Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron.

Sándor Varró
Extrem Light Infrastructure, ERIC,
ALPS Facility, Szeged, Hungary.

Varró S, Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron. [Invited talk presented at PP2024. Particles & Plasmas Symposium 2024. 10-12 June 2024, Budapest, Hungary.] [File:"Varro_S_X17_PP2024_11_June_11h45.ppt"]

Outline.

On the ATOMKI anomaly. Measured invariant masses of the hypothetical X17 and E38 paticles.

Quantized Volkov states applied to protons and quarks. Outline of the derivation of the mass formulae for the hypothetical X17 and E38 particles.

„Statistical size" of the proton (neutron). Connection with the measured invariant mass of the X17 particle (E38 particle).

[„Fifth force interpretation"; vector bosons. „Mass generation" of electromagnetic radiation. Emission and propagation of „massive photons". Constancy of the speed of light.]

On the ATOMKI anomaly; Measured invariant mass of the hypothetical „X17 particle”. The case of the „E38 paticle”.

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Since the first publications ~2016, new and improved experimental results (2021-2023) are also pointing towards the existence of the X17.

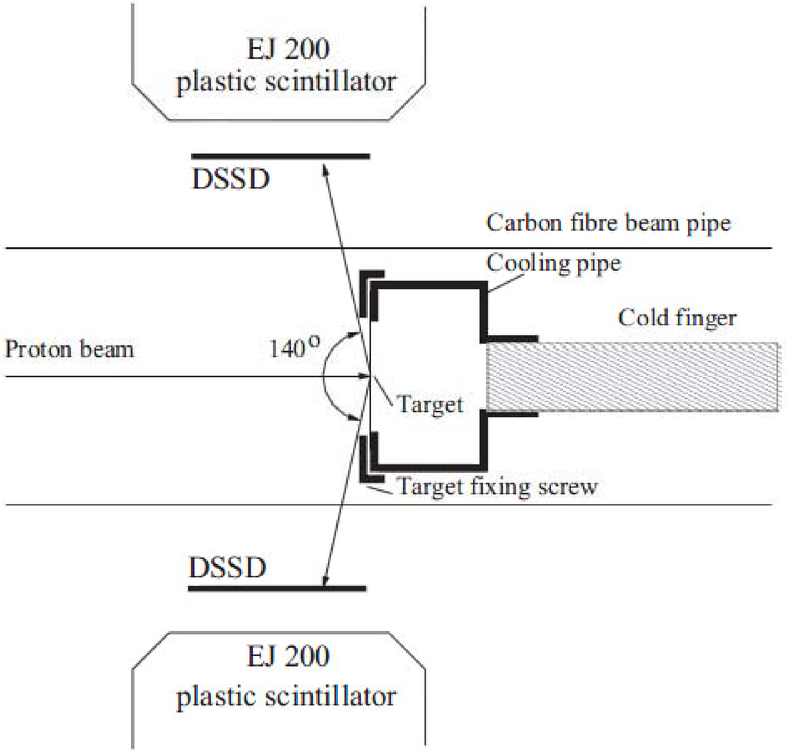


FIG. 1. Schematical drawing of the target cooling system.

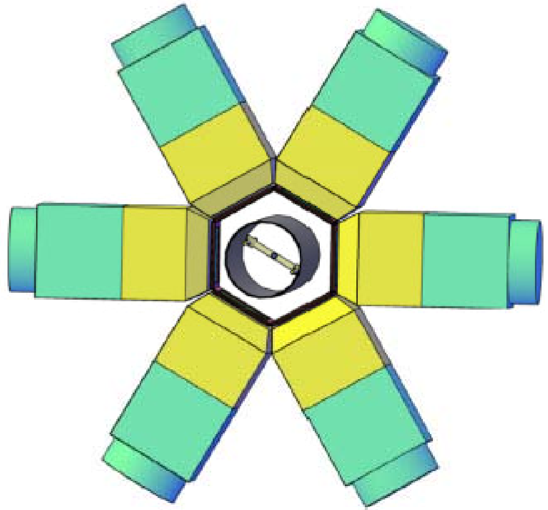
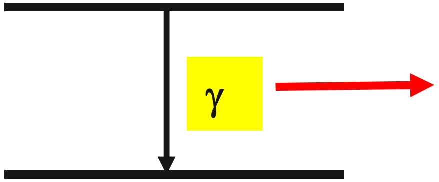


FIG. 2. CAD drawing of the e^+e^- spectrometer. The target (black/blue spot in the center of the figure) is evaporated onto 10 μm Al strip foil spanned between 3 mm thick Perspex rods to minimize the scattering and external pair creation in the vicinity of the target. The beam pipe is shown in black around which the DSSD detectors are arranged. The scintillators are shown in yellow while their light guides are in green. The PMT tubes are not shown.

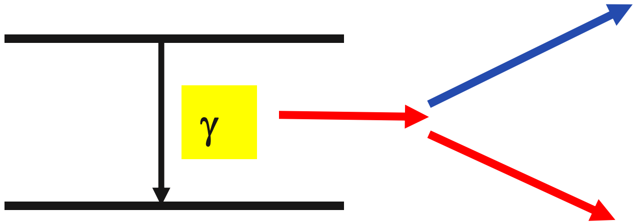
Figures copied from: [1e] Krasznahorkay A J, Csatlós M, Csige L, Gulyás J, Krasznahorkay A, Nyakó B M, Rajta I, Timár J, Vajda I and Sas N J, New anomaly observed in 4He supports the existence of the hypothetical X17 particle. Phys. Rev. C 104, 044003 (2021).

The 7Li(p,γ)8Be Experiment. Anomaly in IPCC (Internal Pair Creation Coefficient). The angular correlation anomaly corresponds to an invariant mass ~ 17 MeV / c^2.

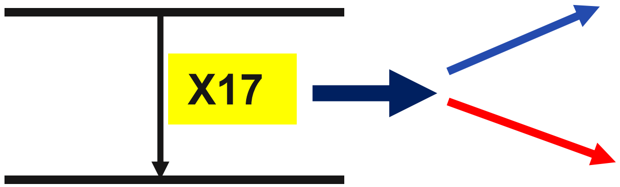
Excited Be decay:



Spontaneous emission of gamma radiation.



Gamma decays to electron – positron pair.



„X17" decays to electron – positron pair.

From the conservation of energy and momentum:

$$E_+ + E_- = E_X$$

$$\boldsymbol{p}_+ + \boldsymbol{p}_- = \boldsymbol{p}_X$$

assuming: $(m_e << m_X)$

$$\cos\Theta = 1 - \frac{(m_X c^2)^2}{E_+ E_-}$$

(Θ is the angle between p$_+$ and p$_-$)

Krasznahorkay, A. J.; Csatlós, M.; Csige, L.; Firak, D. S.; Gulyás, J.; Nagy, Á.; Sas, N. J.; Timár, J.; Tornyi, T. G.; Krasznahorkay, A. On the X(17) light-particle candidate observed in nuclear transitions. *Act. Phys. Polon. B* 2019, *50*, 675-684.

The 7Li(p,γ)8Be Experiment. Anomaly in IPCC (Internal Pair Creation Coefficient).
The angular correlation anomaly corresponds to an invariant mass ~ 17 MeV / c^2.

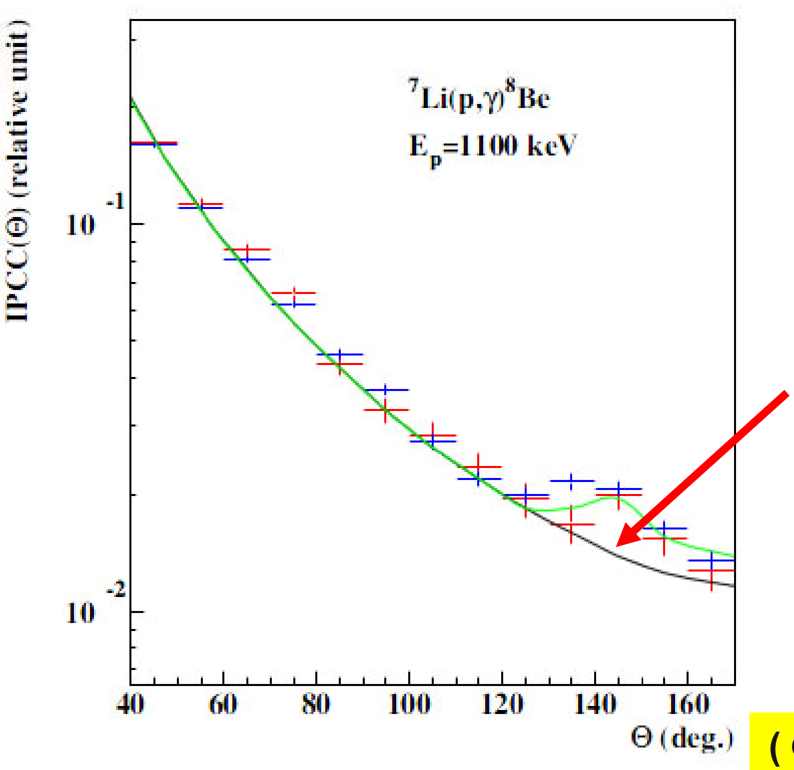


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Figure 3. *Measured angular correlations published previously [3] (blue) and the present results (red) of the e^+e^- pairs originated from the decay of the 18.15 MeV ground state transition in ^{8}Be. The black line represents the background, while the green one is the sum of the signal and background.*

Krasznahorkay, A. J.; Csatlós, M.; Csige, L.; Firak, D. S.; Gulyás, J.; Nagy, Á.; Sas, N. J.; Timár, J.; Tornyi, T. G.; Krasznahorkay, A. On the X(17) light-particle candidate observed in nuclear transitions. *Act. Phys. Polon. B* 2019, *50*, 675-684.

New experiments are also pointing to the existence of the X17 vector boson. Considerations on the fifth force explaining the ATOMKI anomaly...

PHYSICAL REVIEW C **106**, L061601 (2022)

Letter

New anomaly observed in ^{12}C supports the existence and the vector character of the hypothetical X17 boson

A. J. Krasznahorkay,* A. Krasznahorkay,† M. Begala, M. Csatlós, L. Csige, J. Gulyás, A. Krakó, J. Timár, I. Rajta, and I. Vajda
Institute for Nuclear Research (ATOMKI), P.O. Box 51, H-4001 Debrecen, Hungary

N. J. Sas
University of Debrecen, Faculty of Science and Technology, Physics Institute, 4010 Debrecen, PO Box 105, Hungary

PHYSICAL REVIEW D **102**, 036016 (2020)

Dynamical evidence for a fifth force explanation of the ATOMKI nuclear anomalies

Jonathan L. Feng,* Tim M. P. Tait,† and Christopher B. Verhaaren‡
Department of Physics and Astronomy, University of California, Irvine, California 92697-4575, USA

Photon pair spectra around the invariant mass 38 MeV / c^2.

EPJ Web of Conferences **204**, 08004 (2019) https://doi.org/10.1051/epjconf/201920408004
Baldin ISHEPP XXIV

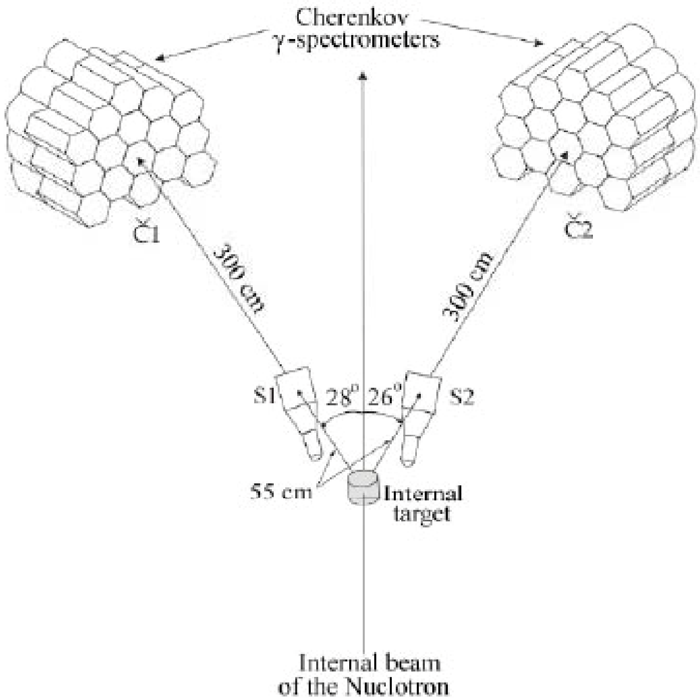


Figure 1. The schematic drawing of the experimental PHOTON-2 setup. The S1 and S2 are scintillation counters

[2] Abraamyan K, Austin C, Baznat M, Gudima K, Kozhin M, Reznikov S and Sorin A, Check of the structure in photon pairs spectra at the invariant mass of about 38 MeV/c2. EJP Web of Conferences 204, 08004 (2019).

