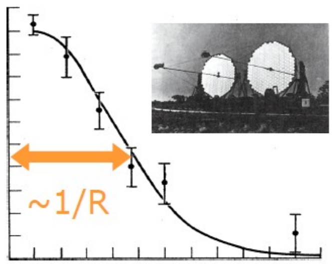
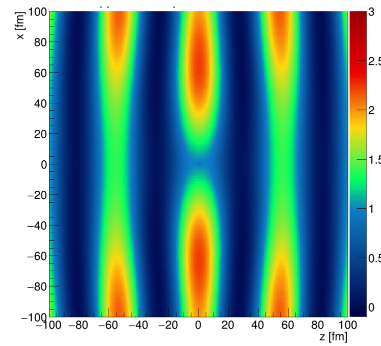
Introduction to HBT correlations in high energy physics

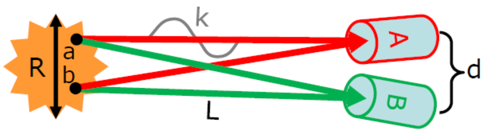
ELFT Particle Physics Summer School

Márton Nagy

May 30, 2024







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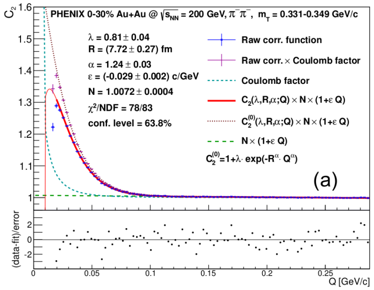


Outline

Well established introduction & recent developments

Intro:

- Heavy-ion physics: motivation, basic results
- Quantum statistical correlations, HBT effect
- Femtoscopy: image reconstruction on the 10^{-15} m scale

Bose-Einstein correlations:

- Source functions; precision treatment
- Final state interactions: Coulomb effect, strong interaction (s-wave)
- New mathematical method for the treatment of Coulomb interaction

Outlook:

New developments in model building

New directions in experimental investigations → see nex talk...

...

Heavy-ion Physics

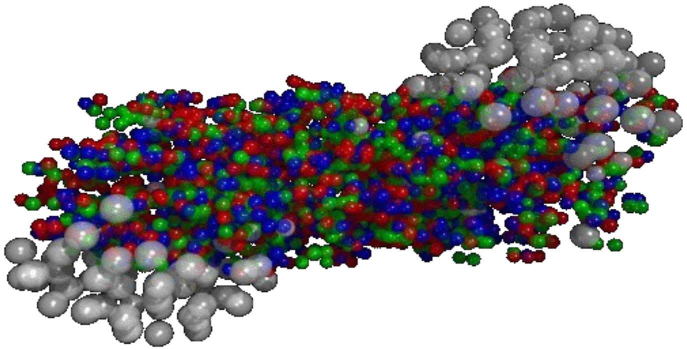
Strong interaction, theory: QCD

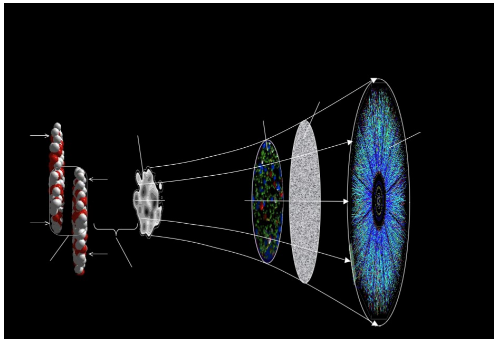
$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_{j=1}^{6} \bar{\psi}^{(j)}_{\alpha,\kappa} \left\{ i\gamma^{\mu}_{\alpha\beta} \left(\delta_{\kappa\lambda} \partial_\mu - i g A_{\mu a} \hat{t}^a_{\kappa\lambda} \right) - m^{(j)} \delta_{\alpha\beta} \delta_{\kappa\lambda} \right\} \psi^{(j)}_{\beta,\lambda}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a_{bc} A^b_\mu A^c_\nu$$

+perturbative solution, lattice QCD, effective theories...

Strong interaction, experiment: *heavy-ion physics* ≡ collisions of heavy nuclei





Phenomenology: connecting experiment to theory; not self-evident

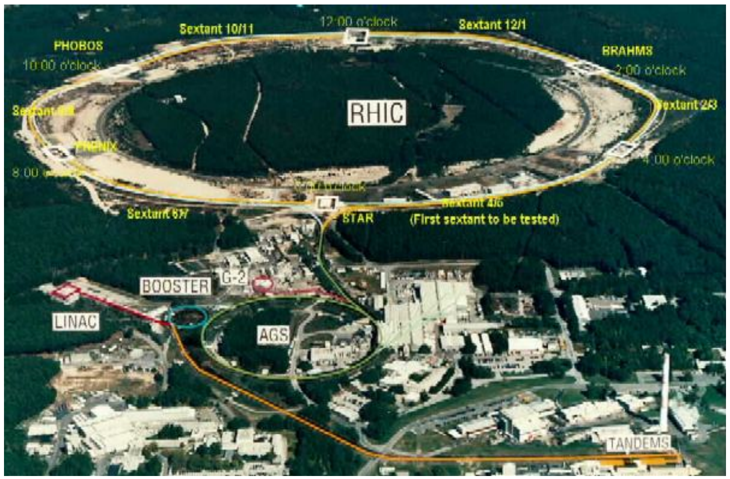
Questions: statistical physical aspects of QCD

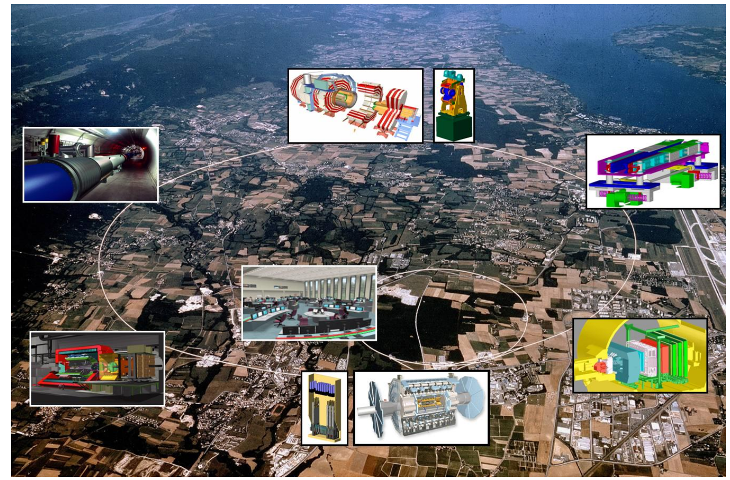
- phases of strong interaction; QGP (**quark gluon plasma**)
- collective properties, critical endpoint (?), ...

Heavy-ion physics

Theory + experiment: big experimental collaborations

- BNL (Brookhaven National Laboratory), RHIC: STAR, *PHENIX,* sPHENIX, ...
  CERN LHC: ALICE, CMS, ATLAS, ...





Various collision energies & systems, similar picture:

RHIC: AuAu @ 200,62.4,54,39,27,19,14.5,7.7,...GeV, UU @ 193 GeV, ...

