Inner structure of Neutron stars

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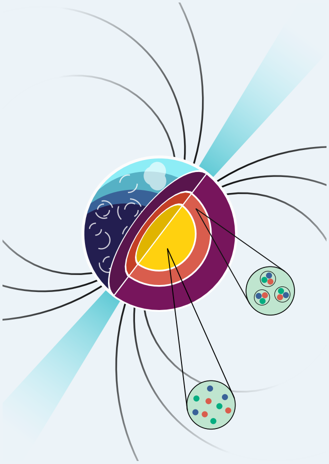
Outline

Pulsars, neutron stars (NSs)

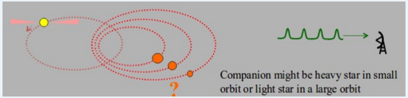


- First pulsar discovered by radio telescope in 1967 (Hewish and Bell), $\nu \approx 1 - 1000$ Hz,
- Pulsars are electromagnetically emitting neutron stars, they can emit in X-rays, γ or even in the visible range
- NSs are formed in supernova explosions, from $10 - 25\, M_\odot$ mass stars
- The densest known matter in our universe, mass of 1cm^3 is $\approx 10^{11} - 10^{12}$ kg (thus about 1 billion tons)
- $M \approx 1.2 - 2.3\, M_\odot$, $R \approx 10 - 14$ km
- NS types: pulsars, magnetars, pulsars+magnetars, non-pulsing NS, altogether about 3000 was discovered
- The possibility of hybrid stars with quark cores is supported by recent studies[a]

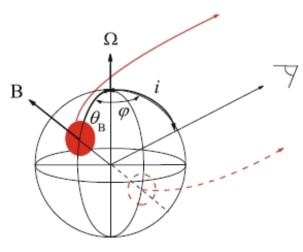
[a] Annala et al., Nature Phys. 16 (2020) 9, 907-910

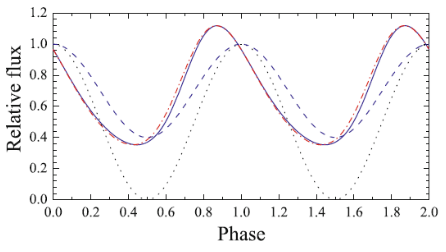
Mass, radius, tidal deformability measurements

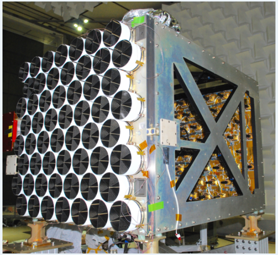
Mass meas. $\longrightarrow$ pulsars in binary systems and Measurement of Doppler shift



Radius measurement $\longrightarrow$ pulse-profile analysis, 'hotspot' observation (NICER)

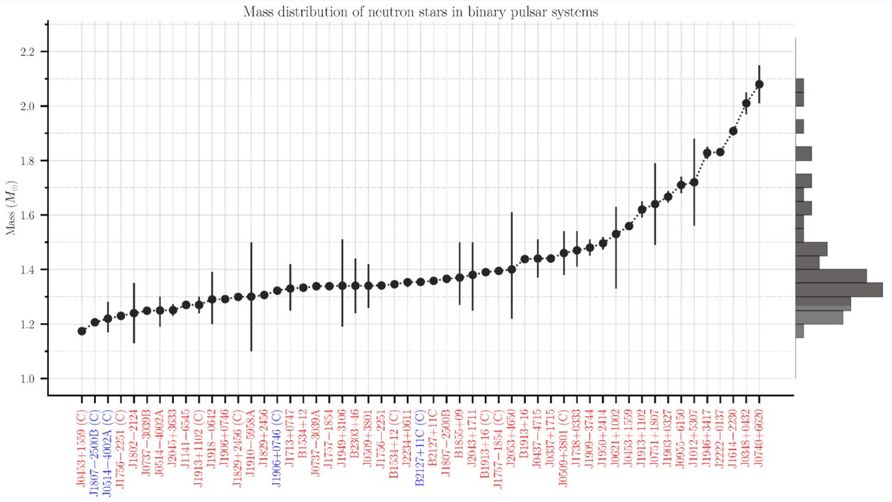






Tidal deformability $\longrightarrow$ Far from merging, tidal deformation causes a phase shift in the gravitational wave signal

