

Deuteron production and elliptic flow in ultrarelativistic heavy-ion collisions

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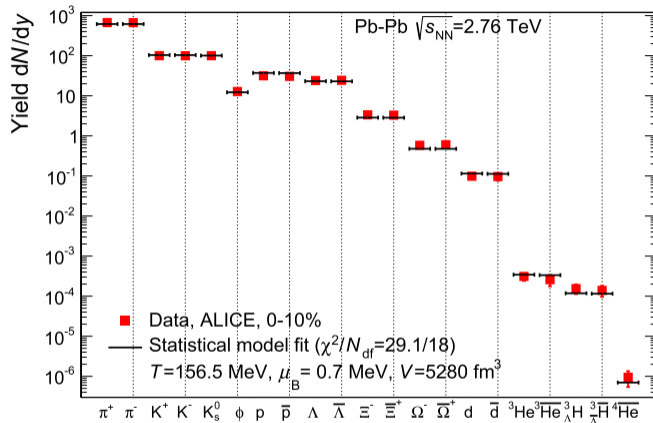
Particles & Plasmas Symposium, Budapest, 2024

12.6.2024

Clusters and statistical model: a coincidence?

Cluster abundancies fit into a universal description with the statistical model

$$T \gg E_b!!!???$$



[A. Andronic *et al.*, J. Phys.: Conf. Ser. 779 (2017) 012012]

This is (a part of the) motivation to look at clusters, although clusters actually carry *femtoscopic* information about the freeze-out.

The deuteron wave function

Wave function(s)

(r is the distance between proton and neutron):

- Hulthen form ($\alpha = 0.23 \text{ fm}^{-1}$, $\beta = 1.61 \text{ fm}^{-1}$)

$$\varphi_d(r) = \sqrt{\frac{\alpha\beta(\alpha + \beta)}{2\pi(\alpha - \beta)^2}} \frac{e^{-\alpha r} - e^{-\beta r}}{r}$$

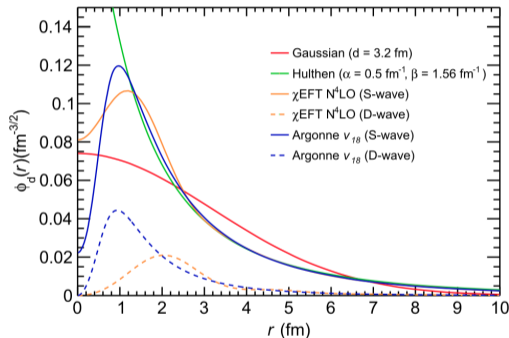
- spherical harmonic oscillator ($d = 3.2 \text{ fm}$)

$$\varphi_d(r) = (\pi d^2)^{-3/4} \exp\left(-\frac{r^2}{2d^2}\right)$$

rms radius: 1.96 fm

Binding energy 2.2 MeV

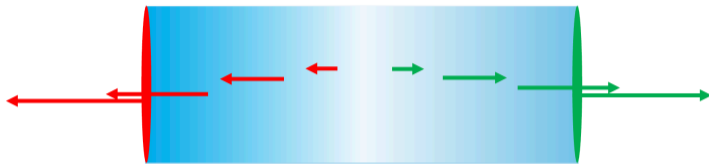
spin 1



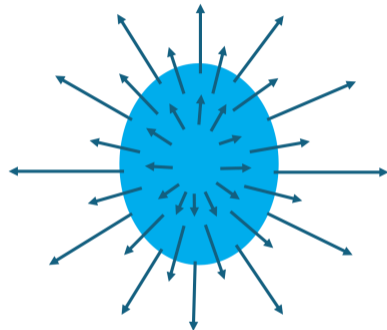
[Figure: M. Mahlein et al., Eur.Phys.J. C 83 (2021) 804]

A short introduction to fireball dynamics (for non-experts)

boost-invariant longitudinal expansion



transverse expansion



- blue-shifted transverse momentum spectra
- azimuthal variation of hadron distribution

