Margaret Island Symposium 2023 on Particles & Plasmas

Instabilities in chiral plasmas

Xu-Guang Huang
Fudan University, Shanghai

June 08, 2023
Budapest, Hungary

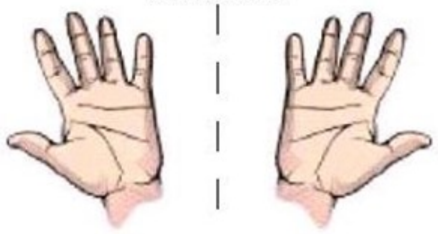
Content

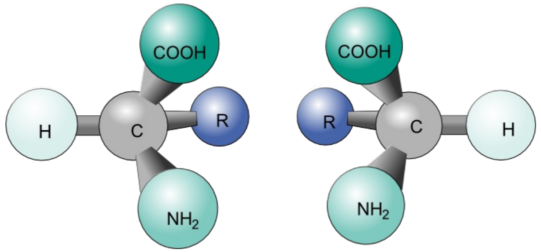
- Introduction: Anomaly-induced transport phenomena
- Instability induced by chiral magnetic effect
- Instabilities induced by chiral vortical effect
  - Chiral vortical instability in background B field
  - Chiral vortical instability at finite viscosity
- Summary

<u>Introduction</u>

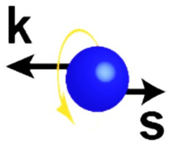
Chiral plasmas: what and where

- Chirality: handedness

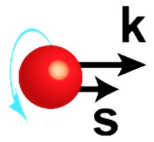




- For massless fermions: helicity



$|L\rangle$



$|R\rangle$

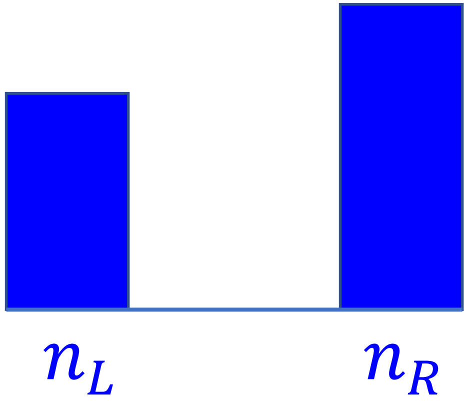
Chirality currents: $J_L^\mu = \bar{\psi}_L \gamma^\mu \psi_L$ $\qquad$ $J_R^\mu = \bar{\psi}_R \gamma^\mu \psi_R$

- Classically

$$\partial_\mu J^\mu = 0 = \partial_\mu J_5^\mu \qquad \text{with} \qquad J^\mu, J_5^\mu = J_R^\mu \pm J_L^\mu$$

Chiral plasmas: what and where

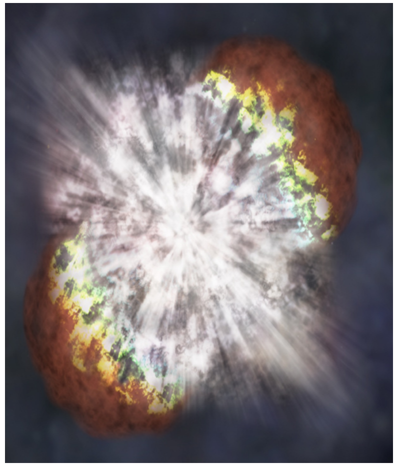
- Chiral plasmas: constituent particles exhibit net chirality

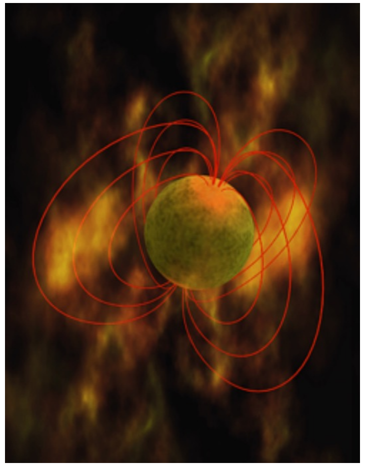


$$n_5 = n_R - n_L > 0$$

$$\text{or } \mu_5 > 0$$

- Example: Core-collapse supernovas and neutron stars





Electron capture $p + e_L^- = n + \nu_L$

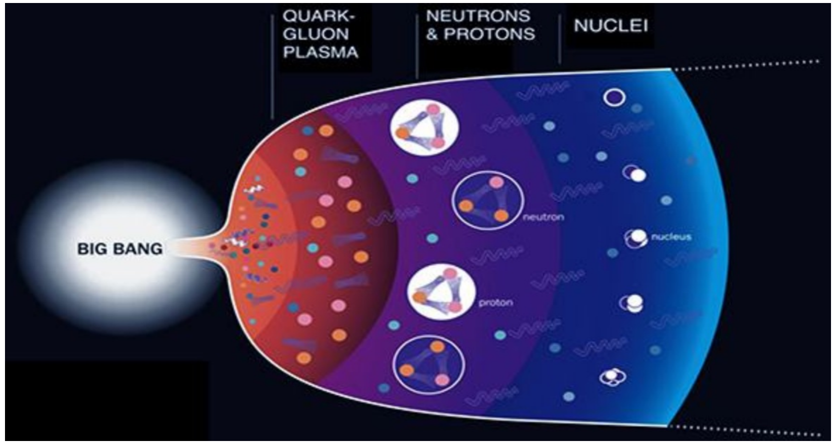
➡ $\mu_5 \sim 100$ MeV (Ohnishi-Yamamoto 2014)

May be strongly suppressed by mass effect

(Grabowska-Kaplan-Reddy 2014)

Chiral plasmas: what and where

- Example: Early Universe

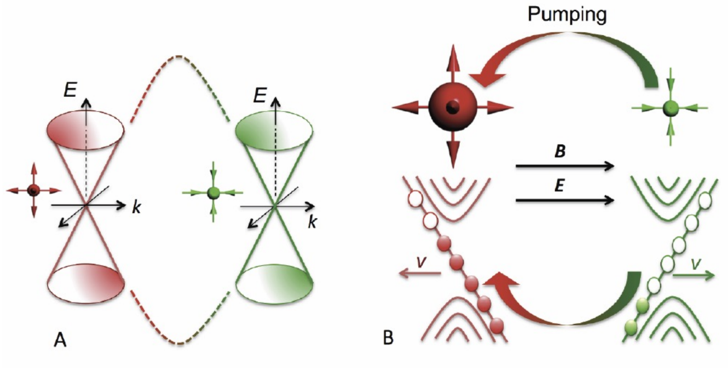


$\mu_5 \ll \alpha_{\text{EM}}\, T$ ~1 GeV for T ~100 GeV

(Brandenburg etal 2017)

