P09

$V_s \ni \vec{v}$

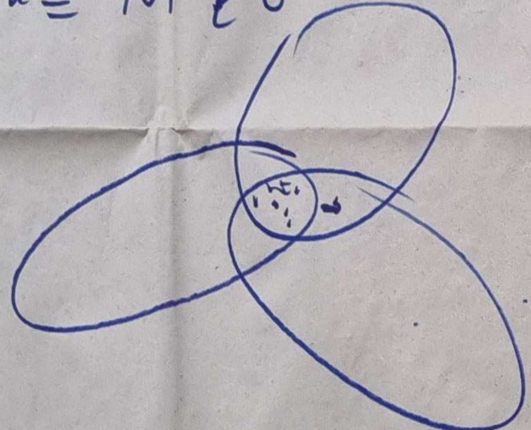
$$\begin{array}{ccc} & V & \\ \swarrow & & \searrow \\ \mathbb{R}^n & & \mathbb{R}^n \end{array} \qquad \begin{array}{ccccc} & & \bar{v}' & & \\ & \swarrow & \downarrow & \searrow & \\ \underline{v} & \Leftrightarrow & \underline{v}' & & \underline{v}'' \end{array}$$

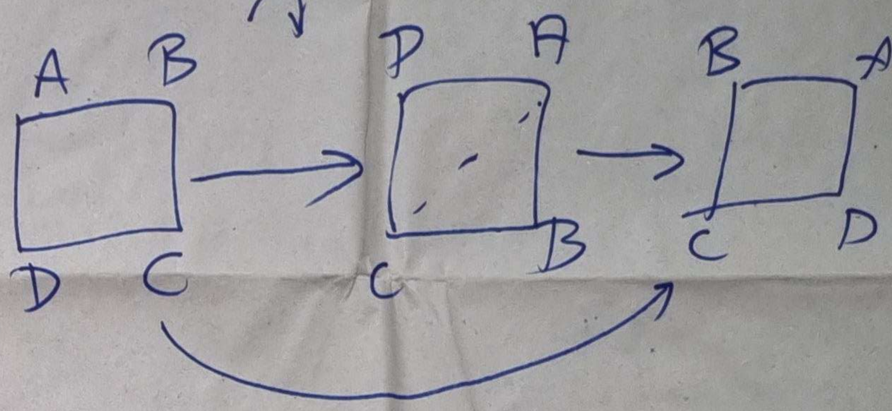
$\vec{v} = v^k \vec{e}_k$

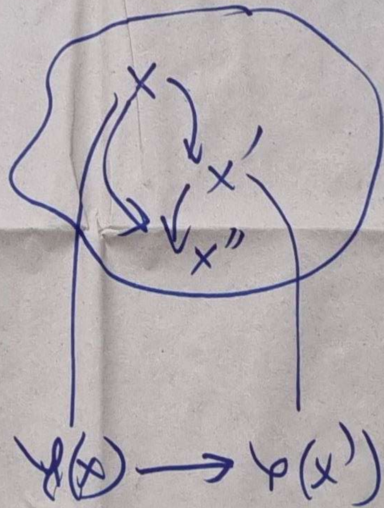
$\underline{v} = \begin{pmatrix} v^1 \\ \vdots \\ v^n \end{pmatrix}$

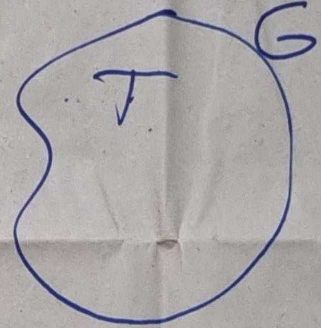
$\vec{v} = v^k \vec{e}_k = v'^l \vec{f}_l$

$v'^k = M^k{}_l v^l$



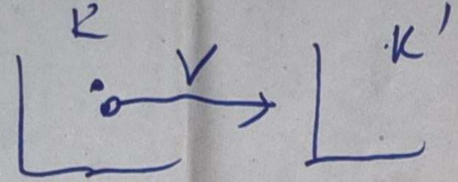






$G \times G \to G$

p10



$x' = f(x,t,V)$

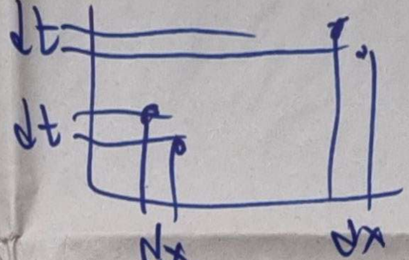
$t' = g(x,t,V)$

$dx' = \frac{\partial f}{\partial x}(x,t,V)\,dx + \frac{\partial f}{\partial t}(x,t,V)\,dt$

