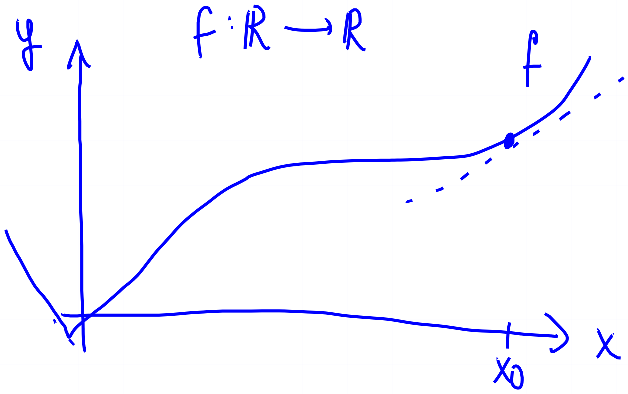
$f: \mathbb{R} \to \mathbb{R}$



$y = f(x_0) + f'(x_0)\cdot(x-x_0)$

$f(x) = f(x_0) + f'(x_0)\cdot(x-x_0) + \text{hiba}(x-x_0)$

affin: $F: V \to W$

$F(x) = F(x_0) + \underbrace{DF[x_0]}_{\in Lin(V,W)}(\underbrace{x-x_0}_{\in V}) + \text{hiba}(x-x_0)$