May be seen in polarization in microwave background or gravitational wave background

- Example : Electron "plasma" in Weyl/Dirac semimetals



$$\partial_\mu J_5^\mu = C_A \vec{E}\cdot\vec{B}$$

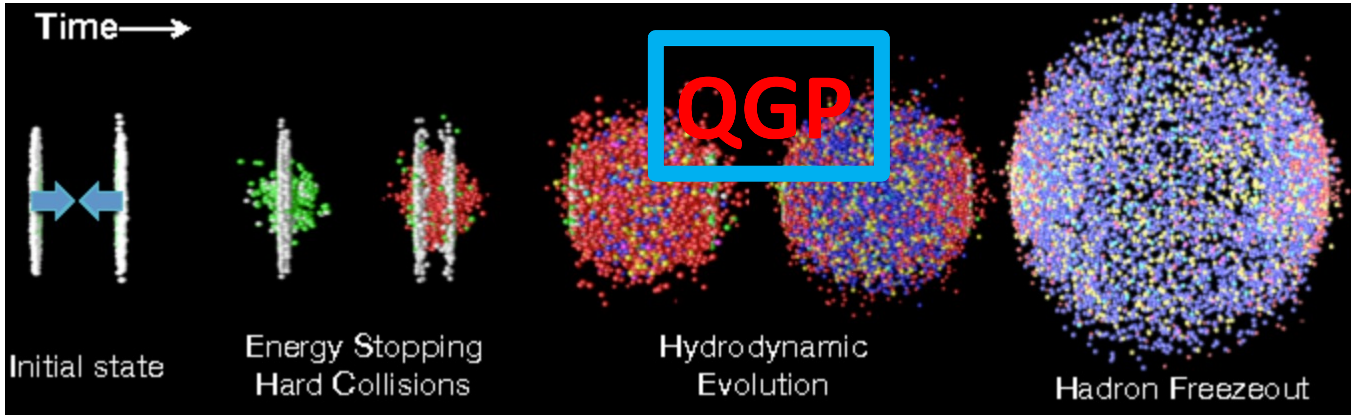
$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B} - \frac{\rho_5}{\tau_V}$$

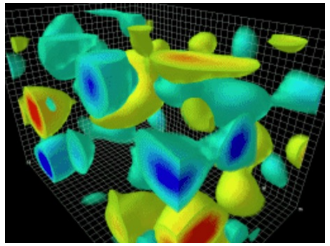
⬇

$$\mu_5 = \frac{3}{4}\frac{v^3}{\pi^2}\frac{e^2}{\hbar^2 c}\frac{\vec{E}\cdot\vec{B}}{T^2 + \frac{\mu^2}{\pi^2}}\tau_V$$ (Li-Kharzeev etal 2014)

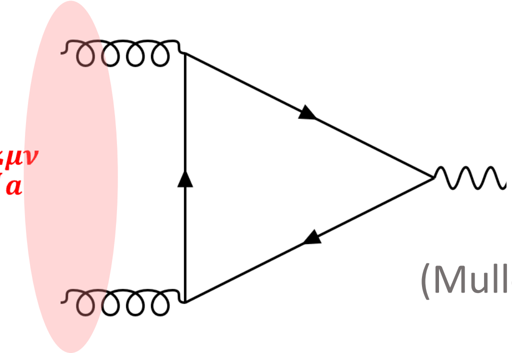
Chiral plasmas: what and where

- Example: Quark gluon plasma (QGP) in heavy ion collisions





$$Q = \frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$



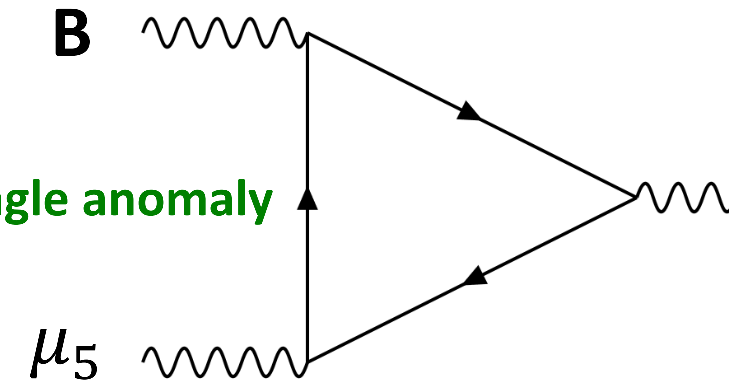
$\mu_5 \sim$10-100 MeV

(Muller-Schafer 2010; Hirono-Hirano-Kharzeev 2014)

Topological transition between different QCD theta vacua

Anomalous transports in chiral plasmas

- Chiral plasmas permit anomalous transport phenomena due to chiral anomaly
- Chiral magnetic effect: Chiral imbalance + B field = vector current



QED triangle anomaly

$$\mathbf{J} = \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

(charge q=1)

Chiral magnetic effect (CME)

(Vilenkin 1980; Kharzeev etal 2007; … …)

- Macroscopic quantum phenomenon
- P- and CP-odd transport
- Time-reversal even, no dissipation
- Fixed by anomaly coefficient, universal

Anomalous transports in chiral plasmas

- Chiral vortical effect: Chiral imbalance + vorticity = vector current
  - Intuitively, this is understood from CME

In magnetic field, Lorentz force:

$$\boldsymbol{F} = e(\dot{\boldsymbol{x}} \times \boldsymbol{B})$$

In rotating frame, Coriolis force:

$$\boldsymbol{F} = 2\varepsilon(\dot{\boldsymbol{x}} \times \boldsymbol{\omega}) + \boldsymbol{O}(\boldsymbol{\omega}^2)$$

