ENTROPIC COMPLEXITY

IN QUARK-GLUON AND OTHER PLASMAS

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ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy ($q = 1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (q *real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable (**unique** trace form; Enciso-Tempesta)

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

C.T., J. Stat. Phys. **52**, 479 (1988)

DEFINITIONS : *q*-*logarithm* : $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q-*exponential* : $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten :

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since $$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\,\frac{S_q(A)}{k}\,\frac{S_q(B)}{k}$$

S_{BG} and $S_q^{Renyi}\ (\forall q)$ are additive, and $S_q\ \ (\forall q \neq 1)$ is nonadditive .

Equivalently, $$S_q(A+B) = S_q(A) + S_q(B) + \boxed{\frac{1-q}{k}}\, S_q(A)\, S_q(B)$$

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \;,\; i.e.,\;\; S(N) \propto N \;\; (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

$W \equiv$ total number of possibilities with nonzero probability, assumed to be equally probable

If $W(N) \sim \mu^N \quad (\mu > 1) \quad \Rightarrow S_{BG}(N) = k \ln W(N) \propto N$ *OK*!

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow S_q(N) = k \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow S_{q=1-1/\rho}(N) \propto N$$ *OK*!

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\ 0 < \gamma < 1)$

$$\Rightarrow S_\delta(N) = k\ [\ln W(N)]^\delta \propto N^{\gamma\,\delta} \Rightarrow S_{\delta=1/\gamma}(N) \propto N$$ *OK*!

If $W(N) \sim D\ \ln N \quad (D > 0)$

$$\Rightarrow S_\lambda^C(N) = k\ [e^{\lambda W(N)} - e^\lambda]\ \Rightarrow S_\lambda^C(N) \sim kN^{\lambda D}$$

$$\Rightarrow S_{\lambda=1/D}^C(N) \propto N$$ *OK*!

IMPORTANT: $\boxed{\mu^N >> \nu^{N^\gamma} >> N^\rho >> \ln N \quad \text{if} \quad N >> 1}$

All happy families are alike; each unhappy family is unhappy in its own way.

Leo Tolstoy (*Anna Karenina*, 1875-1877)

SYSTEMS $W(N)$ $\boxed{(equiprobable)}$	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)	ENTROPY S_λ^{C} $(\lambda > 0)$ (NONADDITIVE)
$\sim A\,\mu^N\ (A>0, \mu>1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$\sim B\,N^\rho\ (B>0, \rho>0)$	NONEXTENSIVE	EXTENSIVE $(q = 1 - 1/\rho)$	NONEXTENSIVE	NONEXTENSIVE
$\sim C\nu^{N^\gamma} (C>0, \nu>1, 0<\gamma<1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\delta = 1/\gamma)$	NONEXTENSIVE
$\sim D\,\ln N\ (D>0)$	NONEXTENSIVE	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\lambda = 1/D)$

SLOW CHEMICAL REACTION

RESEARCH BRIEFINGS | 01 March 2023

Rate of quantum-tunnelling reaction revealed

A physical phenomenon called quantum tunnelling is rare in chemical reactions, making it difficult to study theoretically and experimentally. The measurement of the tunnelling rate in a hydrogen reaction has enabled the verification of quantum-tunnelling calculations, providing a benchmark for testing future quantum calculations.

$$D^- + H_2 \rightarrow H^- + HD$$

Article

Tunnelling measured in a very slow ion–molecule reaction

https://doi.org/10.1038/s41586-023-05727-z

Received: 13 July 2022

Accepted: 12 January 2023

Published online: 1 March 2023

Check for updates

Robert Wild[1], Markus Nötzold[1], Malcolm Simpson[1], Thuy Dung Tran[1,2] & Roland Wester[1]✉

Quantum tunnellin
pathways are energ
or liquid-phase che
calculate theoretic
and also very difficu
allow for accurate f
proton-transfer tun
$H_2 + D^- \rightarrow H^- + HD$, h
Here we present hig
in a cryogenic 22-po
$(5.2 \pm 1.6) \times 10^{-20}$ cm
calculations, servin
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from linear scaling,
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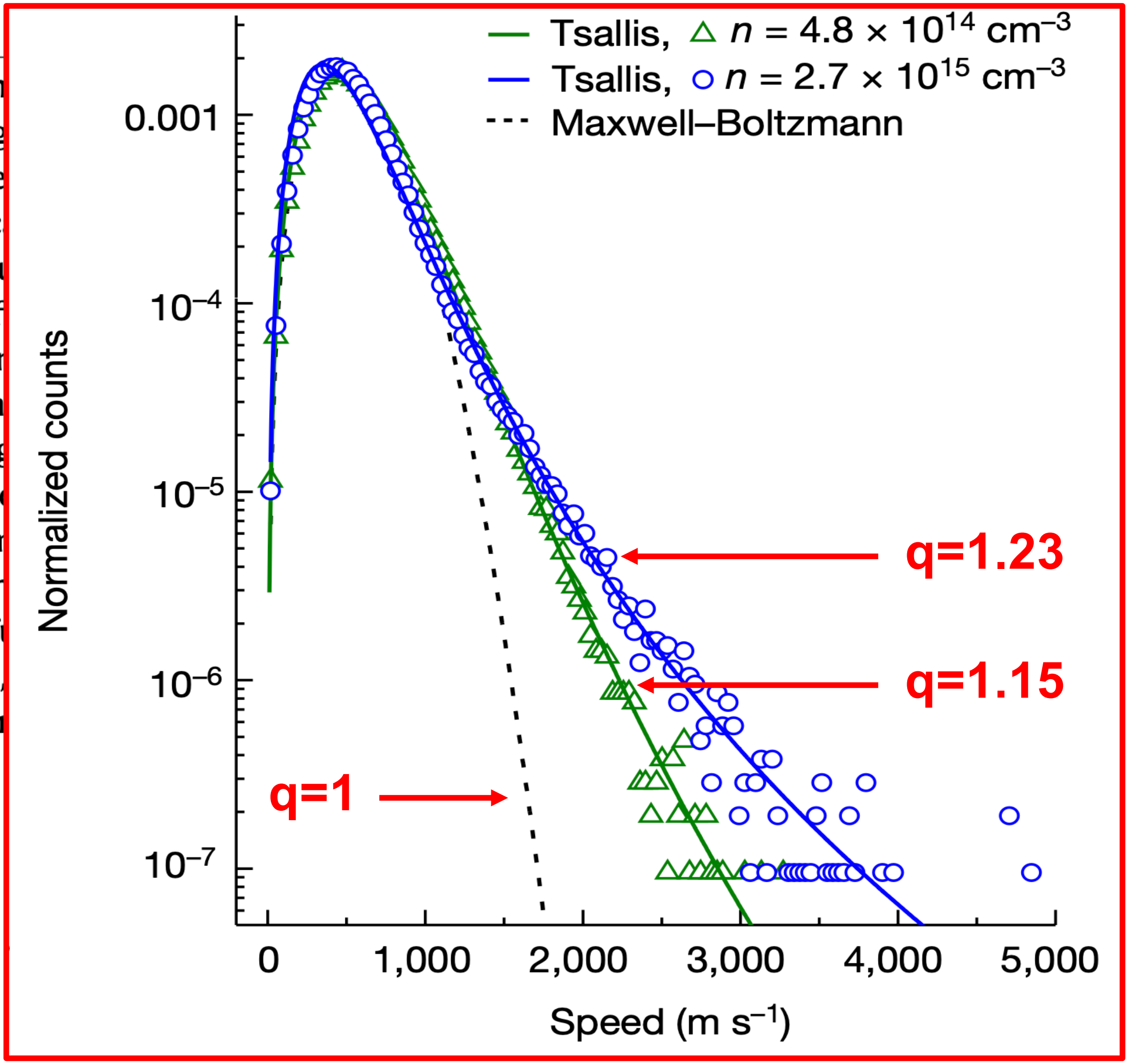
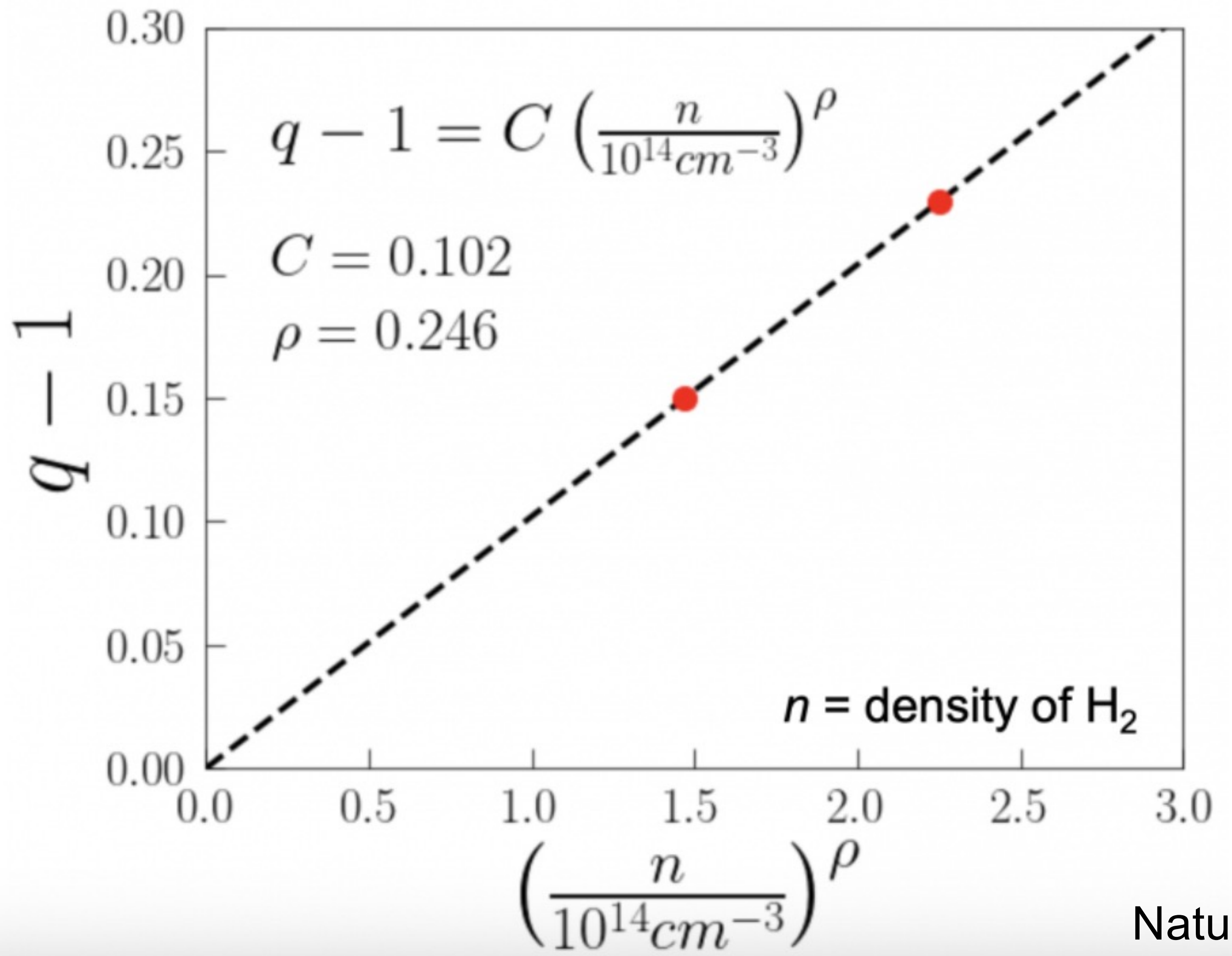