LHC: PbPb @ 2,76 TeV

Observables: Spectrum (yield): $N_1(\mathbf{p}) = E\frac{dn}{d^3\mathbf{p}} = \frac{1}{2\pi p_t}\frac{dn}{dp_t\,dy}\cdot\left[1+2\sum_{n=1}^{\infty} v_n\cos\left[n(\varphi-\Psi_n)\right]\right]$

- Two-particle correlation: $C_2(\mathbf{p}_1,\mathbf{p}_2) = \frac{N_2(\mathbf{p}_1,\mathbf{p}_2)}{N_1(\mathbf{p}_1)N_1(\mathbf{p}_2)}$

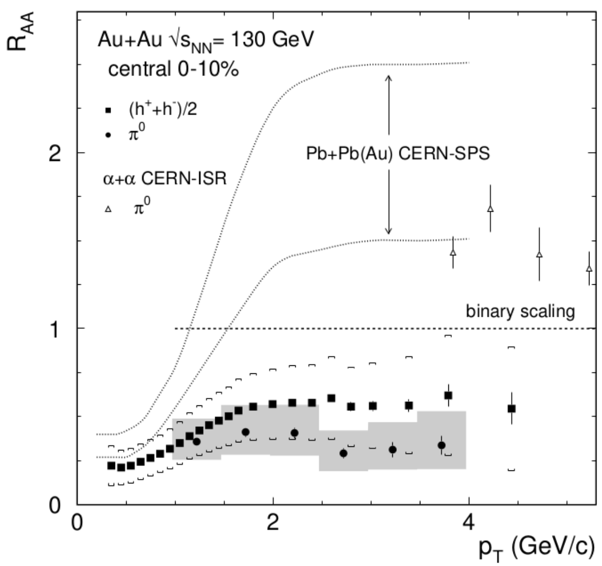
Milestones in heavy-ion physics

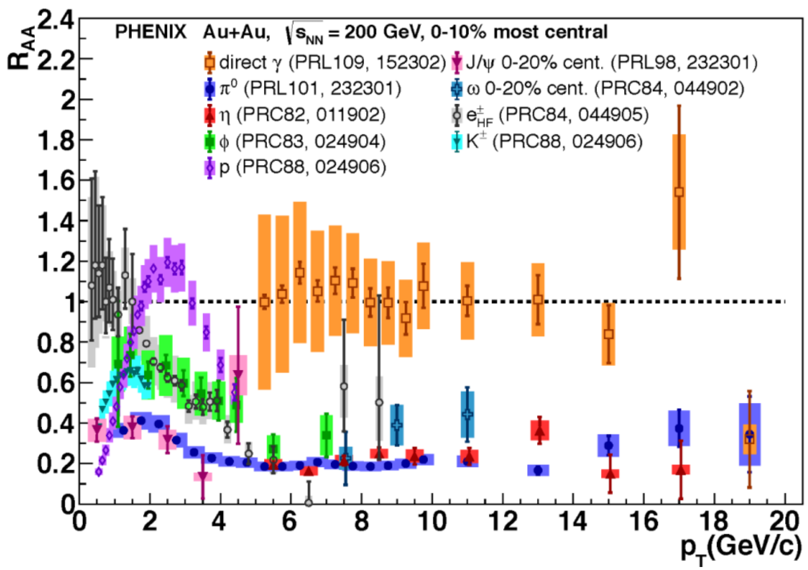
RHIC: from 2000 onwards, LHC: from 2010 onwards

first discoveries → precision measurements; active topics

Suppression at large transverse momentum:

- compared to p+p collisions: R_{AA}<1 in 200 GeV AuAu collisions
- in d+Au no such suppression: *new type of matter*
- at the quark level: different mesons are suppressed similarly
- heavy (c,b) quarks; full *jets*
- Direct photons: no suppression; ,,penetrating probes'' (→ centrality calibration)





Milestones in heavy-ion physics

At low transverse momentum:

Statistical physical processes, thermal distributions → hydrodynamics + *freeze-out*

Azimuthal anisotroies (v_n parameters of spectrum): useful probes at low momentum

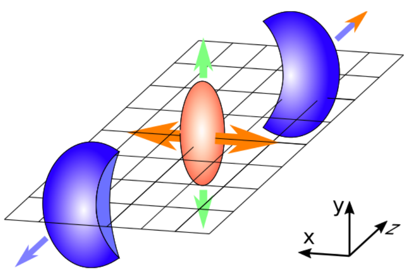
First discovery:

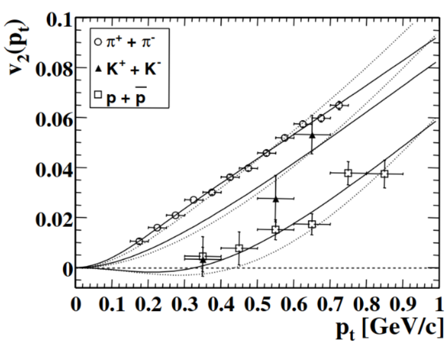
sizeable v_n, especially v_2:

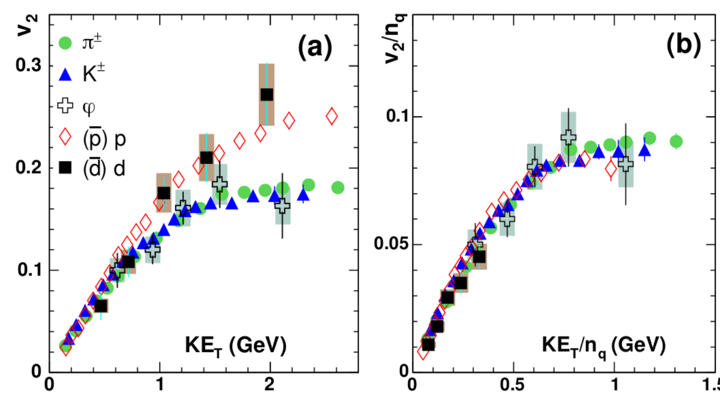
→ *liquid-like phase*

Further refinements:

- collective motion at the quark level
- viscosity (η/s) extremely low...







→ ***sQGP: strongly coupled quark-gluon plasma***

Direct photon spectrum: thermal radiation → $T_0 \gtrsim$ 300-600 MeV

Open questions: direct photon flow...

Milestones in heavy-ion physics

Phase transition:

First order, second order, crossover???

- $R_{\text{out}}^2 - R_{\text{side}}^2 \propto (\Delta\tau)^2$ with R_{out}, R_{side}: sizes of particle source, $\Delta\tau$: freeze-out duration
- a first „input" from femtoscopy (Bose-Einstein correlations, see below)
- experimentally: in high energy (AuAu @ 200GeV) collisions, $R_{out}/R_{side} \approx 1$

  → no latent heat: crossover (in harmony with lattice QCD)

Open questions:

→ details of phase diagram

- New experiments, experimental programs:

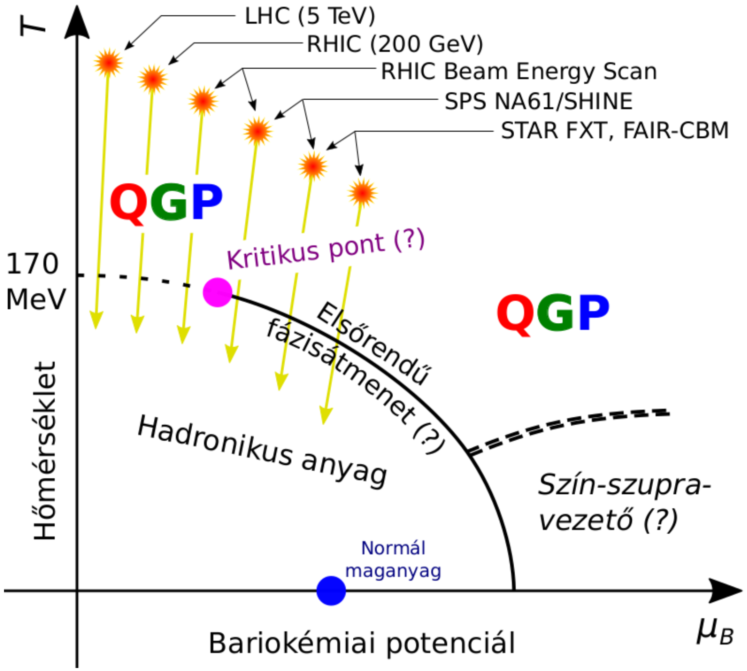
  RHIC Beam Energy Scan (BNL, USA)

  JHITS @ J-PARC (Japan)

  CBM @ FAIR (GSI, Darmstadt)

  MPD @ NICA (Dubna) etc…

→ existence of critical endpoint & first order transition line??