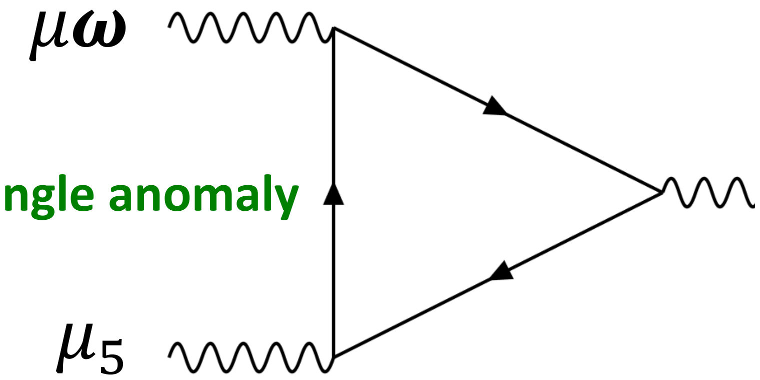
Larmor theorem: $e\boldsymbol{B} \sim 2\varepsilon\boldsymbol{\omega}$

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}/2$$

(Angular velocity of fluid cell)

  - It can be calculated from triangle diagram

QED triangle anomaly



$$\mathbf{J} = \frac{\mu_5}{\pi^2}\mu\boldsymbol{\omega}$$

(charge q=1)

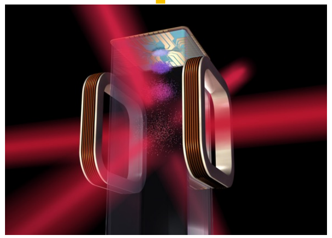
Chiral vortical effect (CVE)

(Vilenkin 1979; Erdmenger etal 2009; Son, Surowka 2009; … …)

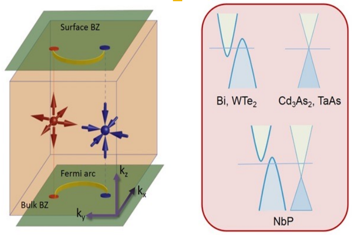
Anomalous transports in chiral plasmas

- Other anomalous transport phenomena:
  - Chiral separation effect (CSE): Charge imbalance + B field = chiral current
    (Son-Zhitnitsky 2004; …)
  - Chiral electric separation effect (CESE): Charge and chiral imbalance + E field = chiral current
    (Huang-Liao 2013; Jiang-Huang-Liao 2014)
  - Chiral torsional effect (CTE): Charge/chiral imbalance + torsion = vector/chiral current
    (Khaidukov-Zubkov 2018; Imaki-Yamamoto 2019; Nissinen-Volovik 2019; …)
  - … …
- Anomalous transports happen across a very broad hierarchy of scales.

$10^{-10}K$ | $1-10^{2}K$ | $10^{10}K$ | $10^{12}K$ | **Temperature**



Cold atomic gases



Weyl/Dirac semimetals

Supernovae

Heavy-ion collisions

CME induced instability

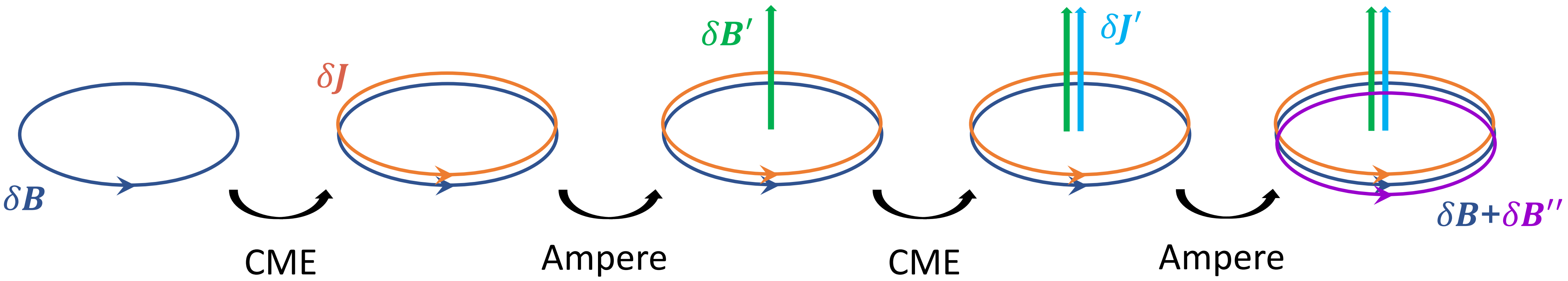
CME induced instability

- Constitutive relation for electric current:

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \xi_B \boldsymbol{B}$$

Ohm current CME $\xi_B \propto \mu_5$

- Interplay of CME and Ampere's law (constant ξ_B):



➡ $\delta\boldsymbol{B}''$ added to $\delta\boldsymbol{B}$ in the same direction

➡ Chiral plasma instability (CPI) or chiral dynamo instability

(Joyce-Shaposhnikov 1997; Boyarsky-Frohlich-Ruchayskiy 2012; Akamatsu-Yamamoto 2013)

CME induced instability

- CPI from modified Maxwell equations

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \sigma \boldsymbol{E} + \xi_B \boldsymbol{B}$$

$$\Rightarrow \quad \partial_t \boldsymbol{B} + \eta \partial_t^2 \boldsymbol{B} = \eta \nabla^2 \boldsymbol{B} + \eta \xi_B \boldsymbol{\nabla} \times \boldsymbol{B}$$

  - High frequency region ($\omega/\sigma \gg 1$): CME-modified electromagnetic waves

$$\partial_t^2 \boldsymbol{B} = \nabla^2 \boldsymbol{B} + \xi_B \boldsymbol{\nabla} \times \boldsymbol{B}$$

  - Low frequency region ($\omega/\sigma \ll 1$): Instability
    Right-handed and left-handed modes $\boldsymbol{B} = B_k^+ \boldsymbol{e}_+(\boldsymbol{k}) + B_k^- \boldsymbol{e}_-(\boldsymbol{k})$:

$$\partial_t B_k^\pm = \boxed{-\eta k^2 B_k^\pm} \pm \boxed{\eta \xi_B B_k^\pm} \quad \Rightarrow \quad B_k^\pm(t) = B_k^\pm(0) e^{\boxed{-\eta k^2 t \pm \eta \xi_B k t}}$$

Magnetic diffusion

Instability

Exponential growth for $k < \xi_B$

- A dynamo action for low-momentum magnetic fields but: $|\xi_B| \propto |\mu_5|$ must be reduced

