From Landau-Zener transitions to Hawking radiation

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Overview

- Landau-Zener transitions
- Logarithmic phase singularity
- Inverted harmonic oscillator
- Experiment

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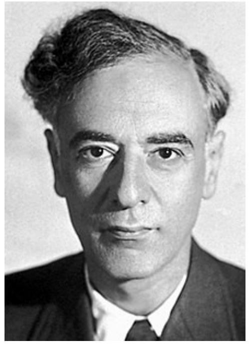
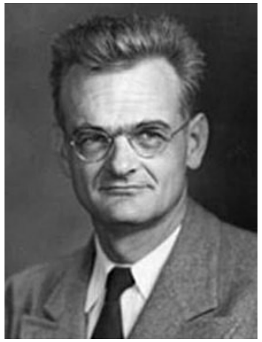
The Landau–Zener formula made simple

Eric P Glasbrenner[1,*] and Wolfgang P Schleich[1,2]



$$H \equiv \hbar \begin{pmatrix} -\alpha t & g \\ g & \alpha t \end{pmatrix}$$



9. A THEORY OF ENERGY TRANSFER. II

The results of I are applied to: 1, predissociation; 2, excitation of oscillations during an optical transition; 3, the case of closely approaching potential curves. The question of the cross-section is discussed.

THE results obtained in I[1] can be applied to a number of effects. It must be added, however, that during the collision of similar atoms it is necessary to add to the selection rules developed in I the postulate of the change in parity which is evident from the fact that the electron-spin components which are normal to the line connecting the nuclei change their sign when mirrored.

1. The most direct generalisation is the predissociation. For the dissociation probability during an oscillation we can use here expression (21) directly. This can also be written in the following form:

$$\omega = \frac{4\pi \hbar D^2}{(F_1 - F_2)\, m^2 r^4} \cdot \frac{j^2}{v}. \qquad (1)$$

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Non-Adiabatic Crossing of Energy Levels.

By CLARENCE ZENER, National Research Fellow of U.S.A.

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1. *Introduction.*

The crossing of energy levels has been a matter of considerable discussion.* The essential features may be illustrated in the crossing of a polar and homopolar state of a molecule.

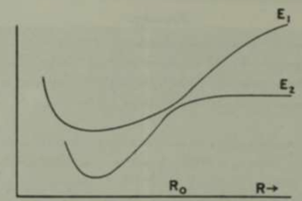


FIG. 1.—Crossing of polar and homopolar states.

Formulation of the problem

$$H \equiv \hbar \begin{pmatrix} -\alpha t & g \\ g & \alpha t \end{pmatrix}$$

$$\dot{a}(t) = -\mathrm{i} g\, \mathrm{e}^{-\mathrm{i}\alpha t^2/2}\, b(t)$$

$$\dot{b}(t) = -\mathrm{i} g\, \mathrm{e}^{\mathrm{i}\alpha t^2/2}\, a(t)$$

$$\dot{a}(t) = -g^2 \mathrm{e}^{-\mathrm{i}\alpha t^2/2} \int_{-\infty}^{t} \mathrm{d}t'\, \mathrm{e}^{\mathrm{i}\alpha t'^2/2}\, a(t')$$

One-line derivation

$$\dot{a}_M(t) = -g^2 e^{-i\alpha t^2/2} \int_{-\infty}^{t} dt' \; e^{i\alpha t'^2/2} \, a_M(t)$$

$$I = \int_{-\infty}^{\infty} dt \; e^{-i\alpha t^2|/2} \int_{-\infty}^{0} dt' \; e^{i\alpha t'^2/2}$$
$$+ \int_{-\infty}^{\infty} dt \; e^{-i\alpha t^2/2} \int_{0}^{t} dt' \; e^{i\alpha t'^2/2}$$

$$a_M(t \to \infty) = \exp\left(-\pi \frac{g^2}{\alpha}\right) = a(t \to \infty)$$

Validity of Markov approximation

$$\dot{a}(t) = -g^2 \mathrm{e}^{-\mathrm{i}\alpha t^2/2} \int_{-\infty}^{t} \mathrm{d}t' \, \mathrm{e}^{\mathrm{i}\alpha t'^2/2} \, a(t')$$