Constantino Tsallis

5 hours ago

Equation (3) [for further details, see for instance C. Tsallis, Introduction to Nonextensive Statistical Mechanics – Approaching a Complex World – Second Edition (Springer, 2023)] is illustrated in Fig. 3 (c). Prof. Roland Webster kindly shared with me the index q corresponding to the values of the hydrogen density n indicated in Fig. 3 (c). With this information, it is possible to construct the attached figure. The ideal gas limit (n=0) corresponds, as expected, to q=1, i.e., to Boltzmann-Gibbs statistical mechanics. Further experimental validation and/or theoretical approaches of the new connection (q-1) proportional n^{1/4} are naturally very welcome.



https://doi.org/10.1038/s41586-023-05727-z

Nature, 17 May 2023

ELECTROENCEPHALOGRAMS

Neural complexity – Non-extensive statistical-mechanical approach of human electroencephalograms

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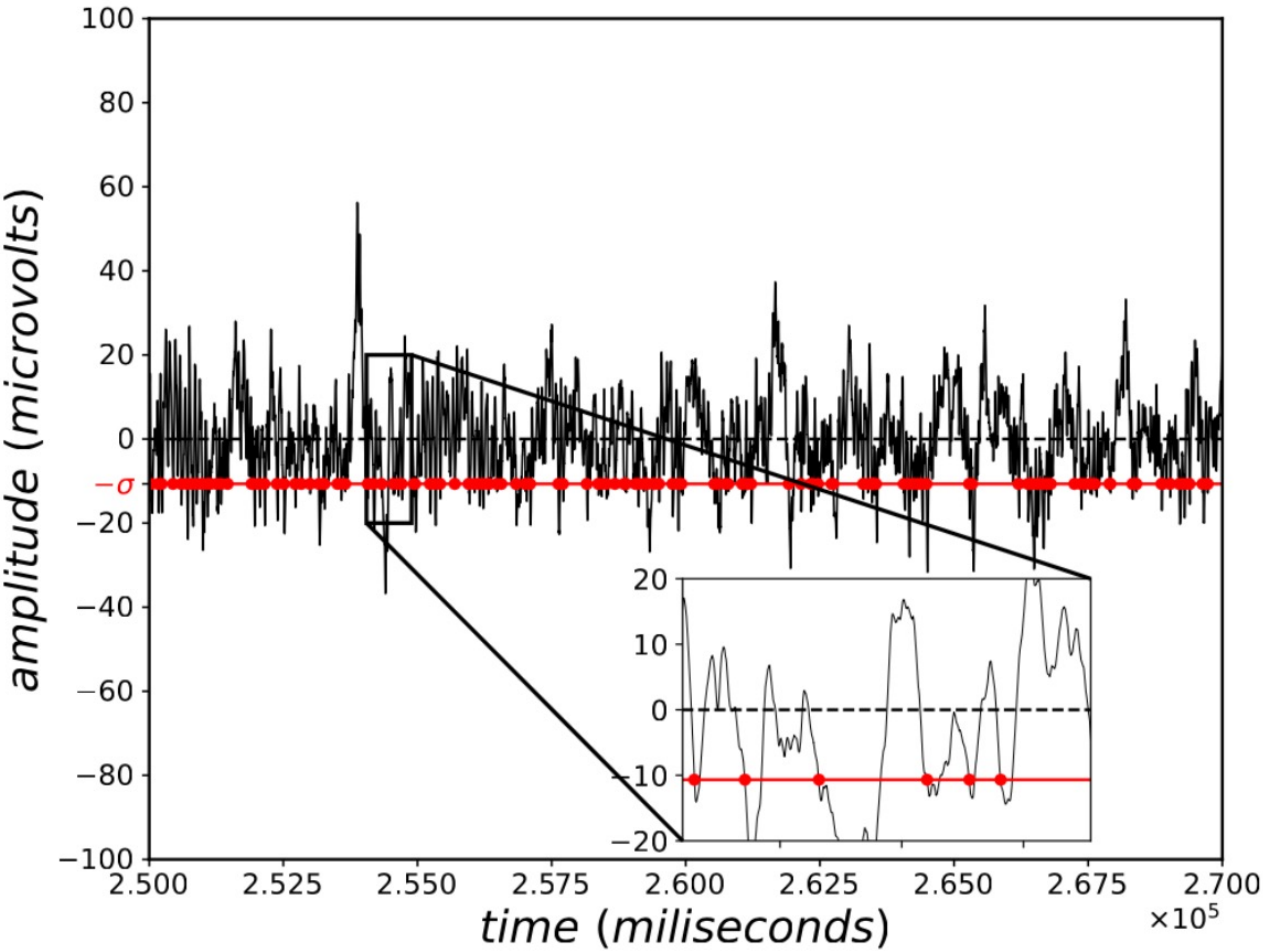
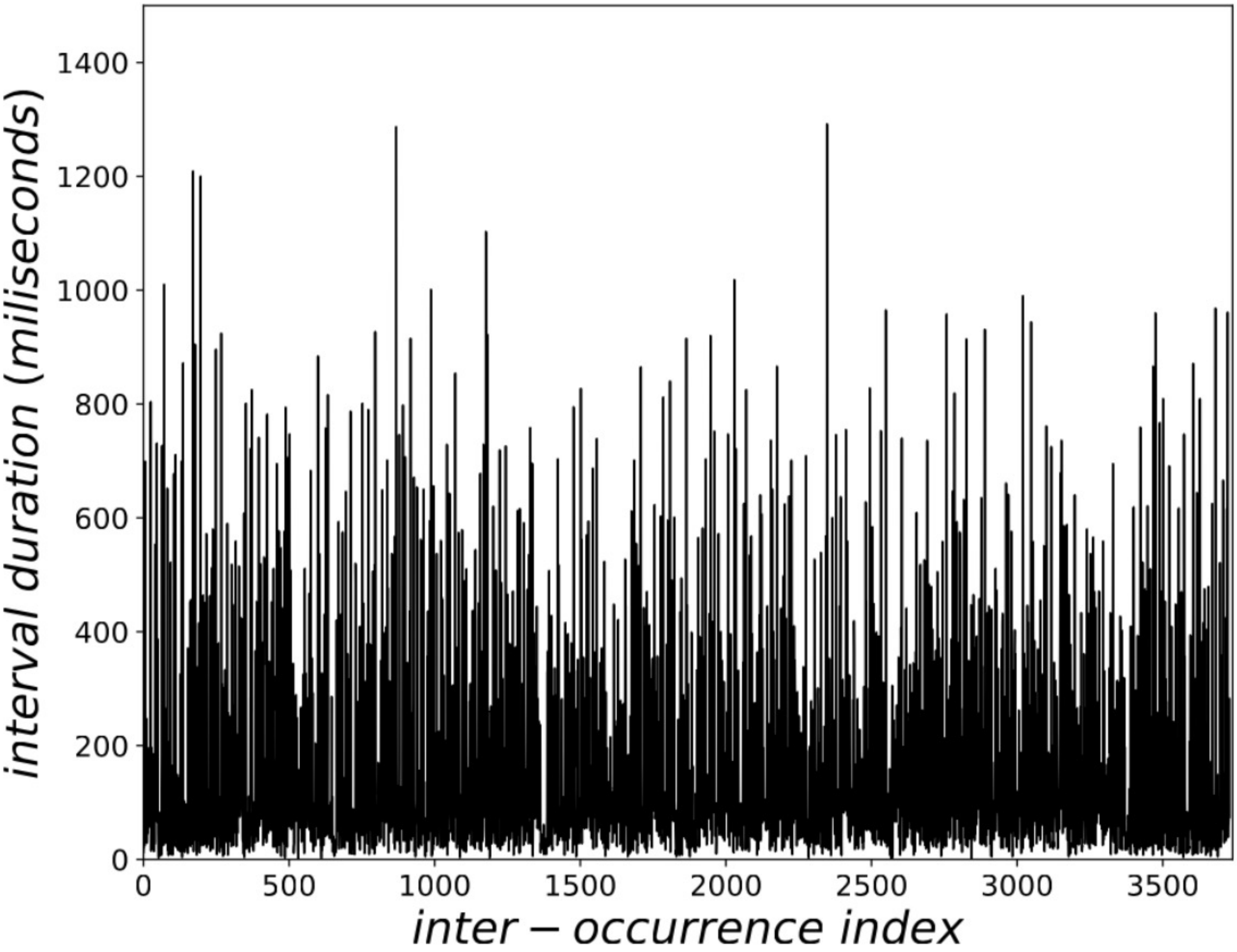
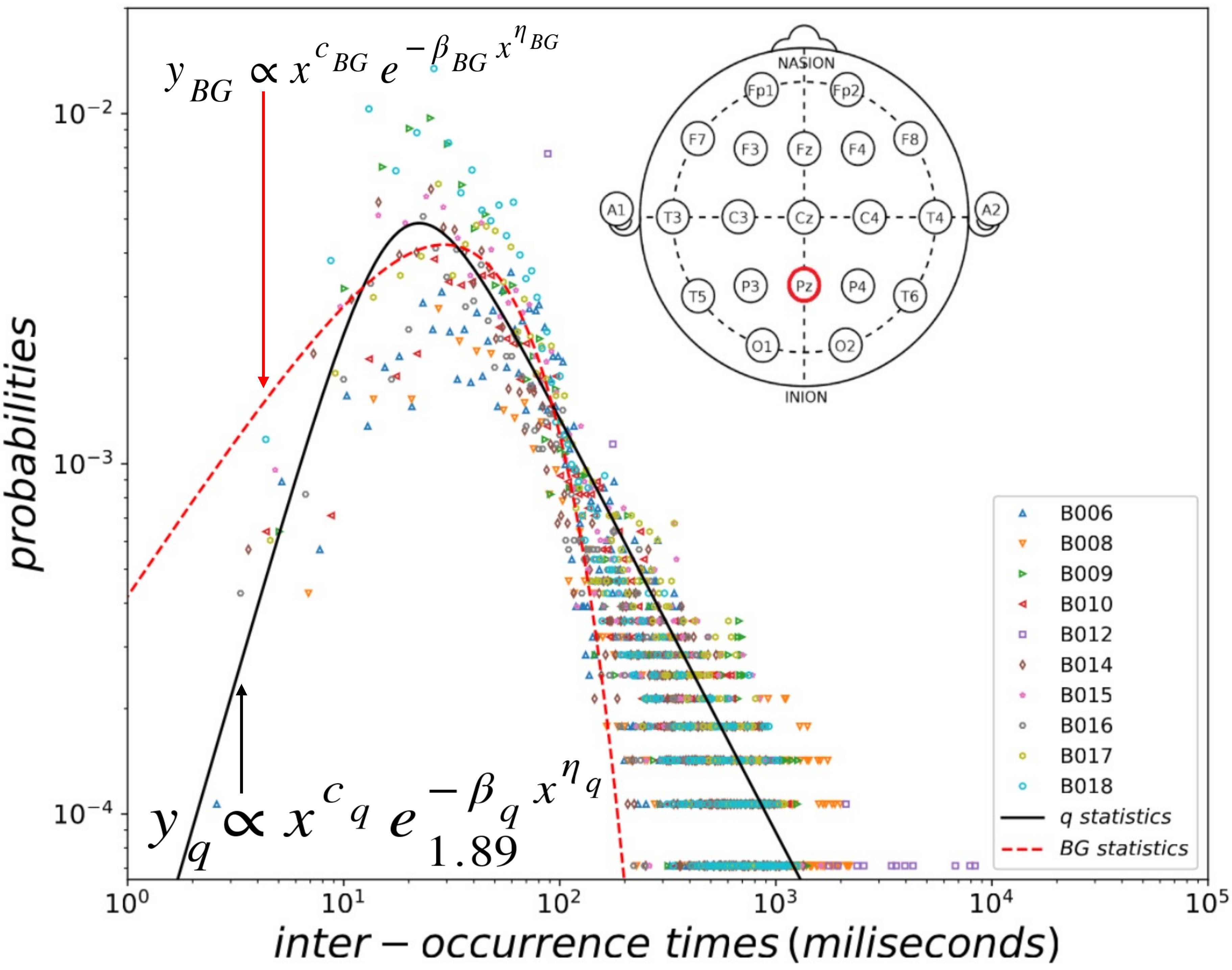
2303.03128 [q-bio.NC]



