

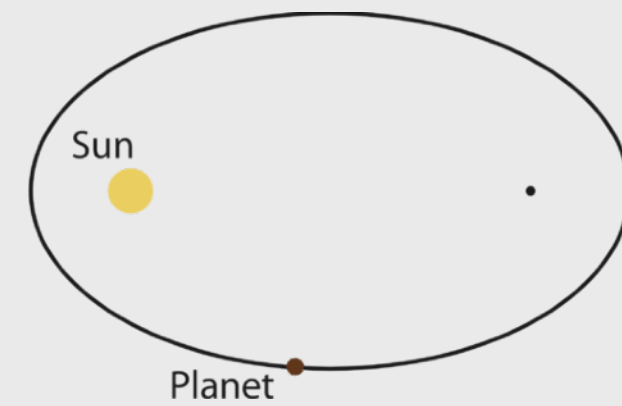
The maximally (super)symmetric 4 dimensional gauge theory

or how to use 2D integrability in 4D quantum gauge theories

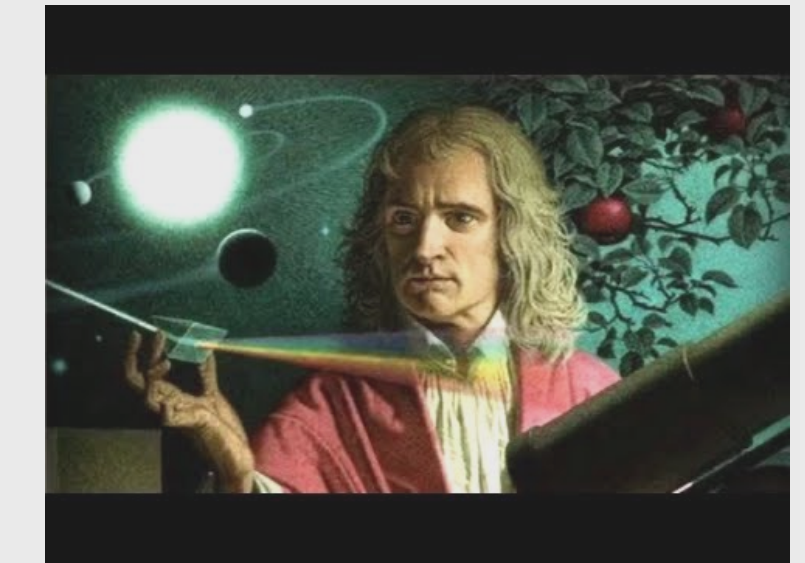
Zoltan Bajnok, HUN-REN Wigner Research Centre for Physics

Exactly soluble problems in physics

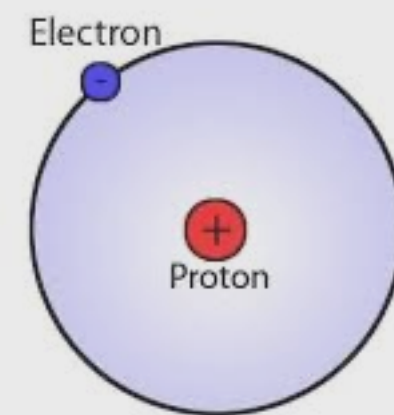
Kepler's problem



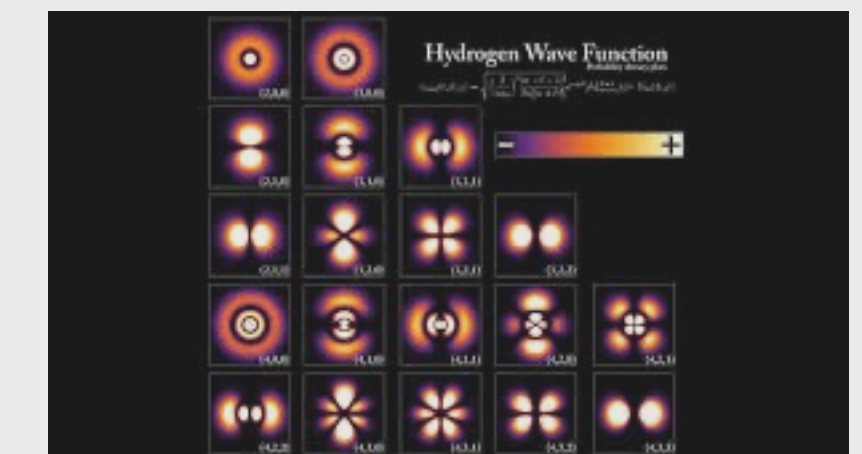
Newtonian mechanics



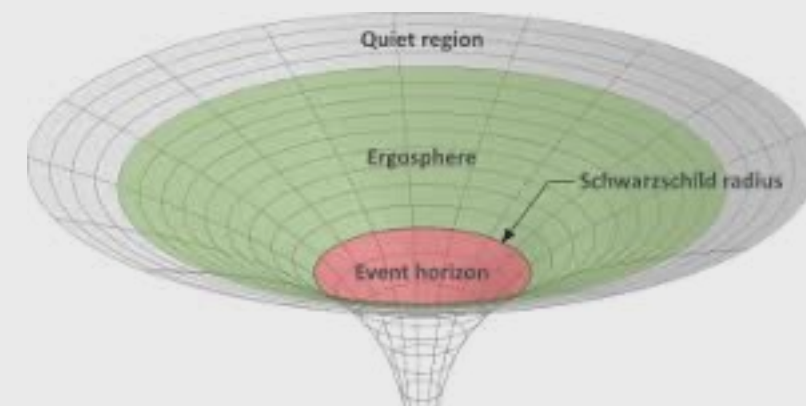
Hydrogen atom



Quantum mechanics

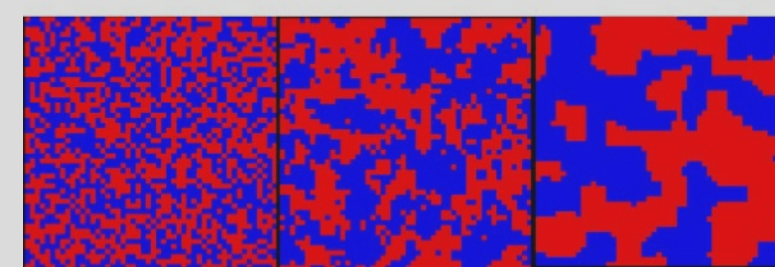


Schwarzschild solution



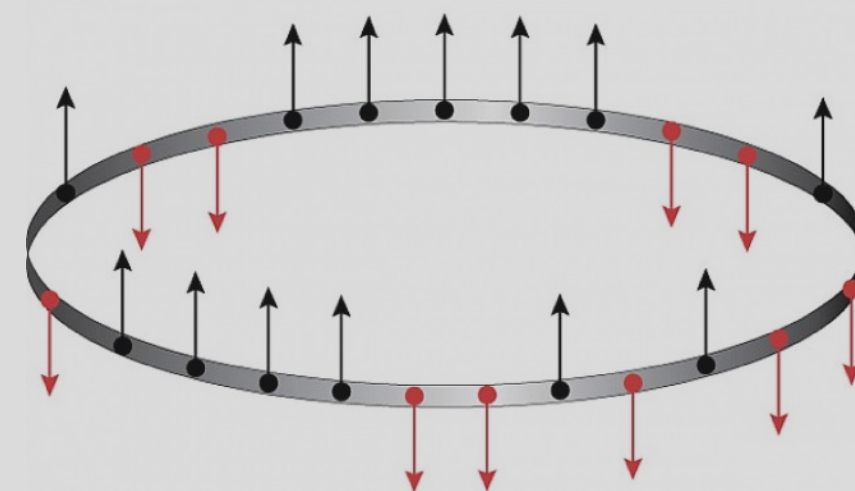
Testing general relativity

Ising model

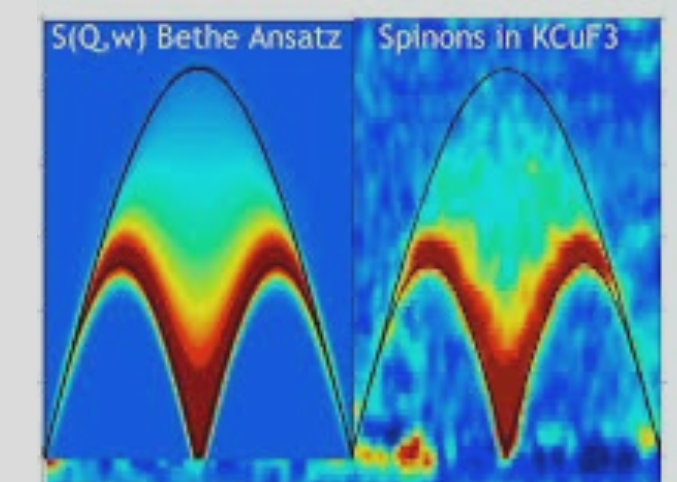


Second order phase transition
2D conformal field theories
statistical field theories

Heisenberg spin chain
Bethe Ansatz



Integrable spinchains
strongly anisotrop solidstate systems
cold atoms

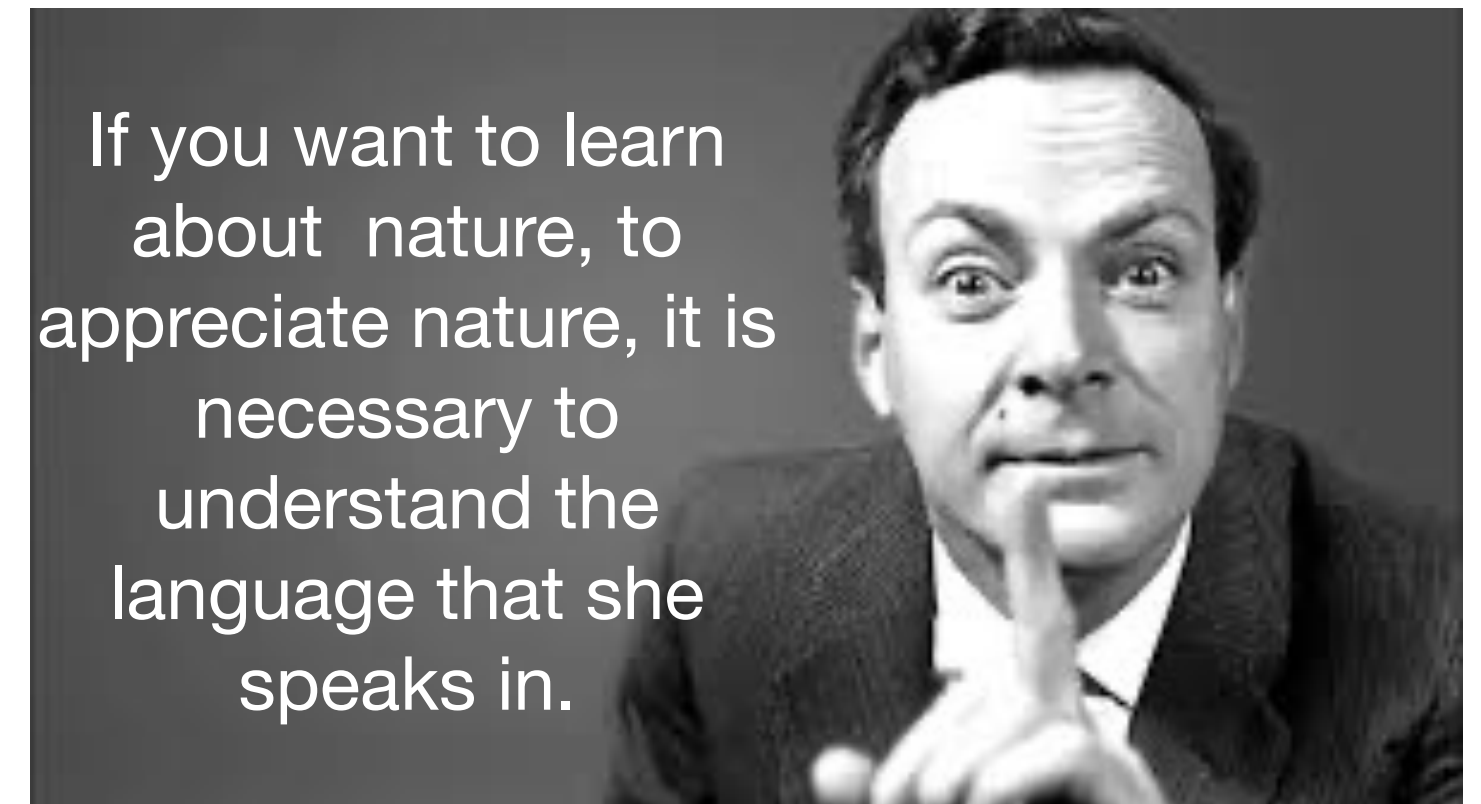


Maximally symmetric
3+1 D gauge theory

potential applications in QCD and
in the standard model

The language of nature is gauge theory

[Feynman]



Gauge group G

compact Lie group $U(1)$ $SU(N)$ $N=2,3,\dots$

particle content

gauge boson



A

matter particles: fermions



Ψ

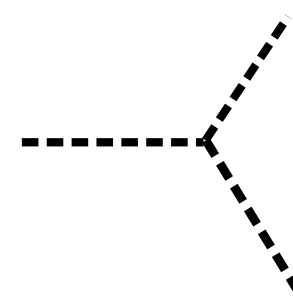
scalar boson



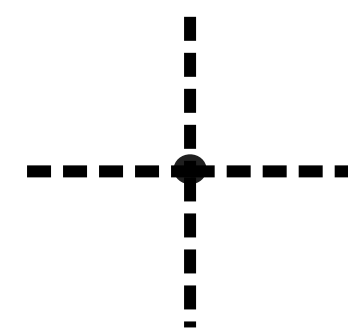
Φ

form representations of G

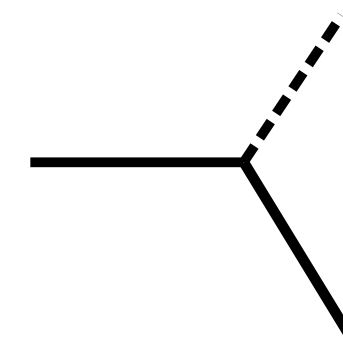
interactions



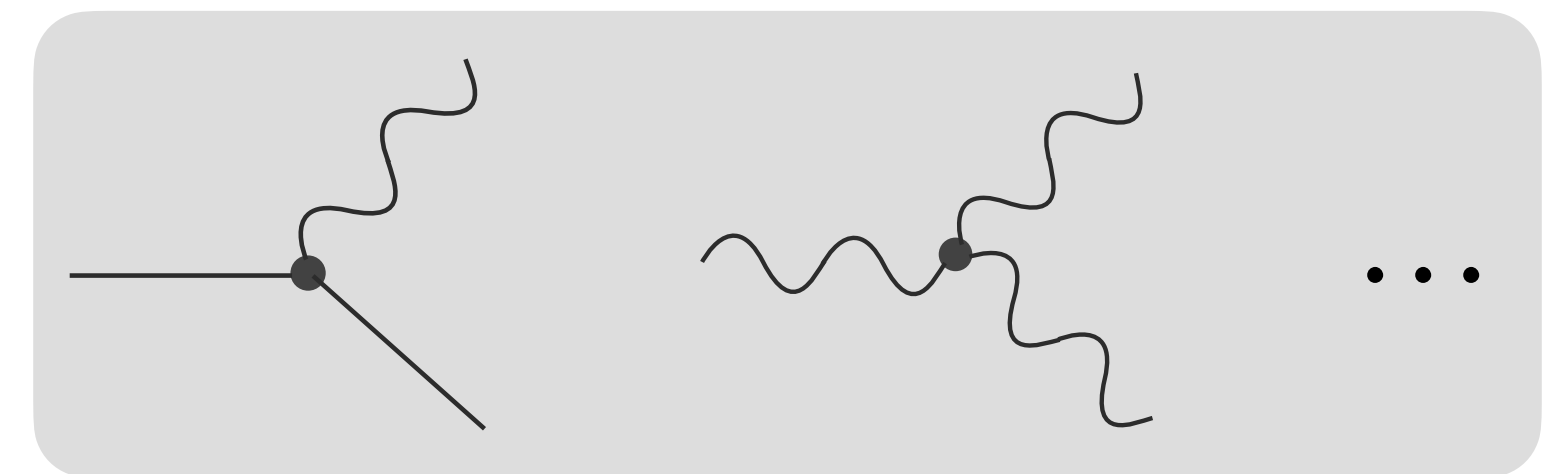
Φ^3



Φ^4



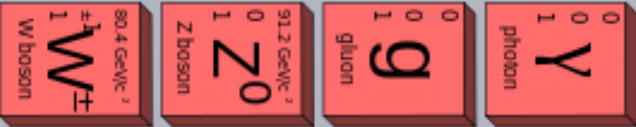

$\Psi\Phi\Psi$



$\Psi A \Psi$

fixed by gauge symmetry

Fundamental interactions of nature

Lie group	Vector boson adjoint	Fermion	scalar	
U(1)	1	electron positron	0	electrodynamics
U(1)xSU(2)	1+3	6 quark 3 neutrino electron, muon, tau	2 Higgs	electroweak
U(1)xSU(2)xSU(3)	 1+3+8		2 Higgs	standard model
SU(N)	$N^2 - 1$	$8(N^2 - 1)$	$6(N^2 - 1)$	all adjoint, most symmetric 4D gauge theory [Brink, Schwartz]

*gravity is a classical gauge theory
but there is no quantum gravity

Maximally symmetric gauge theory

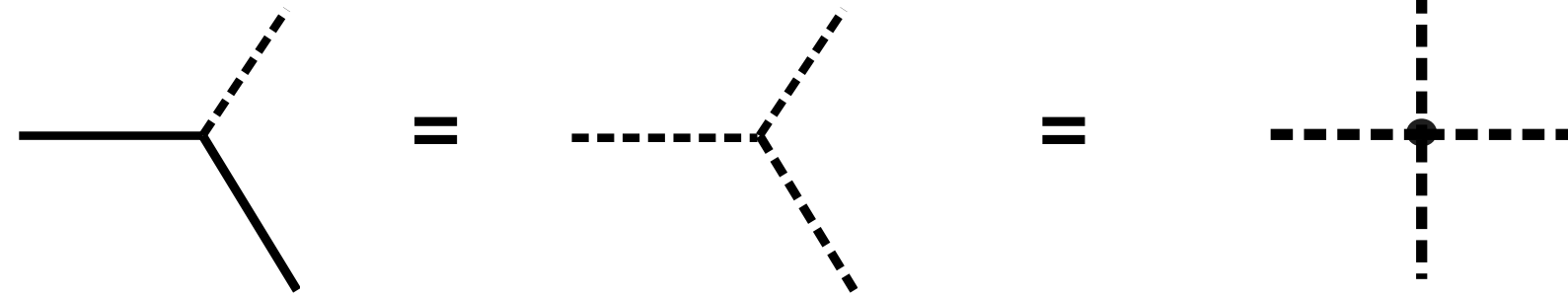
gauge group **SU(N)**
all adjoint $N^2 - 1$

1 gauge boson

8 fermion

6 scalar

Interactions



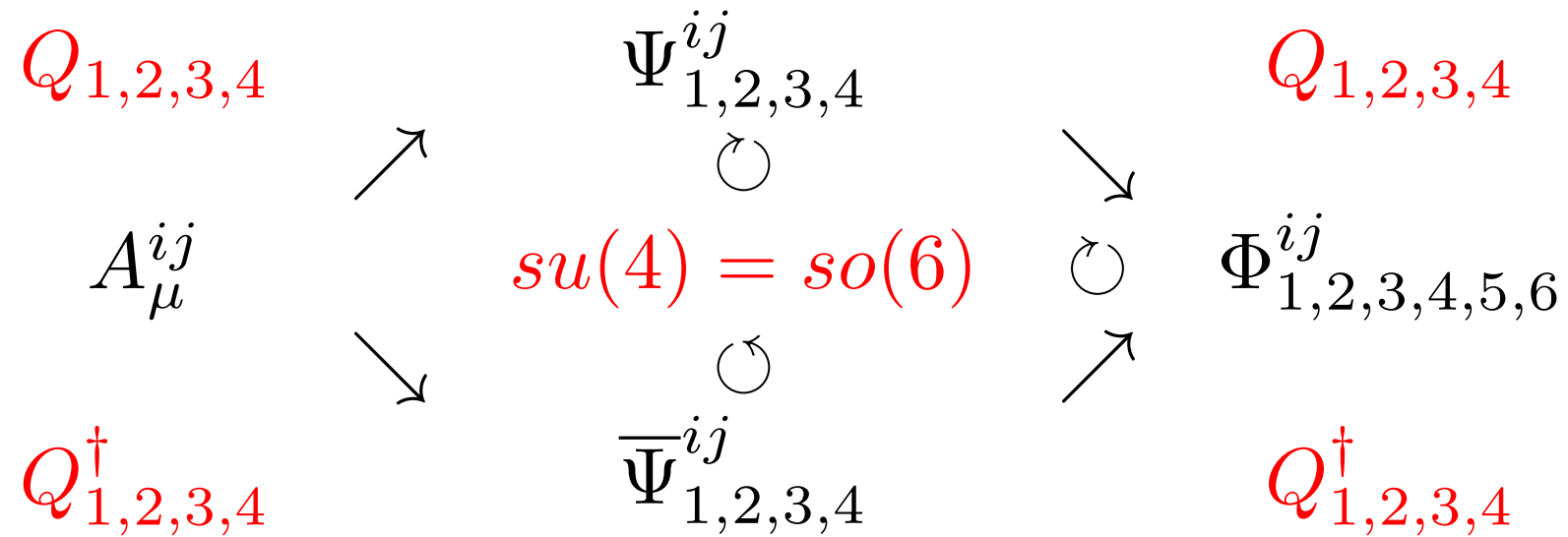
reduction from 10 D $\mathcal{N} = 1$ theory to 4D

$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

Maximally supersymmetric gauge theory

$$so(6) \otimes so(2,4) \subset psu(2,2|4)$$



Perturbation theory

$$E = E_0 + \lambda E_1 + \dots + \frac{1}{N}(1 + \dots)$$

$$\lambda = g_{YM}^2 N$$

$$\frac{1}{N}$$

Standard model

$$\mathcal{L}_{SM} = \mathcal{L}_{Dirac} + \mathcal{L}_{mass} + \mathcal{L}_{gauge} + \mathcal{L}_{gauge/\psi} . \quad (1)$$

