Quantum Chromodynamics and the Quark Model

Particles & Plasmas Symposium

Budapest

June 10th, 2024

Willibald Plessas

Institute of Physics, University of Graz

Contents

- A few historical remarks

  Quarks (1964) and partons ($\gtrsim$ 1968)

- Quest for strong-interaction dynamics ~1972/73
- Quantum chromodynamics (QCD)
- The relativistic constituent quark model (RCQM)
- Hadron spectroscopy
- Hadron reactions

  Electromagnetic, weak, gravitational, strong

- Resonance Description
- Future attempts

1964: Quarks

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

Introduced ***SU(3)*** triplets with fractional charges as fundamental objects: **Quarks**

Murray Gell-Mann, 1929 – 2019

We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (q q q), (q q q q $\bar{q}$), etc., while mesons are made out of (q $\bar{q}$), (q q $\bar{q}$ $\bar{q}$), etc. It is assuming that the lowest baryon configuration (q q q) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration (q $\bar{q}$) similarly gives just **1** and **8**.

6) James Joyce, Finnegan's Wake (Viking Press, New York, 1939) p.383.

„Three quarks for Muster Mark!
Sure he has not got much of a bark.
And sure any he has it's all beside the mark."

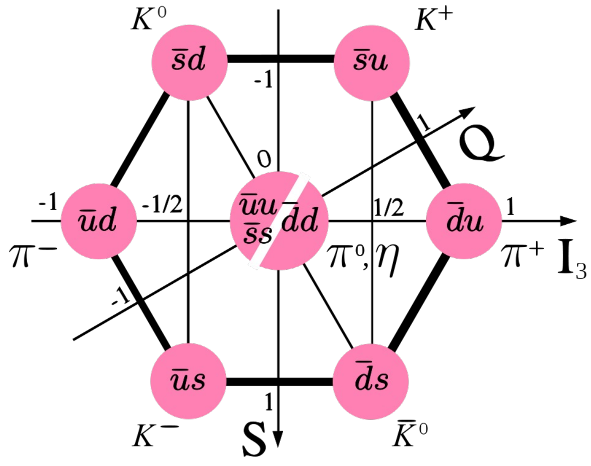
Hadron Multiplets

Mesons

1:

η'

8:

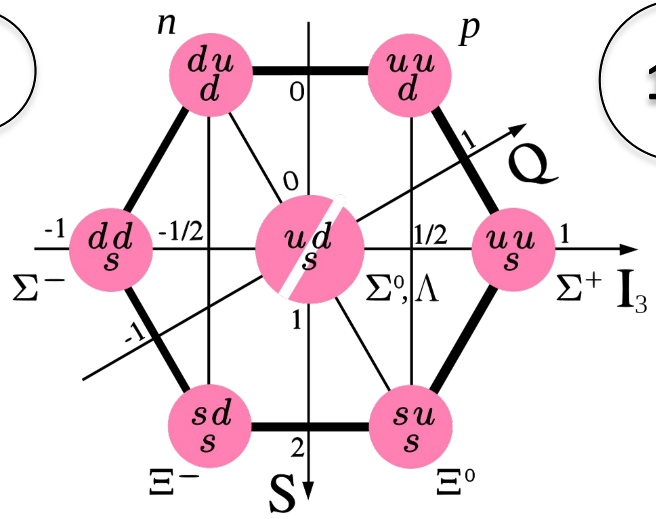


Baryons

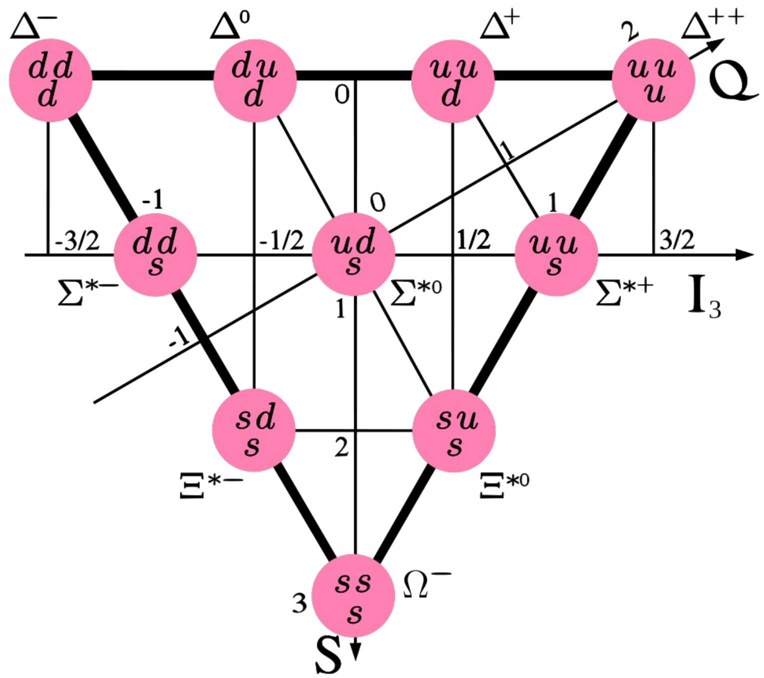
1:

Λ^0 (uds)

8:

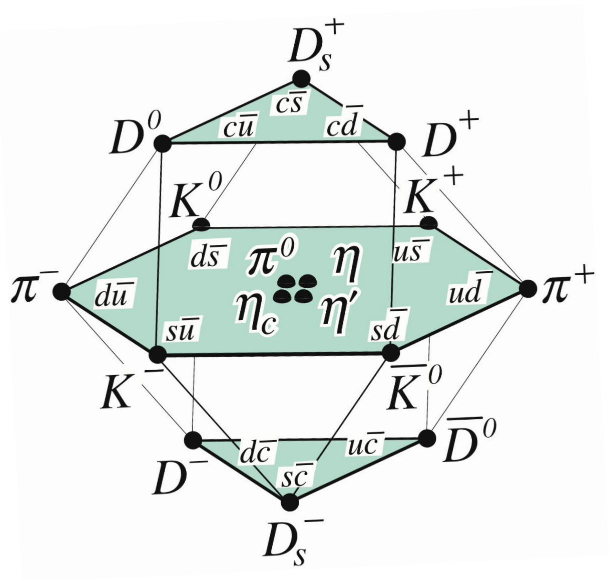


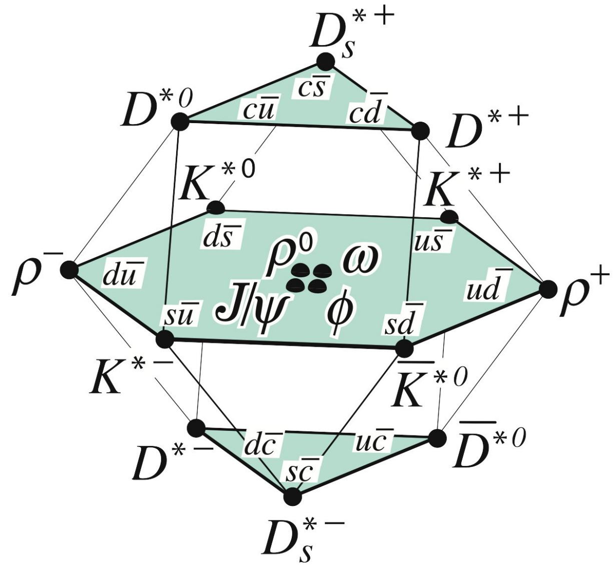
10:

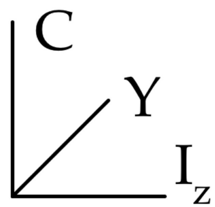


Hadron Multiplets in $SU(4)$ u, d, s, c

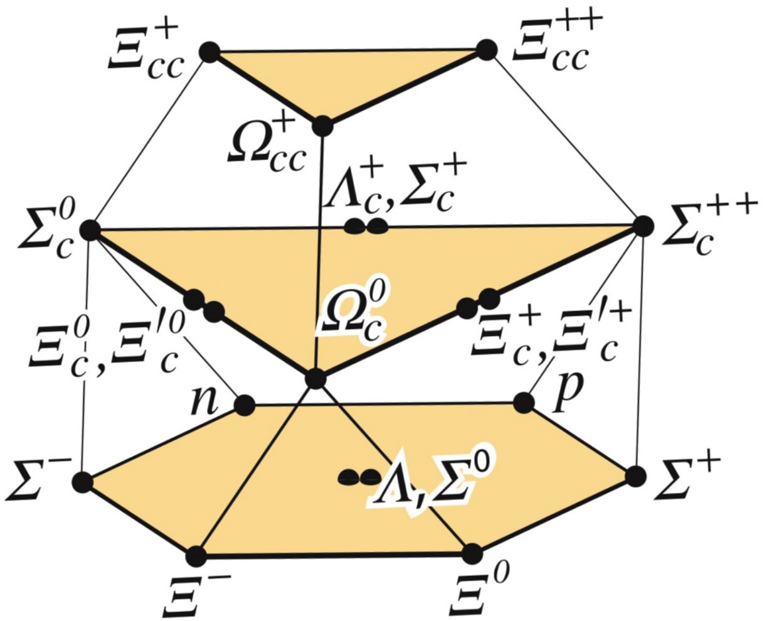
Mesons

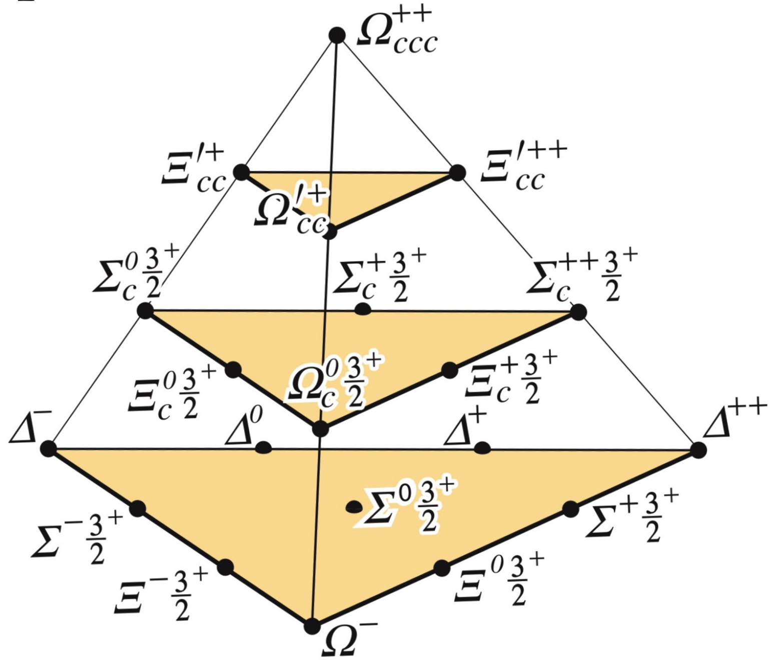






Baryons

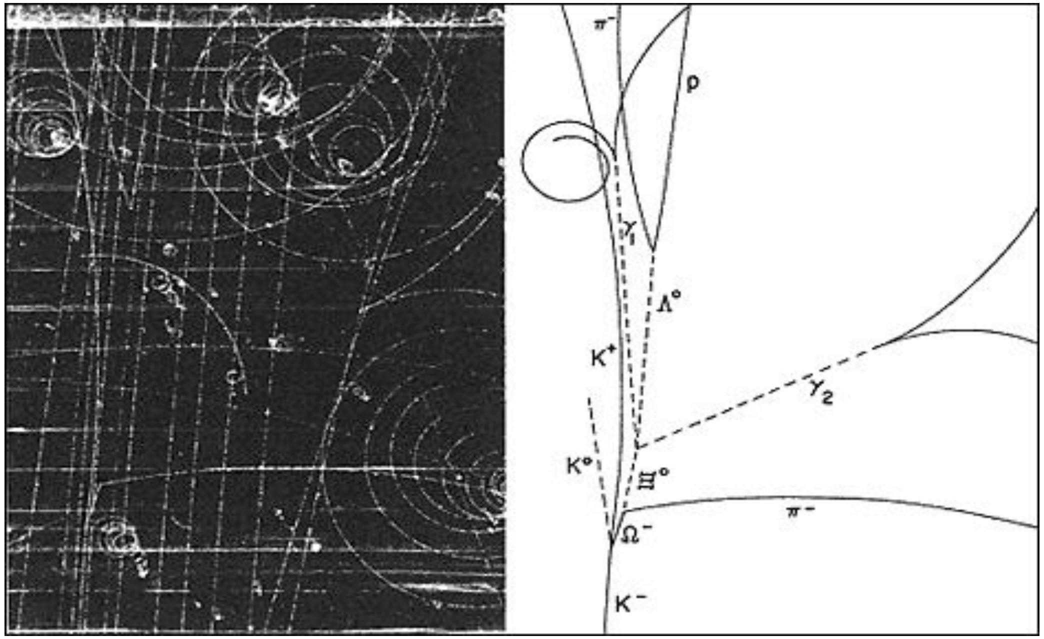




Quarks – Images of Underlying Symmetries

SU(3) symmetry considerations had led Gell-Mann, Ne'eman, and Zweig independently to manifest hadron multiplets – and to predict the Ω^- particle.

Still in 1964 this particle was found by Nicholas Samios et al. at the Brookhaven National Laboratory (BNL):



Nicholas Samios

Murray Gell-Mann received the Nobel Prize in 1969:
"for his contributions and discoveries concerning the classification of elementary particles and their interactions".

George Zweig: “Aces” (1964)

George Zweig @Oberwölz-Symposium
Austria, 2012

ES

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

G. Zweig *)
CERN - Geneva

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number $\frac{1}{3}$ and is consequently fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. An experimental search for the aces is suggested.

1968: Substructure of the Proton

In ~1968 the substructure of the proton was revealed experimentally at the Stanford Linear Accelerator Center (SLAC).