100
80
60
40
20
0
$-\sigma$
−20
−40
−60
−80
−100
amplitude (microvolts)
2.500
2.525
2.550
2.575
2.600
2.625
2.650
2.675
2.700
time (miliseconds)
×10^5
20
10
0
−10
−20

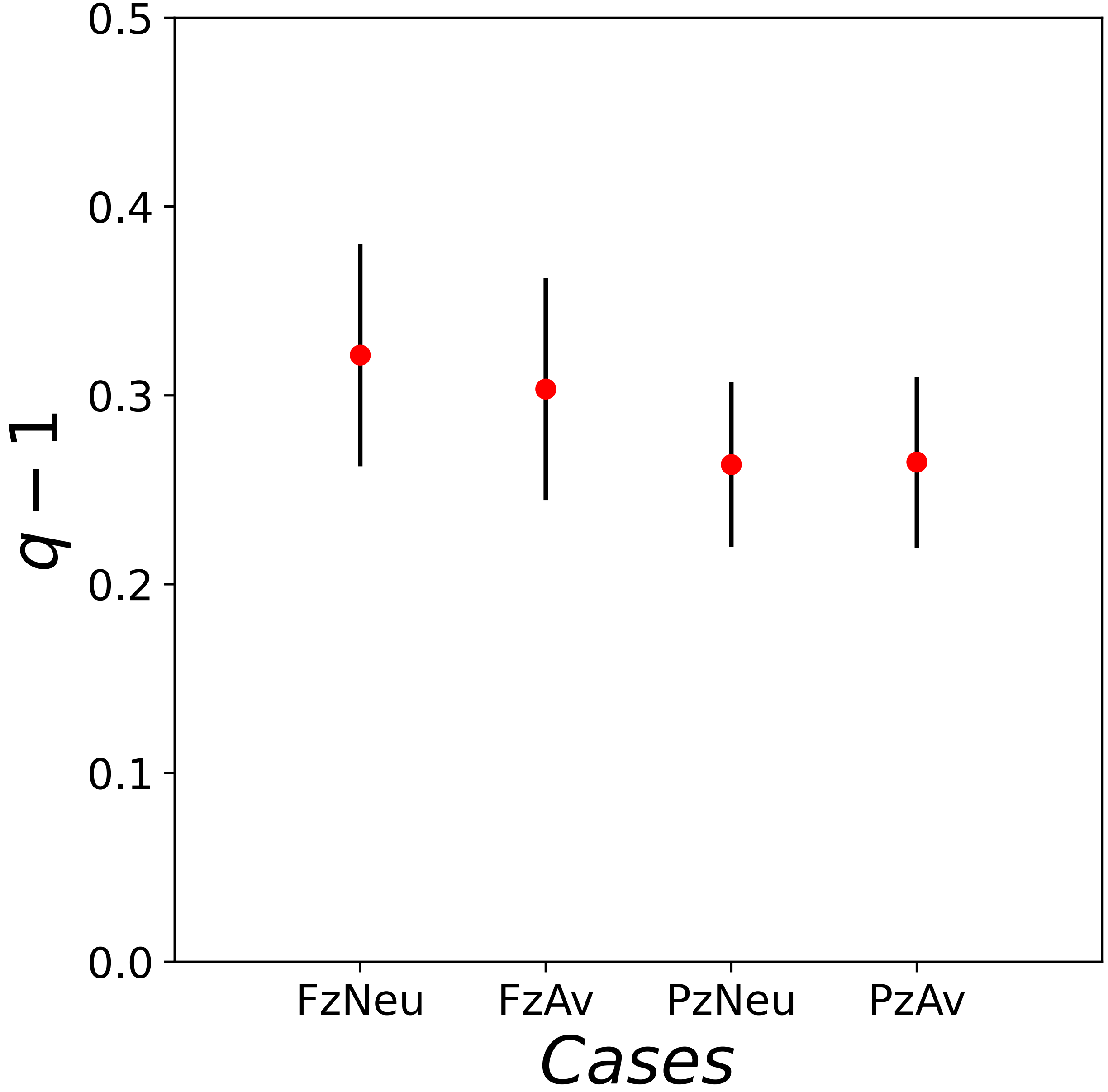


interval duration (miliseconds)
1400
1200
1000
800
600
400
200
0
0
500
1000
1500
2000
2500
3000
3500
inter − occurrence index



$y_{BG} \propto x^{c_{BG}} e^{-\beta_{BG} x^{\eta_{BG}}}$
$y_q \propto x^{c_q} e_{1.89}^{-\beta_q x^{\eta_q}}$
NASION
INION
Fp1
Fp2
F7
F3
Fz
F4
F8
A1
T3
C3
Cz
C4
T4
A2
T5
P3
Pz
P4
T6
O1
O2
B006
B008
B009
B010
B012
B014
B015
B016
B017
B018
q statistics
BG statistics
probabilities
inter − occurrence times (miliseconds)

15 subjects recorded under a cognitive task (neutral or aversive)



ELECTROLYTES

A Letters Journal Exploring the Frontiers of Physics

April 2019

EPL, **126** (2019) 10005
doi: 10.1209/0295-5075/126/10005

www.epljournal.org

New type of equilibrium distribution for a system of charges in a spherically symmetric electric field