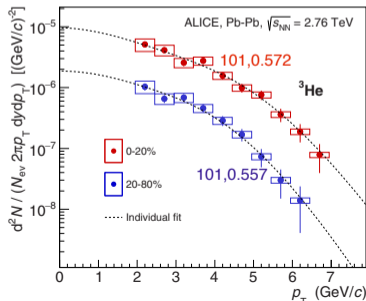
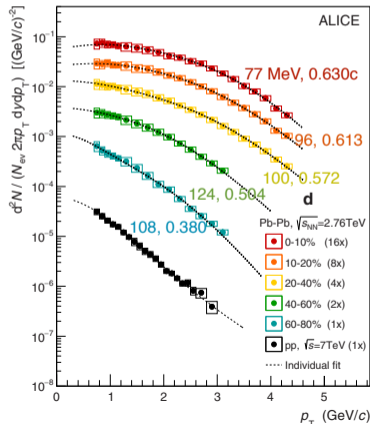
$$\frac{d^2N}{p_T dp_T d\phi} = \frac{dN}{p_T dp_T} \left(1 + \sum_i 2v_i \cos(\phi - \phi_i) \right)$$

Kinetic freeze-out of clusters: ALICE

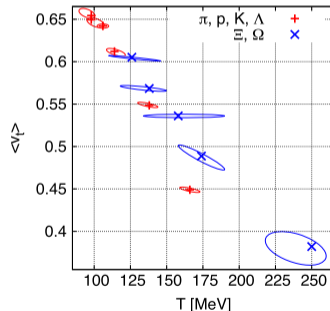
[J. Adam *et al.* [ALICE collab], Phys. Rev. C 93 (2016) 024917]

p_t spectra of d and ^3He fitted individually with the blast-wave formula

$$\frac{dN}{p_t dp_t} \propto \int_0^R r dr m_t I_0 \left(\frac{p_t \sinh \rho(r)}{T} \right) K_1 \left(\frac{m_t \cosh \rho(r)}{T} \right), \quad \rho(r) = \tanh^{-1}(\beta_s (r/R)^n)$$



Fit with MC blast-wave:



[I. Melo, B. Tomášik,
J. Phys. G. 43 (2016) 015102]

Projection of the deuteron density matrix onto two-nucleon density matrix

Deuteron spectrum:

$$E_d \frac{dN_d}{d^3P_d} = \frac{3}{8(2\pi)^3} \int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f_p \left(R_d, \frac{P_d}{2} \right) f_n \left(R_d, \frac{P_d}{2} \right) C_d(R_d, P_d)$$

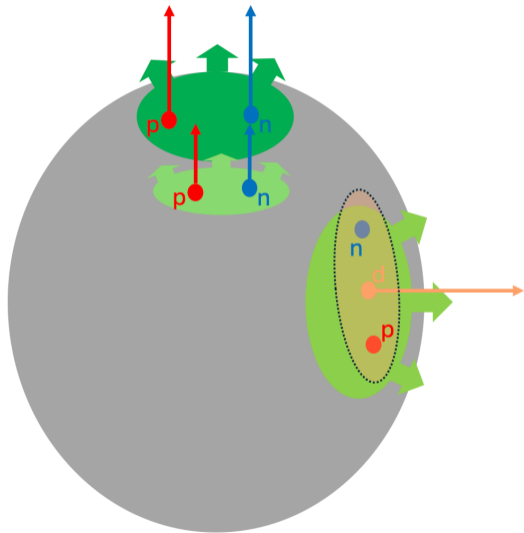
QM correction factor

$$C_d(R_d, P_d) \approx \int d^3r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} |\varphi_d(\vec{r})|^2$$

r relative position, R_+ , R_- : positions of nucleons

approximation: narrow width of deuteron Wigner function in momentum

Homogeneity regions, homogeneity lengths, length scales



- Particles of given momentum are produced only by a portion of fireball with similar velocity—the homogeneity region
- the size of the homogeneity region is given by flow gradients and temperature
- Length scales
 - homogeneity length
 - size of the deuteron

Correction factor: limiting cases

$$C_d(R_d, P_d) \approx \int d^3r \frac{f(R_+, P_d/2)f(R_-, P_d/2)}{f^2(R_d, P_d/2)} |\varphi_d(\vec{r})|^2$$

- **Large homogeneity region** for nucleon momentum $P_d/2$: $L \gg d$
(L is the scale on which $f(R, P_d/2)$ changes)

$$C_d(R_d, P_d) \approx \int d^3r |\varphi_d(\vec{r})|^2 = 1$$

No correction! Just product of nucleon source functions.