First public notice @14th Int. Conf. on High-Energy Physics Vienna 1968

Contribution no. 563 "**Inelastic Electron Scattering from Protons**"
by R.E. Taylor (SLAC), J.I. Friedman, H.W. Kendall (MIT) et al. (unpublished)

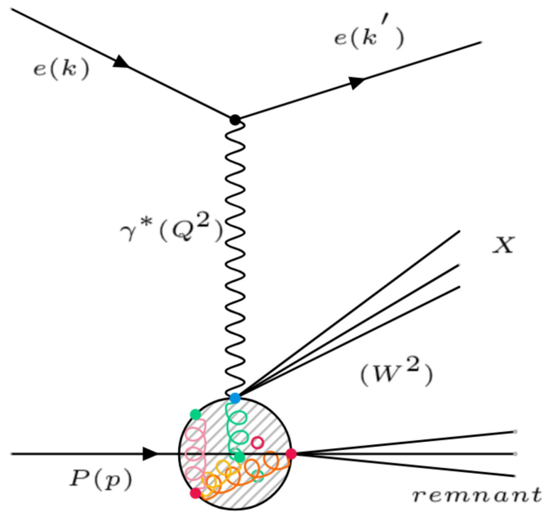
(532)
Photoproduction of π^+ mesons at backward angles between 0.9 GeV and 3.0 GeV
Alvarez R.A., Cooperstein G., Kalata K., Lanza R.C., Luckey D. (MIT)
(Abstract)

(563)
Inelastic scattering from protons
Bloom E., Coward D., DeStaebler H., Drees J., Litt J., Miller G., Mo L., Taylor R.E. (SLAC), Breidenbach M., Friedman J.I., Kendall H.W. (MIT), Loken S. (Cal. Tech.)
(Abstract)

(565)
Multibody photoproduction from hydrogen with 16 GeV bremsstrahlung
Davier M., Derado I., Drickey D., Fries D., Mozley R., Odian A., Villa F., Yount D., Zdanis R. (SLAC)

W.K.H. Panofsky:
Proceedings of the 14th Int. Conf. on High-Energy Physics, Vienna, 1969

Ed. by J. Prentki and J. Steinberger
CERN Sci. Inf. Service, Geneva, 1968, p. 23

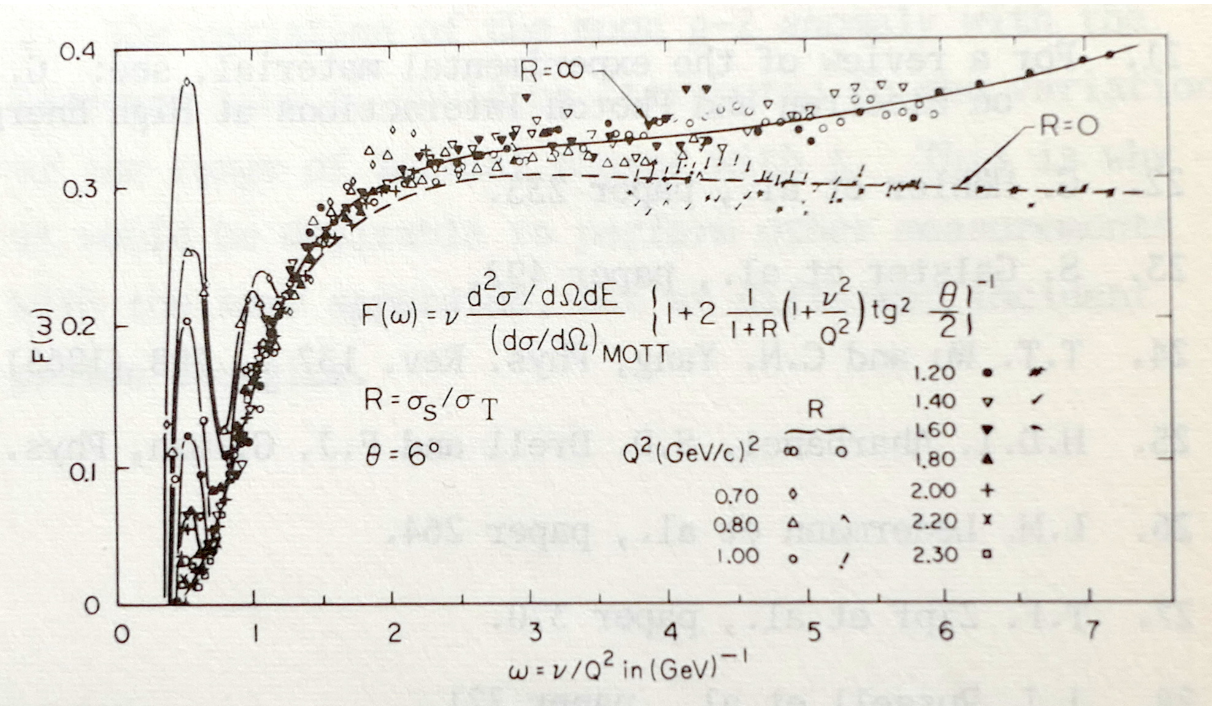


1968: Substructure of the Proton

In ~1968 the substructure of the proton was revealed experimentally at the Stanford Linear Accelerator Center (SLAC).

First public notice @14th Int. Conf. on High-Energy Physics Vienna, August 1968

Contribution no. 563 “**Inelastic Electron Scattering from Protons**”
by R.E. Taylor (SLAC), J.I. Friedman, H.W. Kendall (MIT) et al. (unpublished)



$F(\omega) = \nu W_2(q^2, \nu)$ as a function of $\omega = \nu/q^2$

Rapporteur W.K.H. Panofsky:

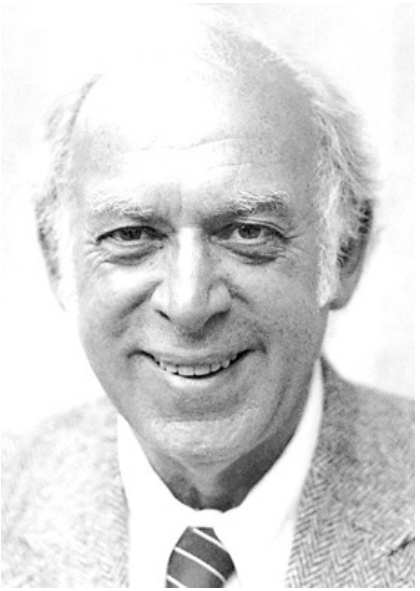
Theoretical speculations are focused on the possibility that these data might give evidence on the point-like charged structures within the nucleon.

However a great deal more fundamental experimental material must be developed before a clear picture can emerge.

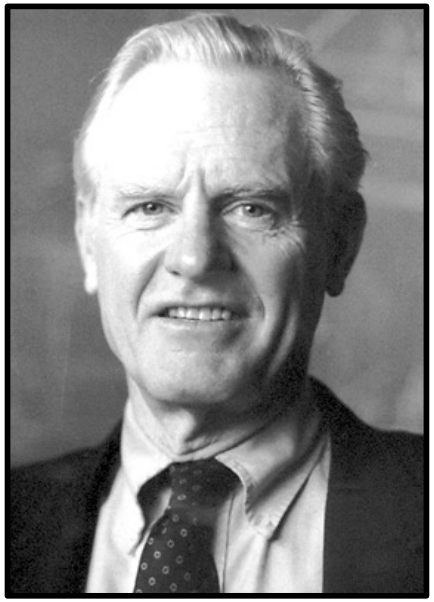
1968: Substructure of the Proton – Partons

The proton (like all other hadrons) consists of parts, the partons (R.P. Feynman and J.D. Bjorken).

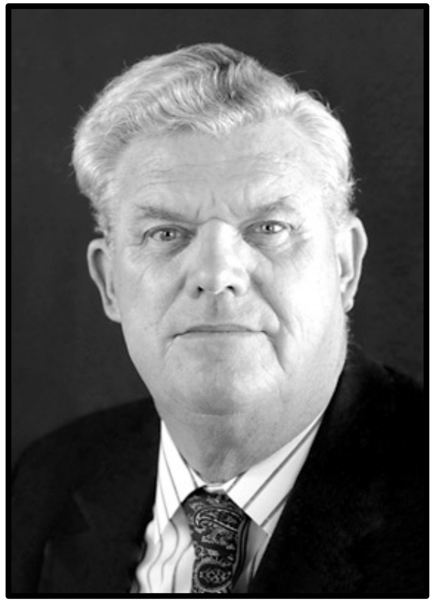
Nobel Prize in 1990 to: J.I. Friedman, H.W. Kendall and R.E. Taylor:
"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics".



J.I. Friedman, 1930 –



H.W. Kendall, 1926 – 1999



R.E. Taylor, 1929 – 2018

1964 – 1968 – 1972 What are Quarks?

- Are quarks particles or just images of symmetries?
- If quarks are particles with spin, which statistics do they follow?
- Can particles/quarks with fractional charges be observed?
- If quarks are particles, what are their interactions?
- Which dynamics keeps hadrons together?

M. Gell-Mann @Schladming Winter School 1972:

Current and constituent quarks

Schladming Winter School 1972

F.l.t.r.: M. Gell-Mann, A. Bartl, P. Breitenlohner, H. Fritzsch, H. Kleinert, P. Urban

1972 – 1973: Emergence of QCD

Current Algebra: Quarks and What Else?

Harald Fritzsch*†

and

Murray Gell–Mann**†

CERN, Geneva, Switzerland

Proceedings of the XVI International Conference on High Energy Physics, Chicago, 1972. Volume 2, p. 135 (J. D. Jackson, A. Roberts, eds.)

Volume 47B, number 4 PHYSICS LETTERS 26 November 1973

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE☆

H. FRITZSCH*, M. GELL-MANN and H. LEUTWYLER**
California Institute of Technology, Pasadena, Calif. 91109, USA

Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

Colored Quarks and Gluons: QCD

$$L = \bar{q}\left[i\gamma_\mu(\partial^\mu + ig\frac{\lambda^a}{2}A_a{}^\mu) - m \right]q - \frac{1}{4}G^a{}_{\mu\nu}G_a{}^{\mu\nu}$$

quark fields

gluon fields

quark fields

gluon field-strength tensor

$$G_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + gf_{abc}A_\mu{}^b A_\nu{}^c$$

gluon self-interaction

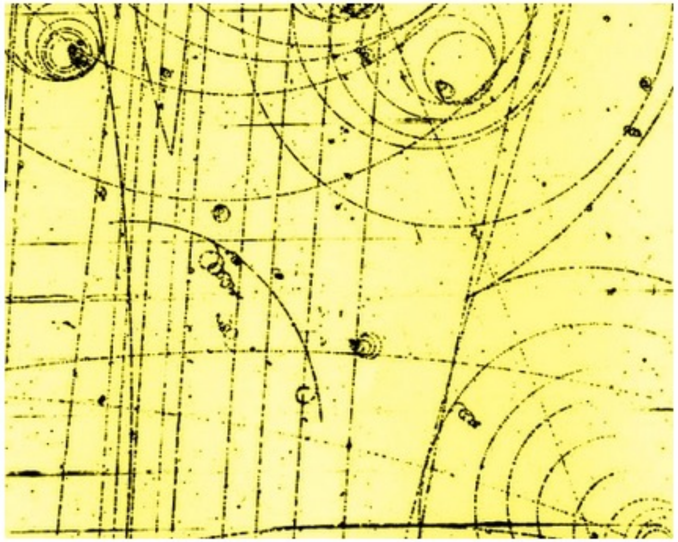
- Non-Abelian gauge theory in $SU(3)_C$

QCD: Millenium Prize Problem

Clay Mathematics Institute

About | Programs & Awards | People | The Millennium Prize Problems | Online resources | Events | News

Quantum Young-Mills theory and the mass gap



The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap": the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang-Mills theory and a mass gap will require the introduction of fundamental new ideas both in physics and in mathematics.

https://www.claymath.org/millennium/yang-mills-the-maths-gap/

QCD Properties and Its Solution

A comprehensive solution of QCD must incorporate the following properties:

- It must have a mass gap, i.e. any vacuum excitation must produce $\Delta > 0$

  (in order to lead to strong but short-ranged nuclear forces).

- It must produce quark confinement

  (in order to guarantee all hadrons to be color singlets – no free quarks).

- It must lead to asymptotic freedom

  (interaction-free for $E \longrightarrow \infty$).

- It must exhibit (spontaneous) chiral-symmetry breaking

  (in order to produce hadrons of non-zero masses)

- It must, of course, describe all hadron phenomenology consistently, at all energies

  (in order to be a practical theory of strong forces).

Solution Methods for QCD

- Perturbative QCD (for $E \longrightarrow \infty$)
- QCD on a space-time lattice
- Effective field theories, in particular, chiral perturbation theory
- Functional methods

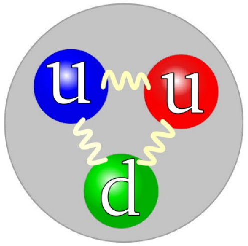
  Dyson-Schwinger equations (DSE)

  Functional renormalization group (FRG)
- Effective / Constituent quark models
- ....

Low-Energy QCD

- Hadrons consist of constituent quarks, e.g. baryons of $\{QQQ\}$,

  such as the (colorless) proton:



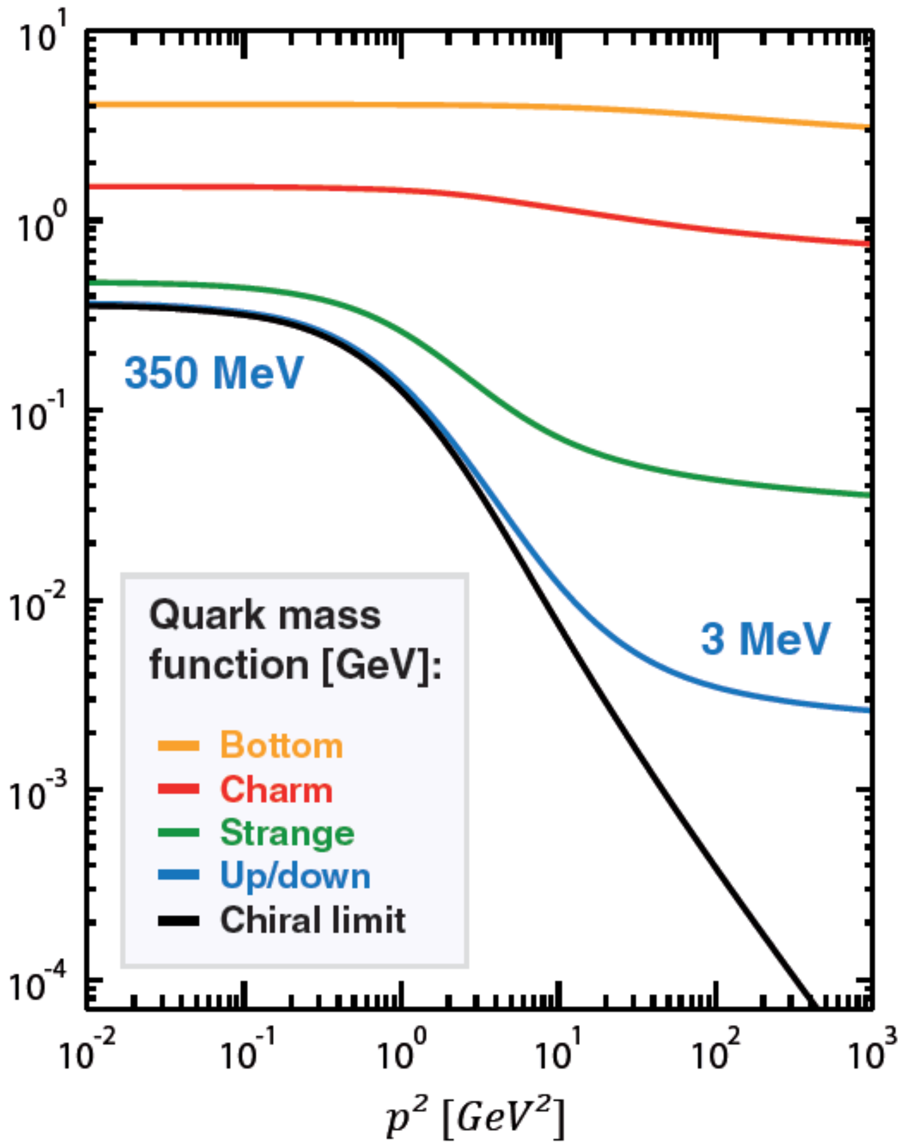
  $\{QQQ\}$ are considered as quasiparticles, confined inside hadrons.

