Synthesis of elements in compact stars:

Pycnonuclear fusion with Carbon isotopes

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Outline

1. General info on pycnonuclear fusion in stars
2. Method of quantum mechanics with high precision for astrophysical tasks
3. Example: fusion in $^{12}C + ^{12}C$ at close distance
4. Systematics for Carbon isotopes: ^{10}C, ^{12}C, ^{14}C, ^{16}C, ^{18}C, ^{20}C, ^{22}C, ^{24}C
5. Plasma screening of nuclear fusion in dense matter
6. Conclusions.

Pycnonuclear reactions in compact stars

Pycnonuclear fusion (Greek: πυκνός (pyknós) – 'dense, compact') is a type of nuclear fusion reaction which occurs due to zero-point oscillations of nuclei around their equilibrium point in crystal lattice sites.

Explanation: In stars, thermal energy of reacting nuclei overcomes the Coulomb repulsion between them because of close distance between nuclei (due to sufficiently high density of stellar matter). In result, fusion can proceed. This is ***pycnonuclear fusion (reaction)*** [1]. It can be at zero temperatures (very low energies).

The term "**pycnonuclear**" was coined by A.G.W. Cameron in 1959 [1], but research showing the possibility of nuclear fusion in extremely dense and cold compositions was published by W. A. Wildhack in 1940 [2].

The phenomenon can be interpreted as overlap of the ***wave function*** of neighboring ***ions***, and is proportional to the overlapping amplitude.

[1] A.G.W.Cameron, *Pycnonuclear reactions and nova explosions*, Astr. J. **130**, 916 (1959).
[2] E.E.Salpeter, H.M.VanHorn, Nuclear reaction rates at high densities, Astr. J. **155**, 183 (1969).
[3] P. Haensel, et al., Astron. Astr. **229**, 117 (1990); **404**, L33 (2003).

Scheme of reactions in neutron stars

Deep from surface		**Density, g cm^{-3}**		**Process**
0 - 1 cm	Atmosphere (gas)	$10^{-3} – 10^{+3}$	electrons, nuclei, atoms, molecules	Thermonuclear H + He burning, *rp*-processes
1 cm - 10 m	Ocean (Coulomb liquid)	10^{3} - 10^{6}	electrons, nuclei	Deep burning (H, C)
10 - 100 m	Outer crust (lattice of nuclei)	10^{6} - 10^{11}	electrons, neutrons, nuclei	Crust processes (pycnonuclear fusion)
100 m – 1km	Inner crust	(4-6) x 10^{11} – 5 x 10^{14}	electrons, neutrons	Separation of neutrons from nuclei
1 km – 10 km	core	> 5 x 10^{14}	nucleons, quarks, leptons, hyperons	

Neutron star:

Masa = 1-2 Solar mas,

Radius – 10-14 km,

Density = 10^{15}g cm^{-3},

Nuclear saturation = 2.8 x 10^{14} g cm^{-3}

[1] A.V. Afanasjev, et al., Report, Univ. of Norte-Dame, USA.

[2] A.Yu. Potekhin, Physics – Uspekhi **53** (12) 1279, (2010).

Pycnonuclear reactions in compact stars

Reactions of neutronization and pycnonuclear fusion can lead to the creation of ***absolutely stable*** environments in superdense substances.

Pycnonuclear burning occurs in ***dense and cold cores of white dwarfs*** [2] and in ***crusts of accreting neutron stars*** [3].

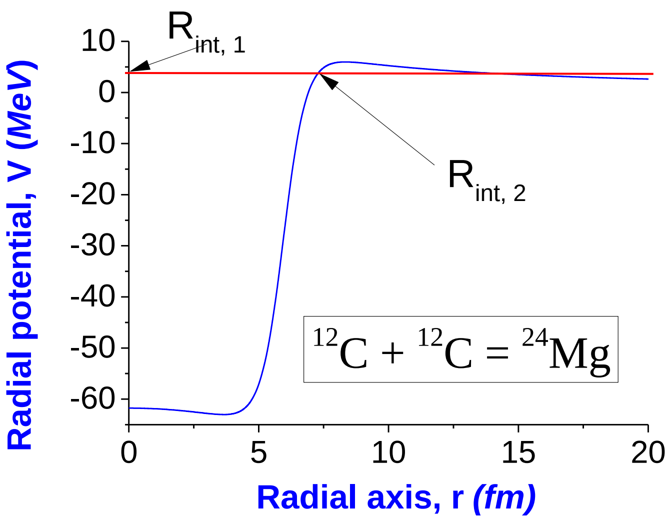
Astrophysical S-factors are estimated for 946 thermonuclear reactions for isitopes C, O, Ne and Mg for energies 2 - 30 MeV [4]. Large database of *S*-factors [5] is formed for isotopes Be, B, C, N, O, F, Ne, Na, Mg, Si (5000 non-resonant thermo-reactions).

[2] E.E.Salpeter, H.M.VanHorn, Nuclear reaction rates at high densities, Astr. J. **155**, 183 (1969).
[3] P. Haensel, et al., Astron. Astr. **229**, 117 (1990); **404**, L33 (2003).
[4] M.Beard, A.V.Afanasjev, et al., At. Dat. Nucl. Dat. Tabl. **96**, 541-566 (2010).
[5] A.V.Afanasjev, M.Beard, et al., Phys. Rev. **C 85**, 054615 (2012).

Approaches to study fusion in stars

Fusion in nuclear reactions in stars is studied on the basis of solution of Schrodinger equation with potential:

$$\hat{H}\Psi = \left(\hat{T} + V(r)\right)\Psi = E\Psi,$$



- Cross-section of fusion reaction:

$$\sigma_{capture}(E) = \frac{\pi\hbar^2}{2mE}\sum_{l=0}^{+\infty}(2l+1)T_l P_l.$$

- Penetrability in WKB-approximation:

$$T_{WKB} = \exp\left\{-2\int_{R_{tp,2}}^{R_{tp,3}}\sqrt{\frac{2m}{\hbar^2}\left(Q_p - V(r)\right)}\,dr\right\}$$

Restrictions:

- WKB-approximation is not applied for energies of pycnonuclear reactions,
- Internal processes in nuclear part of potential were not studied (ignored).
- Tests of QM was not used.

Methodology

Quantum mechanics

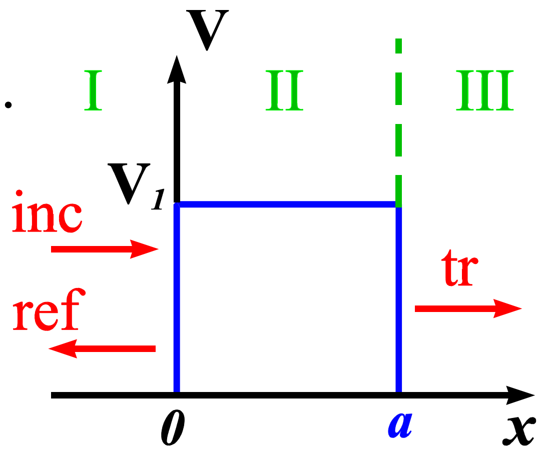
with high precision and tests for astrophysical tasks

Method: 1D tunneling (1)

One can understand idea of method the most clearly in the simplest case – analyzing wave, propagating above rectangular barrier.

- Schrodinger equation:

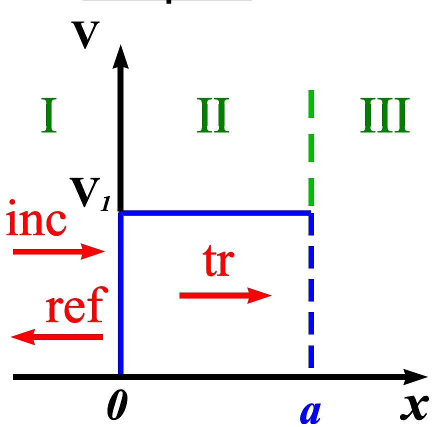
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi(x)+V(x)\varphi(x)=E\,\varphi(x).$$



- Wave function (WF):

$$\varphi(x)=\begin{cases} e^{ikx}+A_R e^{-ikx}, & x\le 0,\\ \alpha e^{-ik_2x}+\beta e^{ik_2x}, & 0\le x\le a,\\ A_T e^{ikx}, & x\ge a \end{cases}$$

- Approach on step-by-step:
- Step 1:



$$\varphi_{inc}^{(1)}=e^{ikx}, \qquad x\le 0,$$
$$\varphi_{tr}^{(2)}=\beta^{(0)}e^{ik_2x}, \qquad 0\le x\le a,$$
$$\varphi_{ref}^{(1)}=A_R^{(0)}e^{-ikx}, \qquad x\le 0.$$

- Continuity condition at $x = 0$:

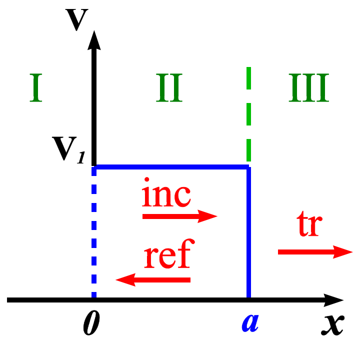
$$\beta^{(0)}=\frac{2k}{k+k_2},\qquad A_R^{(0)}=\frac{k-k_2}{k+k_2}.$$