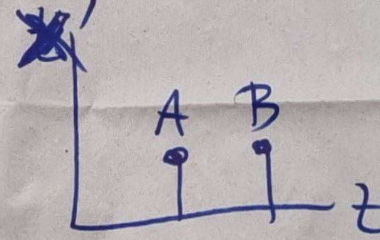
$dt' = \frac{\partial g}{\partial x}\,dx + \frac{\partial g}{\partial t}\,dt$

$$\begin{pmatrix} dx' \\ dt' \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial t} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial t} \end{pmatrix} \begin{pmatrix} dx \\ dt \end{pmatrix}$$

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$



$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \alpha(V) & \beta(V) \\ \gamma(V) & \delta(V) \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} + \begin{pmatrix} x_0' \\ t_0' \end{pmatrix}$$



$dx' = 0$

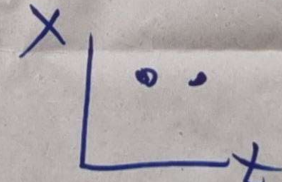
$dt' \neq 0$

$$\begin{pmatrix} 0 \\ dt' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} dx \\ dt \end{pmatrix}$$

$\alpha\,dx + \beta\,dt = 0$

$\frac{dx}{dt} = -\frac{\beta}{\alpha}$

$\beta = -V\alpha = V$



$dx = 0$

$dt = 0$

$\frac{dx'}{dt'} = \frac{\beta}{\delta} = -V$

$\beta = -V\delta$

$$\begin{pmatrix} dx' \\ dt' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} 0 \\ dt \end{pmatrix}$$

$dx' = \beta\,dt$

$dt' = \delta\,dt$

p 11)

$$\begin{pmatrix} dx' \\ dt' \end{pmatrix} = \underline{\underline{M}}(v) \begin{pmatrix} dx \\ dt \end{pmatrix} = \begin{pmatrix} \alpha(v) & -v\alpha(v) \\ \gamma(v) & \alpha(v) \end{pmatrix} \begin{pmatrix} dx \\ dt \end{pmatrix}$$

$$\begin{tikzcd} K \arrow[r, "v"] \arrow[rr, bend right, "u"'] & K' \arrow[r, "w"] & K'' \end{tikzcd}$$

$$\begin{pmatrix} dx'' \\ dt'' \end{pmatrix} = M(u) \begin{pmatrix} dx \\ dt \end{pmatrix} = M(w) \begin{pmatrix} dx' \\ dt' \end{pmatrix} = M(w) M(v) \begin{pmatrix} dx \\ dt \end{pmatrix}$$

$$M(u) = M(w) M(v)$$

$$\begin{pmatrix} \alpha(u) & -u\alpha(u) \\ \gamma(u) & \alpha(u) \end{pmatrix} = \begin{pmatrix} \alpha(w) & -w\alpha(w) \\ \gamma(w) & \alpha(w) \end{pmatrix} \begin{pmatrix} \alpha(v) & -v\alpha(v) \\ \gamma(v) & \alpha(v) \end{pmatrix}$$

① $\alpha(u) = \alpha(w)\alpha(v) - w\alpha(w)\gamma(v)$

② $-u\alpha(u) = -\alpha(w)v\alpha(v) - w\alpha(w)\alpha(v)$

③ $\gamma(u) = \gamma(w)\alpha(v) + \alpha(w)\gamma(v)$

④ $\alpha(u) = -\gamma(w)v\alpha(v) + \alpha(w)\alpha(v)$

$$-w\alpha(w)\gamma(v) = -\gamma(w)v\alpha(v)$$

$$\frac{\gamma(v)}{v\alpha(v)} = \frac{\gamma(w)}{w\alpha(w)} = K$$

$$\gamma(v) = Kv\alpha(v)$$

$$M(v) = \begin{pmatrix} \alpha(v) & -v\alpha(v) \\ Kv\alpha(v) & \alpha(v) \end{pmatrix} = \alpha(v) \begin{pmatrix} 1 & -v \\ Kv & 1 \end{pmatrix}$$

(3) $\gamma(u) = K u \alpha(u) = K w \alpha(w) \alpha(v) + \alpha(w) K v \alpha(v)$