$F(x) - F(x_0) = DF[x_0]\cdot(x-x_0) + \text{ordo}(x-x_0)$

dim V

dim W

V

$DF[x_0] \in Lin(V,W) \equiv W \otimes V^*$

$DF: V \to Lin(V,W) \equiv W \otimes V^*$

$V, W \rightsquigarrow V^*, W^* \rightsquigarrow V^* \times W^* \to \mathbb{R}$

$\boxed{Df[x_0] = f'(x_0)\cdot id_{\mathbb{R}}}$

$V \otimes W$

$v^a \quad w^a \quad p_a \quad r_a \quad t^{ab} \quad t^{ab} p_a r_b$

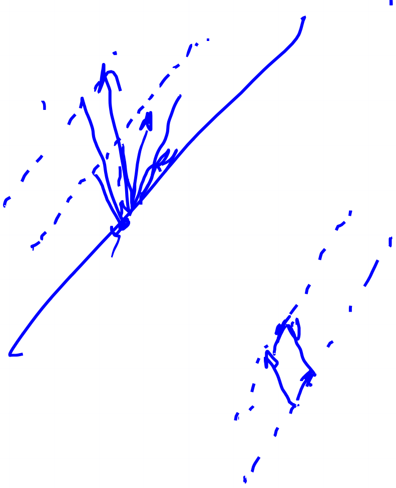
$p(v) = (p|v) = p_a v^a$

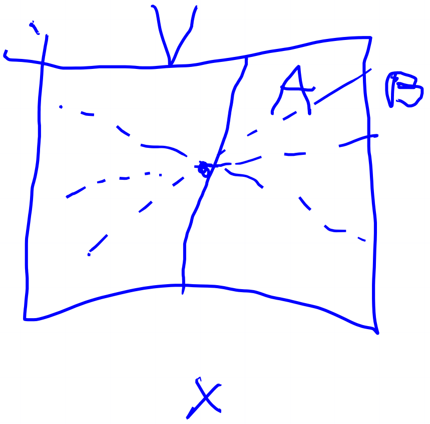
$A^a{}_b \in U \otimes U^*$

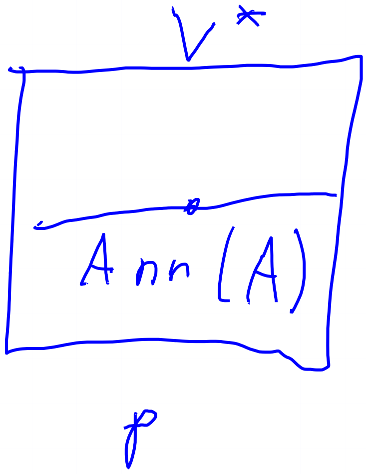
$A^a{}_b u^b \quad p_a A^a{}_b u^b$

$\downarrow \quad \downarrow$

$Lin(U,U) \quad Bilin(U \times U, \mathbb{R})$



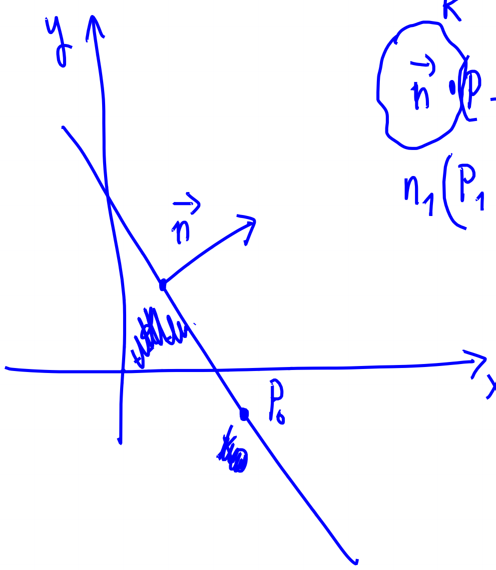




$$\dim(A) + \dim \operatorname{Ann}(A) = \dim(V)$$

$$p(x) = (p \mid x)$$

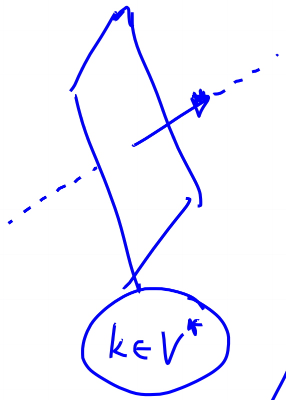
$$r(v) = (r \mid v) \overset{!}{=} 0$$



$$\overset{k}{\vec{n}} \cdot (P - P_0) = 0$$

$$n_1 (P_1 - (P_0)_1) + n_2 (P_2 - (P_0)_2) = 0$$

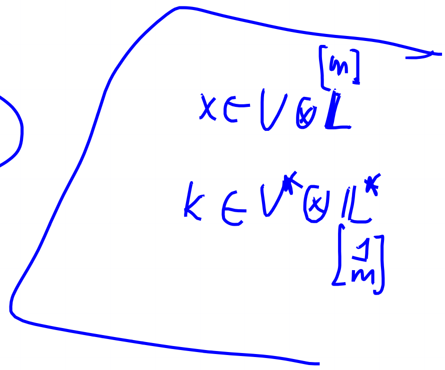
$$k (P - P_0) = 0$$



<u>Fourier-transzformáció</u>

$$(\mathcal{F} f)(k) = \int f(x)\, e^{-i\,kx} \, dx \qquad -i\langle \tilde{k}, x \rangle$$

$$e^y = \sum \frac{1}{k!} y^k \qquad y \in R$$



$$x \in V \otimes L^{[m]}$$

$$k \in V^* \otimes L^{*\left[\frac{1}{m}\right]}$$

téridőkovektor széthasítva

$$\sin(\omega t - kx) = \sin(i, q)$$

téridővektor széthasítva

$P_M : M \to \mathbb{F}$

$P_{\frac{M}{T}} : \frac{M}{T} \to \mathbb{R}$

$P_{\frac{M}{T}}\left(\frac{x}{S}\right) := \underline{P_M(x)}_{S}$

V

$Q : V \to \mathbb{R}$ kvad $\exists B : V \times V \to \mathbb{R}$ szimm.:

$\forall x \in V;\ Q(x) = B(x,x)$

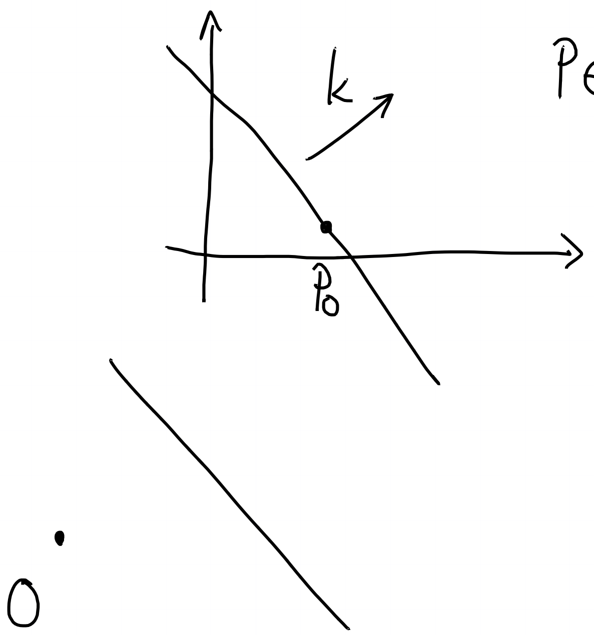
"B"$(x,y) := \frac{1}{2}\left(Q(x+y) - Q(x) - Q(y)\right)$

---

vl. Maxwell $\longrightarrow$ $\begin{cases} D \cdot G = J \\ D \wedge F = 0 \end{cases}$

