01

$\exists\, g : M \times M \to T \otimes T$ bilineáris

$\forall u \in V(1)$ $\quad g(x,y) = -\tau_u(x)\tau_u(y) + l_u(x,y)$

$q \in S_u$ $\quad g(q,q) = -\underbrace{\tau_u(q)^2}_{=0} + l_u(q,q) > 0$

tu $\quad g(tu,tu) = -\underbrace{\tau_u(tu)^2}_{-t^2 < 0} + \underbrace{l_u(tu,tu)}_{=0}$

$x \in \vec{T} \Rightarrow g(x,x) < 0$!

$$\boxed{P(x) = \sqrt{-g(x,x)}}$$

$$g\left(\frac{x}{P(x)}, \frac{x}{P(x)}\right) = \frac{g(x,x)}{P(x)^2} = -1$$

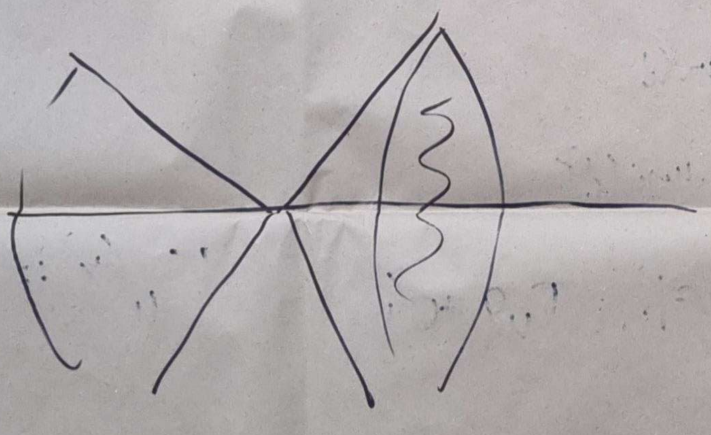
$g(u,y) = -\tau_u(y) + b_u(u,y)$

$\overset{!}{0}$

$$\boxed{b_u(x,y) = g(x,y) + \cancel{b_u t}\, g(u,x)\, g(u,y)}$$

$\mathbb{R}^4$ $\quad g\big((x^0,x^1,x^2,x^3),(x^0,x^1,x^2,x^3)\big) := -x^0x^0 + x^1x^1 + x^2x^2 + x^3x^3$

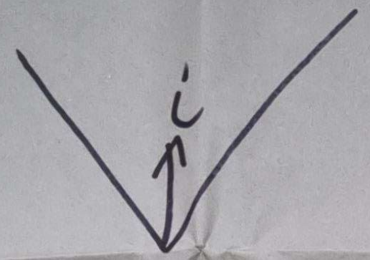
< 0



nyilirányitás

$\boxed{\vec{T}}$

C03



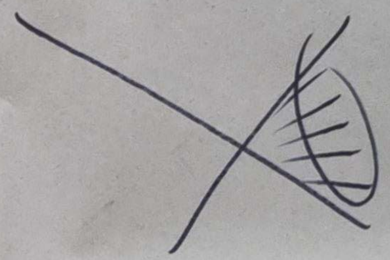
$-x^{\circ} x^{\circ} + \dots$

$(ict)(ict)$

Def. Spec. rel. TM (M, T, g) $\quad g: M \times M \to T \otimes T$
nyílirányítás

$V(1) = \left\{ u \in \frac{M}{T} \;\middle|\; g(u,u) = -1,\ u\ \text{~~jöv~~ poz. nyíli} \right\}$

$\vec{L} = \{ x \in M \mid g(x,x) = 0,\ x\ \text{poz. nyíli} \}$



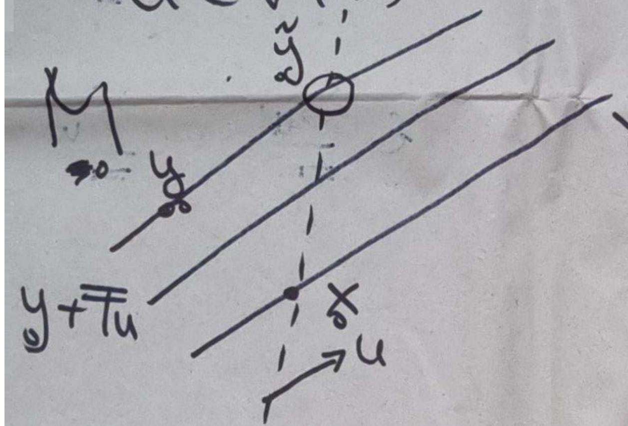
$S_u = \{ q \in M \mid g(u,q) = 0 \} = \{ q \in M \mid \tau_u q = 0 \}$

std. u-térvektorok

$\tau_u(x) := -g(u,x)$

(C04) **Teljességi mf std térvektorai**

$u \in V(1)$ th. mf. $\cancel{\text{krekt.}}$ $\underset{\circ}{M} / T_u$



$x_0 + S_u$.

