

H01

S1 EGYENLESEK

S2) 3 DIMENZÍÓS

S3 IRÁNYÍTOTT

S4) EUKLIDESZI

T1 EGY DIMENZÍÓS

T2 IRÁNYÍTOTT

M1 EGYENES: PÁLYA

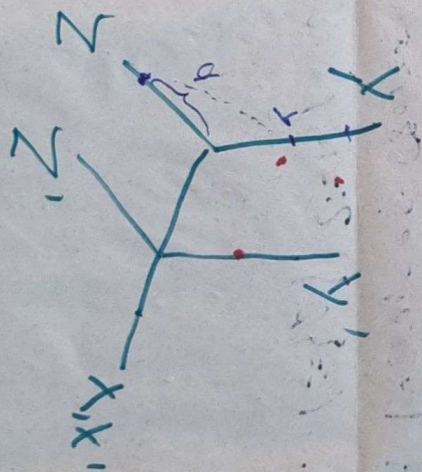
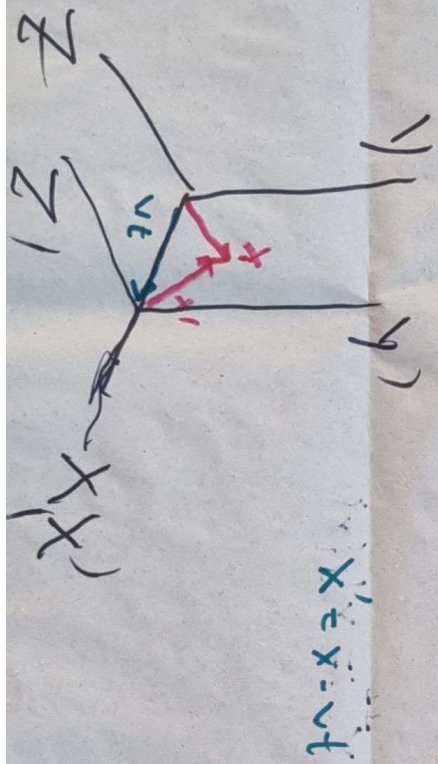
M2 TEGHETLEN PÁLYA

EGYENES

u (TÍRÜKK)

M3 VAN GYORSABB

M4 MINDEEN LASSABB



$$\sqrt{y^2 + z^2}$$

H02

- standard logika

Objektumok, kifejezések, logikai fu. $\rightarrow \neg, \wedge, \vee, \rightarrow$ - ZFC: halmaz \in $A := \dots =$ $(a, b) := \{\{a\}, \{a, b\}\}$ $A \times B$ - Zorn-lemma: (S, \leq) kivál $a \times \exists$ Descartes-szorzat- term: $0 := \emptyset, 1 := \{\emptyset\}, 2 := \{\emptyset, 1\}, \dots$
 $X^{++} := X \cup \{x\}$
 \mathbb{N}

H02

03

② (v, t_1) vektor

$$\| \cdot \| : V \rightarrow \mathbb{R}_0^+$$

$$\|c\| = \|k\| \cdot \|1\|,$$

$$\|x+y\| \leq \|x\| + \|y\|, \quad \|x\|_0 \Rightarrow x=0.$$

$$\| \cdot \|_1 \leq C \| \cdot \|_2$$

$$\textcircled{H} \quad \supset \quad \text{def } A := \{ \supset \text{def } X \mid \dots \}$$

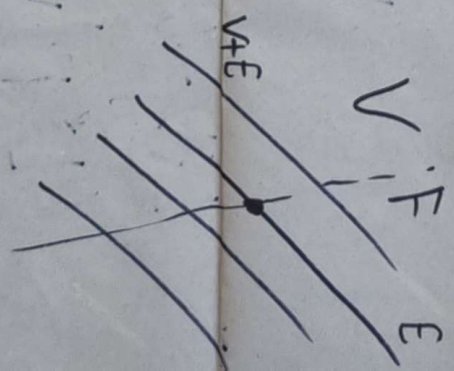
⑦ $E, F \subset V$:
 $E \cap F = \{0\}$
 $E + F = V$

$$\dim V = \dim E + \dim F$$

⑦ $ECV: \therefore v+E: \quad v+E=ntE \Leftrightarrow v-q \in E$

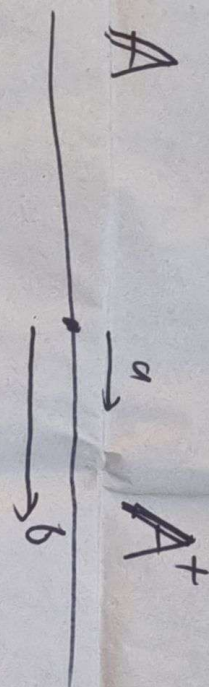
$$v/\epsilon$$

$$(v+\epsilon) + (k+\epsilon) := (v+k) + \epsilon, \quad \alpha(v+\epsilon) = (\alpha v) + \epsilon.$$



H044

I (V, \mathbb{R}^n)



① ~~map~~ $\{V \rightarrow \mathbb{R}\} =: V^*$ $\dim V^* = \dim V$

$p(v) =: (p|_V)$

Vektor

Kovektor

$\{V^* \rightarrow \mathbb{R}\} =: V^{**} = V$ $V^{**} \cong V$ $(p \mapsto (p|_V))$ $V^{**} \cong V$

① $L: V \rightarrow V$ $L^*: U^* \rightarrow V^*$ $p \mapsto p \circ L$

$L^{**} = L$

$(L^* p|_V)_V = (p|_{L(V)})_V$

$(L^* m)^* = m^* L^*$

$\mathbb{R}^* = \mathbb{R}$

① V, U, W .