Mass distribution of NSs



Structure of compact stars

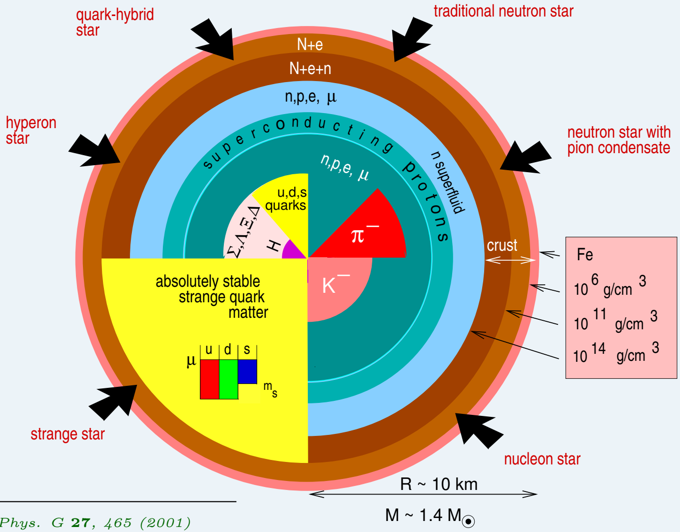
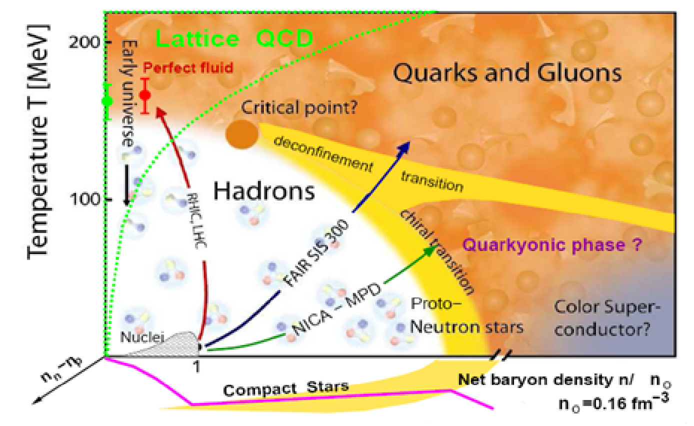


Fig. from *F. Weber, J. Phys. G* **27**, *465 (2001)*

Envisaged phase diagram of QCD



Important details of the phase diagram is still unknown (mainly at large baryon density)

Properties of the phase diagram, especially at finite baryon densities/baryochemical potential can be well studied with using effective field theories of QCD $\longrightarrow$ e.g. details of the phase boundary like existence and location of the CEP, in medium dependence of meson masses, or properties of compact stars etc.

Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

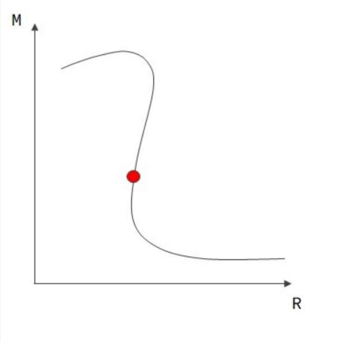
$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \tag{1}$$

where

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

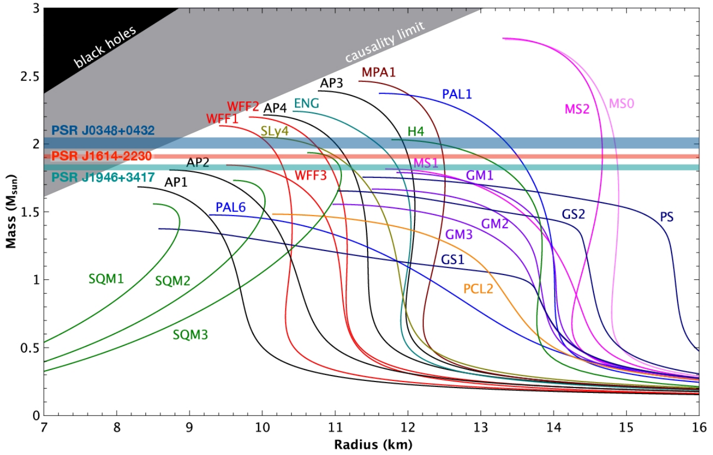
These are integrated numerically for a specific EoS ($p(\varepsilon)$):

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends



$M-R$ curves for different EoSs

- QCD directly unsolvable at finite density
- One can use phenomenological or effective models in the zero temperature finite density region
- Neutron star observations restrict such models [1,2]