$\underset{\circ}{x} + T_u$ affin tér

$(\underset{\circ}{x} + T_u) - (\underset{\circ}{y} + T_u) := (\underset{\circ}{\tilde{y}} - \underset{\circ}{x}) \in S_u$

$\pi_u : M \longrightarrow S_u \qquad x \longmapsto x - \tau_u(x)u$

$= x + g(u,x)u$

$\pi_u = id_M - u \otimes \tau_u$

$\underset{\circ}{\pi}_u : \underset{\circ}{M} \longrightarrow \underset{\circ}{M} / T_u \qquad \underset{\circ}{\pi}_u(\underset{\circ}{x}) := \underset{\circ}{x} + T_u$

<u>áll</u>. $\underset{\circ}{\pi}_u$ affin lek. π_u fölött

$\underset{\circ}{\pi}_u(\underset{\circ}{x}) - \underset{\circ}{\pi}_u(\underset{\circ}{y}) = \pi_u(x_0 - \underset{\circ}{y})$

$\underset{\circ}{M} / T_u$ affin. $M / T_u \equiv S_u$

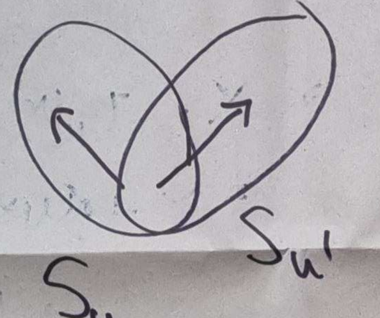
$(\underset{\circ}{x} + T_u) - (\underset{\circ}{y} + T_u) := (\underset{\circ}{x} - \underset{\circ}{y}) + T_u$

$\underset{\circ}{x}$ $\ominus$ $\underbrace{x_0 - \underset{\circ}{\tilde{y}}}_{T_u} + x_0$

CO5

$$V_{u'u} := \frac{\pi_u u'}{\tau_u u'} = \frac{u' - (\tau_u u')u}{\tau_u u'} = \frac{u'}{-g(u,u')} - u \in \frac{S_u}{T} \setminus \frac{S_{u'}}{T}$$

$$V_{uu'} := \frac{\pi_{u'} u}{\tau_{u'} u} \in \frac{S_{u'}}{T} \setminus \frac{S_u}{T}$$



$$\dim(S_u \cap S_{u'}) = 2$$

$$\boxed{x \cdot y := g(x,y)}$$

<u>áll.</u> $$|V_{u'u}|^2 = \underbrace{|V_{uu'}|^2}_{v^2} = 1 - \frac{1}{(-g(u,u'))^2} = 1 - \frac{1}{(\tau_{u'} u)^2}$$

$\frac{S_u}{T}$-beli $\qquad \frac{S_{u'}}{T}$-beli

<u>Relativisztikus faktor</u> $$\tau_{u'} u = -u' \cdot u = \frac{1}{\sqrt{1 - v^2}} \cdot \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

(06)

