P01

<u>STD. Rel. Szb.</u>

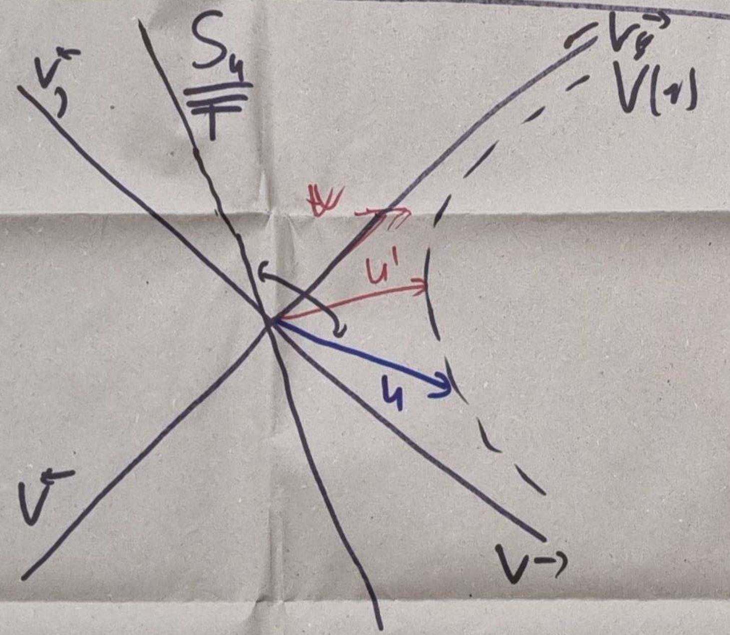
Megf. + Szinkr = Vonatk RSZ.

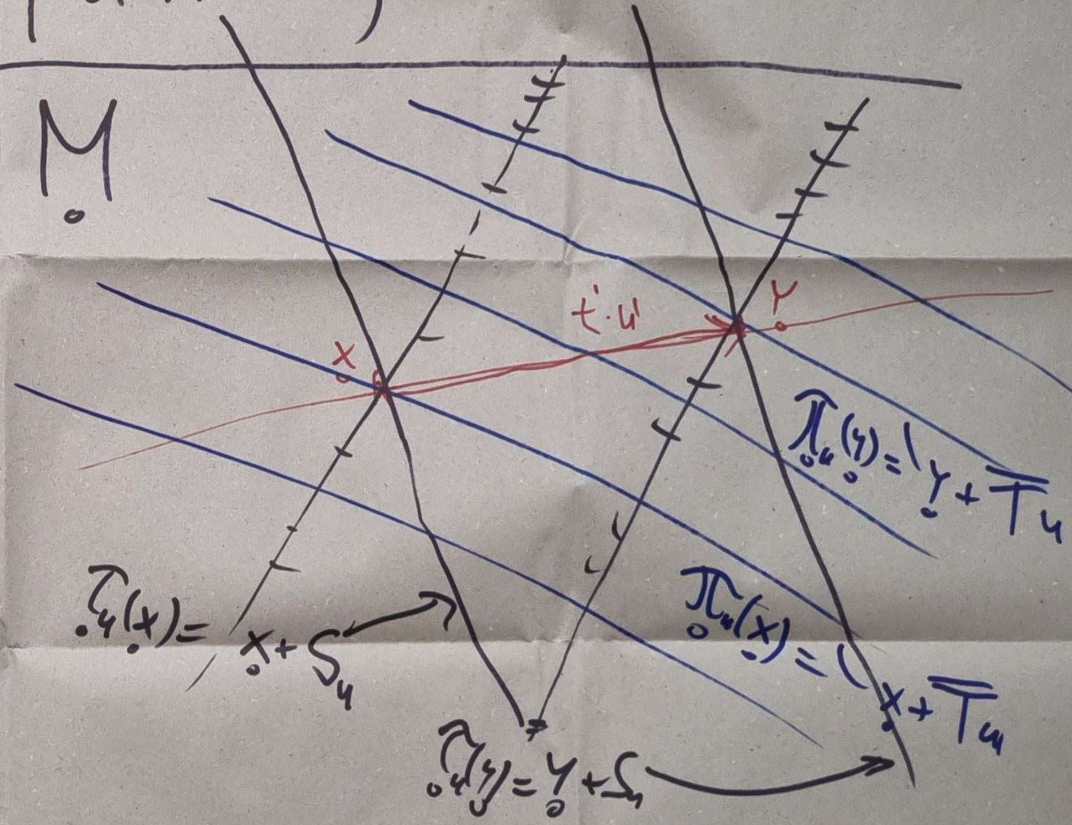
Teh. — " — + Egyenl — " — = Teh. RSZ.