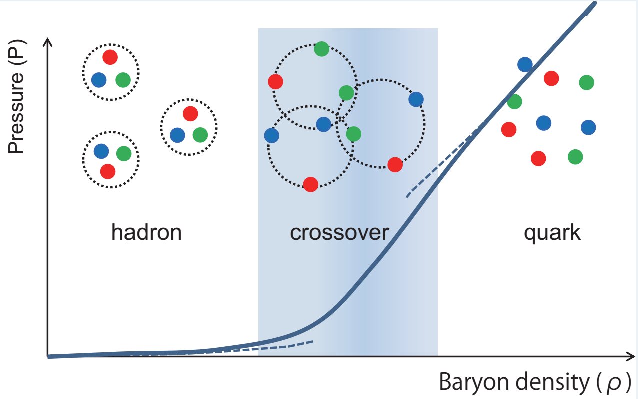


[1] *P. Demorest et al. (2010), Nature* ***467****, 1081*

[2] *J. Antoniadis et al. (2013), Science* ***340****, 6131*

Hybrid EoS for hybrid stars

K. Masuda et al., PTEP **2013** *073D01*



In the crossover region hadrons starts to overlap

⟶ both low and high ρ_B models loose their validity

Gibbs condition (extrapolation from the dashed lines) can be misleading

EoS part I (hadronic part)

Hybrid stars also have a hadronic crust and outer core

- at low densities we use hadronic EoSs:
  - the SFHo to represent soft hadronic EoSs (less steep $p(\varepsilon)$ curve)
  - the DD2 to represent stiff hadronic EoSs
- we apply a smooth connection between the two phases: $\varepsilon(n_B)$ interpolation with a polynomial

$$\varepsilon(n_B) \quad = \quad \sum_{m=0}^{N} C_m n_B^m \,, \qquad n_{BL} < n_B < n_{BU},$$

  the C_m coefficients given by the matching of ε and its derivatives on the boundary
- we have 4 adjustable parameters:
  - 2 from the constituent quark model: m_σ, g_V
  - 2 describing the concatenation: $\bar{n}$, Γ

EoS part II (quark part) - our contribution

For large density the e(P)QM model is used

- nonzero scalar condensates: $\phi_N = \langle \sigma_N \rangle$, $\phi_S = \langle \sigma_S \rangle$
- nonzero vector condensates: $\langle (\rho^0)^0 \rangle = \phi_\rho$, $\langle (\omega)^0 \rangle = \phi_\omega$, $\langle (\Phi)^0 \rangle = \phi_\Phi$
- free electron gas + β-equilibrium
- modified chemical potentials:

$$\mu_u = \mu_q - \frac{2}{3}\mu_e - \frac{1}{2} g_V (\phi_\omega + \phi_\rho)$$
$$\mu_d = \mu_q + \frac{1}{3}\mu_e - \frac{1}{2} g_V (\phi_\omega - \phi_\rho)$$
$$\mu_s = \mu_q + \frac{1}{3}\mu_e - \frac{1}{\sqrt{2}} g_V \phi_\Phi$$

- charge neutrality: $\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0$
- field equations (FEs):

$$\frac{\partial \Omega_{tot}}{\partial \phi_N} = \frac{\partial \Omega_{tot}}{\partial \phi_S} = \frac{\partial \Omega_{tot}}{\partial \phi_\rho} = \frac{\partial \Omega_{tot}}{\partial \phi_\omega} = \frac{\partial \Omega_{tot}}{\partial \phi_\Phi} = 0 \quad \rightarrow p(\varepsilon)\text{curve}$$

Lagrangian of the ePQM model

$U(3)_L \times U(3)_R$ chiral symmetric $\mathcal{L}$-density + explicit sym. breaking + $U(1)_A$ anomaly term + Yukawa-coupling + SSB

$$
\begin{aligned}
\mathcal{L} = & \mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} \mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \mathrm{Tr}(\Phi^\dagger \Phi) \mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2 \mathrm{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + \bar{\Psi}\left(i\gamma^\mu D_\mu - g_F(S - i\gamma_5 P)\right)\Psi - g_V \bar{\Psi}\left(\gamma^\mu (V_\mu + \gamma_5 A_\mu)\right)\Psi,
\end{aligned}
$$

$$
\begin{aligned}
\Phi &= S + iP \equiv \sum_{a=0}^{8} (S_a \lambda_a + i P_a \lambda_a) \\
D^\mu \Phi &= \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}, \\
D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}
$$

Particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar ($\sim \bar{q}_i q_j$) and pseudoscalar ($\sim \bar{q}_i \gamma_5 q_j$) meson nonets