- **Small homogeneity region** for nucleon momentum $P_d/2$: $L \ll d$
 $f(R, P_d/2)$ effectively limits the integration to Ω

$$C_d(R_d, P_d) \approx \int_{\Omega} d^3r |\varphi_d(\vec{r})|^2 = C < 1$$

Interesting regime: $L \approx d$

Analytical approximation of the (average) correction factor

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$\langle C_d \rangle(P_d) = \frac{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2 \left(R_d, \frac{P_d}{2} \right) C_d(R_d, P_d)}{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2 \left(R_d, \frac{P_d}{2} \right)}$$

Approximations:

- Gaussian profile in rapidity and in the transverse direction,
- **weak transverse expansion**
- saddle point integration

$$\langle C_d \rangle \approx \left\{ \left(1 + \left(\frac{d}{2\mathcal{R}_\perp(m)} \right)^2 \right) \sqrt{1 + \left(\frac{d}{2\mathcal{R}_\parallel(m)} \right)^2} \right\}^{-1}$$

Homogeneity lengths:

$$\mathcal{R}_\perp = \frac{\Delta\rho}{\sqrt{1 + (m_t/T)\eta_f^2}} \quad \mathcal{R}_\parallel = \frac{\tau_0 \Delta\eta}{\sqrt{1 + (m_t/T)(\Delta\eta)^2}}$$

The invariant coalescence factor B_2

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$E_d \frac{dN_d}{d^3 P_d} = B_2 E_p \frac{dN_p}{d^3 P_p} E_n \frac{dN_n}{d^3 P_n} \Big|_{P_p=P_n=P_d/2}$$

(Approximations with corrections for box profile)

$$B_2 = \frac{3\pi^{3/2} \langle C_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)} e^{2(m_t - m)(1/T_p^* - 1/T_d^*)}$$

where the effective temperatures are

$$T_p^* = T + m_p \eta_f^2 \quad T_d^* = T + \frac{M_d}{2} \eta_f^2$$

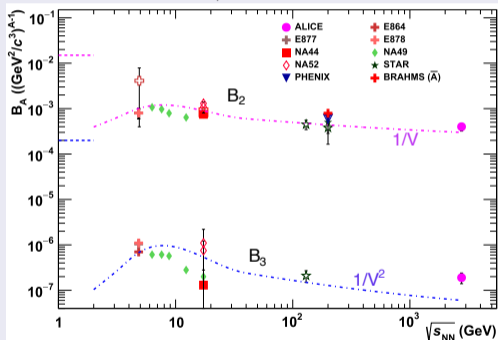
Approximate behaviour: $B_2 \approx 1/\text{volume}$

It works!

B_2 as function of $\sqrt{s_{NN}}$

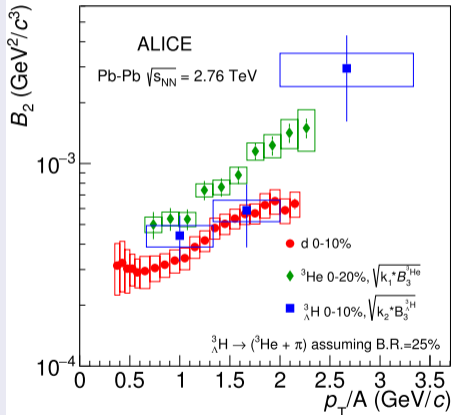
[P. Braun-Munzinger, B. Dönigus, Nucl. Phys. A987 (2019) 144]

Compared with $\propto 1/V$



B_2 as function of p_t

[J. Adam, *et al.* [ALICE coll.], Phys. Lett. B754 (2016) 360]



consistent with decreasing homogeneity volume

Distinguishing coalescence: deuteron number fluctuations

[Z. Fecková, *et al.*, Phys. Rev. C 93 (2016) 054906]

Measure the fluctuations of deuteron number.

- Thermal model prediction: Poissonian fluctuations
- Coalescence:
 - protons and neutrons fluctuate according to Poissonian
 - deuteron number proportional to $p \cdot n$
 - Enhanced fluctuations in case of coalescence

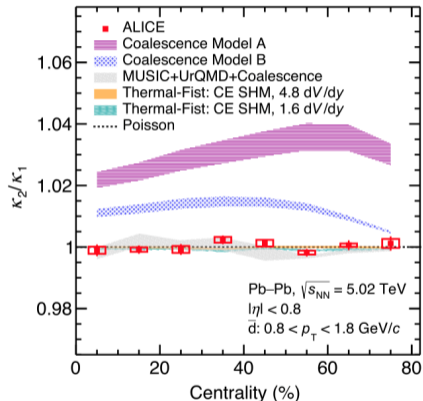
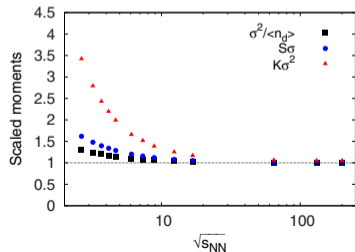


figure: [ALICE collab: Phys. Rev. Lett. 131 (2023) 037864]

How to get the yields consistent with the statistical model?