CME induced instability

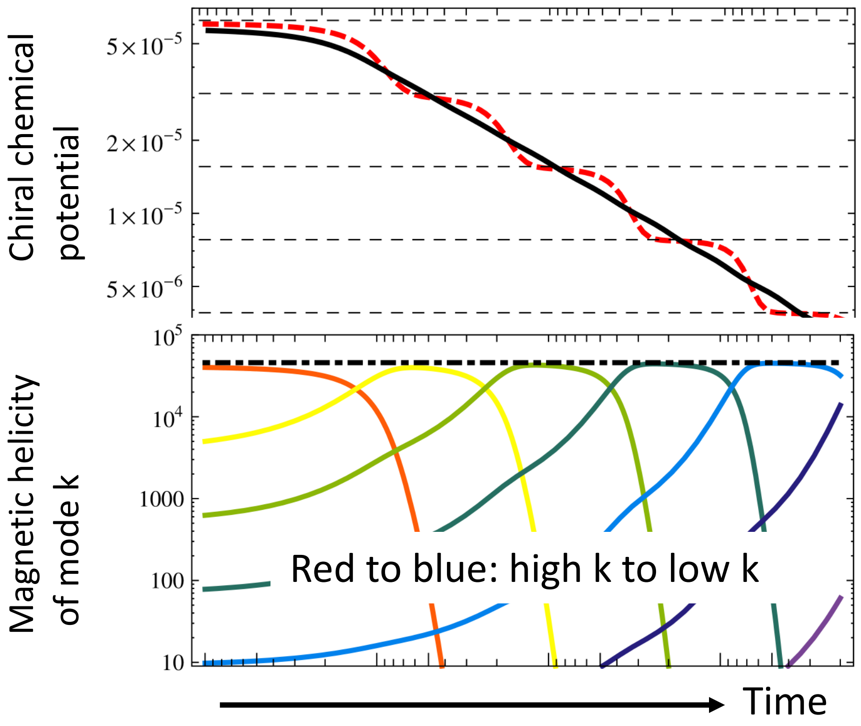
- Dynamics of ξ_B: chiral anomaly

$$\frac{d}{dt}\int d^3\boldsymbol{x}\, n_5 = C\int d^3\boldsymbol{x}\,\boldsymbol{E}\cdot\boldsymbol{B} = -\frac{C}{2}\frac{d}{dt}\int d^3\boldsymbol{x}\,\boldsymbol{A}\cdot\boldsymbol{B}$$

← Magnetic helicity $\mathcal{H}_b$

$$\frac{d\xi_B}{dt} = -C_{\xi_B}\frac{d}{dt}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{k}\left(|B_k^+|^2 - |B_k^-|^2\right)$$

← Interpreted as a helicity conservation law



- CPI drives ξ_B to decay while the magnetic helicity to grow
- For mode $k < \xi_B$: Instability stops when $\xi_B = k$ reached
- Chirality transferred into magnetic helicity
- At mean time, magnetic helicity transferred from high-k modes to low-k modes
- The magnetic energy also transferred from high-k modes to low-k modes (Inverse cascade)
- Profound consequences in astrophysics and cosmology

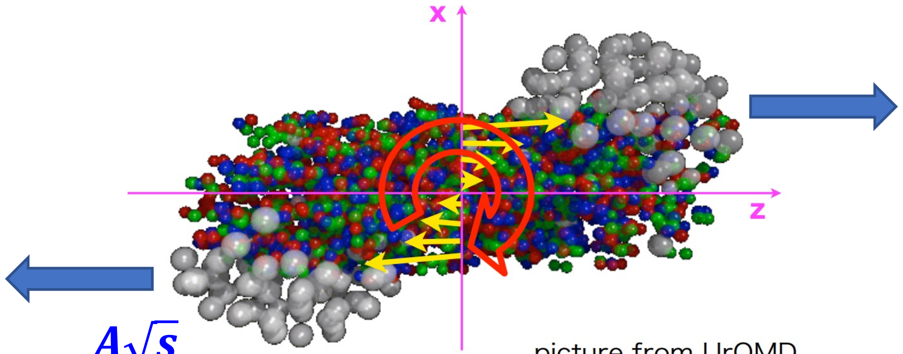
(Boyarsky-Frohlich-Ruchayskiy 2012; and many others)

<u>CVE induced instability I:</u>
<u>Chiral magnetovortical instability</u>

(Wang-Huang, In preparation)

Vorticity in quark gluon plasma

- CVE's role in chiral plasma evolution is less considered. But it may be important when:
  - Vorticity leads to new instability
  - The plasma has very strong vorticity
- Example: "Most vortical fluid" in heavy ion collisions



picture from UrQMD

$$P_z \sim \frac{A\sqrt{s}}{2}$$

Angular momentum

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)