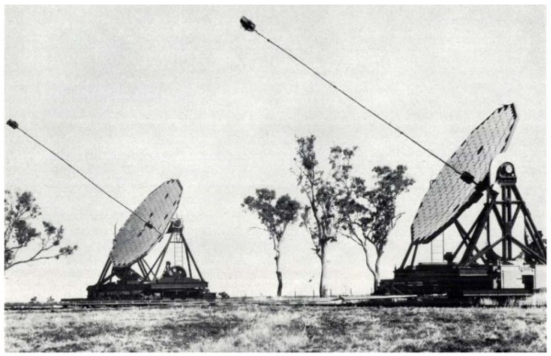


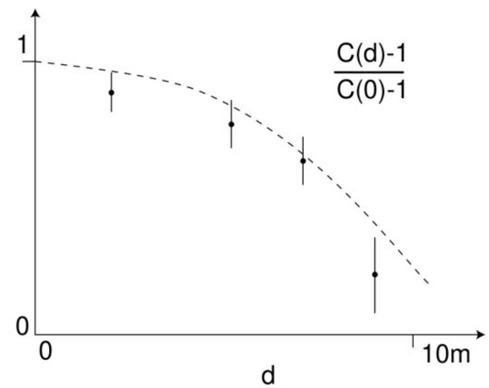
Bose-Einstein correlations

First: in (radio) astronomy

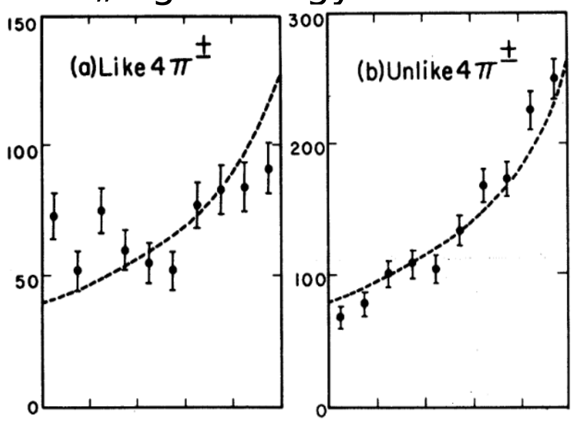
R. Hanbury Brown & R.Q. Twiss, 1954-1956: correlation between *different* photons

→ “interference”; surprise... *HBT effect*





In particle physics: observed correlation between identical pions *($\pi^+\pi^+$, $\pi^-\pi^-$* pairs) in „high energy” reactions (*„GGLP effect”, 1960)*



Basic explanation (Hanbury-Brown, Twiss, U. Fano...): identical bosons → symmetric wave function

From this: correlation as *Fourier transform* of source

- astronomy: good angular resolution, $\Delta\alpha \approx \lambda/d$
- high energy physics: coordinate resolution $\Delta x \approx \hbar/\Delta Q$

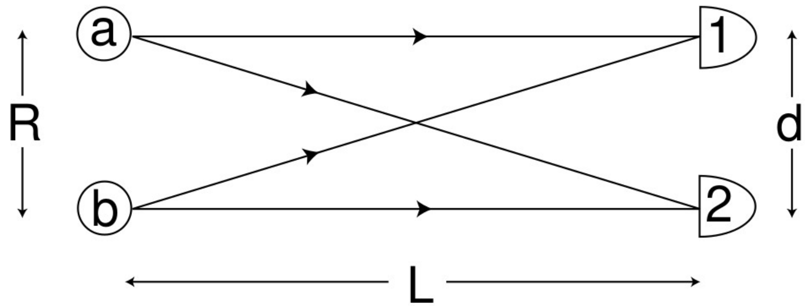
typically: $\Delta Q \approx$10-100 MeV/c → $\Delta x \approx$1 fm

Femtoscopy

Interlude: classical

Simple classical description: (e.g. Baym, Acta Phys Polon. B29 (1997) 1839)

point-like sources



detectors

Assume spherical waves with random phases:

$$A_1 = \frac{1}{L}\left(\alpha e^{ikr_{1a}+i\phi_a} + \beta e^{ikr_{1b}+i\phi_b}\right)$$

$$\Longrightarrow \quad I_1 = \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2 + \alpha^*\beta e^{i(k(r_{1b}-r_{1a})+\phi_b-\phi_a)} + \alpha\beta^* e^{-i(k(r_{1b}-r_{1a})+\phi_b-\phi_a)}\right)$$

$$\Longrightarrow \quad \langle I_1\rangle = \langle I_2\rangle = \frac{1}{L^2}\left(\langle|\alpha|^2\rangle + \langle|\beta|^2\rangle\right)$$

However, $$\langle I_1 I_2\rangle = \frac{1}{L^4}\left[(|\alpha|^4 + |\beta|^4) + 2|\alpha|^2|\beta|^2(1+\cos(k(r_{1a}-r_{2a}-r_{1b}+r_{2b})))\right]$$

$$\Longrightarrow \quad C(\vec{d}\,) - 1 \sim \left|\int d^3r\rho(\vec{r})e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}}\right|^2$$

Bose-Einstein correlations in heavy-ion collisions

Basic formulas & concepts:

$$\mathbf{K} := \frac{\mathbf{p}_1 + \mathbf{p}_2}{2}$$

$$\mathbf{k} := \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}$$

source function: $S(x, \mathbf{p})$

momentum distribution: $N_1(\mathbf{p}) = \int \mathrm{d}x\, S(x, p)$

pair wave function: $\psi^{(2)}(x_1, x_2)$

two-particle distribution: $N_2(\mathbf{p}_1, \mathbf{p}_2) = \int \mathrm{d}x_1 dx_2\, S(x_1, p_1) S(x_2, p_2) \left|\psi^{(2)}(x_1, x_2)\right|^2$

correlation: $C(\mathbf{p}_1, \mathbf{p}_2) = \dfrac{N_2(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1) N_1(\mathbf{p}_2)}$

rel. coordinate distr.: $D(\mathbf{r}, \mathbf{K}) = \int \mathrm{d}^4\rho\, S(\rho + \frac{r}{2}, \mathbf{K}) S(\rho - \frac{r}{2}, \mathbf{K})$

→ **Approximately thus:**

(Koonin-Pratt formula)

$$C(\mathbf{k}, \mathbf{K}) = \frac{\int D(\mathbf{r}, \mathbf{K}) |\psi_{\mathbf{k}}(\mathbf{r})|^2 \mathrm{d}\mathbf{r}}{\int D(\mathbf{r}, \mathbf{K}) \mathrm{d}\mathbf{r}}$$

If final state particles move freely:

→ plane wave, _symmetrized_

$$\Psi^{(2)}_{\mathbf{p}_1\mathbf{p}_2}(x_1, x_2) = \frac{1}{\sqrt{2}}\left[e^{-ip_1\cdot x_1} e^{-ip_2\cdot x_2} + e^{-ip_1\cdot x_2} e^{-ip_2\cdot x_1}\right]$$

$$\Longrightarrow \quad \left|\Psi^{(2)}_{\mathbf{p}_1\mathbf{p}_2}(x_1, x_2)\right|^2 = 1 + \cos\left[(p_1 - p_2)\cdot(x_1 - x_2)\right]$$

Bose-Einstein correlations

For the case of no Final State Interactions:

$$C_2^{(0)}(\mathbf{Q},\mathbf{K}) = 1 + \frac{\widetilde{D}(Q,\mathbf{K})}{\widetilde{D}(0,\mathbf{K})} \qquad \widetilde{D}(Q,\mathbf{K}) := \int \mathrm{d}^4x\, D(x,\mathbf{K}) e^{-iQ\cdot x}$$

→ correlation: essentially Fourier transform of source (*Qx≡ Qx/ħ)*

Core-halo model: explanation for empirical λ „intercept parameter”

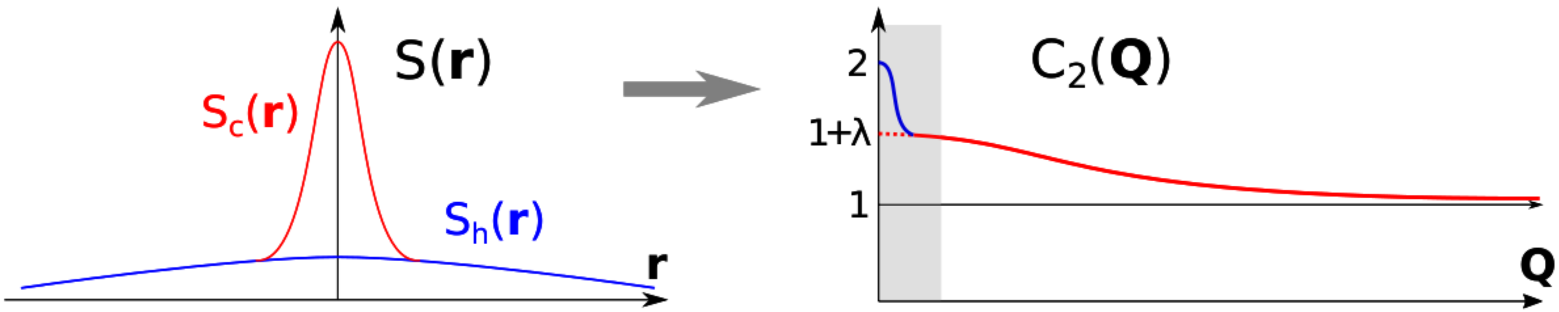
- „***core***”: few fm in size; collision region (QGP)
- „***halo***”: resonance decay contribution (e.g. in case of $\pi^+\pi^+$: η, η’, $K^0{}_S$)

$$S(\mathbf{r},\mathbf{K}) = \sqrt{\lambda}\cdot S_c(\mathbf{r},\mathbf{K}) + \left(1-\sqrt{\lambda}\right)\cdot S_h(\mathbf{r},\mathbf{K})$$

$$D(\mathbf{r},\mathbf{K}) = \lambda\cdot D_{cc}(\mathbf{r},\mathbf{K}) + 2\sqrt{\lambda}\left(1-\sqrt{\lambda}\right)\cdot D_{ch}(\mathbf{r},\mathbf{K}) + \left(1-\sqrt{\lambda}\right)^2\cdot D_{hh}(\mathbf{r},\mathbf{K})$$

⇒ With no FSI: $C_2^{(0)}(\mathbf{Q},\mathbf{K}) = 1 + \lambda\cdot\widetilde{D}_{cc}(\mathbf{Q},\mathbf{K})$

⇒ With FSI:
Bowler-Sinyukov formula: $C_2(\mathbf{Q},\mathbf{K}) = 1-\lambda+\lambda\int \mathrm{d}^3\mathbf{r}\, D_{cc}(\mathbf{r},\mathbf{K})|\psi_{\mathbf{k}}^{(2)}(\mathbf{r})|^2$

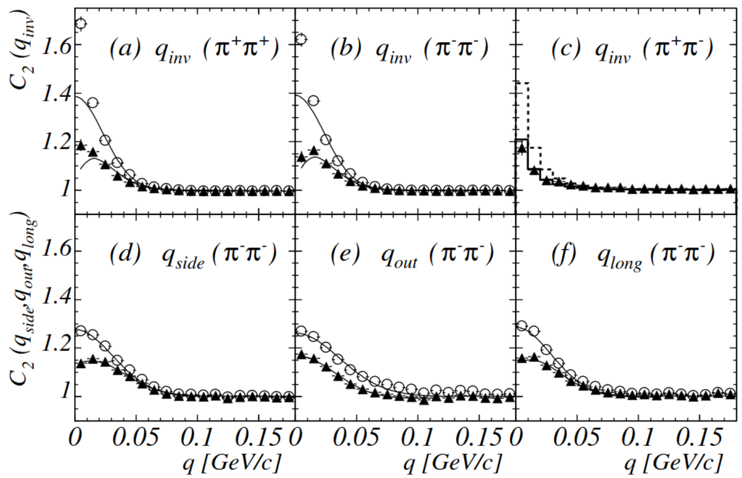


Measurements abound:

At RHIC: from 2002 onwards; *see next talk as well*

Assuming Gaussian sources: $D_{cc}(\mathbf{r}) \propto \exp\left(-r_k r_l (\mathbf{R}^2)^{-1}_{kl}\right)$

Parameters: λ intercept + 3D: R_{out}, R_{long}, R_{side}



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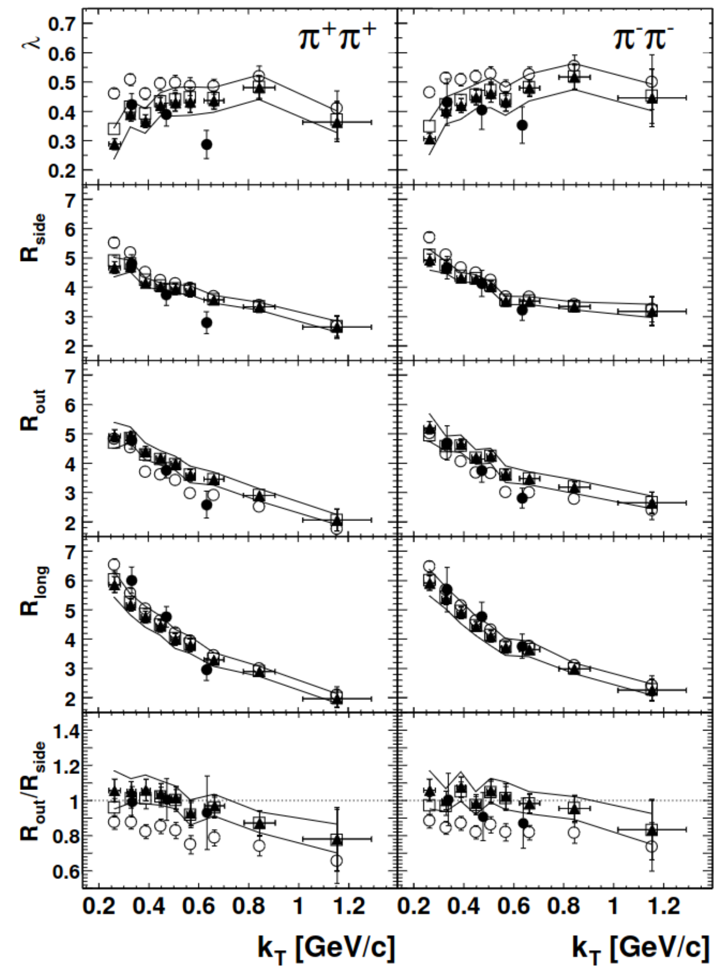
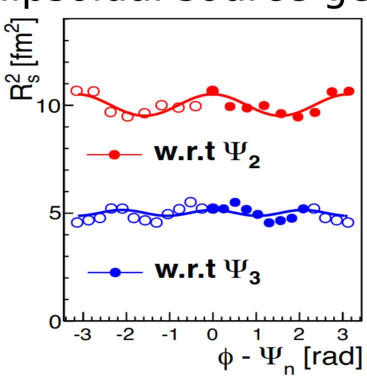
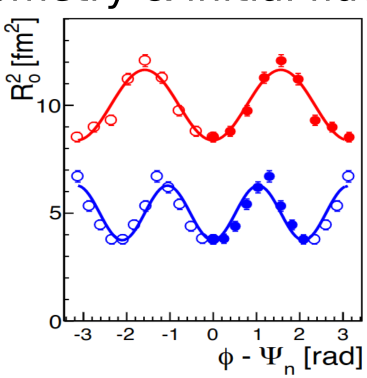