Here,

$$\mathcal{L}_{Dirac} = i\bar{e}_L^i \not{\partial} e_L^i + i\bar{\nu}_L^i \not{\partial} \nu_L^i + i\bar{e}_R^i \not{\partial} e_R^i + i\bar{u}_L^i \not{\partial} u_L^i + i\bar{d}_L^i \not{\partial} d_L^i + i\bar{u}_R^i \not{\partial} u_R^i + i\bar{d}_R^i \not{\partial} d_R^i ; \quad (2)$$

$$\mathcal{L}_{mass} = -v \left(\lambda_e^i \bar{e}_L^i e_R^i + \lambda_u^i \bar{u}_L^i u_R^i + \lambda_d^i \bar{d}_L^i d_R^i + \text{h.c.} \right) - M_W^2 W_\mu^+ W^{-\mu} - \frac{M_W^2}{2 \cos^2 \theta_W} Z_\mu Z^\mu ; \quad (3)$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} (G_{\mu\nu}^a)^2 - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} , \quad (4)$$

where

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c \\ W_{\mu\nu}^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{L}_{WZA} &= ig_2 \cos \theta_W \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu Z^\nu + W_{\mu\nu}^+ W^{-\mu\nu} Z^\nu - W_{\mu\nu}^- W^{+\mu\nu} Z^\nu \right] \\ &+ ie \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu A^\nu + W_{\mu\nu}^+ W^{-\mu\nu} A^\nu - W_{\mu\nu}^- W^{+\mu\nu} A^\nu \right] \\ &+ g_2^2 \cos^2 \theta_W \left(W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W^{-\mu\nu} Z_\nu Z^\nu \right) \\ &+ g_2^2 \left(W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W^{-\mu\nu} A_\nu A^\nu \right) \\ &+ g_2 e \cos \theta_W \left[W_\mu^+ W_\nu^- (Z^\mu A^\nu + Z^\nu A^\mu) - 2W_\mu^+ W^{-\mu\nu} Z_\nu A^\nu \right] \\ &+ \frac{1}{2} g_2^2 \left(W_\mu^+ W_\nu^- \right) \left(W^{+\mu\nu} W^{-\nu\mu} - W^{+\nu\mu} W^{-\mu\nu} \right) ; \end{aligned} \quad (6)$$

and

$$\mathcal{L}_{gauge/\psi} = -g_3 A_\mu^a J_{(3)}^{\mu a} - g_2 \left(W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu \right) - e A_\mu J_A^\mu , \quad (7)$$

where

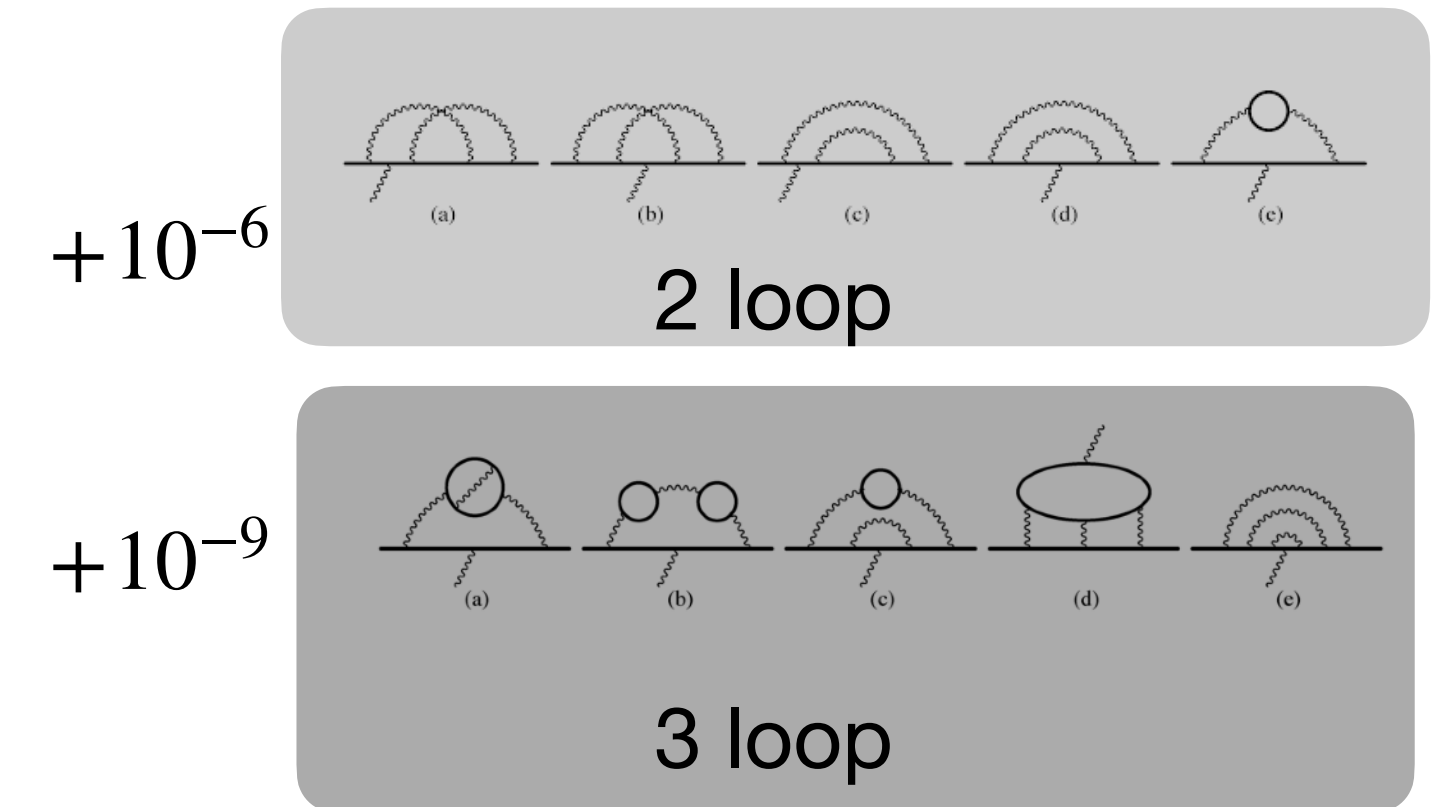
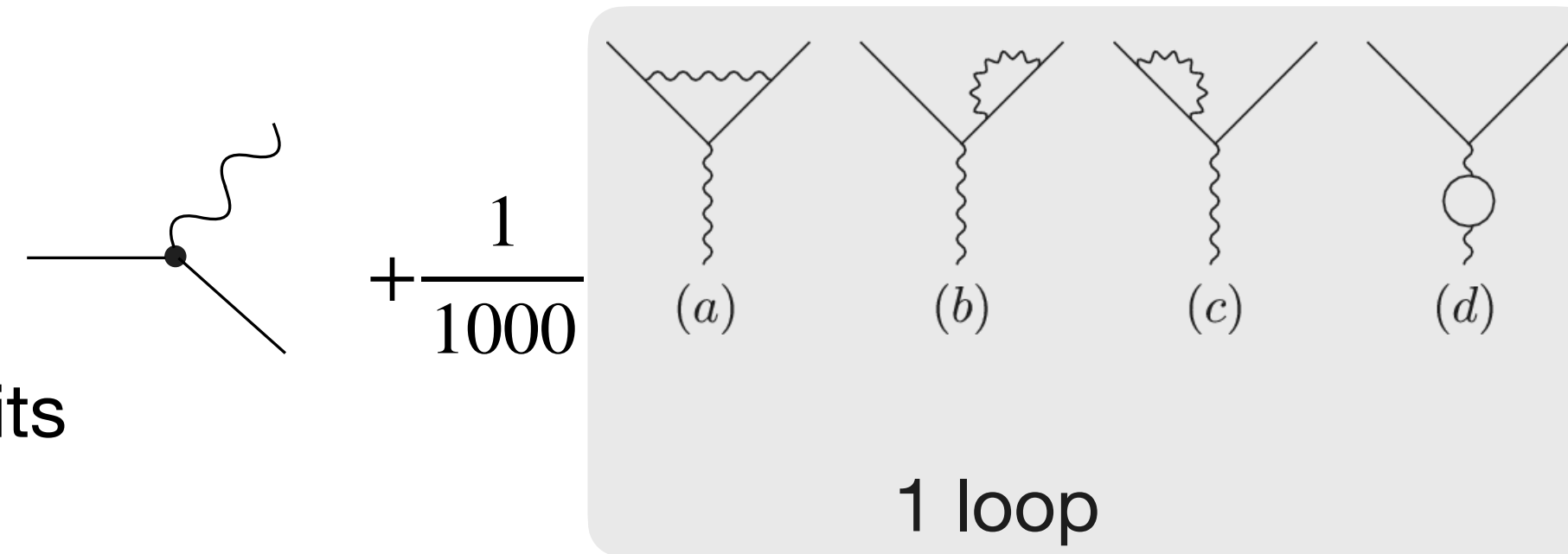
$$\begin{aligned} J_{(3)}^{\mu a} &= \bar{u}^i \gamma^\mu T_{(3)}^a u^i + \bar{d}^i \gamma^\mu T_{(3)}^a d^i \\ J_{W^+}^\mu &= \frac{1}{\sqrt{2}} \left(\bar{\nu}_L^i \gamma^\mu e_L^i + V^{ij} \bar{u}_L^i \gamma^\mu d_L^j \right) \\ J_{W^-}^\mu &= (J_{W^+}^\mu)^* \\ J_Z^\mu &= \frac{1}{\cos \theta_W} \left[\frac{1}{2} \bar{\nu}_L^i \gamma^\mu \nu_L^i + \left(-\frac{1}{2} + \sin^2 \theta_W \right) \bar{e}_L^i \gamma^\mu e_L^i + (\sin^2 \theta_W) \bar{e}_R^i \gamma^\mu e_R^i \right. \\ &\quad \left. + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^i \gamma^\mu u_L^i + \left(-\frac{2}{3} \sin^2 \theta_W \right) \bar{u}_R^i \gamma^\mu u_R^i \right. \\ &\quad \left. + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_L^i \gamma^\mu d_L^i + \left(\frac{1}{3} \sin^2 \theta_W \right) \bar{d}_R^i \gamma^\mu d_R^i \right] \\ J_A^\mu &= (-1) \bar{e}^i \gamma^\mu e^i + \left(\frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left(-\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i . \end{aligned} \quad (8)$$

Perturbation theory

Quantum electrodynamics

Magnetic moment of the electron

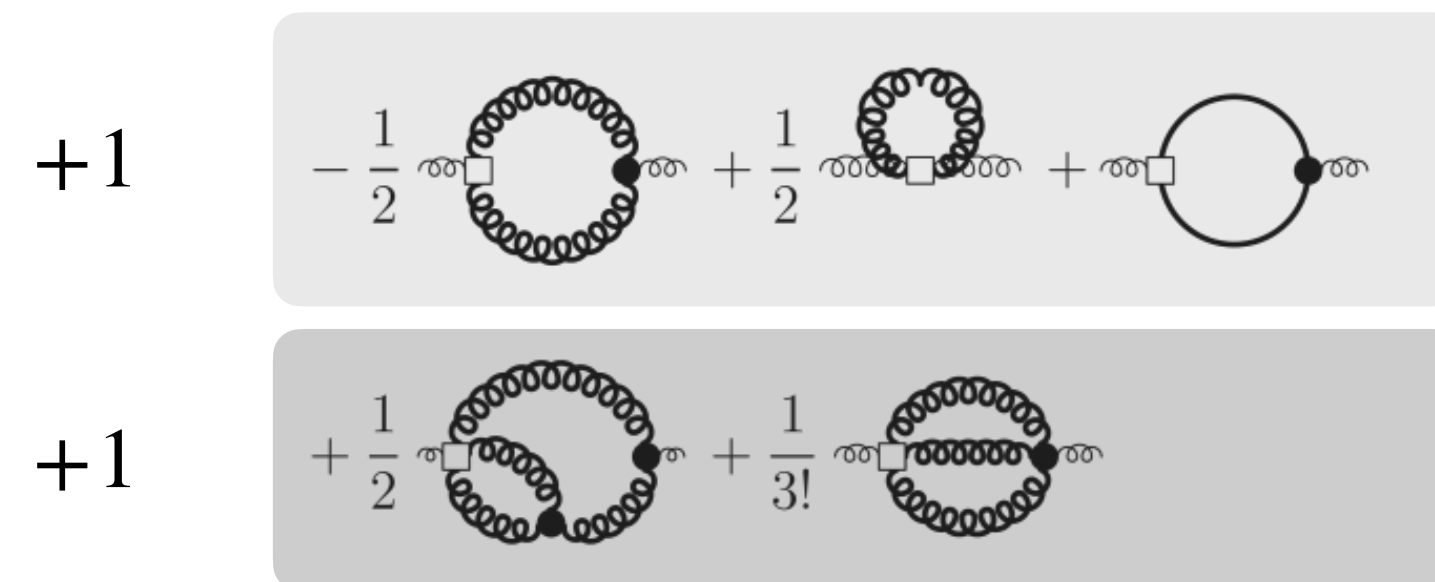
agrees with experiment for 10 digits



QCD: strong interaction

gluon propagator

$$\text{gluon propagator}^{-1} = \text{tree-level} + \text{1-loop}$$



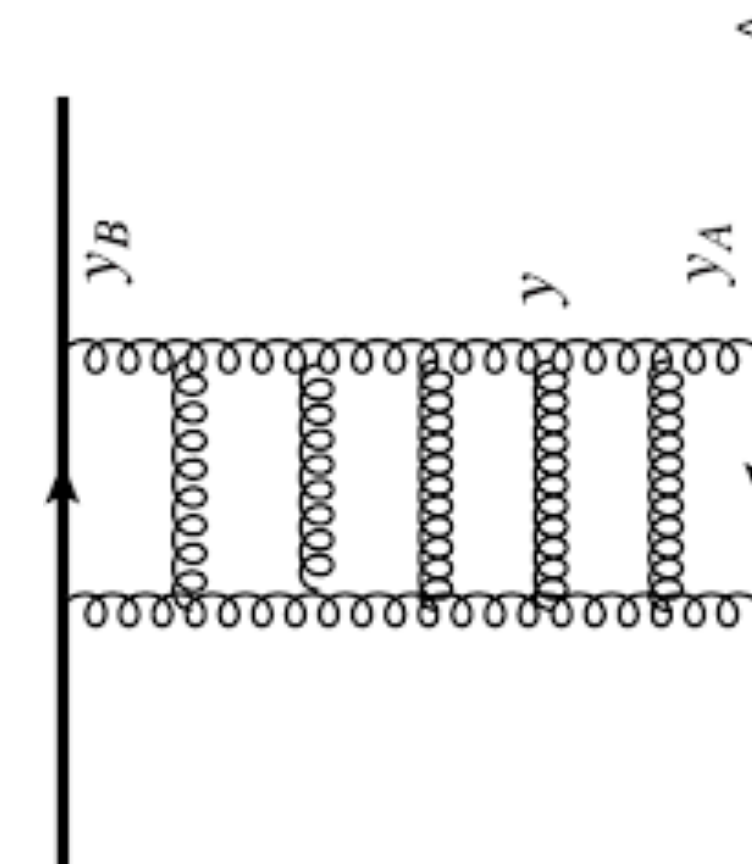
Masses of the mesons

β function in gauge theories

$$\beta_1(g_{YM}) = \mu \partial_\mu g_{YM} = -\frac{g_{YM}^3}{16\pi^2} \left(\frac{11}{3}N - \frac{1}{6} \sum C_{scalars} - \frac{1}{3} \sum C_{fermions} \right)$$

mass of the glueball?

Millenium prize: 1 million dollar



$\mathcal{N} = 4$ SYM is conformal

Maximally symmetric theory

**is conformal
coupling is not running**

masses are zero

$$\beta_1(g_{YM}) = -\frac{g_{YM}^3}{16\pi^2} \left(\frac{11}{3}N - \frac{1}{6} \sum N - \frac{1}{3} \sum N \right) = 0$$

$$\beta = \beta_1 = 0$$

all fields in the adjoint

consequence of $\mathcal{N} = 1$ SUSY

consequence of $\mathcal{N} = 2$ SUSY

calculate correlators of gauge invariant operators

gauge transformations

$$\chi(x) \rightarrow g(x)\chi(x)g^{-1}(x) \quad \chi = \{ \mathcal{F}_{\mu\nu}, \Psi_\alpha, \bar{\Psi}_{\dot{\alpha}}, \Phi_a \}$$

gauge invariant operators

$$\mathcal{O}(x) = \text{Tr}(\chi_1(x)\chi_2(x)\dots\chi_n(x))$$

Conformal invariance

translations

rotations

boosts

scale transformations

inversion

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{|x|^{2\Delta(\lambda,N)}}$$

scaling dimension

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3) \rangle = \frac{C_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3} x_{13}^{\Delta_1+\Delta_3-\Delta_2} x_{23}^{\Delta_2+\Delta_3-\Delta_1}}$$

3pt couplings

$$x_{ij} = |x_i - x_j|$$

Operators product expansion

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\dots\mathcal{O}_n(x_n) \rangle = \sum_k C_{12k} \langle \mathcal{O}_k(x_2)\dots\mathcal{O}_n(x_n) \rangle$$

Scaling dimensions

Conformal invariance

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda, N)}}$$

$$\mathcal{O}(x) = \text{Tr}(\chi_1(x) \chi_2(x) \dots \chi_n(x))$$

$$\chi = \{ \mathcal{F}_{\mu\nu}, \Psi_\alpha, \bar{\Psi}_{\dot{\alpha}}, \Phi_a \}$$

protected operators

$$\mathcal{O} = \text{Tr}(Z^L) = Z_i^j Z_j^k \dots Z_n^i$$

$$X = \Phi_1 + i\Phi_2$$

$$Y = \Phi_3 + i\Phi_4$$

$$Z = \Phi_5 + i\Phi_6$$

$$\langle \mathcal{O}(x) \bar{\mathcal{O}}(0) \rangle = \frac{1}{x^{2\Delta_{\mathcal{O}}}} \quad \Delta_{\mathcal{O}} = L$$

some non-protected operator

$$\mathcal{O} = \text{Tr}(Z^{L-2} X^2)$$

$$\Delta_{\mathcal{O}}(\lambda) = L + \gamma(\lambda)$$

anomalous dimension

$$N \rightarrow \infty$$

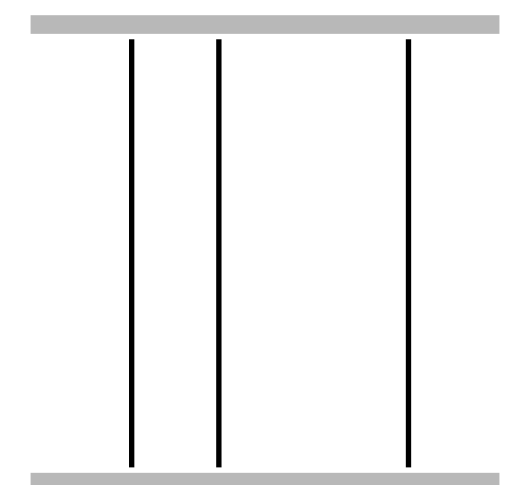
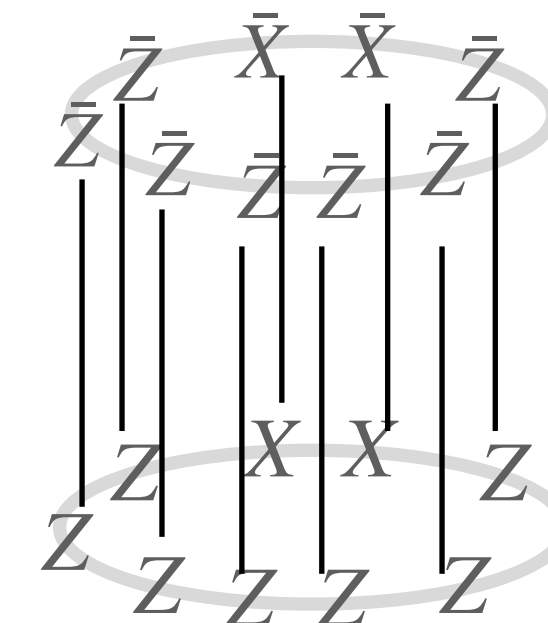
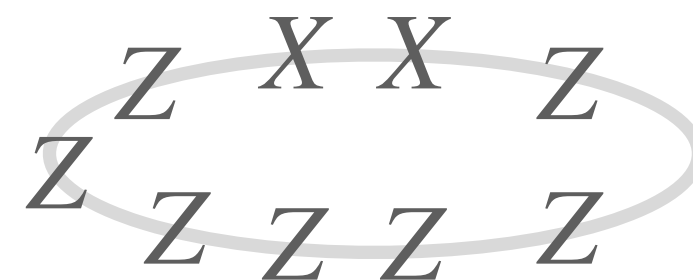
perturbation theory

$$\langle \mathcal{O}(x) \bar{\mathcal{O}}(0) \rangle = \frac{1}{x^{2L}} (1 - \gamma \ln \Lambda^2 x^2 + \dots)$$

$$\langle Z_i^j(x) \bar{Z}_k^l(y) \rangle \sim \frac{\delta_{il} \delta_{jk}}{|x-y|^2}$$

