Wigner 121
Scientific Symposium

Wigner Research Centre for Physics
Institute for Particle and Nuclear Physics
Department of Theoretical Physics
Gravitational Physics Research Group

Mission:

The discovery of gravitational waves emitted by merging compact binaries calls for a better understanding of the gravitational origins of these extreme astrophysical processes. Besides, an increasing number of astronomical observations provide important insights into the formation of black holes and neutron stars, also their evolution over time, on energetic cosmic ray processes, the properties of matter at densities beyond normal, together with information on the structure and evolution of the Universe. Our research aims to provide accurate answers to these cutting-edge questions in gravitational theory.

Among the most important theory-motivated research problems we emphasize our work on modeling neutron stars, analyzing supermassive black hole jet evolutions and in relation with them cylindrically symmetric spacetimes, improving the post-Newtonian description of binary systems, discussing various cosmological models and the backreaction of gravitational waves in strong gravity, developing radically new ways to obtain adequate initial data, finally the analytical and numerical study of their time evolution in Einstein's theory of gravity.

Mátyás Vasúth, László Árpád Gergely, Dániel Barta, and Balázs Kacskovics are also involved in gravitational wave experiments, participating in the Virgo, LIGO and Einstein Telescope Scientific Collaborations.

Theoretical aspects

Considering a spacetime that occurs as a maximum time evolution we show that if the weakest form of Penrose's cosmic censor hypothesis holds, then the curvature cannot remain bounded along incomplete timelike geodesics. Assuming that the currently known weakest form of Penrose's cosmic censor hypothesis is satisfied, we show that the curvature along the geodesics of the spacetime that occurs as a maximum time evolution along the geodesics of non-full-time geodesics cannot remain bounded. [Rácz, I. Spacetime singularities and curvature blow-ups. *Gen Relativ Gravit* **55**, 3 (2023)]

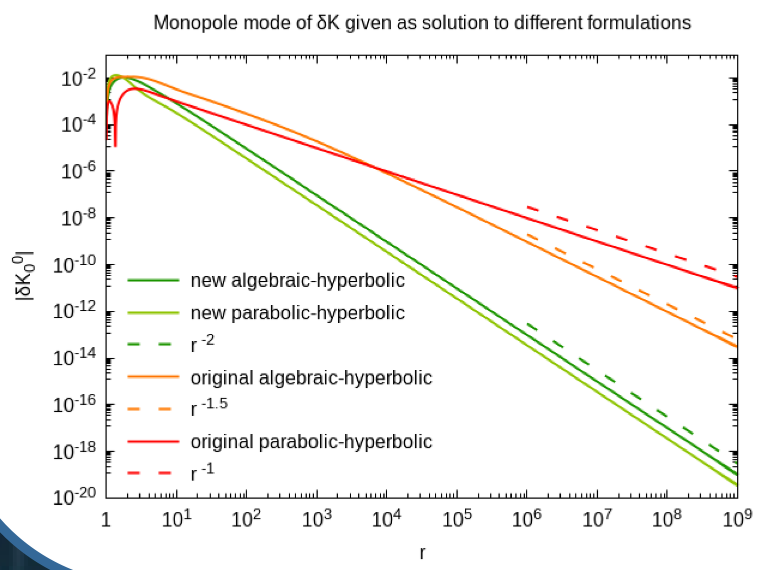
We showed that in all vacuum spacetimes with a compact Cauchy horizon and a non-zero constant surface gravity, there is necessarily a spacetime symmetry. [Petersen, O., Rácz, I. Symmetries of Vacuum Spacetimes with a Compact Cauchy Horizon of Constant Nonzero Surface Gravity. *Ann. Henri Poincaré* (2023). https://doi.org/10.1007/s00023-023-01335-9]

On surfaces with S^3 topology, which play an important role in closed cosmological models, the generation of initial data leads to new solutions of the parabolic-hyperbolic forms of the constraint equations with non-negligible gravitational wave content.

In our mathematical formalism for spacetimes foliated by topological 2-spheres, we use spin-weighted spherical harmonics to analytically define the trapped and marginally trapped surfaces (the latter being the boundaries of black holes and black hole doublets, respectively, in dynamical processes).