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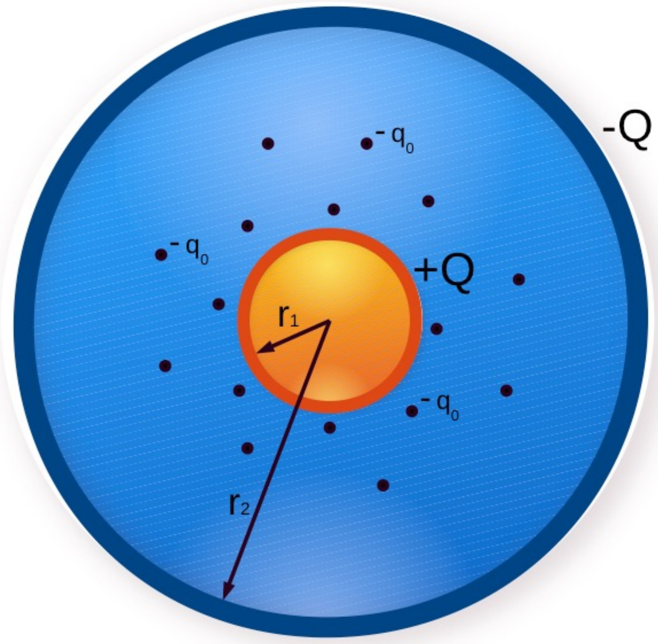
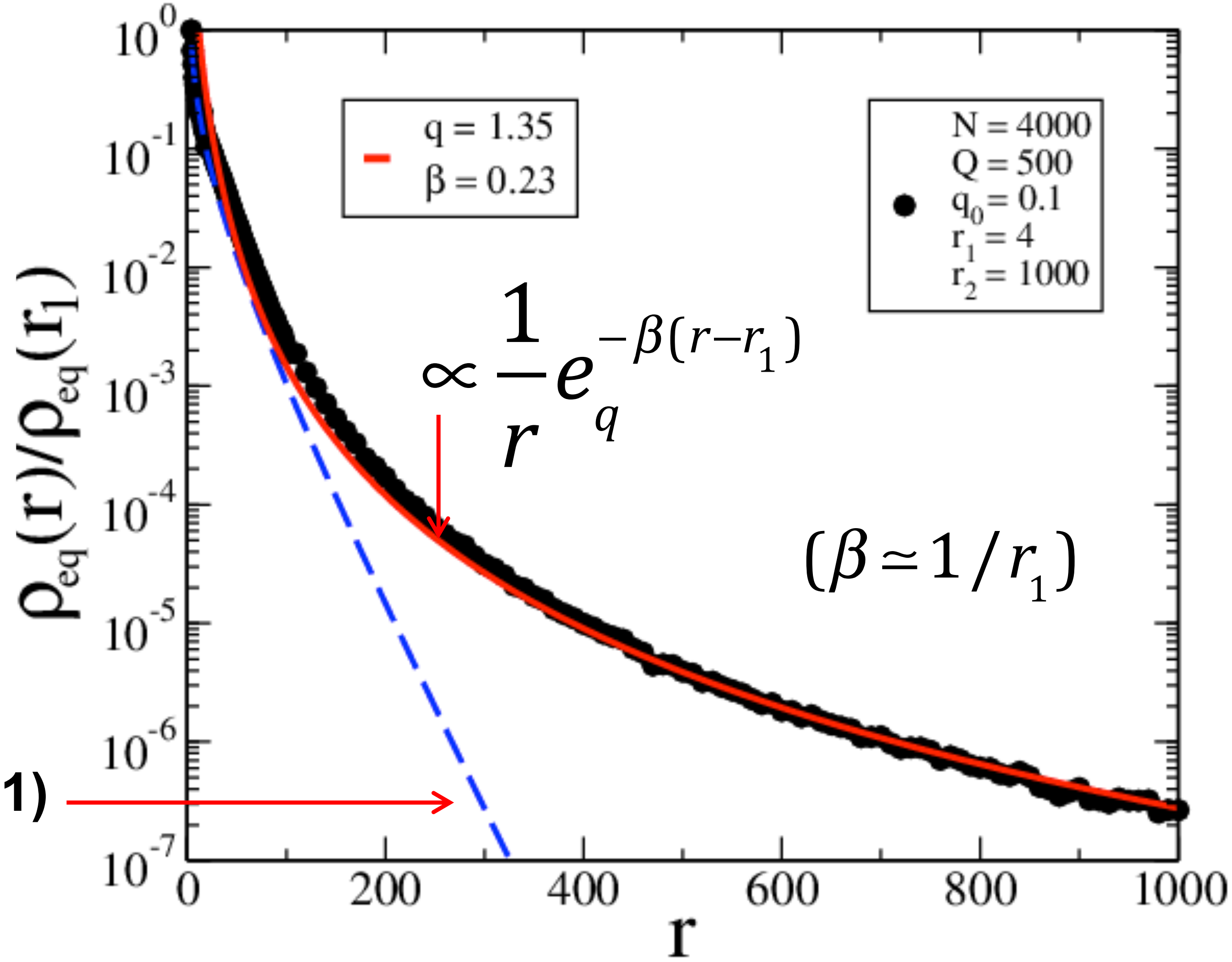
Spherical capacitor (overdamped colloid)





Debye-Hückel (Yukawa) (q=1)

G.A. Casas, F.D. Nobre and E.M.F. Curado (2019)

[see also P. Quarati and A. Scarfone, Astrophys. J. **666**, 1303 (2007)]

Curado and Nobre PRE 2003
Nobre, Curado and Rowlands PRE 2004
Schwammle, Nobre and Curado PRE 2007
Schwammle, Curado and Nobre EPJB 2007

Trace-form entropic functional:

$$S[P]=k\int dx\; g(P(x,t)) \;\; with \;\; g(0)=g(1)=0; \;\; \frac{d^2 g}{dP^2}\leq 0$$

Ansatz for nonlinear Fokker-Planck equation:

$$\frac{\partial P(x,t)}{\partial t}=\frac{\partial}{\partial x}\left\{\frac{d\phi(x)}{dx}\Psi\left[P(x,t)\right]+\Omega\left[P(x,t)\right]\frac{\partial P(x,t)}{\partial x}\right\}$$

$$with \;\Psi>0,\; \Omega>0,\; and\; \phi(x)\equiv confining\; potential$$

Free energy: $$F[P]\equiv U[P]-T_{eff}S[P]=\int dx\;\phi(x)\,P(x,t)-kT_{eff}\int dx\; g(P(x,t))$$

A sufficient condition:

$$\frac{d^2 g[P]}{dP^2} = -\frac{1}{kT_{eff}} \frac{\Omega[P]}{\Psi[P]} \quad (\textit{connection between entropic functional and nonlinear FP equation})$$

$$\Rightarrow \; (i) \; \frac{dF(t)}{dt} = -\int dx\, \Psi[P] \left\{ \frac{d\phi(x)}{dx} + \frac{\Omega[P]}{\Psi[P]} \frac{\partial P(x,t)}{\partial x} \right\}^2 < 0 \quad (H-\textit{Theorem})$$

(*ii*) *one and the same* $P(x)$

simultaneously optimizes the entropy $S|P|$ *with linear constraints*

and yields the stationary state $P(x,\infty)$ *of the associated nonlinear FP equation*

$$S_{q,\delta} = \sum_{i=1}^{W} p_i \left(\ln_q \frac{1}{p_i} \right)^{\delta}$$

C. T. and L.J.L. Cirto 2013

$$S_{q,1} = k_B \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} = k_B \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$$

C. T. 1988

$$S_{1,\delta} = S_{\delta} \equiv k \sum_{i=1}^{W} p_i \left[\ln \frac{1}{p_i} \right]^{\delta}$$

C. T. 2009

$$S_{1,1} = S_{BG}$$

$$\ln_{q,\delta} z \equiv (\ln_q z)^{\delta} = \left(\frac{z^{1-q} - 1}{1-q} \right)^{\delta} \qquad e^{z}_{q,\delta} \equiv e_q^{z^{1/\delta}} = \left[1 + (1-q)\, z^{1/\delta} \right]^{\frac{1}{1-q}}$$

$$S_{q,\delta} = k \sum_i p_i \ln_{q,\delta} \frac{1}{p_i}$$

(C. T. and L.J.L. Cirto 2013, H.S. Lima and C. T. 2020)