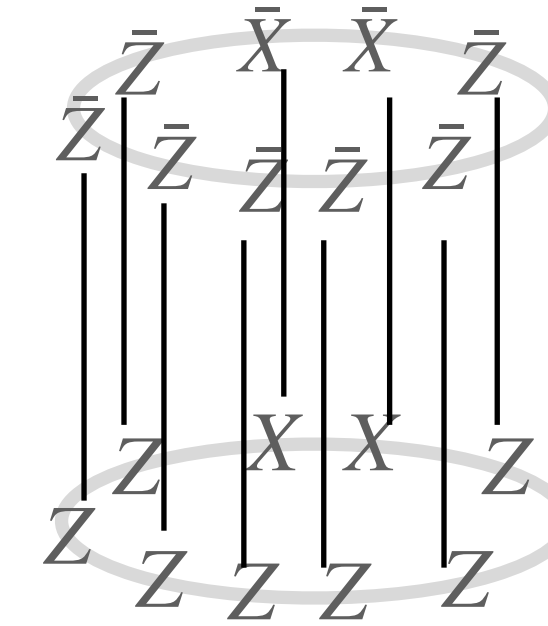
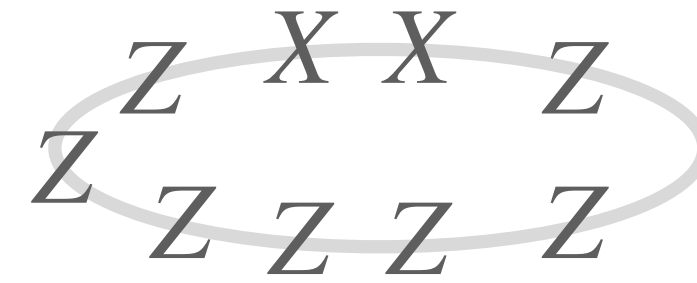
$$\text{UV regularization} \int \frac{dz^4}{|z-x|^4 |z-y|^4} \sim \frac{2}{|x-y|^4} \int_{\Lambda^{-1}}^{|x-y|} \frac{dr d\Omega_3}{r} \sim \frac{2\pi^2}{|x-y|^4} \ln(\Lambda^2 |x-y|^2)$$

$$\mathcal{O} = \text{Tr}(Z^{L-2} X X) = Z_i^j Z_j^k \dots Z_l^m X_m^n X_n^i$$



Scaling dimensions

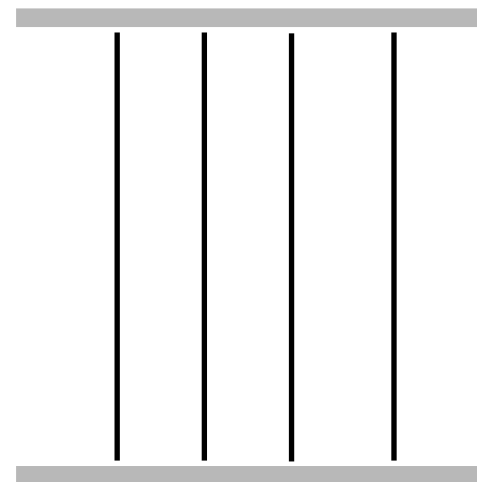
$$\mathcal{O} = \text{Tr}(Z^{L-2}XX) = Z_i^j Z_j^k \dots Z_l^m X_m^n X_n^i$$



$$\langle Z_i^j(x) \bar{Z}_k^l(y) \rangle \sim \frac{\delta_{il} \delta_{jk}}{|x-y|^2}$$

$$\langle Z_i^j(x) \bar{X}_k^l(y) \rangle = 0$$

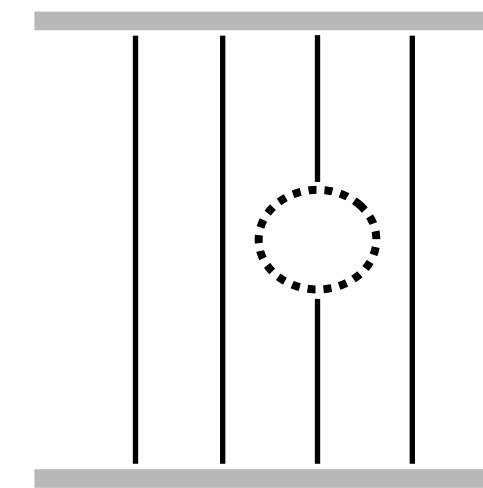
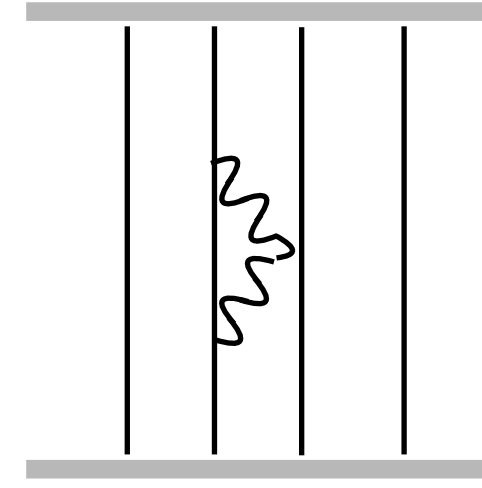
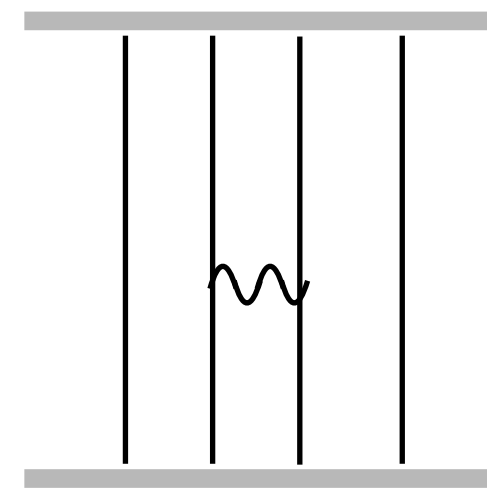
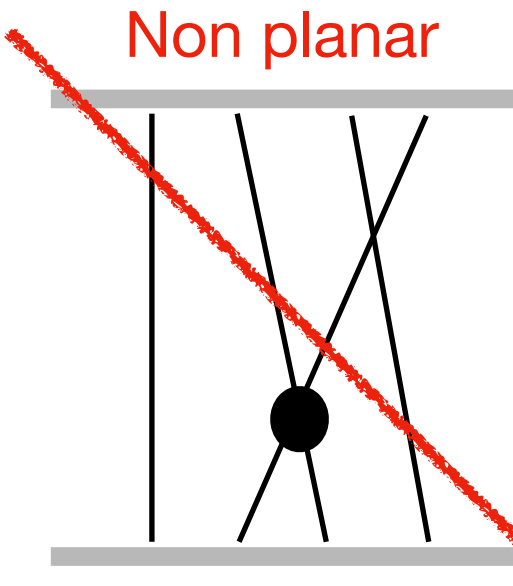
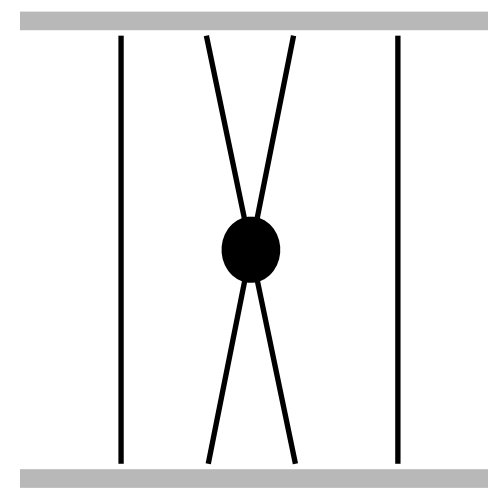
tree level



$$N \rightarrow \infty$$

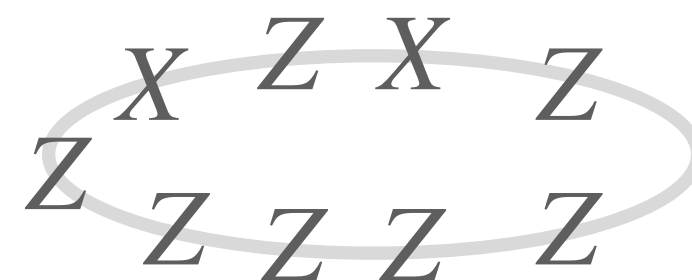
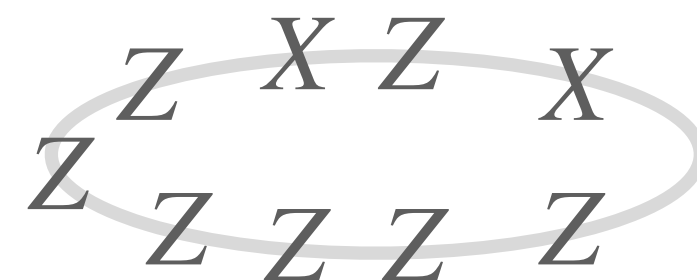
only planar diagrams

one loop



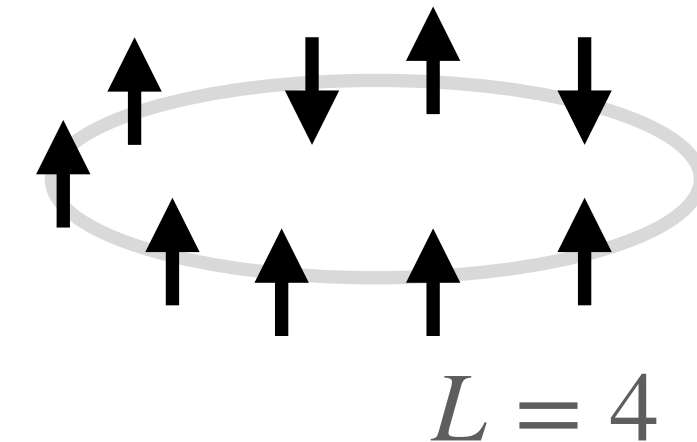
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

mixing with other operators



$$\langle \mathcal{O}(x) \bar{\mathcal{O}}(0) \rangle \propto \frac{1}{x^{2L}} (1 - \gamma_{ij} \ln \Lambda^2 x^2 + \dots)$$

mixing matrix:
Integrable XXX spin chain
Hamiltonian



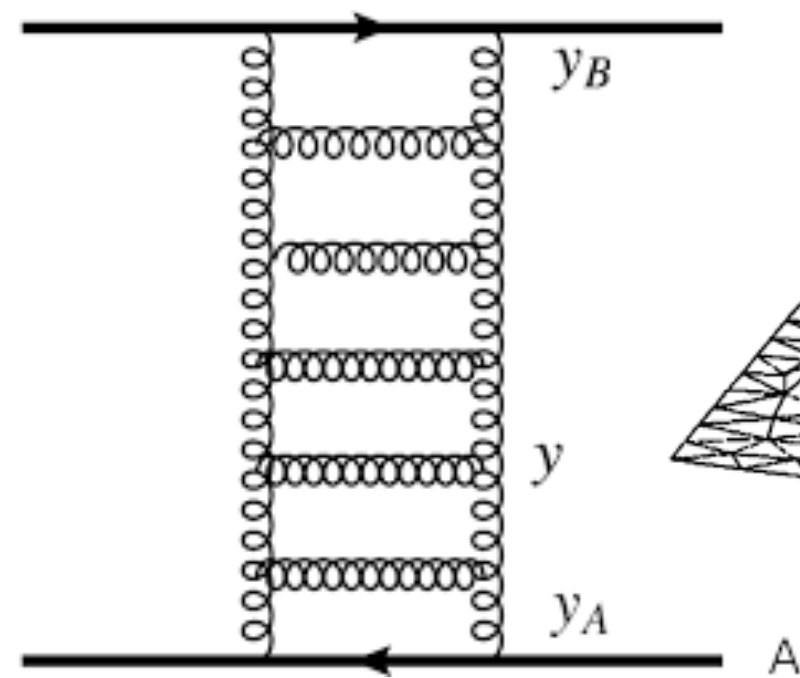
diagonalization

$$\Delta(\lambda) = 4 + \frac{3}{4}\lambda - \frac{3}{16}\lambda^2 + \frac{21}{256}\lambda^3 + \frac{324 + 864\zeta(3) - 1440\zeta(5)}{16384}\lambda^4$$

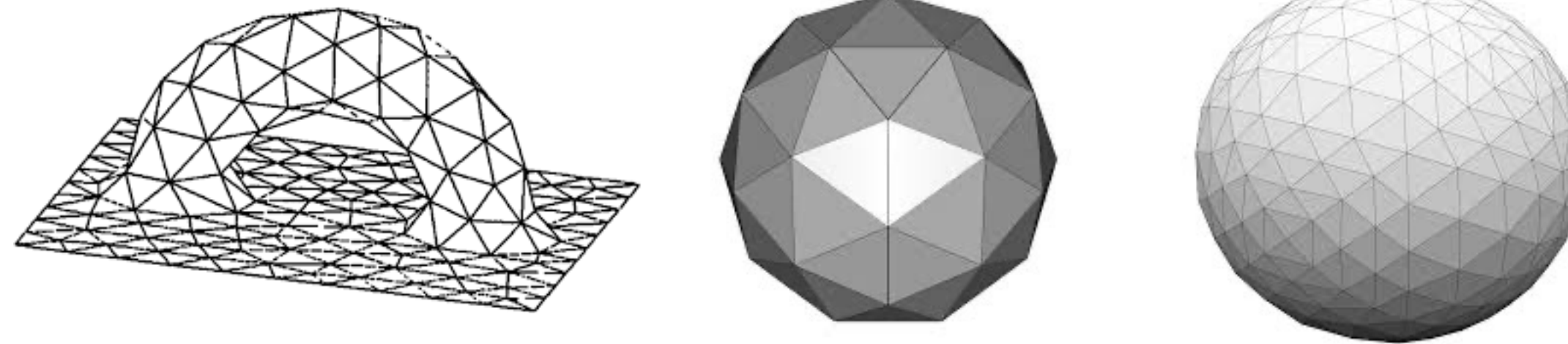
anomalous dimension
matrix

Strong coupling limit

quark-antiquark interaction

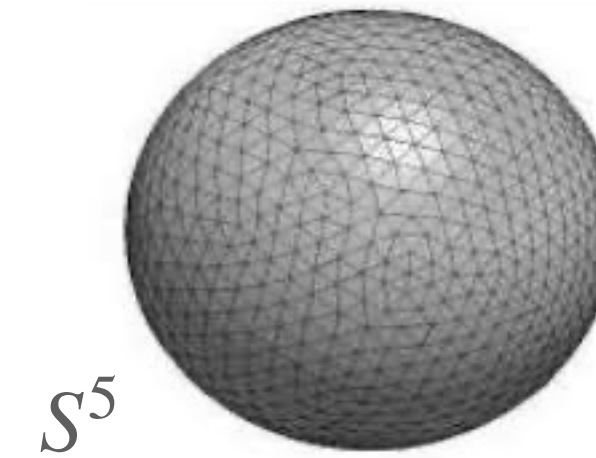


Very dense Feynmann graphs draw a surface

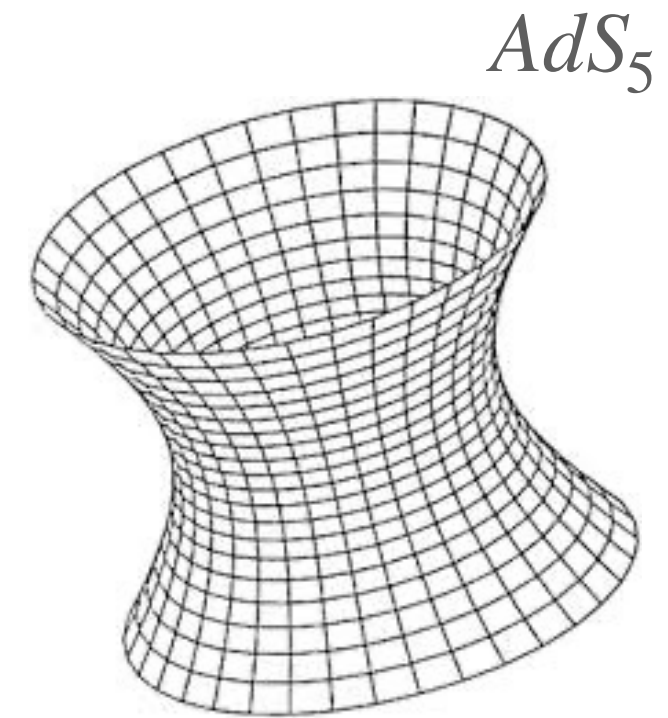


[t Hooft] $\frac{1}{N}$ genus expansion

$$X_1^2 + X_2^2 + \dots + X_6^2 = R^2$$



S^5



AdS_5

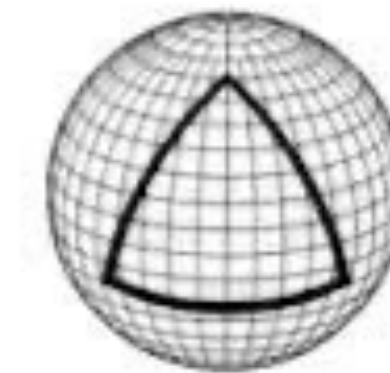
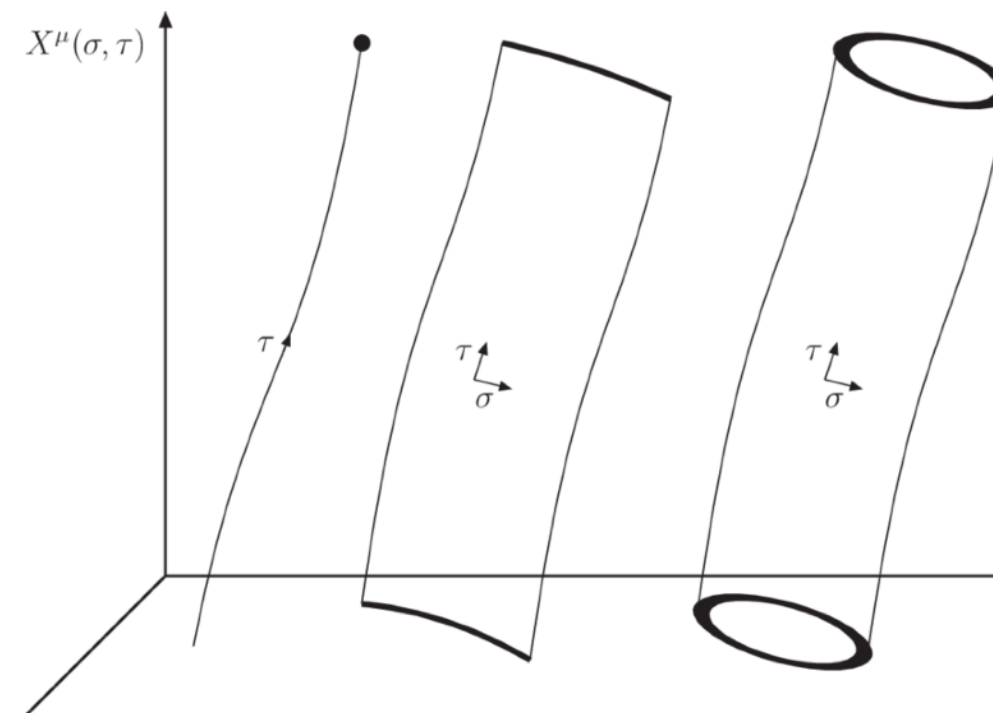
$$-Y_1^2 - Y_2^2 + \dots + Y_5^2 = -R^2$$

The world sheet is swept by a string

string action $\sqrt{\lambda} \int d\tau d\sigma \text{Area}(\tau, \sigma)$

semiclassical expansion

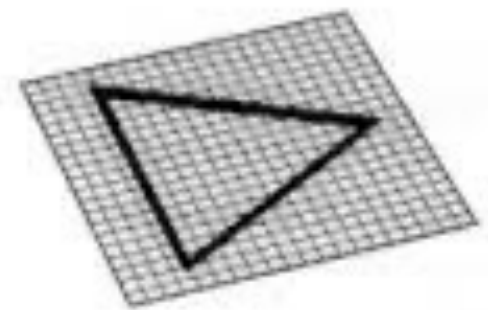
$$E = E_\infty + \frac{1}{\sqrt{\lambda}} E_{\frac{1}{2}} + \dots$$



Positive Curvature



Negative Curvature

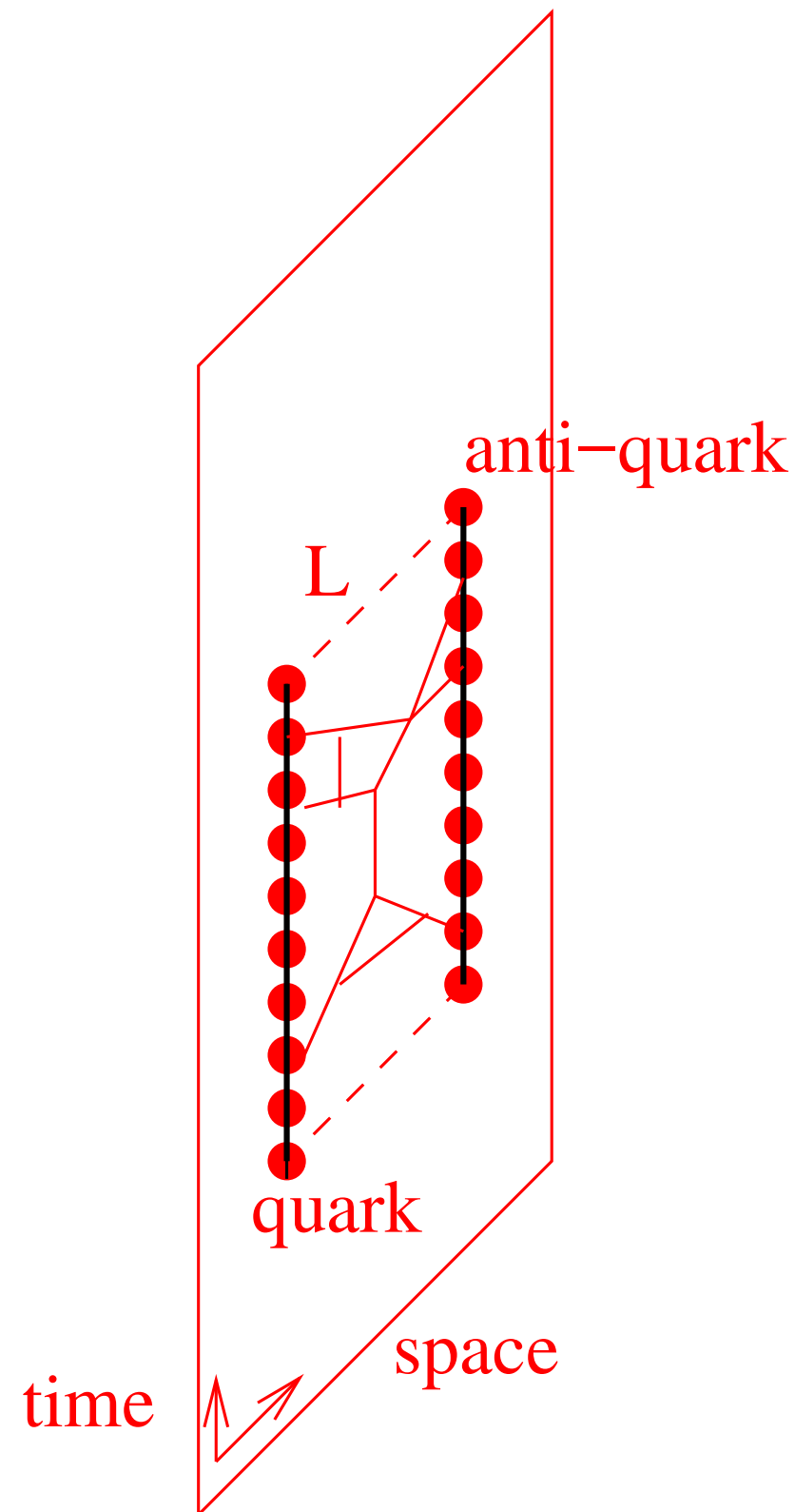


Flat Curvature

AdS/CFT duality conjecture

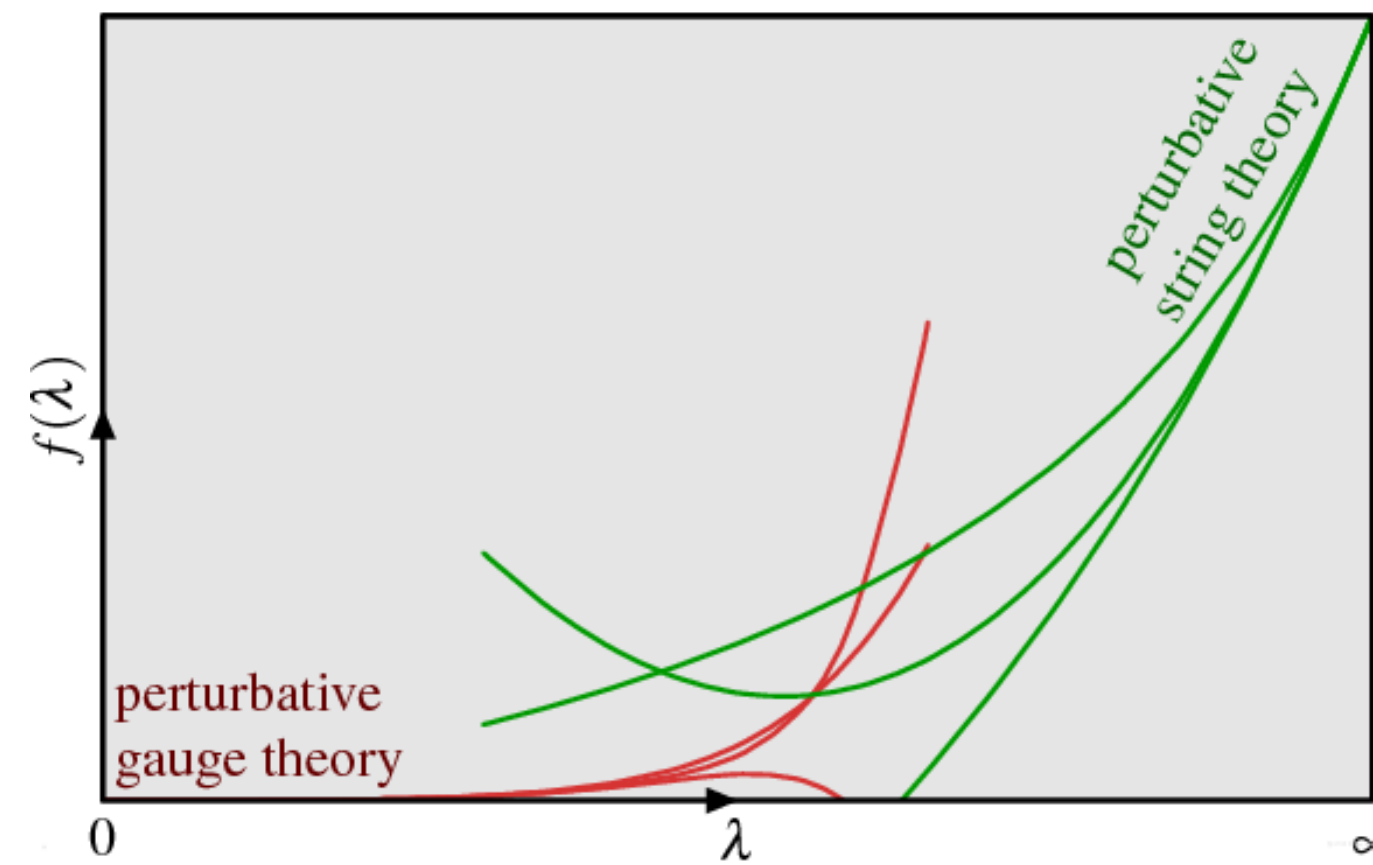
[Maldacena]