Event plane sensitive measurement: explores ellipsoidal source geometry & initial fluctuations





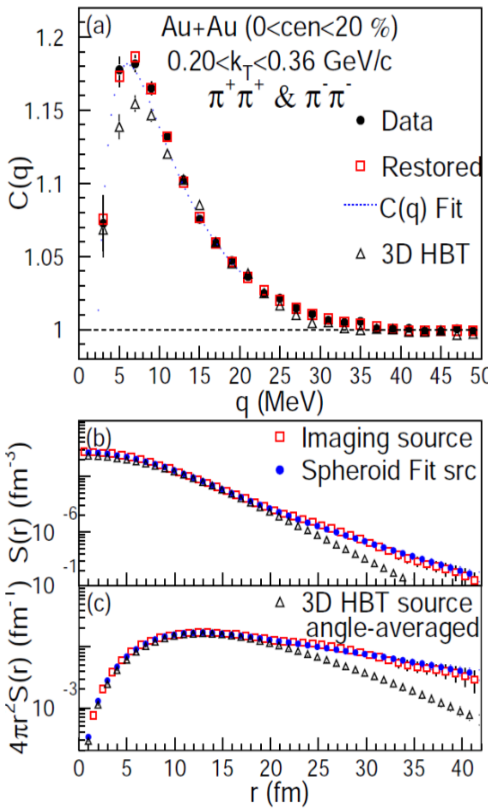
R_{out}/R_{side} consistent with 1

Digression: Gauss → Lévy

Gaussian: $D_{cc}(\mathbf{r}) \propto \exp\left(-r_k r_l (\mathbf{R}^2)^{-1}_{kl}\right)$

Experimentally (from PHENIX; using *imaging* technique

Brown, Danielewicz, PLB 398, 252 (1997)):



Theoretical idea: Csörgő, Hegyi, Zajc, EPJ C 36, 67 (2004)

Generalized Gaussians → **Lévy distributions** as source model

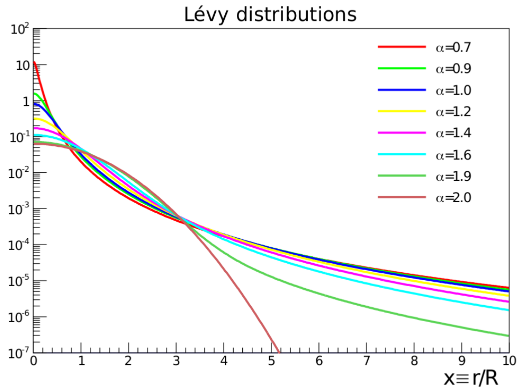
$$\mathcal{L}(\alpha,R,r) = \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\mathbf{r}} e^{-\frac{1}{2}|\mathbf{q}R|^\alpha}$$

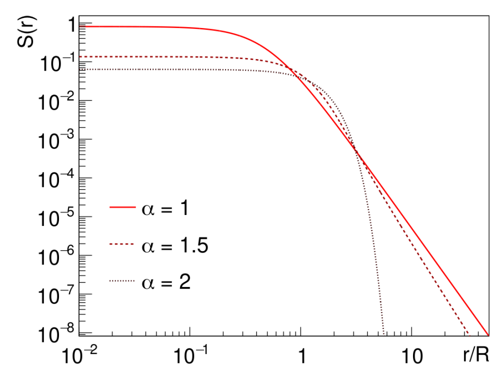
New parameter: α („Lévy index”);

α=2: Gaussian; in other cases:

$$4\pi^2\,\mathcal{L}(\alpha,1,r) \approx \sin\left(\frac{\pi\alpha}{2}\right)\Gamma(\alpha+2)\cdot r^{-3-\alpha}$$

Stable (w.r.t convolution)





Coulomb interaction

Usual treatment: scattering state solutions to NR Schrödinger equation

→ transform into PCMS (Pair Co-Moving System); non-trivial modification of source parameters B. Kurgyis, D. Kincses, M. Nagy, M. Csanád, Universe 9, 328 (2023)

→ Sommerfeld parameter: $\eta := z_1 z_2 \frac{\alpha_{\text{em}} \cdot \mu c^2}{\hbar k \cdot c}, \qquad \alpha_{em} \equiv \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{\hbar c} \approx \frac{1}{137{,}036}$

$$\psi^{(2)}_{\mathbf{k}}(\mathbf{r}) = \frac{\psi_{\mathbf{k}}(\mathbf{r}) + \psi_{\mathbf{k}}(-\mathbf{r})}{\sqrt{2}} = \frac{\mathcal{N}^*}{\sqrt{2}} e^{-ikr}\Big[M\big(1-i\eta, 1, i(kr+\mathbf{kr})\big) + M\big(1-i\eta, 1, i(kr-\mathbf{kr})\big)\Big]$$

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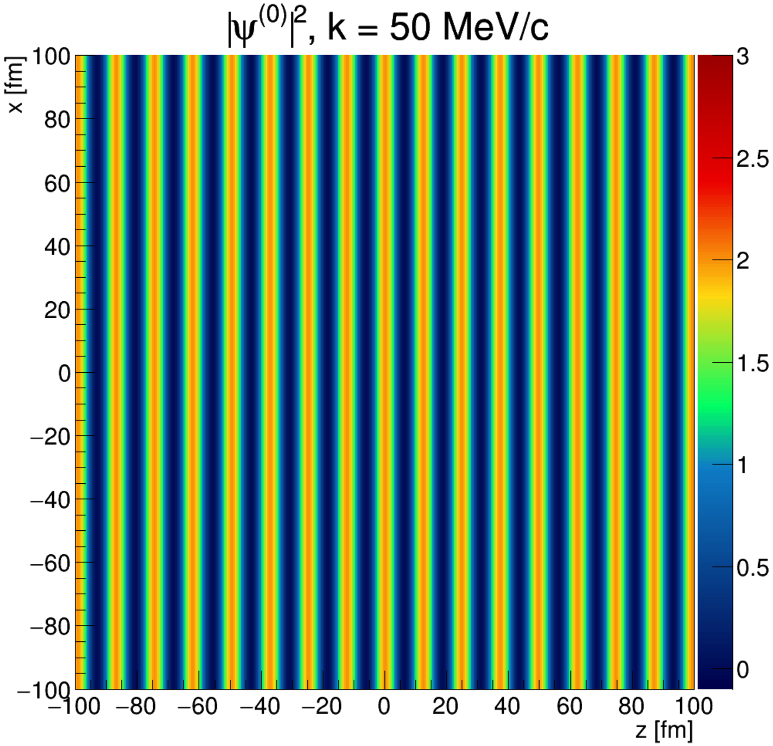