- Transition to under-barrier tunneling:

$$k_2\Rightarrow i\xi,\qquad k_2=\frac{1}{\hbar}\sqrt{2m(E-V_1)}$$

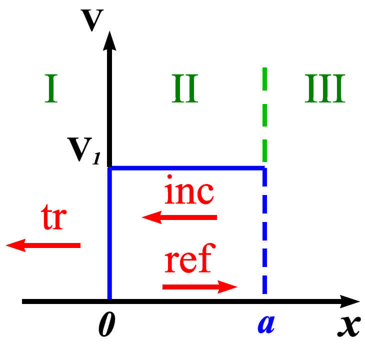
Method: 1D tunneling (2)

- Step 2:



$$\varphi_{inc}^{(2)} = \varphi_{tr}^{(1)} = \beta^{(0)} e^{ik_2 x},$$
$$\varphi_{tr}^{(2)} = A_T^{(0)} e^{ikx},$$
$$\varphi_{ref}^{(2)} = \alpha^{(0)} e^{-ik_2 x}.$$

- Step 3:



$$\varphi_{inc}^{(3)} = \varphi_{ref}^{(2)},$$
$$\varphi_{tr}^{(3)} = A_R^{(1)} e^{-ikx},$$
$$\varphi_{ref}^{(3)} = \beta^{(1)} e^{ik_2 x}.$$

- Continuity WF at $x = 0, a$: $\alpha^{(n)}, \beta^{(n)}, A_T^{(n)}, A_R^{(n)}$

- Amplitudes of transmission, reflection:

$$A_T = T_2^+ T_1^- \left(1 + \sum_{m=1}^{+\infty} (R_2^+ R_1^-)^m\right) = \frac{i4k\xi e^{-\xi a - ika}}{F_{sub}},$$
$$A_R = R_1^+ + T_1^+ R_2^+ T_1^- \left(1 + \sum_{m=1}^{+\infty} (R_2^+ R_1^-)^m\right) = \frac{k_0^2 D_-}{F_{sub}}$$

$$F_{sub} = (k^2 - \xi^2) D_- + 2ik\xi D_+, \quad D_\pm = 1 \pm e^{-2\xi a}$$
$$k_0^2 = k^2 + \xi^2 = \frac{2mV_1}{\hbar^2}$$

- Coefficients:

$$T_1^+ = \beta^{(0)}, \qquad R_1^+ = A_R^{(0)},$$
$$T_2^+ = \frac{A_T^{(n)}}{\beta^{(n)}}, \qquad R_2^+ = \frac{\alpha^{(n)}}{\beta^{(n)}},$$
$$T_1^- = \frac{A_R^{(n+1)}}{\alpha^{(n)}}, \qquad R_1^- = \frac{\beta^{(n+1)}}{\alpha^{(n)}}$$

- Test: $|A_T|^2 + |A_R|^2 = 1$

Method: Arbitrary number of barriers

Calculation of penetrability for arbitrary number of barriers is essentially more complicated, it has been solved.

- Wave function:

$$\varphi(x)=\begin{cases} e^{ikx}+A_R e^{-ikx}, & x\le x_1; \\ \alpha_2 e^{ik_2x}+\beta_2 e^{-ik_2x}, & x_1\le x\le x_2; \\ \dots & \dots \\ \alpha_{N-1} e^{ik_{N-1}x}+\beta_{N-1} e^{-ik_{N-1}x}, & x_{N-2}\le x\le x_{N-1}; \\ A_T e^{ikx}, & x_{N-1}\le x \end{cases}$$

- Calculation of coefficients:

$$T_j^+=\frac{2k_j}{k_j+k_{j+1}}e^{i(k_j-k_{j+1})x_j},\qquad T_j^-=\frac{2k_{j+1}}{k_j+k_{j+1}}e^{i(k_j-k_{j+1})x_j},$$

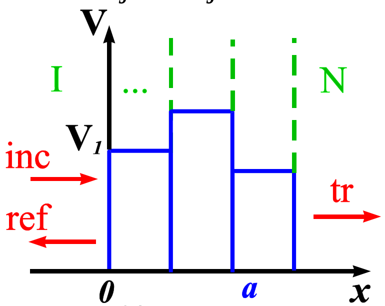
$$R_j^+=\frac{k_j-k_{j+1}}{k_j+k_{j+1}}e^{2ik_jx_j},\qquad R_j^-=\frac{k_{j+1}-k_j}{k_j+k_{j+1}}e^{-2ik_{j+1}x_j}.$$

- Recurrent relations:

$$\tilde{R}_{j-1}^+=R_{j-1}^+ + T_{j-1}^+\tilde{R}_j^+T_{j-1}^-\left(1+\sum_{m=1}^{+\infty}(\tilde{R}_j^+R_{j-1}^-)^m\right)=R_{j-1}^+ + \frac{T_{j-1}^+\tilde{R}_j^+T_{j-1}^-}{1-\tilde{R}_j^+R_{j-1}^-},$$

$$\tilde{R}_{j+1}^-=R_{j+1}^- + T_{j+1}^-\tilde{R}_j^-T_{j+1}^+\left(1+\sum_{m=1}^{+\infty}(R_{j+1}^+\tilde{R}_j^-)^m\right)=R_{j+1}^- + \frac{T_{j+1}^-\tilde{R}_j^+T_{j-1}^-}{1-\tilde{R}_j^-R_{j+1}^+},$$

$$\tilde{T}_{j+1}^+=\tilde{T}_j^+T_{j+1}^+\left(1+\sum_{m=1}^{+\infty}(R_{j+1}^+\tilde{R}_j^-)^m\right)=\frac{\tilde{T}_j^+T_{j+1}^+}{1-\tilde{R}_j^-R_{j+1}^+}.$$



$$\tilde{R}_{N-1}^+=R_{N-1}^+,\quad \tilde{R}_1^-=R_1^-,\quad \tilde{T}_1^+=T_1^+$$

- Amplitudes: $A_T=\tilde{T}_{N-1}^+,\quad A_R=\tilde{R}_1^+.$

- Penetrability, reflection: $T=\frac{k_N}{k_1}|A_T|^2,\quad R=|A_R|^2.$

Cross-section of capture

- Cross-section of capture:

$$\sigma_{\text{capture}}(E) = \frac{\pi \hbar^2}{2mE} \sum_{l=0}^{+\infty} (2l+1) T_l P_l .$$

- Penetrability in WKB-approximation:

$$T_{WKB} = \exp\left\{ -2 \int_{R_{\text{tp},2}}^{R_{\text{tp},3}} \sqrt{\frac{2m}{\hbar^2}\left(Q_p - V(r)\right)}\, dr \right\}$$

Here, E is kinetic energy of relative motion of two nuclei in lab. frame, E_1 is kinetic energy of relative motion of two nuclei in the center-of-mass frame (we use $E = E_1$), m is reduced mass of two nuclei, P_l is probability of fusion of two nuclei, T_l is penetrability of barrier.

- Penetrability and reflection for method MR:

$$T_{MIR} = \frac{k_1}{k_N} \left| A_T \right|^2, \quad R_{MIR} = \left| A_R \right|^2 .$$

- Test for method MR
- (it is absent in WKB-calc.):

$$T_{MIR} + R_{MIR} + M_{MIR} = 1.$$

- Connection with *S*-factor in astrophysics:

$$\sigma(E) = \frac{S(E)}{E} \times T_{full} .$$

Cross-section of α-capture: method MIR & WKB

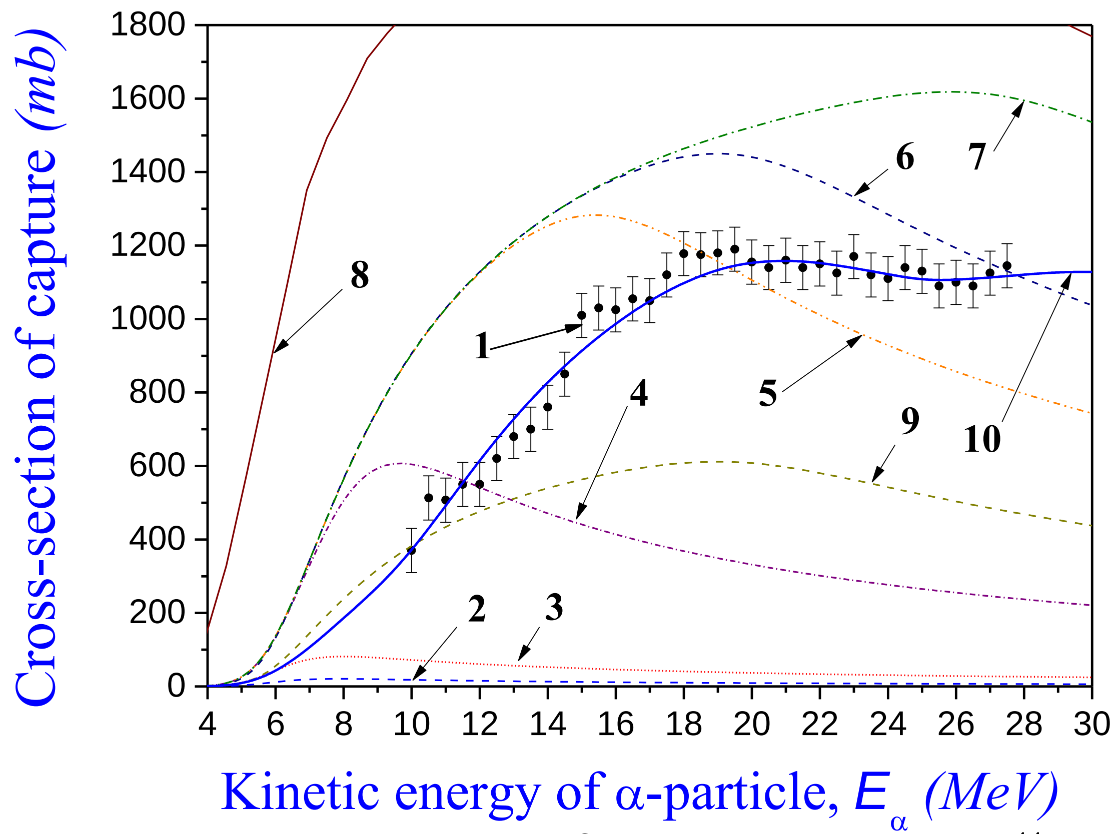


Fig.2. Capture cross-sections of α-particle by nucleus ^{44}Ca, obtained by method MIR (lines 2-7, 9-10) and WKB-approach (line 8). *Line 10 is obtained at inclusion of probabilities of fusion, lines 2-9 are without fusion prob.* [1].

***Conclusion:** Method MIR with included probabilities of fusion (line 10) is in higher agreement with experimental data, than WKB-approach without fusion (line 8).*

Black circles 1 is experimental data, dashed blue line 2 is cross-section at l_{max}=0, short dashed red line 3 is cross-section at l_{max}=1, short dash-dotted purple line 4 is cross-section at l_{max}=5, dash-double dotted orange line 5 is cross-section at l_{max}=10, dashed dark blue line 6 is cross-section at l_{max}=12, dash-dotted green line 7 is cross-section at l_{max}=15, solid brown line 8 is cross-section at l_{max}=20, dashed dark yellow line 9 is renormalized cross-section at l_{max}=17, solid blue line 10 is cross-section at l_{max}=17.

- Cross-section of capture:

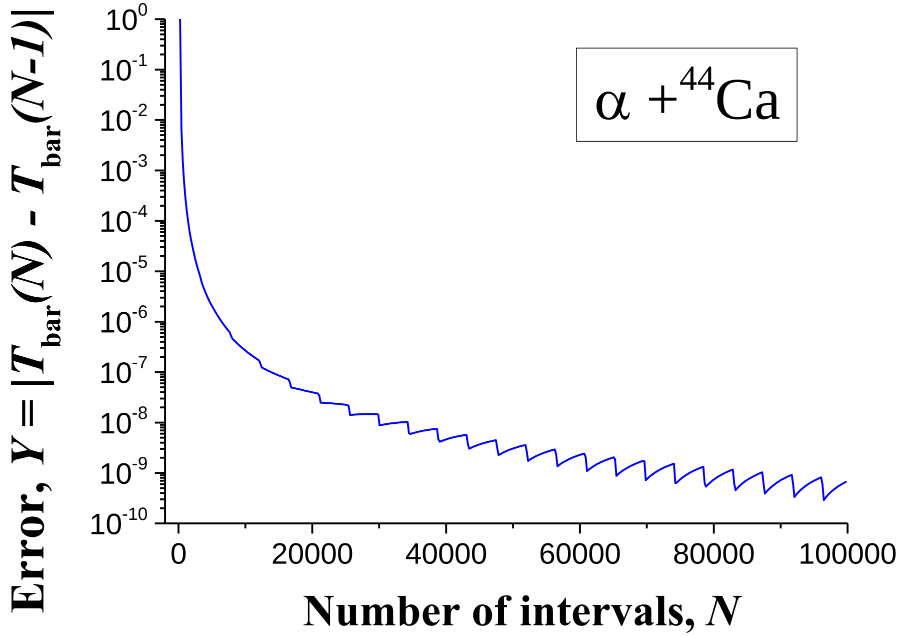
$$\sigma_{capture}(E) = \frac{\pi \hbar^2}{2mE} \sum_{l=0}^{l_{max}} (2l+1) T_l P_l.$$

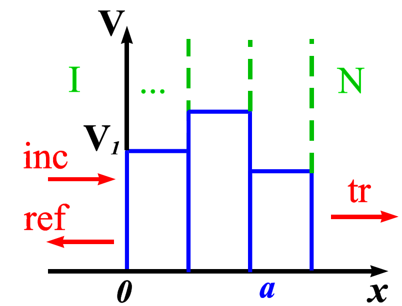
Here, E is kinetic energy of α-particle in lab. frame, E_1 is kinetic energy of relative motion of α-particle and nucleus, m is reduced mass of α-particle and nucleus, P_l is probability of fusion of α-particle and nucleus, T_l is penetrability of barrier.

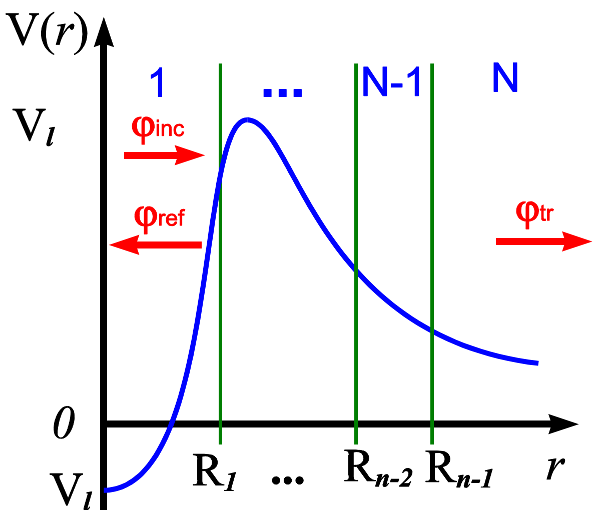
- Test of method: $T_{MIR} + R_{MIR} = 1.$

[1] Maydanyuk S. P., Zhang P.-M., et al. Nucl. Phys. **A940,** 89-118 (2015).

Accuracy of MIR method in capture task





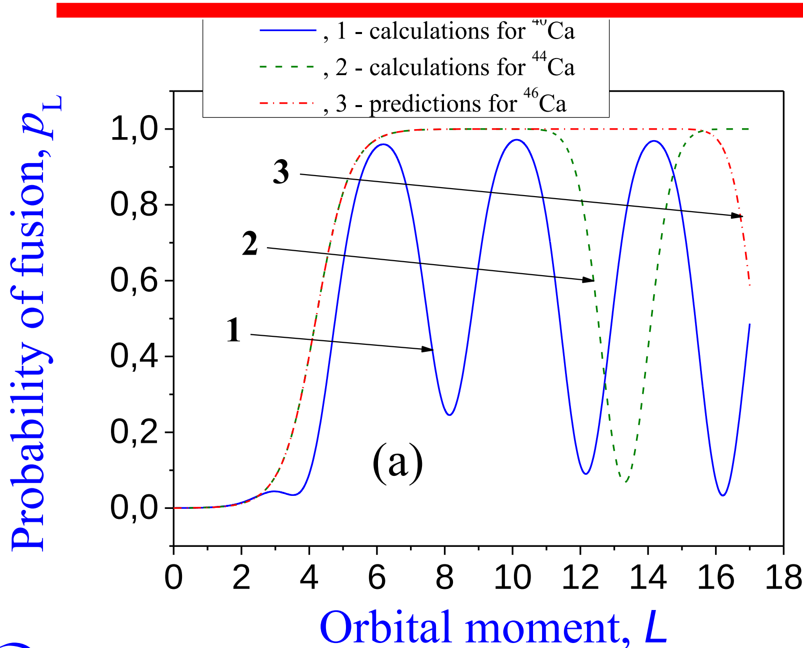


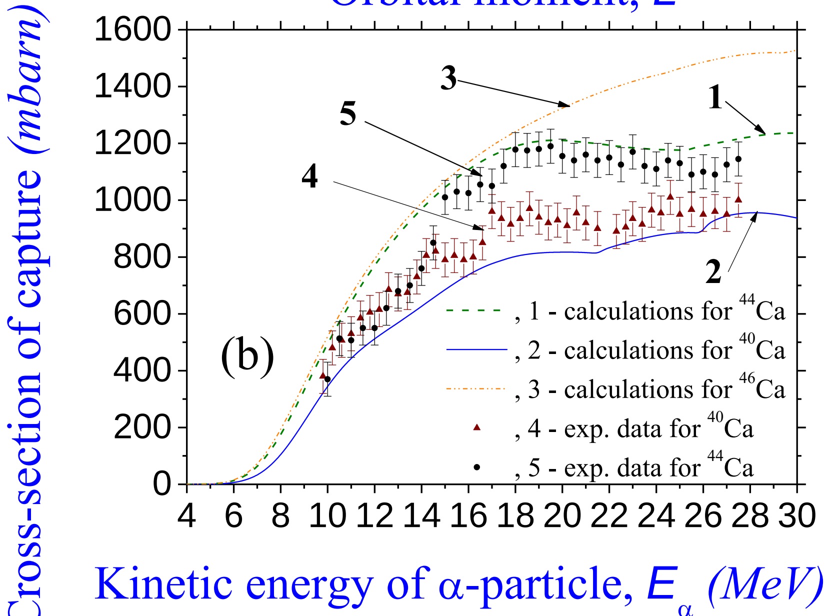
Test of method: $$T_{\text{bar}} + R_{\text{bar}} = 1.$$

Accuracy of method:

- Our method of Mult. Int. Refl.: 10^{-15} ;
- WKB-method (semiclassical, 1 order): 10^{-3}.