$\delta = 1$ $\qquad$ $q = 1$

$$S_{q,1} \equiv S_q = k \sum_i p_i \ln_q \frac{1}{p_i}$$

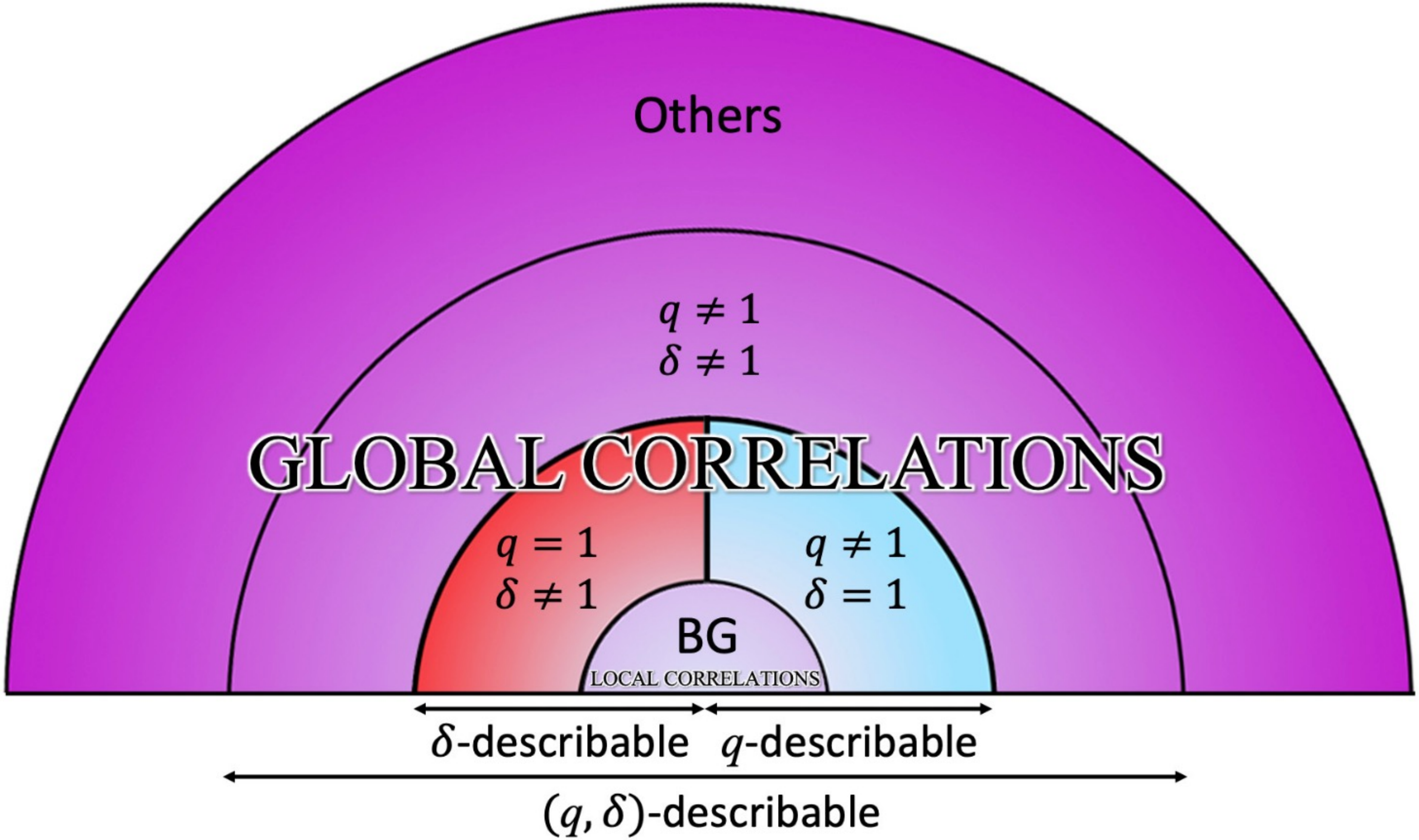
(C. T. 1988)

$$S_{1,\delta} \equiv S_{\delta} = k \sum_i p_i \left(\ln \frac{1}{p_i} \right)^{\delta}$$

(C. T. 2009)

$q = 1$ $\qquad$ $\delta = 1$

$$S_{BG} = k \sum_i p_i \ln \frac{1}{p_i}$$


Others
$q \neq 1$
$\delta \neq 1$
GLOBAL CORRELATIONS
$q = 1$
$\delta \neq 1$
$q \neq 1$
$\delta = 1$
BG
LOCAL CORRELATIONS
δ-describable
q-describable
(q, δ)-describable

Probability distributions extremizing the nonadditive entropy S_δ and stationary states of the corresponding nonlinear Fokker-Planck equation

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(Received 10 July 2013; published 6 November 2013)

Under the assumption that the physically appropriate entropy of generic complex systems satisfies thermodynamic extensivity, we investigate the recently introduced entropy S_δ (which recovers the usual Boltzmann-Gibbs form for $\delta = 1$) and establish the microcanonical and canonical extremizing distributions. Using a generalized version of the H theorem, we find the nonlinear Fokker-Planck equation associated with that entropic functional and calculate the stationary-state probability distributions. We demonstrate that both approaches yield one and the same equation, which in turn uniquely determines the probability distribution. We show that the equilibrium distributions asymptotically behave like stretched exponentials, and that, in appropriate probability-energy variables, an interesting return occurs at $\delta = 4/3$. As a mathematically simple illustration, we consider the one-dimensional harmonic oscillator and calculate the generalized chemical potential for different values of δ.

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial\{A(x)P(x,t)\}}{\partial x} + D\delta\frac{\partial}{\partial x}\left\{\left(\left[\ln\frac{1}{P(x,t)}\right]^{\delta-1} - (\delta-1)\left[\ln\frac{1}{P(x,t)}\right]^{\delta-2}\right)\frac{\partial P(x,t)}{\partial x}\right\}, \quad (14)$$

PHYSICAL REVIEW E **89**, 052135 (2014)

Probability distributions and associated nonlinear Fokker-Planck equation for the two-index entropic form $S_{q,\delta}$

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The probability distributions and associated Fokker-Planck equation of the recently postulated entropic form, $S_{q,\delta}$, are investigated. This entropy was proposed as an unification of the well-known S_q of nonextensive-statistical mechanics and S_δ, which appeared lately as a possibly appropriate candidate for the black-hole entropy. The connection between $S_{q,\delta}$ and a nonlinear Fokker-Planck equation, such as to satisfy an H-theorem, is explored. The stationary-state probability distribution follows a transcendental equation, which is solved numerically for typical values of q and δ. The same transcendental equation is obtained through the maximum-entropy principle, showing that the two procedures are equivalent.

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial\{A(x)P(x,t)\}}{\partial x} + D\delta\frac{\partial}{\partial x}\left\{\left[q\ln_{q,\delta-1}\left(\frac{1}{P}\right)P^{q-1} - (\delta-1)\ln_{q,\delta-2}\left(\frac{1}{P}\right)P^{2q-2}\right]\frac{\partial P(x,t)}{\partial x}\right\}$$

$$A(x) = -\frac{d\phi(x)}{dx} \quad (\phi(x) \text{ is a confining potential})$$

$$g(z) = z \ln \frac{1}{z}$$

$$\Rightarrow (i) \quad S_{BG}[P]$$

$$(ii) \quad \frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left\{ \frac{d\phi(x)}{dx} P(x,t) + D \frac{\partial P(x,t)}{\partial x} \right\}$$

Linear Fokker-Planck Equation

$$(iii) \quad P(x,\infty) = e^{-[\alpha + \beta\phi(x)]}$$

$$g(z) = z \ln_q \frac{1}{z}$$

$$\Rightarrow (i) \quad S_q[P]$$

$$(ii) \quad \frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left\{ \frac{d\phi(x)}{dx} P(x,t) + D_q \frac{\partial [P(x,t)]^{2-q}}{\partial x} \right\}$$

$$(iii) \quad P(x,\infty) = e_q^{-[\alpha_q + \beta_q \phi(x)]}$$

Plastino and Plastino Equation 1995

$$A(x) = -\frac{d\phi(x)}{dx} \quad (\phi(x) \text{ is a confining potential})$$

$$g(z) = z\left(ln\frac{1}{z}\right)^{\delta}$$

$$\Rightarrow (i) \quad S_{\delta}[P]$$

$$(ii) \quad \frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left\{\frac{d\phi(x)}{dx}P(x,t) + D_{\delta}\,\delta\left\{\left(\left[\ln\frac{1}{P(x,t)}\right]^{\delta-1} - (\delta-1)\left[\ln\frac{1}{P(x,t)}\right]^{\delta-2}\right)\frac{\partial P(x,t)}{\partial x}\right\}\right\}$$

$$(iii)\, P(x,\infty) = \text{transcendental function of } \left[\alpha_{\delta} + \beta_{\delta}\,\phi(x)\right]$$

$$g(z) = z\, ln_{q,\delta}\frac{1}{z}$$

$$\Rightarrow (i) \quad S_{q,\delta}[P]$$

$$(ii) \quad \frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left\{\frac{d\phi(x)}{dx}P(x,t) + D_{q,\delta}\,\delta\left\{\left(\left[q\,(P(x,t))^{q-1}\ln_{q,\delta-1}\frac{1}{P(x,t)}\right] - (\delta-1)\left[(P(x,t))^{2q-2}\ln_{q,\delta-1}\frac{1}{P(x,t)}\right]\right)\frac{\partial P(x,t)}{\partial x}\right\}\right\}$$

$$(iii)\, P(x,\infty) = \text{transcendental function of } \left[\alpha_{q,\delta} + \beta_{q,\delta}\,\phi(x)\right]$$

DARK MATTER NEUTRINOS

IceCube Neutrino Obervatory (South Pole)

Eur. Phys. J. C (2022) 82:1123
https://doi.org/10.1140/epjc/s10052-022-11113-2

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Tsallis cosmology and its applications in dark matter physics with focus on IceCube high-energy neutrino data