Invariant mass distribution of γ γ pairs from decay of E38; d + Cu events

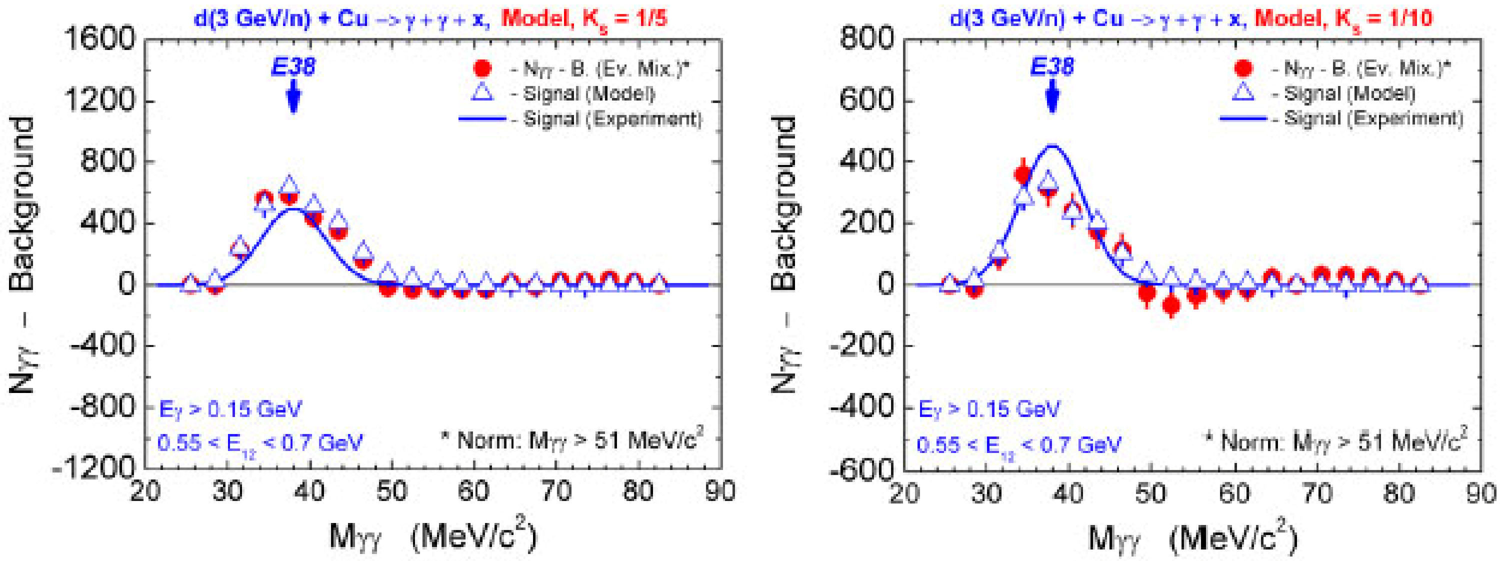


Figure 6. Invariant mass distributions (per 3 MeV/c^2) of $\gamma\gamma$ pairs from the decay of E(38) under the real experimental conditions: △ - pairs $\gamma\gamma$, selected by means of labels of photons applied in the model; • - the result of processing the simulated data with the method applied to the experimental data. The curves are the experimental signal after the background subtraction (Gaussian approximation) proportional to the number of d + Cu events containing a pair of $\gamma\gamma$ in the required energy range. The signal in the simulated data reduced by a factor $K_S = 1/5$ (among the events, containing a photon from the decay of E(38), each 5th event was taken) (left figure) and by a factor $K_S = 1/10$ (right figure)

[2] Abraamyan K, Austin C, Baznat M, Gudima K, Kozhin M, Reznikov S and Sorin A, Check of the structure in photon pairs spectra at the invariant mass of about 38 MeV/c2. EJP Web of Conferences 204, 08004 (2019).

Ch-Y Wong, QED meson description of the anomalouos particles and the X17 particle.

Article

QED Meson Description of the Anomalous Particles at ~17 and ~38 MeV †

Cheuk-Yin Wong
Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA; wongc@ornl.gov

† Based on a talk presented at the 52nd International SymposiumonMultiparticle Dynamics, Gyongyos, Hungary, 20–26 August 2023;

Abstract: The Schwinger confinement mechanism stipulates that a massless fermion and a massless antifermion are confined as a massive boson when they interact in the Abelian QED interaction in (1+1)D.If we approximate light quarks as massless and apply the Schwinger confinement mechanism to quarks, we can infer that a light quark and a light antiquark interacting in the Abelian QED interaction are confined as a QED meson in (1+1)D. Similarly, a light quark and a light antiquark interacting in the QCD interaction in the quasi-Abelian approximation will be confined as a QCD meson in (1+1)D. The QED and QCD mesons in (1+1)D can represent physical mesons in (3+1)D when the flux tube radius is properly taken into account. Such a theory leads to a reasonable description of the masses of *π0, η, and η' , and its extrapolation to the unknown QED sector yields an isoscalar* QED meson at about 17 MeV and an isovector QED meson at about 38 MeV. The observations of the anomalous soft photons, the hypothetical X17 particle, and the hypothetical E38 particle bear promising evidence for the possible existence of the QED mesons. Pending further confirmation, they hold important implications on the properties on the quarks and their interactions.

[4b] Wong Ch-Y, QED meson description of the anomalous particles and the X17 particle. ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics, Aug 21-25 2023 - Gyöngyös, Hungary. https://indico.cern.ch/event/1258038/timetable/#20230822.detailed

Varró S, Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron. [Invited talk presented at PP2024. Particles & Plasmas Symposium 2024. 10-12 June 2024, Budapest, Hungary.] [File:"Varro_S_X17_PP2024_11_June_11h45.ppt"]

Semi-classical and quantized Volkov states.

[16] Varró, S. Theoretical study of the interaction of free electrons with intense light. (Ph.D. dissertation, University of Szeged, 1981). *Hung. Phys. J.* 1983, *31*, 399-454. [17] Bergou, J.; Varró, S. Nonlinear scattering processes in the presence of a quantized radiation field: I. Nonrelativistic treatment. *J. Phys. A Math. Gen.* 1981, *14*, 1469–1482. [18] Bergou, J.; Varró, S. Nonlinear scattering processes in the presence of a quantized radiation field: II. Relativistic treatment. *J. Phys. A Math. Gen.* 1981, *14*, 2281–2303. [21] Varró, S. Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8*, 269.

Terminology; Volkov states [1935]. Exact solutions of the Dirac equation of a charged particle interacting with a *Classical Plane Electromagnetic Field*.

$$[\gamma_\mu (i\partial - \varepsilon A)^\mu - \kappa]\left|\Psi\right\rangle = 0$$

$$\varepsilon \equiv e/\hbar c$$

$$\kappa \equiv mc/\hbar$$

$$A(\xi) = e_x A_0 f(\xi)$$

$$\xi = k_\mu x^\mu = \omega(t - z/c)$$

$$\Psi_{ps}^{(\pm)}(x) = \left[1 \pm \frac{\varepsilon \not{k} \not{A}(\xi)}{2k \cdot p}\right] u_{ps}^{(\pm)} \exp(\mp i S_p^{(\pm)})$$

$$S_p^{(+)}(x) = p \cdot x + \int I_p^{(\pm)}(\xi) d\xi$$

$$I_p^{(\pm)}(\xi) = (1/2k \cdot p)[\pm 2\varepsilon p \cdot A(\xi) - \varepsilon^2 A^2(\xi)$$

Wolkow D M, Über eine Klasse von Lösungen der Diracschen Gleichung. *Zeitschrift für Physik* 94, 250-260 (1935). [Application to strong-field and multiphoton processes: from ~1960...]

Exact solutions of the Dirac equation in a quantized plane wave. Nonlinear Compton scattering (HHG) <u>beyond the semiclassical description</u> (1981). The generalization of the Klein–Nishina formula. [The effect of depletion of the laser field; e.g. altered kinematics (spectrum) .]

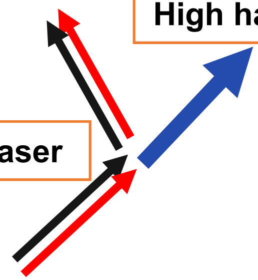
The calculation of the nonlinear Compton process was based on the
Exact solutions for the 'Dirac electron + quantized e.m. radiation mode' system [1-3]:

$$\psi_{E,\boldsymbol{P}} = \left[1 + g\frac{\not{k}\not{\varepsilon}}{2Qk}\left(\hat{a}\mathrm{e}^{ik\cdot r} + \hat{a}^{+}\mathrm{e}^{-ik\cdot r}\right)\right]u_Q e^{i[\boldsymbol{Q}+kg(n)]\cdot\boldsymbol{r}}\,\hat{S}_\theta \hat{D}_\tau \left|n\right\rangle$$

$$\omega'_n = \frac{n\omega_0 + \omega_C \mu_0^2 \Delta}{1 + \left(2\frac{n\omega_0}{\omega_C} + \frac{\mu_0^2}{2}\right)\sin^2\frac{\theta}{2}}$$

$$\Delta = \frac{n_0 - n}{n_0}$$

‚depletion factor' [1-3]



The simplest generalization of the Klein–Nishina formula (complete depletion):

$$|t_{fi}^{(n)}|_{av}^2 = \frac{1}{4}\left[\frac{n\omega_0}{\omega'} + \frac{\omega'}{n\omega_0} - 2 + 4(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}')^2\right]\frac{(nb)^n}{n!}e^{-nb}$$

$$b = \tfrac{1}{2}\mu_0^2\,|\hat{\boldsymbol{k}}'\cdot\boldsymbol{\varepsilon}|^2$$

[1] S. Varró, Theoretical study of the interaction of free electrons with intense light. (Ph.D. dissertation, 1981). (in Hungarian) Hungarian Physical Journal XXXI, 399-454 (1983). [2] J. Bergou, S. Varró:J. Phys. A: Math. Gen. 14, l469-l482 (1981) . [3] J. B. S. V., Nonlinear scattering processes in the presence of a quantised radiation field: II. Relativistic treatment. *Ibid.* 14, 2281 (1981).

The matrix elements of the squeezing operator between photon number eigenstates. Expression in terms of classical Gegenbauer polynomials.

$$S = \exp(\tfrac{1}{2}\xi a^{+2} - \tfrac{1}{2}\xi^* a^2) \qquad H_{\text{int}} = g(a^{+2}\mathrm{e}^{i\varphi} + a^2\mathrm{e}^{-i\varphi})$$

$$\langle m|S(\xi)|n\rangle = \mathrm{e}^{-\eta/4}(-2\mathrm{e}^{-i\varphi}\tanh|\xi|)^{\lambda}\frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\frac{1}{2})}\sqrt{\frac{m!}{n!}}C_m^{(\lambda+\frac{1}{2})}\left(\frac{1}{\cosh|\xi|}\right) \qquad m \le n, \quad \lambda = \tfrac{1}{2}(n-m)$$

$$\langle m|S(\xi)|n\rangle = \mathrm{e}^{-\eta/4}(2\mathrm{e}^{i\varphi}\tanh|\xi|)^{\alpha}\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\frac{1}{2})}\sqrt{\frac{n!}{m!}}C_n^{(\alpha+\frac{1}{2})}\left(\frac{1}{\cosh|\xi|}\right) \qquad m \ge n, \quad \alpha = \tfrac{1}{2}(m-n)$$

Remark: SU(1,1); SL(2,R) group. Lorentz group in 2 + 1 dimensions (osc. repr.).

$$K_+ = \tfrac{1}{2}(a^+)^2, \quad K_- = \tfrac{1}{2}(a)^2, \quad K_0 = \tfrac{1}{4}(aa^+ + a^+a), \quad [K_0, K_\pm] = \pm K_\pm \quad [K_-, K_+] = 2K_0$$

General transformation:

$$U(g) = \exp(-itK_0)S(\xi), \qquad U(g_1)U(g_2) = U(g), \qquad g = g_1 \circ g_2$$

Varró S, Coherent and incoherent superposition of transition matrix elements of the squeezing operator. *New Journal of Physics* 24, 053035 (2021). *E-print*: arXiv: 2112.08430 [quant-ph] . 29th Int. Las. Phys. (LPHYS'21). J. Phys. Conf. Ser. 2249 012013 (2022).

Quantized Volkov states applied to protons and quarks.

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Derivation of the electromagnetic mass formula on the basis of quantized Volkov states. Contributions of the sub-systems to the energy and momentum.