Formula for probability of fusion





Using fitting procedure, we found probabilities of fusion and described them as

$$\sigma_{capture}(E) = \frac{\pi \hbar^2}{2mE} \sum_{l=0}^{+\infty} (2l+1) T_l P_l .$$

$$p_{full}(L) = 1 - p_1(L) - p_2(L),$$

$$p_1(L) = \frac{c_1}{1+\exp\left[\frac{(L-c_2)}{c_3}\right]}, \quad p_2(L) = f_2(L) \cdot \sum_{n=1} \exp\left\{-\frac{(L-n\cdot\Delta)^2}{c_{4n}}\right\},$$

$$f_2(L) = 1 - \exp\{-c_5 \cdot (L-c_6)\}, \quad \Delta = a \cdot (N - N_{magic}) + b,$$

$$a = 2.31, \quad b = 4.05, \quad c_1 = 1, \quad c_2 = 4.2, \quad c_3 = 0.5.$$

Fig.7. Probabilities of fusion (a) calculated by formulas above and cross-sections (b) for capture of α-particles by ^{40}Ca, ^{44}Ca, ^{46}Ca, obtained by method MIR [1].

[1] Maydanyuk S. P., Zhang P.-M., Belchikov S. V. Nucl. Phys. A. - 2015. - Vol. 940. - P. 89-118.

Example

Example for study:

pycnonuclear reaction $^{12}C + ^{12}C$

Capture via simple barrier (1)

- Wave function:

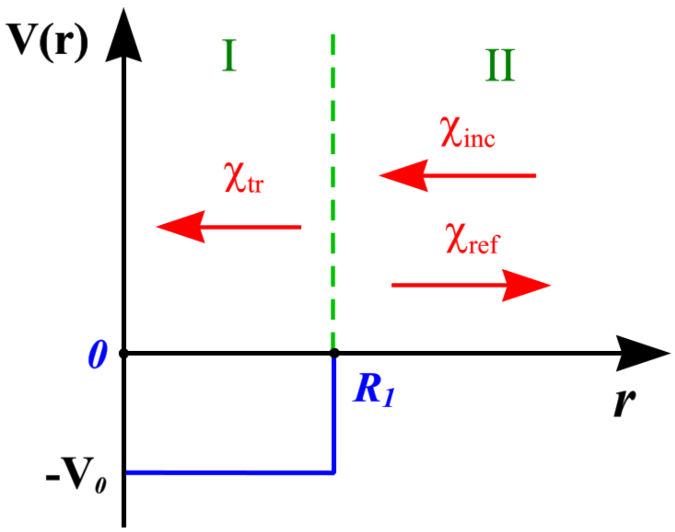
$$\chi(r)=\begin{cases} e^{-ik_2 r}+A_R e^{ik_2 r}, & R_1 \le r, \\ \alpha_1 e^{ik_1 r}+\beta_1 e^{-ik_1 r}, & R_{\min} \le r \le R_1, \end{cases}$$

Schrodinger equation (L=0):

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}\chi(r)+V(r)\chi(r)=E\,\chi(r).$$

Propagation by steps:

- 1 step:



$$\chi_{inc}^{(1)}=e^{-ik_2 r}, \qquad R_1 \le r_1,$$

$$\chi_{tr}^{(1)}=\beta_1^{(1)} e^{-ik_1 r}, \quad R_{\min} \le r \le R_1,$$

$$\chi_{ref}^{(1)}=\alpha_2^{(1)} e^{ik_2 r}, \qquad R_1 \le r.$$

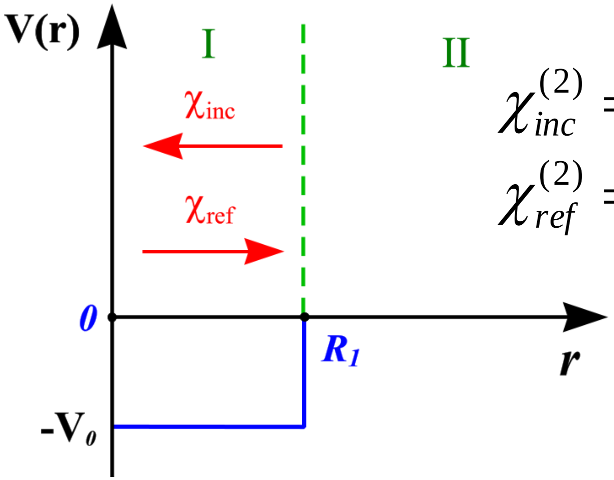
- Continuity of wave function at $r = R_1$:

$$\alpha_2^{(1)}=R_1^{-}=\frac{k_2-k_1}{k_2+k_1}\exp[-2ik_2R_1],$$

$$\beta_1^{(1)}=T_1^{-}=\frac{2k_2}{k_2+k_1}\exp[-i(k_2-k_1)R_1].$$

Capture via simple barrier (2)

- 2 step:



From continuity of wave function:

$$\chi_{inc}^{(2)} = \chi_{tr}^{(1)} = \beta_1^{(1)} e^{-ik_1 r},$$

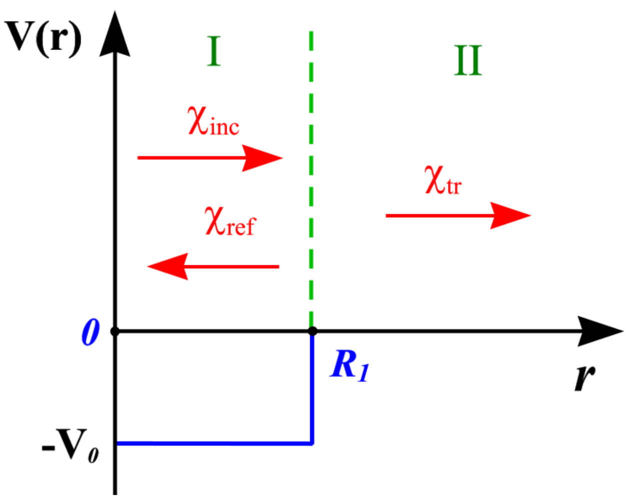
$$\chi_{ref}^{(2)} = \alpha_1^{(2)} e^{ik_1 r},$$

$$\alpha_1^{(2)} = R_0 \cdot \beta_1^{(1)}, \quad R_0 = -1,$$

$$\alpha_2^{(3)} = \alpha_1^{(2)} \cdot T_1^{+}, \quad T_1^{+} = \frac{2k_1}{k_2 + k_1} e^{i(k_1 - k_2)R_1},$$

$$\beta_1^{(3)} = \alpha_1^{(2)} \cdot R_1^{+}, \quad R_1^{+} = \frac{k_1 - k_2}{k_1 + k_2} e^{2ik_1 R_1}.$$

- 3 step:

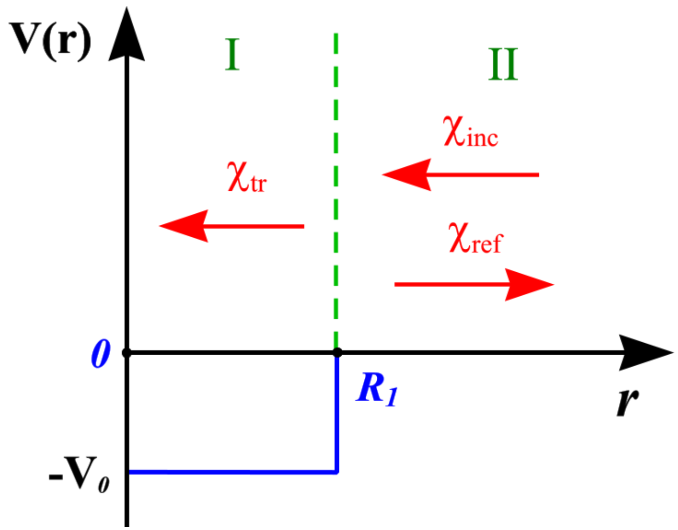


$$\chi_{inc}^{(3)} = \alpha_1^{(2)} e^{ik_1 r}, \quad R_{\min} \le r \le R_1,$$

$$\chi_{tr}^{(3)} = \alpha_2^{(3)} e^{ik_2 r}, \quad R_1 \le r_1,$$

$$\chi_{ref}^{(3)} = \beta_1^{(3)} e^{-ik_1 r}, \quad R_{\min} \le r \le R_1.$$

Capture via simple barrier (3)



