Richardson–Lucy Algorithm Based Image Reconstruction for Proton CT

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GPU Day 2023 – Massive parallel computing
for science and industrial application
Budapest, 16. May 2023

Imaging with protons

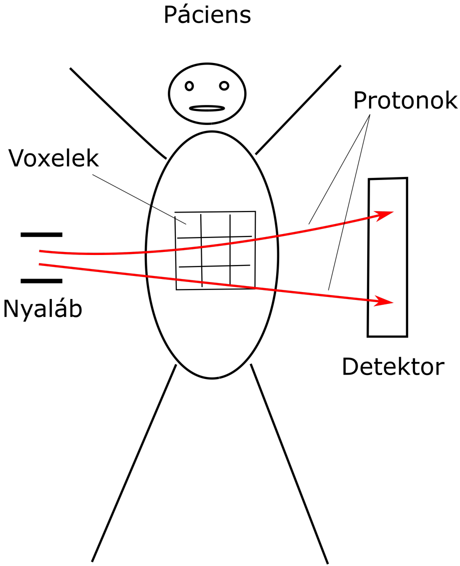


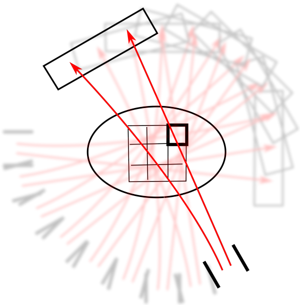
Image Reconstruction – a Huge Linear Problem

Huge linear problem:

$$\mathbf{y} = \mathbf{A}\,\mathbf{x}\,,$$

where:

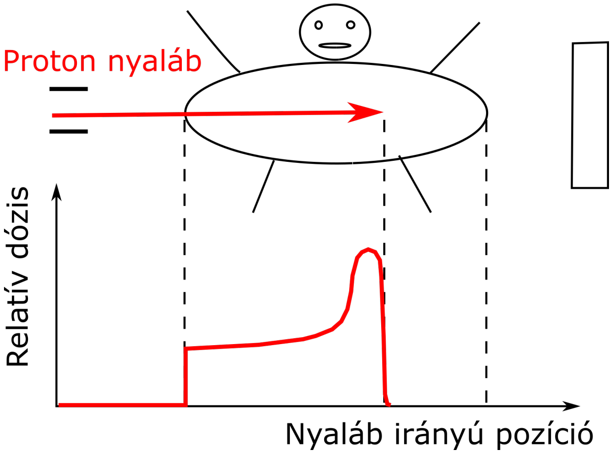
- $\mathbf{y}$ is the energy loss of protons $\Leftrightarrow$ track integral of RSP
- $\mathbf{x}$ RSP value of voxels
- $\mathbf{A}$ proton – voxel interaction coefficients



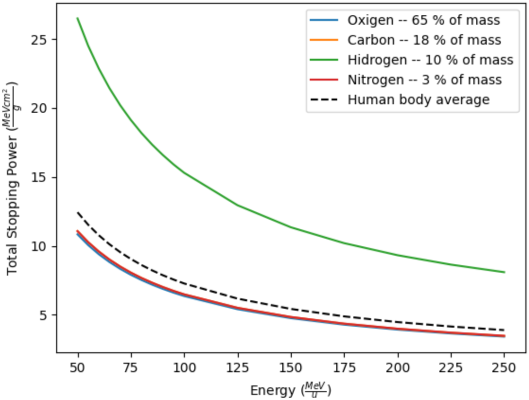
Goal: Solve the linear problem

$$\mathbf{x} = \mathbf{f}\,(\,\mathbf{y},\,\mathbf{A}\,)\,.$$

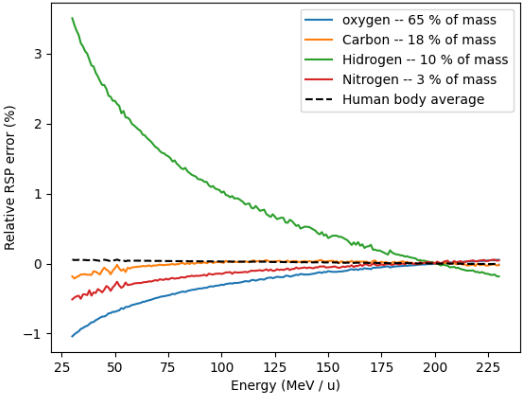
Irradiation with protons



Stopping Power



Relative Stopping Power

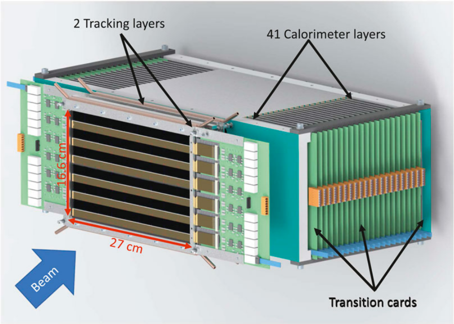


Motivation and role of proton imaging

- Nowadays the importance of the proton therapy is increasing
  $\Rightarrow$ more and more motivation to improve the technology
- The use of proton CT images is a promising direction
  $\Rightarrow$ lower inaccuracy in RSP measurement
  $\Rightarrow$ decreased safety zone around the tumour
- A pCT image measures the relative stopping power (RSP) distribution of the patient