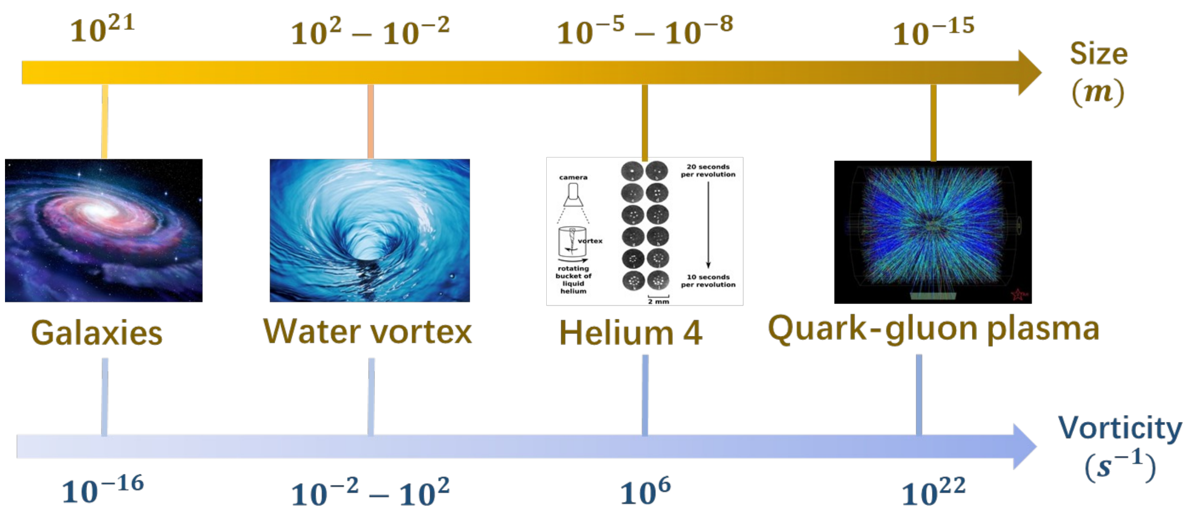
Vorticity in quark gluon plasma

Global angular momentum

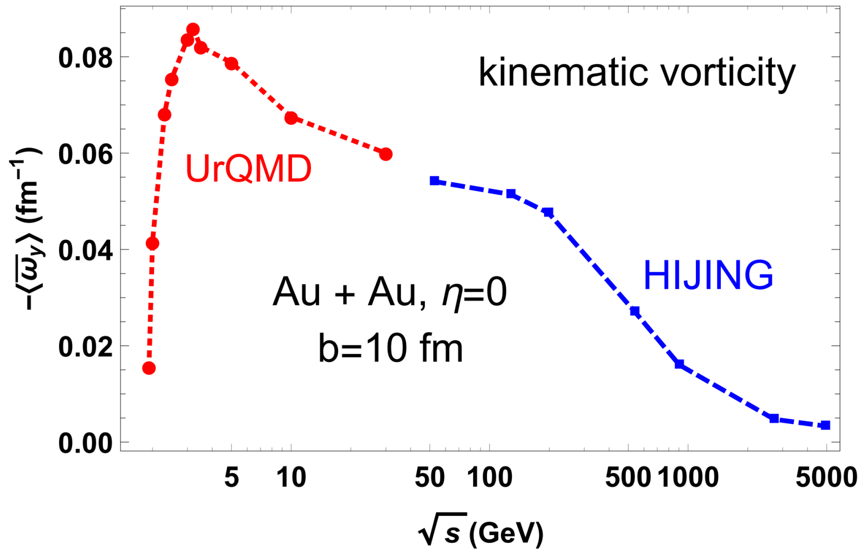
Local fluid vorticity

$$\omega = \frac{1}{2}\nabla\times v$$

(Angular velocity of fluid cell)



Energy dependence



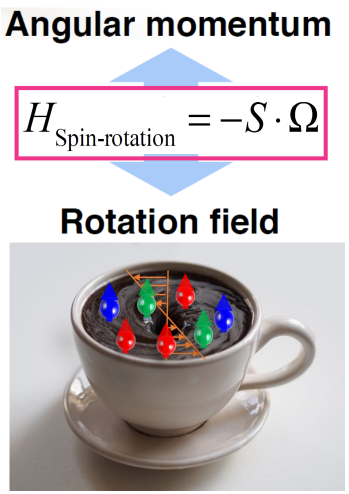
(Deng-Huang 2016; Deng-Huang-Ma-Zhang 2020)

- The most vortical fluid: $\omega \sim 10^{20} - 10^{21}s^{-1}$
- Relativistic suppression at high energies

(See also: Jiang-Lin-Liao 2016; Becattini-Karpenko etal 2015,2016; Xie-Csernai etal 2014,2016,2019; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; … …)

Vorticity in quark gluon plasma

- Spin can thus be polarized by vorticity



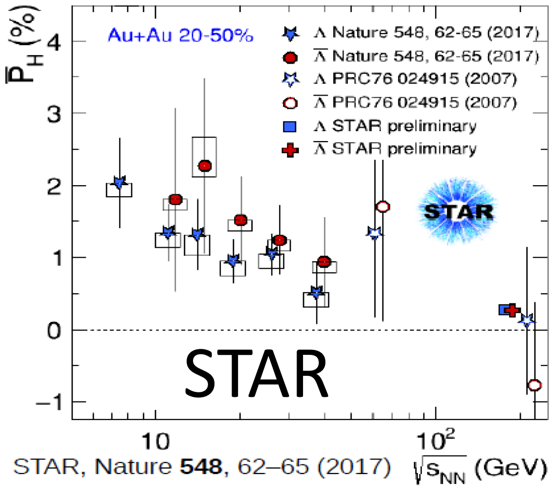
(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{S})/T}$$

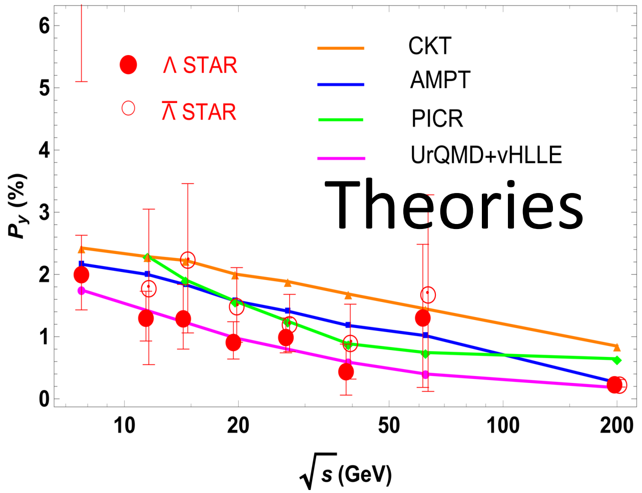
$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

(Original idea:
Liang-Wang 2004)

- Strong vorticity verified by hyperon spin polarization measurements



=



(Li-Pang-Wang-Xia 2017;
Sun-Ko 2017;
Wei-Deng-Huang 2019;
Xie-Wang-Csernai 2017;
Karpenko-Becattini 2016;
… …)

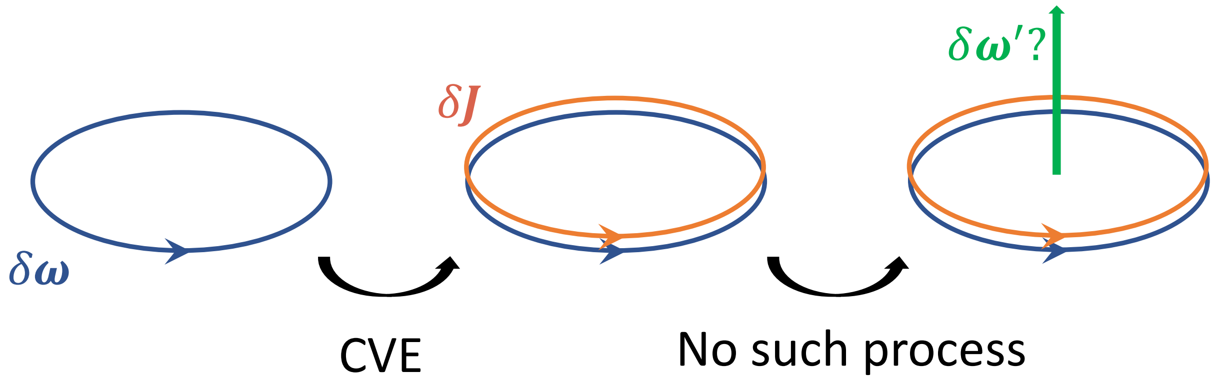
CVE induced instability

- Question: Any CVE induced plasma instability?
- Constitutive relation for electric current:

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \xi_\omega \boldsymbol{\omega}$$

Ohm current CVE $\xi_\omega \propto \mu\mu_5$

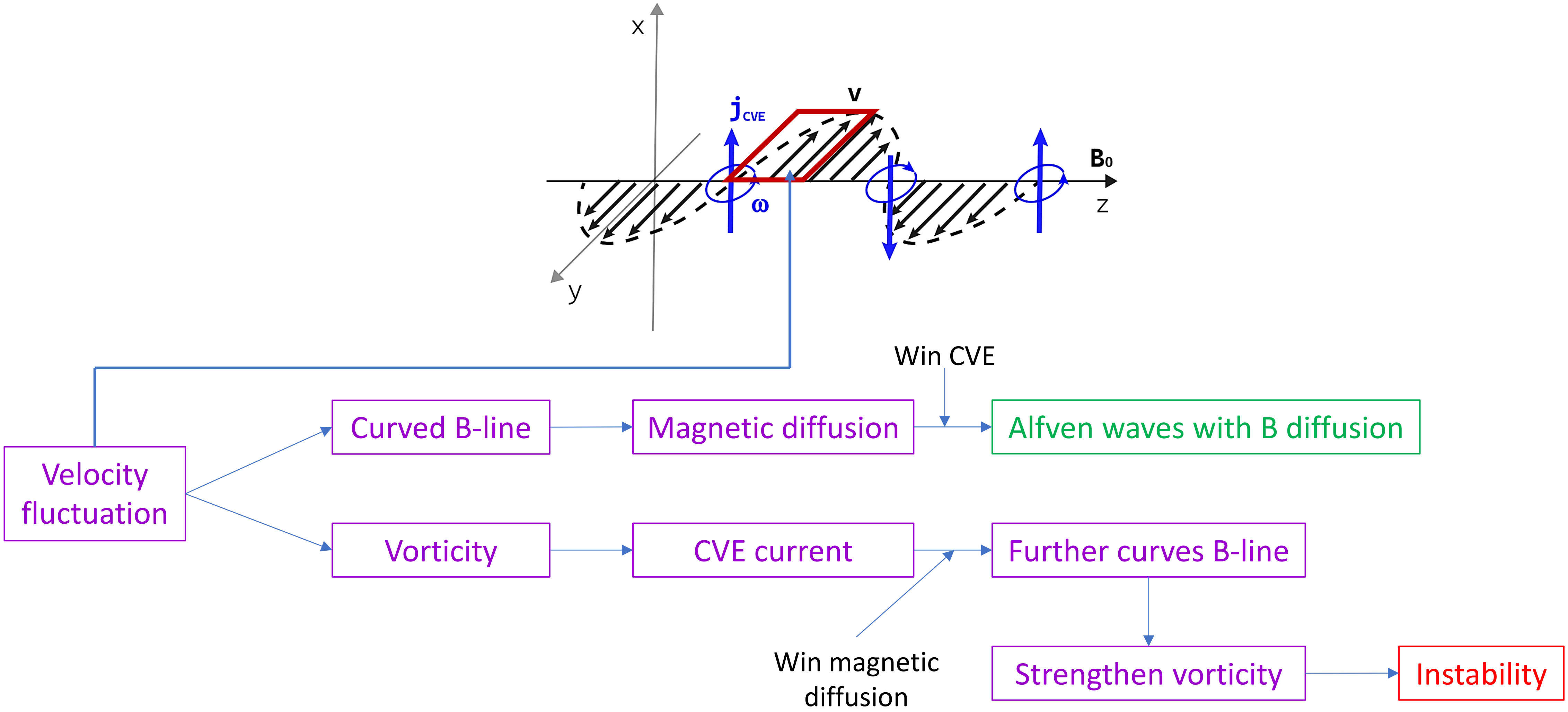
- No direct coupled evolution between fluctuations of vorticity and current



No CVE analog of chiral plasma instability

CVE induced instability

- However, in the presence of external B field: Chiral magnetovortical instability (CMVI)



Chiral magnetovortical instability

- Chiral magnetohydrodynamics (MHD) equations

$$\rho\left(\partial_t+\boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\boldsymbol{v}=-\boldsymbol{\nabla}P+\boxed{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}$$

$$\partial_t\boldsymbol{B}=\boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B})+\boxed{\eta\nabla^2\boldsymbol{B}}+\boxed{\eta\xi_\omega\boldsymbol{\nabla}\times\boldsymbol{\omega}}$$

Lorentz force

Magnetic diffusion

CVE

$$\boldsymbol{\nabla}\cdot\boldsymbol{v}=\boldsymbol{0}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=\boldsymbol{0}$$

- Consider a background $\boldsymbol{B}_0$. Small perturbed $\boldsymbol{v}$ and magnetic field $\boldsymbol{b}$ transverse to $\boldsymbol{B}_0$.