$F = ma$ $\quad | \quad |^2 + | \quad |^2 + \quad = 0$

$\nabla F(\ ) = 0$



$P \in S$ík

$\boxed{(k \mid P - P_0) = 0}$

$\boxed{n \cdot (P - P_0) = 0}$

$n \cdot P = n \cdot P_0$

$(P - 0) - (P_0 - 0) = P - P_0$

$n \cdot ((P - 0) - (P_0 - 0)) = 0$

$n \cdot \underbrace{(P - 0)}_{P} = n \cdot (\underbrace{P_0 - 0}_{P_0})$

$h: S \times S \to L \otimes L$

$q \in S \quad q^\flat := h(q, \cdot) : S \to L \otimes L$

$\frac{q}{m^2} \in \frac{S}{L \otimes L} \quad \left(\frac{q}{m^2}\right)^\flat := h\left(\frac{q}{m^2}, \cdot\right) : S \xrightarrow{\text{lin}} \mathbb{R}$

$\left(\frac{q}{m^2}\right)^\flat \in S^*$

$\flat : \frac{S}{L \otimes L} \longrightarrow S^*$

(1) $\dim\left(\frac{S}{L \otimes L}\right) = 3 \qquad \dim S^* = 3$

(2) $\flat$: inj $\Leftrightarrow \ker \flat = 0 \qquad \frac{q}{m^2} \in \ker \flat$

$\left(\frac{q}{m^2}\right)^\flat (q') = 0 \qquad \forall q' \in S$

Pl. $q' := q$

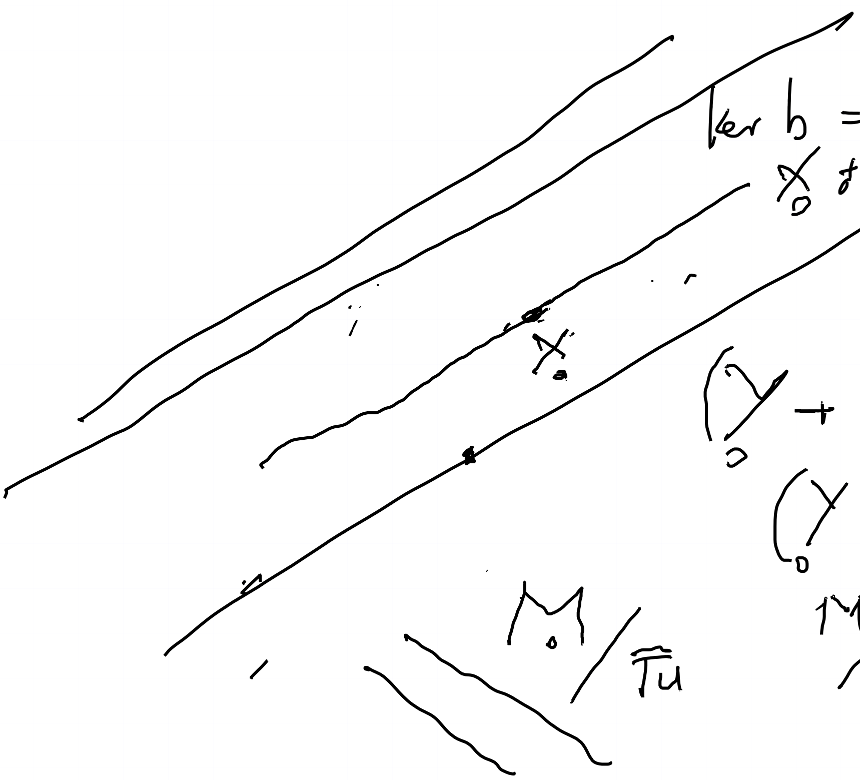
$\left(\frac{q}{m^2}\right)^\flat (q) = h\left(\frac{q}{m^2}, q\right) = \frac{h(q,q)}{m^2}$

$= 0 \Leftrightarrow q = 0$

$\Box$

$\flat^{-1} = \sharp$

$\ker \flat = 0$



$X_0 + T_u \qquad Y_0 + T_u$

$(Y_0 + T_u) - (X_0 + T_u) := (Y_0 - X_0) + T_u$

$M_0 / T_u \qquad M / T_u \qquad M / T_{u'}$

$B_{uu'}$

$B_{u'u} : M \to M \qquad B_{u'u} u = u'$

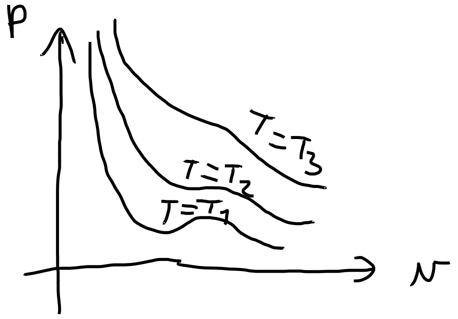
$$\overleftarrow{\sigma}_u : M/_{T_u} \times M/_{T_u} \to \mathbb{L} \otimes \mathbb{L}$$