3+1 D maximally symmetric gauge theory



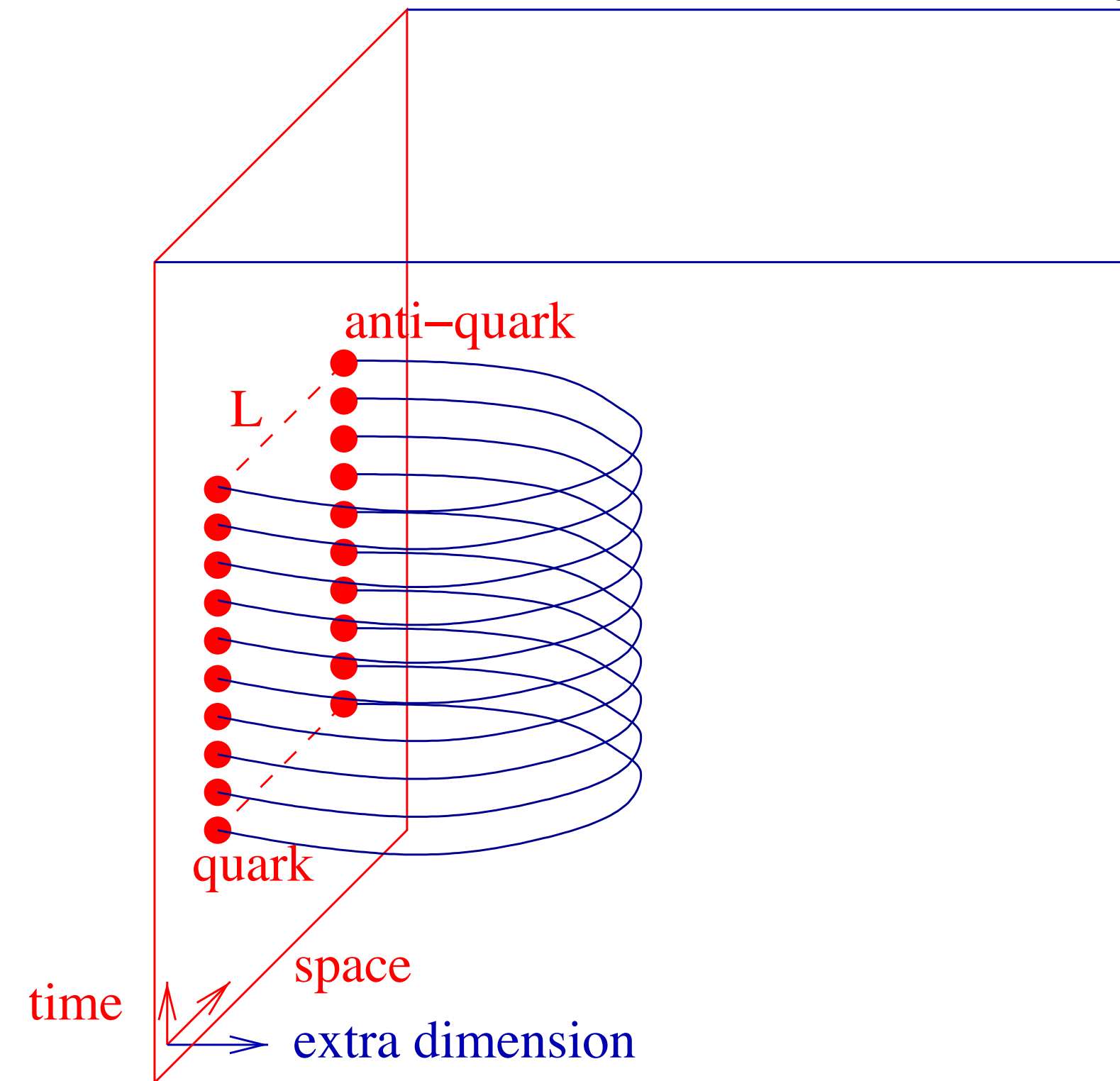
$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$

loop expansion



$$V(L) = \frac{f(\lambda)}{L}$$

9+1 D string theory on $AdS_5 \times S^5$



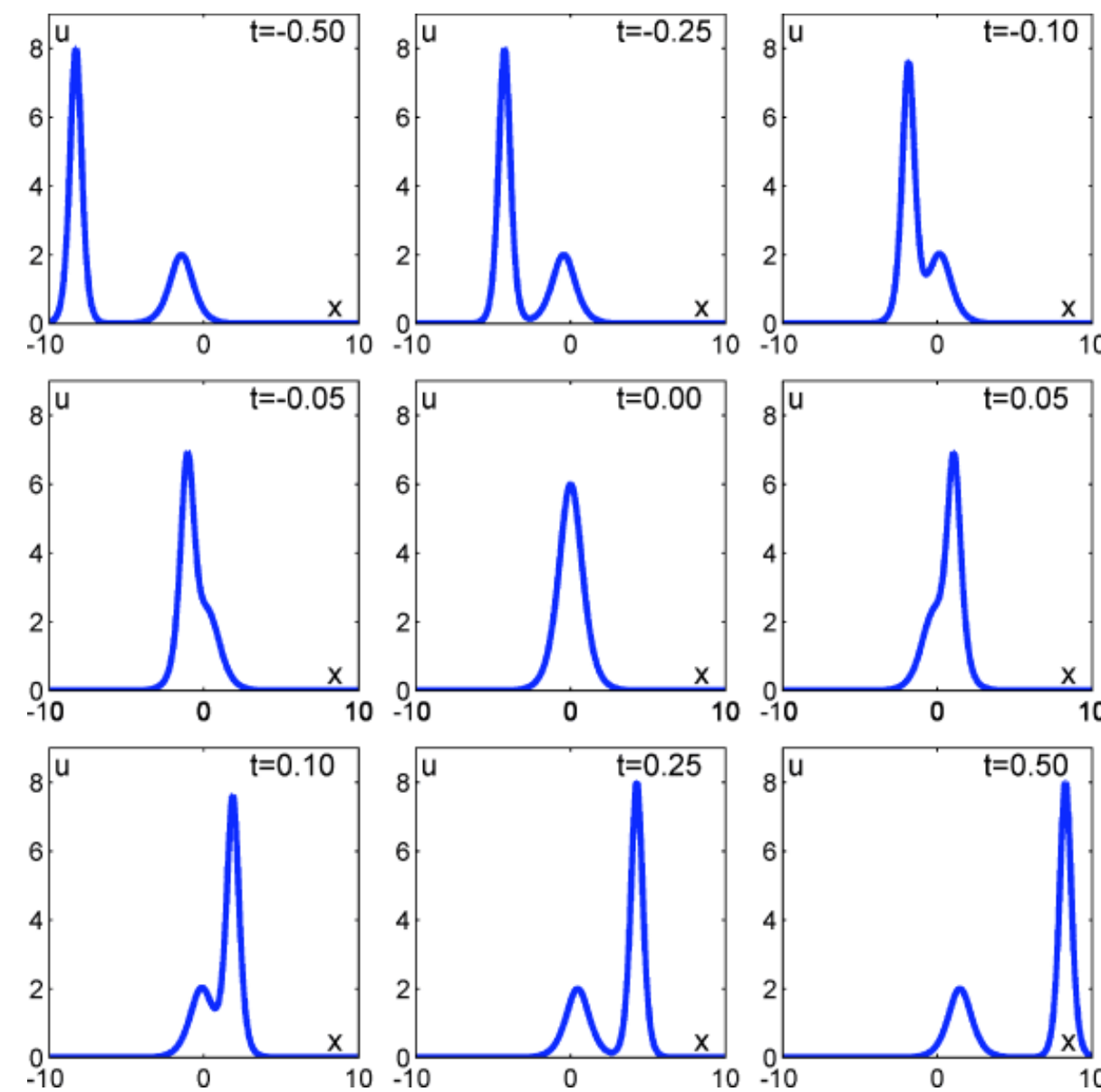
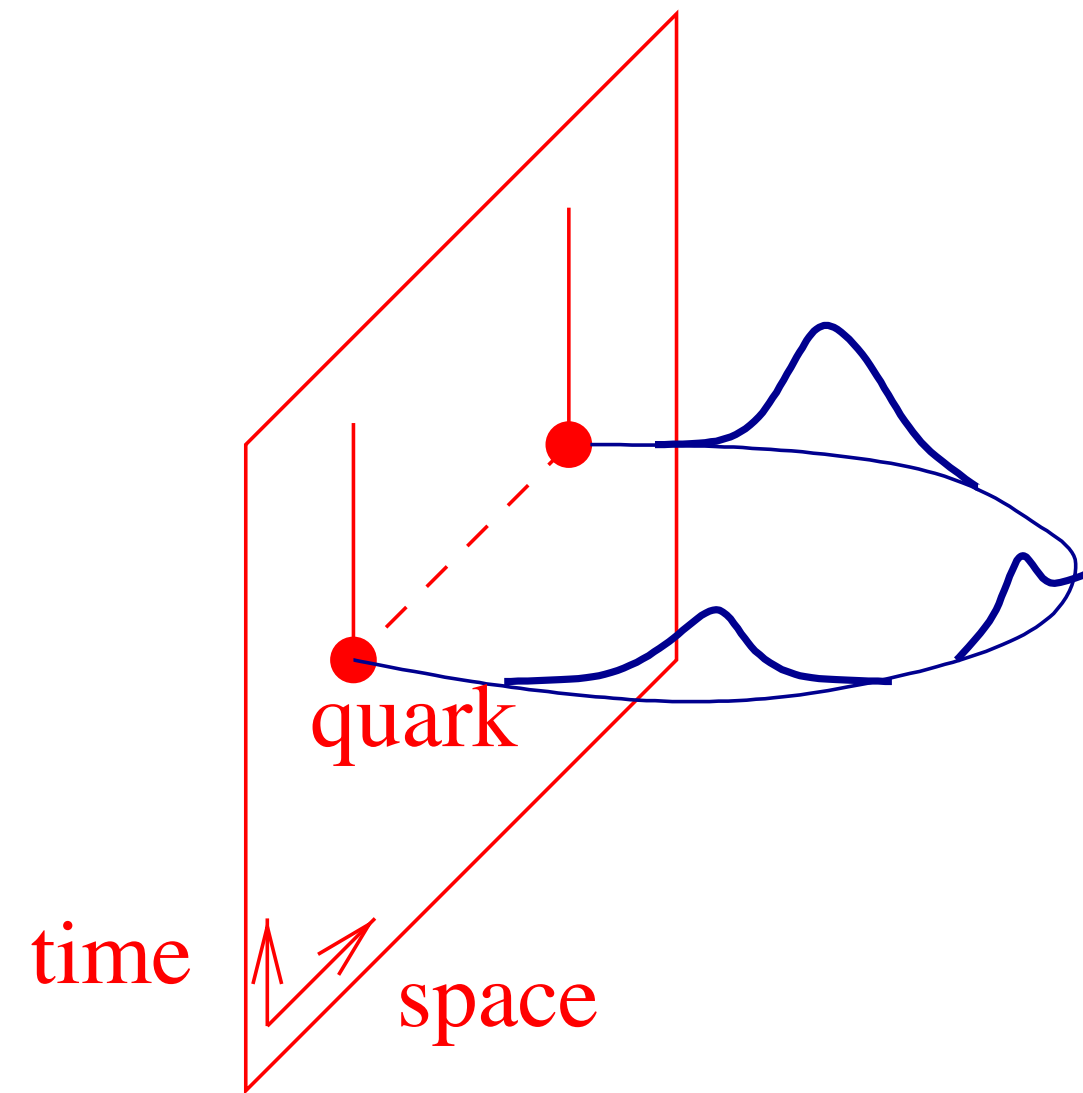
$$V(L) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimal AdS surface

quadratic fluctuations

2D integrable description

1+1 D string theory is
integrable
no particle production

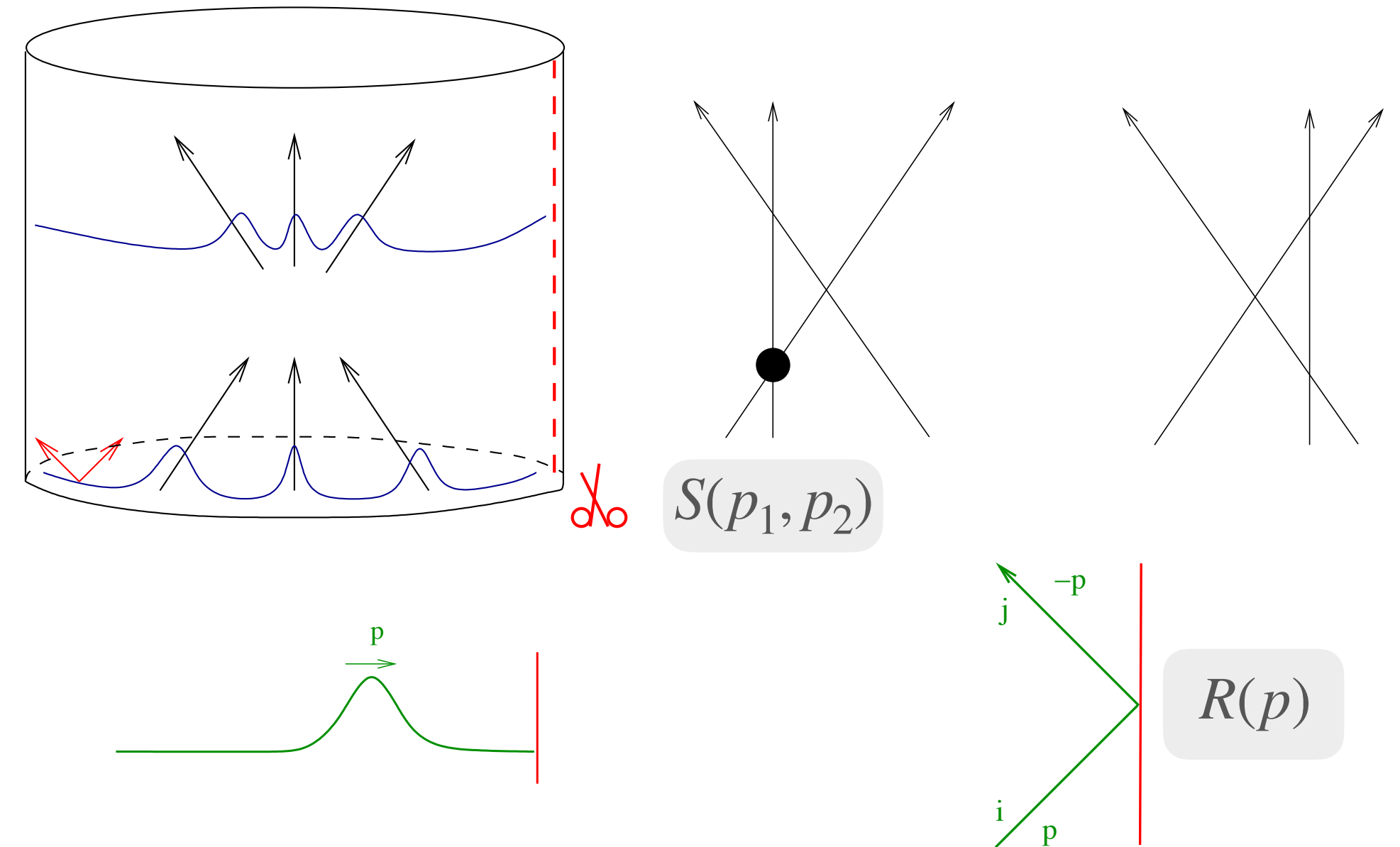


[Bajnok, Palla, Takacs]

Quark antiquark potential = 1+1 D Casimir effect
ground-state energy on the strip

$$E_0 = \int dp \log(1 - R(p)\bar{R}(p)e^{-\epsilon})$$

spacetime diagram



scattering, reflection,
dispersion relation
exact calculation

$$E(p) = \sqrt{1 + \lambda \sin^2 \frac{p}{2}}$$

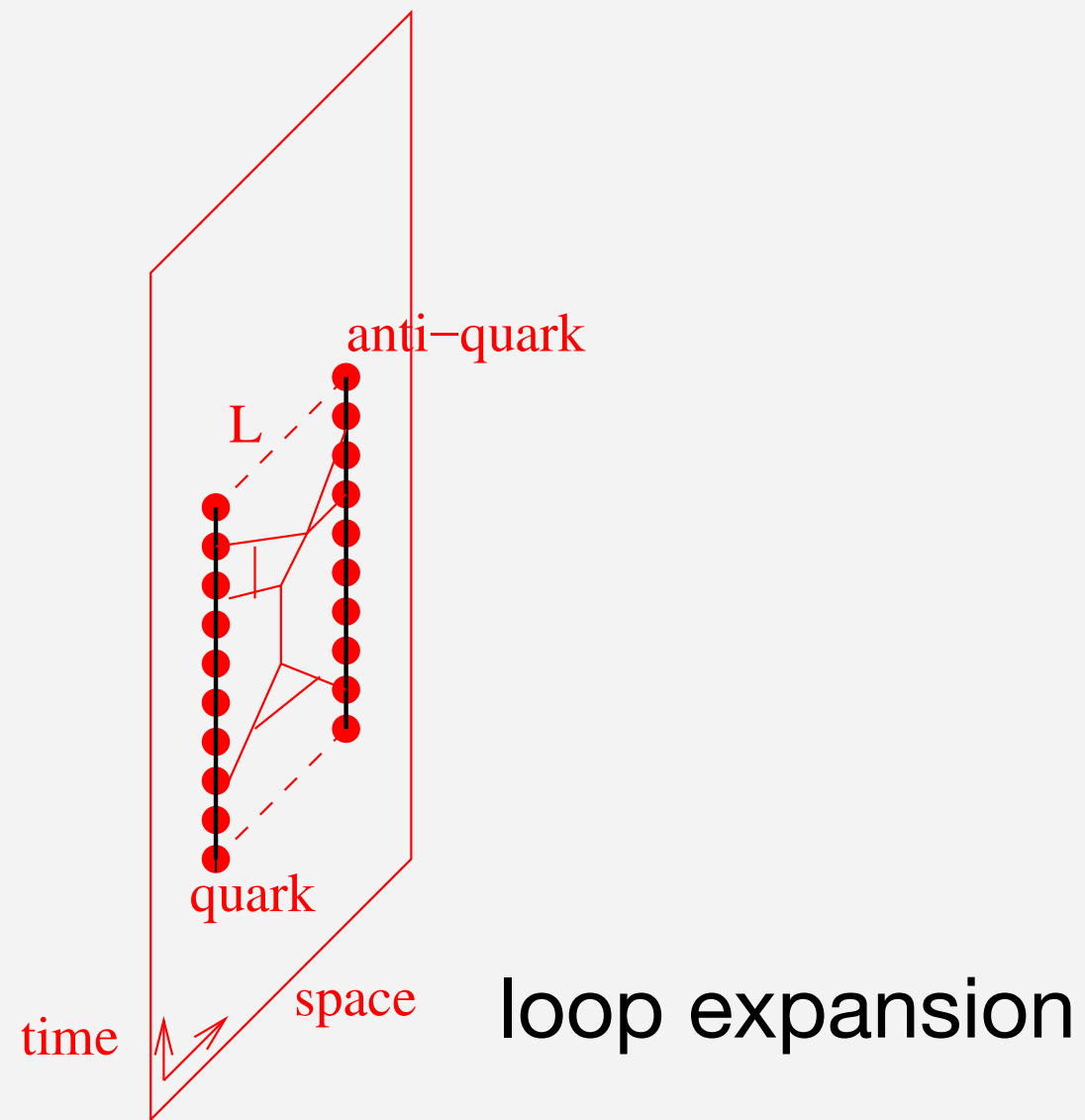
[Correa, Maldacena, Sever][Drukker]