Stationary states of the system:

$$\psi_{E,\boldsymbol{P}} = \exp[-i\boldsymbol{k}\cdot\boldsymbol{r}(a_1^+ a_1 + a_2^+ a_2 + 1)]\left\{1 + g\frac{\not k}{2(k\cdot p)}[\not\epsilon_1(a_1 + a_1^+) + \not\epsilon_2(a_2 + a_2^+)]\right\}u_p$$

$$\times S_1(\theta)S_2(\theta)D_1(\tau_1)D_2(\tau_2)\left|n_1, n_2\right\rangle \exp\{i[\boldsymbol{p} + g_{\boldsymbol{p}}(n)\boldsymbol{k}]\cdot\boldsymbol{r}\}$$

Parameters (p is on the free mass shell):

$$\tau_{1,2} = p_{x,z}\, g/(k\cdot p) \qquad \mathrm{e}^{2\theta} = \sqrt{1+2b} \qquad b = g^2/(k\cdot p) \qquad g_p(n) = \sqrt{1+2b}(n+1-\tau^2)$$

Total energy :

$$E/\hbar c = K_0 = \bar{p}_0 + \sqrt{1+2b}\,k_0(n_1 + n_2 + 1)$$

$$\bar{k}_0$$

$$\bar{k}_0^2 - |\boldsymbol{k}|^2 = 2bk_0^2$$

Total momentum:

$$\boldsymbol{P}/\hbar = \boldsymbol{K} = \bar{\boldsymbol{p}} + \boldsymbol{k}(n_1 + n_2 + 1)$$

„Massive Photon”.

~ (mass)² term of the radiation subsystem

Varró S, Proposal for an electromagnetic mass formula for the X17 particle. [Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.] This article belongs to the Special Issue “Multiparticle Dynamics”, Edited by Csörgő T, Csanád M and Novák T; https://www.mdpi.com/journal/universe/special_issues/3R2XYDMCV9]. Varró S, Proposal for an electromagnetic mass formula for the X17 particle. *Universe* 2024, *10*, 86 (2024). [https://doi.org/10.3390/universe10020086].

Derivation of the electromagnetic mass formula from
Summing up with respect to the charged particle's quantum numbers.

$$\bar{k}_0^2 - |\boldsymbol{k}|^2 = 2bk_0^2$$

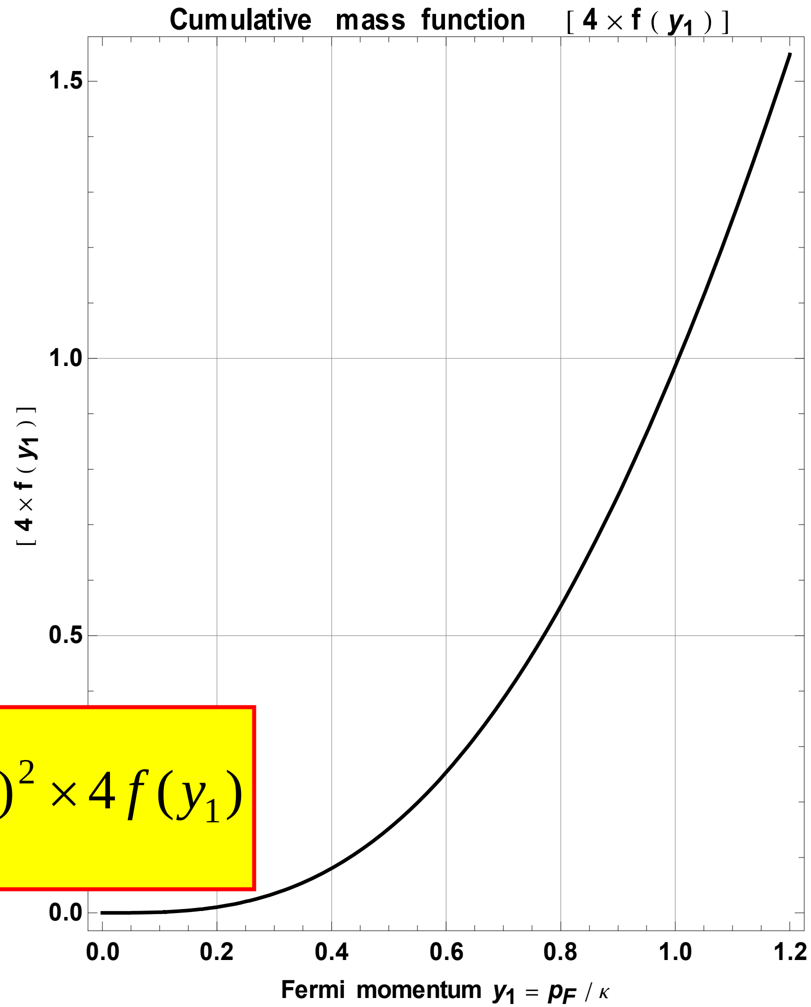
The mass term $2b(k_0)^2$ does not depend on the frequency ω, and ('of course') it is the same for any occupation numbers n_1, n_2 = 0, 1, 2, ... ('multiplicity') of the dressed radiation quanta.

$$2bk_0^2 = \frac{e^2}{\hbar c}\frac{4\pi}{V}\frac{1}{p_0 - p_z}$$

$$p_0 = \sqrt{\boldsymbol{p}^2 + \kappa^2}$$

$$\boldsymbol{y} := \boldsymbol{p}/\kappa \qquad y := |\boldsymbol{y}|$$

$$\hbar^2 c^2 \times \sum_{spin,\ momentum} \frac{\kappa}{p_0} 2bk_0^2 = \frac{1}{\pi}\frac{e^2}{\hbar c}(mc^2)^2 \times 4f(y_1)$$



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Determination of the Fermi momentum from the normalization: One particle in the 'interaction sphere' of radius r_s.

Particle density is obtained by integrating the usual (Lorentz invariant) phase-space density with respect to the momentum parameter, and summing up with respect to the spin.

$$n = (\kappa^3/\pi^2) g(y_1)$$

$$g(y) = \tfrac{1}{2}[y\sqrt{1+y^2} - \log(y + \sqrt{1+y^2})]$$

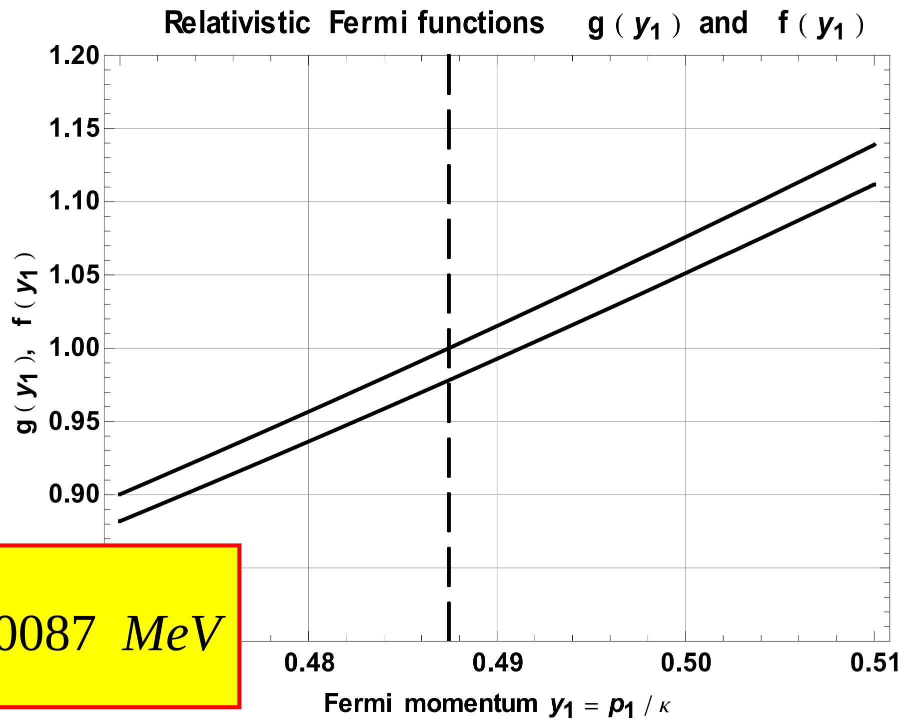
[We have plot the functions multiplied by 27.6560 !]

Normalization to one particle in the interaction sphere:

$$V = \frac{4\pi}{3} r_s^3$$

$$\frac{4(\kappa r_s)^3}{3\pi} g(y_1) = 1$$

We have taken $r_s = 4/\kappa$. From the normalization condition we have determined the Fermi momentum, y_1 = 0.487446, which we put to the mass formula for the X17:



$$m_X c^2 = \sqrt{\frac{e^2}{\pi\hbar c}}(m_p c^2)\sqrt{4 f(y_1)} = 17.0087 \ \ MeV$$

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Derivation of the electromagnetic mass formula for X17 on the basis of quantized Volkov states. Sum over the charged particle's momentum and spin.

$$(m'_\gamma c^2)^2 = \hbar^2 c^2 \times 2\sum \Delta n_x \Delta n_y \Delta n_z \frac{\kappa}{q_0} k_0^2 2b = \frac{1}{\pi}\frac{e^2}{\hbar c}(mc^2)^2 \times 4 f(y_1)$$

$y_1 = p_F / \kappa$ is the Fermi momentum, and f(y) is given as:

$$\sqrt{\frac{1}{\pi}\frac{e^2}{\hbar c}}(m_p c^2) = 45.2206 \;\; MeV$$

$$f(y) = \sqrt{1+y^2}\log(\sqrt{1+y^2}+y) - y$$

Normalization: one particle in the interaction sphere:

$$V = \frac{4\pi}{3} r_s^3$$

$$\frac{4(\kappa r_s)^3}{3\pi} g(y_1) = 1$$

Here we have taken m_p as the proton mass. This is 'Wentzel-type' mass term, though we did not consider vacuum polarization. Rather, we have taken a real positive energy charge.

Particle density:

$$n = (\kappa^3/\pi^2) g(y_1)$$

$$g(y) = \tfrac{1}{2}[y\sqrt{1+y^2} - \log(y+\sqrt{1+y^2})]$$

From the normalization condition (see previous and right hand side of this dia) we have determined the Fermi momentum, y_1 = 0.487446. In the non-relativistic approximation, f (y) = y3 / 3 and g (y) = y^3 / 3, and then the proposed mass formula for the X17 comes out:

$$m_X c^2 = \frac{2}{3\pi}\sqrt{\frac{e^2}{\hbar c}}(m_p c^2) = 17.0087 \;\; MeV$$

The Compton frequency of the X17 field is completely analogous to the classical plasma frequency (f/g=0.98):

$$\omega_X^2 = \frac{4\pi e^2 n_p}{m_p} \times \frac{f(y_1)}{g(y_1)}$$

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Wentzel's deduction of the 'non-vanishing self-energy of the photon' (1948).

PHYSICAL REVIEW VOLUME 74, NUMBER 9 NOVEMBER 1, 1948

New Aspects of the Photon Self-Energy Problem

GREGOR WENTZEL
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received June 29, 1948)

A finite but non-vanishing value for the self-energy of the photon, corresponding to a finite rest-mass, can be deduced from the new invariant formulation of quantum electrodynamics developed by Tomonaga and Schwinger, in the e^2 order approximation. The implications of this result are discussed.

state considered, divided by κ_0^2. Therefore, a photon of momentum $\hbar\boldsymbol{\kappa}$, having the total energy $\hbar c\kappa_0 = \hbar c|\boldsymbol{\kappa}|$ in the zero-order approximation, appears in the second-order approximation as having the energy

$$\hbar c\kappa_0\left(1+\frac{1}{8\pi^2}\frac{e^2}{\hbar c}\frac{\mu^2}{\kappa_0^2}+\cdots\right) = c\left[(\hbar|\boldsymbol{\kappa}|)^2+\frac{1}{4\pi^2}\frac{e^2}{\hbar c}(mc)^2\right]^{\frac{1}{2}}. \quad (25)$$

This corresponds to a "photon rest-mass" amounting to one electron mass divided by