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star -} & \bar{K}_0^{\star 0} & \sigma_S \end{pmatrix} \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

multiple possible assignments
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548), \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$
fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

In case of compact stars, also nonzero vector condensates

Determination of the parameters

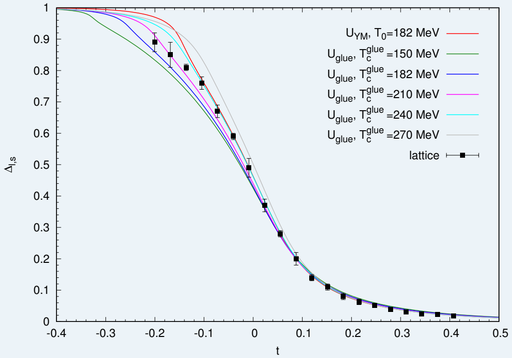
14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

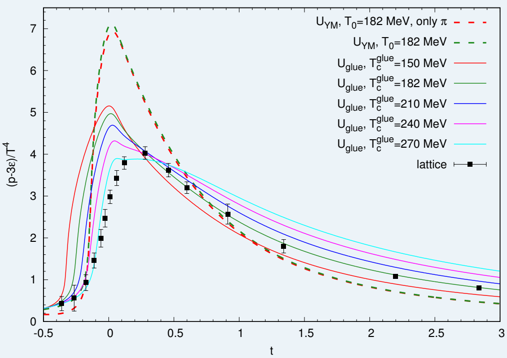
$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i}\right]^2,$$

$(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N) \longrightarrow$ from the model, $Q_i^{\mathrm{exp}} \longrightarrow$ PDG value,
$\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi},$
  $\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $\mu_B = 0$

Comparison with lattice results at $\mu_B = 0$





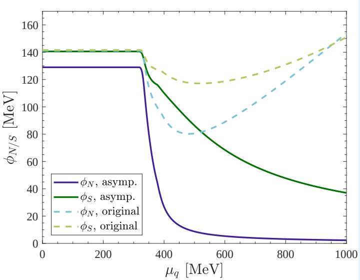
Lattice: Borsányi *et al.*, JHEP 1009, 073 (2010)

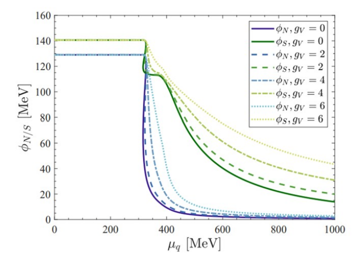
Left: subtracted condensates, right: scaled interaction measure both as a function of reduced temperature ⟶ very good match

Asymptotic behaviour in μ_B at $T = 0$

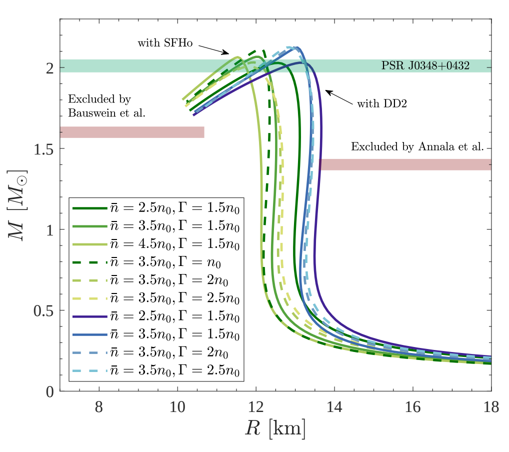
- not all parameterizations give acceptable behaviour for high baryon chemical potential → chiral symmetry is not restored
- investigation of asymptotic behaviour gives extra condition among certain parameters: $3h_1 + 2h_2 + 2h_3 < 0$
- $m_\sigma = 290$ MeV is given in the best fit

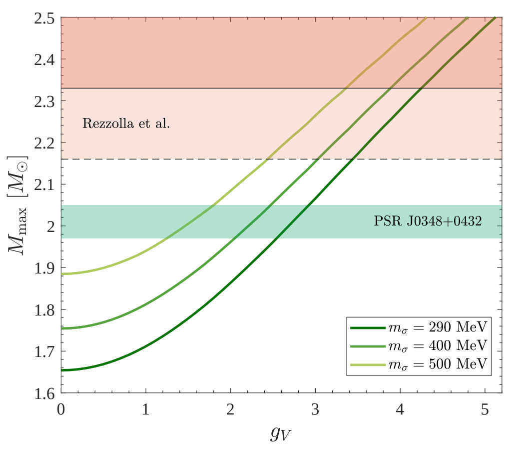
Common belief that as a function of μ_B the transition is first order → however for $g_v > 3.1$ the transition becomes crossover