[Bajnok, Balog et al]

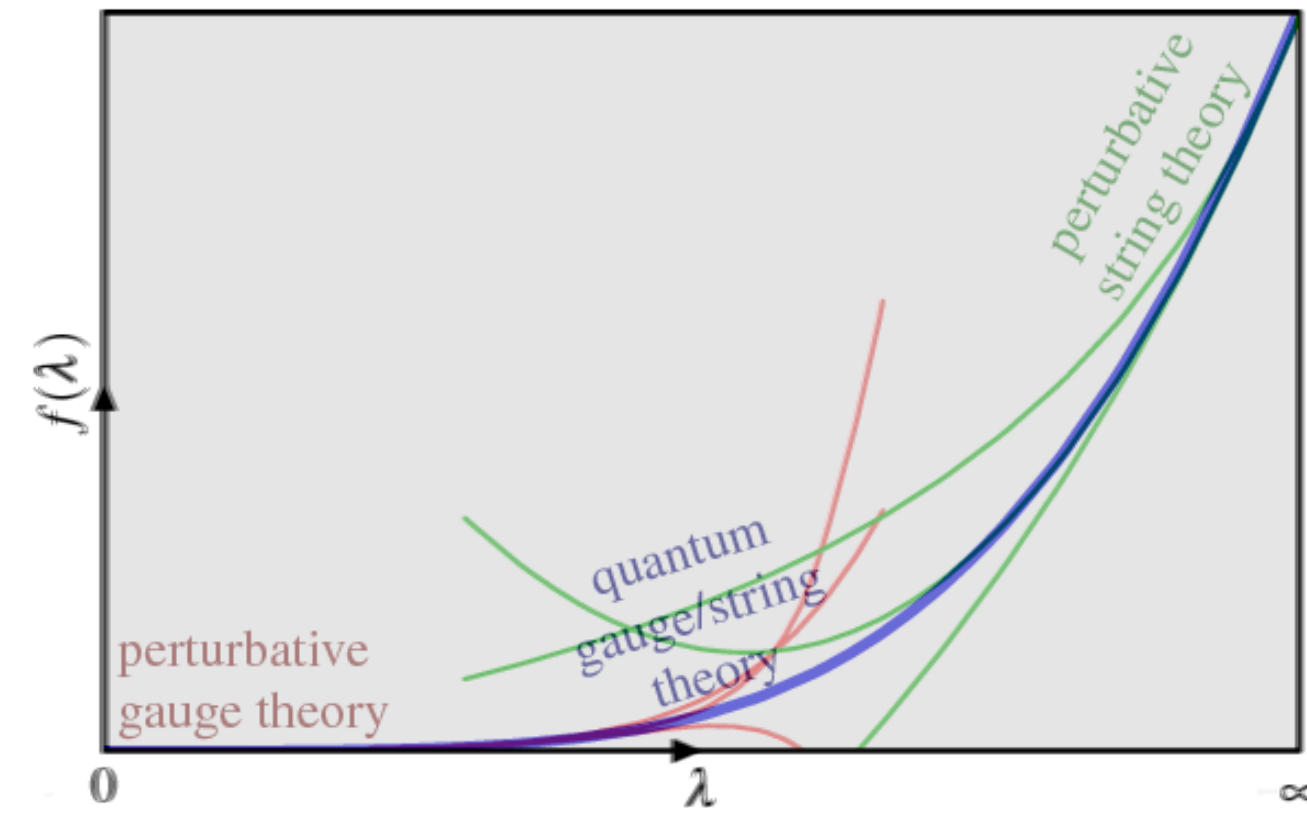
\mathcal{E} : egzakt TBA integrálegyenlet

Triality

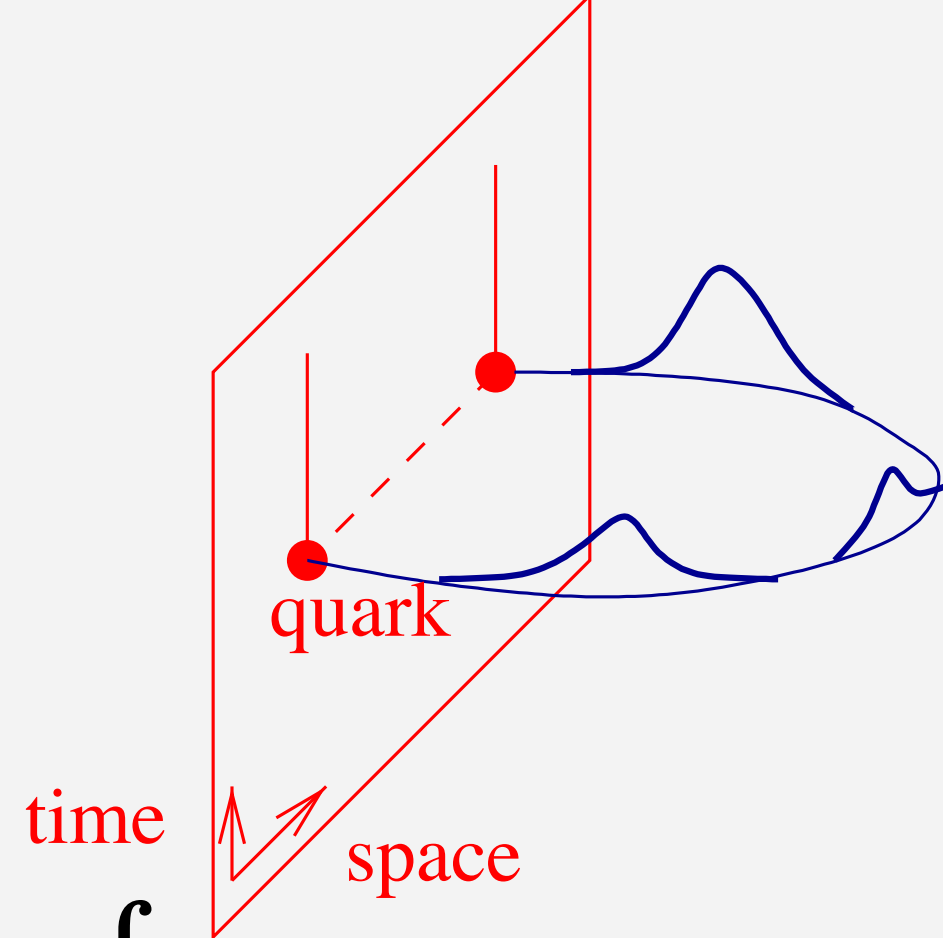
3+1 D gauge theory



$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$



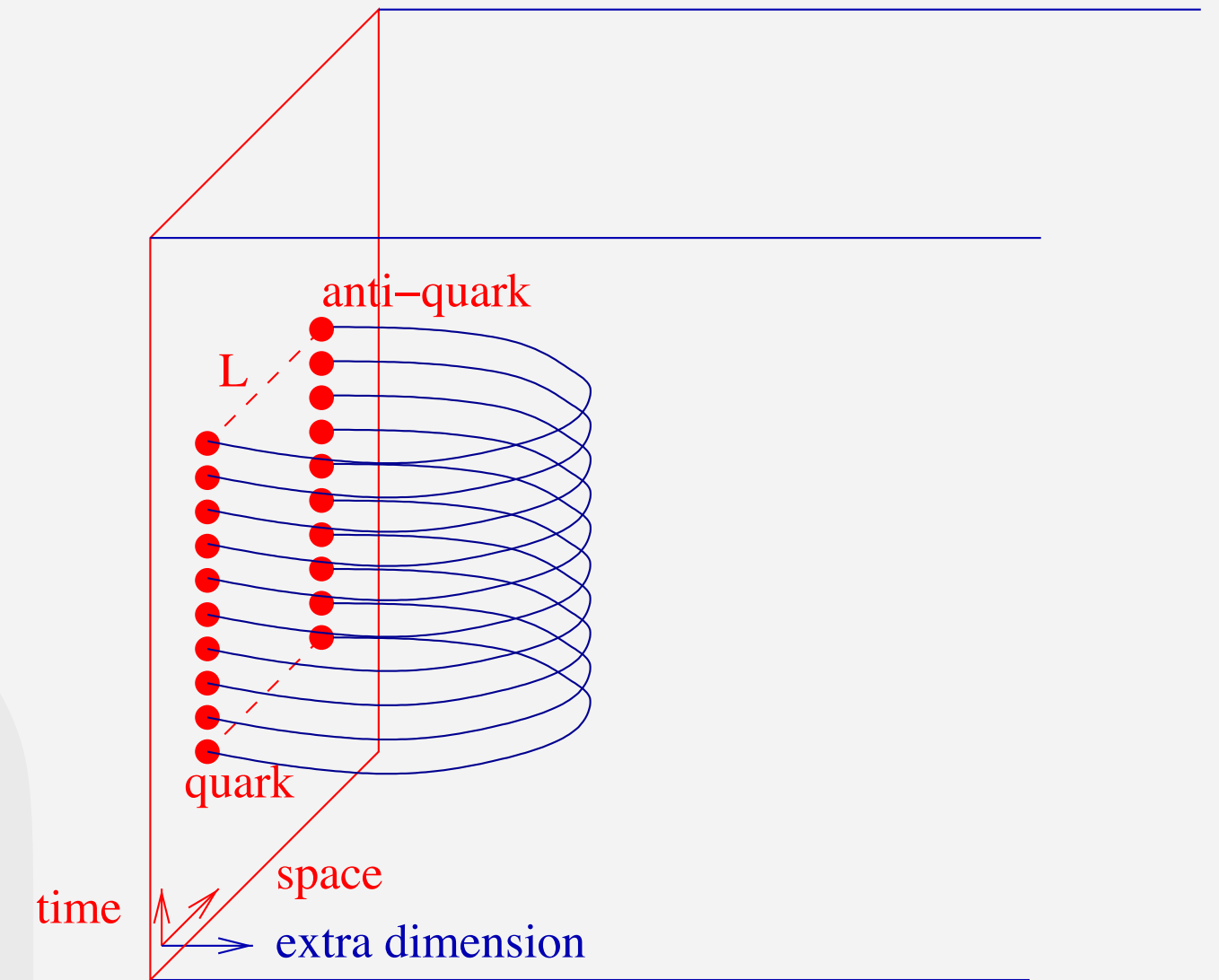
1+1 D integrable QFT



$$V(L) = L^{-1} \int dp \log(1 - R(p)\bar{R}(p)e^{-\epsilon(p)})$$

Exact groundstate energy

1+9 D string theory



$$V(L) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimal surface

fluctuations

Spectral problem

3+1 D gauge theory

Conformal Field Theory

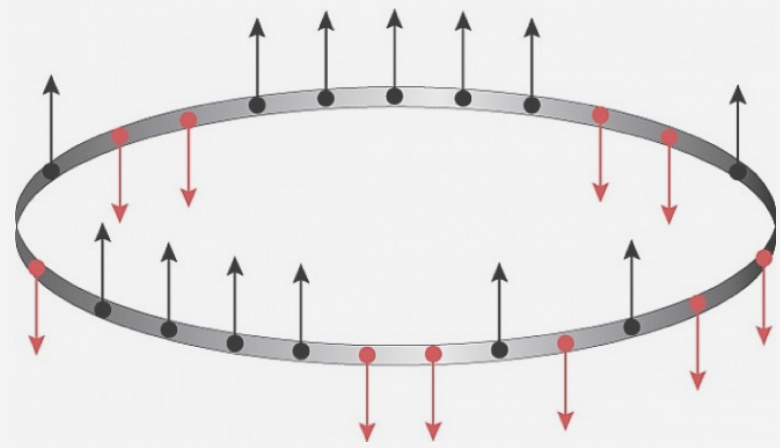
$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

$$\mathcal{O} = \text{Tr}(Z^J)$$

$$Z = \Phi_1 + i\Phi_2$$

$$X = \Phi_3 + i\Phi_4$$

$$\mathcal{O} = \text{Tr}(Z^{J-M} X^M) \quad |\uparrow\uparrow \cdot \downarrow\downarrow\rangle$$

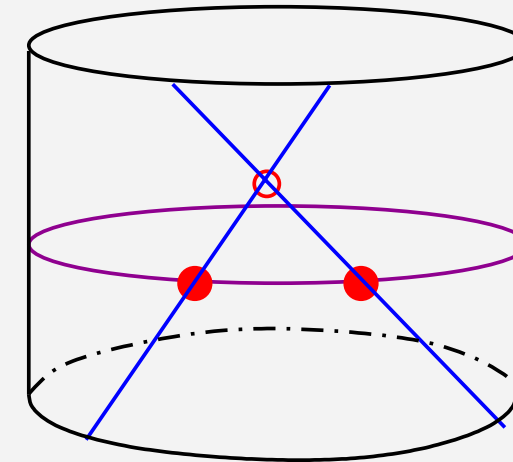


Integrable spin chain

1+1 D integrable QFT

Finite size spectrum of multiparticle states

$$E(J) = 2E(p) + \delta E(p)$$



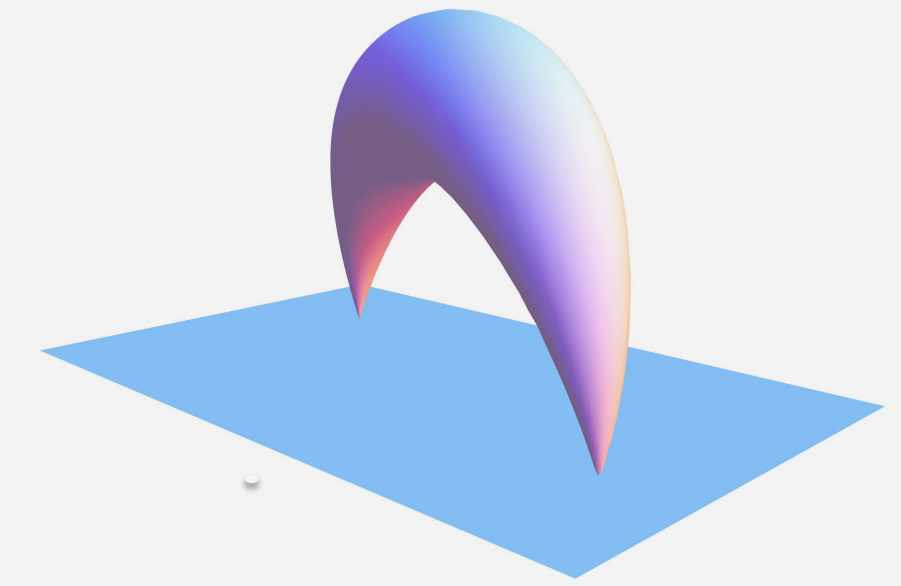
Bethe ansatz $e^{ipJ} S(p, -p) = 1$

vacuum polarisation

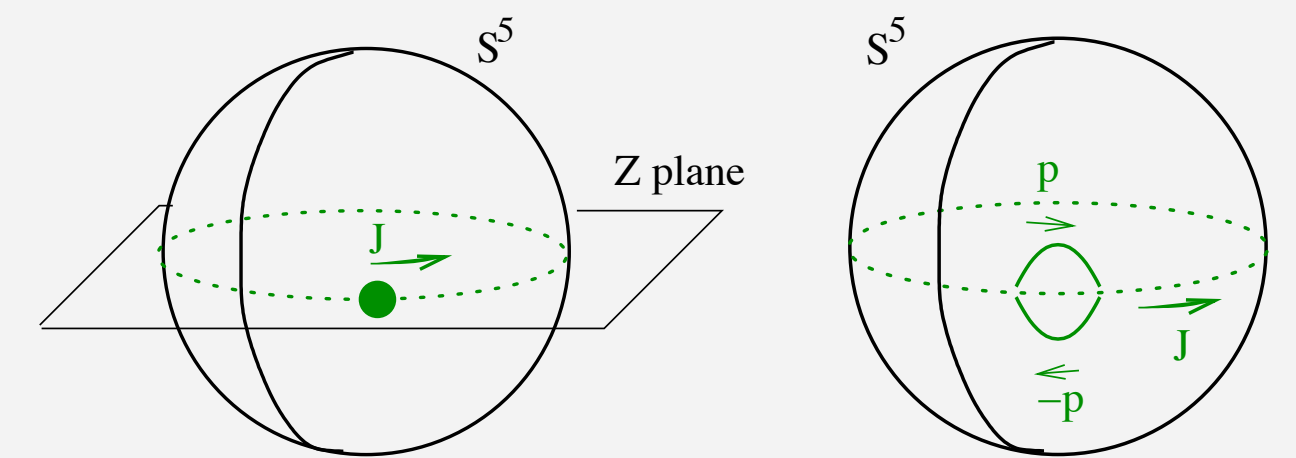
[Bajnok, Janik]

$$\delta E(p) = \int dq S(q, p) S(q, -p) e^{-\epsilon(q)}$$

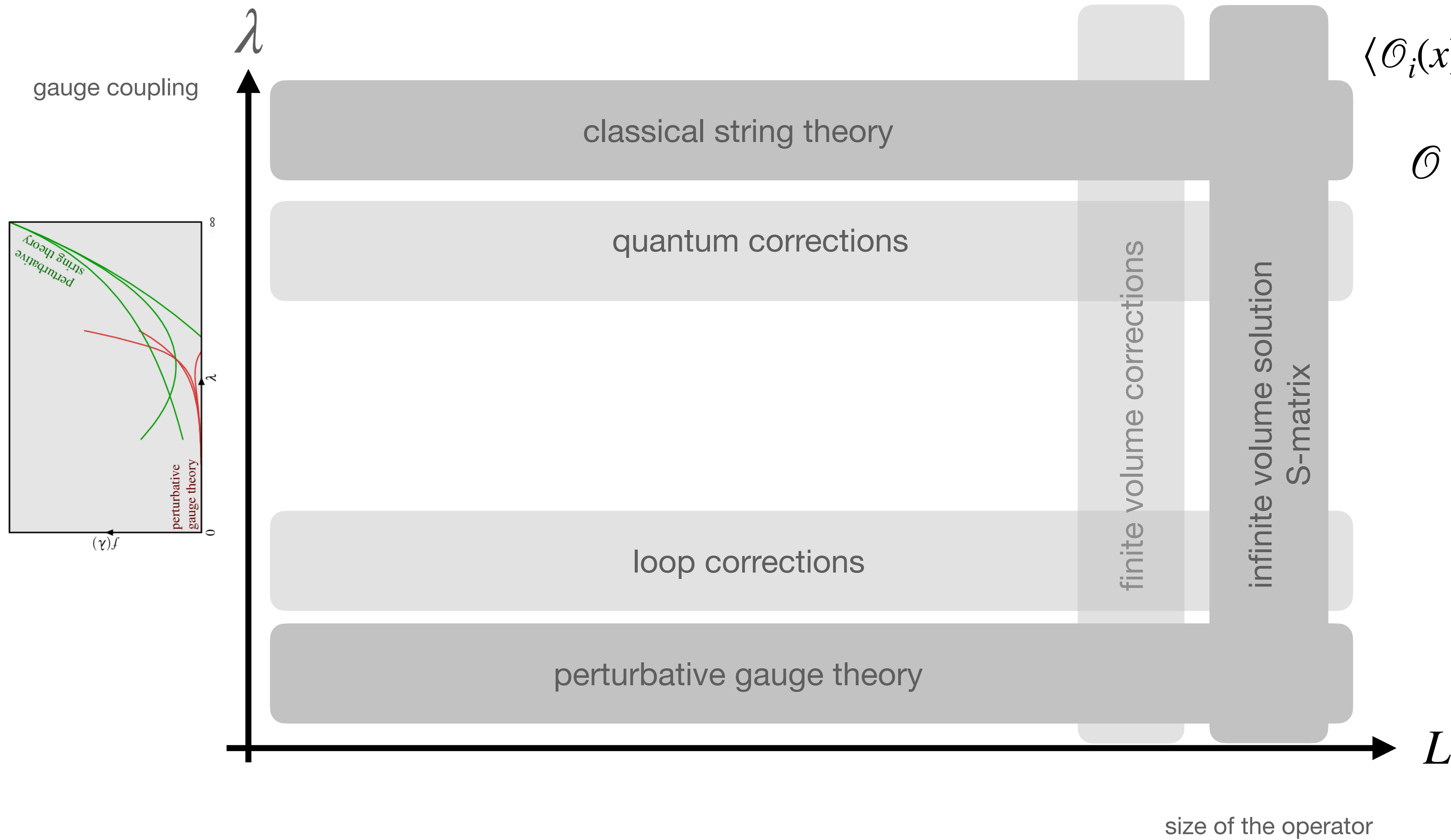
9+1 D string theory



Spinning strings' energy

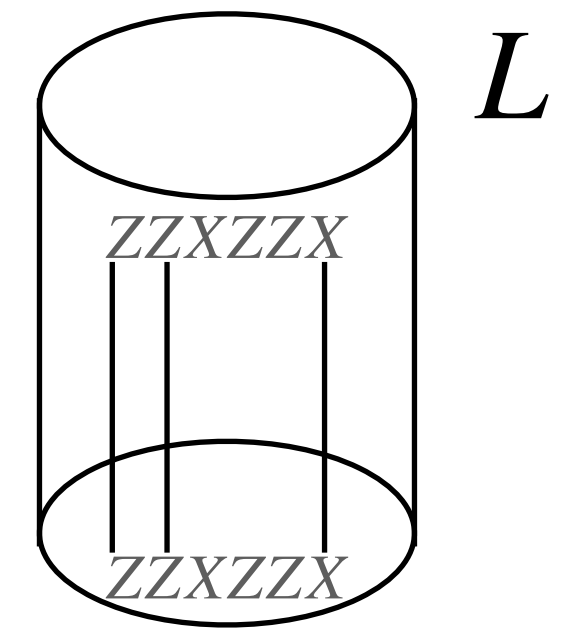


Spectral problem: how integrability works



$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

$$\mathcal{O} = \text{Tr}(Z^{L-M} X^M)$$

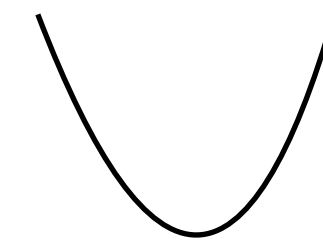


How integrability works: diagonal case

UV description

1+1 d scalar: sinh-Gordon theory

$$\mathcal{L} = \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 - \frac{m^2}{b^2} (\cosh b\varphi - 1) \quad [\text{Dorey}] [\text{Bajnok}]$$