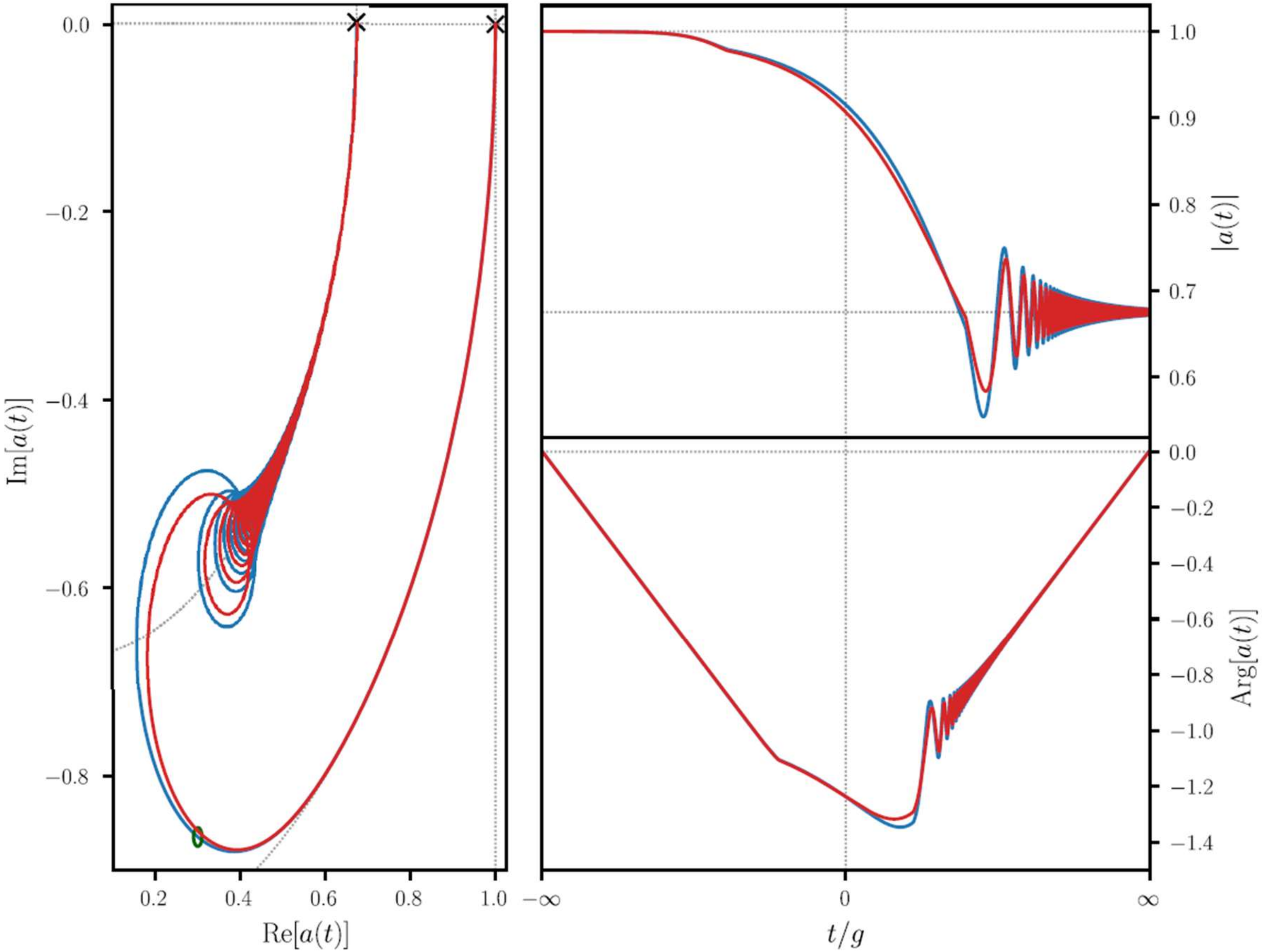
$$\dot{a}(t) = -g^2 \int_{0}^{\infty} \mathrm{d}\tau \, \mathrm{e}^{-\mathrm{i}\alpha\tau t} \mathrm{e}^{\mathrm{i}\alpha\tau^2/2} \, a(t-\tau)$$

Landau-Zener transition: jump between circular paths

$$\dot{a}(t) = -g^2 \int_0^\infty \mathrm{d}\tau \, \mathrm{e}^{-\mathrm{i}\alpha\tau t} \mathrm{e}^{\mathrm{i}\alpha\tau^2/2} \, a(t-\tau)$$

$$\dot{a}_S(t) \approx -g^2 \int_0^\infty \mathrm{d}\tau \, \mathrm{e}^{-\mathrm{i}\alpha t\tau} \, a_S(t)$$

$$a_S(t) = \exp\left[-\pi \frac{g^2}{\alpha} \Theta(t)\right] \exp\left[\mathrm{i}\frac{g^2}{\alpha} \mathcal{P} \int_{-T}^{t} \mathrm{d}t' \frac{1}{t'}\right]$$


0.0
−0.2
−0.4
−0.6
−0.8
Im[$a(t)$]
0.2
0.4
0.6
0.8
1.0
Re[$a(t)$]
1.0
0.9
0.8
0.7
0.6
$|a(t)|$
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
Arg[$a(t)$]
$-\infty$
0
∞
t/g

Stückelberg oscillations

$$\dot{a}(t) = -g^2 \int_0^\infty \mathrm{d}\tau \; \mathrm{e}^{-\mathrm{i}\alpha\tau t} \, \mathrm{e}^{\mathrm{i}\alpha\tau^2/2} \; a(t-\tau)$$