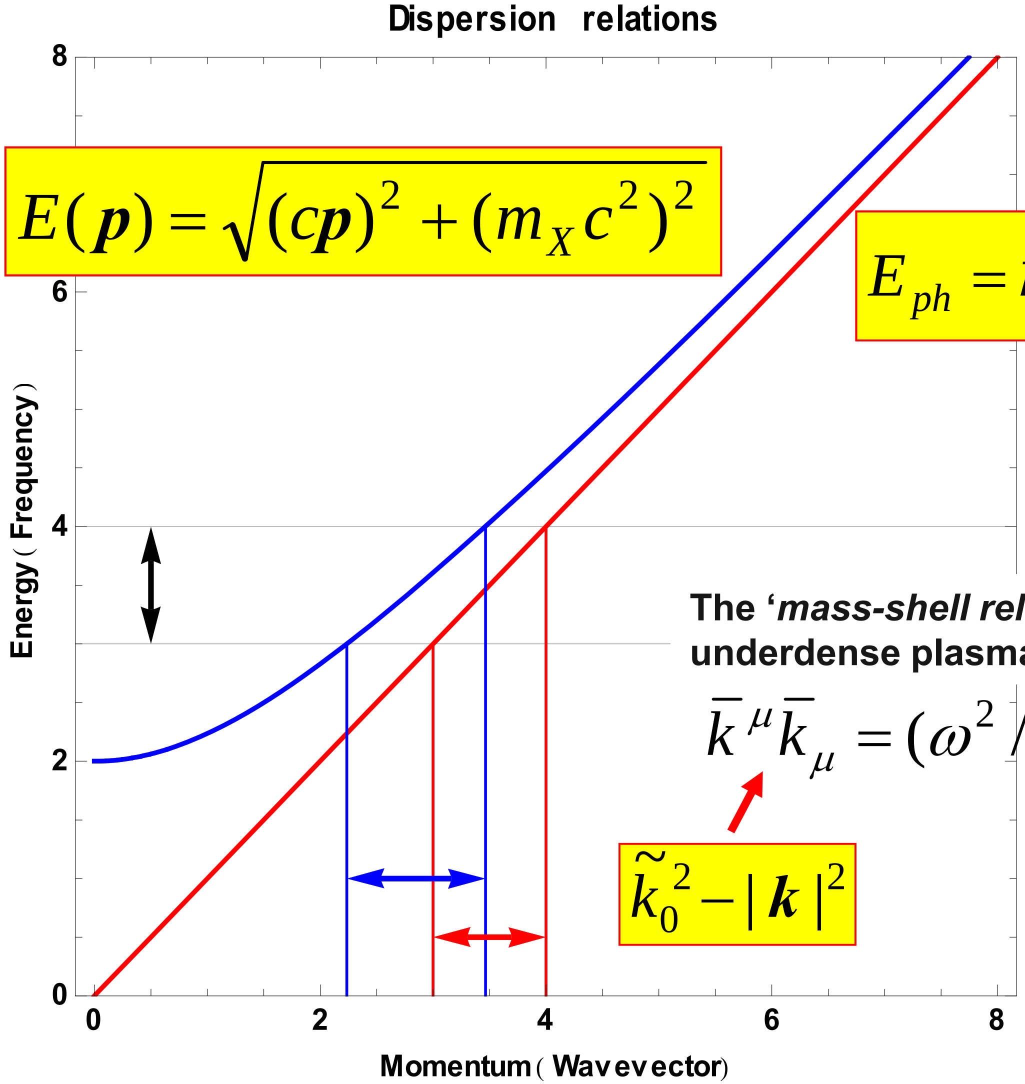
"CONCLUSION. The results of the last section are hardly encouraging in view of higher approximations. We have tried to take the quantum theory of fields seriously, without admitting any ad hoc subtractions inconsistent with the principles of quantum mechanics. The outcome shows that the empirical fact, that the photon has no rest-mass, does not fit naturally into the framework of quantum electrodynamics. It seems questionable to what extent the predictions of such a theory in higher order effects are trustworthy.
Finally, it should be remembered that the pair creation of other charged particles (mesons, protons) is likely to contribute to the photon selfenergy. Therefore, the phenomena involving electrons, positrons, and photons only, can hardly be expected to be quite independent, in the higher order effects, of the existence of other particles and their nature."

Perturbation theory plus an extrapolation:

$$m_\gamma = \sqrt{\frac{e^2}{\pi\hbar c}}m_e = \frac{m_e}{\sqrt{\pi 137}}$$

Wentzel G, New aspects of the photon self-energy problem. Physical Rewiev 74 (1), 1070-1075 (1948).

Dispersion relations, rest mass; electrons, photons, plasmons.



$$E(\boldsymbol{p}) = \sqrt{(c\boldsymbol{p})^2 + (m_X c^2)^2}$$

$$E_{ph} = \hbar c \sqrt{|\boldsymbol{k}_{ph}|^2 + 0^2}$$

The '*mass-shell relation*' of e.m. radiation in an underdense plasma *does not depend on the frequency* !

$$\bar{k}^{\mu}\bar{k}_{\mu} = (\omega^2 / c^2)(1 - n_m^2(\omega)) = (\omega_X^2 / c^2)$$

$$\tilde{k}_0^2 - |\boldsymbol{k}|^2$$

$$\omega(\boldsymbol{k}) = \sqrt{\omega_X^2 + c^2 |\boldsymbol{k}|^2}$$

'rest mass': $m_X = \hbar \omega_X / c^2$

[In the above figure the energy unit is one half of the rest energy. In the formula κ is the Compton wave number.]

Simple plasma and statistical consideration on E38. We take the Neutron = udd. All these results can also be derived from the Dirac equation for a single charge, as above.
[This procedure can be justified the Dirac-Fock-Podolsky many-time formalism (equivalent with the Tomonaga-Schwinger eq.)]

Composit plasma:

$$e \rightarrow e\sqrt{(2/3)^2 + (1/3)^2 + (1/3)^2} = e\sqrt{2/3}$$

Additional statistical factors: 1 singlet, and Sqrt [3] for triplet of the dd quarks.

$$m_E c^2 = (1+\sqrt{3})\frac{2}{3\pi}\sqrt{\frac{\frac{2}{3}e^2}{\hbar c}}(m_n c^2) = 37.9937 \quad MeV$$

Nete that: The difference in comparison to the functional form of the mass formula of the X17 particle is only the factor:

$$(1+\sqrt{3})\sqrt{2/3}$$

Varró S, Proposal for an electromagnetic mass formula for the X17 particle. [Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.] This article belongs to the Special Issue “Multiparticle Dynamics”, Edited by Csörgő T, Csanád M and Novák T; https://www.mdpi.com/journal/universe/special_issues/3R2XYDMCV9]. Varró S, Proposal for an electromagnetic mass formula for the X17 particle. *Universe* 2024, *10*, 86 (2024) [https://doi.org/10.3390/universe10020086].

Newton’s Fifth Rule of Reasoning.

>>> We are to admit no more causes of natural things, than such as are both true and sufficient to explain their appearance.<<<

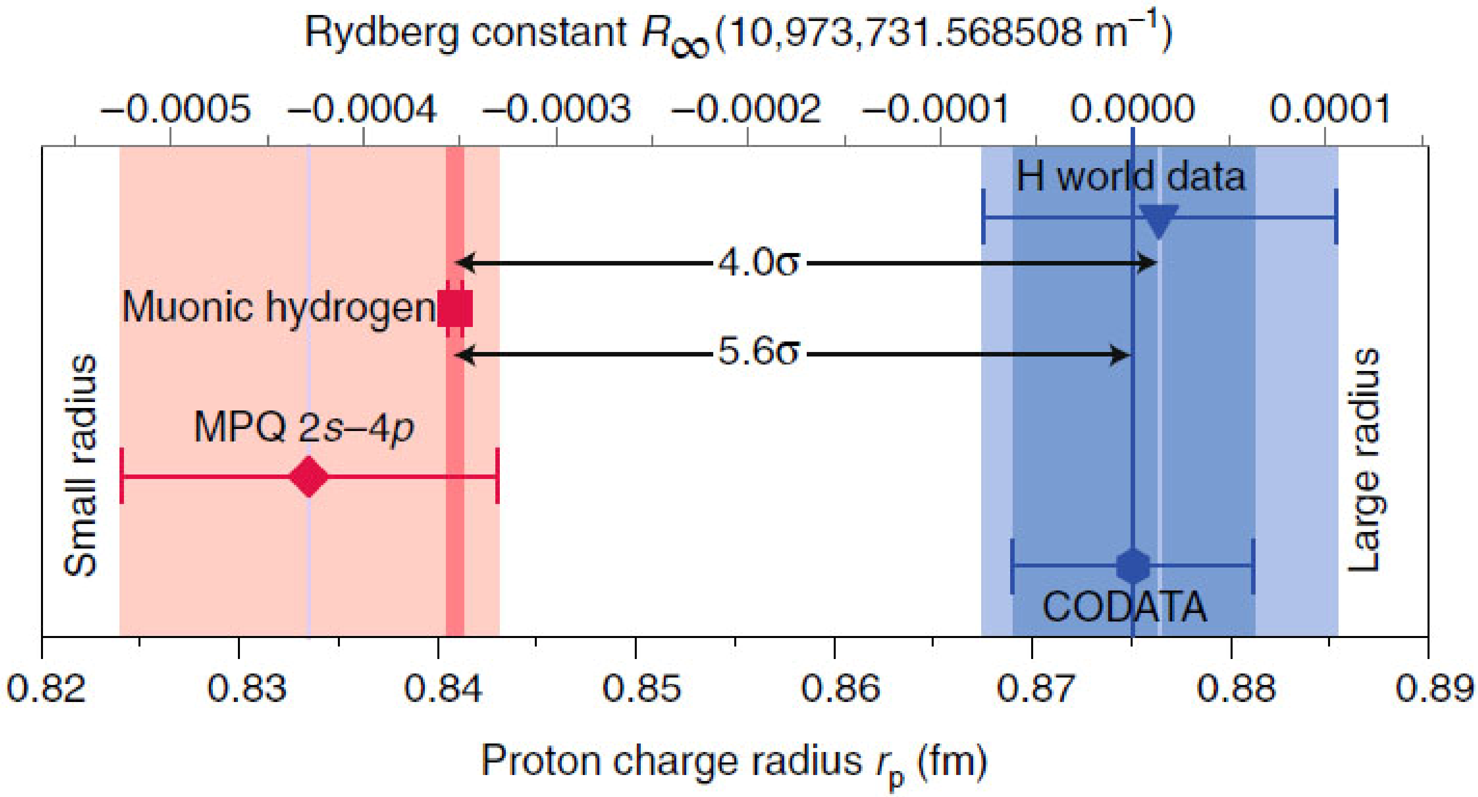
[Occam’s razor.]

Varró S, Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron. [Invited talk presented at PP2024. Particles & Plasmas Symposium 2024. 10-12 June 2024, Budapest, Hungary.] [File:”Varro_S_X17_PP2024_11_June_11h45.ppt”]

Proton size puzzle. „small radius” and „large radius”

$r_p = 0.84087(39)$ fm

$r_p = 0.8764(89)$ fm



Udem Th, Quantum electrodynamics and the proton size. Nature Physics 14, 632 (2018).

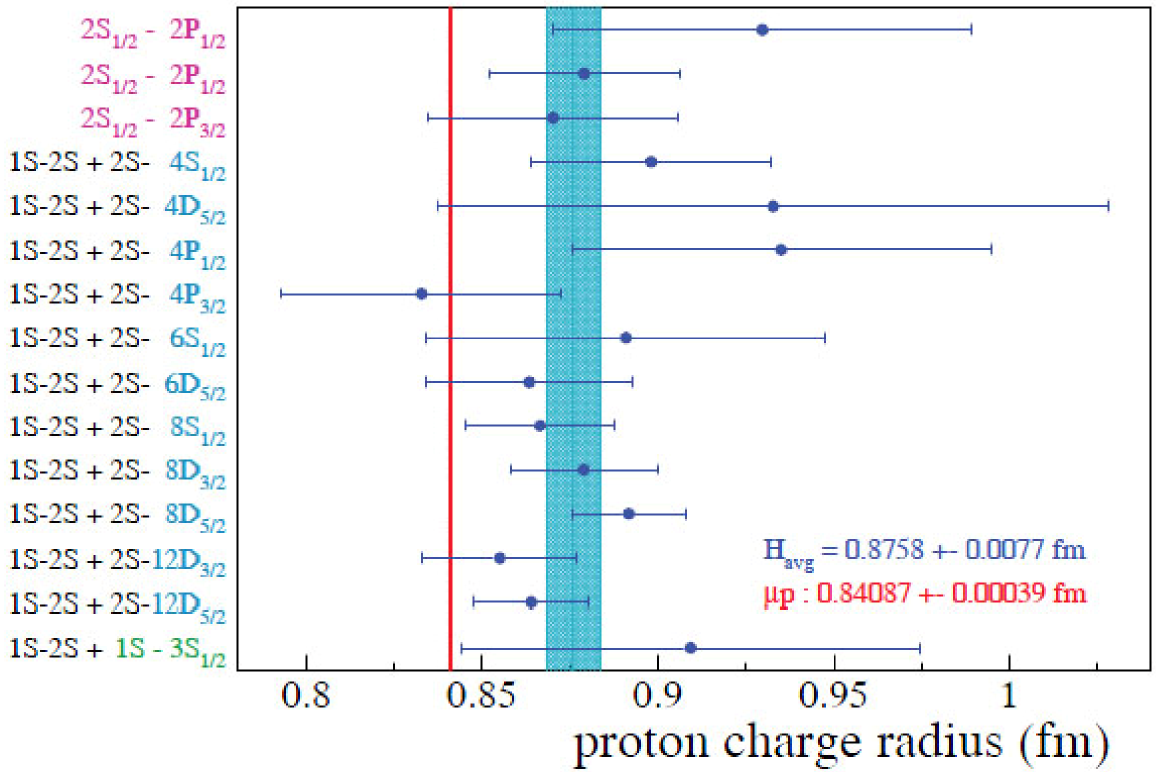


Figure 2: Proton charge radii r_p obtained from hydrogen spectroscopy. According to Eq. (4), r_p can best be extracted from a combination of the 1S-2S transition frequency [25] and one of the 2S-8S,D or 12D transitions [26,27]. The value from muonic hydrogen [1,2] is shown with its error bar.