LSZ reductions formula

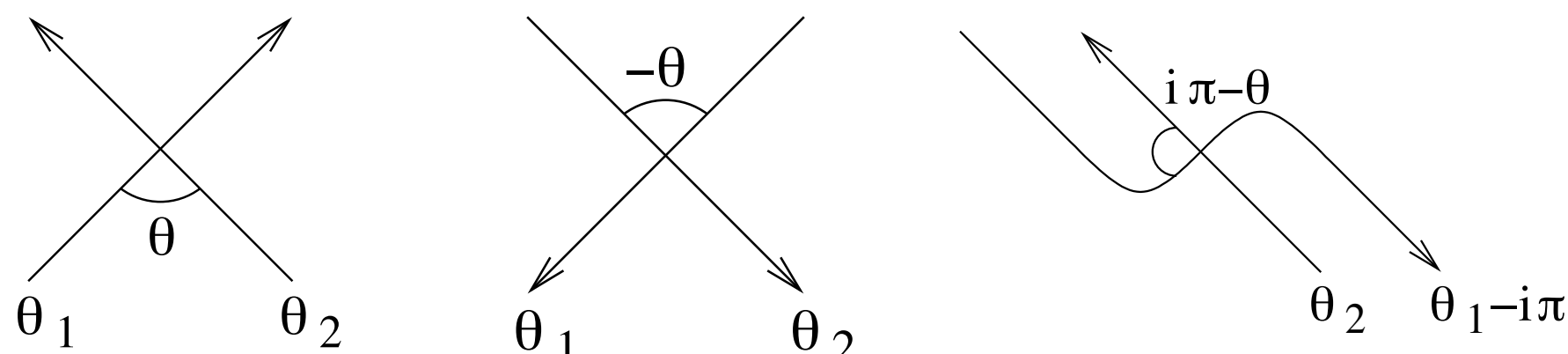
$$\mathcal{D}_j = i \int d^2 x_j e^{ip_j x - i\omega_j t} \{ \partial_t^2 - \partial_x^2 + m^2 \}$$

$$\langle p'_1, p'_2 | p_1, p_2 \rangle = \bar{\mathcal{D}}'_1 \bar{\mathcal{D}}'_2 \mathcal{D}_1 \mathcal{D}_2 \langle 0 | T(\varphi(1)\varphi(2)\varphi(3)\varphi(4)) | 0 \rangle = S(\theta_1 - \theta_2)$$

S-matrix perturbatively

$$S(\theta) = 1 - \frac{ib^2}{4 \sinh \theta} - \frac{b^4(\theta(\pi/\sinh \theta - i))}{32\pi \sinh \theta} + \frac{ib^6(\pi/\sinh \theta - i)^2}{256\pi^2 \sinh \theta} + O(b^8)$$

all perturbative orders: unitarity and crossing symmetry



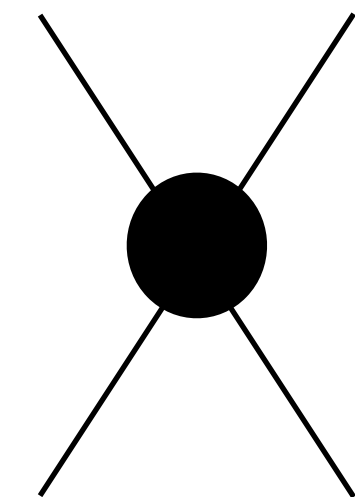
$$S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$$

IR description

one massive particle

$$p = m \sinh \theta$$

∞ many conserved charges
factorised scattering



unitarity, crossing symmetry

$$S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$$

$$S(\theta) = \frac{\sinh \theta - i \sin \pi a}{\sinh \theta + i \sin \pi a}$$

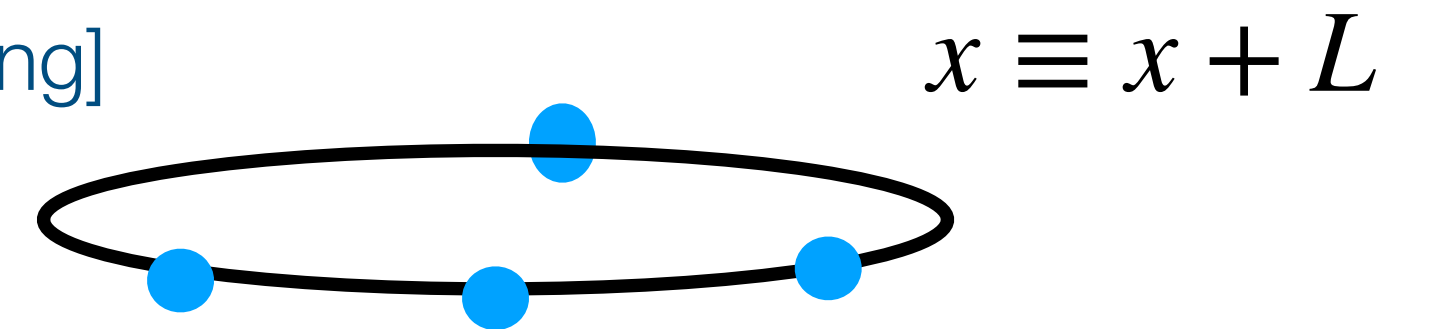
$$a = \frac{b^2}{8\pi + b^2}$$

Large volume spectrum: diagonal case

multiparticle state on the circle
momentum quantization

$$p = m \sinh \theta$$

[Bethe] [Yang]



Periodicity of wave function

$$e^{imL \sinh \theta_j} \prod_{k:k \neq j} S(\theta_j - \theta_k) = 1$$

$$\Phi_j = mL \sinh \theta_j - i \sum_{k:k \neq j} \log S(\theta_j - \theta_k) = 2\pi n_j$$

Energy

$$E(\{\theta\}) = \sum_k m \cosh \theta_k$$

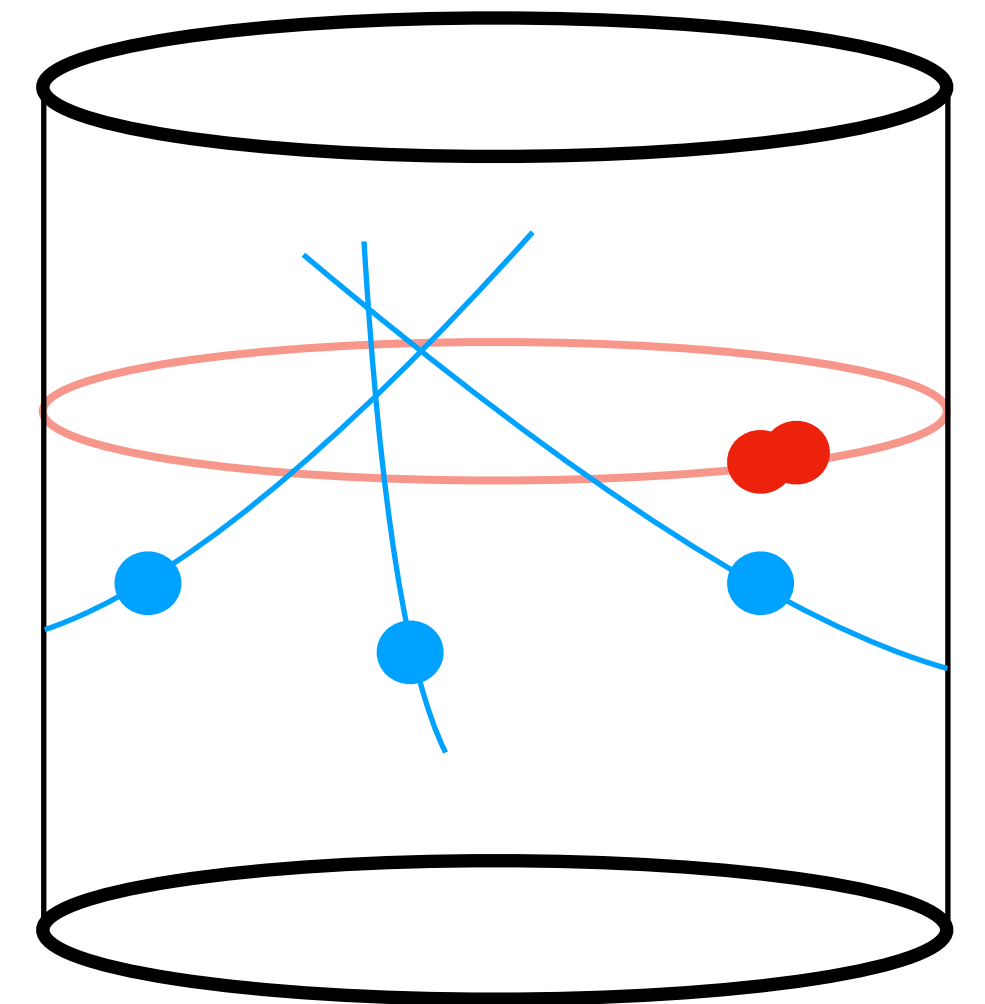
Finite size corrections

direct corrections
to energy

$$\delta E(\{\theta\}) = \int \frac{du}{2\pi} \prod_i S\left(u + \frac{i}{2\pi} - \theta_i\right) e^{-mL \cosh u}$$

corrections
to momentum quantization

$$\delta \Phi(\{\theta\}) = \int \frac{du}{2\pi} \prod_i S'\left(u + \frac{i}{2\pi} - \theta_i\right) e^{-mL \cosh u}$$



all virtual effects can be summed up by the Thermodynamic Bethe Ansatz

How integrability works: non-diagonal case

UV description

1+1 d scalar: sine-Gordon theory

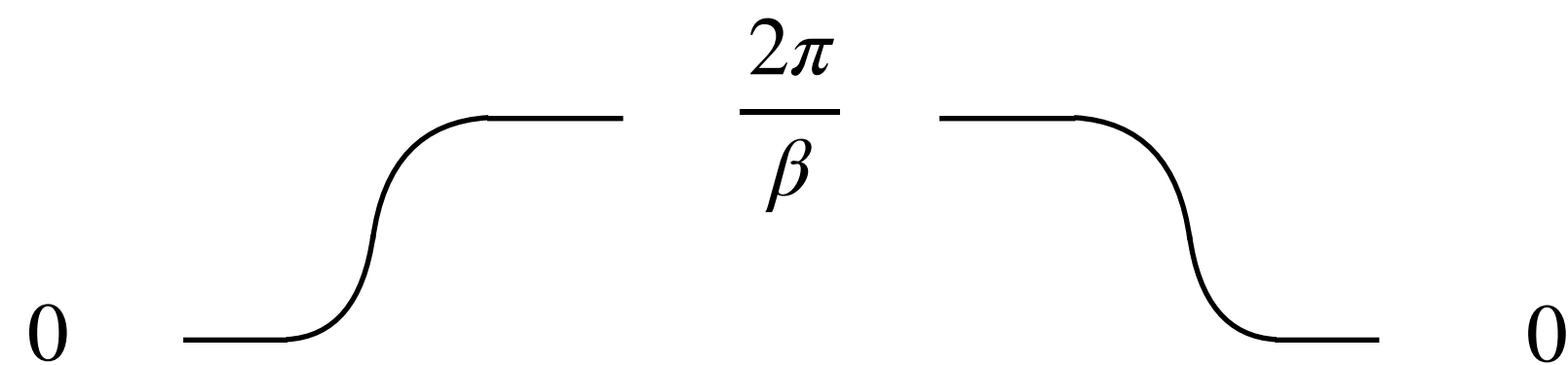
IR description

$$\mathcal{L} = \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 + \frac{m^2}{\beta^2} (\cos \beta \varphi - 1)$$

analytic continuation

$$b \rightarrow i\beta$$

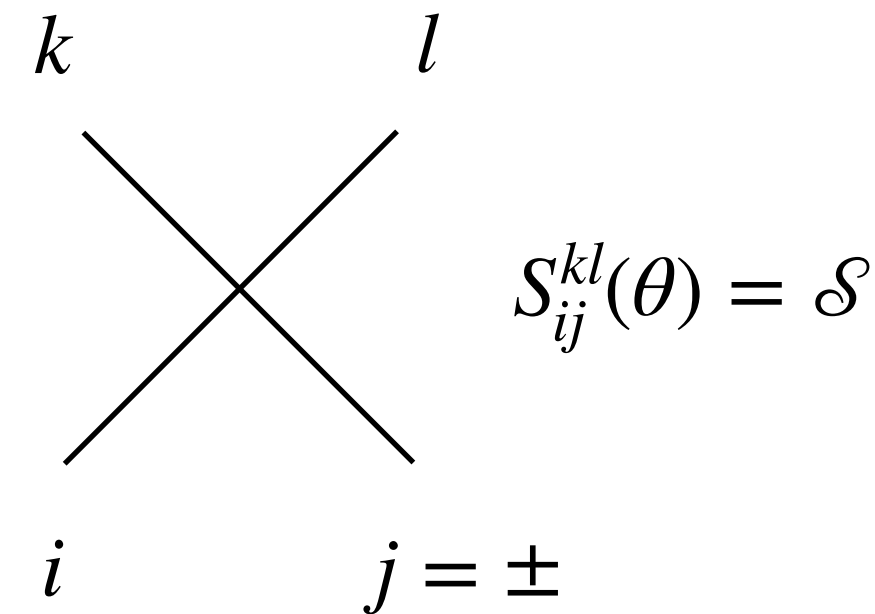
topological excitations: soliton and anti-soliton



classical scatterings: time delays

quantum symmetry $U_q(sl_2)$

[Felder, LeClair]



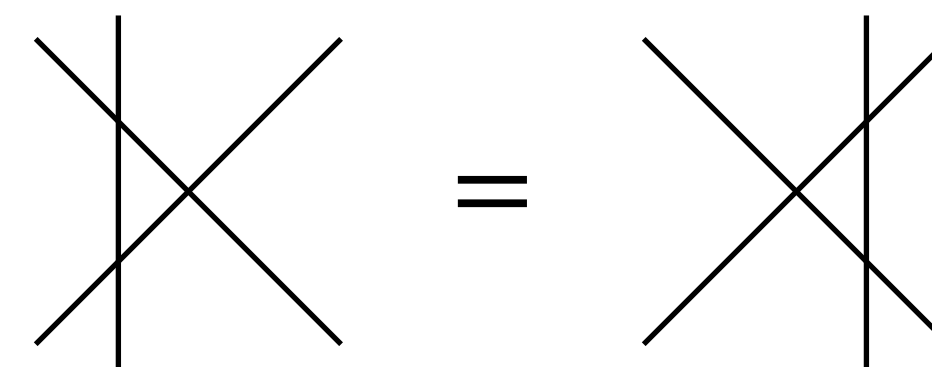
doublet of massive particles

$$p = m \sinh \theta$$

unitarity, crossing symmetry

$$\mathcal{S}(\theta) = \mathcal{S}(-\theta)^{-1} = \mathcal{C} \mathcal{S}(i\pi - \theta) \mathcal{C}^{-1}$$

**∞ many conserved charges
factorised scattering**



YBE, symmetry

$$S(\theta) = S_{XXZ} S_0(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} S_0(\theta)$$

[Zamolodchikov, Zamolodchikov]

$$b = \frac{\sin(i\lambda\theta)}{\sin(\lambda(\pi + i\theta))}$$

$$c = \frac{-\sin \lambda\pi}{\sin(\lambda\pi + i\theta)}$$

How integrability works: AdS5/CFT4

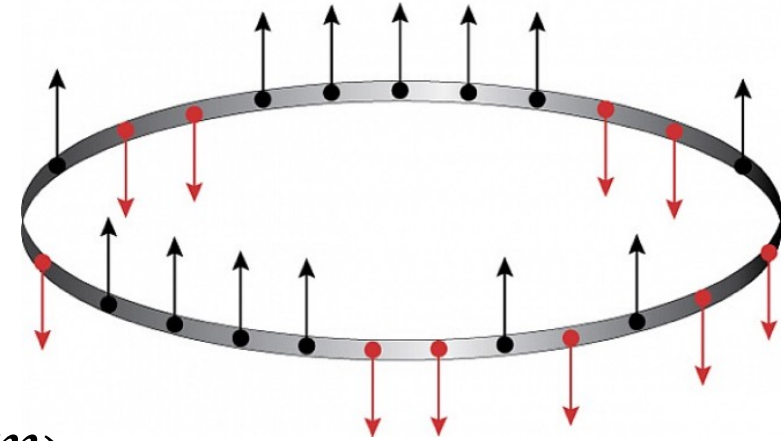
UV description

IR description

1+1 d scalar: sine-Gordon theory

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

magnonic excitations



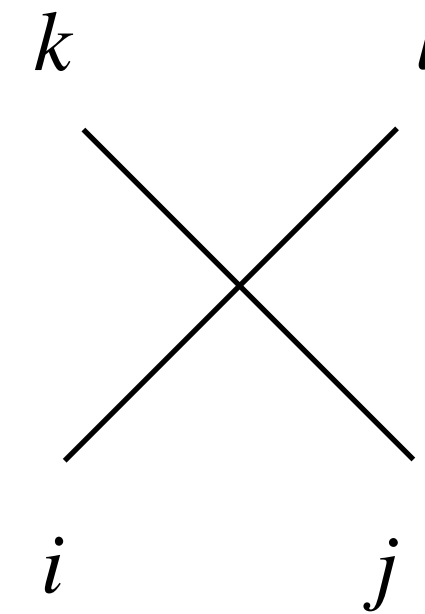
$$\mathcal{O} = \text{Tr}(Z^j \Psi Z^k \bar{\Psi} Z^l X Y D Z^m)$$

$$\sum_{n,m} e^{i(pn+qm)} \text{Tr}(\underbrace{Z \dots Z}_n X \underbrace{Z \dots Z}_{n+m} X Z \dots Z) + S(p,q) e^{i(qn+pm)} ()$$

8 boson + 8 fermion particles

quantum symmetry $psu(2|2)_c \otimes psu(2|2)_c$

dispersion relation $E(p) = \sqrt{1 + \lambda \sin^2(p/2)}$



$$S_{ij}^{kl}(\theta) = \mathcal{S}$$

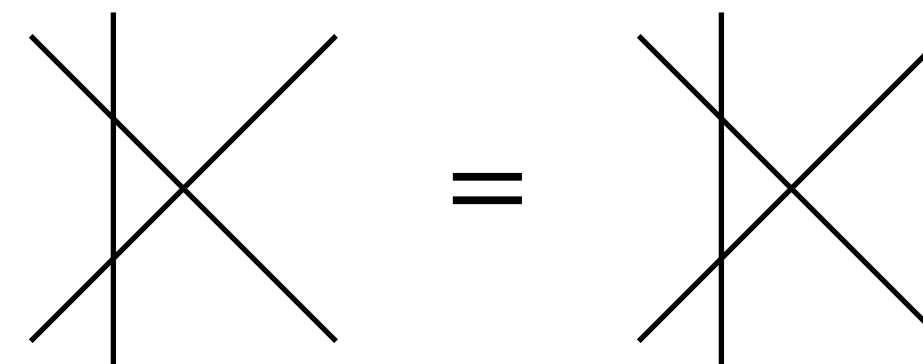
massive particles

$$E(p) = \sqrt{1 + \lambda \sin^2(p/2)}$$

unitarity, crossing symmetry

$$\mathcal{S}(p_1, p_2) = \mathcal{S}(p_2, p_1)^{-1} = \mathcal{C} \mathcal{S}(p(z_1), p(z_2 + \omega)) \mathcal{C}^{-1}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ f_1 \\ f_2 \end{pmatrix} \otimes \begin{pmatrix} \dot{b}_1 \\ \dot{b}_2 \\ \dot{f}_1 \\ \dot{f}_2 \end{pmatrix}$$



YBE, symmetry

$$\mathcal{S} = \mathcal{S}_{Hubbard} \otimes \mathcal{S}_{Hubbard} \mathcal{S}_0$$

**∞ many conserved charges
factorised scattering**

$\mathcal{S}_{Hubbard}$

16x16 sparse matrix

[Beisert]

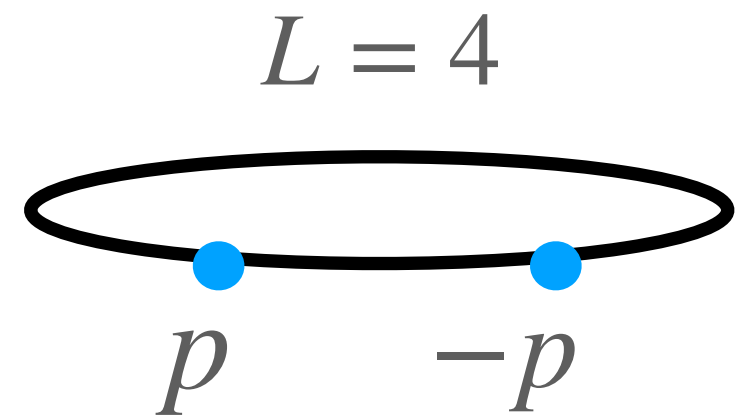
The Konishi dimension

Konishi is the simplest gauge invariant non-protected single trace operator

$$\mathcal{O} = \text{Tr}(\Phi_i^2) \sim \text{Tr}(ZZXX + \dots) \sim \text{Tr}(D^2Z^2)$$

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

asymptotic Bethe Ansatz



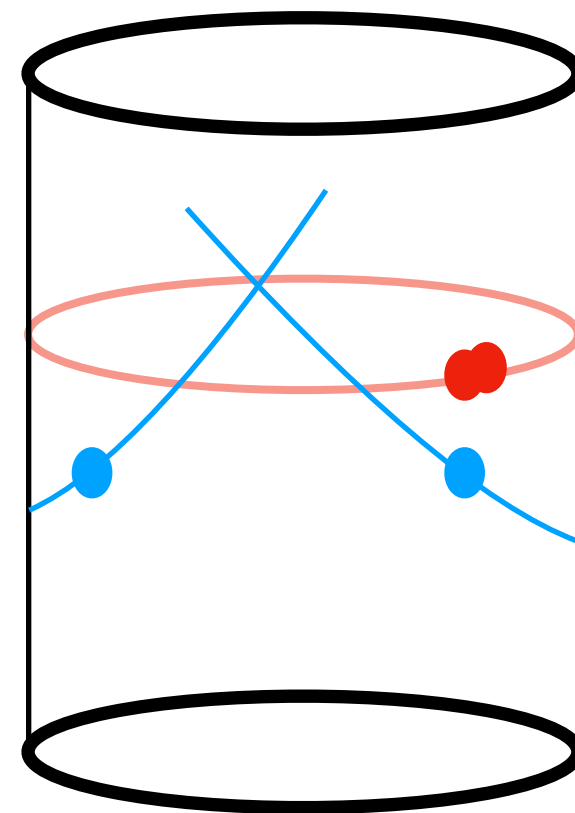
$$e^{ipL} S(p, -p) = 1 \quad p = \frac{2\pi}{3} - \frac{\sqrt{3}}{16}\lambda + \frac{9\sqrt{3}}{512}\lambda^2 + \frac{9\sqrt{3}(1 + \zeta(3))}{512}\lambda^6 + \dots$$

$$\Delta(\lambda) = 2\sqrt{1 + \lambda \sin^2(p/2)} = 4 + \frac{3}{4}\lambda - \frac{3}{16}\lambda^2 + \frac{21}{256}\lambda^3 - \frac{2820 + 288\zeta(3)}{16384}\lambda^4$$

vacuum polarisation effects

$$\delta E(\{\theta\}) = \sum_Q \int \frac{u}{2\pi} \prod_i S_Q(\tilde{u}, p) e^{-\tilde{E}(u)L}$$