— " — STD — " — = Std teh. RSZ.

↳ $u \in V(1)$ ↳ $S_u = \{x \in M \mid u \cdot x = 0\}$

$$\frac{\underline{\underline{M}}}{T}$$





P02 u-std. Rel. seb

$$V_{u'u} = \frac{\Pi_u(y) - \Pi_u(x)}{\mathcal{T}_u(y) - \mathcal{T}_u(x)} = \frac{\Pi_u(\overbrace{y - x}^{t'u'})}{\mathcal{T}_u(y-x)} \underset{\uparrow}{=} \frac{t'u' - t'\tilde{\tau}_u(u')u}{t'\tilde{\tau}_u(u')} = \frac{u' - (-u\cdot u')u}{-u\cdot u'}$$

$\Pi_u = 1 - u \otimes \tilde{\tau}_u$

$$\boxed{V_{u'u} = \frac{u'}{-u\cdot u'} - u \in \underset{\mp}{S_u}}$$

$$0 \le |V_{u'u}|^2 = 1 - \frac{1}{(-u'u)^2} = |V_{uu'}|^2 \le 1$$

$$V_{uu'} = -B_{u'u}V_{u'u}, \quad V_{u'u} = -B_{uu'}V_{uu'} \quad 1 \le -u'u = \gamma(v) = \frac{1}{\sqrt{1-v^2}} < \infty$$

$v = |V_{uu'}|$

$v \in \underset{\mp}{S_u}$ sel seb $|v| < 1$ $\quad v \longmapsto u_v = \dfrac{u+v}{\sqrt{1-|v|^2}} \in V(1)$ ✓

$w \in \vec{V} \subset \underset{\mp}{M}$ $\quad V_{w,u} = \dfrac{w}{-u\cdot w} - u \quad |V_{w,u}| = 1$ $\quad V_{u_v u} = v$ ✓

$v \in \underset{\mp}{S_u}$ $|v| = 1$

$v \longmapsto w_v = u + v \in \vec{V}$ ✓

$V_{w_v,u} = v$ ✓

P∅3

$\tau_u(x) = -u \cdot x \qquad u = (u^0, u^1, u^2, u^3) \qquad \tau_u = -(-u^0, u^1, u^2, u^3)$

$\pi_u = 1 - u \otimes \tau_u = 1 + u \otimes u \ : \quad \begin{pmatrix} u^0 \\ u^1 \\ u^2 \\ u^3 \end{pmatrix} (-u^0, u^1, u^2, u^3) \qquad \begin{bmatrix} 1 - u^0u^0 & u^1u^0 & \\ -u^0u^1 & 1 + u^1u^1 & \\ -u^0u^2 & u^1u^2 & \\ -u^0u^3 & u^1u^3 & \end{bmatrix}$

$\pi_{(1,0,0,0)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \pi_{\gamma(1,v,0,0)} = \begin{bmatrix} 1-\gamma^2 & \gamma^2 v & 0 & 0 \\ -\gamma^2 v & 1+\gamma^2 v^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\gamma = \cancel{\frac{1}{\sqrt{1-v^2}}} \quad \frac{1}{\sqrt{1-v^2}}$

$B_{u'u} = 1 + \frac{(u'+u) \otimes (u'+u)}{1 - u' \cdot u} - 2u' \otimes u$

$B_{\gamma\begin{bmatrix}1\\v\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}} = \begin{bmatrix} \gamma & \gamma v & & \\ \gamma v & \gamma & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$

$B_{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\gamma\begin{bmatrix}1\\v\\0\\0\end{bmatrix}} = \begin{bmatrix} \gamma & -\gamma v & & \\ -\gamma v & \gamma & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$