- The $\{Q\text{–}Q\}$ interaction is furnished by the low-energy d.o.f. of QCD, resulting from the spontaneous breaking of chiral symmetry ($SB\chi S$), i.e. for N_F flavors $SU(N_F)_L \times SU(N_F)_R \rightarrow SU(N_F)_V$, leading to the appearance of Goldstone bosons.

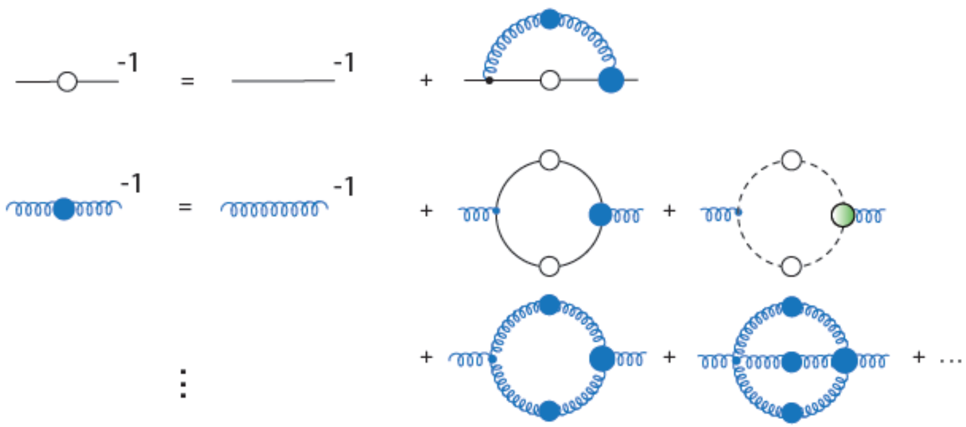
- Construct a Poincaré-invariant interacting mass operator based on

$$\mathcal{L}_{\text{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^F \cdot \vec{\phi}\psi$$

Dynamical Mass Generation



Dynamical quark masses generated by Dyson-Schwinger equations (DSE):



Slide by courtesy from G. Eichmann

Relativistic Quantum Mechanics

Relativistic quantum mechanics (RQM)

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M}\,|P, J, \Sigma\rangle = M\,|P, J, \Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu\,|P, J, \Sigma\rangle = P^\mu\,|P, J, \Sigma\rangle \quad , \qquad \hat{P}^\mu = \hat{M}\hat{V}^\mu$$

Relativistic Constituent Quark Model (RCQM)

Interacting mass operator

$$
\begin{aligned}
\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\
\hat{M}_{free} &= \sqrt{\hat{H}^2_{free} - \hat{\vec{P}}^2_{free}} \\
\hat{M}^{rest\ frame}_{int} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} [\hat{V}^{conf}_{ij} + \hat{V}^{hf}_{ij}]
\end{aligned}
$$

fulfilling the **Poincaré algebra**

$$
\begin{array}{lll}
[\hat{P}_i, \hat{P}_j] = 0, & [\hat{J}_i, \hat{H}] = 0, & [\hat{P}_i, \hat{H}] = 0, \\
[\hat{K}_i, \hat{H}] = -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\
[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}
\end{array}
$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Universal Goldstone-Boson-Exchange RCQM

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow \quad H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)

UGBE RCQM Parametrization

$$
\begin{aligned}
V^{conf}(\vec{r}_{ij}) &= B + C\, r_{ij} \\
V_\beta(\vec{r}_{ij}) &= \frac{g_\beta^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\} \\
&= \frac{g_\beta^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2 \frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right\}
\end{aligned}
$$

$B = -402$ MeV, $C = 2.33\ \mathrm{fm}^{-2}$

$\beta = 24:$ $\frac{g_{24}^2}{4\pi} = 0.7,$ $\mu_{24} = \mu_\pi = 139$ MeV, $\Lambda_{24} = 700.5$ MeV

$\beta = 0:$ $\left(\frac{g_0}{g_{24}}\right)^2 = 1.5,$ $\mu_0 = \mu_{\eta'} = 958$ MeV, $\Lambda_0 = 1484$ MeV

$m_u = m_d = 340$ MeV, $m_s = 480$ MeV,

$m_c = 1675$ MeV, $m_b = 5055$ MeV

Solution of Mass-Operator Eigenvalue Problem

$$\hat{M}\,|P,J,\Sigma,F_{abc}\rangle = M\,|P,J,\Sigma,F_{abc}\rangle$$
$$= M\,|M,V,J,\Sigma,F_{abc}\rangle$$

$\rightarrow$ baryon wave functions (initially in rest frame)

$$\Psi_{PJ\Sigma F_{abc}}(\vec{\xi},\vec{\eta}) = \left\langle \vec{\xi},\vec{\eta} \middle| P,J,\Sigma,F_{abc} \right\rangle \ ,$$

where $\vec{\xi}$ and $\vec{\eta}$ are the usual Jacobi coordinates and

P	.....	momentum eigenvalues
$(M\,,V$	.....	mass resp. velocity eigenvalues)
J	.....	intrinsic spin $\hat{=}$ total angular momentum)
Σ	.....	z-component of J
F_{abc}	.....	flavor content

Advanced Few-Body Methods: A) SVM

A) Stochastic Variational Method (SVM)

$$\Psi_{PJ\Sigma F_{abc}}(\mathbf{x}) = \sum_i c_i \left\{ e^{-\frac{1}{2}\tilde{\mathbf{x}}A\mathbf{x}} \left[\Theta_{LM_L}(\hat{\mathbf{x}})\chi_S\right]_{J\Sigma} \phi_{F_{abc}} \right\}_i$$

with linear and nonlinear variational parameters

$$c_i\,, \qquad A = \{\beta, \delta, \nu, n, \lambda, l, L, s, S, F_{abc}, d\}$$

searched by a generalized Rayleigh-Ritz principle through a stochastic selection of basis states

V.I. Kukulin and V.M. Krasnopol'sky: J. Phys. G **3**, 795 (1977)

Y. Suzuki and K. Varga: *Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems* (Springer, Berlin, 1998)

Advanced Few-Body Methods: B) FIE

B) Modified Faddeev Integral Equations (FIE)

$$
\begin{aligned}
H &= H_0 + v_\alpha + v_\beta + v_\gamma = \\
& H_0 + v_\alpha^{\mathrm{conf}} + v_\beta^{\mathrm{conf}} + v_\gamma^{\mathrm{conf}} + \tilde{v}_\alpha + \tilde{v}_\beta + \tilde{v}_\gamma = \\
& H^{\mathrm{conf}} + \tilde{v}_\alpha + \tilde{v}_\beta + \tilde{v}_\gamma \,,
\end{aligned}
$$

with $H^{\mathrm{conf}} = H_0 + v_\alpha^{\mathrm{conf}} + v_\beta^{\mathrm{conf}} + v_\gamma^{\mathrm{conf}}$

$$
\begin{aligned}
\Psi_{PJ\Sigma F_{abc}}(\mathbf{k}) &= \left(\tilde{\psi}_\alpha + \tilde{\psi}_\beta + \tilde{\psi}_\gamma\right)_{PJ\Sigma F_{abc}}(\mathbf{k}) \\
\tilde{\psi}_\alpha &= G_\alpha^{\mathrm{conf}}(E)\tilde{v}_\alpha\left(\tilde{\psi}_\beta + \tilde{\psi}_\gamma\right)
\end{aligned}
$$

$$
G_\alpha^{\mathrm{conf}}(E) = \left(E - H^{\mathrm{conf}} - \tilde{v}_\alpha\right)^{-1}
$$

Z. Papp: Few-Body Syst. **26**, 99 (1999)

Z. Papp, A. Krassnigg, and W. Plessas: Phys. Rev. C **62**, 044004 (2000)

J. McEwen, J. Day, A. Gonzalez, Z. Papp, and W. Plessas: Few-Body Syst. **47**, 225 (2010)

Solution Accuracy FIE vs. SVM

Baryon	J^P	Faddeev GBE	Faddeev OGE	SVM GBE	SVM OGE	Experiment
N(939)	$\frac{1}{2}^+$	939	940	939	939	938-940
N(1440)	$\frac{1}{2}^+$	1459	1578	1459	1577	1420-1470
N(1520)	$\frac{3}{2}^-$	1520	1521	1519	1521	1515-1525
N(1535)	$\frac{1}{2}^-$	1520	1521	1519	1521	1525-1545
N(1650)	$\frac{1}{2}^-$	1646	1686	1647	1690	1645-1670
N(1675)	$\frac{5}{2}^-$	1646	1686	1647	1690	1670-1680
Δ(1232)	$\frac{3}{2}^+$	1240	1229	1240	1231	1231-1233
Δ(1600)	$\frac{3}{2}^+$	1718	1852	1718	1854	1550-1700
Δ(1620)	$\frac{1}{2}^-$	1640	1618	1642	1621	1600-1660
Δ(1700)	$\frac{3}{2}^-$	1640	1618	1642	1621	1670-1750
Λ(1116)	$\frac{1}{2}^+$	1133	1127	1136	1113	1116
Λ(1405)	$\frac{1}{2}^-$	1561	1639	1556	1628	1401-1410
Λ(1520)	$\frac{3}{2}^-$	1561	1639	1556	1628	1519-1521
Λ(1600)	$\frac{1}{2}^+$	1607	1749	1625	1747	1560-1700
Λ(1670)	$\frac{1}{2}^-$	1672	1723	1682	1734	1660-1680
Λ(1690)	$\frac{3}{2}^-$	1672	1723	1682	1734	1685-1695

Z. Papp, A. Krassnigg, and W. Plessas: Phys. Rev. C **62**, 044004 (2000)

J.P. Day: PhD Thesis, Univ. Graz (2013)

