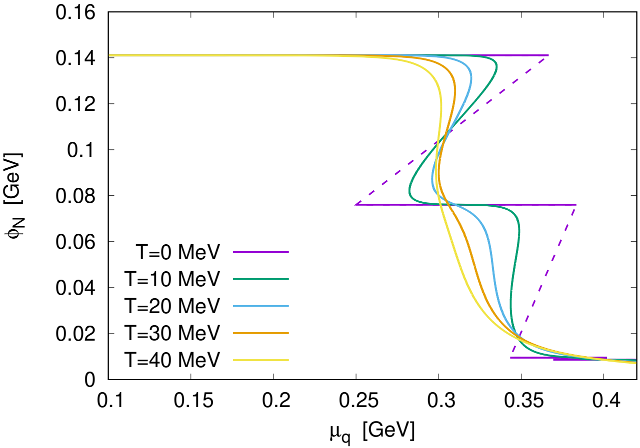
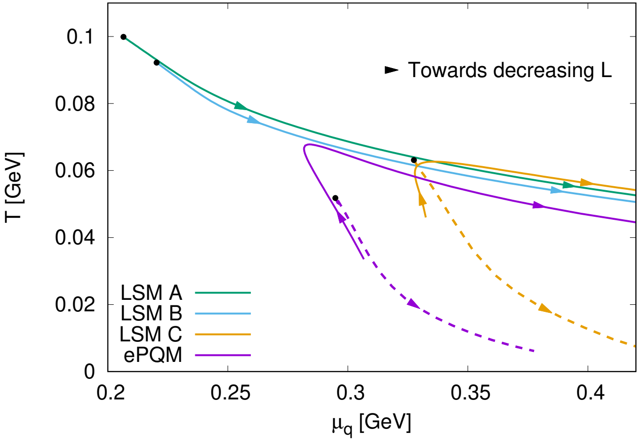


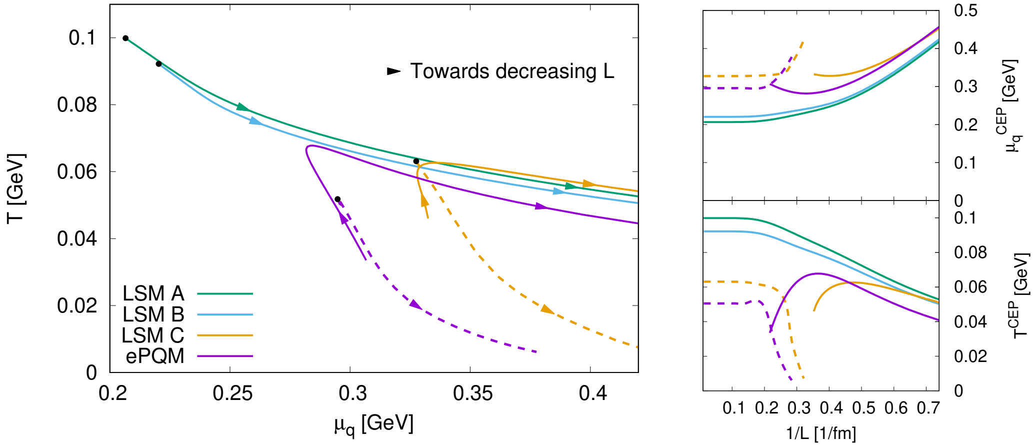




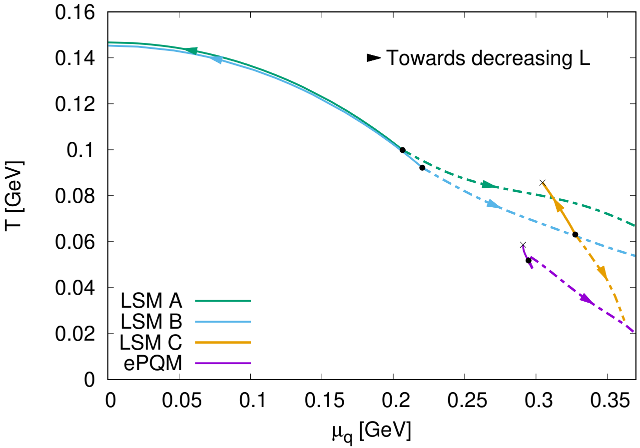
LSM A: J. Phys. G **38**, 085101 (2011)

LSM B, C: Phys. Rev. D **79**, 014018 (2009)

- For the "low lying" models the $L \to \infty$ CEP and the $L \to 0$ CEP is not continuously connected
- For "higher lying" models there is a continuous path
- At $L < 2$ fm the CEP is governed by the first mode entering below the Fermi surface


Towards decreasing L
T [GeV]
μ_q [GeV]
0.1
0.08
0.06
0.04
0.02
0
0.2
0.25
0.3
0.35
0.4
LSM A
LSM B
LSM C
ePQM
μ_q^{CEP} [GeV]
T^{CEP} [GeV]
1/L [1/fm]
0.5
0.4
0.3
0.2
0.1
0
0.1
0.08
0.06
0.04
0.02
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7

CEP with PBC (and the vacuum zero mode)



- With vacuum part of $L = \infty$ new unphysical first-order transition below $L \approx 5.5$ fm
- With (partially included) size-dependent vacuum part the trend is reversed.
- LSM A with zero mode added: J. Phys. G **38**, 085101 (2011)

- The CEP (probably) moves to lower T and higher μ_q with the decreasing size in most scenarios.
- **The details of the finite size effects depend on the used momentum space constraint, the boundary condition, and the treatment of the vacuum part.**
- With low momentum cutoff, the size-dependent vacuum term leads to the reduction of the chirally broken phase, and the disappearance of the CEP when the size decreases, contrary to the case with infinite size vacuum.
- With discretization the CEP will be determined by the modes entering below the Fermi surface. The location of the $L \to \infty$ CEP strongly affects its trajectory with the decreasing size. The vacuum part has an especially strong effect in the case of PBC.

Thank you!