Assume thermal source function (Boltzmann)

$$f_N(p_N, x) = 2 \exp\left(-\frac{p_n \cdot u + \mu_N}{T}\right) H(r, \phi, \eta)$$

coalescence:

$$E_d \frac{dN_d}{d^3P_d} = \frac{3}{4} \int_{\Sigma_f} \frac{P_d \cdot d\Sigma_f(R_d)}{(2\pi)^3} \left(2 \exp\left(-\frac{p_n \cdot u + \mu_N}{T}\right)\right)^2 (H(r, \phi, \eta))^2 C_d(R_d, P_d)$$

thermal production:

$$E_d \frac{dN_d}{d^3P_d} = 3 \int_{\Sigma_f} \frac{P_d \cdot d\Sigma_f(R_d)}{(2\pi)^3} \exp\left(-\frac{P_d \cdot u + \mu_d}{T}\right) H(r, \phi, \eta)$$

they are equal if

- volume is large, i.e. $C_d(R_d, P_d) = 1$
- $\mu_d = 2\mu_N$, and μ_N guarantees right number of nucleons - Partial Chemical Equilibrium
- $H^2(r, \phi, \eta) = H(r, \phi, \eta)$, fulfilled for box profile

see also [X. Xu, R. Rapp, Eur. Phys. J. 55 (2019) 68]

- deuteron spectrum sensitive to the shape of the density profile, through $(H(r, \phi, \eta))^2$
- proton spectrum sensitive to $H(r, \phi, \eta)$
- effects for homogeneity lengths comparable with the size of the cluster

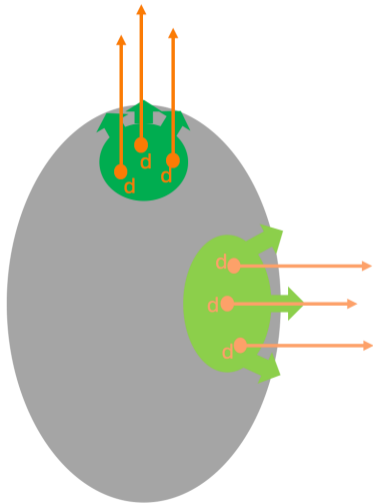
⇒ femtoscopy probe

- elliptic flow of deuterons - probes finer changes in homogeneity lengths

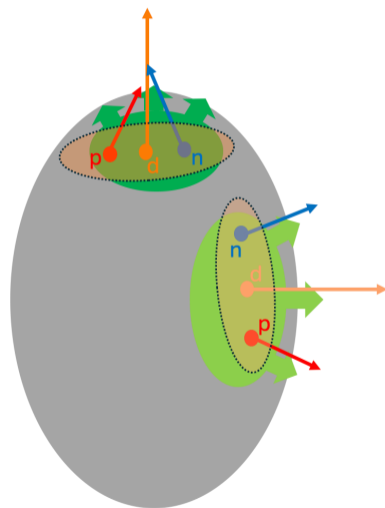
see also [A. Polleri, *et al.*, Phys. Lett. B 473 (2000) 193]

Homogeneity lengths and the elliptic flow of clusters

direct thermal production of deuterons



coalescence production of deuterons



Simulate v_2 of deuterons—the strategy

- set-up Blast Wave model with azimuthal anisotropy
- assume Partial Chemical Equilibrium (lower FO temperature than T_{ch})
- the model **must** reproduce p_t -spectra and $v_2(p_t)$ of **protons and pions**
- **simulate p_t spectra and $v_2(p_t)$ of deuterons in blast-wave model and in coalescence, and look for differences**
- features of the model:
 - includes resonance decays
 - Monte Carlo simulation (SMASH: modified HadronSampler and decays)
 - built-in anisotropy in expansion flow and in fireball shape
 - includes modification of distribution function due to viscosity
 - freeze-out time depending on radial coordinate
- obtain T and transverse expansion from fitting p_t spectra of p and π (and K , Λ)
- then obtain anisotropy parameters from $v_2(p_t)$
- simulate thermal production of deuterons
- simulate coalescence of deuterons (by proximity in phase-space)