[Bajnok, Janik]



$$\frac{324 + 864\zeta(3) - 1440\zeta(5)}{16384}\lambda^4$$

agrees with 4 loop gauge theory calculations

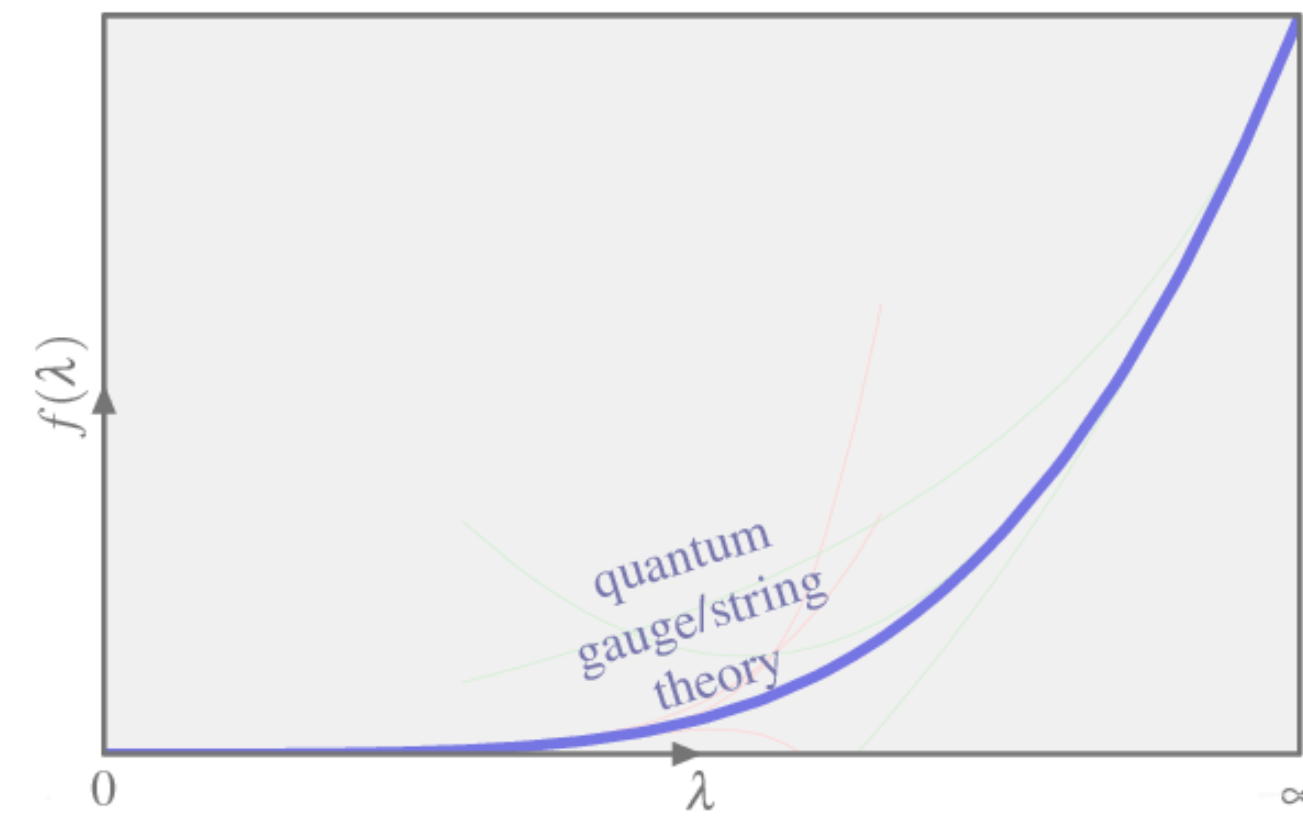
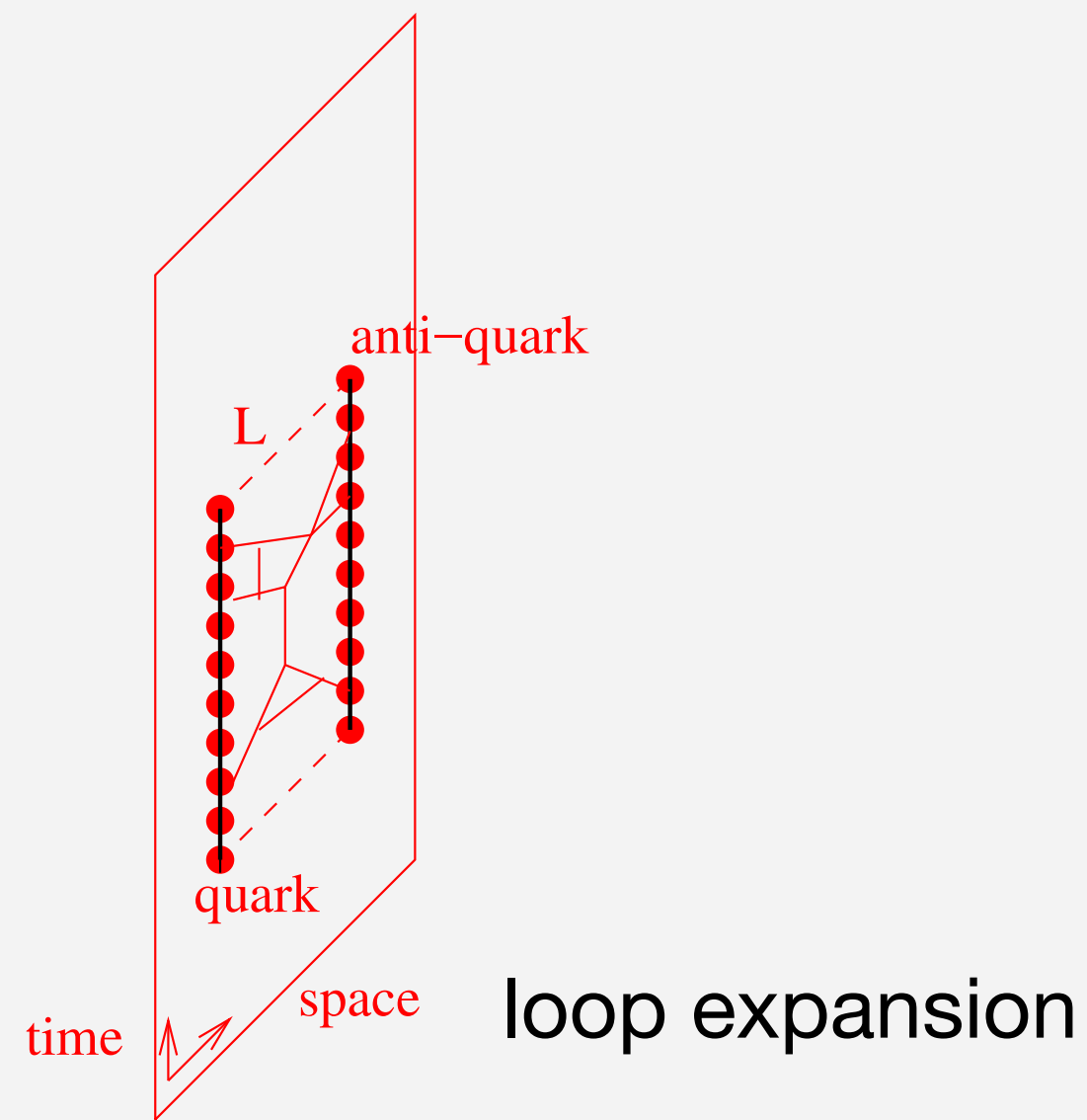


[Fiamberti et al]

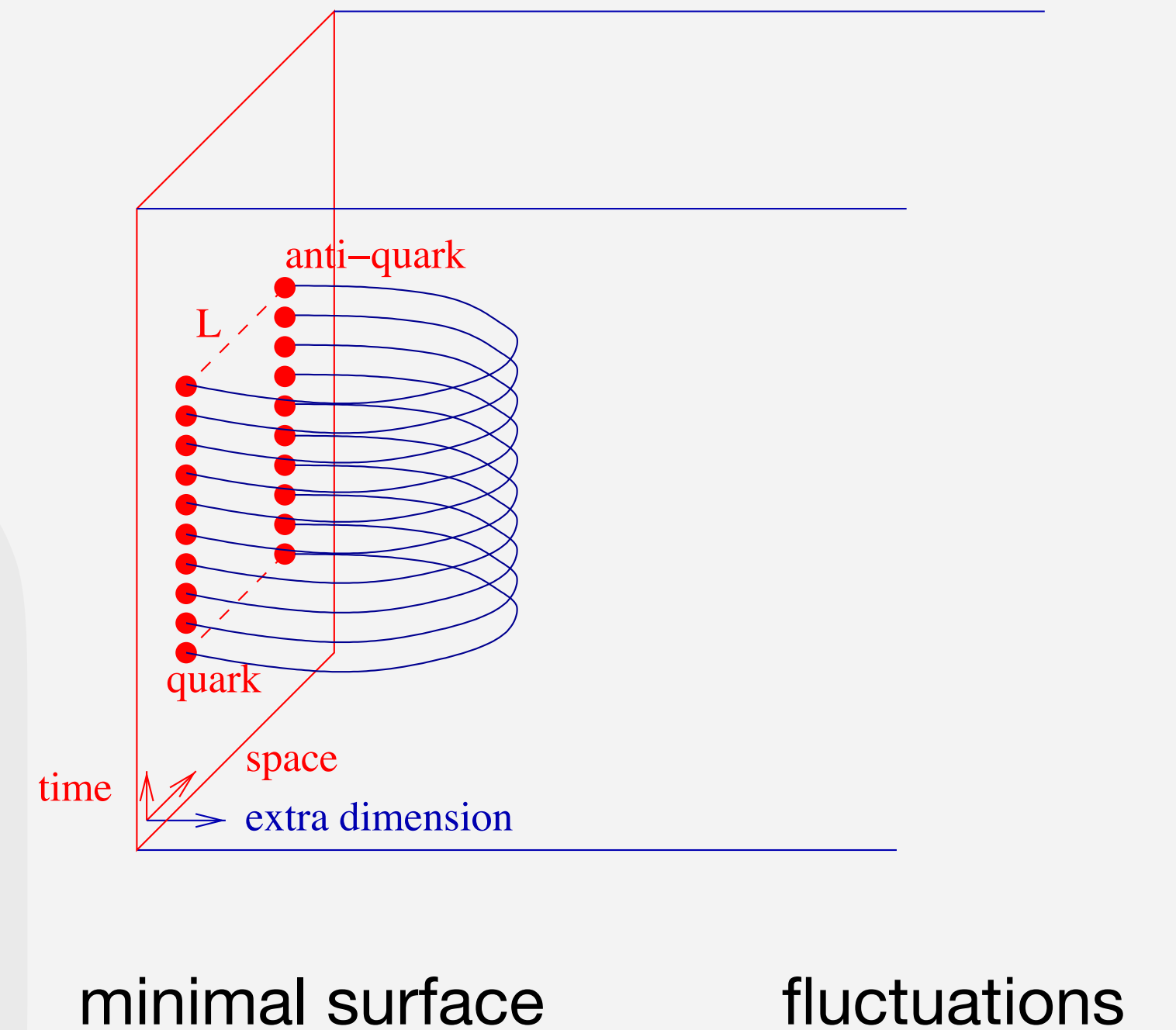
recently 11 loop result

Conclusion

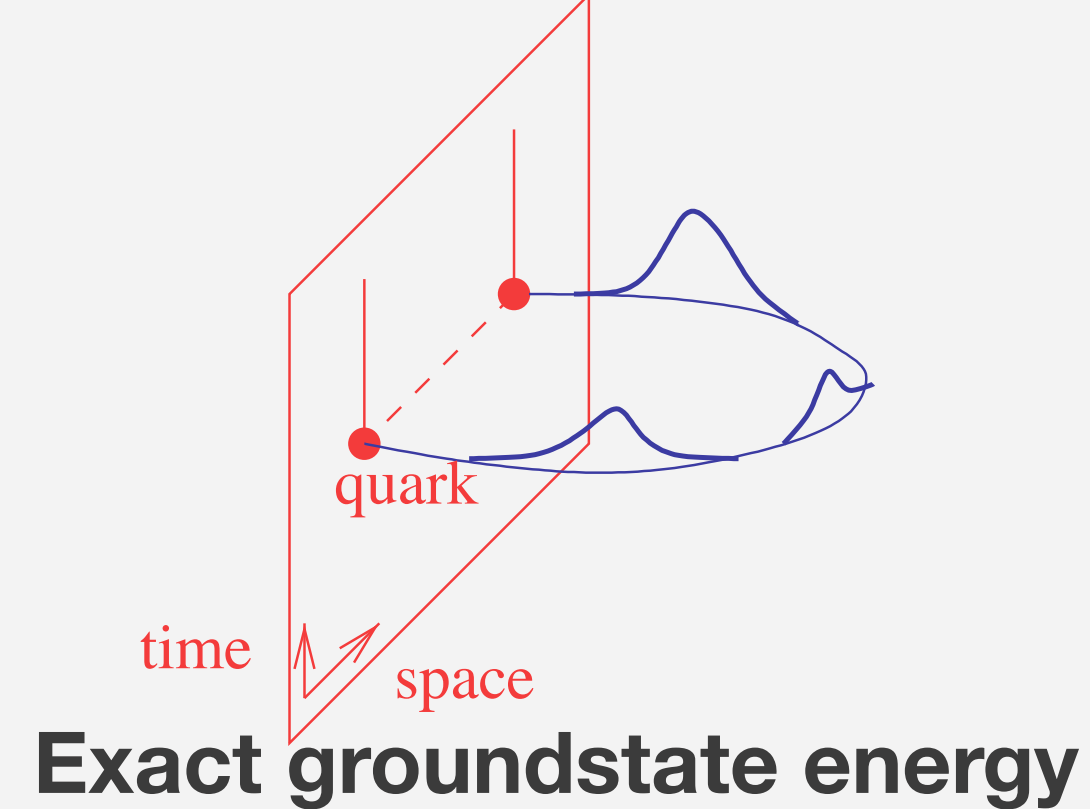
3+1 D gauge theory



1+9 D string theory



1+1 D integrable QFT



Heroic efforts to extend to many more observables, including correlations function, 1,2,3,4,..., gluon scattering amplitudes, ..., heavy ion collisions

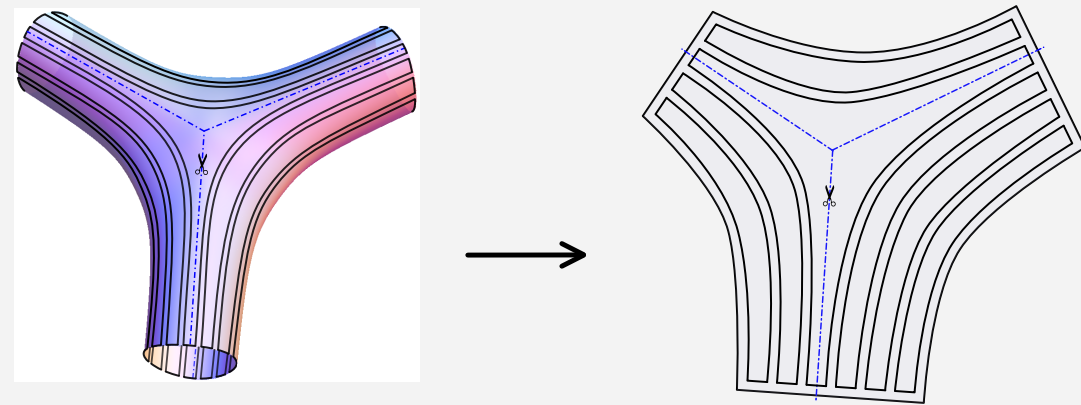
String interactions

3+1 D gauge theory

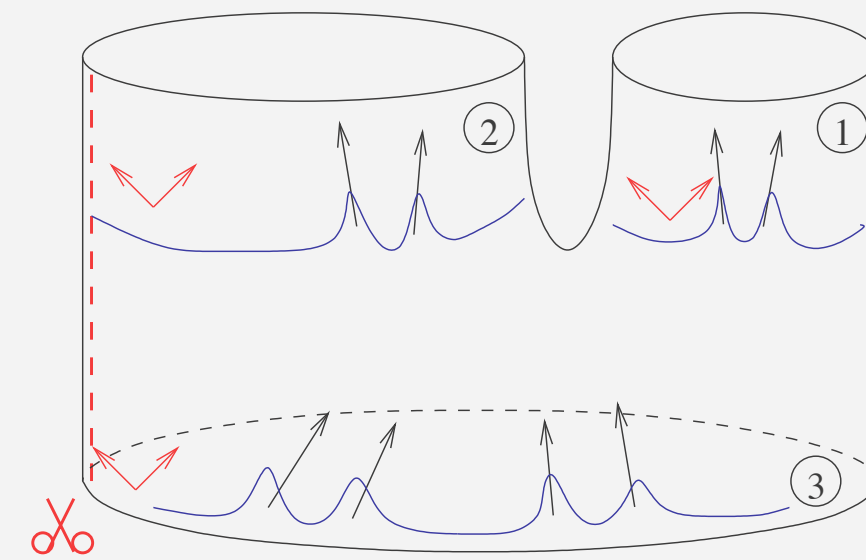
3-point function

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = C_{ijk}(\lambda)$$

spin-chain overlaps

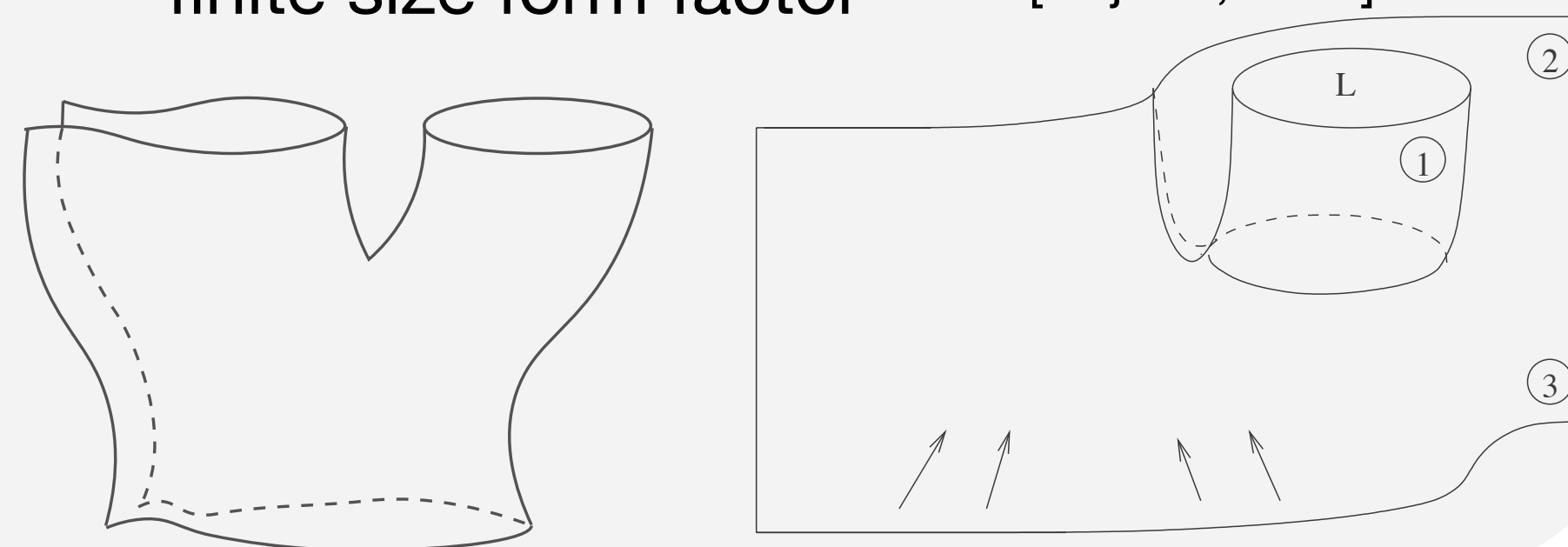


1+1 D integrable QFT

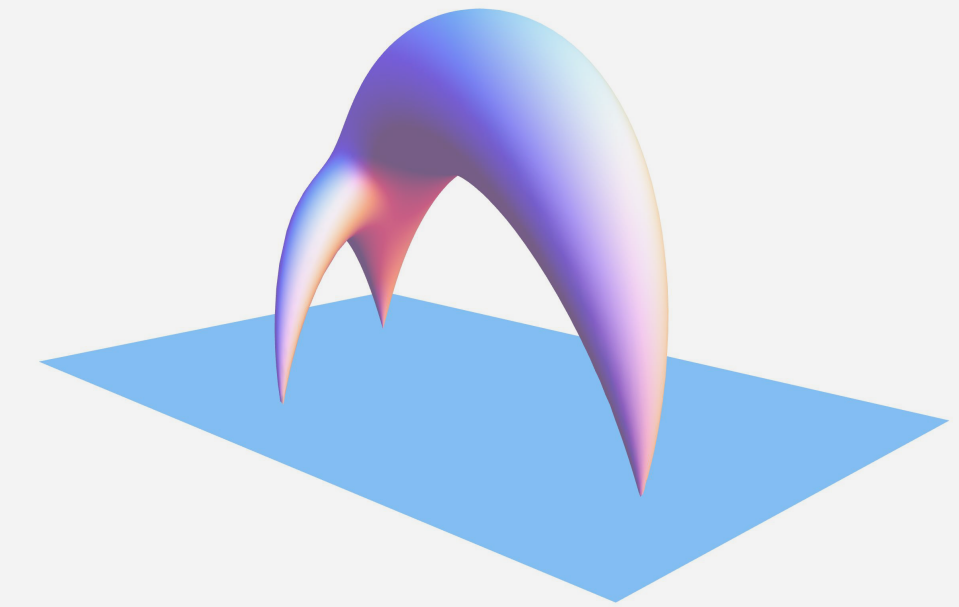


finite size form factor

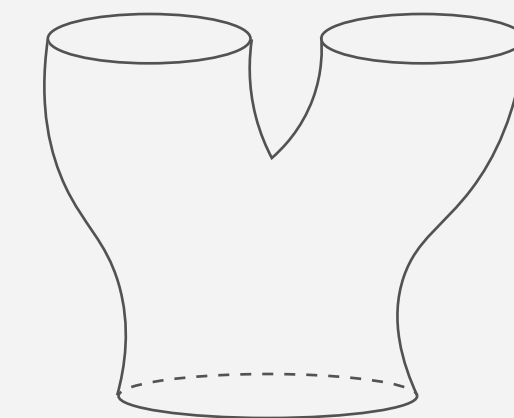
[Bajnok, Janik]