Asymptotically flat solutions of the constraints:

Currently, all simulations of compact binaries are contaminated by non-physical radiation already present in the initial data. Our group is working on an alternative formulation of the Einstein constraint equations in the hope of ameliorating this problem. A major obstacle to the practical use of the evolutionary formulation has been the lack of understanding how to control the asymptotic behavior of the initial data obtained. In particular, in the naive approach, the monopole part of the scalar **K** was not following the required decay rates. As the figure shows, our new method achieves the desired $O(r^{-2})$ fall-off rate compatible with asymptotic flatness. This result allows us, using the evolutionary formulation of the constraints, to obtain initial data for binary systems with physically adequate gravitational wave content. [Károly Csukás and István Rácz: Phys. Rev. D 107, 084013 (2023)]



Timespace

... Modern physics is almost on the scent. Time has already been recast as part of space, but soon they will also be able to characterize space in terms of time. Then they will realize that the two are interchangeable. If time is nothing but the fourth dimension of space, then space cannot be more than the interrelation of the surface and depth of time that is interpreted as present. This will create an imperishable moment. The two are one. And when they find this, all that remains is to formulate the process of imagination. Do you know what is that? What the Upanishads taught. The MAYA. We will be able to distinguish illusion from reality. We will be able to see –...

Hamvas Béla, Karnevál II. p. 432. (1948-51)

Radial oscillation of relativistic compact stars

We consider a static spherically symmetric star where one can introduce small, adiabatic perturbations in the fluid 4-velocity, expressed by $\delta u^{\mu}_{\mathrm{radial}} = [e^{\nu_0/2}, -e^{\lambda_0-\nu_0/2}\delta u_r, 0, 0]$ where $\delta u_r = dr/dt$ is associated with a displacement field in the Lagrangian representation: $\partial\xi/\partial t = \delta u_r$.

1. The perturbation equations are obtained from $\delta(\nabla_\mu T^{\mu\nu}) = 0$, $\delta(G_{\mu\nu} - 8\pi T_{\mu\nu}) = 0$. Then, it is straightforward to compute the linear perturbations of any equilibrium quantity $(\delta\rho, \delta p, \ldots)$.

2. With the assumption of harmonic time dependence:

FUNDAMENTAL EQUATION FOR RADIAL PULSATION

$$\omega^2 e^{\lambda_0-\nu_0}(p_0+\varepsilon_0)\xi = \left[\frac{4}{r}\frac{dp_0}{dr} + 8\pi e^{\lambda_0} p_0(p_0+\varepsilon_0) - \frac{1}{p_0+\varepsilon_0}\left(\frac{dp_0}{dr}\right)^2\right]\xi - e^{-(\lambda_0+2\nu_0)/2}\frac{d}{dr}\left[e^{(\lambda_0+3\nu_0)/2}\frac{\Gamma p_0}{r^2}\frac{d}{dr}\underbrace{\left(r^2 e^{-\frac{\nu_0}{2}}\xi\right)}_{X}\right]$$

a) The fluid at the center of the star is assumed to remain at rest: $X = 0$ at $r = 0$
b) The Lagrangian change in the pressure vanishes at the surface: $\delta p = e^{\nu_0/2} r^{-2}\Gamma p_0 \frac{d}{dr}\left(r^2 e^{-\nu_0/2}X\right) \equiv 0$ at $r = R$