CamScannerrel szkennelve

P04

$u = \gamma(1, v, 0, 0)$ $\quad \gamma = \frac{1}{\sqrt{1-v^2}}$

$u' = \gamma'(1, 0, v', 0)$ $\quad \gamma' = \frac{1}{\sqrt{1-v'^2}}$

$\frac{1}{\gamma} - \gamma = \frac{1-\gamma^2}{\gamma} = \underbrace{-\gamma v^2}$

$$V_{u'u} = \frac{u'}{-uu'} - u = \frac{\gamma'(1, 0, v', 0)}{\gamma\gamma'} - \gamma\frac{\gamma(1, v, 0, 0)}{\gamma} = \left(\frac{1}{\gamma} - \gamma, -\gamma v, \frac{v'}{\gamma}, 0\right)$$

$$V_{u,u'} = \left(-\gamma' v'^2, \frac{v}{\gamma'}, -\gamma' v', 0\right)$$

$$V_{u'u}\Big|_{v'=0} = (-\gamma v^2, -\gamma v, 0, 0)$$

$$V_{uu'}\Big|_{v'=0} = (0, v, 0, 0)$$

$B \;\; \gamma\begin{pmatrix}1\\v\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad V \;\; \gamma\begin{pmatrix}1\\v\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad = \gamma\begin{bmatrix}1 & +v\\ +v & 1\end{bmatrix}\begin{pmatrix}0\\v\end{pmatrix} = \gamma\begin{bmatrix}v^2\\v\end{bmatrix}$

$B \;\; \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \;\; \gamma\begin{pmatrix}1\\v\\0\\0\end{pmatrix} \quad V \;\; \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}1\\v\\0\\0\end{pmatrix} \quad = \gamma\begin{bmatrix}1 & -v\\ -v & 1\end{bmatrix}\begin{bmatrix}-\gamma v^2\\ -\gamma v\end{bmatrix} = \begin{bmatrix}0\\v\end{bmatrix}$

POS) $V_{u''u} \neq V_{u''u'} + V_{u'u}$

$$\begin{array}{ccc} V_{u''u} & & V_{u''u'} & & V_{u'u} \\ \cap & & \cap & & \cap \\ \frac{S_u}{\mp} & & \frac{S_{u'}}{\mp} & & \frac{S_u}{\mp} \end{array}$$

$$\left.\begin{array}{l} V_1 := V_{u'u} \\ V_2 := B_{uu'} V_{u''u'} \end{array}\right\} \in \frac{S_u}{\mp}$$