The Cooper-Frye formula:

$$E \frac{d^3 N_i}{dp^3} = \int_{\Sigma} d^3 \sigma_{\mu} p^{\mu} f(x, p),$$

The freeze-out hypersurface:

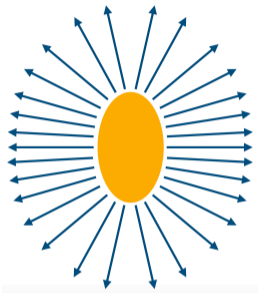
$$x^{\mu} = (\tau(r) \cosh \eta_s, r \cos \Theta, r \sin \Theta, \tau(r) \sinh \eta_s)$$

$$\tau(r) = s_0 + s_2 r^2, \quad \eta_s = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right)$$

$$d^3 \sigma^{\mu} = (\cosh \eta_s, 2s_2 r \cos \Theta, 2s_2 r \sin \Theta, \sinh \eta_s) r \tau_f(r) d\eta_s dr d\Theta,$$

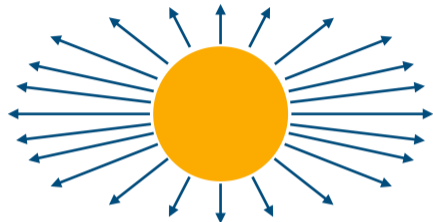
Extended Monte Carlo Blast-Wave model: azimuthal anisotropies

Shape anisotropy:



$$R(\Theta) = R_0 (1 - a_2 \cos(2\Theta))$$

Flow anisotropy:



$$u^\mu = (\cosh \eta_s \cosh \rho(r), \sinh \rho(r) \cos \Theta_b, \\ \sinh \rho(r) \sin \Theta_b, \sinh \eta_s \cosh \rho(r))$$

$$\bar{r} = r/R(\Theta)$$

$$\rho(\bar{r}, \Theta_b) = \bar{r} \rho_0 (1 + 2\rho_2 \cos(2\Theta_b))$$

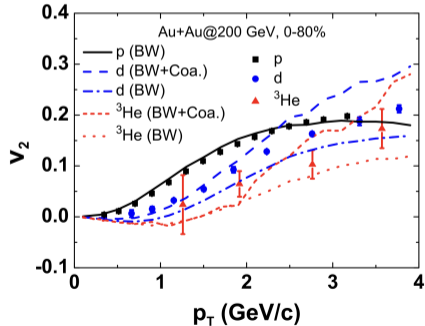
Identified $v_2(p_t)$ for different species allows resolving them.

Simple BW does not reproduce $v_2(p_t)$

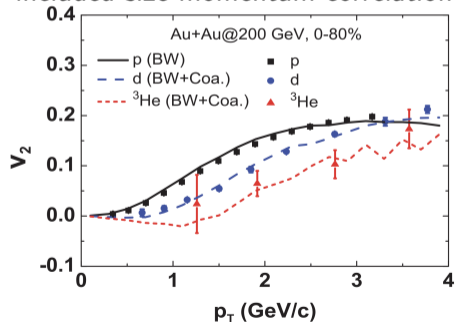
[X. Yin, C.-M. Ko, Y. Sun, L. Zhu, Phys.Rev. C 95 (2017) 054913]

Instead used a source with radius depending on p_t : $R_0 = 10e^{0.23(p_t-0.9)}$ fm

vanilla blast-wave model with coalescence



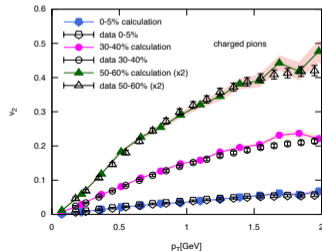
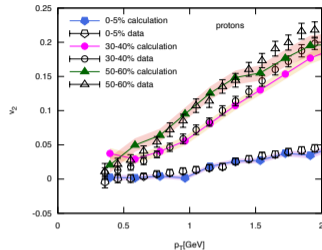
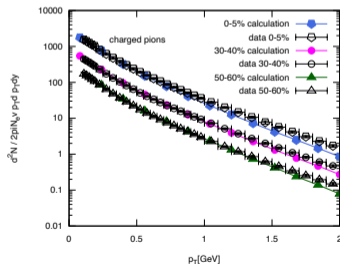
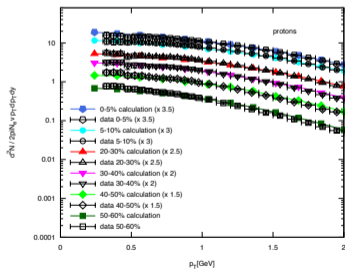
included size-momentum correlation