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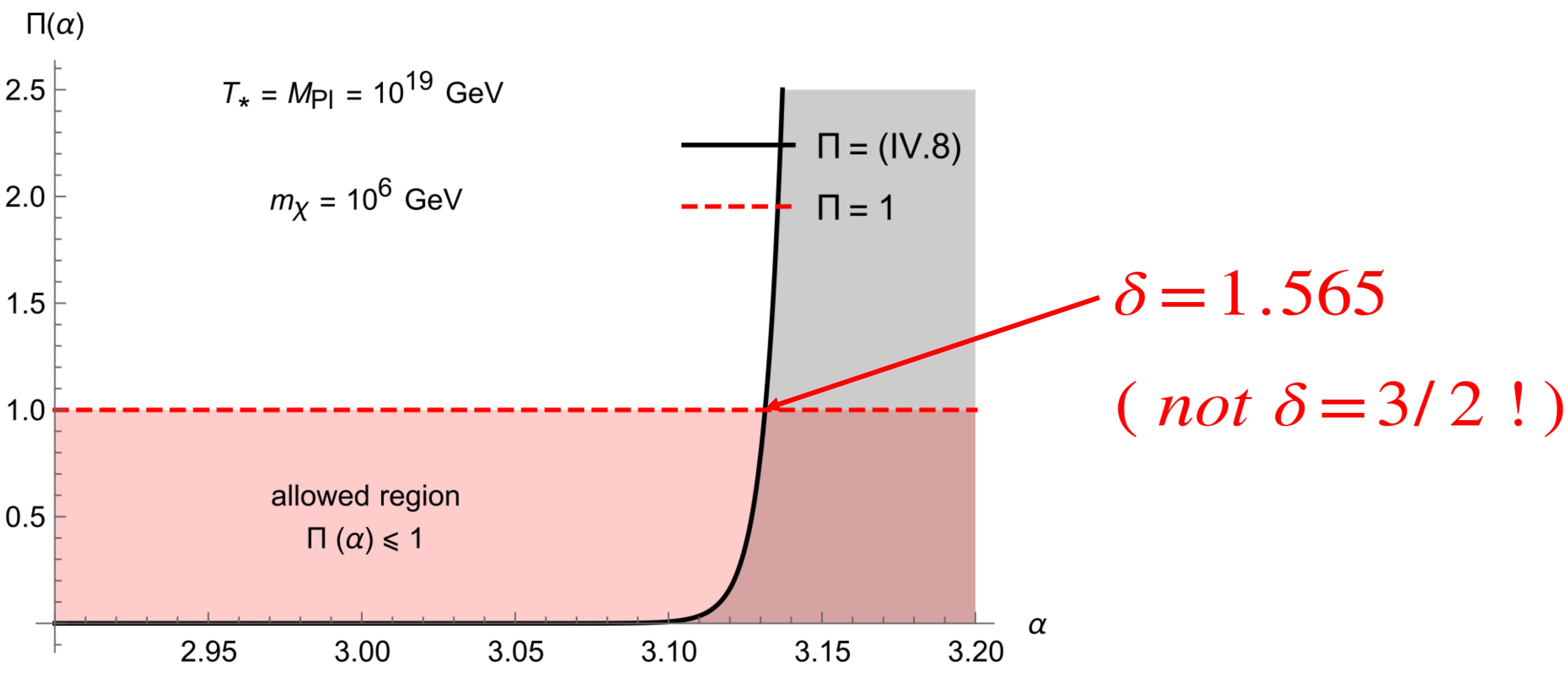
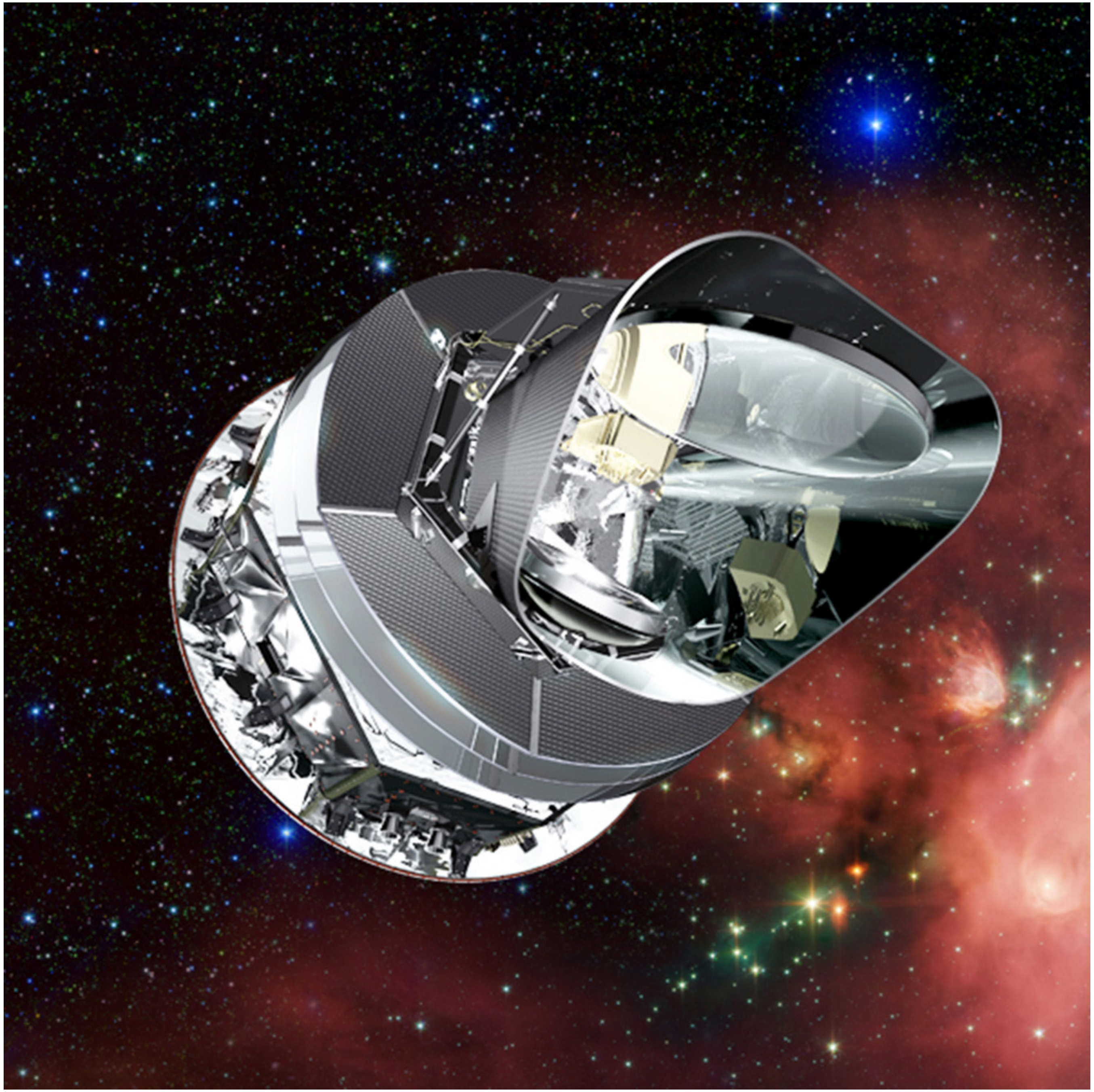
[4] INFN-Gruppo Collegato di Salerno, Salerno, Italy



Planck Observatory / ESA



General Relativity and Gravitation (2023) 55:57
https://doi.org/10.1007/s10714-023-03104-9

RESEARCH ARTICLE

Search for neutrino masses in the Barrow holographic dark energy cosmology with Hubble horizon as IR cutoff

Amin Salehi[1] · M. Pourali[2] · Y. Abedini[2]

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Table 1 Observational constraints at 68% on main and derived parameters of the IDE+ $\sum m_\nu$ scenario

Model	Δ	H_0	Ω_m	$\sum m_\nu$	N_{eff}	h
Pantheon	$1.82^{+.17}_{-.42}$	$69.62^{+0.14}_{-0.14}$	$0.288^{+0.029}_{-0.028}$	< 0.183	$3.02^{+0.17}_{-0.17}$	$0.6962^{+0.0014}_{-0.0014}$
Union2	$1.83^{+.18}_{-.43}$	$69.68^{+0.26}_{-0.26}$	$0.289^{+0.041}_{-0.025}$	< 0.161	$2.83^{+0.2}_{-0.2}$	$0.6968^{+0.0026}_{-0.0026}$
CC	$1.31^{+1.41}_{-0.51}$	66.71^{+1}_{-1}	$0.288^{+0.008}_{-0.008}$	< 0.121	$2.95^{+0.11}_{-0.12}$	$0.6671^{+0.01}_{-0.01}$
Pantheon+cc+Union2	$1.74^{+.25}_{-.17}$	$69.86^{+0.17}_{-0.17}$	$0.280^{+0.1}_{-0.1}$	< 0.134	$2.92^{+0.12}_{-0.12}$	$0.6986^{+0.0017}_{-0.0017}$

The parameter H_0 is in the units of $km/sec/Mpc$, whereas $\sum m_\nu$ reported in the 95% CL, is in the units of eV

$$\textit{Planck Collaboration}\ (2018) + \textit{Approach A} \Rightarrow \Delta = 1.74 \Rightarrow \delta = 1 + \frac{\Delta}{2} = 1.87 > \frac{3}{2}$$

Table 2 Observational constraints at 68% on main and derived parameters of the IDE+ $\sum m_\nu$ scenario

Model	Δ	H_0	Ω_m	$\sum m_\nu$	N_{eff}	β	h
Pantheon	$.25^{+.12}_{-.19}$	$69.51^{+0.3}_{-0.5}$	$0.298^{+0.01}_{-0.011}$	< 0.153	$3.01^{+0.17}_{-0.12}$	$-.2$	$0.6951^{+0.003}_{-0.005}$
Union2	$.32^{+.18}_{-.12}$	$69.98^{+0.15}_{-0.13}$	$0.296^{+0.016}_{-0.013}$	< 0.165	$2.98^{+0.2}_{-0.2}$	$-.19$	$0.6998^{+0.0015}_{-0.0013}$
CC	$.3^{+.12}_{-0.12}$	$67.21^{+.3}_{-.3}$	$0.285^{+0.03}_{-0.02}$	< 0.275	$2.8^{+0.22}_{-0.22}$	$-.14$	$0.6721^{+0.003}_{-0.003}$
Pantheon+cc+Union2	$.52^{+.1}_{-.08}$	$69.46^{+0.4}_{-0.4}$	$0.276^{+0.006}_{-0.005}$	< 0.152	$3.05^{+0.13}_{-0.13}$	$-.15$	$0.6946^{+0.004}_{-0.004}$