Bergen pCT collaboration

- Goal: reach the clinical research with a pCT prototype
- Apply monolithic active pixel sensors (MAPS)
- Use pencil beam for imaging
- Measure 10^6 proton / second
- Reach $<$ 1 % RSP error



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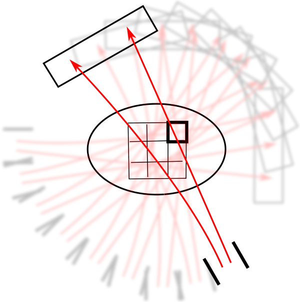

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Image reconstruction – Richardson – Lucy algorithm

- Originally introduced for astrophysics application
- It is a fixed point iteration for large and sparse linear problems
- Initialization: arbitrary positive vector
- Init: unit vector or precalculated approximate solution

The formula for the i^{th} element of the next image vector:

$$x_i^{k+1} = x_i^k \frac{1}{\sum_j A_{i,j}} \sum_j \frac{y_j}{\sum_l A_{l,j} x_l^k} A_{i,j} \ ,$$

where k is the number of iteration. 20-300 iteration is typical.

Avoid the multiply calculations

Calculated for every voxel in every iteration:

$$x_i^{k+1} = x_i^k \frac{1}{\sum_j A(i,j)} \sum_j \frac{y_j}{\sum_l A(l,j) x_l^k} A(i,j) \;\Rightarrow\; x_i^{k+1} = x_i^k N_i \sum_j H_j^k A(i,j)$$

Calculated for every proton in every iteration:

H_j^k is called Hadamard ratio in the k^{th} iteration and defined as:

$$H_j^k = \frac{y_j}{\sum_l A(l,j) x_l^k} \tag{1}$$

Calculated for every voxel only once:

N_i is the normalization of the i^{th} voxel:

$$N_i = \frac{1}{\sum_j A(i,j)} \tag{2}$$

GPU algorithm

Algorithm 1 GPU algorithm

GPU: calculate voxel normalization
for needed number of iterations **do**
    while end of proton histories **do**
        CPU: read certain amount of proton histories
        GPU: calculate Hadamard ratio:

- parallel calculation of proton histories
- serial calculation of voxel interactions

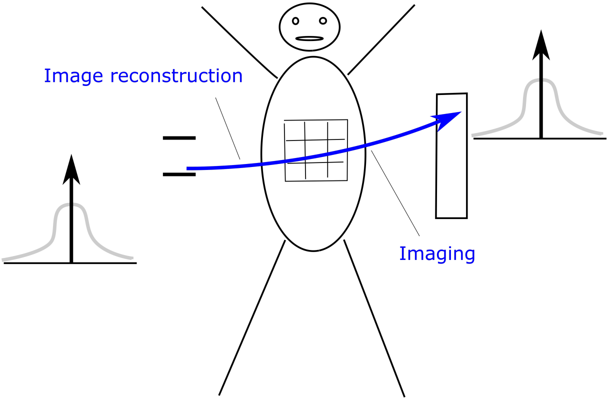
        GPU: calculate voxel contribution

- serial calculation of proton histories
- parallel calculation of voxel interactions

        GPU: Update the image vector
    end while
end for
CPU: Save the image vector

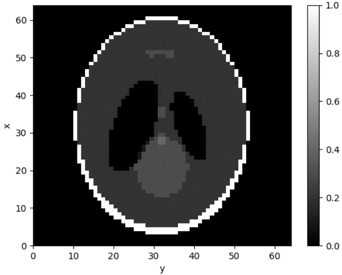
Ideal imaging

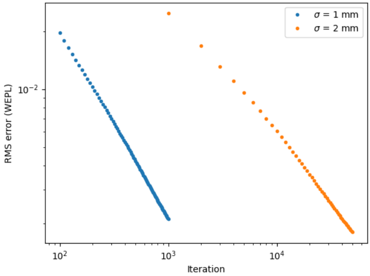
Proton incoming and outgoing path is included without measurement errors