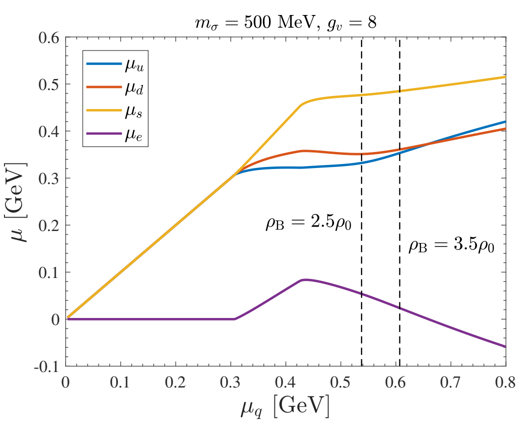
Restriction by maximum mass of NSs



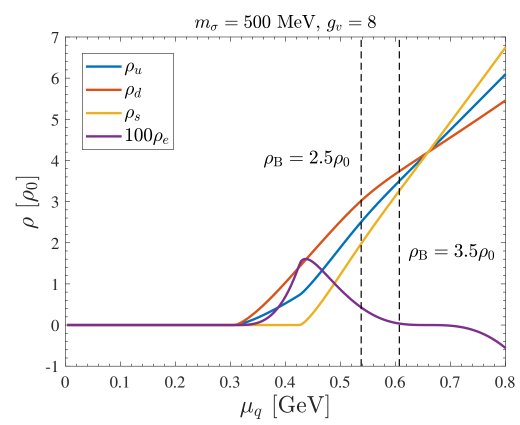


M_{max} depends mostly on quark model parameters with $m_\sigma = 290$ MeV g_V is constrained to $2.5 < g_V < 4.3$

Quark chemical potentials and densities



1. ábra. μ of quarks and the electron



2. ábra. number densities of quarks and the electron (enlarged by 100×)

Bayesian analysis

Bayes' theorem:

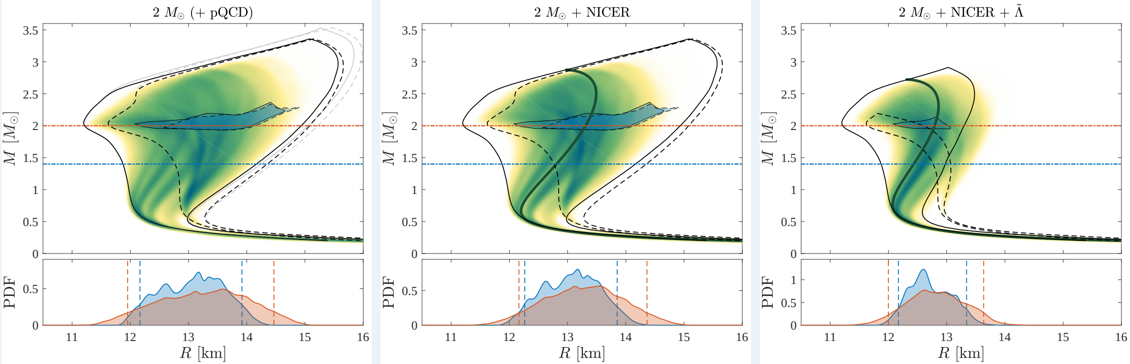
$$p(\vartheta|\text{data}) = \frac{p(\text{data}|\vartheta)p(\vartheta)}{p(\text{data})}$$

$$p(\text{data}|\vartheta) = p(M_{\text{max}}|\vartheta)p(\text{NICER}|\vartheta)p(\tilde{\Lambda}|\vartheta)$$