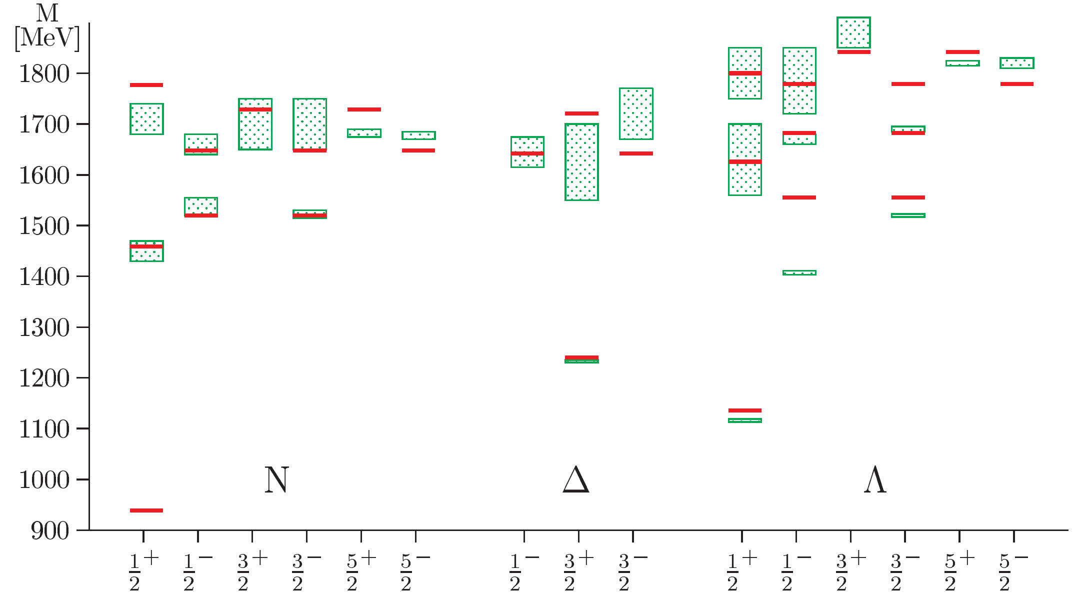
Spectroscopy of Baryons

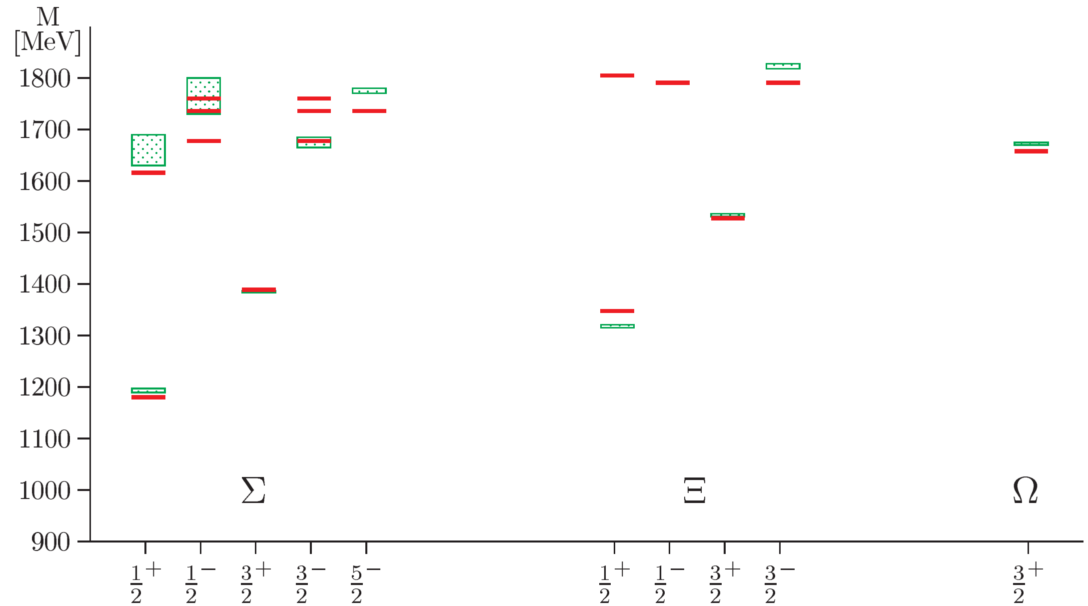
Excitation Spectra

of Baryons with ALL Flavors

u, d, s, c, b

Excitation Spectra of Baryons with *u*, *d*, *s* Flavors





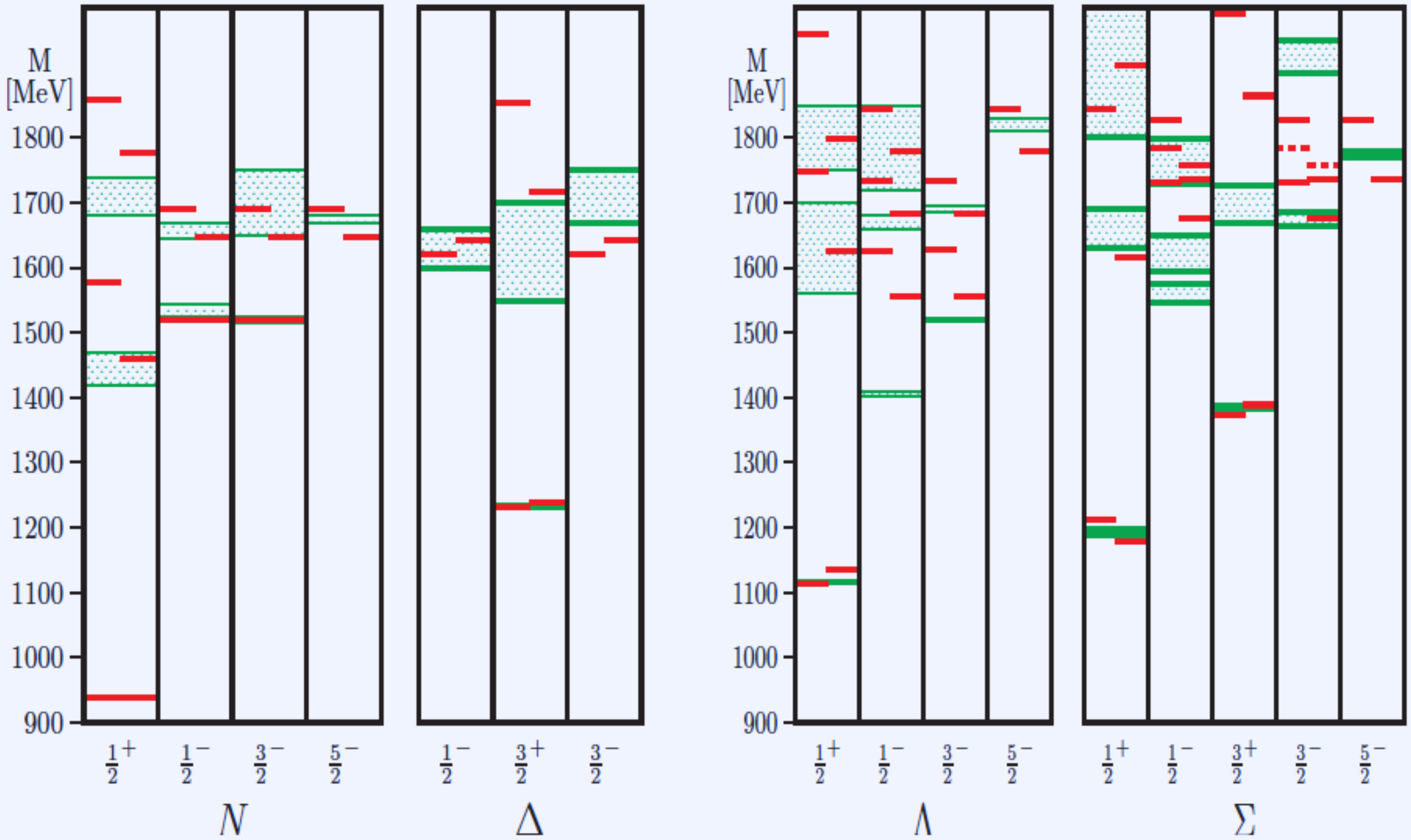
red levels = theoretical prediction

green boxes = exp. data
with uncertainties

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn:

Phys. Rev. D 58, 094030 (1999)

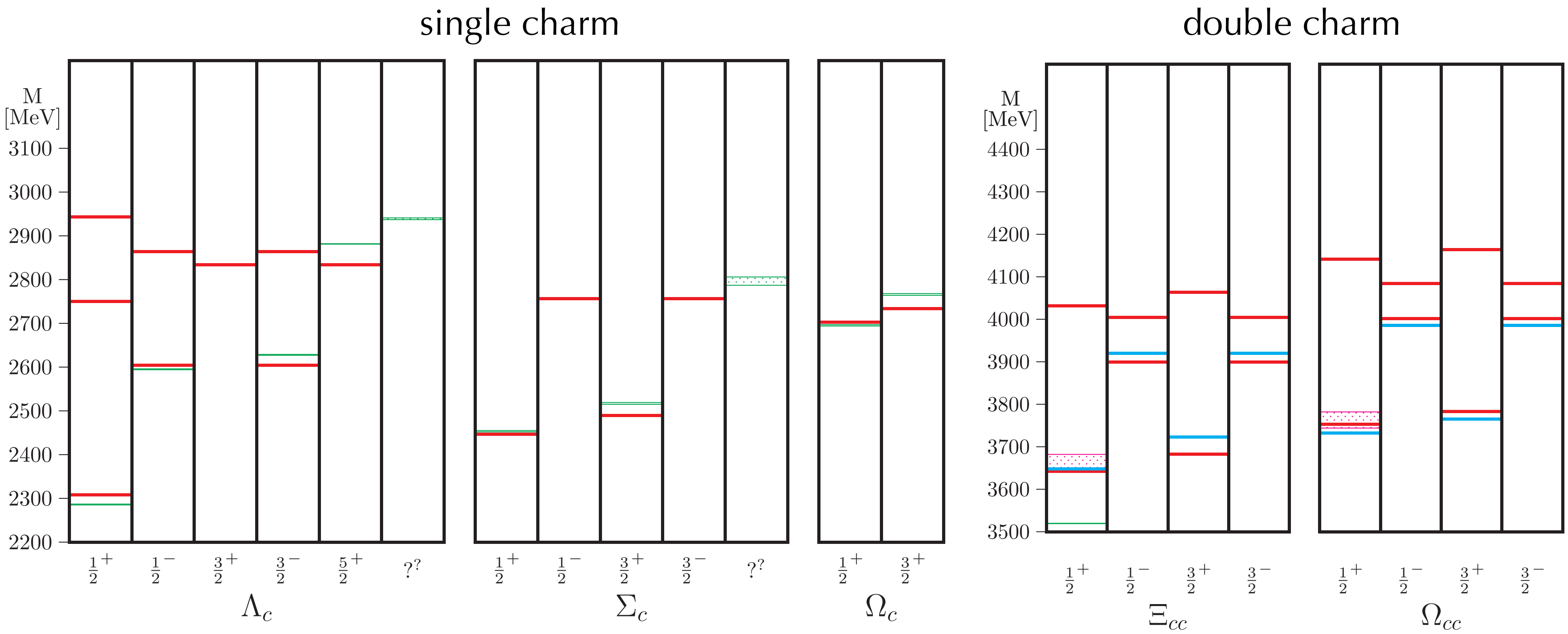
Comparison of N and Λ Level Orderings



left **levels**: **One-gluon-exchange** RCQM
right **levels**: **Goldstone-boson-exchange** RCQM

W. Plessas: Few-Body Syst. Suppl. **15**, 139 (2003)

Excitation Spectra of Charm Baryons



↑ our value m(Ξ_{cc}) = 3642 MeV

Left panel – single charm:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

Right panel – double charm:

green M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

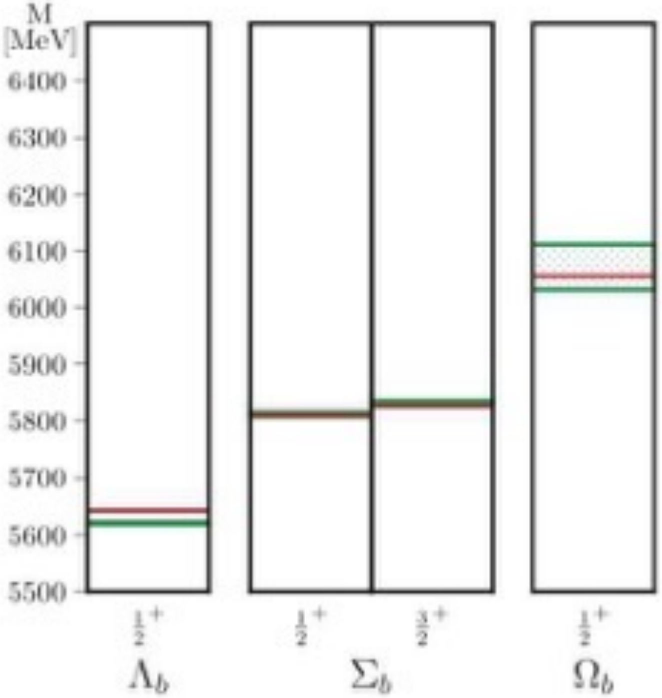
New datum from LHCb 2017: m(Ξ_{cc})= 3621.40$\pm$0.72(stat.)$\pm$0.27(syst.)$\pm$0.14(Λ_c) MeV

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

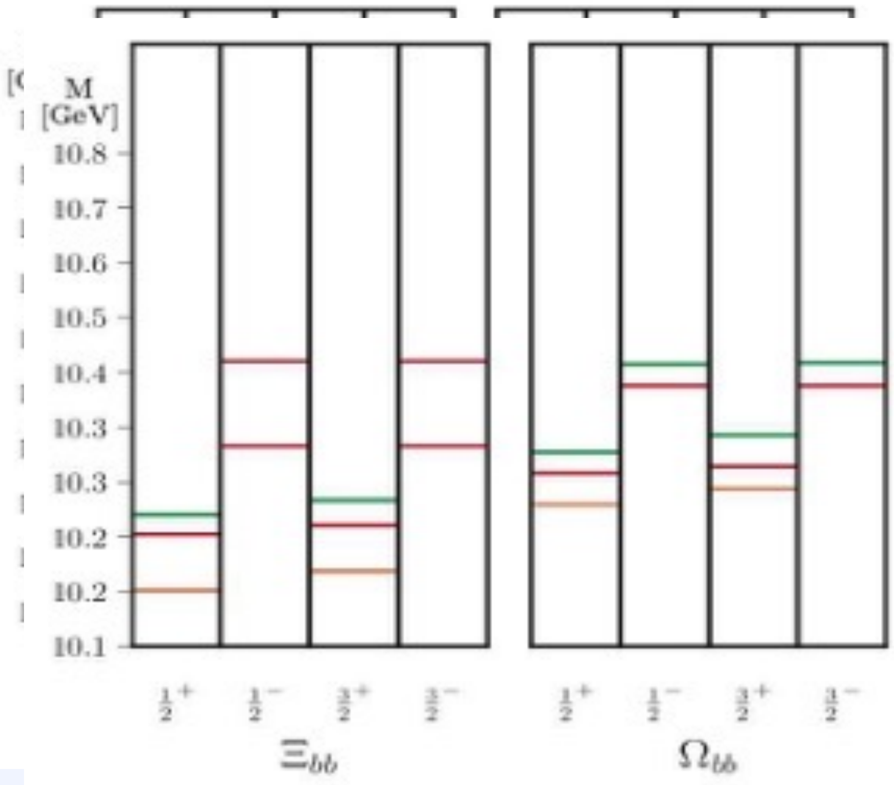
magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Excitation Spectra of Bottom Baryons

single bottom



double bottom



Left panel – single bottom:

red Universal GBE RCQM prediction

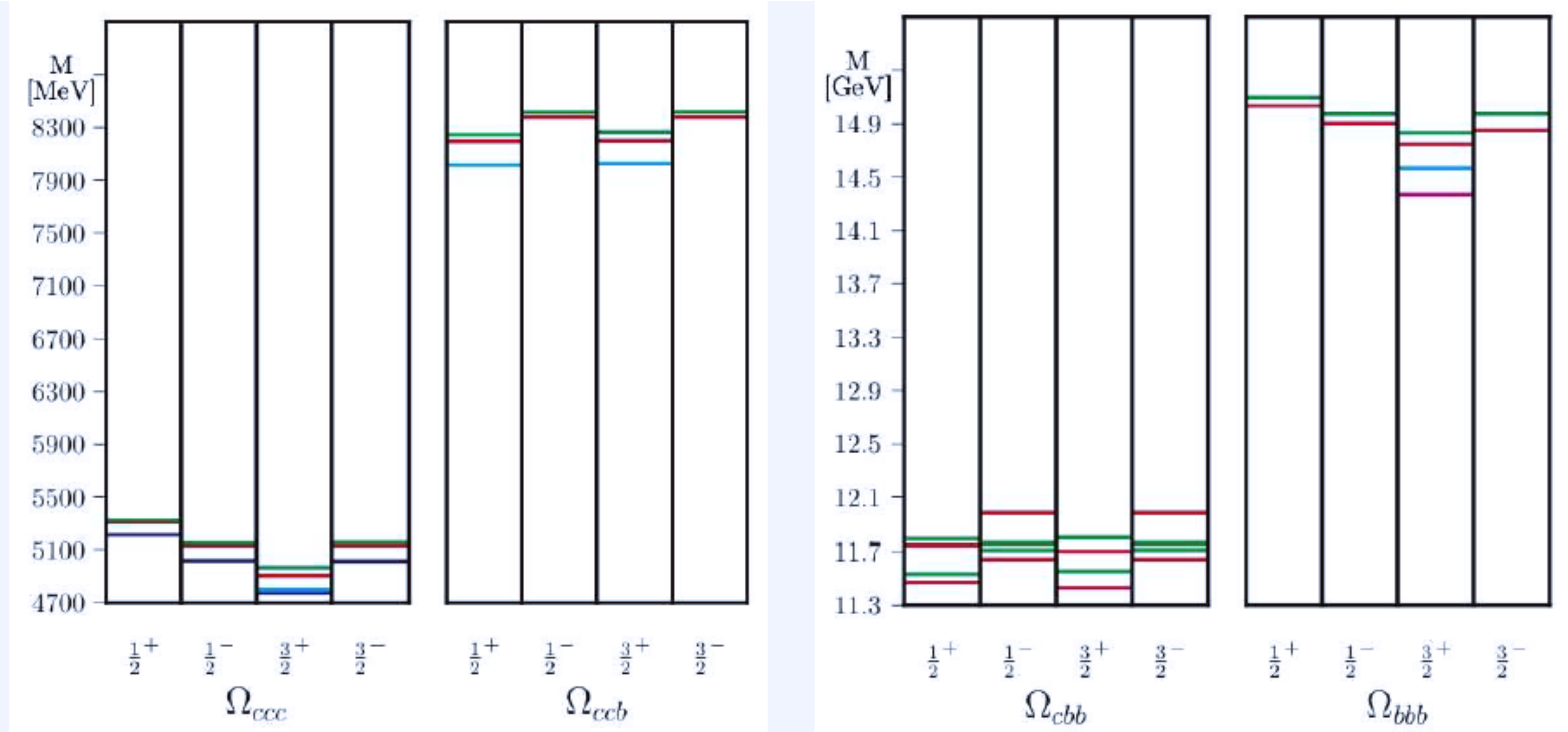
green PDG 2013 (experiment)