Linearized chiral MHD:

$$\rho\partial_t\boldsymbol{v}=\boldsymbol{B}_0\cdot\boldsymbol{\nabla}\boldsymbol{b}$$

$$\partial_t\boldsymbol{b}=\boldsymbol{B}_0\cdot\boldsymbol{\nabla}\boldsymbol{v}+\eta\nabla^2\boldsymbol{b}-\eta\xi_\omega\nabla^2\boldsymbol{v}$$

➡ Dispersion relation of eigenmodes: $\omega_\pm\equiv-i\frac{\eta}{2}\boldsymbol{k}^2\pm\sqrt{\underline{\frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\rho}-\frac{\eta^2\boldsymbol{k}^4}{4}-i\eta\xi_\omega\boldsymbol{k}^2\frac{\boldsymbol{B}_0\cdot\boldsymbol{k}}{\rho}}}$

Instability ⬅ Negative when $|\xi_\omega|>\sqrt{\rho}$

Chiral magnetovortical instability

- Long-wavelength (low-k) modes $k_z, k_t \ll v_A/\eta$:

$$\omega_\pm \approx \pm \frac{\boldsymbol{B}_0 \cdot \boldsymbol{k}}{\sqrt{\rho}} - i\frac{\eta}{2}\left(1 \pm \frac{\xi_\omega}{\sqrt{\rho}}\right)\boldsymbol{k}^2$$

Alfven waves $v_A = |\boldsymbol{B}_0|/\sqrt{\rho}$

Magnetic diffusion

CVE

- Short-wavelength (high-k) modes $v_A/\eta \ll k_z, k_t \ll 1/\eta$:

$$\omega_\pm = \pm \frac{\xi_\omega}{\rho}\boldsymbol{B}_0 \cdot \boldsymbol{k} - \frac{i}{2}[1 \mp (-1)]\eta \boldsymbol{k}^2 \mp i\frac{\boldsymbol{B}_0^2(\xi_\omega^2 - \rho)}{\eta\rho^2}$$

Chiral Alfven waves

(Yamamoto 2015): No dynamical magnetic field

- Similarly with CPI, CMVI would drive $|\mu_5|$ to drop and finally be tamed

The fate of CMVI

- Dynamics of ξ_ω:

$$\partial_t J_5^0 + \nabla \cdot \boldsymbol{J}_5 = C\boldsymbol{E}\cdot\boldsymbol{B}$$

$$J_5^0 = n_5 + \kappa_B \boldsymbol{v}\cdot\boldsymbol{B} + \kappa_\omega \boldsymbol{v}\cdot\boldsymbol{\omega}$$

Chiral separation effect $\kappa_B \propto \mu$

Axial CVE $\kappa_\omega \propto T^2 + \#\mu^2$

Magnetic helicity $\mathcal{H}_b = \langle \boldsymbol{A}\cdot\boldsymbol{b}\rangle$

Kinetic helicity $\mathcal{H}_v = \langle \boldsymbol{v}\cdot\boldsymbol{\omega}\rangle$

Cross helicity $\mathcal{H}_c = \langle \boldsymbol{v}\cdot\boldsymbol{b}\rangle$

$$\chi_5 \partial_t \mu_5 = -\frac{C}{2}\partial_t \mathcal{H}_b - \kappa_B \partial_t \mathcal{H}_c - \kappa_\omega \partial_t \mathcal{H}_v - \Gamma \chi_5 \mu_5$$

Chirality-flipping rate (Γ)

- Eigenmodes of Chiral MHD: CVE-modified Elsasser fields $\boldsymbol{z}_{1,2} = \sum_{s=\pm} z_{1,2s}\boldsymbol{e}_s(\boldsymbol{k})$

$$z_{1\pm} \approx \left(1 - \frac{i\eta\xi'_\omega \boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm - \left(1 - \frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm$$

$$z_{2\pm} \approx \left(1 - \frac{i\eta\xi'_\omega \boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm + \left(1 + \frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm$$

$\Rightarrow$

$$\partial_t z_{1\pm}(t,\boldsymbol{k}) = -i\omega_+ z_{1\pm}(t,\boldsymbol{k})$$

$$\partial_t z_{2\pm}(t,\boldsymbol{k}) = -i\omega_- z_{2\pm}(t,\boldsymbol{k})$$

Primed: scale by mass density ρ

$\boldsymbol{B}'_0 = \boldsymbol{B}_0/\sqrt{\rho}, \boldsymbol{b}' = \boldsymbol{b}/\sqrt{\rho}$, and $\xi'_\omega = \xi_\omega/\sqrt{\rho}$

The fate of CMVI

- Consider $\xi_\omega > 0$:

$$\begin{cases} \boldsymbol{z}_{1,2}(t,\boldsymbol{k}) = exp\left[-i\int_0^t \omega_\pm(t',\boldsymbol{k})dt'\right]\boldsymbol{z}_{1,2}(0,\boldsymbol{k}) \\ \omega_\pm \approx \pm\frac{\boldsymbol{B}_0\cdot\boldsymbol{k}}{\sqrt{\rho}} - i\frac{\eta}{2}\left(1\pm\frac{\xi_\omega}{\sqrt{\rho}}\right)\boldsymbol{k}^2 \end{cases}$$

$$\begin{cases} z_{1\pm} \approx \left(1-\frac{i\eta\xi'_\omega\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm - \left(1-\frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm \\ z_{2\pm} \approx \left(1-\frac{i\eta\xi'_\omega\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm + \left(1+\frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm \end{cases}$$

- CVE catalyzes magnetic diffusion of $\boldsymbol{z}_1$ modes

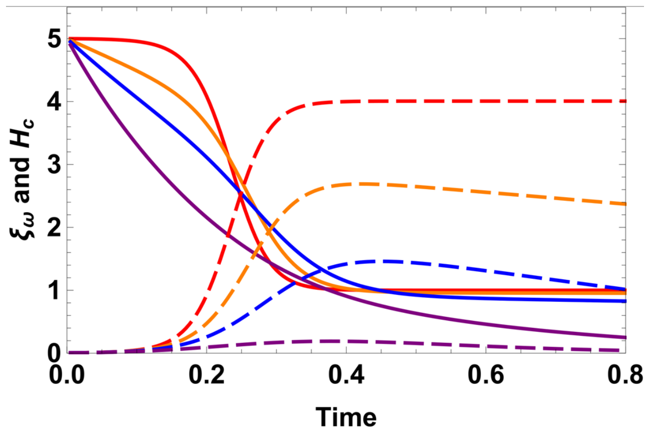
➡ A new mechanism for rapid alignment of velocity and magnetic field (Alfvenic state) (Sudan 1979; Matthaeus etal 2008)