Pohl R, Antognini A, Nez F, et al., The size of the proton. Nature 466, 213-216 (2010).. Picture copied from: Pohl R et alMuonic hydrogen and the proton radius puzzle (2016)

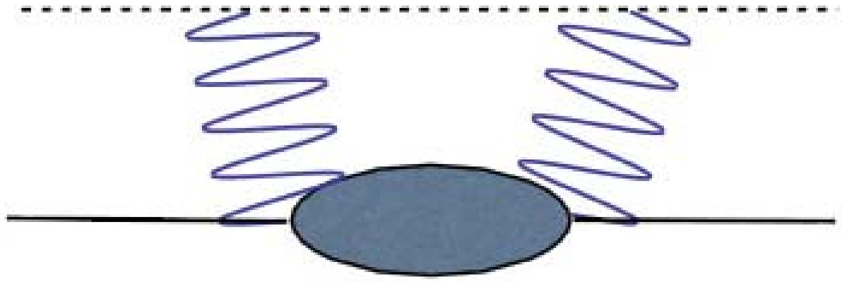


Figure 5: The box diagram for the $\mathcal{O}(\alpha^5 m^4)$ corrections. The graph in which the photons cross is also included in the calculation. The blob represents all possible excitations of the proton, the wiggly lines represent the exchanged photons. The solid line represents the proton and the dashed line represents the muon.

Pohl R, Antognini A, Nez F, et al., The size of the proton. Nature 466, 213-216 (2010).. Picture copied from: Pohl R et alMuonic hydrogen and the proton radius puzzle (2016)

„Statistical proton size”.

$$\frac{4(\kappa r_s)^3}{3\pi} g(y_1) = 1$$

$$r_s = 4/\kappa = \frac{2}{\pi}\lambda_p$$

$$\lambda_p = \frac{h}{m_p c} = 1.32141 \text{ fm}$$

$$r_s = \frac{2}{\pi}\lambda_p = 0.8412356 \text{ fm}$$

$$r_s \to r_s' = r_s \times 1.00602 = 0.846299 \text{ fm}$$

This is quite close to the „smaller” proton charge radius (see references in the Appendix).

According to our above reasonings, one may even say that the precise measurement of the X17's invariant mass supports the „smaller” proton (charge) radius.

Similarly, with the derivation of the E38's invariant mass, a „statistical neutron radius” comes out, for which we have received the value 0.845135 fm.

V. S, Proposal for an electromagnetic mass formula for the X17 particle. *Universe* 2024, *10*, 86 (2024).

Varró S, Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron. [Invited talk presented at PP2024. Particles & Plasmas Symposium 2024. 10-12 June 2024, Budapest, Hungary.] [File:"Varro_S_X17_PP2024_11_June_11h45.ppt"]

Additional note.
[In the context of the so-called „Fifth Force Interpretation”]

Elementary considerations on plasmons: „mass generation” of elelectromagnetic radiation. Emission an propagation of true „massy photons”.

Origin of the dielectric function and index of refraction. Elementary (textbook) considerations.

Displacement vector:

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P} = (1 + 4\pi\chi_e)\boldsymbol{E} = \varepsilon\boldsymbol{E}$$

Induced polarization density:

$$m\ddot{\boldsymbol{r}} = e\boldsymbol{E}_0 e^{-i\omega t} \qquad \boldsymbol{r} = -\frac{e}{m\omega^2}\boldsymbol{E}_0 e^{-i\omega t} \qquad \boldsymbol{P} = e\boldsymbol{r}n_e = -\frac{e^2 n_e}{m\omega^2}\boldsymbol{E}$$

Electric susceptibility, plasma frequency, dielectric function:

$$4\pi\chi_e = -\frac{\omega_p^2}{\omega^2} \qquad \omega_p = \sqrt{\frac{4\pi n_e e^2}{m}} \qquad \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Index of refraction, propagation of e.m. radiation in an underdense plasma:

$$n_m^2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad e^{-i\zeta}, \qquad \zeta = k_0(x_0 - n_m z) = \omega_0(t - n_m z/c)$$

‘Equivalence’ of the Maxwell equations in a plasma medium with the field equations of a massive boson in vacuum; ‘mass’: $\kappa \equiv \omega_p / c$

Maxwell fields

Gauss

$$\nabla \cdot \boldsymbol{D}_\omega = 4\pi\rho_\omega ,$$

No magnetic charge:

$$\nabla \cdot \boldsymbol{B}_\omega = 0,$$

Faraday:

$$\nabla \times \boldsymbol{E}_\omega = -\frac{1}{c}\frac{\partial \boldsymbol{B}_\omega}{\partial t},$$

Ampére:

$$\nabla \times \boldsymbol{B}_\omega = \frac{1}{c}\frac{\partial \boldsymbol{D}_\omega}{\partial t} + \frac{4\pi}{c}\boldsymbol{j}_\omega .$$

Massive vector fields

$$\nabla \cdot \boldsymbol{E} = -\kappa^2 V + 4\pi\rho,$$

$$\nabla \cdot \boldsymbol{B} = 0,$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \kappa^2 \boldsymbol{A} + \frac{4\pi}{c}\boldsymbol{j}.$$

Potentials; **Lorentz gauge;** **Klein-Gordon equation:**

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \quad \boldsymbol{E} = -\partial_0 \boldsymbol{A} - \nabla V, \quad \partial_0 V + \nabla \cdot \boldsymbol{A} = 0, \quad (\partial^2 + \kappa^2) A_\mu = 0$$

Covariant formalism for the massive vector field $\kappa \equiv \omega_p / c$

Lagrange density:

$$\mathscr{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa^2}{8\pi} A_\mu A^\mu - \frac{1}{c} j_\mu A^\mu ,$$

Field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Field equations (Euler-Lagrange equations):

$$\partial^\mu F_{\mu\nu} + \kappa^2 A_\nu = \frac{4\pi}{c} j_\nu , \qquad \partial^\alpha F^{\beta\gamma} + \partial^\gamma F^{\alpha\beta} + \partial^\beta F^{\gamma\alpha} = 0$$

No gauge freedom (only Lorentz gauge) if the current is conserved (continuity).

$$\partial_\mu j^\mu = \frac{c\kappa^2}{4\pi} \partial_\alpha A^\alpha$$ **That is, if** $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$ **, then** $\partial_0 V + \nabla \cdot \mathbf{A} = 0.$

Inhomogeneous Klein-Gordon equation for each components:

$$(\partial^2 + \kappa^2) A_\mu = \frac{4\pi}{c} j_\mu , \qquad \partial^2 = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 .$$

Lánczos C, Die tensoranalytischen Beziehungen der Diracschen Gleichung. *Z. für Physik* **57**, 447 (1929). Proca A, Sur la théorie ondulatoire des électrons positifs et négatifs. *J. Phys. Radium* **7**, 347-353 (1936). See recent review Goldhaber A S and Nieto M N, Photon and graviton mass limits. RMP **82,** 939 (2010).

Radiation modes of the free massive vector field.

Two transverse, and one longitudinal polarization.

$$\omega_k / c = \sqrt{\kappa^2 + \boldsymbol{k}^2}$$

$$\kappa \equiv \omega_p / c$$

$$\boldsymbol{A}^{(r)} = \sum_{\boldsymbol{k}} (2L^3\omega_k)^{-\frac{1}{2}} \left[\left(\sum_{\alpha=1,2} a_{\boldsymbol{k},\alpha} \boldsymbol{e}_{\boldsymbol{k},\alpha} + \frac{c\kappa}{\omega_k} a_{\boldsymbol{k},3} \boldsymbol{e}_{\boldsymbol{k},3} \right) e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + c.c. \right]$$

Energy density, energy flux density:

$$u = \frac{1}{8\pi}[\boldsymbol{E}^2 + \boldsymbol{B}^2 + \kappa^2(V^2 + \boldsymbol{A}^2)], \qquad \boldsymbol{S} = \frac{1}{4\pi}[\boldsymbol{E} \times \boldsymbol{B} + \kappa^2 V \boldsymbol{A}]$$

Green's function of the Klein-Gordon equation; Invariant functions Δ, introduced by Jordan and Pauli (1928). The wake-field described by the Bessel function J_1.

$$\Delta_P(\boldsymbol{r},t;\boldsymbol{r}',t') = \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\delta\left[\left(t - \frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}'|\right) - t'\right] - \Theta c\kappa \frac{J_1(\kappa s)}{s},$$

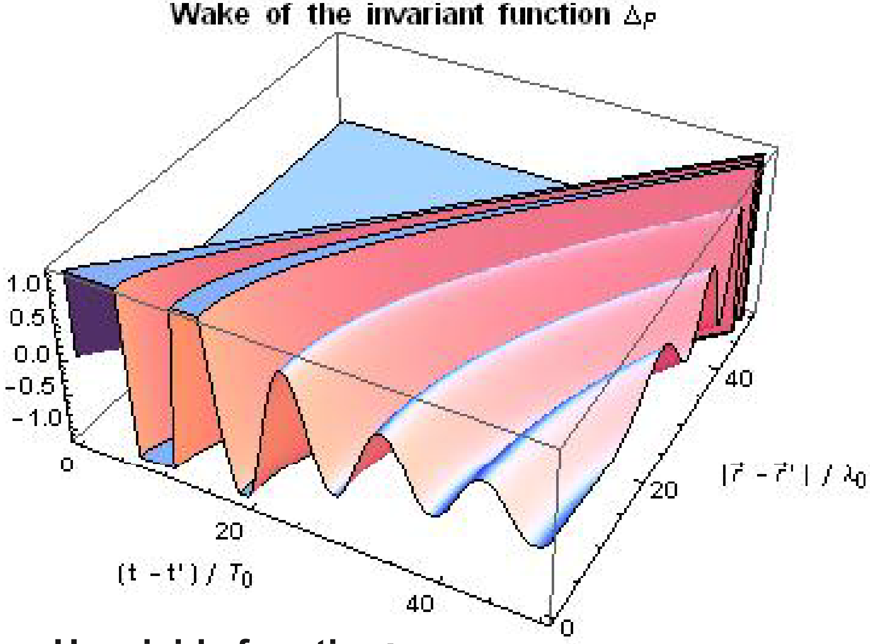
$$\Theta \equiv \Theta\left[\left(t - \frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}'|\right) - t'\right], \qquad s^2 \equiv c^2(t-t')^2 - (\boldsymbol{r}-\boldsymbol{r}')^2.$$

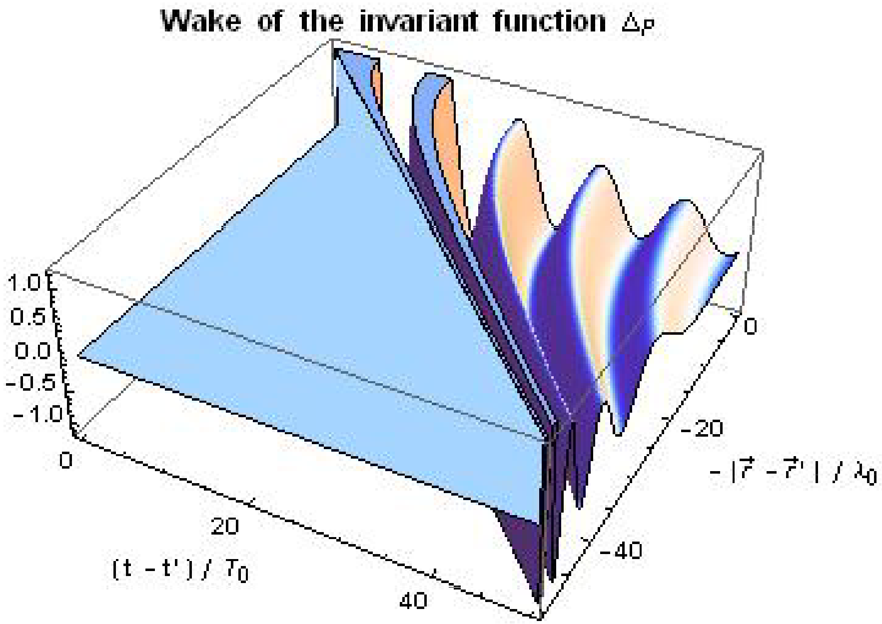
Stueckelberg E C G, Théorie de la Radiation de Photons de Masse Arbitrairement petite. *Helv. Phys. Acta* 30, 209 (1957). Krause D E, Kloor H T and Fischbach E, Multipole radiation from massive fields: Application to binary pulsars. *Phys. Rev. D* 49, 6892 (1994).