Potential and resonant scattering:

$$\chi_{pot} = S_{pot} e^{ik_2 r} = \alpha_2^{(1)} e^{ik_2 r}, \quad \chi_{res} = S_{res} e^{ik_2 r} = \sum_{n=2}^{+\infty} \alpha_2^{(n)} e^{ik_2 r}.$$

$$\beta_1 = -\alpha_1 = A_{osc} \cdot T_1^-, \qquad T_{bar} = \frac{k_1}{k_2} \left| \beta_1^1 \right|^2 = \frac{k_1}{k_2} \left| T_1^- \right|^2,$$

$$S_{res} = \sum_{n=2}^{+\infty} \alpha_2^{(n)} = A_{osc} \cdot T_1^- R_0 T_1^+, \quad R_{bar} = \left| \alpha_2^{(1)} \right|^2 = \left| R_1^- \right|^2.$$

Summation of amplitudes:

$$\sum_{n=1}^{+\infty} \beta_1^{(n)} = -\sum_{n=1}^{+\infty} \alpha_1^{(n)} = \left\{ 1 + R_0 R_1^+ + (R_0 R_1^+)^2 + (R_0 R_1^+)^3 + \ldots \right\} T_1^- = A_{osc} T_1^-,$$

$$\sum_{n=2}^{+\infty} \alpha_2^{(n)} = \left\{ 1 + R_0 R_1^+ + (R_0 R_1^+)^2 + (R_0 R_1^+)^3 + \ldots \right\} T_1^- R_0 T_1^+ = A_{osc} T_1^- R_0 T_1^+,$$

$$A_{osc} = 1 + R_0 R_1^+ + (R_0 R_1^+)^2 + (R_0 R_1^+)^3 + \ldots = \frac{1}{1 - R_0^- R_1^+}.$$

Tests:

$$T_{bar} + R_{bar} = 1,$$

$$|S| = \left| S_{res} + S_{pot} \right| = \left| \alpha_2^{(1)} + \sum_{n=2}^{+\infty} \alpha_2^{(n)} \right| = 1,$$

Direct QM

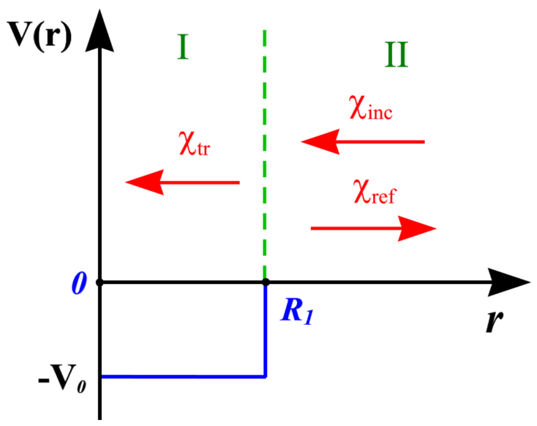
Capture via simple barrier (4)

Probability of existence of compound nucleus [1]:

$$P_{cn} = \int_0^{r_1} |\chi(r)|^2 dr = \int_0^{r_1} \left|\alpha_1 e^{ik_2 r} + \beta_1 e^{-ik_2 r}\right|^2 dr = P_{osc} T_{bar} P_{loc}.$$

$$P_{osc} = |A_{osc}|^2, \quad T_{bar} = \frac{k_1}{k_2} |T_1^-|^2, \quad P_{loc} = 2\frac{k_2}{k_1}\left(r_1 - \frac{\sin(2k_1 r_1)}{2k_1}\right).$$

$$A_{osc} = \left(1 + \sum_{i=1}^{+\infty} (R_0 R_1^+)^i\right) = \frac{k + k_1}{(k + k_1) + (k_1 - k)\exp(2ik_1 r_1)}.$$



- Exact analytical solution of Gamow's idea;
- Appearance of new factor P_{loc}
- Modern half-lives in decay tasks are calculated without this factor

Half-life of decay:

$$\tau = \hbar \ln 2/\Gamma, \quad \Gamma = P_p F \frac{\hbar^2}{4m} T, \quad F_1 = \left\{\int_{R_1}^{R_2} \frac{dr}{2k(r)}\right\}^{-1}$$

Γ – width of decay;

F_1 – factor of oscillating behavior;

P – spectroscopic factor.

1. S.P.Maydanyuk, P.M.Zhang, L.P.Zou, Phys. Rev. **C96,** 014602 (2017).

New quasi-bound states in scattering

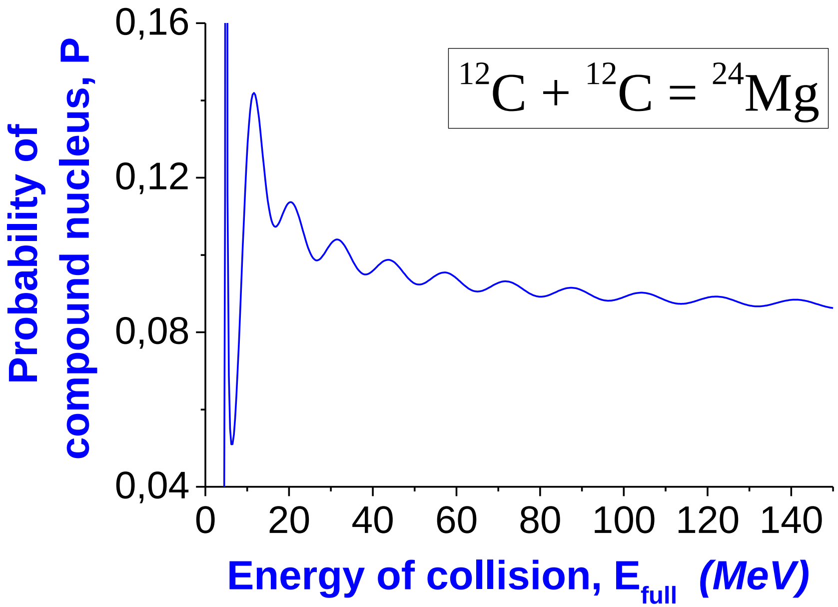
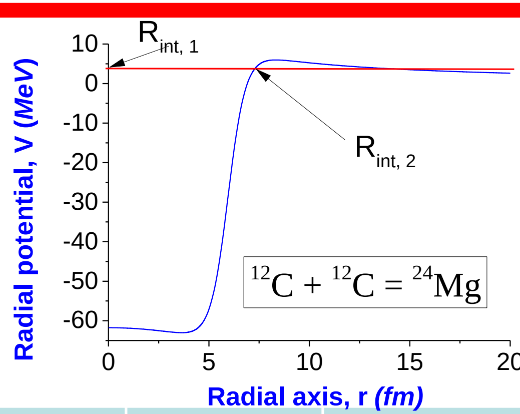


Fig.3. Maximums are clearly visible. These are states of the most probable existence of compound nucleus. We called them as ***quasi-bound states in pycnonuclear reactions***.



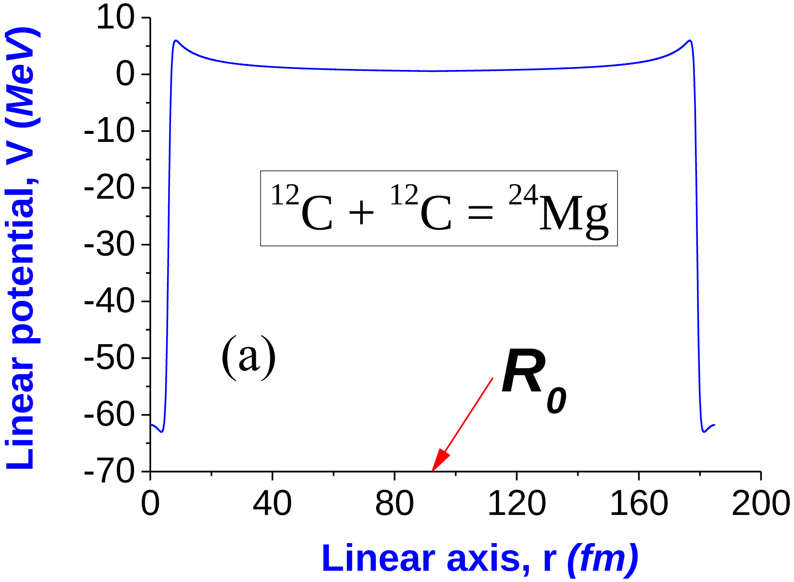
N	E, MeV	P_{cn}	R_{pot}	R_{res}
1	5.0032	0.7805	0.03581	0.6116
2	11.607	0.1419	6.24E-5	0.0038
3	20.313	0.1136	2.07E-6	2.25E-5
4	31.420	0.1040	2.31E-7	1.92E-6
5	43.729	0.0987	4.99E-8	1.71E-8
6	57.238	0.0954	1.57E-8	3.23E-8
7	71.647	0.0931	5.70E-9	3.75E-9