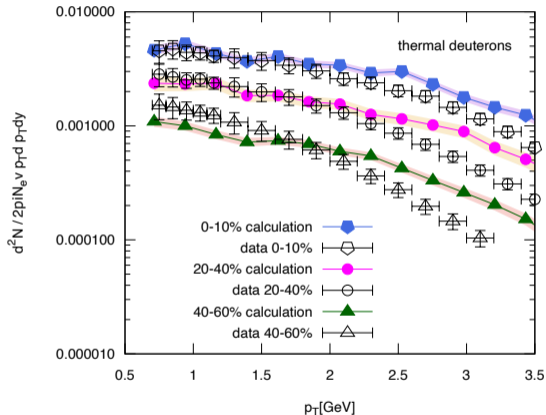
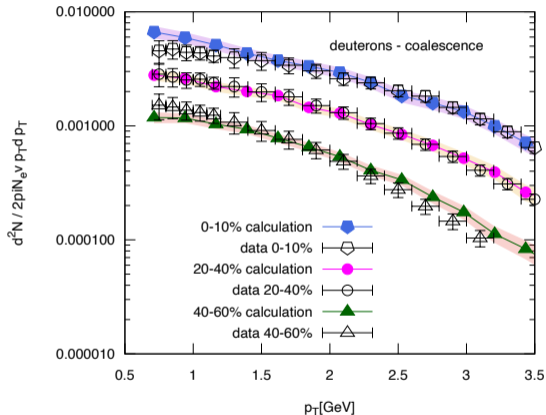
Model calibration

centrality	T [MeV]	ρ_0	R_0 [fm]	s_0 [fm/c]	a_2	ρ_2
0-5%	95	0.98	15.0	21 ± 2	0.016	0.008
30-40%	106	0.91	10.0	9 ± 1	0.085	0.03
50-60%	118	0.80	6.0	6 ± 0.5	0.15	0.02

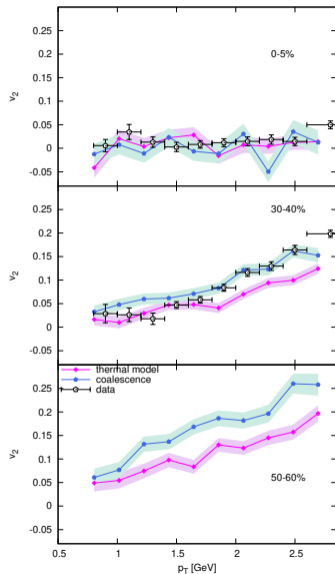
$$s_2 = -0.02 \text{ fm}^{-1}$$



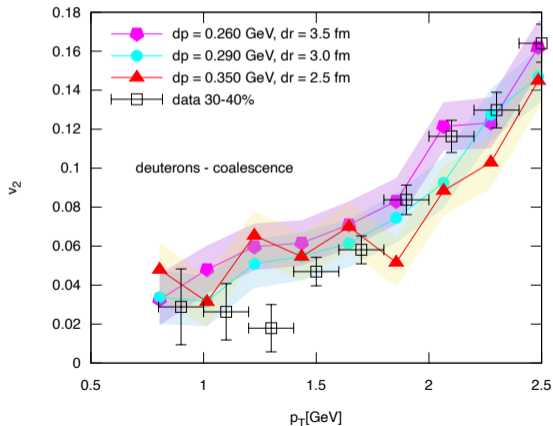
Results for deuterons: p_t spectra



Results for deuterons: v_2



Coalescence for $\Delta p < \Delta p_{max}$ and $\Delta r < \Delta r_{max}$.



Conclusions and outlook

- Deuteron (and cluster) production is a femtosopic probe
- Elliptic flow in more peripheral collisions can help resolving the mechanism of deuteron production: coalescence vs. thermal production
- Data on hypertriton $v_2(p_t)$ are upcoming
- Is there sensitivity on the deuteron wave function?

[R. Vozábová, B. Tomášik, Phys. Rev. C 109, 064908 (2024), arXiv:2402.06327]