$$\text{Lin}(V, U) := \{L: V \rightarrow U \mid L \text{ lin}\}$$

$$\text{Bilin}(V \times U, W) := \{B: V \times U \rightarrow W \mid B \text{ bilinear}\}$$

$$V^* = \text{Lin}(V, \mathbb{R})$$

$$B \in \text{Bilin}(U \times V, \mathbb{R})$$

$$L \in \text{Lin}(U, U^*)$$

$$v \mapsto B(\cdot, v) \quad \text{Lin}(V, U^*)$$

$$(u, v) \mapsto \underbrace{(u | Lv)}_{= u(Lv)}$$

$$\text{Bilin}(U \times V, \mathbb{R}) \cong \text{Lin}(V, U^*) = U^* \otimes V^*$$

$$U \otimes V := \text{Bilin}(U^* \times V^*, \mathbb{R})$$

$$\begin{matrix} u \\ r \end{matrix} \quad \begin{matrix} v \\ p \end{matrix}$$

$$r \otimes p: U \times V \rightarrow \mathbb{R}, \\ (u, v) \mapsto (u | r)(p | v)$$

$$r \otimes p: U \rightarrow U^*$$

$$r \otimes p(u) = r(p | u)$$

$$V \otimes V, \quad V^* \otimes V, \quad V \otimes V^*, \quad V^* \otimes V^*$$

$$\sum_{i=1}^n v_i \otimes v_i$$

$$\sum_i p_i^1 v_i$$

$$T_r$$

$$T_r$$

$$V \wedge V$$

$$\sum_i (p_i | v_i)$$

$$V^* \wedge V^*$$

$$V^* \wedge V^*$$

(K)

$$V(e_1, \dots, e_n)$$

$$V = \sum_{i=1}^n v_i^1 e_i$$

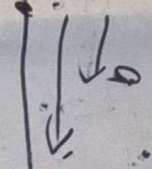
$$V^* (p_1^1, \dots, p_n^1)$$

$$(p_i^1 | e_j) = \delta_{ij}^1$$

$$V = \sum_{i=1}^n (p_i^1 | v) e_i$$

$V \wedge V = 0$

④ A



(A, a)

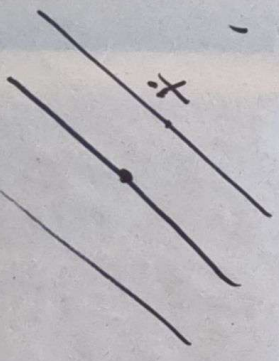
$A \succ B$
 $A \otimes B$

$A^* \otimes B \otimes A^* =: \frac{B}{A}$

$A^+ \rightarrow (A \otimes A)^+, a \mapsto a \otimes a$

$(V, V, -) \quad E \subset V,$

$\dot{x} + E$

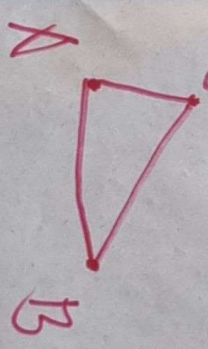
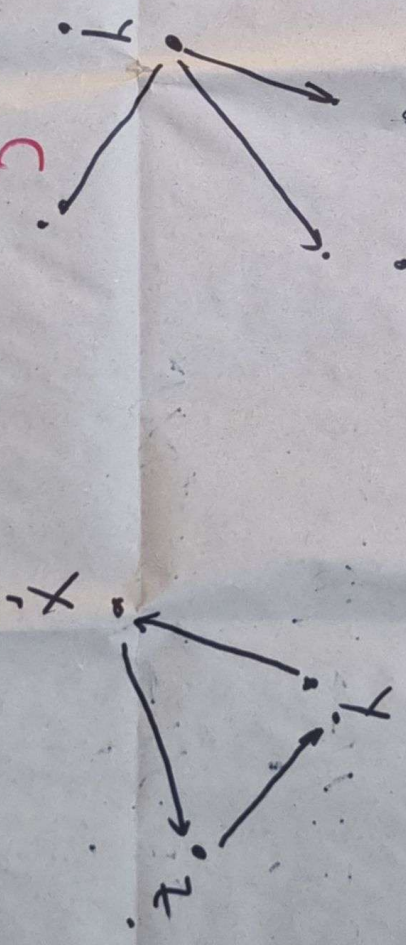


$V, V \otimes A$

④ $(V, V, -) \rightarrow V \times V \rightarrow V$

① $\forall y \in V: O_y: V \rightarrow V, \dot{x} \mapsto \dot{x} - y$
 $(\dot{x}, y) \mapsto \dot{x} - y$

② $\forall \dot{x}, y, z \in V: (\dot{x} - y) + (y - z) + (z - \dot{x}) = 0$



$O_y^{-1} x =: y + x$

①

$$V, A, f: A \rightarrow V$$

$$f'(a) := \lim_{b \rightarrow a} \frac{f(b) - f(a)}{b - a} \in V$$

$$U, V \quad \text{ordo: } U \rightarrow V$$

$$\lim_{x \rightarrow 0} \frac{\text{ordo}(x)}{\|x\|} = 0$$

$$F: U \rightarrow V$$

$$\exists D[F]x: U \rightarrow V:$$

$$F(y) - F(x) = D[F]x (y - x) + \text{ordo}(y - x)$$

$$\left(\overset{\text{lin}}{\in \text{Lin}(U, V)} \right)$$

$$+ \text{ordo}(y - x)$$

$$DF: U \rightarrow V \otimes U^*$$

$$D^2 F: U \rightarrow V \otimes U^* \otimes U^*$$