Probability of existence of compound nucleus:

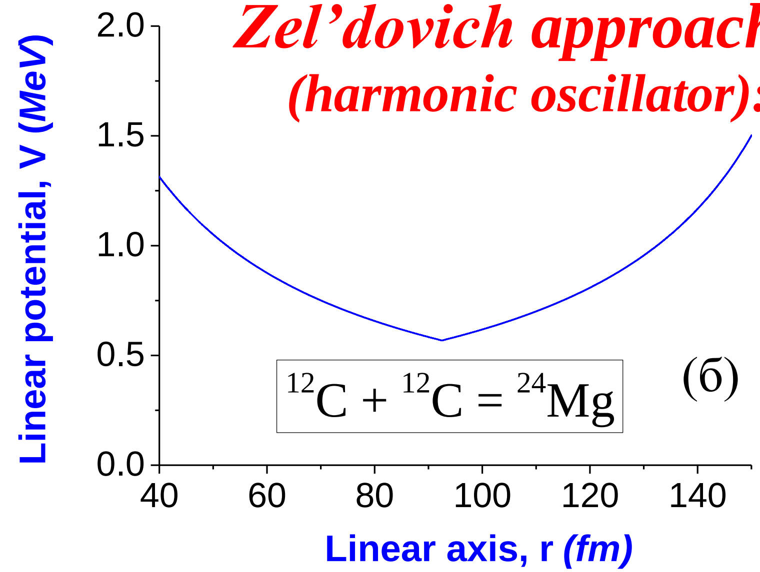
$$P_{\rm cn}(E) = \int_{r_{\rm int,1}}^{r_{\rm int,2}} |\chi(r)|^2 dr = \sum_{j=1}^{n} \left\{ \left(|\alpha_j|^2 + |\beta_j|^2 \right) \Delta r + \frac{\alpha_j \beta_j^*}{2ik_j} e^{2ik_j r} \Big|_{r_{j-1}}^{r_j} + c.c. \right\}.$$

Energy levels of zero-point vibrations (1)

Our method:



Zel'dovich approach (harmonic oscillator):



$$E_0 = \frac{\hbar w}{2} = \frac{\hbar Z e}{\sqrt{m R_0^3}},$$

$$\Delta E = \frac{2Z^2 e^2}{R_0}, \quad E_{full} = E_0 + \Delta E.$$

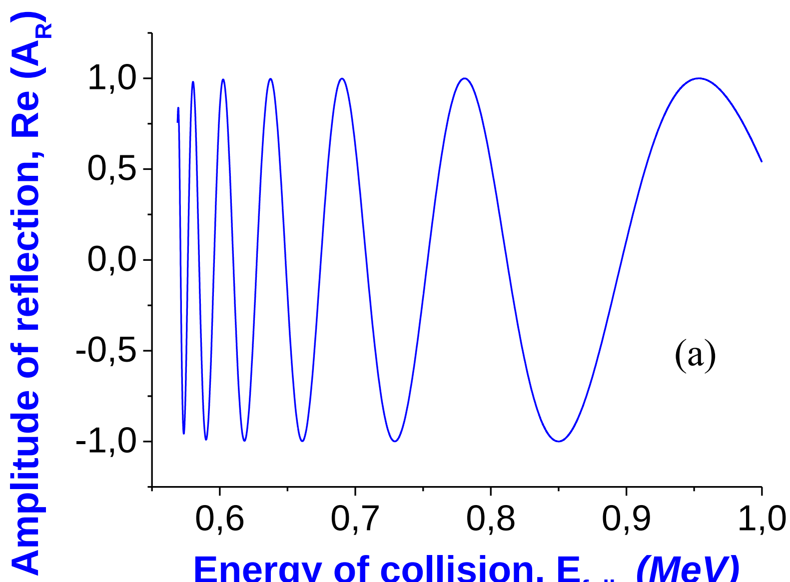
Determination of energy levels:

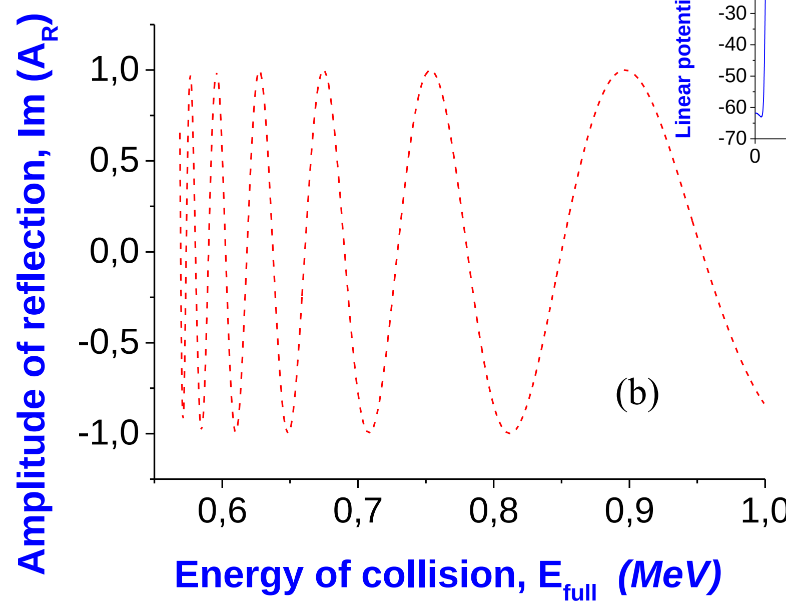
Using method MR, energy levels are calculated, where modulus of WF is minimal or maximal at point R_0 .

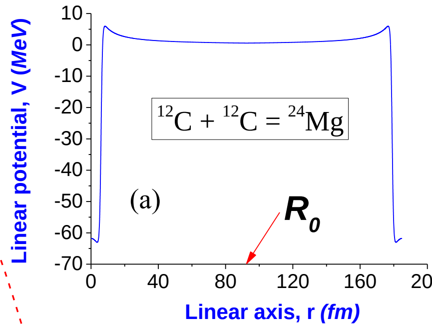
$$R_0 = 92.4\, fm.$$

$$E_0 = 0.021 MeV,$$
$$\Delta E = 0.567 MeV,$$
$$E_{full} = 0.589 MeV.$$

Energy levels of zero-point vibrations (2)







Condition for determination of states:

$$\text{even states:}\quad \chi(R_0) = e^{-ikR_0} + A_R e^{ikR_0} = e^{-ikR_0} + e^{ikR_0}, \quad A_R = +1,$$

$$\text{odd states:}\quad \chi(R_0) = e^{-ikR_0} + A_R e^{ikR_0} = e^{-ikR_0} - e^{ikR_0}, \quad A_R = -1.$$

$$\mathrm{Re}(A_R) = \pm 1,$$
$$\mathrm{Im}(A_R) = 0.$$

Idea of determination of levels: Using method MR, energy levels are determined, where condition of amplitude A_R at point R_0 is fulfilled.

Energy levels of zero-point vibrations (3)

No.	Energy, MeV	Amplitude AR, Re	Amplitude AR , Im
1	0.569699398797595	0.933197319275621	-0.359364387908423
2	0.574108216432866	-0.929937621506901	0.367717310044127
3	0.580280561122244	0.999976682784241	-0.006828900923611
4	0.589979959919840	-0.999804559327251	0.019769753373290
5	0.603206412825651	0.987566383351778	-0[illegible]
6	0.619078156312625	-0.987872204000573	0.155269148780593
7	0.637595190380762	0.999675512392435	-0.025472925291808
8	0.661402805611222	-0.997827712405446	0.065877586140615
9	0.690501002004008	0.999520247643956	-0.030972157654334
10	0.729298597194389	-0.999921682875423	0.012515115484128
11	0.781322645290581	0.999300653371264	-0.037392568402883
12	0.850100200400802	-0.999977543965837	0.006701609064401
13	0.954148296593186	0.999861191695802	-0.016661252673514