Ernst Carl Gerlach
Stückelberg v. Breidenbach

$$\dot{a}(t) = -g^2 \mathrm{e}^{-\mathrm{i}\alpha t^2/2} \int_0^\infty \mathrm{d}\tau \; \mathrm{e}^{\mathrm{i}\alpha(\tau-t)^2/2} \; a(t-\tau)$$

Alternative approach

$$\dot{a}(t) = -\mathrm{i}g\,\mathrm{e}^{-\mathrm{i}\alpha t^2/2}\,b(t)$$

$$\dot{b}(t) = -\mathrm{i}g\,\mathrm{e}^{\mathrm{i}\alpha t^2/2}\,a(t)$$

$$\ddot{a}(t) + \mathrm{i}\alpha t\dot{a}(t) + g^2 a(t) = 0$$

$$\dot{a}_S(t) = -g^2\frac{1}{\varepsilon + \mathrm{i}\alpha t}a_S(t) = -g^2\frac{\varepsilon - \mathrm{i}\alpha t}{\varepsilon^2 + (\alpha t)^2}a_S(t)$$

Black hole explosions?

From this it follows that the number of particles emitted in this wave packet mode is $(\exp(2\pi\omega/\kappa) - 1)^{-1}$ times the number of particles that would have been absorbed from a similar wave packet incident on the black hole from I^{-}. But this is just the relation between absorption and emission cross sections that one would expect from a body with a temperature in geometric units of $\kappa/2\pi$. Similar results hold for

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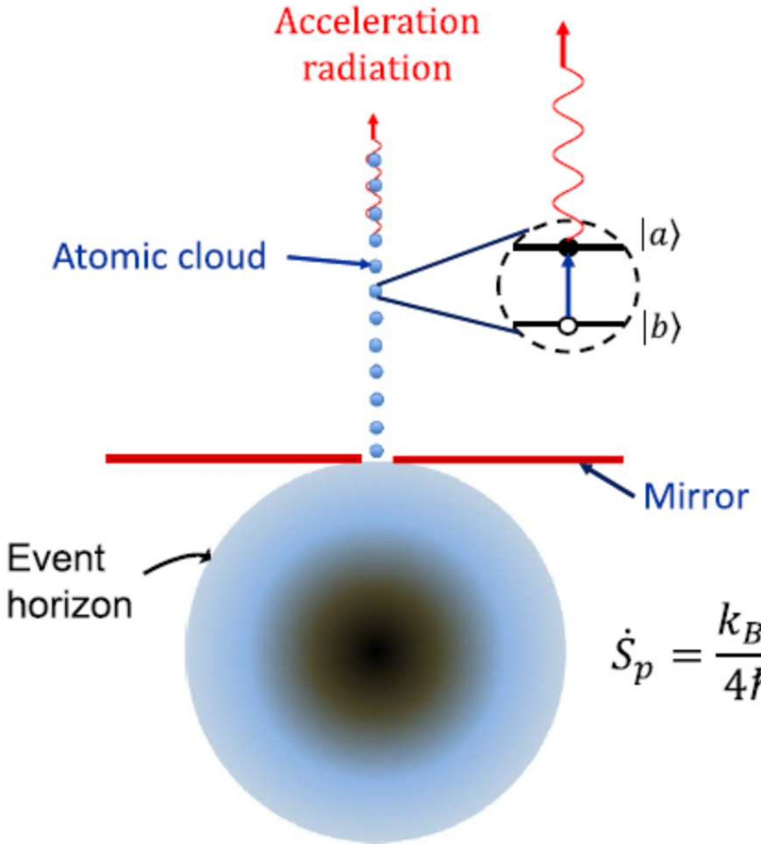
Quantum optics approach to radiation from atoms falling into a black hole

Marlan O. Scully[a,b,c,1], Stephen Fulling[a,d], David M. Lee[a], Don N. Page[e], Wolfgang P. Schleich[a,f], and Anatoly A. Svidzinsky[a]



$$\dot{S}_p = \frac{k_B c^3}{4\hbar G}\dot{A}_p$$

Quantum optics approach to radiation from atoms falling into a black hole

Marlan O. Scully[a,b,c,1], Stephen Fulling[a,d], David M. Lee[a], Don N. Page[e], Wolfgang P. Schleich[a,f], and Anatoly A. Svidzinsky[a]

$$P_{exc} = \frac{g^2}{\omega^2}\left|\int_0^\infty dx e^{-2i\nu \ln x} e^{-ix\left(1+\frac{2\nu}{\omega}\right)}\right|^2$$

$$= \frac{4\pi g^2 \nu}{\omega^2\left(1+\frac{2\nu}{\omega}\right)^2}\frac{1}{e^{4\pi\nu}-1}$$

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Logarithmic phase singularity

$$\psi(r) = (r - r_0)^{i\kappa} = e^{i\kappa \ln(r - r_0)}$$

$$(r - r_0)\frac{d\psi}{dr} = i\kappa\psi$$

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Tunneling of an energy eigenstate through a parabolic barrier viewed from Wigner phase space