The wake-field part of the Jordan-Pauli invariant function Δ_P. [$\omega_p / \omega_0 = 0.1$]

Besides the usual (retarded) Liénart – Wiechert term, there is a wake-field in the Geen's function:

$$\Theta \frac{J_1\left(2\pi(\omega_p/\omega_0)\sqrt{(T/T_0)^2-(R/\lambda_0)^2}\right)}{2\pi(\omega_p/\omega_0)\sqrt{(T/T_0)^2-(R/\lambda_0)^2}}$$





Heaviside function:

$$\Theta\left[\left(t-\frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}'|\right)-t'\right]c\kappa\frac{J_1(\kappa s)}{s},$$

$$s^2 \equiv c^2(t-t')^2-(\boldsymbol{r}-\boldsymbol{r}')^2$$

Varró S, Plasmons, Massive Photons and the Hypothetical X17 Particle. [SZTE seminar. TTIK FI, Budó auditórium. 10. April 2024, Wed. 3pm.]
[File:"Varro_S_X17_SZTE_FIZ_15h_10_Apr_2024_v_1.ppt"]

The question of the constancy of the speed of light. The work of Zoltán Bay; the new standard for the meter (1983).

PHYSICAL REVIEW D VOLUME 5, NUMBER 4 15 FEBRUARY 1972

Frequency Dependence of the Speed of Light in Space

Z. Bay and J. A. White*

National Bureau of Standards, Washington, D. C. 20234

(Received 16 August 1971)

To characterize the possible dispersion of the velocity of light in space (vacuum) a Cauchy-type formula, $n^2 = 1 + A/\nu^2 + B\nu^2$, is used. It is shown that relativity only allows a nonzero A term, independent of the nature of the waves or a quantization thereof. Recent experimental data provide upper bounds for A and B, limiting thereby the dispersion in the microwave, infrared, visible, and ultraviolet regions of the spectrum to less than one part in 10^{20}.

„From this it follows that $(n^2-1)\nu^2=A$, where A is a relativistic invariant and therefore the only possible dispersion formula for vacuum is

$$n^2 = 1 + \frac{A}{\nu^2}$$

if the special theory of relativity is correct. Causality requires <u>A < 0</u>. (For A >0 the group velocity exceeds c_0 and a signal may appear to propagate backwards in time to some observer.)"

Bay Z and White J A, Frequency dependence of the speed of light in space. PRD 5, 976 (1972)

Varró S, Klein-Gordon radio and attosecond light pulses.
[Talk presented at Seminar 9 of LPHYS'21. [29th International Laser Physics Workshop, 19-23 July 2021, Zoom Meeting].

The question of the constancy of the speed of light. The work of Zoltán Bay; the new standard for the meter (1983). Remark on the dispersion formulas in Aether models (1836-1842).

»This fact provides additional experimental support for the **suggestion made recently that c be used in metrology to connect the unit of time (the second) and the unit of length (the meter).** The connection is made by assigning an agreed upon value to c in m/sec. The results of this paper show that in the above-mentioned broad spectrum this assignment can be made without reference to frequency.«

Bay Z and White J A, Frequency dependence of the speed of light in space. PRD 5, 976 (1972).

[Note that in our notation the group velocity is $v_{group} = cn_m < c$, and $v_{phase} = c/n_m > c$.]

Remark on the first dispersion theory based ***on mechanical Aether models***. [Cauchy, Sur la dispersion de la lumiére (1836), Neumann F E (1841), O'Brien M (1842)

$$\rho\frac{\partial^2 \boldsymbol{e}}{\partial t^2} = n\frac{\partial^2 \boldsymbol{e}}{\partial x^2} - C\boldsymbol{e}$$

$$\boldsymbol{e} \sim \exp[i\omega(t - x/V)]$$

$$\frac{n}{V^2} = \rho - \frac{C}{\omega^2}$$

'mass term' in the 'Klein-Gordon equation' (1841) responsible for dispersion

Sir Edmund Whittaker, *A History of the Theories of Aether and Electricity*. (Thomas Nelson and Sons Ltd, London, 1951).

Varró S, Klein-Gordon radio and attosecond light pulses. [Talk presented at Seminar 9 of LPHYS'21. [29th International Laser Physics Workshop, 19-23 July 2021, Zoom Meeting].

“Tief ist der Brunnen von Vergangenheit. Sollte man ihn nich unergründlich nennen?”
[Thomas Mann, Joseph und seine Brüder.]

Summary.

● We have shown that, based on the quantized Volkov states, the „proton-dressed radiation" is a massive boson field with (rest mass)c^2 = 17.0087 MeV, which may perhaps be identified with the X17 particlee, found by the ATOMKI group in several internal pair creation experiments.

● The „neutron-dressed radiation" (quark-photon plasmon) is also a massive boson field with (rest mass)c^2 = 37.9937 MeV, corresponding to the nvariant mass of $\gamma\,\gamma$ pairs from decay of E38, appearing e.g. in d + Cu events

● In our considerations a natural connection appears between the mass of the X17 particle and the proton size, the latter also being a subject of intensive research, with some puzzling outcomes. At the end of the talk we discussed this connection, which delivers 0.846299 fm for the proton radius. For the neutron radius we have received 0.845135 fm.

[● By an elementary consideration we have introduced the plasmon dispersion relation and the „mass generation" of electromagnetic radiation. Fifth force interpretation and true massive photons.]

Acknowledgement. I thank several interesting and useful discussions with
Prof. A. J. Krasznahorkay, Prof. T. Csörgő, and Prof. Ch-Y Wong at the at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.

Appendix.

Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron.

Sándor Varró
Extrem Light Infrastructure, ERIC, ALPS Facility, Szeged, Hungary.

Abstract. Recent observations of anomalous angular correlations of electron-positron pairs, stemming from the products of several nuclear reactions, have been interpreted by assuming the existence of a neutral boson of rest mass of about 17 MeV/c^2, called the X17 particle [1,2]. The results of a different set of experiments on gamma photon pair spectra, around the invariant mass 38 MeV/c^2, indicates the existence of the so-called E38 particle [3], which may mediate energy and momentum from the excited products.
In the present talk we rely on quantum electrodynamics, in order to show a possible derivation of the above-mentioned masses. We use the exact solutions of the Dirac equation of the system of a charged particle and quantized electromagnetic plane waves, which we apply to a proton or to the udd quarks of the neutron. These non-perturbative solutions contain a frequency change of the radiation quanta, which results in the appearance of an effective mass of plasmons as massive photons. In the first case the derived formula yields the invariant mass 17.0087 MeV/c^2 for the dressed radiation [4]. In the second case (udd quark-photon plasma) the received analytic expression gives 37.9937 MeV/c^2 for the invariant mass [4]. Besides the Sommerfeld fine structure constant and the nucleon masses, the derived formulas contain merely some first-principle statistical factors. In our formalism the mass of the X17 particle and the proton size are inherently connected. We have derived the numerical value 0.846299 fm for the proton radius [4], which is quite close the the („smaller”) experimental value [5]. For the neutron radius we have received 0.845135 fm. These radii are smaller by a factor of 0.640452 than the respective Compton wavelengths.

Varró S, Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron. [Invited talk presented at PP2024. Particles & Plasmas Symposium 2024. 10-12 June 2024, Budapest, Hungary.] [File:”Varro_S_X17_PP2024_11_June_11h45.ppt”]

Mass Formulas for the Hypothetical X17 and E38 Particles, and the Sizes of the Proton and Neutron.

Sándor Varró
Extrem Light Infrastructure, ERIC, ALPS Facility, Szeged, Hungary

References.

[1] Krasznahorkay A J, Csatlós M, Csige L, et al., Observation of anomalous internal pair creation in 8Be : a possible indication of a light, neutral boson. Phys. Rev. Lett. 116, 042501 (2016).
[2] Ahn T T, Trong T D, Krasznahorkay A J, et al., Checking the anomaly with a two-arm electon positron pair spectrometer. Universe 2024, 10, 168.
[3] Abraamyan K, Austin C, Baznat M, Check of the structure in photon pairs spectra at the invariant mass of about 38 MeV/c2. EJP Web of Conferences 204, 08004 (2019).
[4] Varró S, Proposal for an electromagnetic mass formula for the X17 particle. Universe 2024, 10, 86.
[5] Pohl R, Antognini A, Nez F, et al., The size of the proton. Nature 466, 213-216 (2010).

Refined study of the „8Be anomaly" with the 7Li(p,γ)8Be reaction.

New results on the ^{8}Be anomaly

A J Krasznahorkay*, M Csatlós, L Csige, Z Gácsi, J Gulyás, Á Nagy, N Sas, J Timár, T G Tornyi, I Vajda

Institute for Nuclear Research, Hungarian Academy of Sciences, MTA Atomki

A Krasznahorkay

CERN, Geneva, Switzerland

E-mail: * kraszna@atomki.hu

Abstract.

Recently, we observed anomalous internal pair creation for the M1 transition depopulating the 18.15 MeV isoscalar 1^+ state in ^{8}Be. We observed a significant (σ = 7.37) peak-like deviation from the predicted angular correlation of the e^+e pairs at $\theta = 140^o$. To the best of our knowledge no nuclear physics related description of such deviation can be made. However, the deviation can be described by assuming the creation and subsequent decay of a boson with mass of ≈17 MeV. In order to clarify the interpretation, we re-investigated the ^{8}Be anomaly with an improved, and independent setup. We have confirmed the signal of the assumed X(17) particle and constrained its mass (m_0c^2 = 17.01(16) MeV) and branching ratio compared to the γ-decay ($B_x = 6(1) \times 10^{-6}$).

Figures copied from: A J Krasznahorkay∗, M Csatl´os, L Csige, Z G´acsi, J Guly´as, ´A Nagy, N Sas, J Tim´ar, T G Tornyi, I Vajda, New results on the 8Be anomaly. IOP Conf. Series: Journal of Physics: Conf. Series 1056 (2018) 012028. Conference on Neutrino and Nuclear Physics (CNNP2017).

The 7Li(p,γ)8Be Experiment.

2. Experiments

To populate the 18.15 MeV 1^+ state in ^{8}Be selectively, we used the ^{7}Li(p,γ)^{8}Be reaction at the E_p=1030 keV resonance [8]. The average energy loss of protons in the target was 70 keV, so the actual proton energy was 1100 keV. The experiment was performed at the new 2-MV Tandetron accelerator in Debrecen. A proton beam with a typical current of 1.0 μA impinged on 300 μg/cm^2 thick Li target evaporated on 20 $\mu g/cm^2$ thick carbon foil. In contrast to our previous experiment [3, 9], we used a much thinner ^{12}C backing and we increased the number of telescopes (from 5 to 6), which resulted in different electron detection efficiency as a function of the correlation angle. As a considerable improvement, we replaced the gas-filled MWPC detectors with a double-sided silicon strip detector (DSSSD) array.

The e^+e^- pairs were detected by six plastic scintillator + DSSSD detector telescopes placed perpendicularly to the beam direction at azimuthal angles of 0°, 60°, 120°, 180° 240° and 300°. The size of the scintillators is 82×86× 80 mm^3. The positions of the hits were registered by the double-sided silicon strip detectors having strip widths of 3 mm. The telescope detectors were placed around the vacuum chamber made of a carbon fibre tube with a wall thickness of 1 mm.

γ rays were also detected for monitoring purposes. A ϵ_{rel}=100% HPGe detector was used at 25 cm from the target to detect the 18.15 MeV γ rays produced in the ^{7}Li(p,γ)^{8}Be reaction.

Figures copied from: A J Krasznahorkay∗, M Csatl´os, L Csige, Z G´acsi, J Guly´as, ´A Nagy, N Sas, J Tim´ar, T G Tornyi, I Vajda, New results on the 8Be anomaly. IOP Conf. Series: Journal of Physics: Conf. Series 1056 (2018) 012028. Conference on Neutrino and Nuclear Physics (CNNP2017).

Experiments and simulations on the „E38 particle".