$X, Y \in T^{\to}$ $\quad |X \cdot X| = |X|^2$ $\quad |X \cdot Y| \geq |X||Y|$

$u \neq u'$ $\quad |X + Y| \geq |X| + |Y|$

$|u \cdot u'| \geq 1$

$B_{u'u} : M \to M$

$X \mapsto X + \frac{((u'+u)\cdot X)(u'+u)}{1 - u' \cdot u}$ ~ $A_{u,\dots}$

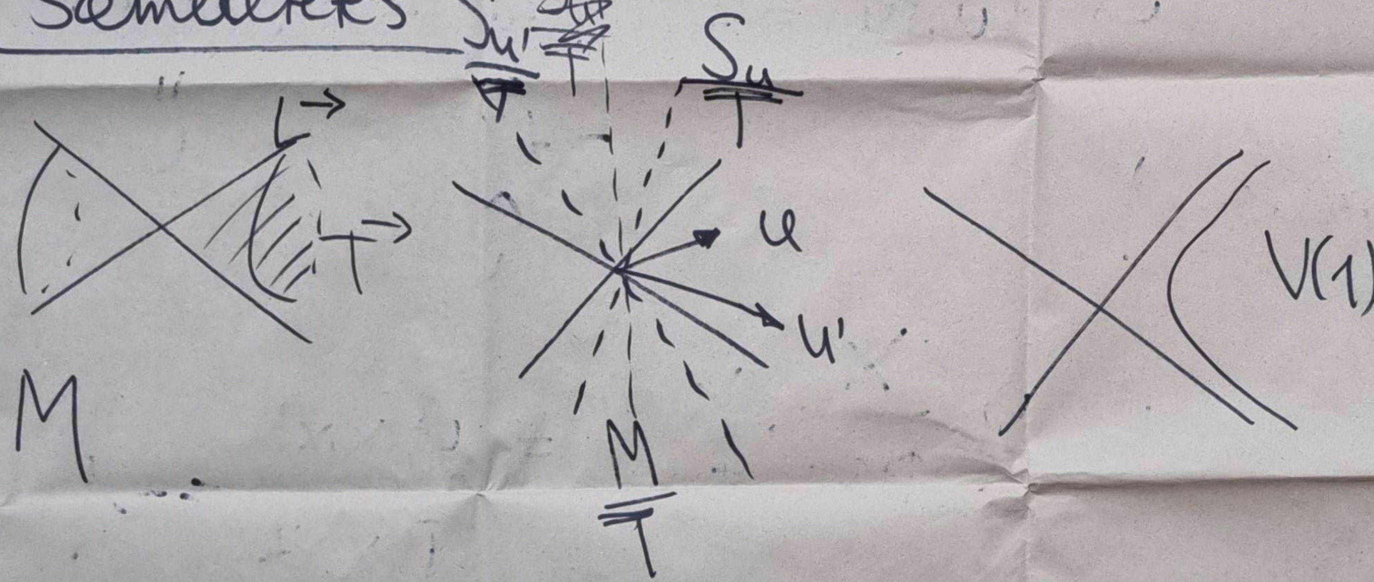
$id_M + \frac{(u'+u)\otimes(u'+u)}{1 - u' \cdot u} - 2u' \otimes u$

1. $B_{uu} = id_M$
2. $(B_{u'u})^{-1} = B_{uu'}$
3. $B_{u''u'} B_{u'u} \neq B_{u''u}$

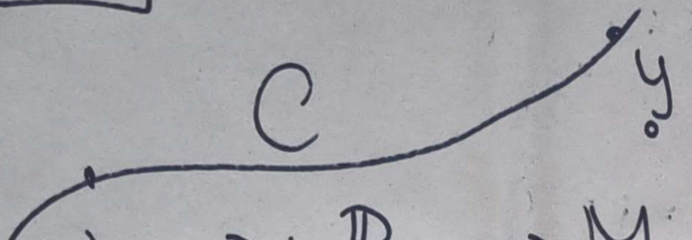
és $=$ $u\ u'\ u''$ egy síkban van

Thomas-forg. $R_{uu'u''} := B_{uu''} B_{u''u'} B_{u'u}$

Szemléletes



C07



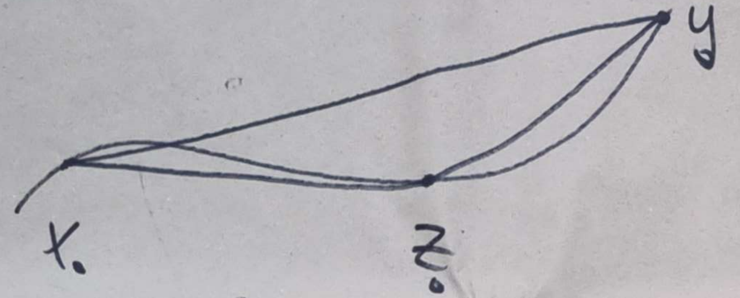
$\underset{\circ}{x}$ $p : \mathbb{R} \to \underset{\circ}{M}$

Sajátidő

$$t_C(\underset{\circ}{x}, \underset{\circ}{y}) = \int_{p^{-1}(x)}^{p^{-1}(y)} |\dot{p}(a)| \, da$$

pseudohossz

$$|\dot{p}(a)| = \sqrt{-\dot{p}(a)\cdot\dot{p}(a)}$$



áll. két időszerűen szep. téridőpont között a legtöbb sajátidő a ~~kheteteuség~~ világvonalon telik el