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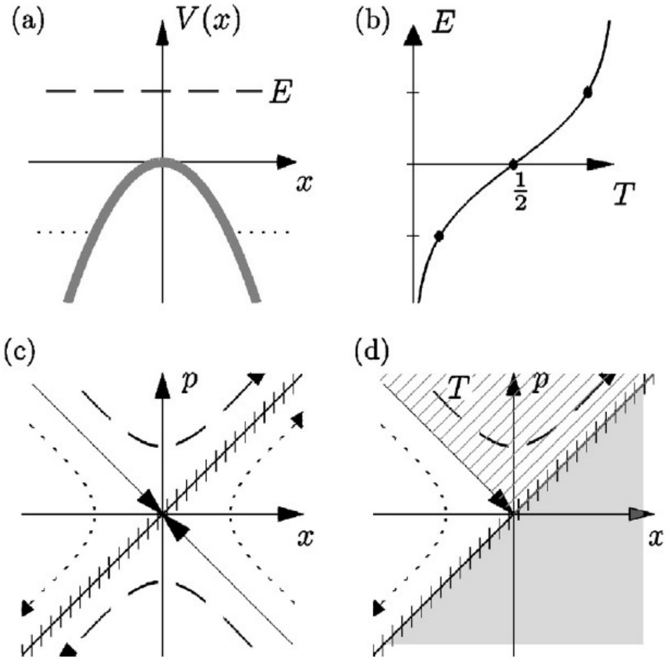
[d] Wigner Research Centre for Physics, Hungarian Academy of Sciences, Institute for Solid State Physics and Optics, 1525 Budapest, Hungary



Inverted oscillator: energy eigenstates

$$\hat{H} \equiv \frac{\hat{p}^2}{2m} - \frac{m\omega^2}{2}\hat{x}^2$$

$$\hat{\xi} \equiv \frac{1}{\sqrt{2}}\left(\hat{x} - \frac{\hat{p}}{m\omega}\right) \qquad \hat{\pi} \equiv \frac{1}{\sqrt{2}}\left(\hat{p} + m\omega\hat{x}\right)$$

$$\hat{H} = -\omega\hat{\xi}\hat{\pi} + \frac{i}{2}\hbar\omega$$

$$u(\xi) = \frac{N_E}{\sqrt{|\xi|}} \exp\left(-i\frac{E}{\hbar\omega}\ln|\xi|\right)$$

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The logarithmic phase singularity in the inverted harmonic oscillator

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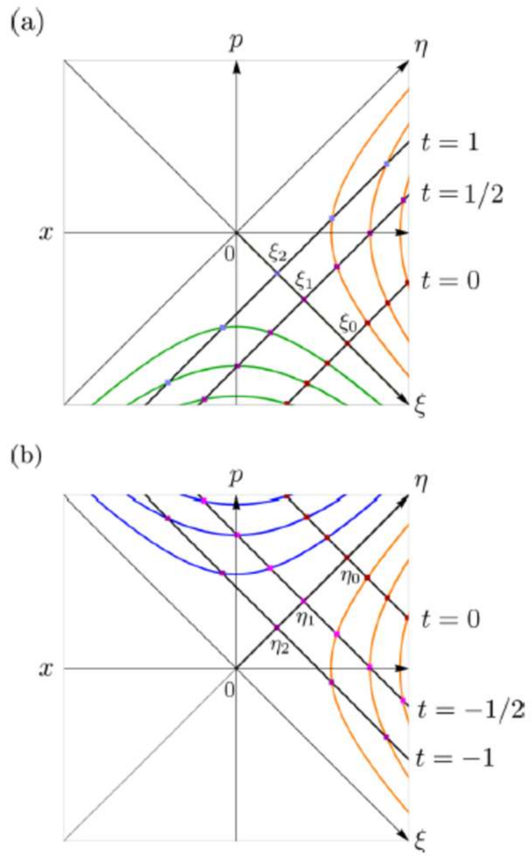
Freyja Ullinger,[1,2] Matthias Zimmermann,[2] and Wolfgang P. Schleich[1,3]



Fermi-Dirac statistics

$$\langle \eta | \Psi_\varepsilon^+ \rangle = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\xi \, \frac{\exp\left(-\mathrm{i}\varepsilon \ln|\xi|\right)}{\sqrt{\xi}} \exp\left(-\mathrm{i}\xi\eta\right)$$

$$|\langle \eta | \Psi_\varepsilon^+ \rangle|^2 = \frac{1}{2\pi|\eta|} \left[\frac{\Theta(\eta)}{1 + \mathrm{e}^{2\pi\varepsilon}} + \frac{\Theta(-\eta)}{1 + \mathrm{e}^{-2\pi\varepsilon}} \right]$$