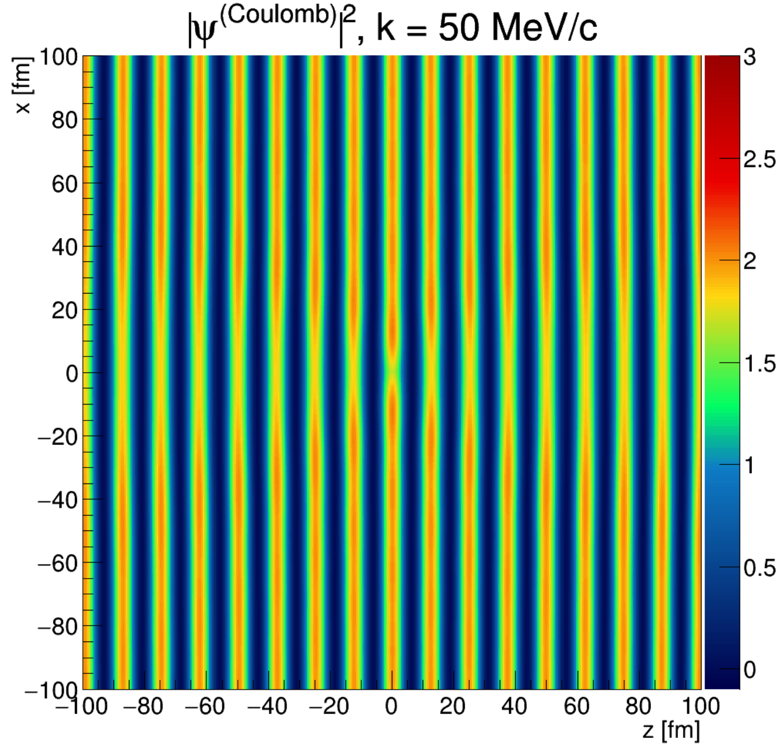

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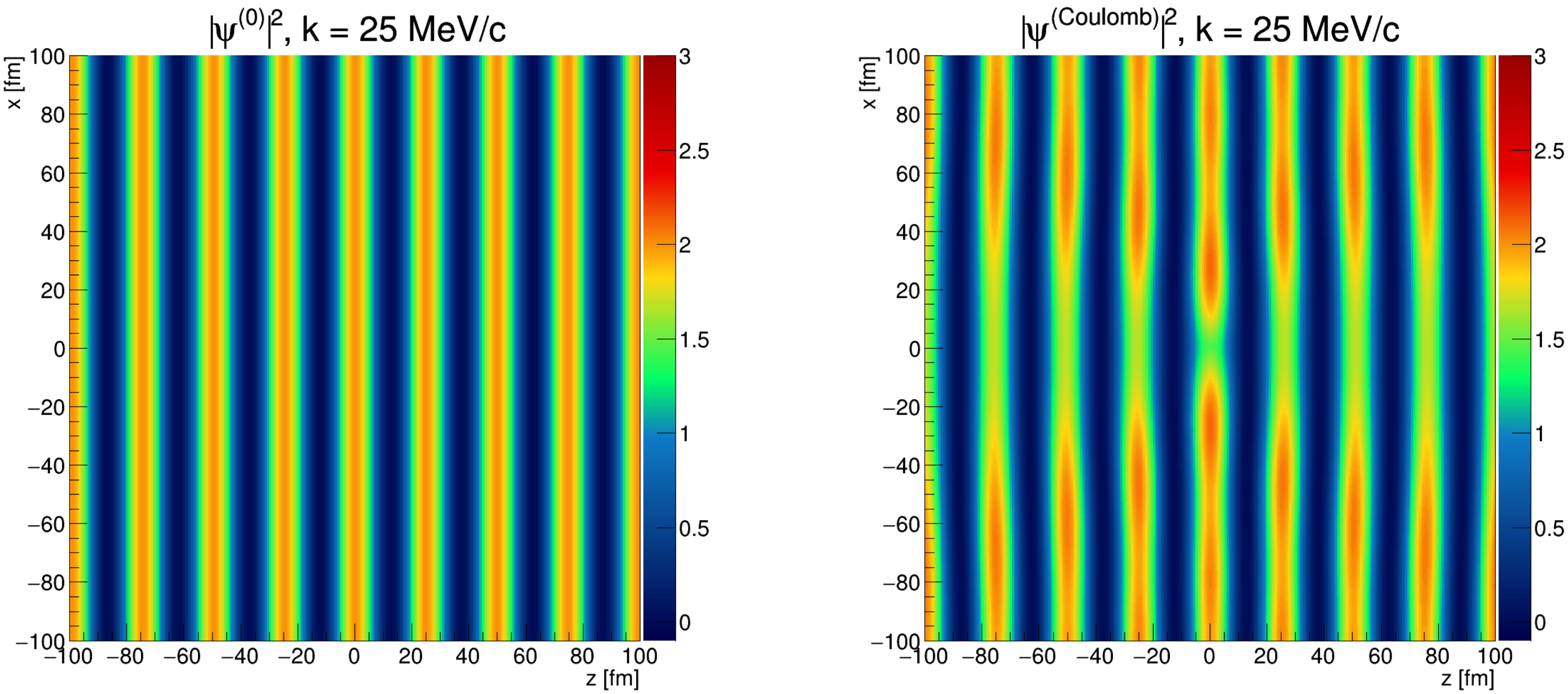

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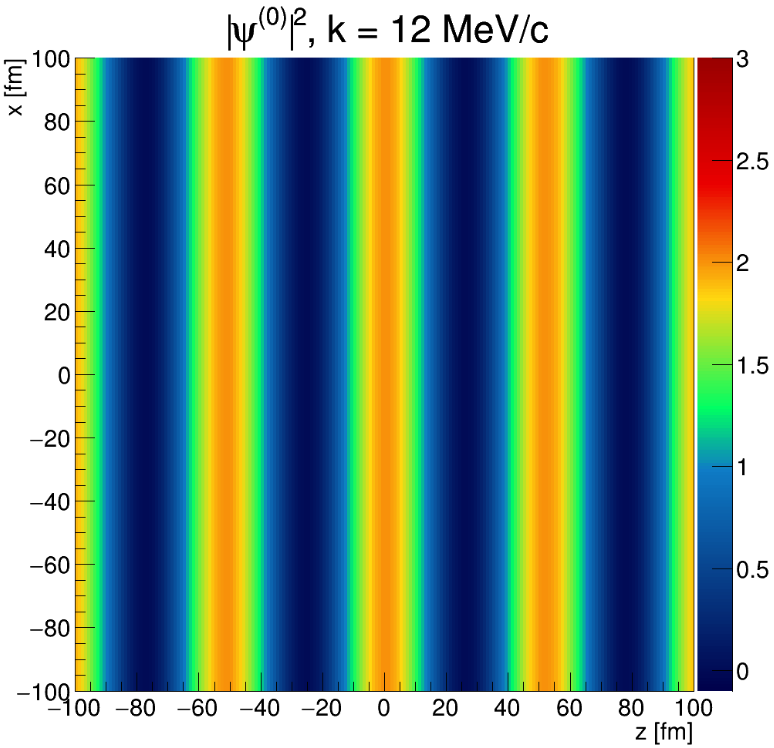