Izomorfia

$$(\underset{\circ}{M}, \overline{\overline{T}}, g)$$

$$\underset{\circ}{L} \downarrow \quad \downarrow F \text{ lin. bij}$$

aff. bij $(\underset{\circ}{\tilde{M}}, \tilde{\overline{\overline{T}}}, \tilde{g})$

$x, y \in M$

$s \in T$

$$\frac{\tilde{g}(Lx, Ly)}{(Fs)^2} = \frac{g(x,y)}{s^2}$$

$\exists L : M \to \tilde{M}$ lin

$\underset{\circ}{L}(\underset{\circ}{x}) - \underset{\circ}{L}(\underset{\circ}{y}) = L(\underset{\circ}{x} - \underset{\circ}{y})$

C08

ARITMETIKAI TERIDŐMODELL : $(\underset{0}{M}, T, g)$

$\underset{0}{M} := \mathbb{R}^4$ $\quad (M := \mathbb{R}^4$ szintén$)$

$T := \mathbb{R}$

$g\big((x^0, x^1, x^2, x^3), (y^0, y^1, y^2, y^3)\big) := -x^0y^0 + x^1y^1 + x^2y^2 + x^3y^3$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\underbrace{(x^0, x^1, x^2, x^3)^b}_{\in M^*}(y^0, y^1, y^2, y^3) = -x^0y^0 + x^1y^1 + x^2y^2 + x^3y^3$

$(x_0, x_1, x_2, x_3) := (x^0, x^1, x^2, x^3)^b = (-x^0, x^1, x^2, x^3)$

SOR

$g(x, y) = x_i y^i$

$T^{\to} = \{(x^0, x^1, x^2, x^3) \mid x_i x^i < 0, \ x^0 > 0\}$ NY/UR

$V(1) = \{(u^0, u^1, u^2, u^3) \mid u_i u^i = -1, \ u^0 > 0\}$

Pl.

$B \begin{pmatrix} \gamma \\ \gamma v \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$u := (1, 0, 0, 0)$

$u' := \gamma(1, v, 0, 0)$

C09 áll. $(\underset{\circ}{M}, \mathbb{T}, g) \sim \left(\mathbb{R}^4, \mathbb{R}, g\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\right)$

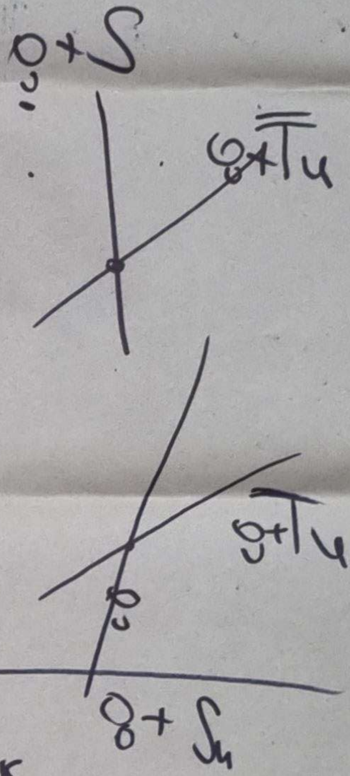
$s \in \mathbb{T}$ $\quad F(s) := 1$

(e_0, e_1, e_2, e_3) $\quad e_i \cdot e_j = \begin{cases} -s^2 & i=j=0 \\ +s^2 & i=j>0 \\ 0 & \text{egyébként} \end{cases}$

$\underset{\circ}{o} \in \underset{\circ}{M}$

$$\underset{\circ}{x} = \underset{\circ}{o} + (\underset{\circ}{x} - \underset{\circ}{o}) = \underset{\circ}{o} + \sum_{k=0}^{3} \frac{g(\underset{\circ}{x} - \underset{\circ}{o}, e_k)}{g(e_k, e_k)} e_k$$

$$L(\underset{\circ}{x}) := \left(g(\underset{\circ}{x} - \underset{\circ}{o}, e_0), g(\underset{\circ}{x} - \underset{\circ}{o}, e_1), \dots, g(\underset{\circ}{x} - \underset{\circ}{o}, e_3)\right)$$



SRTM $\longleftrightarrow$ ARIT $\longleftrightarrow$ SRTM'

$(\underset{\circ}{M}, \mathbb{T}, g) \circlearrowleft$ automorfizmus

szimmetria, ha $F = id_{\mathbb{T}}$ $\quad$ Poincaré-csop.

$\mathcal{P} := \{ L : \underset{\circ}{M} \to \underset{\circ}{M}$ affin bij. $\mid Lx \cdot Ly = x \cdot y \}$,

Lorentz-csop.