9+1 D string theory

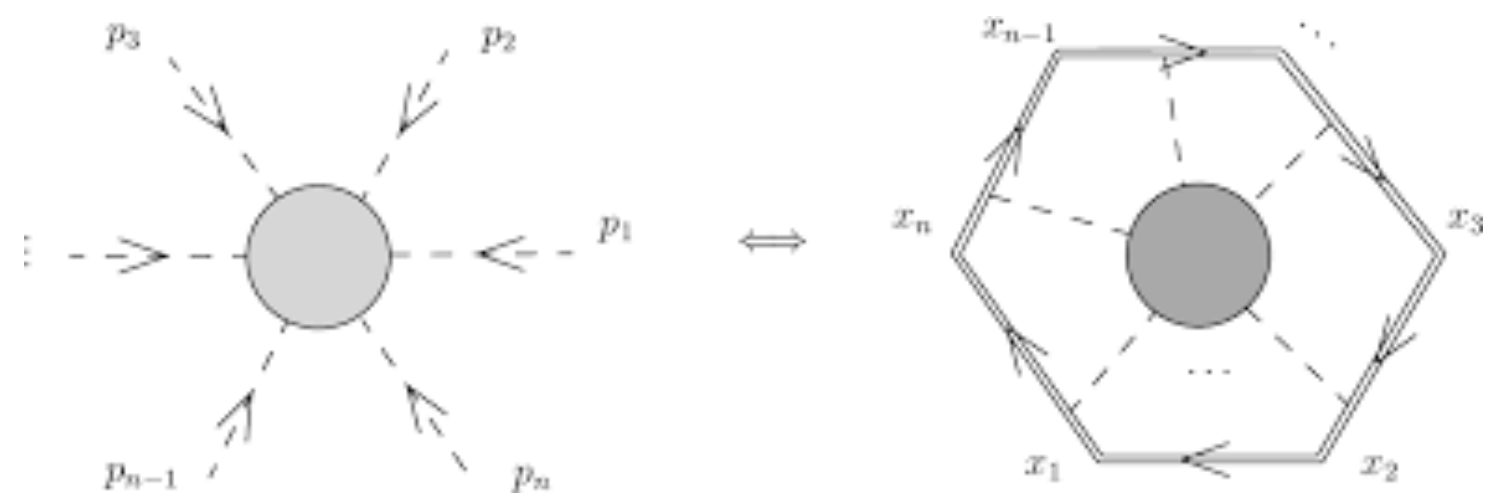


string interaction vertex

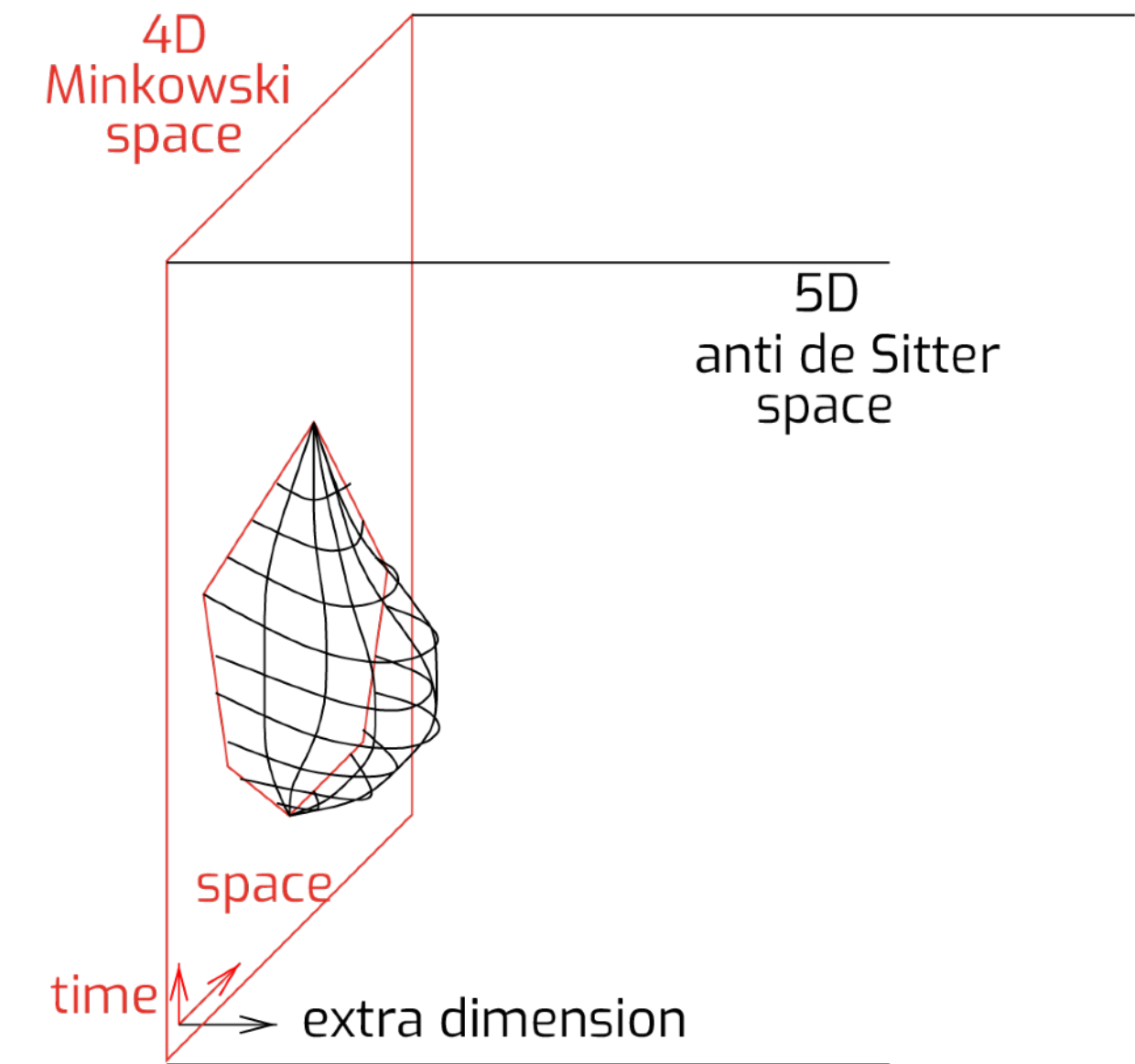
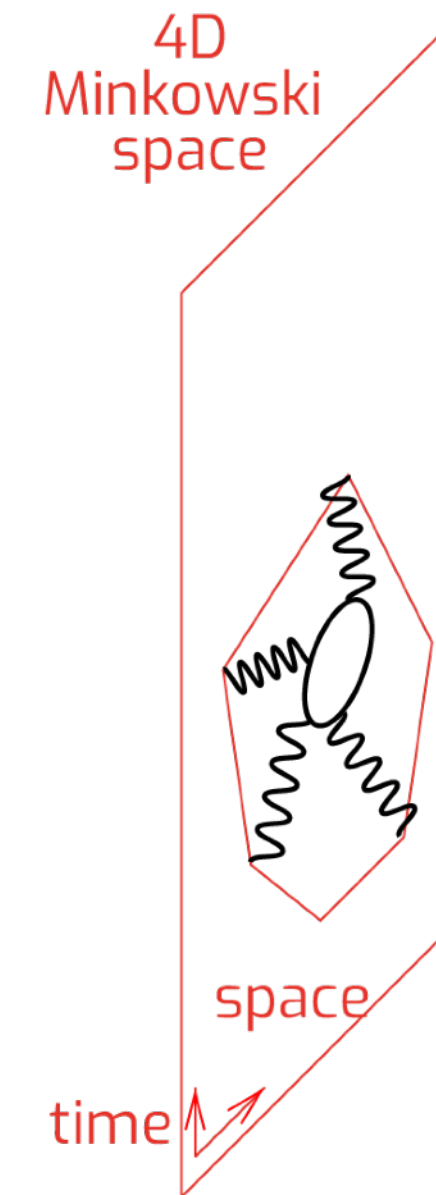


Other observables

gluon scattering amplitudes = light-like Wilson lines



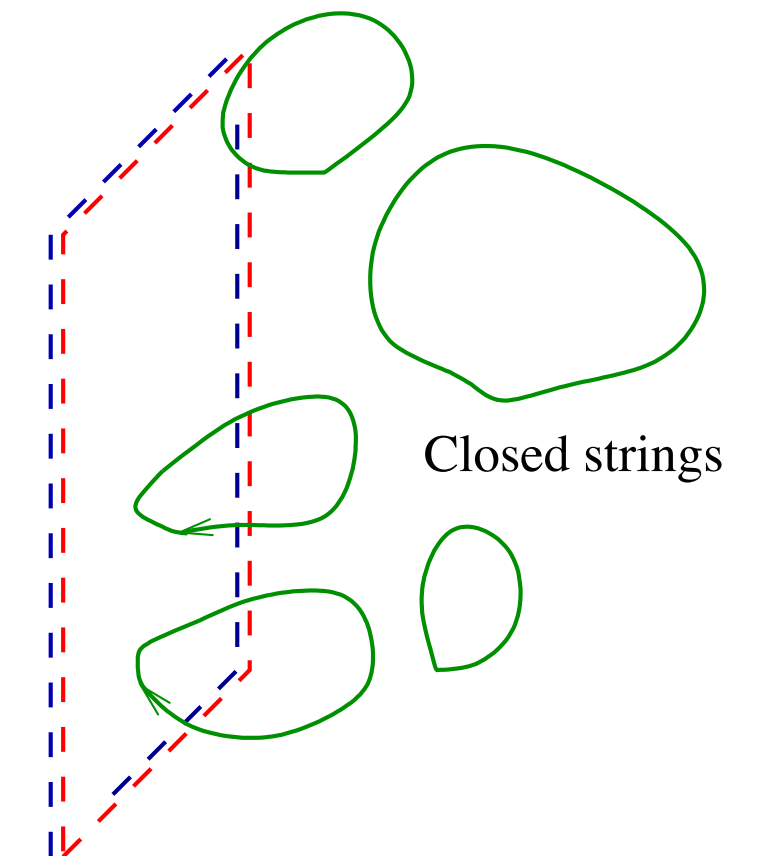
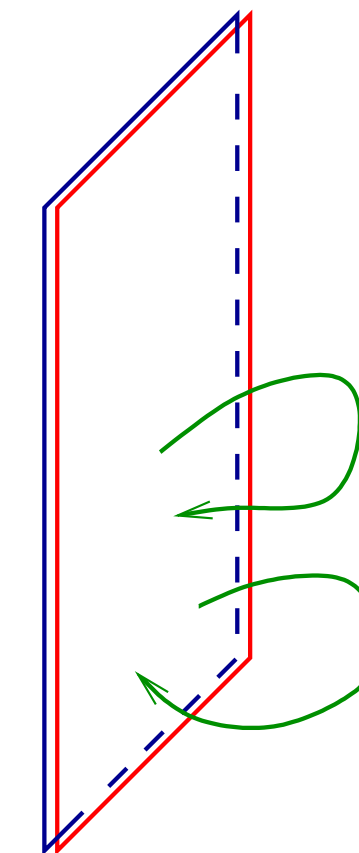
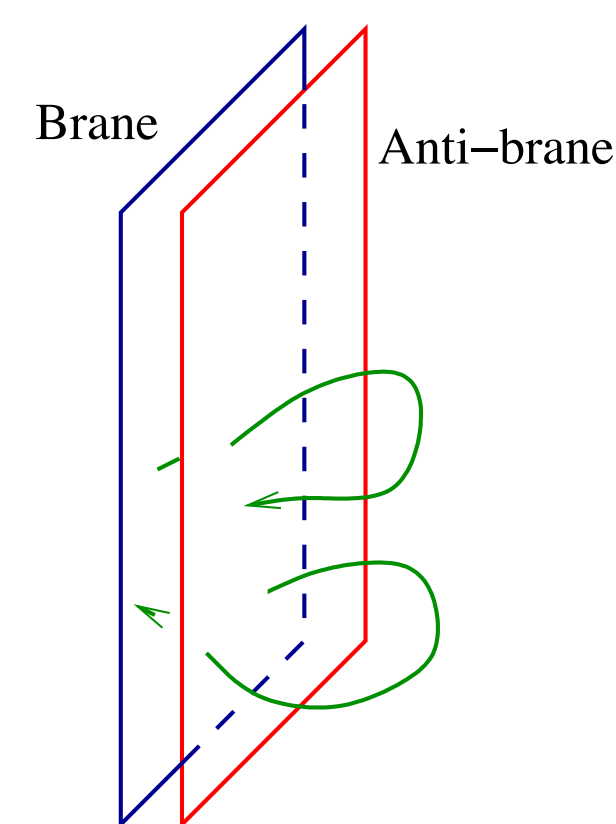
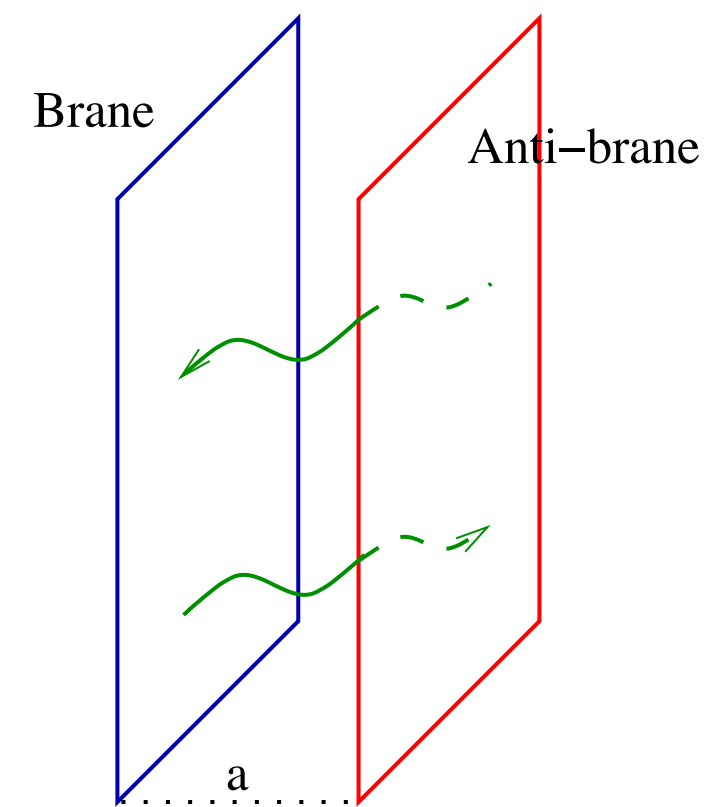
[Bajnok,Balog,Ito,Satoh,Toth]



Tachyon condensation

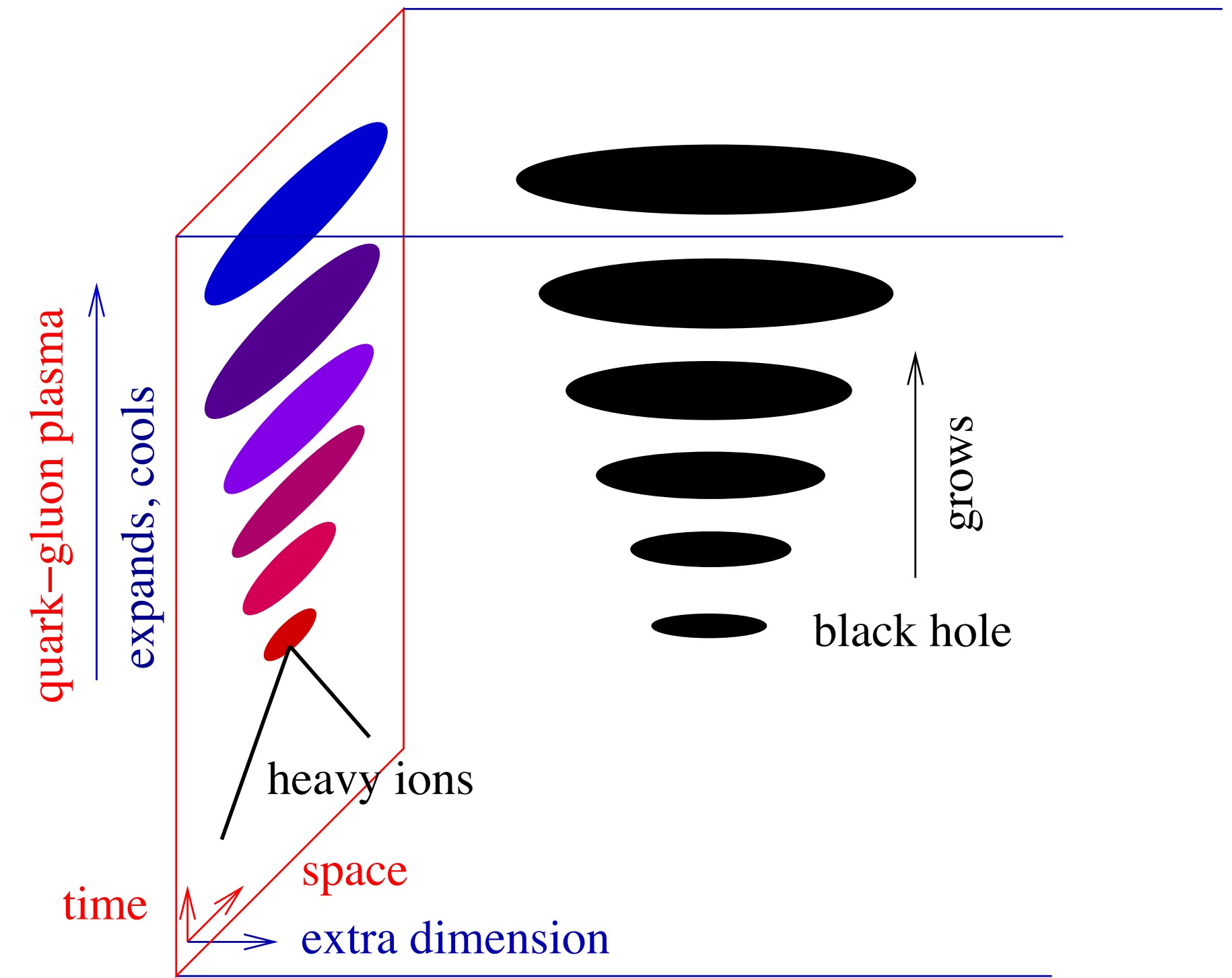
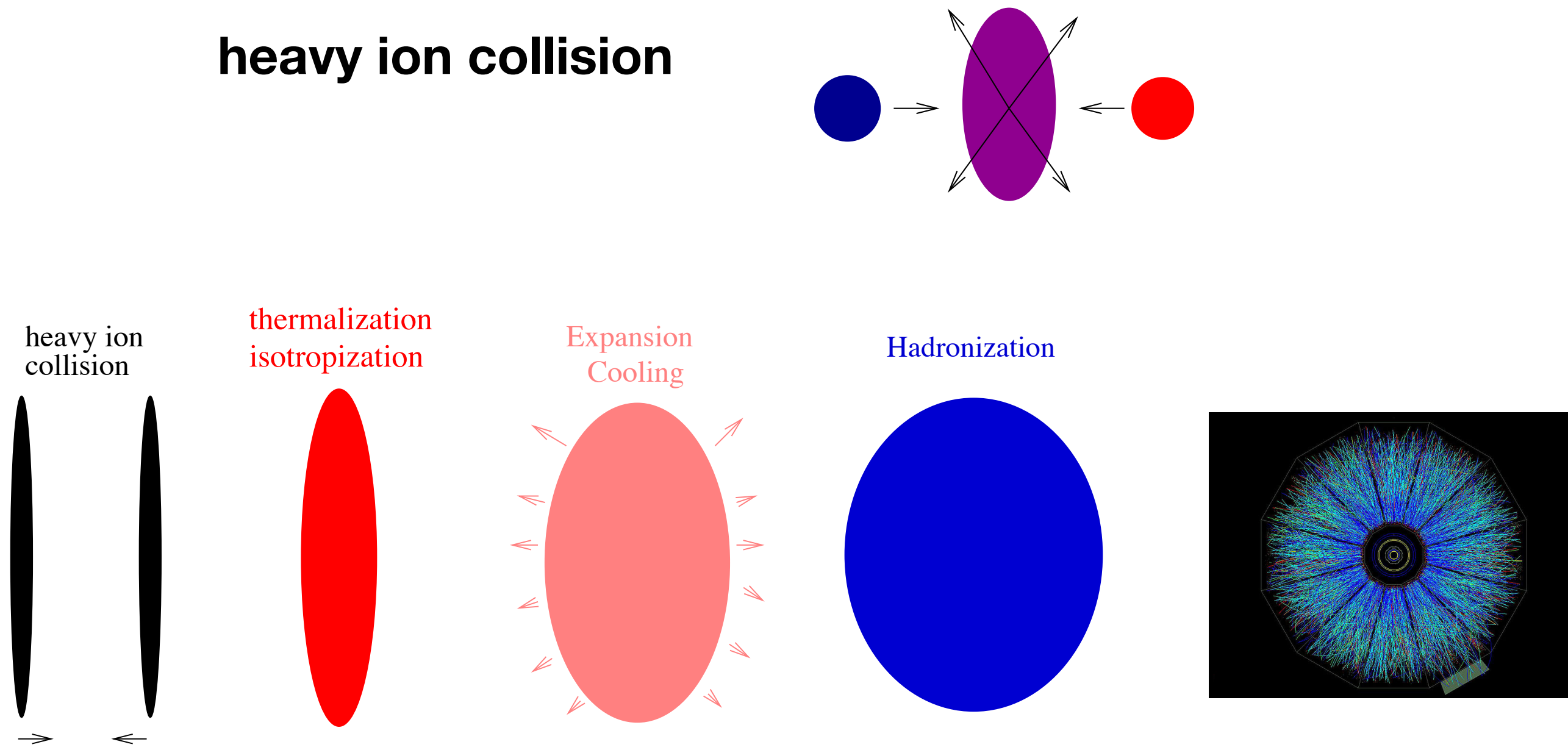
$$\mathcal{O} = \text{Det}(Z \dots ZXZ \dots ZX \dots Z)$$

[Bajnok,Drukker et al.]



Quark-gluon plasma

heavy ion collision



Gauge theory at finite temperature

Relativistic hydrodynamics

perfect fluid $\frac{\eta}{s} = \frac{1}{4\pi}$

low energy string theory = gravitation

Einstein equations

growing black hole

[Janik et al]

Qualitatively similar phenomena in accelerators in the heavy ion collision