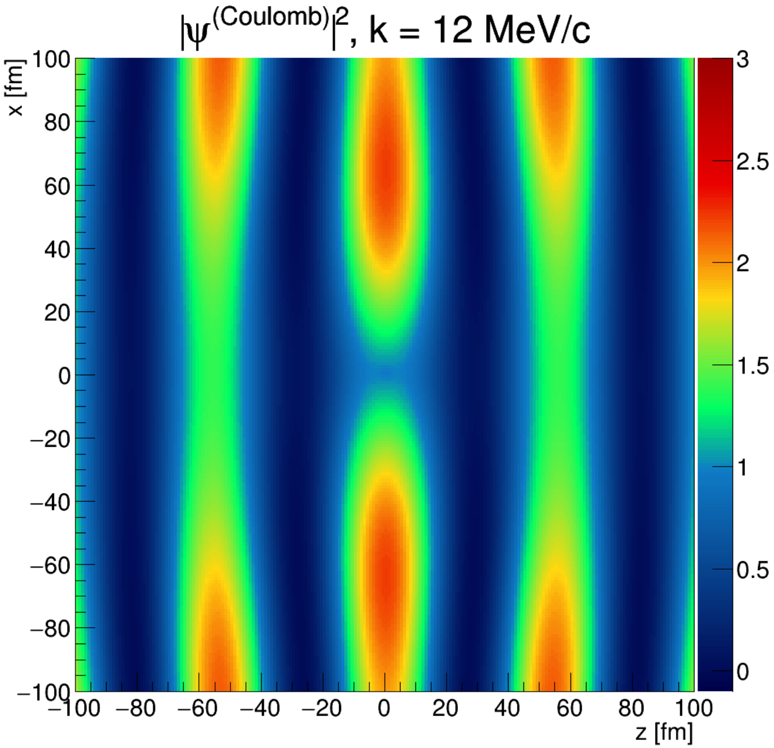

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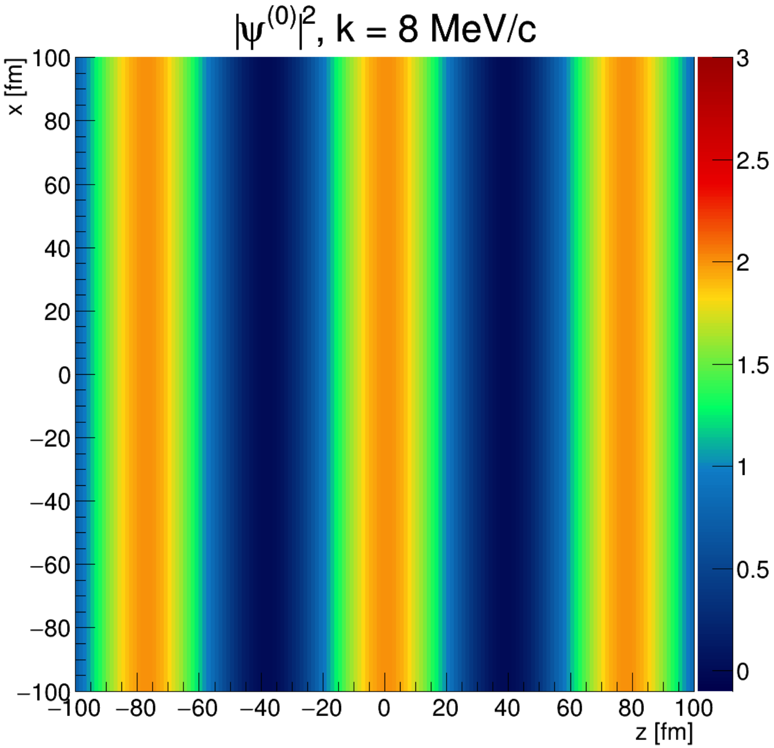

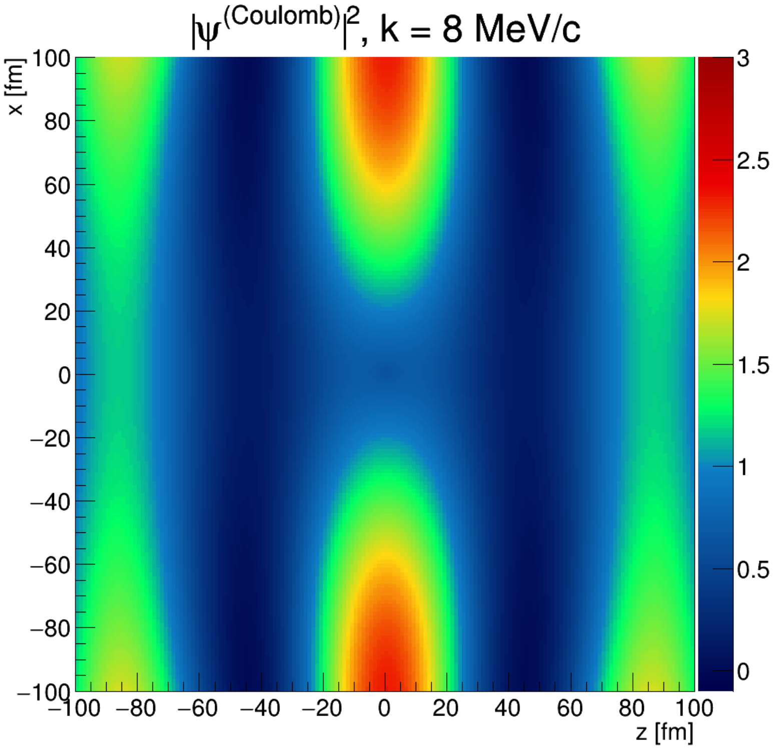

Coulomb interaction

Usual treatment: scattering state solutions to NR Schrödinger equation

→ transform into PCMS (Pair Co-Moving System); non-trivial modification of source parameters B. Kurgyis, D. Kincses, M. Nagy, M. Csanád, Universe 9, 328 (2023)

→ Sommerfeld parameter: $\eta := z_1 z_2 \frac{\alpha_{\rm em} \cdot \mu c^2}{\hbar k \cdot c}, \qquad \alpha_{em} \equiv \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{\hbar c} \approx \frac{1}{137{,}036}$

$$\psi_{\mathbf{k}}^{(2)}(\mathbf{r}) = \frac{\psi_{\mathbf{k}}(\mathbf{r}) + \psi_{\mathbf{k}}(-\mathbf{r})}{\sqrt{2}} = \frac{\mathcal{N}^*}{\sqrt{2}} e^{-ikr} \Big[M\big(1-i\eta, 1, i(kr+\mathbf{kr})\big) + M\big(1-i\eta, 1, i(kr-\mathbf{kr})\big) \Big]$$

Several possibilities:

→ Direct fitting wih (numerically calculated) “Coulomb transform”

→ Iterative Coulomb correction: C(Q) & $C^{(0)}$(Q) from S(r) source with given parameters, ratio: Coulomb correction; multiply measured C(Q) with this, fit with $C^{(0)}$(Q), re-calculate… → convergent, less calculation needed

→ Apply $C^{(0)}$(Q)/C(Q) calculated for a fix parameter set

not self-consistent; however, still applied extensively

→ Gamow correction: valid for point-like source

$$\mathcal{N} = e^{-\pi\eta/2}\Gamma(1+i\eta) \qquad \Rightarrow \qquad |\mathcal{N}|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$$