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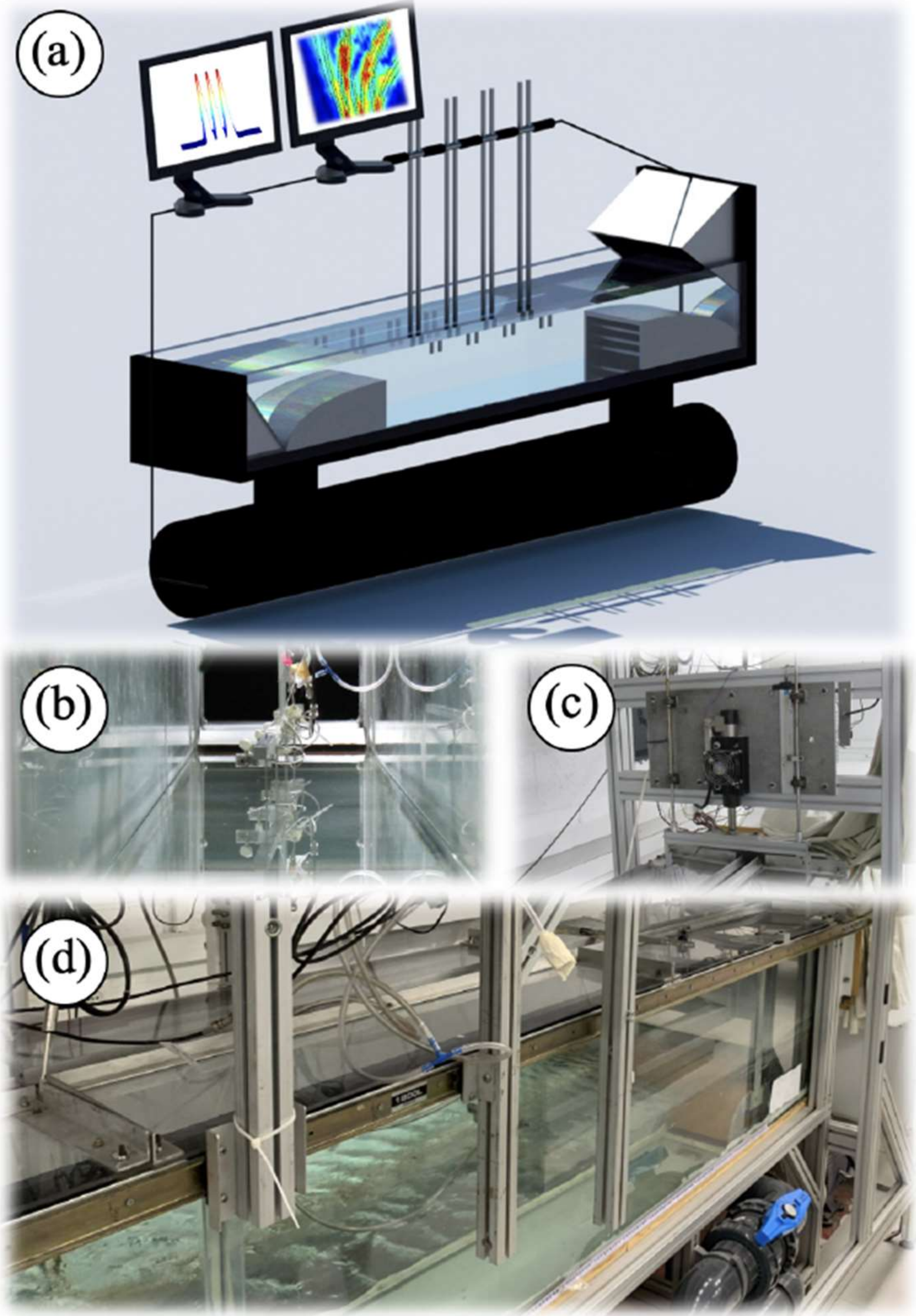
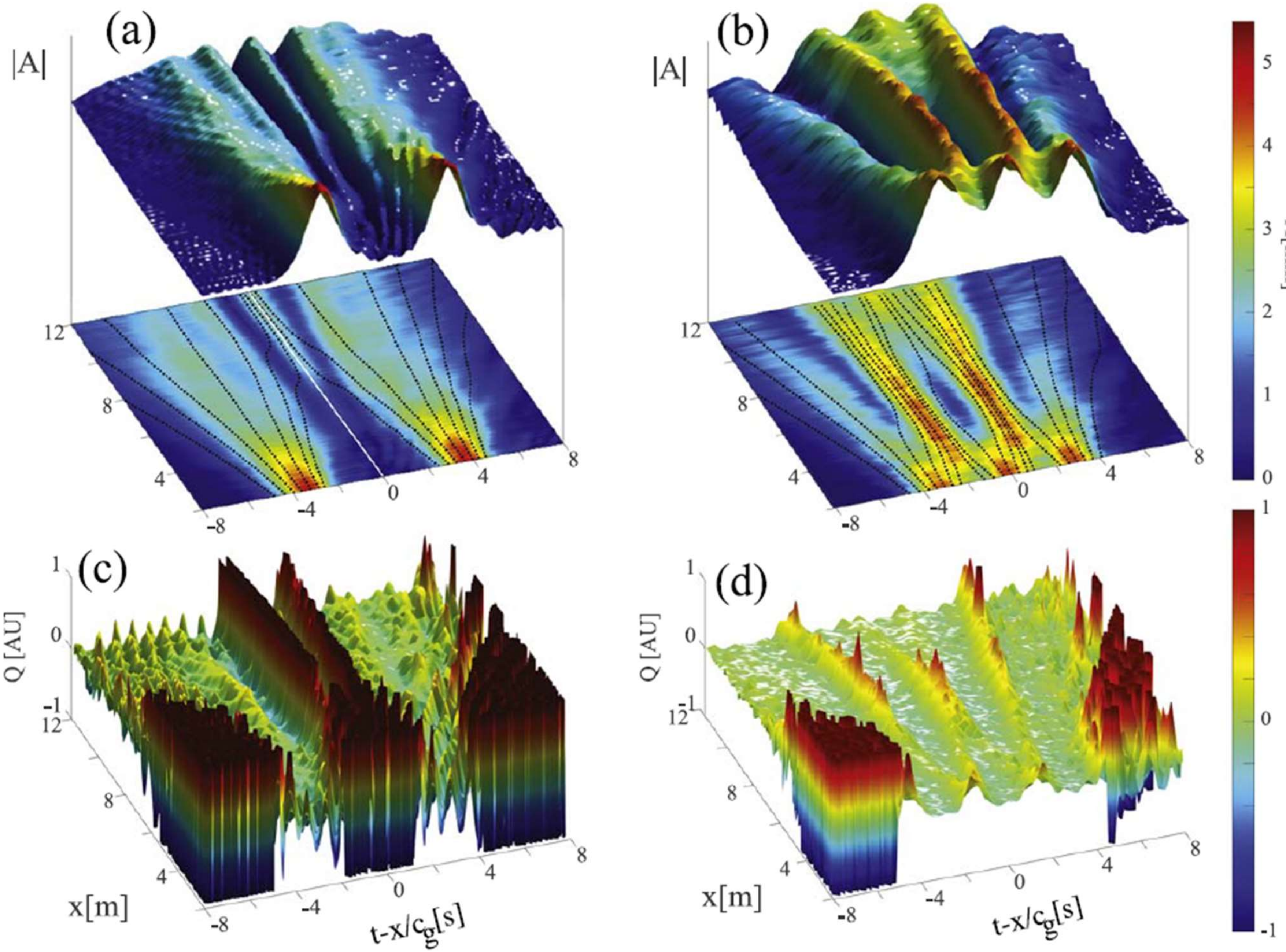
PAPER