Check of the structure in photon pairs spectra at the invariant mass of about 38 MeV/c^2

Khachik Abraamyan[1,2,*], *Chris* Austin[3], *Mircea* Baznat[4], *Konstantin* Gudima[4], *Mikhail* Kozhin[1], *Sergey* Reznikov[1], and *Alexander* Sorin[1,5]

[1]VBLHEP JINR, 141980 Dubna, Moscow region, Russia

[2]Yerevan State University, 0025, Yerevan, Armenia

[3]33 Collins Terrace, Maryport, Cumbria CA15 8DL, England

[4]Institute of Applied Physics, MD-2028 Kishinev, Moldova

[5]BLTP JINR, 141980 Dubna, Moscow region, Russia

Abstract. Results of analysis of the effective mass spectra of photon pairs produced in *d*C, *d*Cu and *p*C interactions at momenta of 2.75, 3.83 and 5.5 GeV/c per nucleon, respectively, are presented. A structure at effective mass of about 38 MeV/c^2 is observed. The results of testing the observed signal are presented. The test results support the conclusion that the observed signal is the consequence of detection of a particle with a mass of about 38 MeV/c^2 decaying into a pair of photons.

[2] Abraamyan K, Austin C, Baznat M, Gudima K, Kozhin M, Reznikov S and Sorin A, Check of the structure in photon pairs spectra at the invariant mass of about 38 MeV/c2. EJP Web of Conferences 204, 08004 (2019).

KINEMATIC EFFECTS DUE TO QUANTIZATION. This is an example which shows that the nonclassical nature of the strong light field can even manifest itself in the kinematics of the HHG spectrum. The reaction additionally modifies the frequency.

J. Phys. A: Math. Gen. **14** (1981) 2281–2303. Printed in Great Britain

$$[\gamma_\mu(i\partial - \varepsilon\hat{A}_{laser})^\mu - \kappa]\Psi = \varepsilon\gamma_\mu \hat{A}^\mu_{scatt}\Psi$$

Relative depletion. From the quantized pondero-motive energy shift.

Nonlinear scattering processes in the presence of a quantised radiation field: II. Relativistic treatment

J Bergou and S Varró

Central Research Institute for Physics, H-1525 Budapest 114, POB 49, Hungary

Received 22 October 1980

$$\omega'_n = \frac{n\omega_0 + \omega_C\mu_0^2\Delta}{1+\left(2\frac{n\omega_0}{\omega_C}+\frac{\mu_0^2}{2}\right)\sin^2\frac{\theta}{2}}$$

Abstract. The Dirac equation of a rel
quantised mode of the electromagnetic
the corresponding eigenvalues (the spe
linear and circular polarisation. The stat
of which, corresponding to the four-m
corresponding to the photon content, di
nonlinear Compton scattering is calculated. It is shown explicitly that the results reduce to the semiclassical one in the limit of high intensity and small depletion, while in the large (complete) depletion limit they contain a depletion factor which ensures convergence of the highly nonlinear processes.

Gordon-Volkov states (1927, 1935): Exact solutions of the Klein–Gordon and Dirac equations of an electron in an arbitrary intense 'laser field' propagating in vacuum. After ~ 80 years; the only new exact, closed form solutions for the same problem in a medium [S. V. (2013, 2014)]

Der Comptoneffekt nach der Schrödingerschen Theorie.

Von **W. Gordon** in Berlin.

(Eingegangen am 29. September 1926.)

Über eine Klasse von Lösungen der Diracschen Gleichung.

Von **D. M. Wolkow** in Leningrad.

(Eingegangen am 12. Februar 1935.)

IOP PUBLISHING — LASER PHYSICS LETTERS

Laser Phys. Lett. **10** (2013) 095301 (13pp) — doi:10.1088/1612-2011/10/9/095301

LETTER

New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium

Sándor Varró

$$\mu''' \equiv \mu_0 (mc^2 / \hbar\omega_p)$$

Nuclear Instruments and Methods in Physics Research A 740 (2014) 280–283

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ELSEVIER

New exact solutions of the Dirac and Klein–Gordon equation of a charged particle propagating in a strong laser field in an underdense plasma

Sándor Varró

IOP Publishing | Astro Ltd — Laser Physics Letters

Laser Phys. Lett. **11** (2014) 016001 (14pp) — doi:10.1088/1612-2011/11/1/016001

Letter

A new class of exact solutions of the Klein–Gordon equation of a charged particle interacting with an electromagnetic plane wave in a medium

Sándor Varró

Varró S, New concepts, selected open questions in strong-field laser-matter interaction and/or ultrafast science at ELI-ALPS. [ELI-ALPS Workshop talk. 23 Januar 2024. Tuesday, 10:00 – 12:30h] [File:"Varro_S_Concepts_ELI_23_Jan_2024_v_1.ppt"]

Illustration. Green's function of a (scalar) charged particle interacting with a quantized electromagnetic plane wave field. Position representation.

Az imént bemutatott diagonalizálási eljárással a megfelelő, inhomogén Dirac-, ill. Klein—Gordon-egyenlet is megoldható. Illusztrációképpen most megadjuk a „skalár elektron + kvantált e. m. módus" rendszer propagátorát,

$$\hat{\mathcal{G}}(x, y) = e^{ik\cdot x\left(a^+a - \frac{1}{2}\right)} \left(\int \frac{d^4 p}{(2\pi)^4} D_\sigma \frac{e^{-ip\cdot(x-y)}}{p^2 - 2p\cdot k f_p(a^+a) - \varkappa^2} D_\sigma^{-1} \right) e^{-ik\cdot y\left(a^+a + \frac{1}{2}\right)}, \quad (7.8)$$

ahol $f_p(a^+a) \equiv \left(1 + \frac{g^2}{p\cdot k}\right)\left(a^+a + \frac{1}{2} - |\sigma|^2\right)$, a többi jelölés ugyanaz mint fentebb volt.

$$D_\sigma = \exp(\sigma a^+ - \sigma^* a)$$

$$\sigma = -p\cdot\varepsilon^* /(kp + g^2)$$

[[1] S. Varró, Theoretical study of the interaction of free electrons with intense light. (Ph.D. dissertation, 1981). (in Hungarian)
Hungarian Physical Journal XXXI, 399-454 (1983)..

Illustration. Green's function of a (scalar) charged particle interacting with a quantized electromagnetic plane wave field. Momentum representation.

$\hat{\mathscr{G}}$ impulzusreprezentációbeli $\langle l+n|\hat{\mathscr{G}}(p,q)|n\rangle$ mátrix elemeiről a (4.17) Hilb-formula alkalmazásával belátható, hogy az n, $V\to\infty$ ($(n/V)\equiv\varrho$ állandó) határesetben megegyeznek a megfelelő szemiklasszikus Green-függvény ((6.23)) nemdiagonális mátrixelemeivel

$$\lim_{\substack{n,\,V\to\infty\\ \varrho=\text{állandó}}} \langle l+n|\hat{\mathscr{G}}(p,q)|n\rangle = (2\pi)^4\delta_2(p+lk-q)e^{il\varphi}\cdot\sum_{r=-\infty}^{\infty}\frac{J_{l+r}(\zeta)J_r(\zeta)}{(p-rk)^2-\varkappa_*^2}, \quad (7.9)$$

$$\zeta,\varphi,\varkappa_*\equiv(6.23\text{a—b}),\quad a_0\longleftrightarrow\frac{c}{\omega}(2\pi\varrho\hbar\omega)^{1/2}. \quad (7.9\text{a})$$

$$\sigma = -p\cdot\varepsilon^*/(kp+g^2)$$

A másik szélsőséges eset a vákuum—vákuum átmenet, amelynek leírására (7.8) szintén alkalmas (ellentétben a szemiklasszikus Green-függvénnyel). A vákuumban mozgó skalár elektron propagátora

$$\langle 0|\hat{\mathscr{G}}(p,q)|0\rangle = (2\pi)^4\delta_4(p-q)e^{-|\sigma|^2}\sum_{m=0}^{\infty}\frac{(|\sigma|^{2m}/m!)}{(p-m\tilde{k})^2-\varkappa_{**}^2}, \quad (7.10)$$

ahol

$$\tilde{k}\equiv k\left(1+\frac{g^2}{p\cdot k}\right),\quad \varkappa_{**}^2=\varkappa^2+2(p\cdot k+g^2)|\sigma|^2+g^2. \quad (7.10\text{a})$$

(7.10) alapján a $(p-m\tilde{k})^2=\varkappa_{**}^2$ egyenlet által meghatározott pólusok a virtuális fotonokkal való kölcsönhatásról adnak számot.

[[1] S. Varró, Theoretical study of the interaction of free electrons with intense light. (Ph.D. dissertation, 1981). (in Hungarian)
Hungarian Physical Journal XXXI, 399-454 (1983)..

Varró S, Derivation of electromagnetic mass formulas for the X17 and E38 particles from quantized Volkov states.
[Seminar talk at Uni-Ulm, October 2023] [File:"Varro_S_X17_Ulm_Talk_Oct_2023_v_1.ppt"]

The 'List' is now complete: Classical polynomials in the expansions of generalized coherent states; (generalized) squeezed coherent states; and squeezed number states.

(Gen.) Coherent States (Bloch, Nordsieck 1937, Feynman 1951, Schwinger 1953). Laguerre polynomialsk.

$$D(\sigma) = \exp(\sigma a^{+} - \sigma^{*} a) \qquad a \to D^{+}(\sigma) a D(\sigma) = a + \sigma$$

$$\langle m | D(\sigma) | n \rangle = \sqrt{\frac{n!}{m!}} \sigma^{m-n} L_n^{m-n}(|\sigma|^2) \exp(-\tfrac{1}{2}|\sigma|^2)$$

Squeezed Coherent States (Popov and Perelomov 1969). Stoler (1970) Equation (15'). Hermite polynomials.

$$\langle m | S(z) D(\beta) | 0 \rangle = \frac{t^m}{\sqrt{n!}} H_m\left(\frac{\beta}{2 c_r t}\right) \exp(-\tfrac{1}{2}|\beta|^2 + \beta^2 t^{*2}), \qquad t = \sqrt{s_r \mathrm{e}^{-\varphi} / 2 c_r}$$

Squeezed Number Eigenstates (present work, 2021-2022). Gegenbauer polynomials.

$$S(\xi) = \exp[\tfrac{1}{2}(\xi \hat{a}^{+2} - \xi^{*} \hat{a}^{2})] \qquad a \to b = S^{+}(\xi) a S(\xi) = a \cosh r + a^{+} e^{i\varphi} a \sinh r$$

$$\langle m | S(\xi) | n \rangle = \mathrm{e}^{-\eta/4} (2 \mathrm{e}^{i\varphi} \tanh|\xi|)^{\alpha} \sqrt{\frac{n!}{m!}} \frac{\Gamma(\alpha + \frac{1}{2})}{\sqrt{\pi}} C_n^{\alpha+\frac{1}{2}}\left(\frac{1}{\cosh|\xi|}\right), \qquad \alpha = \tfrac{1}{2}(m-n) \geq 0$$

Varró S, Coherent and incoherent superposition of transition matrix elements of the squeezing operator. *New Journal of Physics 24, 053035* (2021). *E-print*: arXiv: 2112.08430 [quant-ph] . 29th Int. Las. Phys. (LPHYS'21). J. Phys. Conf. Ser. 2249 012013 (2022).
Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, 8 (7), 269 (2021). [https://doi.org/10.3390/photonics8070269].

The »Lánczos–Proca vector field«

[A] „Neither the titles nor the detailed texts of Proca's papers indicate explicitly that this is an equation for the electromagnetic field. Indeed, from the context it is clear that he was thinking of a charged massive spin-1 field. The idea that this could be identified with a massive photon came later." [A] Goldhaber and Nieto (2010).

[B] „Since 1928, Dirac's relativistic wave-equation has been written in various forms in order to facilitate its interpretation. For example, in 1930 Sauter [1] and Proca [2] rewrote it using Clifford numbers. However, the most direct road was taken by Lanczos in 1929 [3].... Hence, (1) [Lánczos's coupled quaternionic equations, SV] can be seen as Maxwell's equations with feed-back. This was a very important idea, and precisely the one that lead Proca in 1936 to discover the correct equation for the massive spin one particle [5]. The concept was easily generalized, and Kemmer finally wrote down the wave-equation of the (peudo-) scalar and (pseudo-) vector particles in the form we still use today [6]. Later, Gürsey [7] showed that Proca's and Kemmer's equations are just degenated cases of Lanczos's." [B] Gsponer and Hurni (1993).

[A] Goldhaber A S and Nieto M M, Photon and graviton mass limits. *Rev. Mod. Phys*. 82, 939-978 (2010). Quote; de Broglie L, *La Méchanique Ondulatoire du Photon, Une Novelle Théorie de la Lumiere* (Hermann, Paris, 1940).