$= K \alpha(v) \alpha(w) (w + v)$

(2) $u \alpha(u) = \alpha(v) \alpha(w) (v + w)$

(1) $\alpha(u) = \alpha(v)\alpha(w) - w \alpha(w) K v \alpha(v)$

$= \alpha(v)\alpha(w)(1 - Kvw)$

$$\boxed{u = \frac{v + w}{1 - Kvw}}$$

$$\alpha\left(\frac{v+w}{1-Kvw}\right) = \alpha(v)\alpha(w)(1 - Kvw)$$

A) $K = 0$

$M(v) = \alpha(v) \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} x' \\ t' \end{pmatrix} = M(v) \begin{pmatrix} x \\ t \end{pmatrix}$

$x' = \alpha(v)(x - vt)$

$t' = \alpha(v) t$

$M(-v)$

$= \alpha(-v) \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$

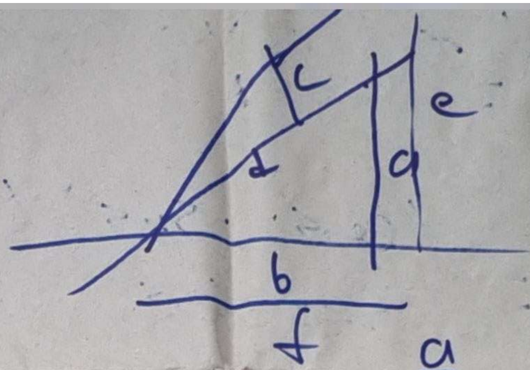
CamScannerrel szkennelve

13)

$K > 0 \quad K = \frac{1}{c^2}$

$g(\varepsilon)\, g(\varphi) = g(\varepsilon + \varphi)$

$M(V) = \begin{pmatrix} 1 & -V \\ \frac{V}{c^2} & 1 \end{pmatrix} \alpha(V)$



$\alpha(V) = \alpha(-V)$

$\frac{a}{b}$

$u = \frac{V + W}{1 - \frac{VW}{c^2}}$

$M(V)M(-V) = \alpha(V)^2 \begin{pmatrix} 1 & -V \\ \frac{V}{c^2} & 1 \end{pmatrix} \begin{pmatrix} 1 & V \\ -\frac{V}{c^2} & 1 \end{pmatrix}$

$\frac{e}{f} = \frac{\frac{a}{b} + \frac{c}{d}}{1 - \frac{a}{b}\frac{c}{d}}$

$\frac{u}{c} = \frac{\frac{V}{c} + \frac{W}{c}}{1 - \frac{V}{c}\frac{W}{c}}$

$= \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$

$\alpha(V)^2 \left(1 + \frac{V^2}{c^2}\right) \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$

$\varepsilon = \operatorname{arctg} \frac{a}{b}$

$\alpha^2(V) = \frac{1}{1 + \frac{V^2}{c^2}}$

$\varphi = \operatorname{arctg} \frac{c}{d}$

$\eta = \operatorname{arctg} \frac{e}{f}$

$\operatorname{tg} \varepsilon = \frac{V}{c}$

$\operatorname{tg} \varphi = \frac{W}{c}$

$\frac{1}{1 + \operatorname{tg}^2 \varepsilon} = \cos^2 \varepsilon$

$\eta = \varepsilon + \varphi$

$\frac{u}{c} = \operatorname{tg}(\varepsilon + \varphi)$

$\alpha(\ ) = \cos \varepsilon$

$M = \cos\varepsilon \begin{pmatrix} 1 & -c \operatorname{tg}\varepsilon \\ \frac{1}{c}\operatorname{tg}\varepsilon & 1 \end{pmatrix} = \begin{pmatrix} \cos\varepsilon & -c\sin\varepsilon \\ \frac{1}{c}\sin\varepsilon & \cos\varepsilon \end{pmatrix}$

(p 14) $\binom{x}{t}' = \begin{pmatrix} & \\ & \end{pmatrix}\binom{x}{t}$

$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cos\varepsilon & -\sin\varepsilon \\ \sin\varepsilon & \cos\varepsilon \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$

$\varepsilon = \frac{\pi}{3} \quad \varphi = \frac{\pi}{6}$

$\varepsilon + \varphi = \frac{\pi}{2}$

$\operatorname{th}(\varepsilon+\varphi) = \dfrac{\operatorname{th}\varepsilon + \operatorname{th}\varphi}{1 + \operatorname{th}\varepsilon \operatorname{th}\varphi}$

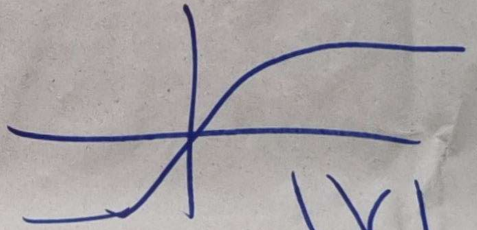
$\frac{v}{c} = \operatorname{th}\varepsilon$