Observation of Bohm trajectories and quantum potentials of classical waves

Georgi Gary Rozenman[1,2], Denys I Bondar[3], Wolfgang P Schleich[4,5], Lev Shemer[6] and Ady Arie[2]

Quantity	Quantum mechanics	Surface gravity water waves
Complex-valued function	wave function $\psi(x, t)$	surface envelope $A(\tau, \xi)$
Propagation coordinate	t	ξ
Transverse coordinate	x	τ
Wave equation	$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$	$i\frac{\partial A}{\partial\xi} = \frac{\partial^2 A}{\partial\tau^2}$
Guiding equation	$\frac{d}{dt}\overline{x}(t) = \frac{\hbar}{m} Im\left\{\frac{1}{\psi(\overline{x}(t), t)}\frac{\partial}{\partial\overline{x}}\psi\right\}$	$\frac{d}{d\xi}\overline{\tau}(\xi) = 2\,Im\left\{\frac{1}{A(\xi, \overline{\tau}(\xi))}\frac{\partial}{\partial\overline{\tau}}A\right\}$
Quantum potential	$Q = -\frac{\hbar^2}{2m}\frac{1}{\mid\psi\mid}\frac{\partial^2}{\partial x^2}\lvert\psi\rvert$	$Q = -\frac{1}{\mid A\mid}\frac{\partial^2}{\partial\tau^2}\lvert A\rvert$