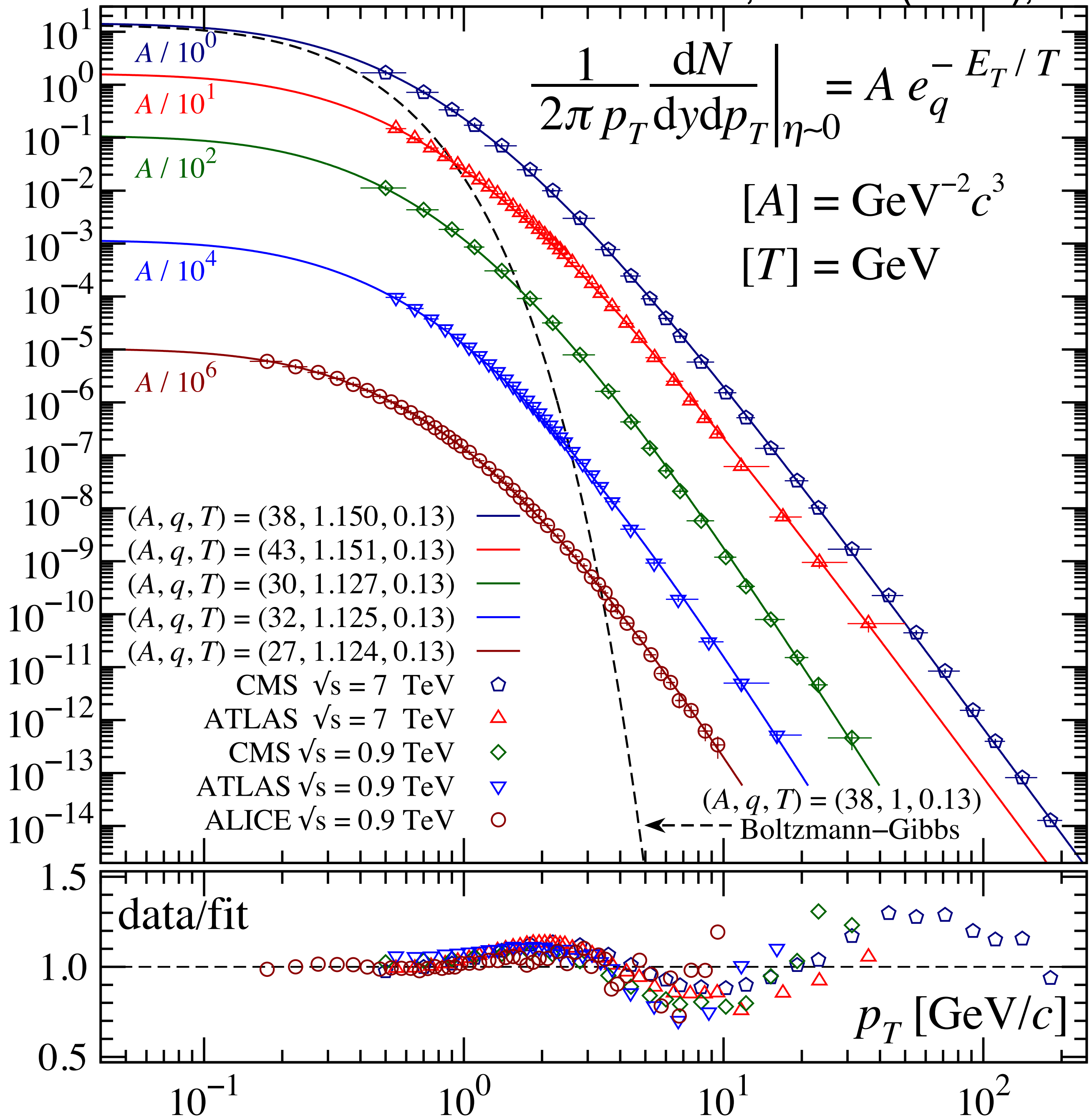
The parameter H_0 is in the units of $km/sec/Mpc$, whereas $\sum m_\nu$ reported in the 95% CL, is in the units of eV

$$\textit{Planck Collaboration}\ (2018) + \textit{Approach B} \Rightarrow \Delta = 0.52 \Rightarrow \delta = 1 + \frac{\Delta}{2} = 1.26 < \frac{3}{2}$$

HIGH ENERGY COLLISIONS

SIMPLE APPROACH: TWO-DIMENSIONAL SINGLE RELATIVISTIC FREE PARTICLE

C.Y. Wong, G. Wilk, L.J.L. Cirto and C. T.,
EPJ Web of Conferences **90**, 04002 (2015), and PRD **91**, 114027 (2015)



$$E_T = \sqrt{m^2c^4 + p_T^2c^2}$$

LHC/CERN proton-proton collisions

$q = 1.14 \pm 0.015$

$T = 0.13$ GeV

pion π^+ mass $= 0.1396$ GeV

pion π^0 mass $= 0.1350$ GeV

Equilibrium Distribution of Heavy Quarks in Fokker-Planck Dynamics

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[2]Physics Department, University of Arizona, Tucson, Arizona 85721
(Received 8 July 1999)

We obtain an explicit generalization, within Fokker-Planck dynamics, of Einstein's relation between drag, diffusion, and the equilibrium distribution for a spatially homogeneous system, considering both the transverse and longitudinal diffusion for dimension $n > 1$. We provide a complete characterization of the equilibrium distribution in terms of the drag and diffusion transport coefficients. We apply this analysis to charm quark dynamics in a thermal quark-gluon plasma for the case of collisional equilibration.

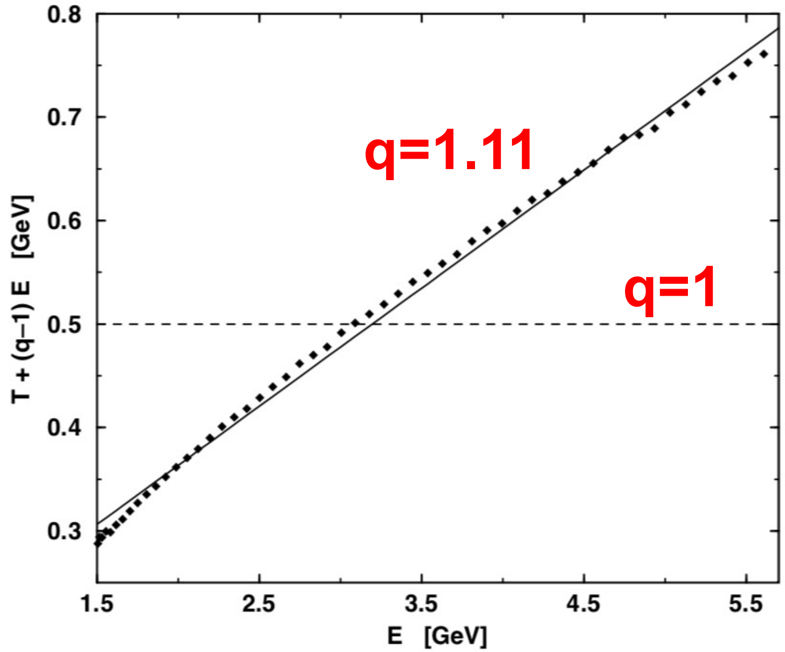

FIG. 1. Calculated data (diamonds) and linear fit for the ratio in Eq. (25) for a charmed quark $m_c = 1.5$ GeV thermalizing in gluon background at $T_b = 500$ MeV. Dashed line: result expected for a Boltzmann-Jüttner distribution, $T = T_b$.

PHYSICAL REVIEW D **101,** 034019 (2020)

Fractals, nonextensive statistics, and QCD

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In this work, we analyze how scaling properties of Yang-Mills field theory manifest as self-similarity of truncated n-point functions by scale evolution. The presence of such structures, which actually behave as fractals, allows for recurrent nonperturbative calculation of any vertex. Some general properties are indeed independent of the perturbative order, what simplifies the nonperturbative calculations. We show that for sufficiently high perturbative orders a statistical approach can be used, the nonextensive statistics is obtained, and the Tsallis index, q, is deduced in terms of the field theory parameters. The results are applied to QCD in the one-loop approximation, where q can be calculated, resulting in a good agreement with the value obtained experimentally. We discuss how this approach allows us to understand some intriguing experimental findings in high energy collisions, as the behavior of multiplicity against collision energy, long-tail distributions, and the fractal dimension observed in intermittency analysis.

First-principle Yang-Mills/QCD grounds yields

$$\frac{1}{q-1} = \frac{11}{3}N_c - \frac{2}{3}N_f$$ (Deppman, Megias and Menezes PRD 2020)

where $N_c \equiv$ number of colors

$N_f \equiv$ number of flavors

hence

$$(N_c, N_f) = (3,6) \Rightarrow q = \frac{8}{7} \simeq 1.14$$ **SU(6)**

(Deppman, Megias and Menezes PRD 2020)

$$(N_c, N_f) = (3,3) \Rightarrow q = \frac{10}{9} \simeq 1.11$$ **SU(3)**

(Walton and Rafelski PRL 2000; C.T. 2022)

Comparative study of the heavy-quark dynamics with the Fokker-Planck Equation and the Plastino-Plastino Equation