$\alpha(v) = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \operatorname{ch}\varepsilon$

3) $K < 0 \quad K = -\frac{1}{c^2}$

$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \operatorname{ch}\varepsilon & -\operatorname{sh}\varepsilon \\ -\operatorname{sh}\varepsilon & \operatorname{ch}\varepsilon \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$

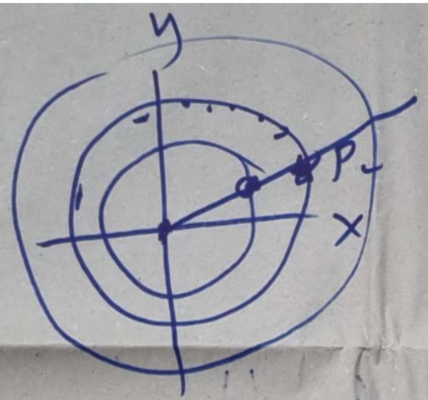
$M(v) = \alpha(v) \begin{pmatrix} 1 & -v \\ -\frac{v}{c^2} & 1 \end{pmatrix}$

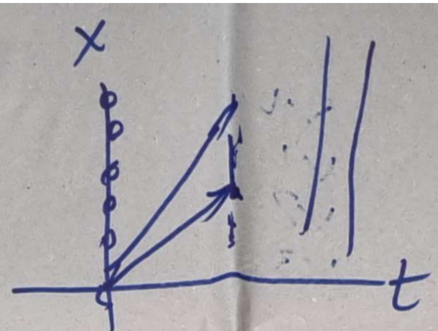


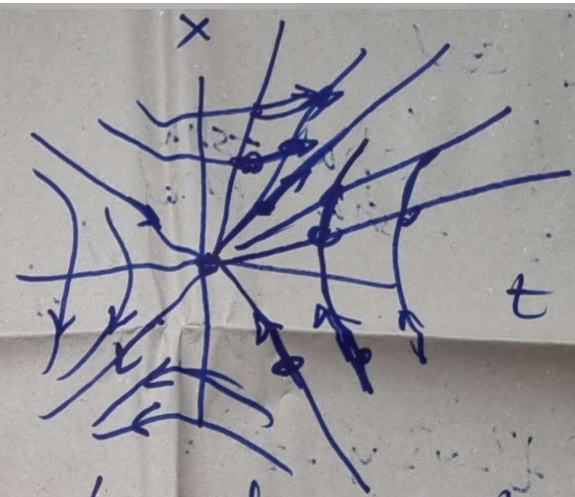
$\operatorname{th}\varepsilon \in]-1, 1[$

$\left|\frac{v}{c}\right| < 1$

$u = \dfrac{v+w}{1+\frac{vw}{c^2}} \qquad \dfrac{u}{c} = \dfrac{\frac{v}{c}+\frac{w}{c}}{1+\frac{v}{c}\frac{w}{c}}$







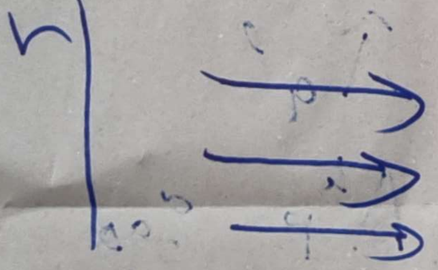
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$x = r\cos\alpha$

$y = r\sin\alpha$

$r \to r' = r$

$\alpha \to \alpha' = \alpha + \varphi$



$x'^2 + y'^2 = x^2 + y^2$

$x' = x\cos\varphi - y\sin\varphi$

$y' = x\sin\varphi + y\cos\varphi$

$\frac{x}{t} \to \frac{x'}{t'} = \frac{x}{t} + v$

$t \to t' = t$

$\begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x' = x\,\mathrm{ch}\,w - t\,\mathrm{sh}\,w$