$$\sigma_u : M \times M \to \mathbb{L} \otimes \mathbb{L}$$

$$\sigma_{u'}(B_{u'u} x, B_{u'u} y) = \sigma_u(x, y)$$

$\hbar \in \mathbb{M} \otimes \frac{\mathbb{L} \otimes \mathbb{L}}{\mathbb{T}} \qquad \hbar = 1.05 \dots 10^{-34}\, kg\, \frac{m^2}{s}$

$\Rightarrow kg = \dots$



$$p(T, v) = \frac{RT}{v-b} - \frac{a}{v^2}$$

$R, a, b \to p_n = \frac{a}{b^2}$

$v_n = b$

$T_n = \frac{a}{bR}$

$$\hat{p}(\hat{T}, \hat{v}) = \frac{\hat{T}}{\hat{v} - 1} - \frac{1}{\hat{v}^2}$$

$\hat{p} := \frac{p}{p_n} \qquad \hat{T} = \frac{T}{T_n}$

$\hat{v} := \frac{v}{v_n}$

$$K : \overset{\circ}{M} \longrightarrow M^*$$

$$K(y) - K(x_0) = \underbrace{DK[x_0] \underbrace{(y - x_0)}_{M}}_{M^*} + \text{hiba}(y - x_0)$$

$$DK[x_0] \in \mathrm{Lin}(M, M^*) = M^* \otimes M^*$$

$$DK[x_0](\underset{M}{u}, \underset{M}{v}) \in \mathbb{R}$$

HA $\quad DK[x_0](v, u) = - DK[x_0](u, v) \quad \forall u, v$

antiszimm

$$B(v,u) = \underbrace{\frac{1}{2}\left(B(v,u) - B(u,v)\right)}_{\text{antiszimm.}} + \underbrace{\frac{1}{2}\left(B(v,u) + B(u,v)\right)}_{\text{szimm}}$$

$$(\wedge B)(v,u)$$

$$\left(\wedge\, DK[x_0]\right) =: (D \wedge K)[x_0]$$

$$F : \overset{\circ}{M} \longrightarrow M^* \otimes M^* \qquad M^* \wedge M^*$$

$$DF : \overset{\circ}{M} \longrightarrow M^* \otimes M^* \otimes M^* \qquad (M^* \wedge M^*) \otimes M^*$$

$$[DF][x_0](u, v, w) := \frac{1}{3!} \sum (\pm 1) [DF][x_0](u, v, w)$$

$$(D \wedge F)[x_0](u_1, u_2, u_3) = \frac{1}{3!} \sum_{\sigma \in S_3} (-1)^{|\sigma|} DF[x_0](u_{\sigma(1)}, u_{\sigma(2)}, u_{\sigma(3)})$$

permutációk

(differenciál 3-forma)

$\left(\mathbb{R}^3, \begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix}\right)$

$$\begin{array}{ccc}
\mathbb{R}^3 \longrightarrow \mathbb{R} & \text{skalar} & V \to \mathbb{R} \quad 0\text{-forma} \\
\big\downarrow \text{grad} & & \big\downarrow D\wedge \\
\mathbb{R}^3 \longrightarrow \mathbb{R}^3 & \text{vektor} & V \to V^* \quad 1\text{-forma kovektor} \\
\big\downarrow \text{rot} & & \big\downarrow D\wedge \\
\mathbb{R}^3 \longrightarrow \mathbb{R}^3 & \text{axialvektor} & V \longrightarrow V^*\wedge V^* \quad 2\text{-forma} \\
\big\downarrow \text{div} & & \big\downarrow D\wedge \\
\mathbb{R}^3 \longrightarrow \mathbb{R} & \text{pseudoskalar} & V \longrightarrow V^*\wedge V^*\wedge V^* \quad 3\text{-forma}
\end{array}$$

$(D\wedge)\circ(D\wedge) = 0$

$N \qquad L : V \otimes L \longrightarrow V$

$L \otimes L : V \otimes V \longrightarrow V \otimes V$

$x \otimes y \longmapsto Lx \otimes Ly$

$-\mathrm{id} \otimes -\mathrm{id}\,(-x) \otimes (-y)) = x \otimes y$

$j : N \wedge N \to N \qquad j(n^1 \otimes m)$