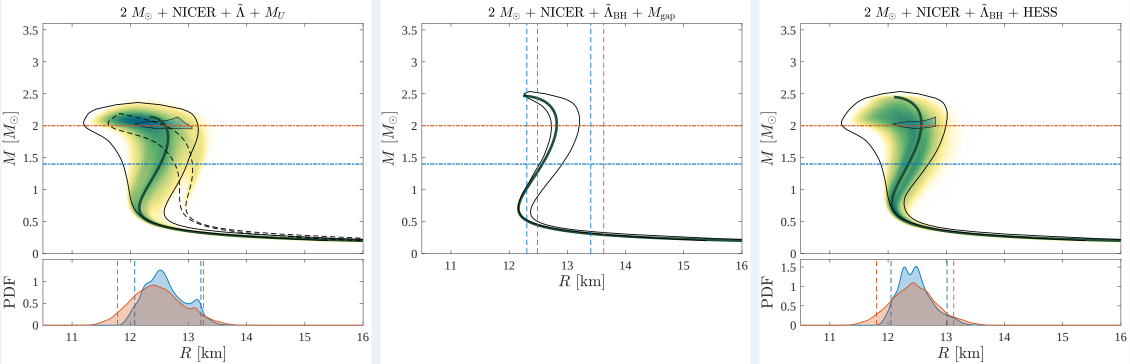
- ϑ set of parameters of our model
- Lower limit on maximum mass from $2\ M_\odot$ NS observations
- Mass-radius probability densities from observations of PSR J0030+0451 and PSR J0740+6620 with NICER
- Tidal deformability data from LVC for GW170817 + constraint from no prompt collapse to BH
- Upper mass constraint from hypermassive NS hypothesis[1]

[1] Rezzolla, Most & Weih, Astrophys. J. Lett. 852 (2018) 2, L25

Restriction of $M - R$ curves with Bayesian analysis I



Restriction of $M - R$ curves with Bayesian analysis II



Conclusion

- Neutron stars are very special celestial objects, whose structure is shaped by strong interactions
- In recent years, more and more accurate observations, e.g. mass, radius, tidal deformability
- Astrophysical observations can be used to infer in-medium properties of strongly interacting matter
- For theoretical description, effective models can be used, e.g. for hybrid stars
- The $M - R$ curves calculated so far are consistent with current observations when certain parameters of the model are constrained (e.g. $2.5 < g_V < 4.3$)
- More details in:
  - Phys.Rev.D 108 (2023) 4, 043002
  - Phys.Rev.D 105 (2022) 10, 103014
  - Phys.Rev.D 102 (2020) 2, 028501

Thank You!

Field equations (FE)

Four coupled field equations are obtained by extremizing the grand potential

$$\Omega(T,\mu_q) = U^{\text{tree}}_{\text{meson}}(\langle M\rangle) + \Omega^{(0)\text{vac}}_{\bar{q}q} + \Omega^{(0)T}_{\bar{q}q}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$$

using $\dfrac{\partial\Omega}{\partial\phi_N} = \dfrac{\partial\Omega}{\partial\phi_S} = \dfrac{\partial\Omega}{\partial\Phi} = \dfrac{\partial\Omega}{\partial\bar{\Phi}} = 0$ $\qquad E_f^{\pm}(p) = E_f(p) \mp \mu_q, \quad E_f^2(p) = p^2 + m_f^2$

$$1)\quad -\frac{1}{T^4}\frac{d\,U(\Phi,\bar{\Phi})}{d\Phi} + \frac{6}{T^3}\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left(\frac{e^{-\beta E_f^-(p)}}{g_f^-(p)} + \frac{e^{-2\beta E_f^+(p)}}{g_f^+(p)}\right) = 0$$

$$2)\quad -\frac{1}{T^4}\frac{dU(\Phi,\bar{\Phi})}{d\bar{\Phi}} + \frac{6}{T^3}\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left(\frac{e^{-\beta E_f^+(p)}}{g_f^+(p)} + \frac{e^{-2\beta E_f^-(p)}}{g_f^-(p)}\right) = 0$$

$$3)\quad m_0^2\phi_N + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - \frac{c_1}{\sqrt{2}}\phi_N\phi_S - h_{0N} + \frac{3}{2}g_F\left(\langle\bar{q}_u q_u\rangle_T + \langle\bar{q}_d q_d\rangle_T\right) = 0$$

$$4)\quad m_0^2\phi_S + (\lambda_1+\lambda_2)\phi_S^3 + \lambda_1\phi_N^2\phi_S - \frac{\sqrt{2}}{4}c_1\phi_N^2 - h_{0S} + \frac{3}{\sqrt{2}}g_F\langle\bar{q}_s q_s\rangle_T = 0$$

renormalized fermion tadpole: $\qquad m_{u,d} = \frac{g_F}{2}\phi_N$ and $m_s = \frac{g_F}{\sqrt{2}}\phi_S$

$$\langle\bar{q}_f q_f\rangle_T = 4m_f\left[-\frac{m_f^2}{16\pi^2}\left(\frac{1}{2} + \ln\frac{m_f^2}{M_0^2}\right) + \int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\left(f_f^-(p) + f_f^+(p)\right)\right]$$

25