[B] Gsponer A and Hurni J-P, Lanczos' equation to replace Dirac's equation? In J. D. Brown et al., eds., Proceedings of the Cornelius Lanczos International Centennary Conference, Raleigh, NC, 12-17 December 1993 (SIAM, Philadelpia, 1994) pp. 509-512.

A remark on „mass generation”. We just mention this point, and shall not consider the more general subject of „symmetry breaking” etc.

»In 1963 the condense matter theorist Phil Anderson pointed out that in a superconductor the Goldstone mode becomes a massive „plasmon” mode due to longrange (Coulomb) forces, and this mode is just the longitudinal partner of transverse electromagnetic modes, which are also massive [6].«

Higgs P W, Evading the Goldstone theorem. *Ann. Phys.* (Berlin) 526, 211-213 (2014). [Nobel Lecture, 8 December 2013.]

[[6] Anderson P W, Plasmons, gauge invariance, and mass. *Phys. Rev*. 130, 439-442 (1963).]

The question of the constancy of the speed of light. The work of Zoltán Bay; the new standard for the meter (1983).

PHYSICAL REVIEW D VOLUME 5, NUMBER 4 15 FEBRUARY 1972

Frequency Dependence of the Speed of Light in Space

Z. Bay and J. A. White*

National Bureau of Standards, Washington, D. C. 20234

(Received 16 August 1971)

To characterize the possible dispersion of the velocity of light in space (vacuum) a Cauchy-type formula, $n^2 = 1 + A/\nu^2 + B\nu^2$, is used. It is shown that relativity only allows a nonzero A term, independent of the nature of the waves or a quantization thereof. Recent experimental data provide upper bounds for A and B, limiting thereby the dispersion in the microwave, infrared, visible, and ultraviolet regions of the spectrum to less than one part in 10^{20}.

„From this it follows that $(n^2-1)\nu^2=A$, where A is a relativistic invariant and therefore the only possible dispersion formula for vacuum is

$$n^2 = 1 + \frac{A}{\nu^2}$$

if the special theory of relativity is correct. Causality requires <u>A < 0</u>. (For A >0 the group velocity exceeds c_0 and a signal may appear to propagate backwards in time to some observer.)"

Bay Z and White J A, Frequency dependence of the speed of light in space. PRD **5**, 976 (1972)

Varró S, Klein-Gordon radio and attosecond light pulses.
[Talk presented at Seminar 9 of LPHYS'21. [29th International Laser Physics Workshop, 19-23 July 2021, Zoom Meeting].

The question of the constancy of the speed of light. The work of Zoltán Bay; the new standard for the meter (1983). [Recent changes in SI; 26th meeting of CGPM, November 2018]

»This fact provides additional experimental support for the **suggestion made recently that c be used in metrology to connect the unit of time (the second) and the unit of length (the meter).** The connection is made by assigning an agreed upon value to c in m/sec. The results of this paper show that in the above-mentioned broad spectrum this assignment can be made without reference to frequency.«

Bay Z and White J A, Frequency dependence of the speed of light in space. PRD **5**, 976 (1972).

The **speed of light in vacuum** c is **299 792 458 m/s**. [declared in 1983]

[Recent changes in SI; 26th meeting of CGPM, November 2018]

„... decides that,effective from 20 May 2019,the International System of Units,the SI,is thesystem of units in which:the unperturbed ground state hyperfine transition frequency of the caesium133 atom $\Delta\nu_{Cs}$ is **9 192 631 770 Hz**,
the **speed of light** in vacuum *c is* ***299792458 m/s***,
the **Planck constant** *h is* $\mathbf{6.62607015\times10^{-34}}$ ***Js***,
the **elementary charge** *e is* $\mathbf{1.602176634\times10^{-19}}$ ***C***,
the **Boltzmann constant** *k is* $\mathbf{1.380649\times10^{-23}}$ ***J/K***,
the **Avogadro constant** N_A *is* $\mathbf{6.02214076\times10^{23}}$ $\mathbf{mol^{-1}}$,
the **luminous efficacy** of monochromatic radiation of frequency 540×10^{12} Hz, $\mathbf{K_{cd}}$**, is 683 lm/W**, ...”

26th meeting of the General Conference on Weights and Measures (CGPM). Versailles, France, 13-16 November 2018.

Proton size puzzle. Electronic measurements before...

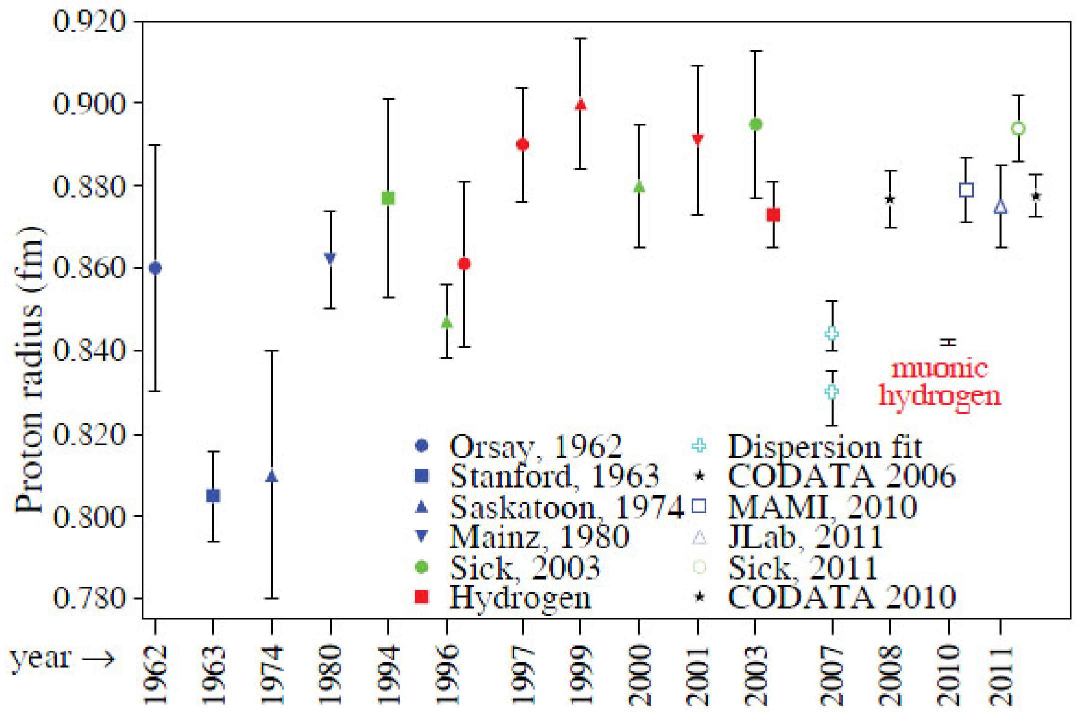

Figure 1: Proton radius determinations over time. Electronic measurements seem to settle around r_p=0.88 fm, whereas the muonic hydrogen value [1,2] is at 0.84 fm.

Pohl R, Antognini A, Nez F, et al., The size of the proton. Nature 466, 213-216 (2010).. Picture copied from: Pohl R et al, Muonic hydrogen and the proton radius puzzle (2016)

Proton size puzzle.

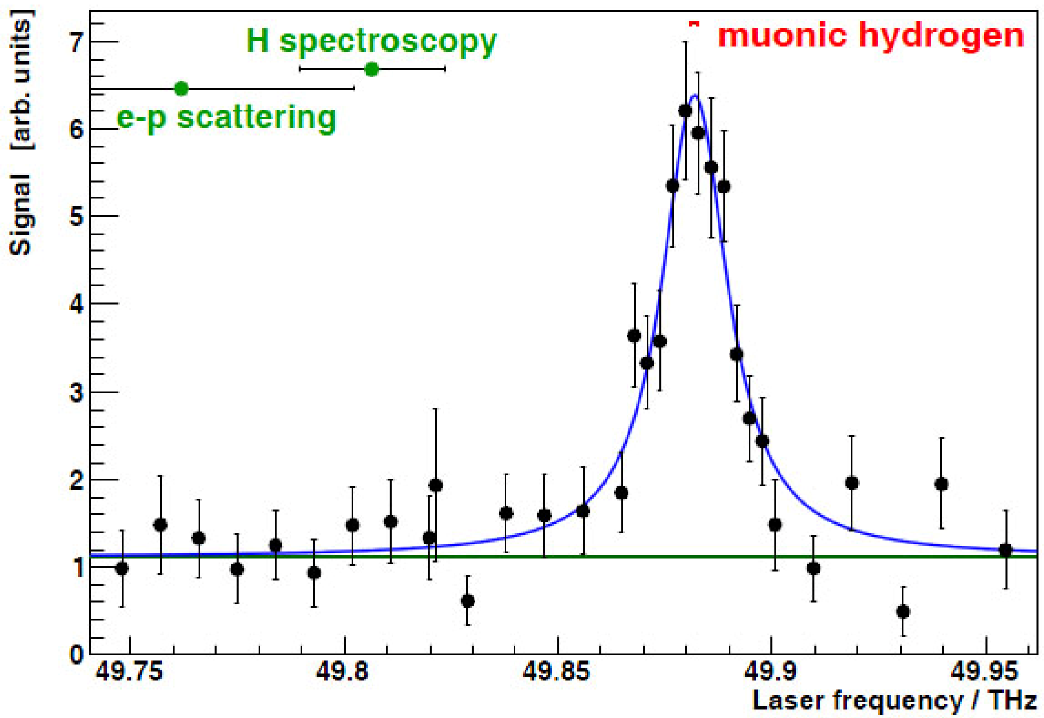

Figure 3: Resonance in muonic hydrogen, together with the positions predicted using the proton radii from elastic electron-proton scattering using pre-2009 world data [19, 23] and the CODATA-2006 value from H spectroscopy [22].

Pohl R, Antognini A, Nez F, et al., The size of the proton. Nature 466, 213-216 (2010).. Picture copied from: Pohl R et al Muonic hydrogen and the proton radius puzzle (2016)

Proton size puzzle.

$$\Delta E_{\mathrm{FS}} = 8.3521\,\mathrm{meV}$$

$$\Delta E_{\mathrm{HFS}}^{2S_{1/2}} = 22.8089(51)\,\mathrm{meV},$$

$$\Delta E_{\mathrm{HFS}}^{2P_{1/2}} = 7.9644\,\mathrm{meV}$$

$$\Delta E_{\mathrm{HFS}}^{2P_{3/2}} = 3.3926\,\mathrm{meV}$$

$$\Delta = 0.1446\,\mathrm{meV}$$

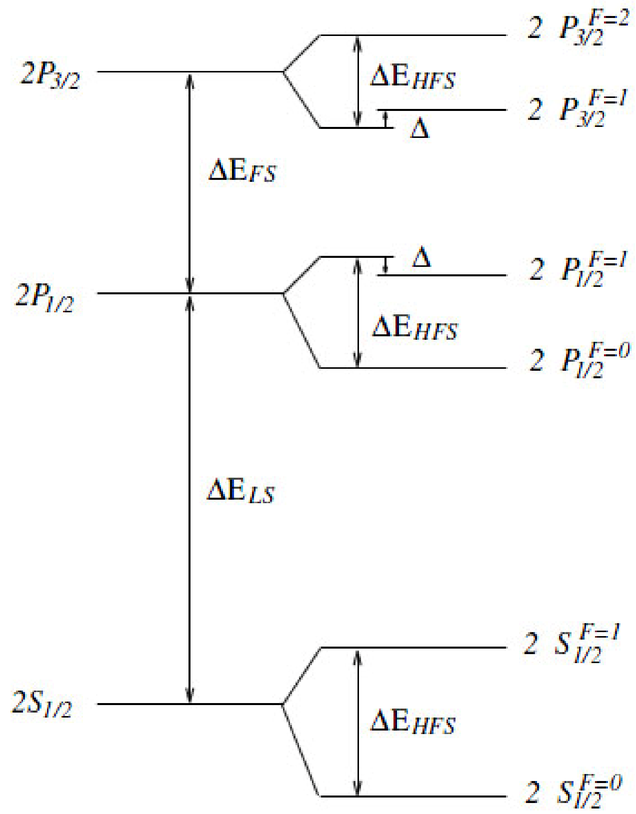

Figure 4: Level scheme of muonic hydrogen for $n = 2$ shell, the artificial $2P_{1/2}$ and $2P_{3/2}$ levels corresponds to centroid for $\Delta = 0$. Numerical values for level

Pohl R, Antognini A, Nez F, et al., The size of the proton. Nature 466, 213-216 (2010).. Picture copied from: Pohl R et alMuonic hydrogen and the proton radius puzzle (2016)