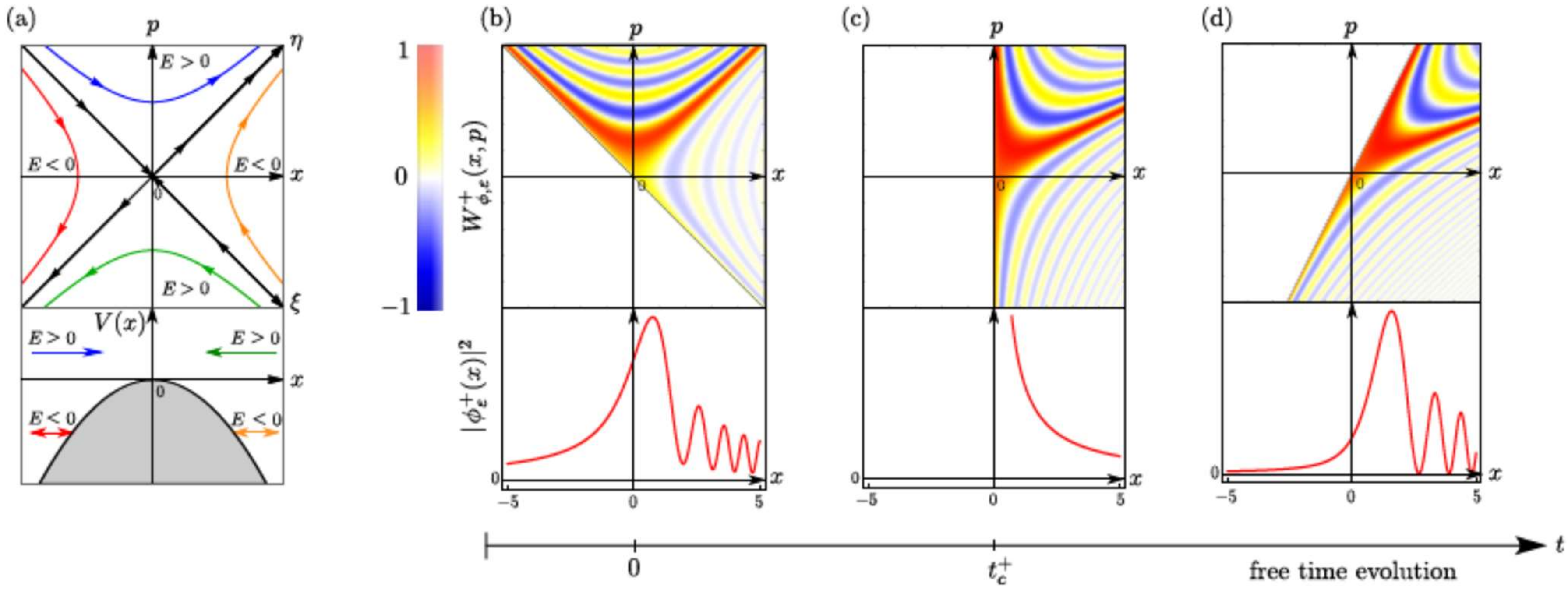

(a)
(b)
(c)
(d)