$$E_0^{(Zel'dovich)} = 0.589 MeV.$$

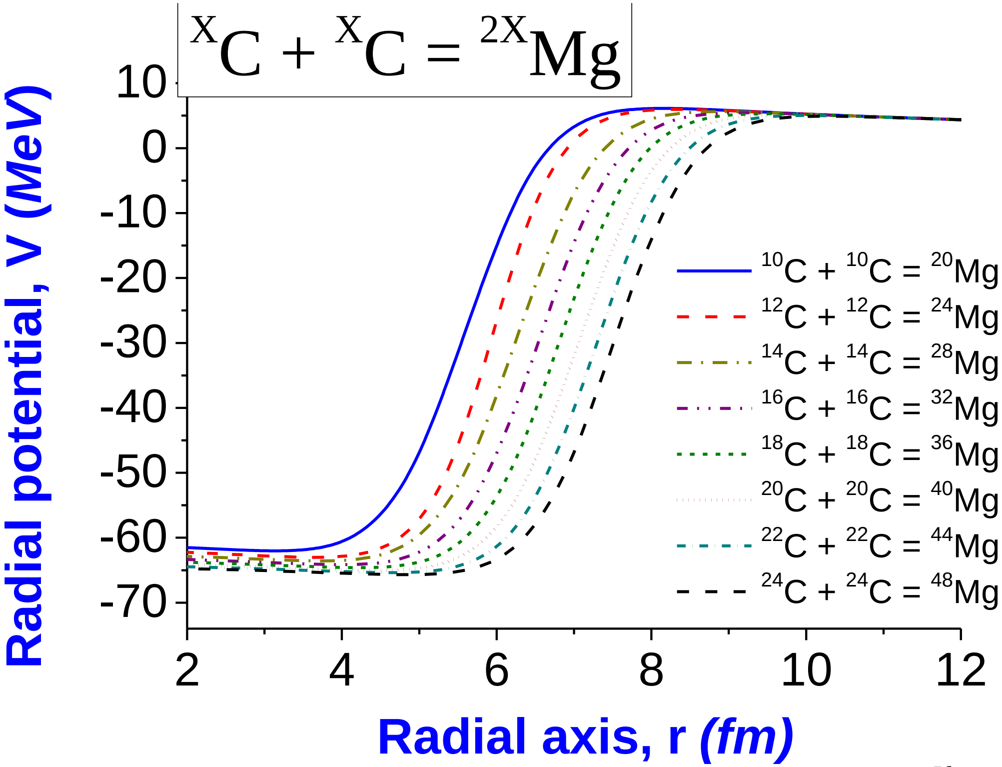
Error in calculation of amplitudes: $\left| |A_T|^2 + |A_R|^2 - 1 \right| < 10^{-14}.$

Systematics

Calculations for isotopes of Carbon:

^{10}C, ^{12}C, ^{14}C, ^{16}C, ^{18}C, ^{20}C, ^{22}C, ^{24}C

Potential of interactions



Quantum mechanical study on the basis of solution of Schrodinger equation with potential:

$$V(r) = V_C(r) + V_N(r) + V_{l=0}(r),$$

$$V_N(r) = -\frac{V_R}{1+\exp\left(\frac{r-R_R}{a_R}\right)},$$

$$R_R = r_R(A_1^{1/3} + A_2^{1/3}),$$

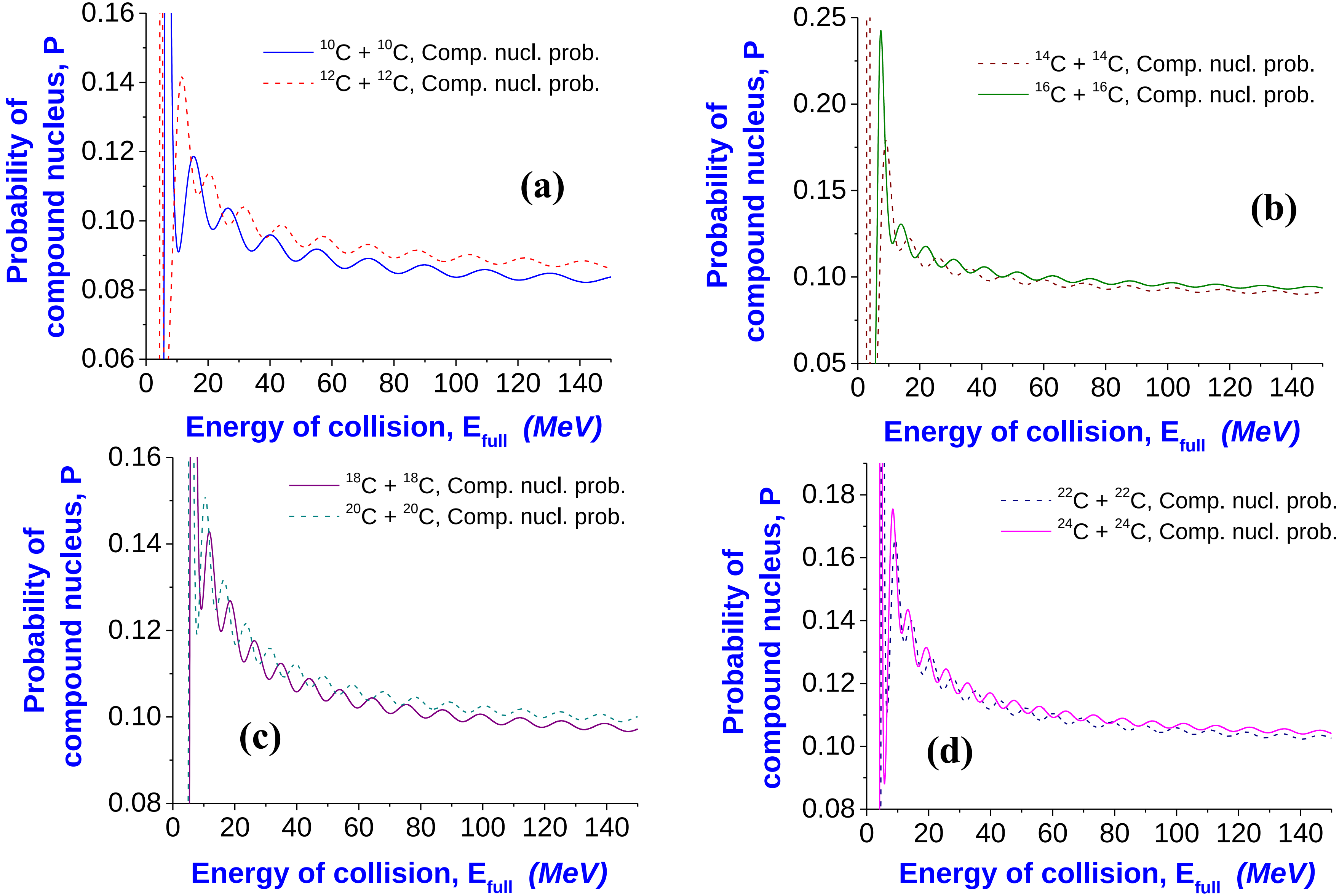
$$r_R = 1.30\, fm, \quad a_R = 0.44\, fm. \; V = -75 MeV.$$

$$\rho_0 = \frac{m_A}{V_A} = \frac{Am_u}{4/3\pi R_0^3},$$

$$R_0 = \left(\frac{Am_u}{4/3\pi\rho_0}\right)^{1/3}. \qquad n_A = \frac{\rho_0}{Am_u}.$$

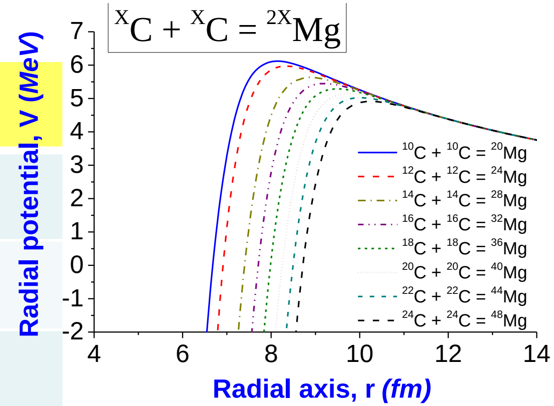
$$\rho_0 = 6\cdot 10^9 \frac{g}{cm^3},$$
$$R_0 = 91\, fm - 105\, fm,$$
$$n_A = 3.014\cdot 10^{-7}\, fm^{-3}.$$

New quasi-bound states for fusion



New quasi-bound energies for fusion

No.	$^{10}C + ^{10}C$	$^{12}C + ^{12}C$	$^{14}C + ^{14}C$	$^{16}C + ^{16}C$
1	0.63471	4.88176	9.0621	7.2705
2	15.3326	11.4509	16.5270	13.8396
3	26.3807	20.4088	25.7835	21.9018



No.	$^{18}C + ^{18}C$	$^{20}C + ^{20}C$	$^{22}C + ^{22}C$	$^{24}C + ^{24}C$
1	6.3747	5.4789	5.1803	4.5831
2	11.7495	10.5551	9.0621	8.4649
3	18.3186	16.5270	14.4368	13.2424

	Barrier max
^{10}C	6.249 MeV
^{12}C	5.972 MeV
^{24}C	5.001 MeV

Table 2. Indication for synthesis. Energies for ground quasibound states only for isotopes ^{10}C, ^{12}C, ^{24}C (marked data) are smaller than barrier maximums. That means for such energies bound system is formed, and for its decay through tunneling phenomenon. Halflives of such systems should be larger essentially. This is indication on synthesis of more heavy nucleus with high probability (which can be estimated).

Plasma screening of pychnonucl. fusion (1)

Strong plasma screening of pycnonuclear reactions with ^{12}C

Plasma screening of pychnonucl. fusion (2)

Atomic nuclei in dense stellar matter are fully ionized by enormous electron pressure, electrons are so energetic that they constitute almost rigid background of negative charge in which ions are located [1].