$t' = t\,\mathrm{ch}\,w - x\,\mathrm{sh}\,w$

$x'^2 - t'^2 = x^2 - t^2$

$t^2 - x^2 > 0$

< 0

8.16] c=1

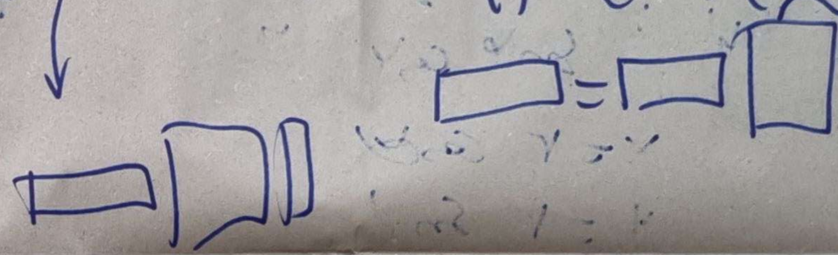
$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} \operatorname{ch} w & -\operatorname{sh} w & & \\ -\operatorname{sh} w & \operatorname{ch} w & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$(t^2 - x^2 - y^2 - z^2) = (\quad) = \text{inv}$$

$$x'^k = \Lambda^k{}_l x^l$$

$$x'^k = \Lambda^k{}_l x^l \qquad () = (\;)()$$



$$g_{kl} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$g_{kl} x^k x^l = -t^2 + x^2 + y^2 + z^2$$

$$g'_{kl} x'^k x'^l = g_{kl} x^k x^0$$

$$g'_{kl} \Lambda^k{}_p x^p \Lambda^l{}_q x^q = g_{pq} x^p x^q$$

$$g'_{kl} \Lambda^k{}_p \Lambda^l{}_q = g_{pq}$$

$$g' = g \qquad \Lambda^l{}_q = \tilde{\Lambda}_q{}^l$$

$$\tilde{\Lambda}_q{}^l g_{lk} \Lambda^k{}_p = g_{qp}$$

$$\boxed{\tilde{\Lambda}\, g\, \Lambda = g}$$

$\underline{\underline{\tilde{\Lambda}}}\,\underline{\underline{g}}\,\underline{\underline{\Lambda}} = \underline{\underline{g}}$ $\qquad g = \boxed{\begin{array}{c|c} -1 & \tilde{\underline{0}} \\ \hline \underline{0} & \underline{\underline{I}} \end{array}}$

$\alpha^2 - |c|^2 = 1$

$\Lambda = \begin{pmatrix} \alpha & \boxed{\tilde{\underline{b}}} \\ \boxed{\underline{c}} & \underline{\underline{A}} \end{pmatrix} \qquad \tilde{\Lambda} = \begin{pmatrix} \alpha & \tilde{\underline{c}} \\ \underline{b} & \tilde{\underline{\underline{A}}} \end{pmatrix}$

$\alpha = \pm \operatorname{ch} w \quad w \in \mathbb{R}$

$\underline{c} = \operatorname{sh} w\, \underline{n} \quad |\underline{n}| = 1$

$\begin{pmatrix} \alpha & \tilde{\underline{c}} \\ \underline{b} & \tilde{\underline{\underline{A}}} \end{pmatrix} \begin{pmatrix} -1 & \underline{0} \\ \underline{0} & \underline{\underline{I}} \end{pmatrix} \begin{pmatrix} \alpha & \tilde{\underline{b}} \\ \underline{c} & \underline{\underline{A}} \end{pmatrix} \overset{!}{=} \begin{pmatrix} -1 & \tilde{\underline{0}} \\ \underline{0} & \underline{\underline{I}} \end{pmatrix}$

$\begin{pmatrix} -\alpha & \tilde{\underline{c}} \\ -\underline{b} & \tilde{\underline{\underline{A}}} \end{pmatrix} \begin{pmatrix} \alpha & \tilde{\underline{b}} \\ \underline{c} & \underline{\underline{A}} \end{pmatrix}$

$-\alpha^2 + \tilde{\underline{c}}\,\underline{c} = -1$

$-\underline{b}\alpha + \tilde{\underline{\underline{A}}}\,\underline{c} = \underline{0}$

$-\underline{b} \circ \tilde{\underline{b}} + \tilde{\underline{\underline{A}}}\,\underline{\underline{A}} = \underline{\underline{I}}$

$\Lambda = \begin{pmatrix} \pm 1 & \boxed{\underline{0}} \\ \boxed{\underline{0}} & \boxed{\pm \underline{\underline{I}}} \end{pmatrix} \begin{pmatrix} 1 & \boxed{\underline{0}} \\ \boxed{\underline{0}} & \underline{\underline{F}} \end{pmatrix} \begin{pmatrix} \operatorname{ch} w & -\tilde{\underline{e}} \operatorname{sh} w \\ -\underline{e} \operatorname{sh} w & \boxed{\begin{array}{l} \underline{\underline{I}} + \\ +(\operatorname{ch} w - 1) \\ \quad n \circ \tilde{n} \end{array}} \end{pmatrix}$

$\tilde{F} = F^{-1} \qquad \underline{\underline{F}}\,\underline{e} = \underline{n}$

$F \in SO(3)$