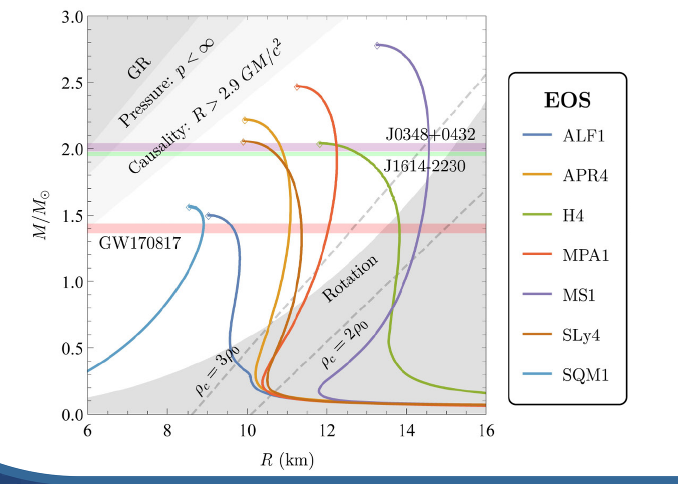
BOUNDARY CONDITIONS

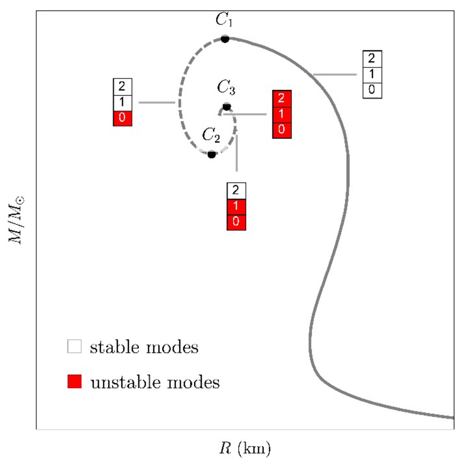
[Barta, CQG **36**, 215012 (2019)]

The fundamental equation together with its boundary conditions constitutes a Sturm–Liouville eigenvalue problem for a discrete set of scalar-valued eigenfunctions of radial displacement $\{X_0(r), X_1(r), \ldots, X_j(r), \ldots\}$ with their respective eigenvalues $\{\omega_0^2, \omega_1^2, \ldots, \omega_j^2, \ldots\}$.

3. To find the eigenfrequencies, we convert the boundary value problem to an initial value problem by "shooting" method!

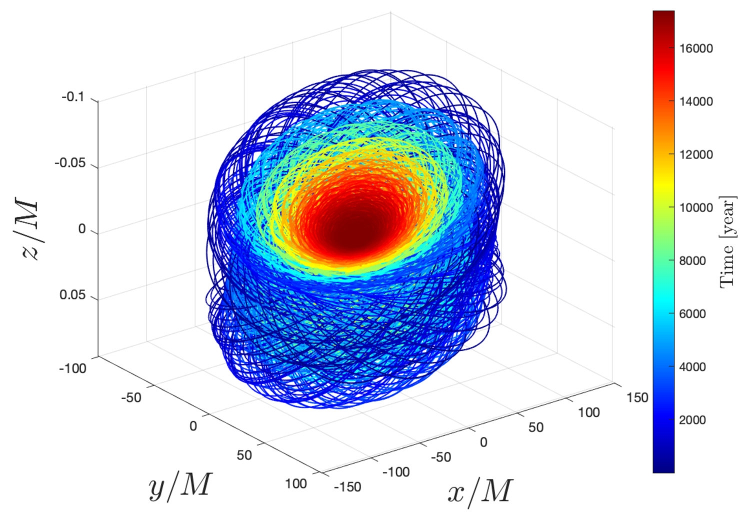
- ***If any of these*** ω_j^2 ***is negative***, the frequency is purely imaginary, any perturbation of the star ($\sim e^{i\omega t}$) will grow exponentially in time. ⇨ ***dynamically unstable***
- If $\omega_j^2 > 0$, ***the star is stable*** against adiabatic radial perturbations (up to the *j*th excited oscillation mode)
- The smallest eigenvalue ω_0^2 is associated with the *fundamental-mode frequency* of radial oscillations which *has no nodes* between the center and the stellar surface, whereas the first excited mode ($j = 1$) has a node, the second one ($j = 2$) has two.