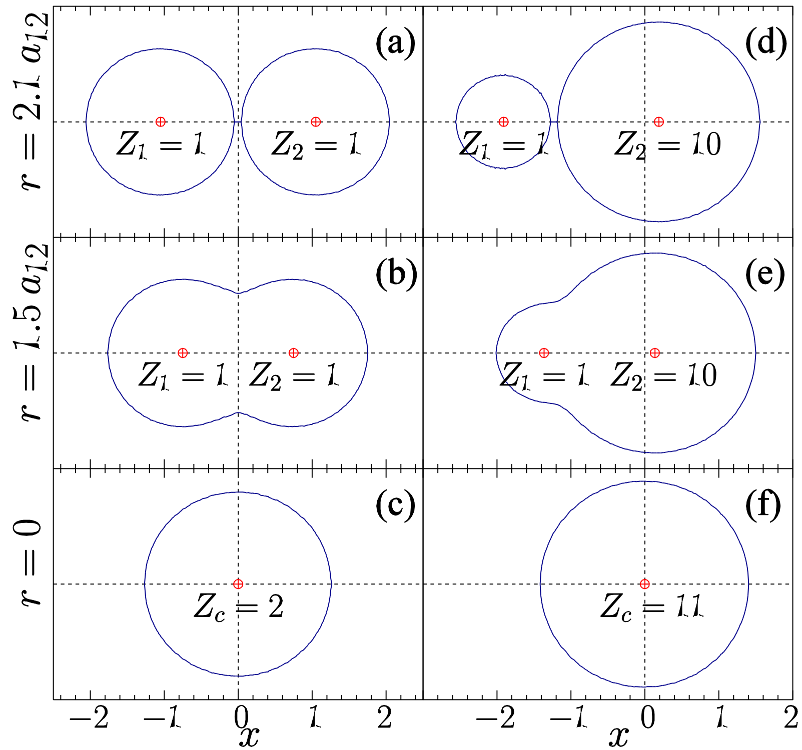


Fig.1: Simulated shapes of electron drops around two colliding nuclei at ion-distance:

for $Z_1 = Z_2$: (a) $2.1a_{12}$, (b) $1.5a_{12}$, (c) 0,

for $Z_2 = 10Z_1$ (d) $2.1a_{12}$, (e) $1.5a_{12}$, (f) 0 [1].

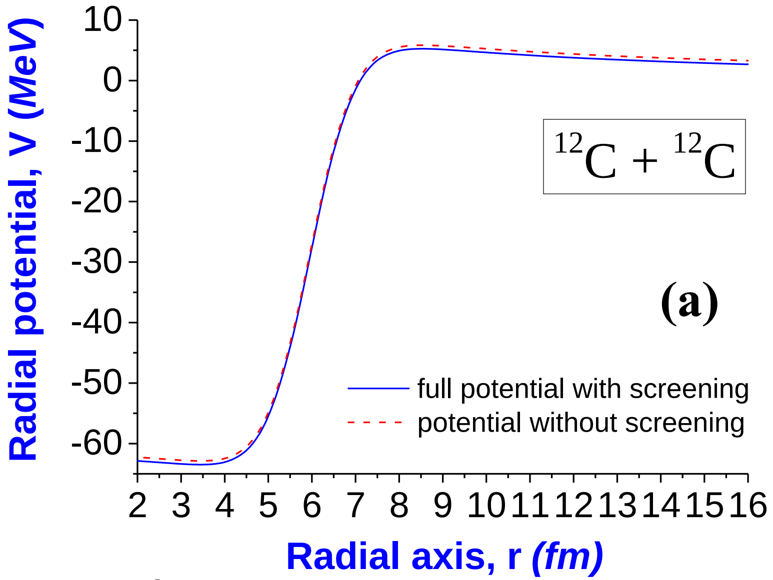
Quantum mechanical study on the basis of solution of Schrodinger equation with potential is

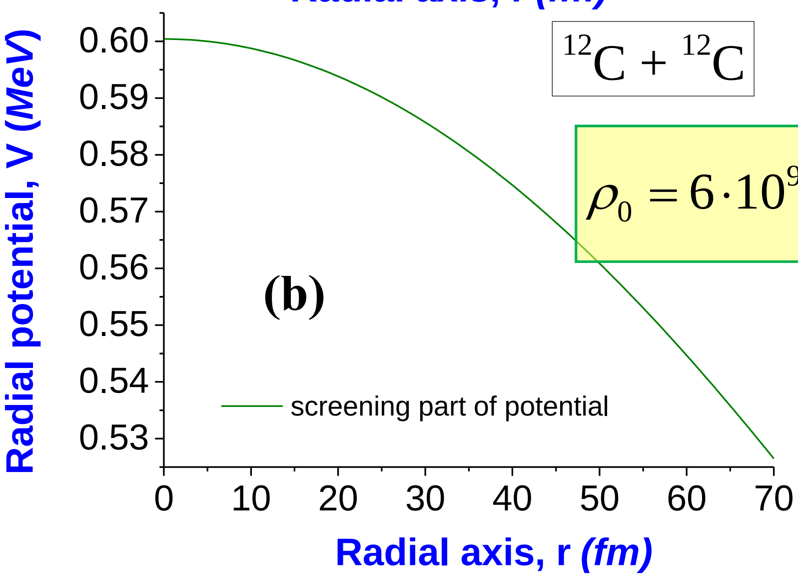
$$\hat{H}\,\Psi = \left(\hat{T} + \hat{V}\right)\Psi = E\,\Psi.$$

$$a_{12} = \frac{a_1 + a_2}{2}, \quad a_j = Z_j^{1/3} a_e, \quad a_e = \left(\frac{3}{4\pi n_e}\right)^{1/3}.$$

1. P.A.Kravchuk, D.G.Yakovlev, Phys. Rev. **C89**, 015802 (2014).

Plasma screening of pychnonucl. fusion (3)





$$\rho_0 = 6 \cdot 10^9 \ \frac{g}{cm^3}$$

Potential of interactions between nuclei with screening:

$$V(r) = U_C(r) + V_N(r) + V_{l=0}(r),$$

Coulomb potential for incident nuclei with screening is [1]:

$$U_{C,\,full}(r) = V_C(r) + H_{screen}(r),$$

$$H(r) = E_{12}\, h(x), \quad x = \frac{r}{a_{12}},$$

$$h(x) = b_0 + b_2 x^2 + b_4 x^4,$$

$$b_0 = 1.0573, \quad b_2 = -0.25, \quad b_4 = 0.0394.$$

$$E_{12} = \frac{Z_1^2 e^2}{a_{12}}, \quad a_{12} = a_1 = Z_1^{1/3} a_e, \quad a_e = \left(\frac{3}{4\pi n_e}\right)^{1/3}.$$

1. P.A.Kravchuk, D.G.Yakovlev, Phys. Rev. **C89,** 015802 (2014).

Plasma screening of pychnonucl. fusion (4)

No.	$^{12}C + ^{12}C$ *with screening, MeV*	$^{12}C + ^{12}C$ *without screening, MeV*
1	0.02096192	0.51743486
2	0.26733466	0.53667334
3	0.34428857	0.54629258
4	0.47895791	0.55591182
5	0.78677354	0.57515030
6	2.21042084	0.59438877
7	3.83607214	0.61362725
8		0.64248496
9		0.68096192
10		0.72905811
11		0.91182364
12		1.11382765
13		2.76833667

Table: Energies for zero-point vibrations (data are below 4 MeV)

Energies for quasibound states are not much different for processes with screening and without it.

Conclusions (1)

1)Formation of compound nuclear systems needed for synthesis of heavy nuclei in pycnonuclear fusion with isotopes of Carbon in compact stars is studied on a quantum mechanical basis.

2)New quantum method for pycnonuclear reactions in compact stars is developed, taking the nuclear potential of interactions between nuclei into account. It gives appearance of new states (called as quasibound states), in which compound nuclear systems of Magnesium are formed from isotopes of Carbon with the largest probability.

3)Rates of pycnonuclear reactions are changed essentially after taking into account nuclear forces and quantum mechanical basis.

4)Energy spectra of zero-point vibrations and spectra of quasibound states are estimated with high precision for reactions with isotopes of Carbon.

5)At the first time influence of plasma screening on quasibound states and states of zero-point vibrations in pycnonuclear reactions has been studied.

Conclusions (2)

6)The probability of formation of compound nuclear system in quasibound states in pycnonuclear reaction is essentially larger than the probability of formation of this system in states of zero-point vibrations studied by Zel'dovich and followers. Synthesis of Magnesium from isotopes of Carbon is more probable through the quasibound states than through the states of zero-point vibrations in compact stars. Energy spectra of zero-point vibrations are changed essentially after taking plasma screening into account.

7)Analysis shows that from all studied isotopes of Magnesium only ^{24}Mg is stable after synthesis at energy of relative motion of 4.881 MeV of incident nuclei ^{12}C.

Thank you for attention!