Right panel – double bottom:

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra



red	Universal GBE RCQM
green	W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrelativistic one-gluon-exchange CQM)
blue	S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)
cyan	A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)
magenta	S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Rest-Frame Baryon States

Mass operator eigenstates

$$\hat{M}\,|P, J, \Sigma, T, M_T\rangle = M\,|P, J, \Sigma, T, M_T\rangle$$

represented in configuration space

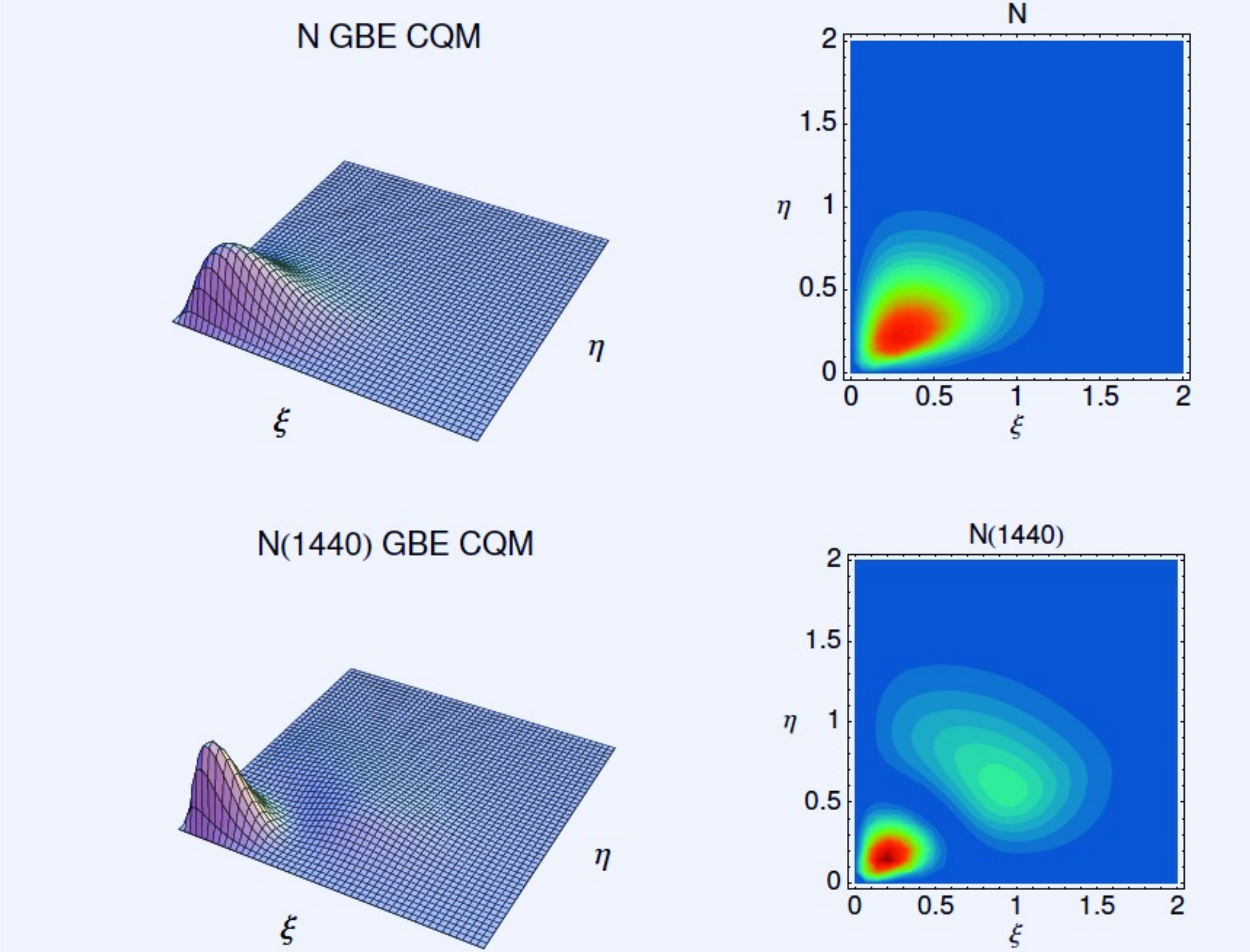
$$\left\langle \vec{\xi}, \vec{\eta} \middle| P, J, \Sigma, T, M_T \right\rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi}, \vec{\eta})$$

with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

Picture the baryon wave functions through
spatial probability density distributions

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta$$
$$\Psi^\star_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$

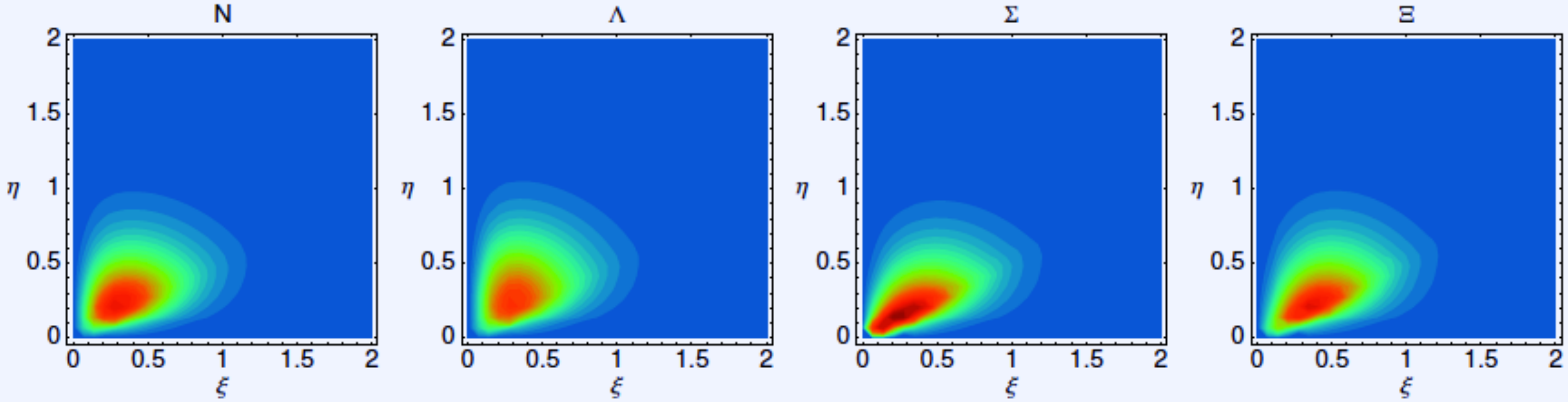
Spatial Probability Density Distributions



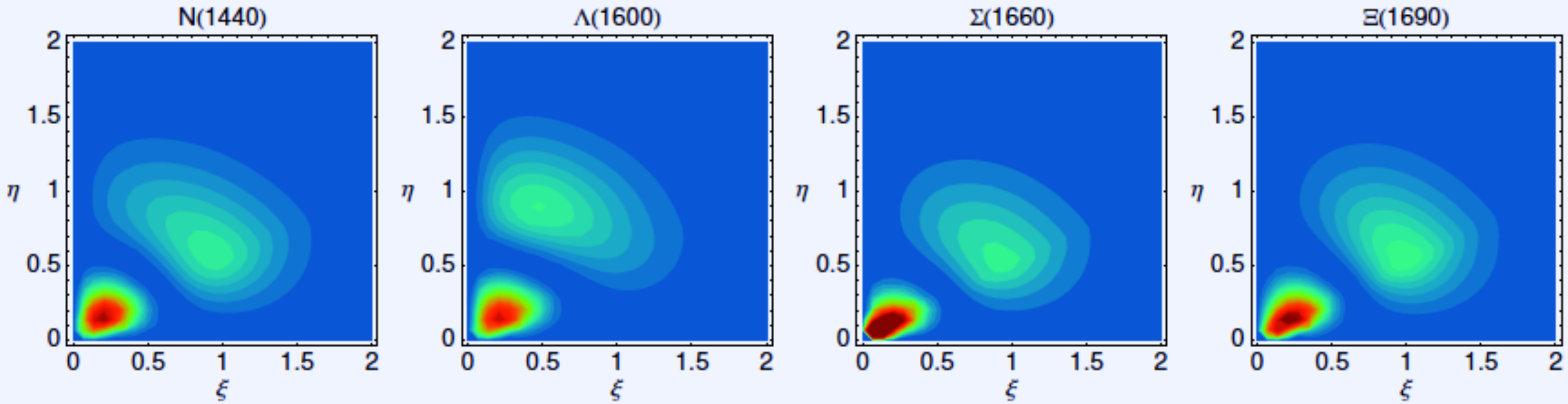
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ **octet** baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:



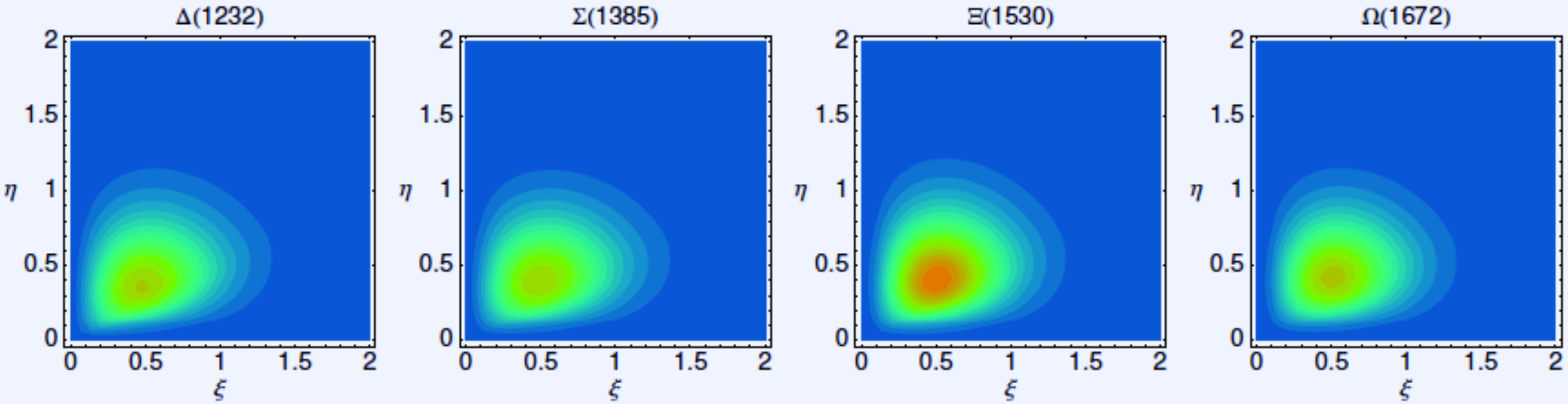
$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ **octet** baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:



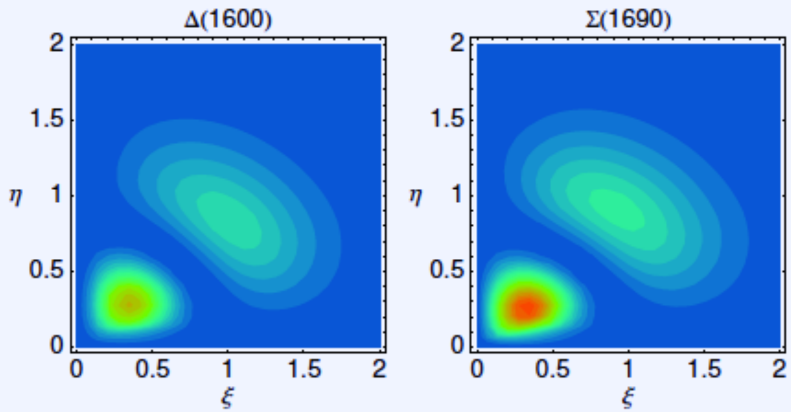
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$:



$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states $\Delta(1600)$, $\Sigma(1690)$:



T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

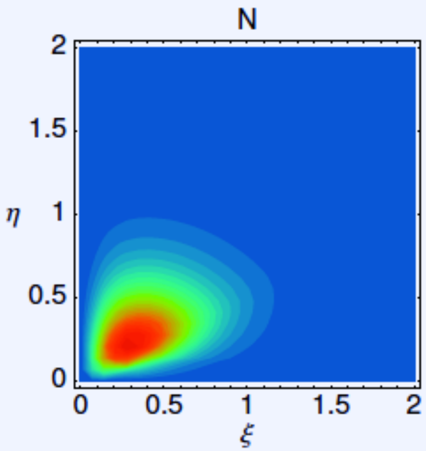
Electric Radii vs. Root-Mean-Square Radii

The **root-mean-square radius** (in the rest frame):

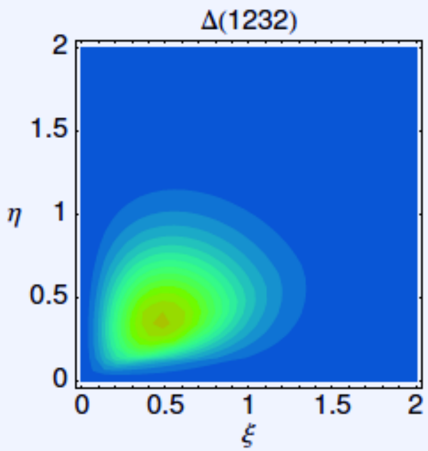
$$r_{\rm rms} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P=0, J, \Sigma \left| \hat{r}_i^{\,2} \right| P=0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

$\rightarrow$ Idea about the spatial distribution of constituent quarks.