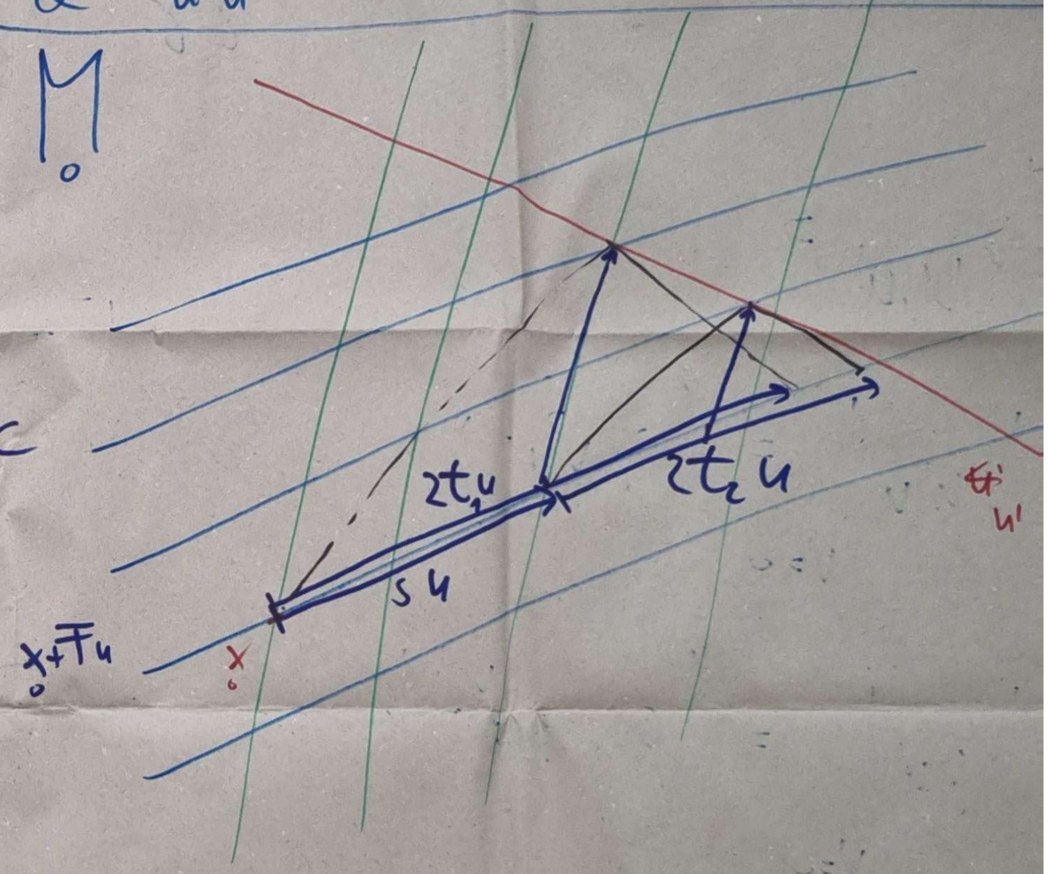
$V_3 := V_{u''u}$

$\alpha = -u' \cdot u$

$$\boxed{V_3 = \frac{V_2 + V_1 - \frac{\alpha}{1+\alpha}(V_2 \times V_1) \times V_1}{1 + V_1 \cdot V_2}}$$

$$|V_{u'u}| = \left| \frac{ct_2 - ct_1}{(\underset{\circ}{t} + s + t_2) - (\underset{\circ}{t} + t_1)} \right| = \frac{t_2 - t_1}{s - (t_1 - t_2)} c$$

$\underset{\circ}{M}$



$\underset{\circ}{t} = \tau_u(\underset{\circ}{x}) = \underset{\circ}{x} + S_u$

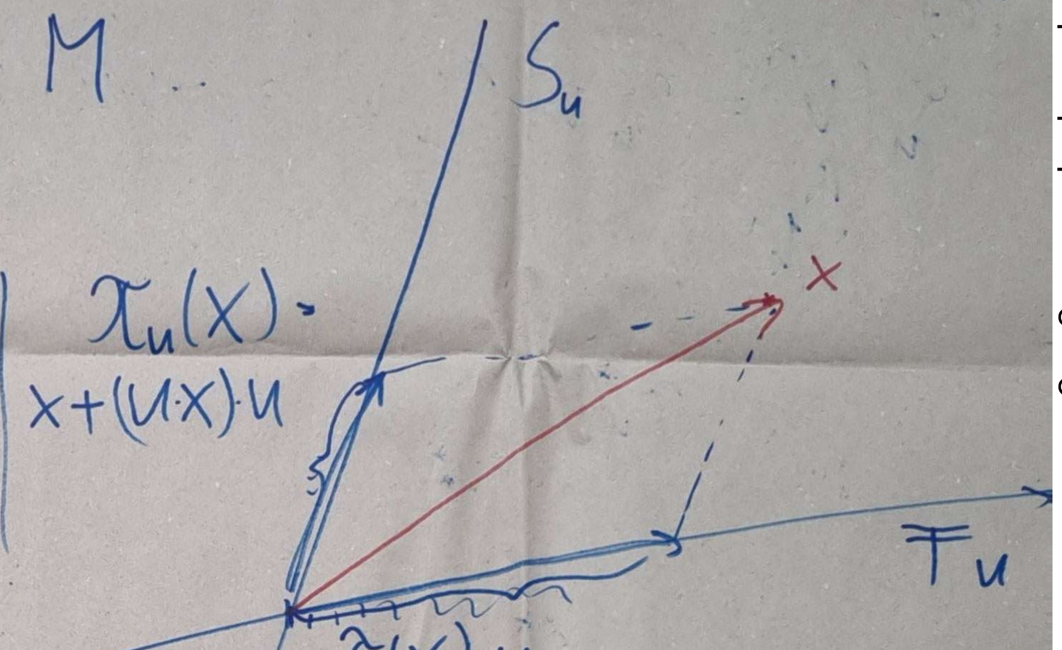
P06 Std. vektori széthasítások

$$M \underset{u\cdot}{\overset{\tau_u}{\rightleftarrows}} \mathbb{T} \qquad \tau_u = -u\cdot$$