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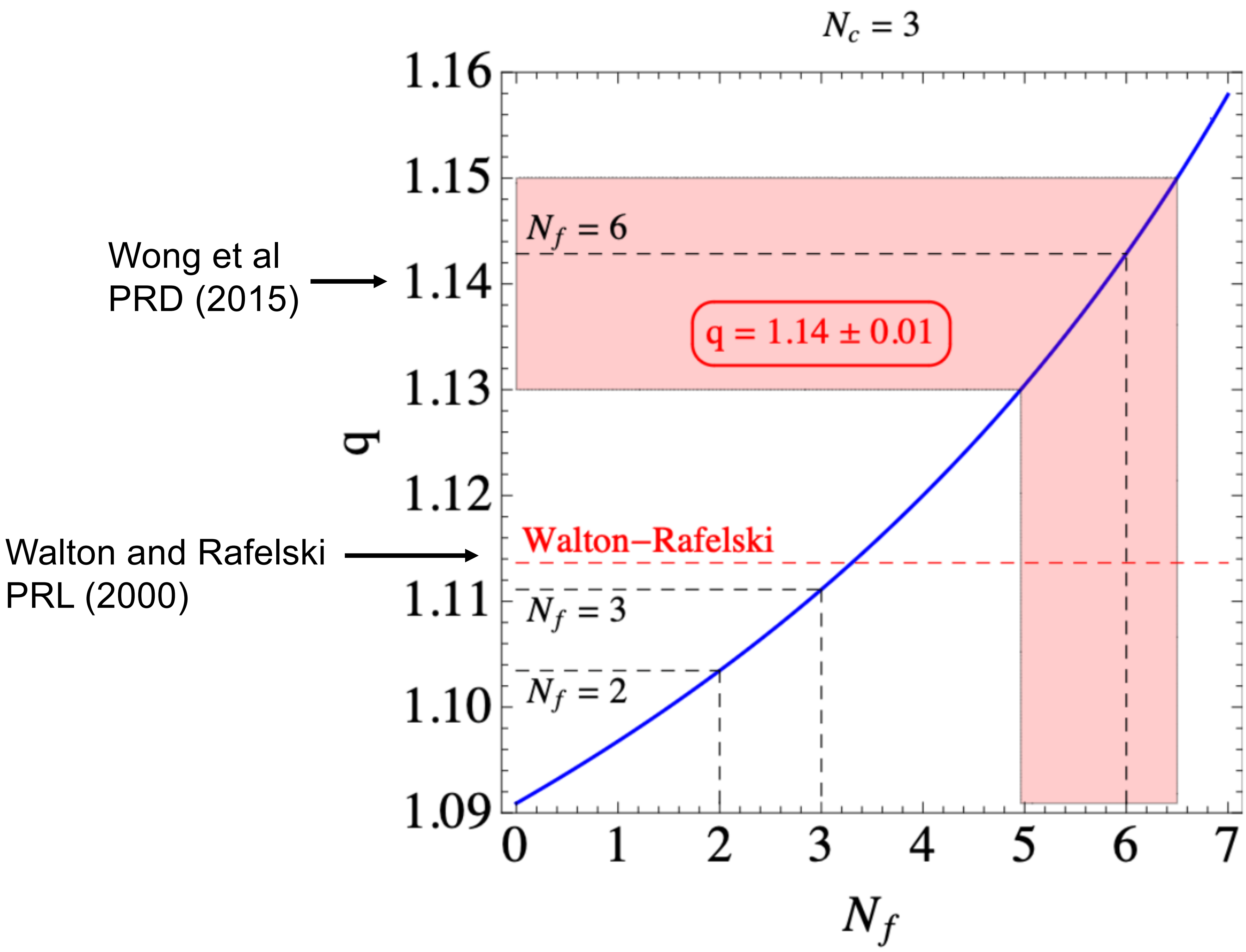
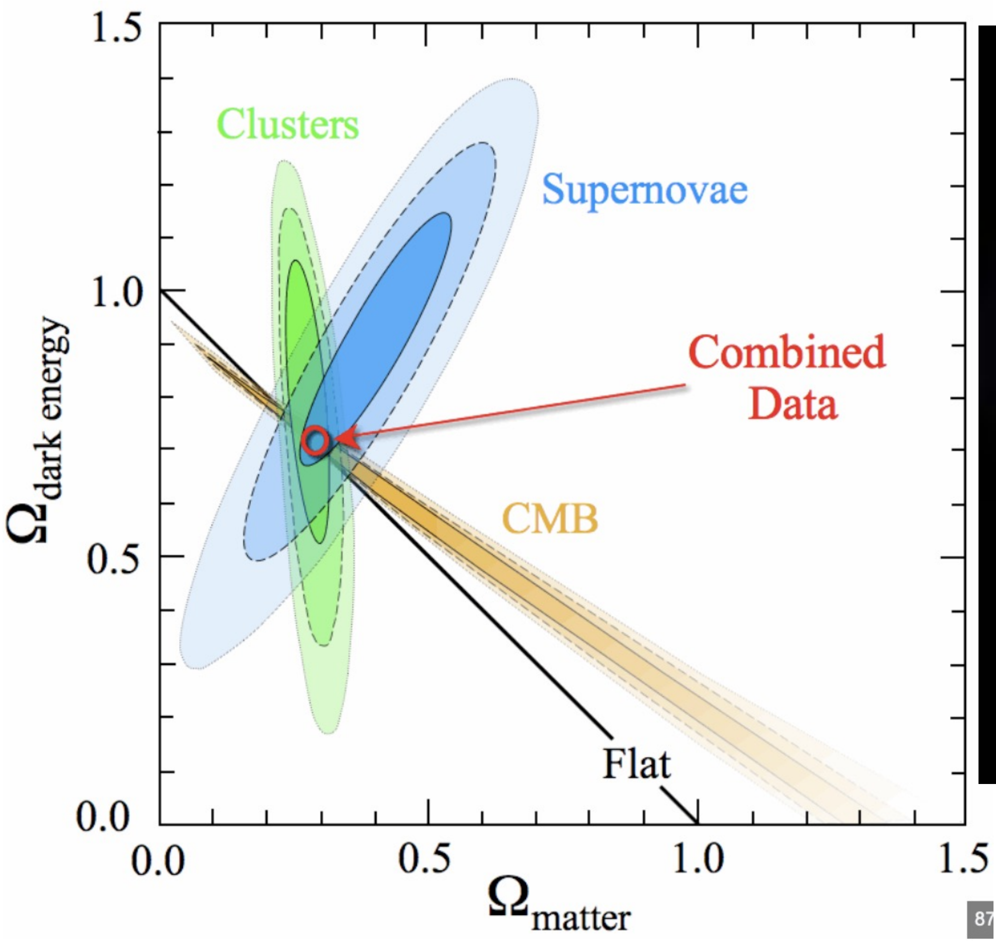
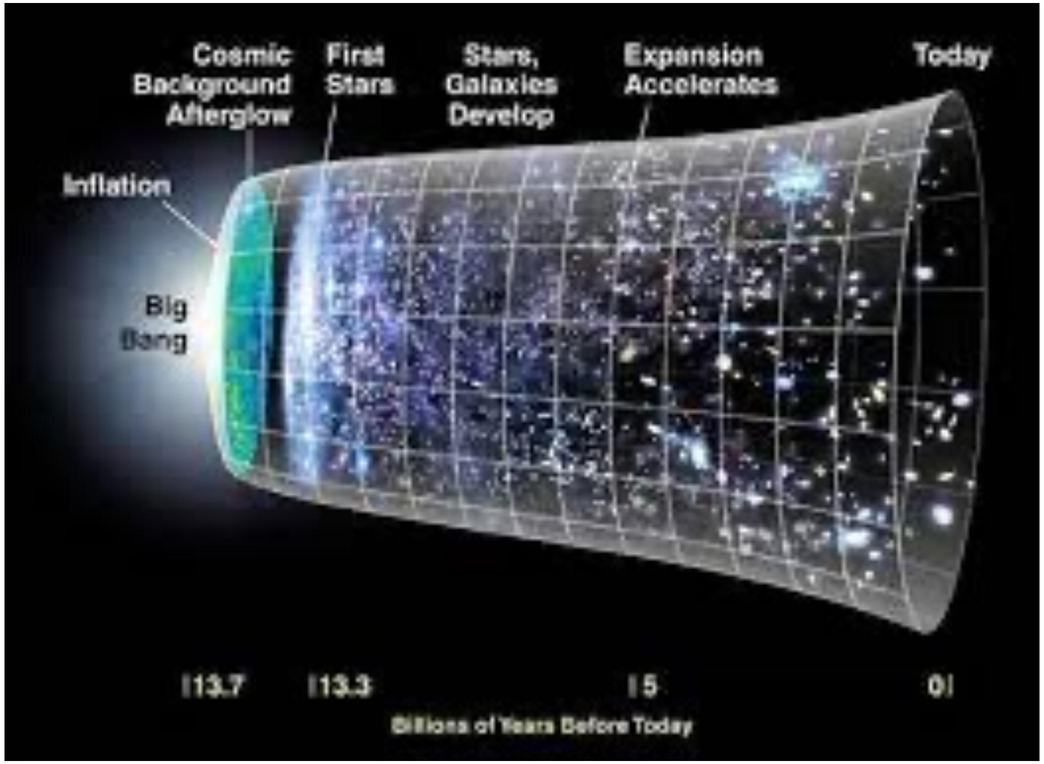
arXiv 2303.03819 [hep-ph]



$N_c = 3$
Wong et al
PRD (2015)
Walton and Rafelski
PRL (2000)
q
$N_f = 6$
q = 1.14 ± 0.01
Walton−Rafelski
$N_f = 3$
$N_f = 2$
1.16
1.15
1.14
1.13
1.12
1.11
1.10
1.09
0
1
2
3
4
5
6
7
N_f

The current Standard Model of Cosmology (SMC), also called the "Concordance Cosmological Model" or the "ΛCDM Model," assumes that the universe was created in the "Big Bang" from pure energy, and is now composed of about 5% ordinary matter, 27% dark matter, and 68% dark energy [1]. Apr 18, 2019





Reviews of Modern Physics, Vol. 54, No. 3, July 1982

Einstein gravity as a symmetry-breaking effect in quantum field theory*

Stephen L. Adler

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$$b_0 = \frac{1}{8\pi^2}\left[\frac{11}{3}n - \frac{2}{3}N_f\right]$$

Universe **2021**, *7*, 402. https://doi.org/10.3390/universe7110402

universe MDPI

Article

Cold Dark Matter: A Gluonic Bose–Einstein Condensate in Anti-de Sitter Space Time †

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† This paper is an extended version from the proceeding paper: Jean-Pierre Gazeau, Gilles Cohen-Tannoudji. The Realization of Some Cosmological Experiments Seems to Favor the Ideas of Sakharov. In Proceedings of the 1st Electronic Conference on Universe, online, 22–28 February 2021.
‡ These authors contributed equally to this work.

$$\left\langle T^{\mu}_{\ \mu} \right\rangle_0 = -\frac{1}{8}\left[11N_c - 2N_f\right]\left\langle \frac{\alpha_s}{\pi}\left(F^a_{\mu\nu}F^{a\mu\nu}\right)^r \right\rangle_0$$

zero with **$N_c = N_f$** leads to **dark / visible = 11/2 = 5.5 ~ 27/5 = 5.4**

THANq