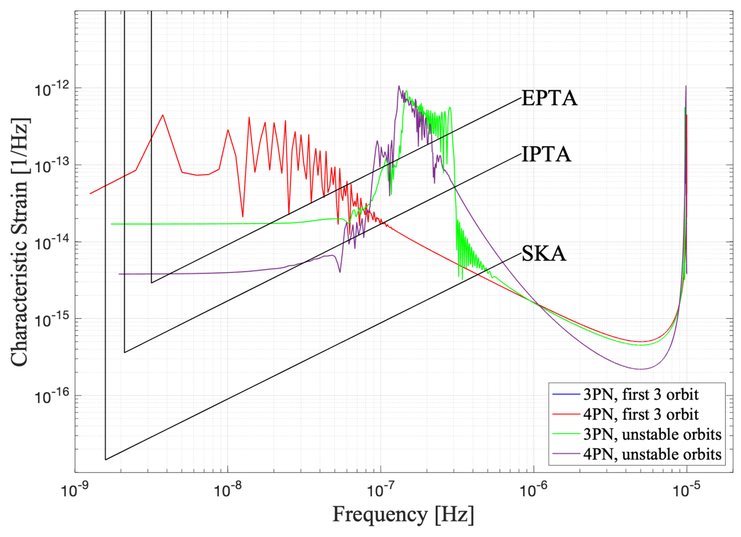




Schematic mass–radius relation illustrating the spiral structure of the unstable branch. [Barta, CQG **38**, 185002 (2021)]

Modelling post-Newtonian sources





terms

$$\boldsymbol{a} = \boldsymbol{a}_{PN} + \boldsymbol{a}_{2PN} + \boldsymbol{a}_{SO} + \boldsymbol{a}_{SS} + \boldsymbol{a}_{RR} + \boldsymbol{a}_{PNSO} + \boldsymbol{a}_{3PN} + \boldsymbol{a}_{RRPN} + \boldsymbol{a}_{3PNSO} + \boldsymbol{a}_{RRSO} + \boldsymbol{a}_{RRSS} + \boldsymbol{a}_{4PN}.$$

The above equation of motion contains the leading order, spin-spin (SS) effects, spin-orbit (SO) effects and radiation-reaction (RR) contributions of post-Newtonian approximation. Simultaneously with the orbital evolution, the radiation field h_{ij} is determined by the analytic waveform contributions up to 2PN order in harmonic coordinates, valid for general eccentric and spinning sources. This can be decomposed as

$$h_{ij} = \frac{2G\mu}{c^4 D}\left(Q_{ij} + P^{0.5}Q_{ij} + PQ_{ij} + P^{1.5}Q_{ij} + P^2Q_{ij} + PQ_{ij}^{SO} + P^{1.5}Q_{ij}^{SO} + P^2Q_{ij}^{SO} + PQ_{ij}^{SS} + P^{1.5}Q_{ij}^{tail}\right),$$

where D is the distance to the source, and $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the binary. Equation of the radiation field contains the quadrupole term and the 1.5PN tail term, terms for higher-order relativistic corrections and for corrections of spin-spin and spin-orbit effects. The spin evolution have been calculated in CBwaves by

$$\dot{\boldsymbol{S}}_i = \frac{G}{c^2 r^3}\left\{\Omega_i \boldsymbol{L}_N - \boldsymbol{S}_j - 3(\hat{\boldsymbol{n}}\boldsymbol{S}_j)\hat{\boldsymbol{n}} + \frac{G\mu m}{c^5 r^2}\left[\frac{2}{3}(\boldsymbol{v}\boldsymbol{S}_j) + 30\dot{r}(\hat{\boldsymbol{n}}\boldsymbol{S}_j)\right]\hat{\boldsymbol{n}}\right\} \times \boldsymbol{S}_i$$

where $i, j = 1,2$ and $i \neq j$. The above equation contains the standard spin--orbit and spin--spin, the 2PN spin--orbit, the 3PN spin--orbit and the 3.5PN radiative spin--spin contributions. Let us note that in CBwaves, we presume the use of covariant spin supplementary condition, $S_A^{\mu\nu}u_{A_\nu} = 0$, where u_A^μ is the 4-velocity of the center-of-mass world line of body A, with $A = 1,2$.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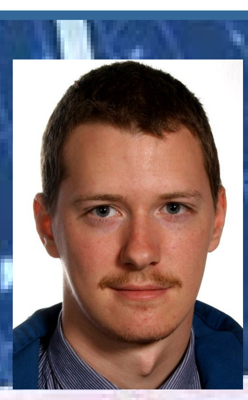
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