$$M \underset{i_u}{\overset{\pi_u}{\rightleftarrows}} S_u \qquad \pi_u = 1 - u \otimes \tau_u$$

$$i_u(q) = q \quad q \in S_u$$

$\pi_u(x) = x + (u\cdot x)\cdot u$



$$\xi_u = (\tau_u, \pi_u) : M \longrightarrow \mathbb{T} \times S_u$$

$$x \longmapsto (\tau_u(x), \pi_u(x)) = (\underbrace{-u\cdot x}_{\text{u-tein.}}, \underbrace{x + (u\cdot x)\cdot u}_{\text{u-ra mer.}})$$

$\tau_u(x)\cdot u$

$-u\cdot x \in \mathbb{T}$

$$\xi_u^{-1}(u, i) : \mathbb{T} \times S_u \longrightarrow M$$

$$(t, q) \longmapsto t\cdot u + i_u q$$

$\mathbb{T} \times S_u$ -n:

$$g(x, x') = g(\xi^{-1}(t,q), \xi^{-1}(t',q')) = g(tu + q, t'u + q') = -tt' + \underbrace{g(q, q')}_{q\cdot q'} = g_u((t,q), (t',q'))$$

P07)

$$\varphi_u(\underset{\substack{\cap \\ V(1)}}{u'}) = (\tau_u(u'), \pi_u(u')) = (-u\cdot u', u' + (u\cdot u')u) = (-u\cdot u')(1, V_{u';u})$$

$$\uparrow \frac{1}{\sqrt{1-|u\cdot u'|^2}}$$

Kovekt:

$$\begin{tikzcd} & \mathbb{T}^* \arrow[dl, bend right, "\tau_u^*"'] \\ M^* \arrow[ur, "u^*"] \arrow[dr, "i_u^*"] & \\ & S_u^* \arrow[ul, bend left, "\pi_u^*"] \end{tikzcd}$$

$$\eta_u = (\varphi_u^{-1})^* = (u^*, i_u^*) : M^* \longrightarrow \mathbb{T}^* \times S_u^*$$

$$k \longmapsto (ku, k\underset{\substack{\uparrow \\ \pi_u}}{i_u}) = (\underbrace{u\cdot k}_{-\tau_u(k)}, \pi_u k)$$

$$\eta_u = (-\tau_u, \pi_u)$$

$$\eta_u^{-1} = (\tau^*, \pi_u^*) : \mathbb{T}^* \times S_u^* \longrightarrow M^*$$

$$(e, p) \longmapsto eu + p$$

P08

$$\begin{tikzcd} & M \arrow[dl, "\xi_u"'] \arrow[dr, "\xi_{u'}"] & \\ \mathbb{T}\times S_u & & \mathbb{T}\times S_{u'} \arrow[ll, "(1, B_{uu'})"'] \end{tikzcd}$$

$v = |V_{u'u}|$

$$V_{u'u} = \frac{u'}{-u\cdot u'} - u$$

$$\xi_{u'u} = (1, B_{uu'})\,\xi_{u'}\xi_u^{-1} = \gamma(v)\begin{bmatrix} 1 & -V_{u'u} \\ -V_{u'u} & \mathcal{J}(V_{u'u}) \end{bmatrix} : \mathbb{T}\times S_u \to \mathbb{T}\times S_u$$

L. TR SZABAL

$\mathbb{T}\times S_u \ni (t, q)$

$$\mathcal{J}(V_{u'u}) = \frac{1}{\gamma(v)}\left(-1\, id_{S_u} + \frac{\gamma(v)^2}{\gamma(v)+1} V_{u'u}\otimes V_{u'u}\right)$$

$S_u \ni q = q_{\parallel} + q_{\perp}$

$$t \longmapsto \xi_{u'u} t = \frac{1}{\sqrt{1-v^2}}\left(t - V_{u'u}\cdot q_{\parallel}\right)$$

$$q_{\perp} \longmapsto \xi_{u'u} q_{\perp} = q_{\perp}$$

$$q_{\parallel} \longmapsto \xi_{u'u} q_{\parallel} = \frac{1}{\sqrt{1-v^2}}\left(q_{\parallel} - t V_{u'u}\right)$$