$r^N_{\rm rms}$= 0.304 fm



$r^\Delta_{\rm rms}$= 0.390 fm

Exp.: $r_E^p \sim 0.88$ fm

$(r_E^n)^2 \sim -0.12$ fm^2

$r_E^{\Delta^{++}} = r_E^{\Delta^+} = r_E^{\Delta^-} = 0.656$ fm

$r_E^{\Delta^0} = 0$ fm

Calculation of Covariant Observables

Matrix elements of a transition operator $\hat{O}$ between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T_3', Y'|\hat{O}|P, J, \Sigma, T, T_3, Y\rangle$$

$\hat{O}$ ... $\hat{J}^{\mu}_{\mathrm{em}}$ $\rightarrow$ electromagnetic FF's

... $\hat{A}^{\mu}_{\mathrm{axial}}$ $\rightarrow$ axial FF's

... $\hat{S}$ $\rightarrow$ scalar FF

... $\hat{\Theta}^{\mu\nu}$ $\rightarrow$ gravitational/tensor FF's

... $\hat{D}^{\mu}_{\lambda}$ $\rightarrow$ hadronic decays

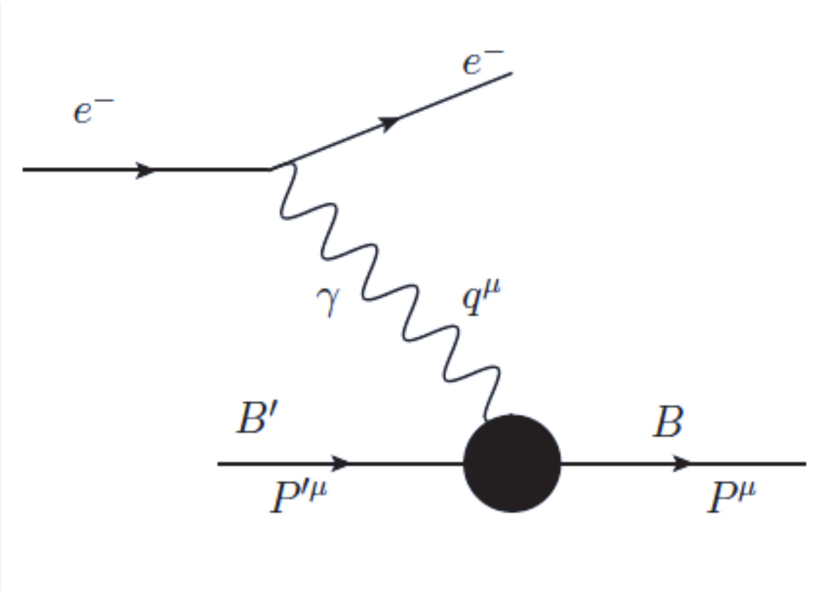
To be calculated from microscopic three-quark ME's

$$\langle \underbrace{p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'; f_{i_1'}, f_{i_2'}, f_{i_3'}}_{\Uparrow} |\hat{O}| \underbrace{p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3}}_{\Uparrow} \rangle$$

boosted 3-body states boosted 3-body states

e^- Scattering – Electromagnetic Form Factors

Elastic electron scattering:



Invariant form factors:

$$F^{\nu}_{\Sigma'\Sigma}(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}^{\nu}_{\mathrm{em}} | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2 \,; \qquad q^\mu = P^\mu - P'^\mu$$

Electromagnetic Sachs Form Factors

Spin-$\frac{1}{2}$ baryons:

$$G_E^B(Q^2) = \frac{1}{2M} F^{\nu=0}_{\frac{1}{2}\frac{1}{2}}(Q^2)$$

$$G_M^B(Q^2) = \frac{1}{Q} F^{\nu=1}_{\frac{1}{2}-\frac{1}{2}}(Q^2)$$

Spin-$\frac{3}{2}$ baryons:

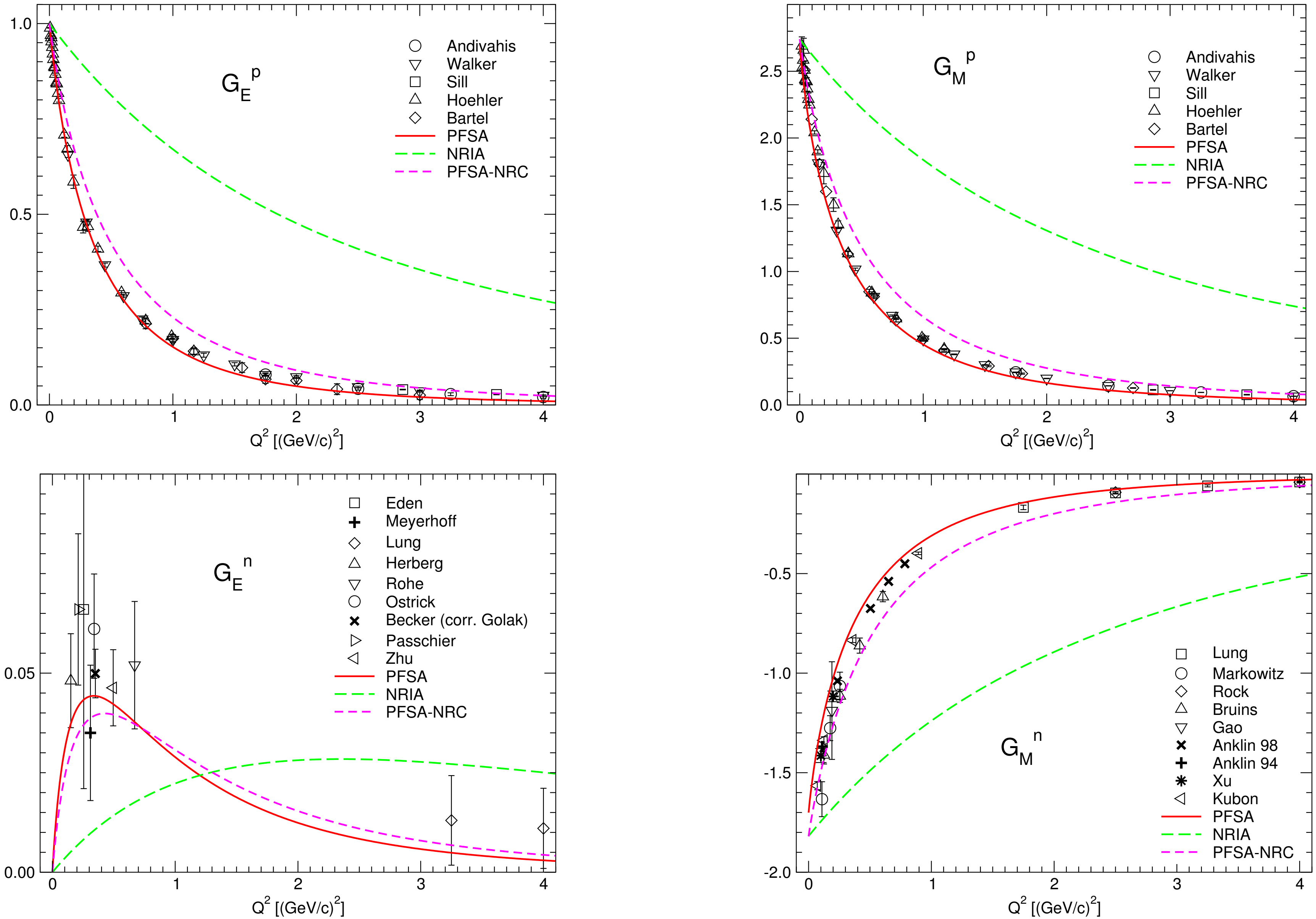
$$G_E^B(Q^2) = \frac{1}{4M}[F^{\nu=0}_{\frac{1}{2}\frac{1}{2}}(Q^2) + F^{\nu=0}_{\frac{3}{2}\frac{3}{2}}(Q^2)]$$

$$G_M^B(Q^2) = \frac{3}{5Q}[F^{\nu=1}_{\frac{1}{2}-\frac{1}{2}}(Q^2) + \sqrt{3}F^{\nu=1}_{\frac{3}{2}\frac{1}{2}}(Q^2)]$$

Electric/charge radius r_E:

$$r_E^2 = -6\frac{d}{d\,Q^2}G_E(Q^2)\Big|_{Q^2=0}$$

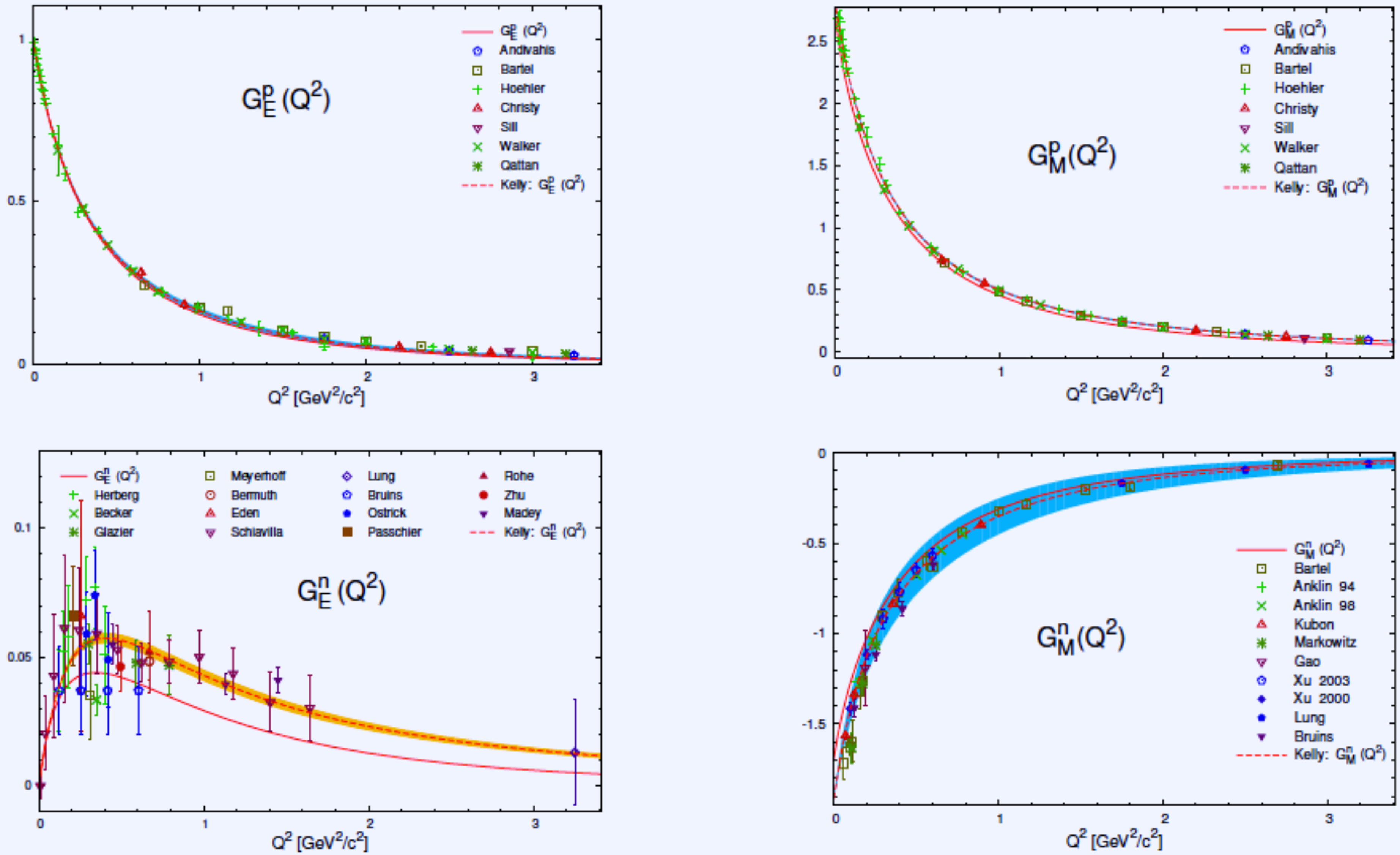
Electromagnetic Nucleon Form Factors



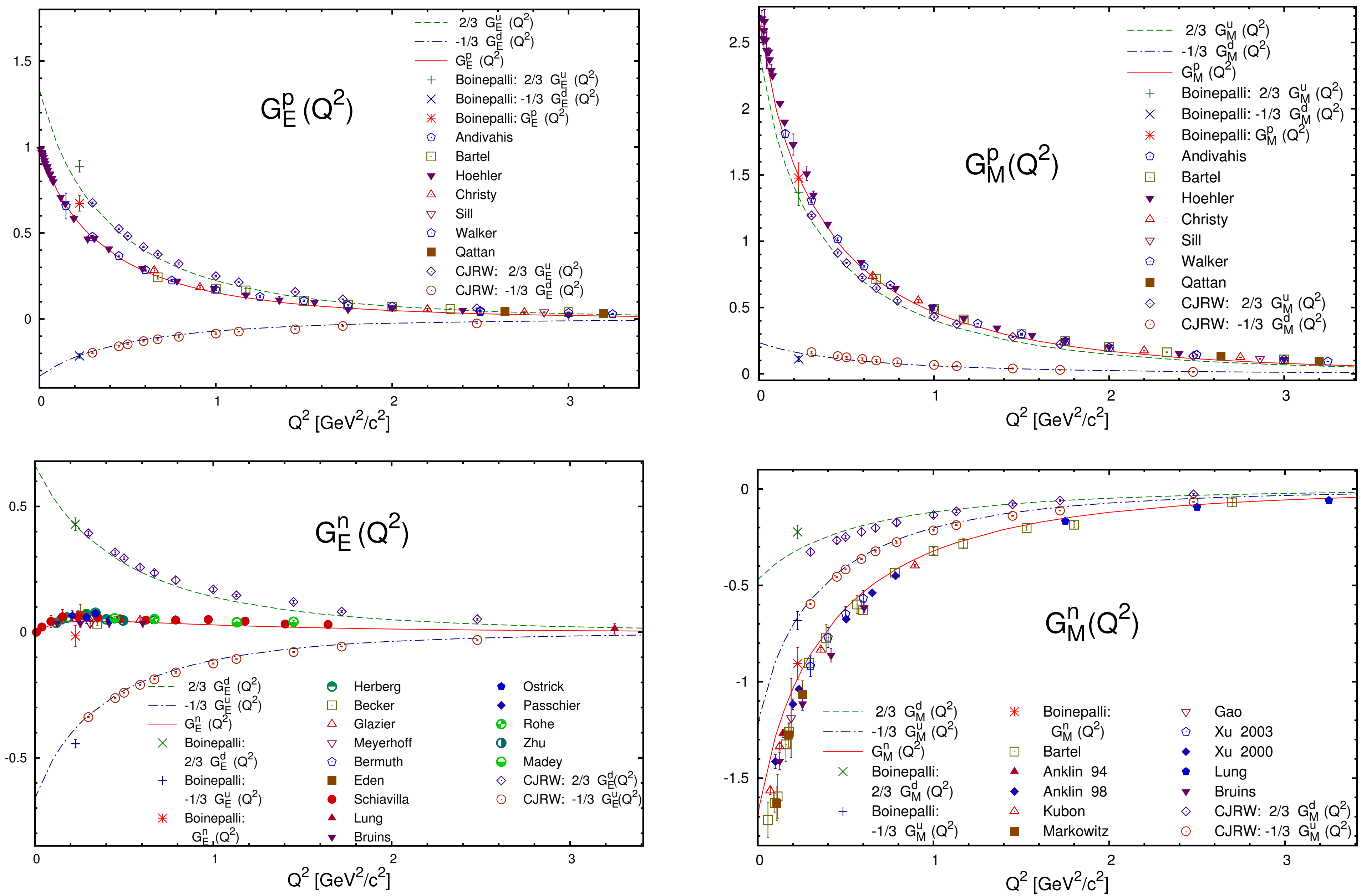
R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. B511 (2001) 33

Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:

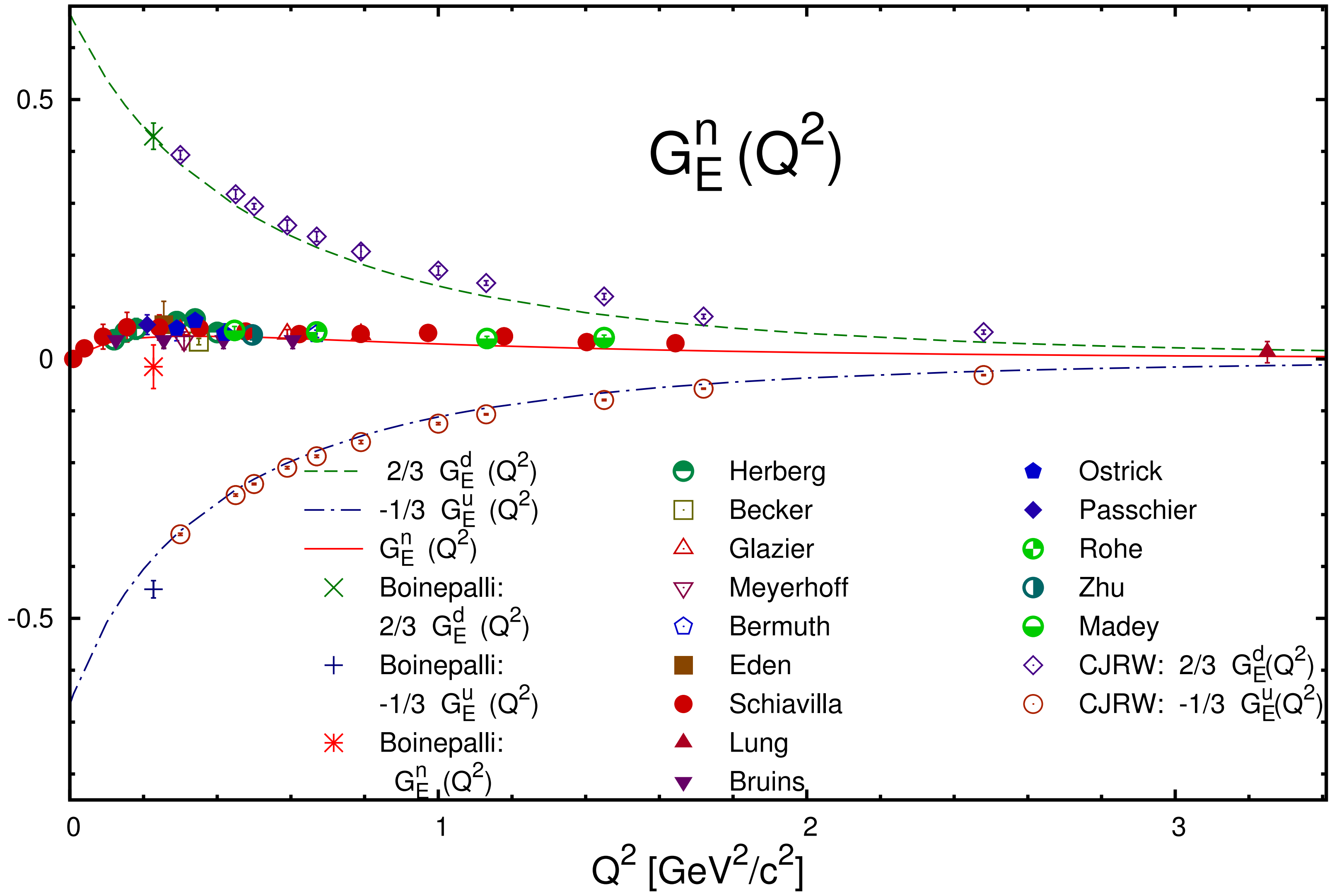


Flavor Decomposition of Nucleon Form Factors



M. Rohrmoser, K.-S. Choi, and W. Plessas: Few-Body Syst. 58 (2017) 83

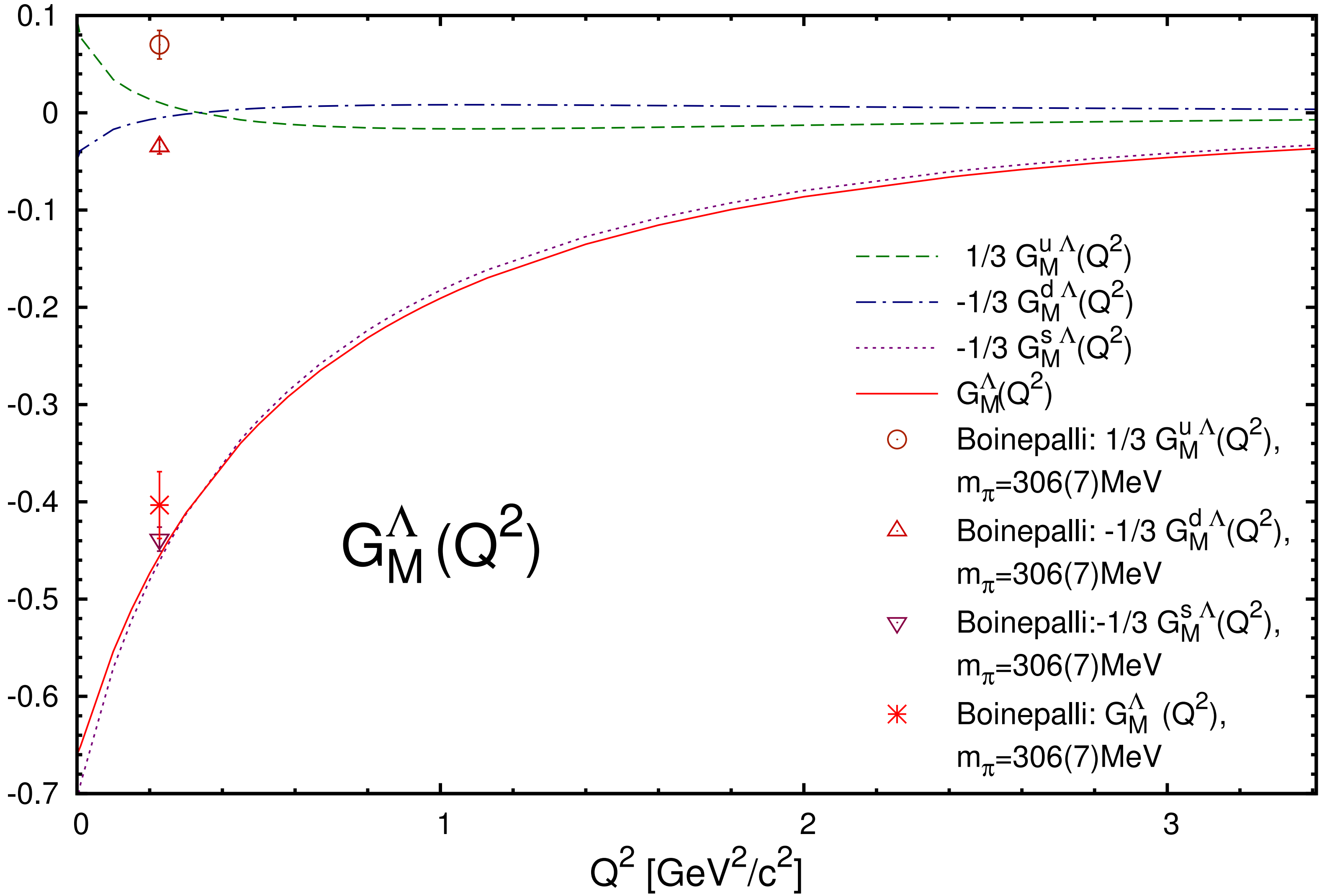
Flavor Decomposition of Neutron $G_E(Q^2)$



M. Rohrmoser, K.-S. Choi, and W. Plessas: Few-Body Syst. 58 (2017) 83

Lattice QCD: S. Boinepalli et al.: Phys. Rev. D74 (2006) 093005

Flavor Components in $\Lambda(1116)$ Magnetic FF $G_M(Q^2)$



M. Rohrmoser, K.-S. Choi, and W. Plessas: Few-Body Syst. 58 (2017) 83

Lattice QCD: S. Boinepalli et al.: Phys. Rev. D74 (2006) 093005

Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123 [1)]
		0.70870 ± 0.00113 [2)]
n	-0.13	-0.1161 ± 0.0022

1) CODATA value (PDG)

2) Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Electric Radii and Magnetic Moments – Nonrelativistic !!

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	0.82	0.10	0.7692 ± 0.0123[1)]
			0.70870 ± 0.00113[2)]
n	-0.13	-0.01	-0.1161 ± 0.0022

[1)] CODATA value (PDG) [2)] Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	2.70	2.74	2.792847356
n	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Baryon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

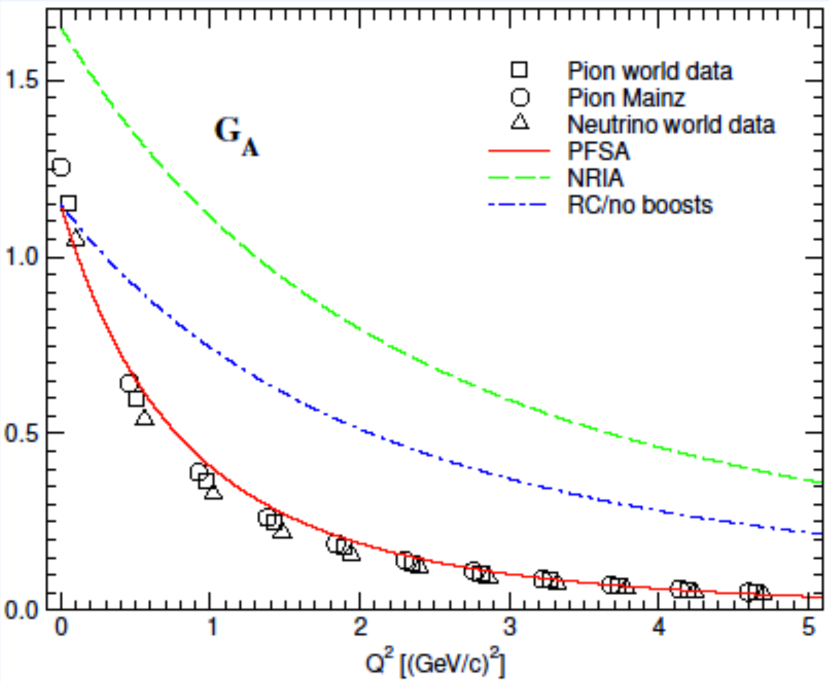
Magnetic moments μ [n.m.]

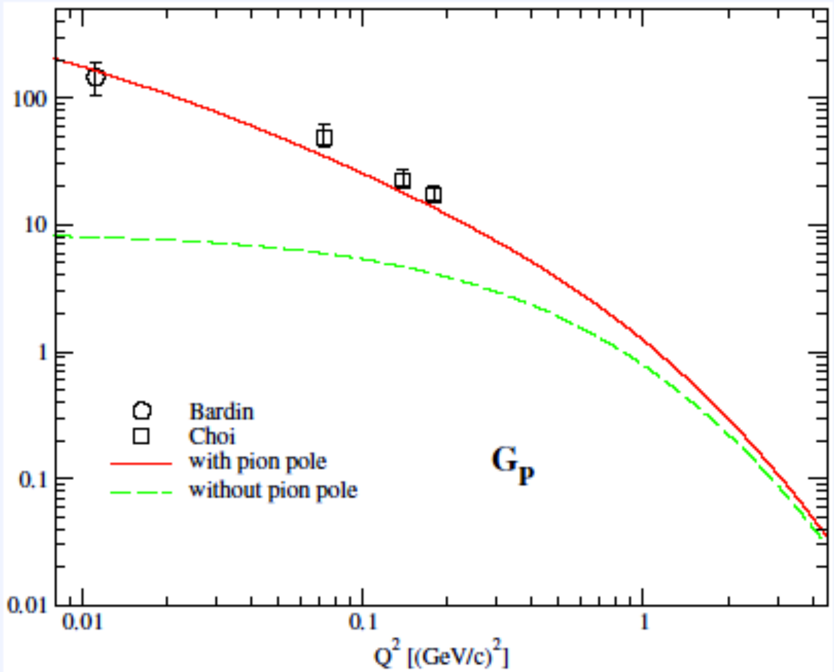
Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
Δ^{++}	4.17	$3.7 - 7.5$
Ω^-	-1.59	-2.020 ± 0.05

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Axial Nucleon Form Factors

Covariant predictions of the GBE RCQM:

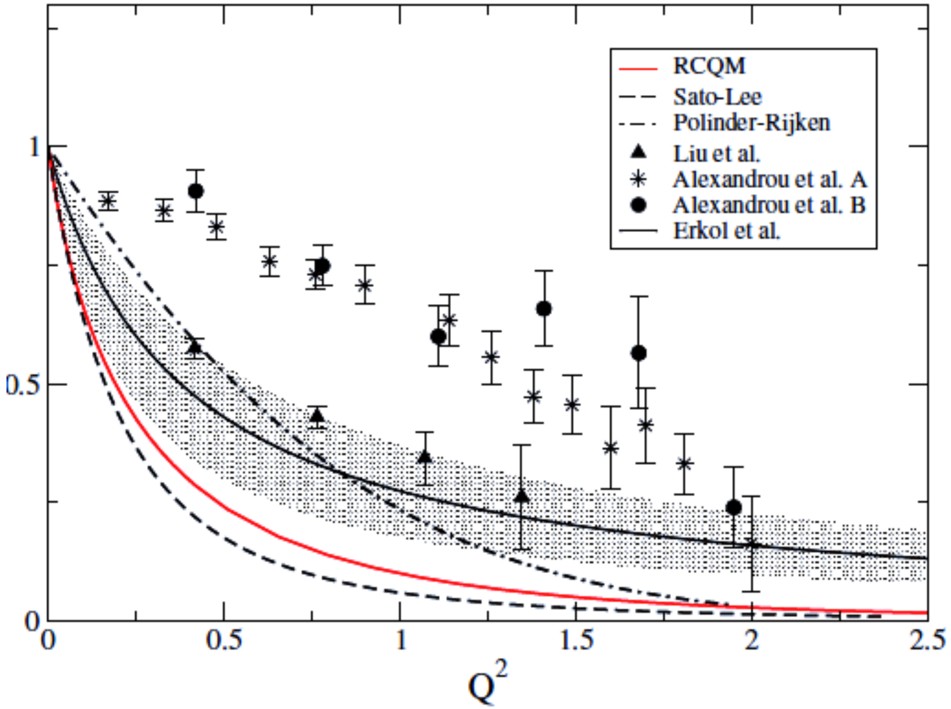




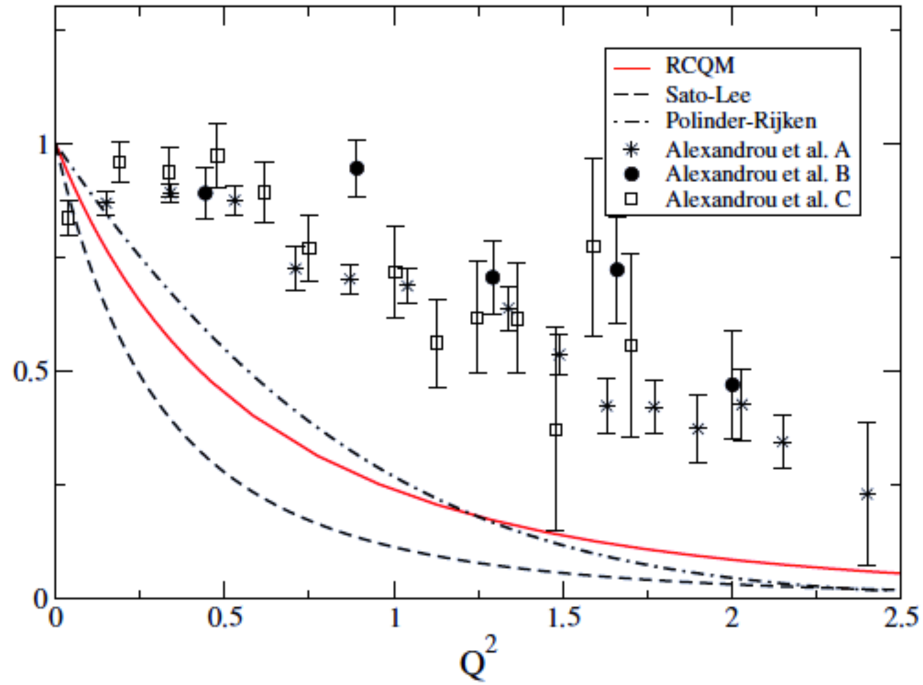
$$g_A^{GBE} = 1.15 \qquad \text{vs.} \qquad g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Meson-Baryon Vertices – Strong FFs



$G_{\pi NN}(Q^2)$

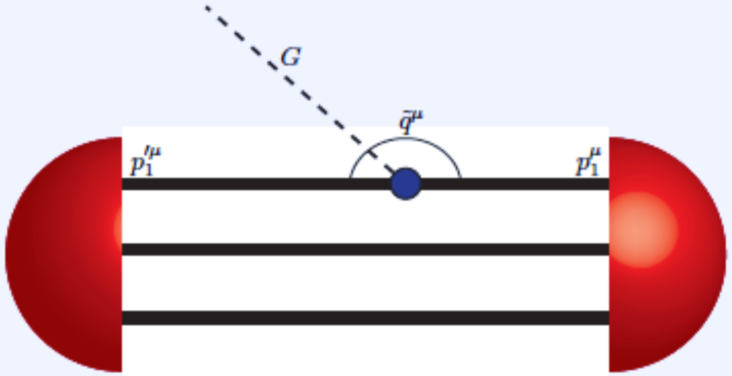


$G_{\pi N\Delta}(Q^2)$

First genuine microscopic predictions of the πNN and $\pi N\Delta$ strong-interaction vertices from the realtivistic Goldstone-boson-exchange constituent quark model

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. 102 (2009) 132002

Gravitational Nucleon Form Factors

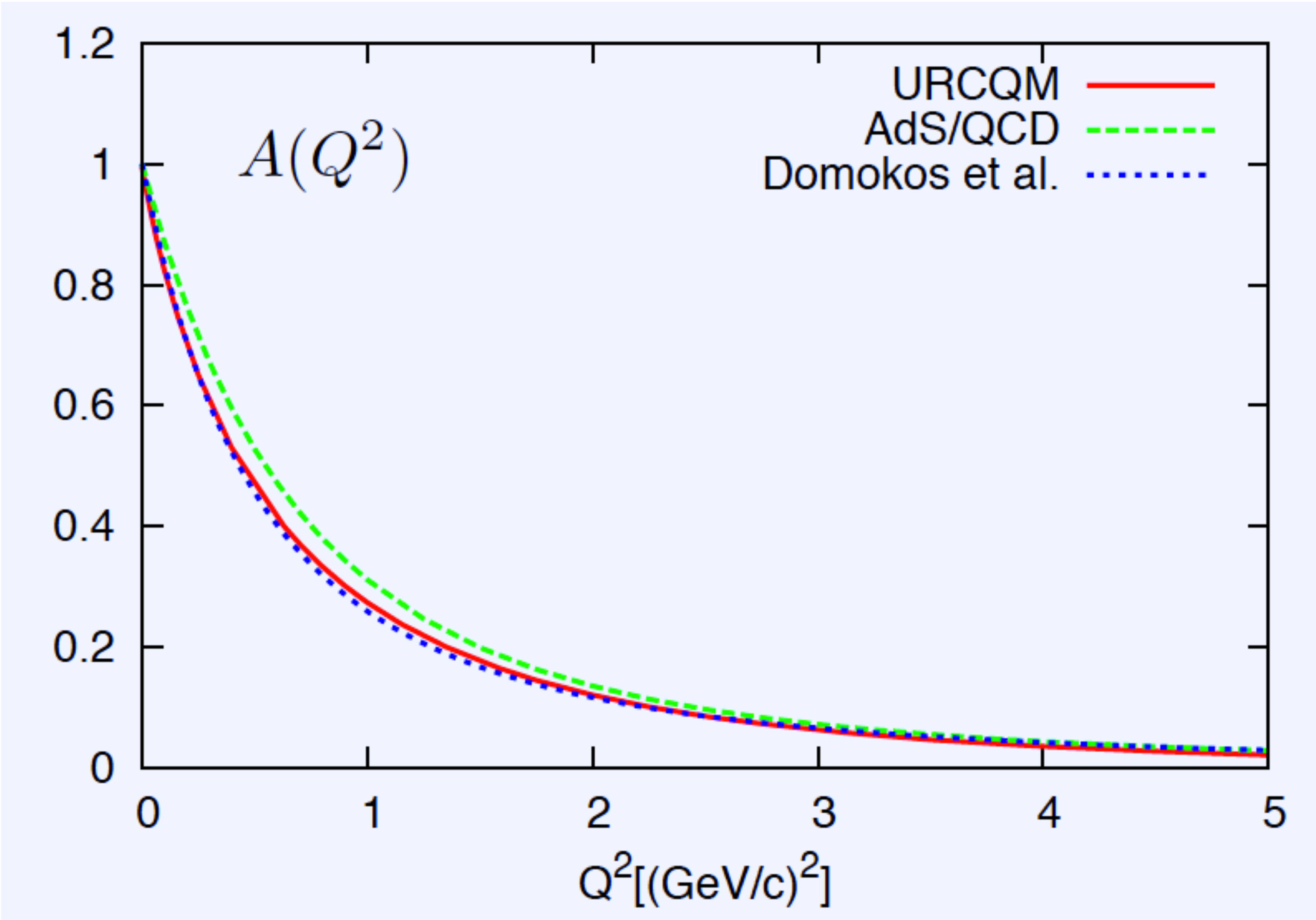


Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P' J \Sigma' | \hat{\Theta}^{\mu\nu} | P J \Sigma \rangle = \bar{U}(P') \Big[\gamma^{(\mu} \bar{P}^{\nu)} A(Q^2) + \frac{i}{2M} \bar{P}^{(\mu} \sigma^{\nu)} B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} C(Q^2) \Big] U(P)$$

$$A(Q^2) \sim \langle P' J \Sigma' | \Theta^{00} | P J \Sigma \rangle$$

Gravitational Nucleon Form Factor $A(Q^2)$



Hadronic Resonance Decays

$N^*, \Delta^* \to N\pi$	Experiment [MeV]	Relativistic GBE	Relativistic OGE	Nonrel. EEM GBE	Nonrel. EEM OGE
$N(1440)$	$(227 \pm 18)^{+70}_{-59}$	30	59	7	27
$N(1520)$	$(66 \pm 6)^{+\ 9}_{-\ 5}$	21	23	38	37
$N(1535)$	$(67 \pm 15)^{+28}_{-17}$	25	39	559	1183
$N(1650)$	$(109 \pm 26)^{+36}_{-\ 3}$	6.3	9.9	157	352
$N(1675)$	$(68 \pm 8)^{+14}_{-\ 4}$	8.4	10.4	13	16
$N(1700)$	$(10 \pm 5)^{+\ 3}_{-\ 3}$	1.0	1.3	2.2	2.7
$N(1710)$	$(15 \pm 5)^{+30}_{-\ 5}$	19	21	8	6

$N \to N\eta$	Experiment [MeV]	Relativistic GBE	Relativistic OGE	Nonrel. EEM GBE	Nonrel. EEM OGE
$N(1520)$	$(0.28 \pm 0.05)^{+0.03}_{-0.01}$	0.1	0.1	0.04	0.04
$N(1535)$	$(64 \pm 19)^{+\ 28}_{-\ 28}$	27	35	127	236
$N(1650)$	$(10 \pm 5)^{+\ 4}_{-\ 1}$	50	74	283	623
$N(1675)$	$(0 \pm 1.5)^{+\ 0.3}_{-\ 0.1}$	1.5	2.4	1.1	1.8
$N(1700)$	$(0 \pm 1)^{+\ 0.5}_{-\ 0.5}$	0.5	0.9	0.2	0.3
$N(1710)$	$(6 \pm 1)^{+\ 11}_{-\ 4}$	0.02	0.06	2.9	9.3

With theoretical masses

T. Melde, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **72**, 015207 (2005); ibid. **74**, 069901 (2006)

Coupled-Channels RCQM with Explicit Mesons

Coupled-channels mass-operator eigenvalue equation

for π-dressing of a given bare $\{\widetilde{QQQ}\}$ cluster state

$$\begin{pmatrix} M_{\widetilde{QQQ}} & K_{\pi\widetilde{QQQ}} \\ K^{\dagger}_{\pi\widetilde{QQQ}} & M_{\widetilde{QQQ+\pi}} \end{pmatrix} \begin{pmatrix} |\psi_{QQQ}\rangle \\ |\psi_{QQQ+\pi}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{QQQ}\rangle \\ |\psi_{QQQ+\pi}\rangle \end{pmatrix},$$

where $M_{\widetilde{QQQ}}$ is the $\{\widetilde{QQQ}\}$ mass operator with confinement.

After Feshbach elimination of the $|\psi_{QQQ+\pi}\rangle$ channel:

$$[M_{\widetilde{QQQ}} + \underbrace{K_{\pi\widetilde{QQQ}}(m - M_{\widetilde{QQQ+\pi}})^{-1}K^{\dagger}_{\pi\widetilde{QQQ}}}_{V_{opt}}]|\psi_{QQQ}\rangle = m|\psi_{QQQ}\rangle.$$

It is an exact eigenvalue equation for $|\psi_{QQQ}\rangle$, yielding in general a complex eigenvalue m of the π-dressed $\{QQQ\}$ system.

π-Dressing Effects on N and Δ

Predictions of the **CC RCQM**

	CC	RCQM	SL	KNLS	PR Gauss	PR Multipole
$\frac{f^2_{\pi\tilde{N}\tilde{N}}}{4\pi}$	0.071	0.0691	0.08	0.08	0.013	0.013
m_N	939	939	939	939	939	939
$m_{\tilde{N}}$	1096	1067	1031	1037	1025	1051
$m_N - m_{\tilde{N}}$	-157	-128	-92	-98	-86	-112

	CC	RCQM	SL	KNLS	PR Gauss	PR Multipole
$\frac{f^2_{\pi\tilde{N}\tilde{\Delta}}}{4\pi}$	0.239	0.188	0.334	0.126	0.167	0.167
m_N	939	939	939	939	939	939
$Re[m_\Delta]$	1232	1232	1232	1232	1232	1232
$m_{\tilde{\Delta}}$	1327	1309	1288	1261	1329	1347
$Re[m_\Delta] - m_{\tilde{\Delta}}$	-95	-77	-56	-29	-96	-115
$2\, Im[m_\Delta] = \Gamma$	67	47	64	27	52	52
$\Gamma_{exp}(\Delta \to \pi N)$			~ 117			

(all values in MeV)

Summary and Some Open Problems

- Baryon spectroscopy of ALL flavors can be consistently described in a universal relativistic constituent quark model (URCQM)
- The nonrelativistic constituent quark model must be discarded
- The covariant structures of the baryon ground states (N, Δ, Λ, Ξ, … form factors) at low momentum transfers result in agreement with experimental observables
- Beyond that the results agree with (reliable) lattice QCD data
- Strong baryon resonance decays fail with $\{QQQ\}$ d.o.f. only
- A realistic description of hadron resonances still represents a formidable challenge (for all QCD-based approaches)
- Inclusion of explicit meson d.o.f. can be achieved in a coupled-channels relativistic constituent quark model
- Calculation of baryon properties in a medium will be an interesting task

The End

Thank you very much for your attention!