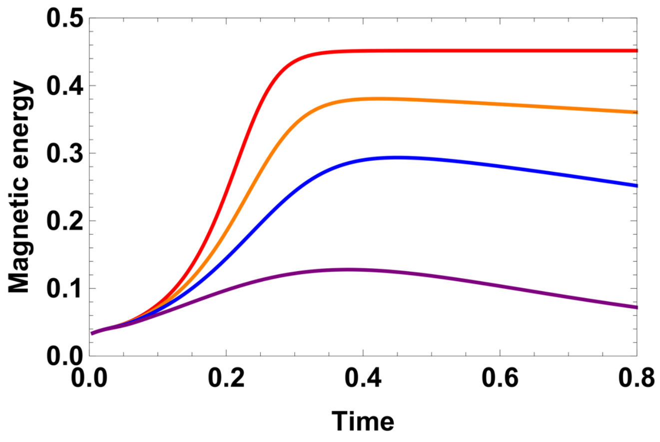
➡ Rapid arising of cross helicity

- Drives ξ_ω to decay to $\xi'_\omega = 1$ stationary state (if $\Gamma = 0$)
- Magnetic energy rapidly arises. A new dynamo action similar to turbulent cross-helicity dynamo (Yoshizawa 1990)

- Numerical simulation with zero initial magnetic and kinetic helicities

Red to purple: Chirality flipping rate from zero to $0.002/\eta$



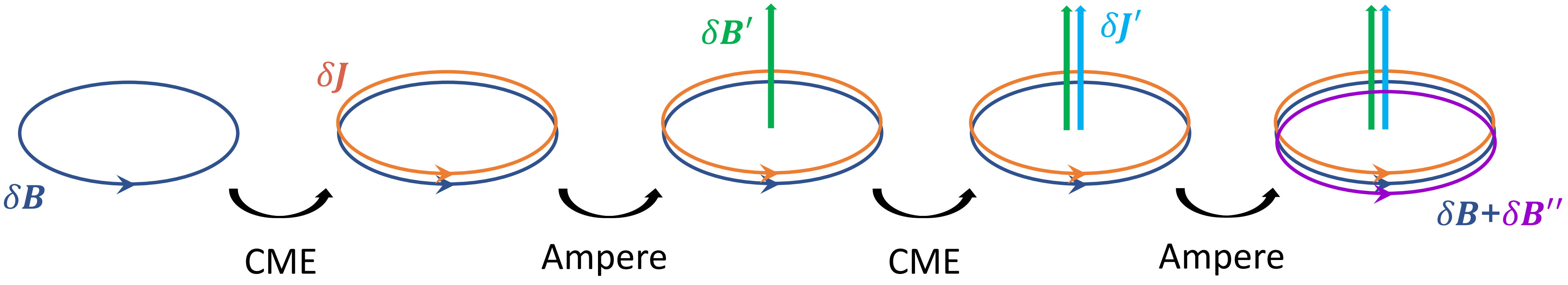


CVE induced instability II: Viscosity induced CVE instability

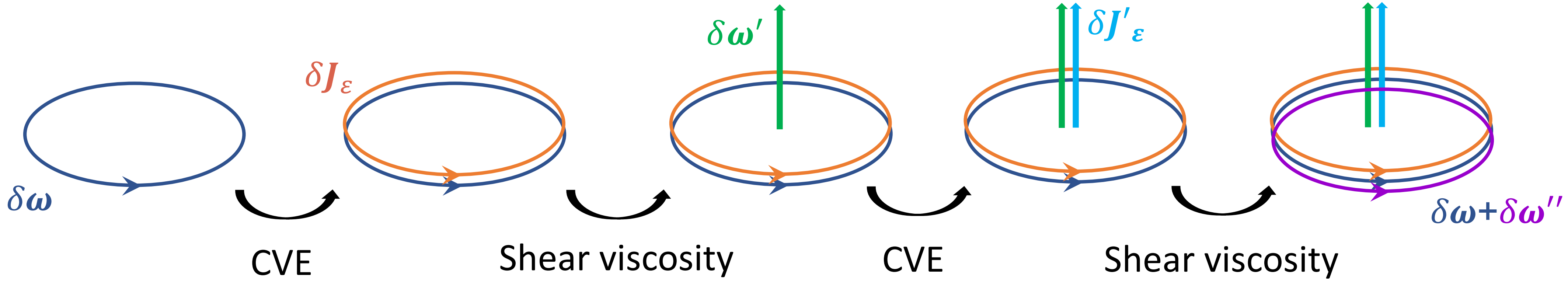
(Wang-Hattori-Huang, In preparation)

CVE instability at finite shear viscosity

- Look at again the CME induced instability



- Consider CVE induced energy current at finite shear viscosity



➡ Thus it is possible for CVE analog of chiral plasma instability but at finite shear viscosity

CVE instability at finite shear viscosity

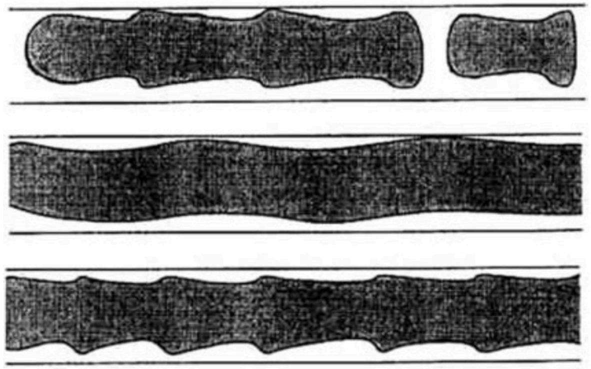
- Indeed, we can find that one of the shear modes is unstable:

$$\omega = -i\nu \left(1 \pm \frac{\lambda_\omega}{\rho}|\boldsymbol{k}|\right) \boldsymbol{k}^2 \longleftarrow \quad \nu = \frac{\mu}{\rho}$$: Kinematic viscosity, $\lambda_\omega \propto \mu_5$: CVE for energy current

Compare with CPI by CME: $\omega = -i\eta\left(|\boldsymbol{k}| \pm \xi_B\right)|\boldsymbol{k}|$

- Unlike CPI, this instability is active for large k mode. It can induce normal cascade of fluid energy and kinetic helicity.
- Note: Though viscosity usually stabilizes the flow, there are known viscous instabilities:

Saffman-Taylor instability



Oil in pipes

Summary

Summary

- There are new instabilities due to anomaly in chiral plasma
- Though CVE-induce instability need large CVE coefficients, they still indicate a tendence to destabilizing the system
- It will be interesting to study their consequences in astrophysics, cosmology, and quark gluon plasma
- Is it possible to produce chiral plasma in laboratory?

Thank you

huangxuguang@fudan.edu.cn