Shepp – Logan phantom – convergence without uncertainties

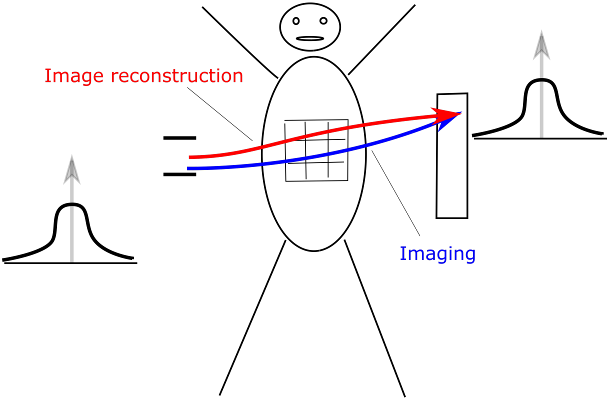
- Reconstructed Shepp-Logan phantom and the convergence
- The algorithm converges slower with wider uncertainty distribution



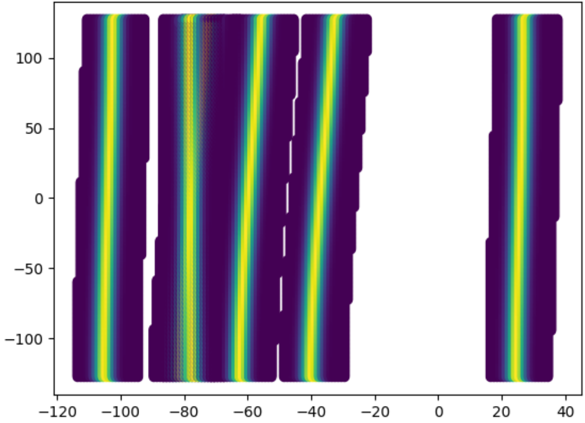


Realistic imaging

The incoming and outgoing pron path includes measurement uncertainties

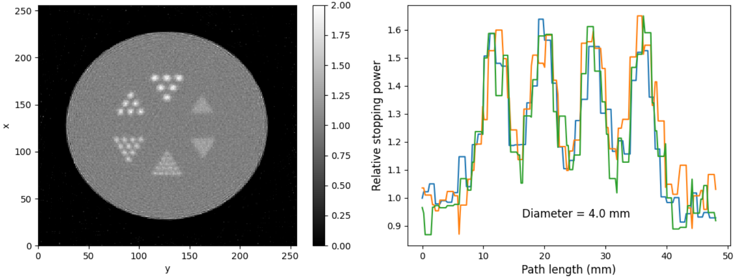


Probability Density Based Proton – Voxel Interaction



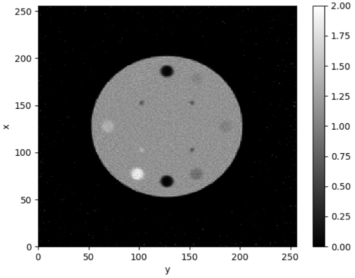
Derenzo Phantom – Spatial Resolution

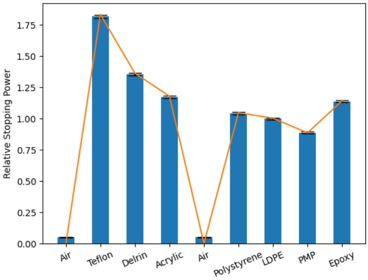
- Reconstructed RSP distribution and valley-to-peak distribution
- Spatial resolution: modulation transfer function at 10%
- My algorithm: 2.1 $\frac{lp}{cm}$ < Proton CT literature: 3.2 $\frac{lp}{cm}$



CTP404 Phantom – RSP Accuracy

- Reconstructed RSP distribution and avg. RSP of the inserts
- RSP accuracy: my algorithm: 0.5% $\sim$ pCT literature: 0.4%





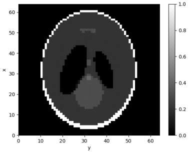
Summary

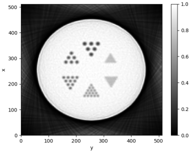
Goal:

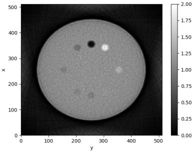
- Apply the Richardson – Lucy algorithm for proton CT image reconstruction

Results:

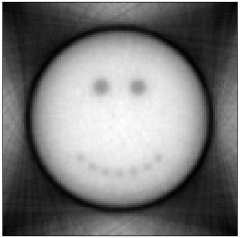
- RSP accuracy such as state of the art results in the literature
- The spatial resolution remains the week point of this algorithm
  $\Rightarrow$ Requires further investigations and revisit of the simplified probability density based interaction calculation

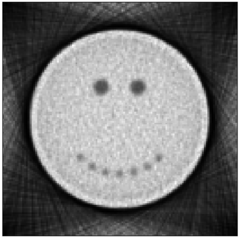


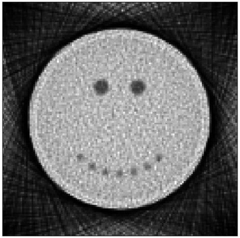




Thank you for your attention!







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