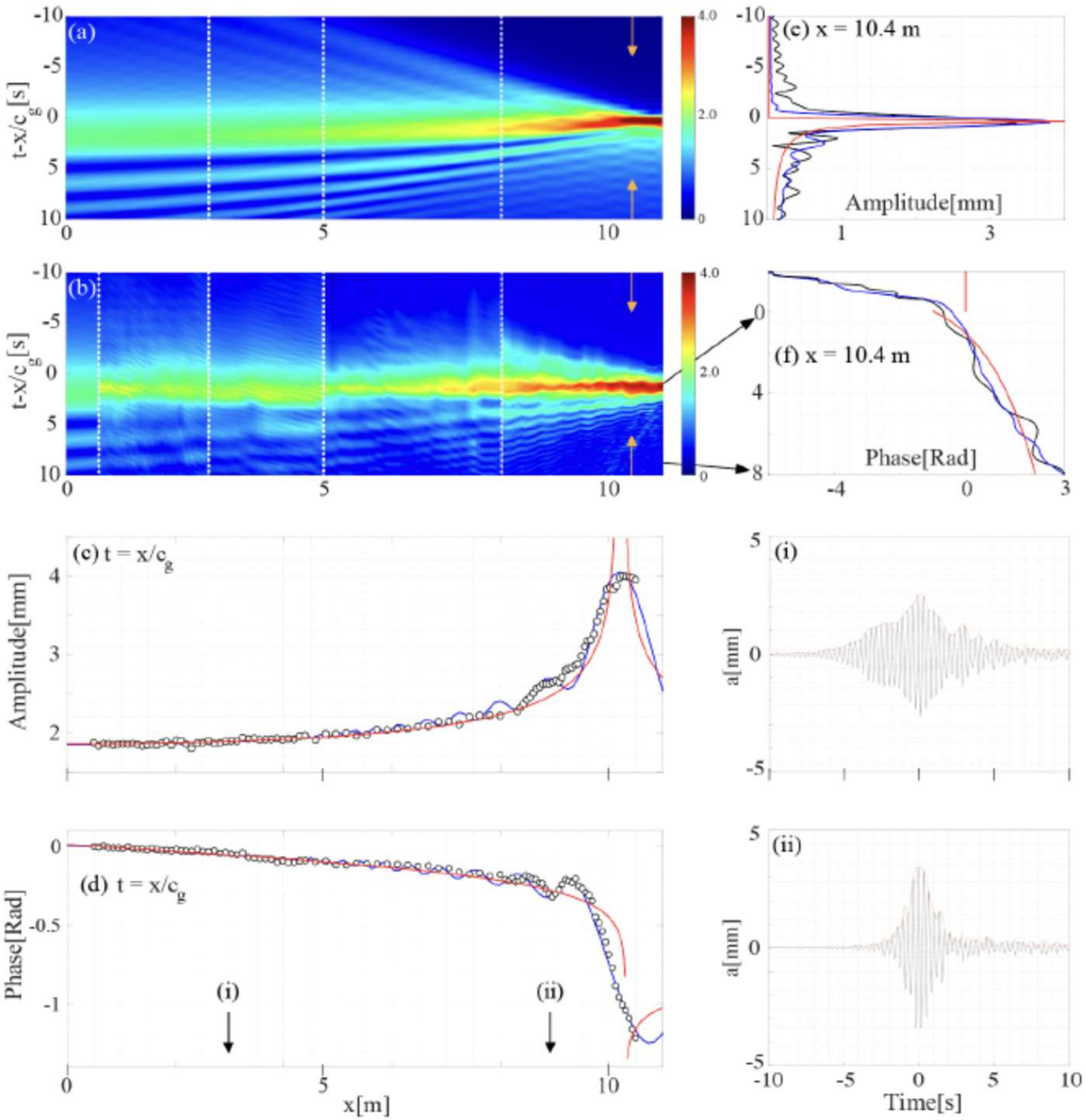


(a)
(b)
(c)
(d)
|A|
|A|
A[mm]
Q [AU]
Q [AU]
x[m]
x[m]
t-x/c_g[s]
t-x/c_g[s]

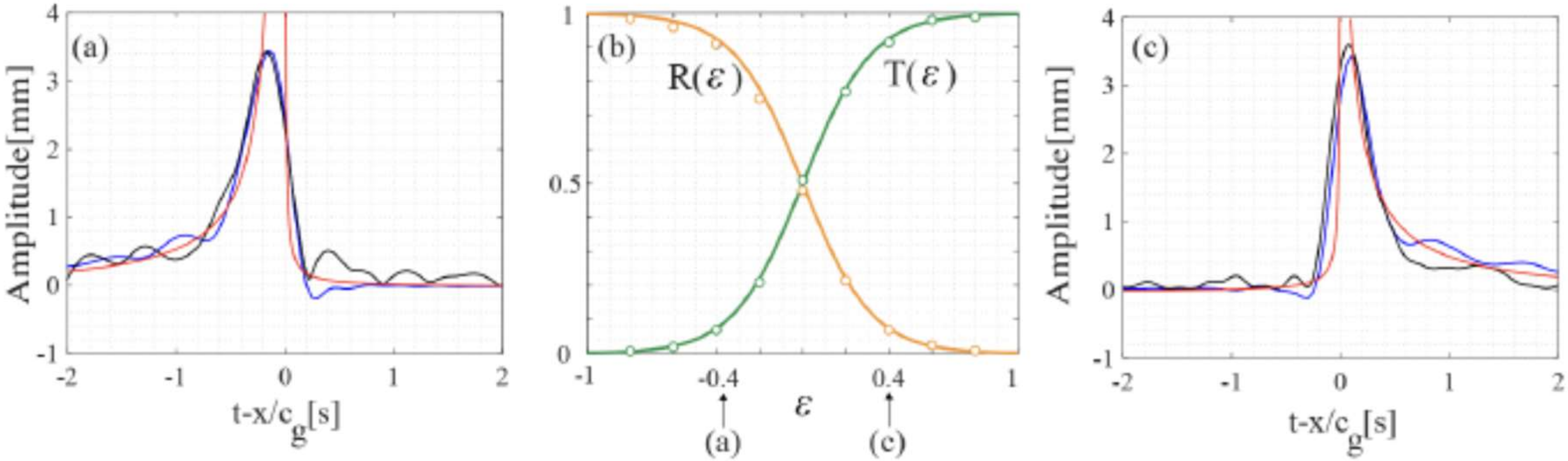
Observation of a phase space horizon with surface gravity water waves

Georgi Gary Rozenman [1,2†], Freyja Ullinger[3,4†], Matthias Zimmermann[3†],
Maxim A. Efremov[3,4], Lev Shemer[5], Wolfgang P. Schleich[4,6], and Ady Arie[2,*]





(a)
(b)
t-x/c_g [s]
(c) x = 10.4 m
Amplitude[mm]
(f) x = 10.4 m
Phase[Rad]
(c) t = x/c_g
(d) t = x/c_g
(i)
(ii)
a[mm]
Time[s]
x[m]



(a)
Amplitude[mm]
t-x/c_g[s]
(b)
R(ε)
T(ε)
ε
(a)
(c)
(c)
Amplitude[mm]
t-x/c_g[s]

Summary

- Landau-Zener transitions
- Logarithmic phase singularity
- Inverted harmonic oscillator
- Experiment