Need for refinement of treatment of Coulomb interaction…

A glimpse of own work: new method for Coulomb effect

Eur. Phys. J. C 83 (2023) 11, 1015

What's needed:

$D(\mathbf{r})|\psi^{(2)}_{\mathbf{k}}(\mathbf{r})|^2$ integrated over space

→ direct calculation *slow* (esp. for slowly decaying source functions)

→ iterative correction: still too slow

→ Gamow approximation: not precise enough

→ lookup table in computer memory… painful

Important case is when source itself is a Fourier transform,

$$D(\mathbf{r}) := \int \frac{d^3\mathbf{q}}{(2\pi)^3} f(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}}$$

→ Sometimes (e.g. in case of Lévy distributions) even this is possible only numerically

→ D($\mathbf{r}$) : slowly decaying; $\psi^{(2)}_{\mathbf{k}}(\mathbf{r})$: oscillates

→ Awkward: Fourier transform then „almost inverse", *numerically…*

Idea: „exchange integrals"

$$C_2(\mathbf{Q}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{r}\,|\psi^{(2)}_{\mathbf{k}}(\mathbf{r})|^2 \int d^3\mathbf{q}\, f(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}} = \frac{1}{(2\pi)^3}\int d^3\mathbf{r} \int d^3\mathbf{q}\, f(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}} |\psi^{(2)}_{\mathbf{k}}(\mathbf{r})|^2 \overset{??}{=}$$

$$\overset{??}{=} \frac{1}{(2\pi)^3}\int d^3\mathbf{q} \int d^3\mathbf{r}\, f(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}} |\psi^{(2)}_{\mathbf{k}}(\mathbf{r})|^2 =$$

$$= \frac{1}{(2\pi)^3}\int d^3\mathbf{q}\, f(\mathbf{q}) \int d^3\mathbf{r}\, e^{i\mathbf{q}\mathbf{r}} |\psi^{(2)}_{\mathbf{k}}(\mathbf{r})|^2 \quad ↯↯↯$$

Not working in this way. Resolution: regularization, λ∈ℝ⁺, then λ→0. Careful with math (once in a lifetime…)

New method for Coulomb effect

Proper way then:

$$C_2(\mathbf{Q})=\int \mathrm{d}^3\mathbf{r}\,|\psi_{\mathbf{k}}^{(2)}(\mathbf{r})|^2 D(\mathbf{r})=\int \mathrm{d}^3\mathbf{r}\,\lim_{\lambda\to 0} e^{-\lambda r}|\psi_{\mathbf{k}}^{(2)}(\mathbf{r})|^2\, D(\mathbf{r})$$

$$=\lim_{\lambda\to 0}\int \mathrm{d}^3\mathbf{r}\, e^{-\lambda r}|\psi_{\mathbf{k}}^{(2)}(\mathbf{r})|^2\, D(\mathbf{r})=$$

$$=\lim_{\lambda\to 0}\int \mathrm{d}^3\mathbf{r}\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} f(\mathbf{q})\, e^{-\lambda r}|\psi_{\mathbf{k}}^{(2)}(\mathbf{r})|^2\, e^{i\mathbf{qr}}=$$

$$=\lim_{\lambda\to 0}\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} f(\mathbf{q})\int \mathrm{d}^3\mathbf{r}\, e^{-\lambda r} e^{i\mathbf{qr}}|\psi_{\mathbf{k}}^{(2)}(\mathbf{r})|^2.$$

Ingredients:

- (Lebesgue) integrability
- *Lebesgue theorem* (lim & integral exchange)
- *Fubini theorem* (for double integral)

→ last sten: *cannot* take limit before integral

Further elaboration: in spherically symmetric case, for now

$$C_2(Q)=\frac{|\mathcal{N}|^2}{2\pi^2}\lim_{\lambda\to 0}\int_0^\infty \mathrm{d}q\, q^2 f_s(q)\left[\mathcal{D}_{1\lambda s}(q)+\mathcal{D}_{2\lambda s}(q)\right]$$

$$\mathcal{D}_{1\lambda s}(q)=\int \mathrm{d}^3\mathbf{r}\frac{\sin(qr)}{qr}e^{-\lambda r}M\left(1+i\eta,1,-i(kr+\mathbf{kr})\right)M\left(1-i\eta,1,i(kr+\mathbf{kr})\right)$$

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New method for Coulomb effect

After performing **r**-integrals (complex analysis needed; A. Nordsieck, Phys. Rev. 93, 785 (1954).)

$$\mathcal{D}_{1\lambda s}(q)=\frac{4\pi}{q}\mathrm{Im}\left[\frac{1}{(\lambda-iq)^2}\left(1+\tfrac{2k}{q+i\lambda}\right)^{2i\eta}\mathcal{F}_+\left(\tfrac{4k^2}{(q+i\lambda)^2}\right)\right]$$

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$$\mathcal{F}_+(x)\equiv {}_2F_1(i\eta,1+i\eta,1,x)$$

For $\lambda\rightarrow 0$: ill-behaved function limits (in integrands)

Goal: find usable formula for the above limits

Result:

$$C_2(Q)=|\mathcal{N}|^2\left(1+f_s(2k)+\frac{\eta}{\pi}\left[\mathcal{A}_{1s}+\mathcal{A}_{2s}\right]\right)$$

$$\mathcal{A}_{1s}=-\frac{2}{\eta}\int_0^\infty \mathrm{d}q\,\frac{f_s(q)-f_s(0)}{q}\mathrm{Im}\left[\left(1+\frac{2k}{q}\right)^{2i\eta}\mathcal{F}_+\left(\frac{4k^2}{q^2}-i0\right)\right],$$

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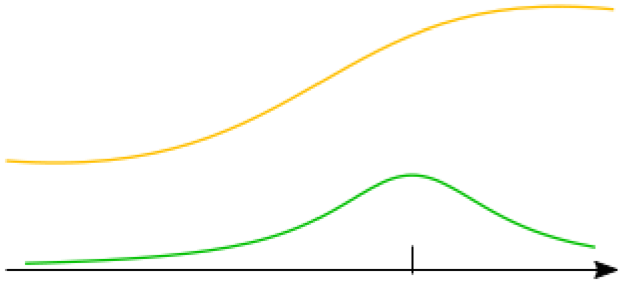

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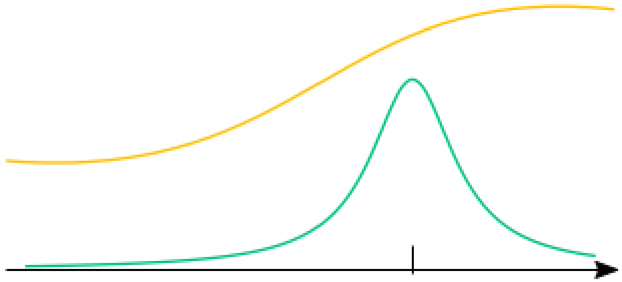

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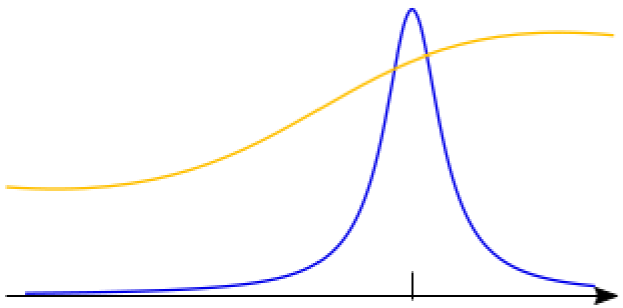

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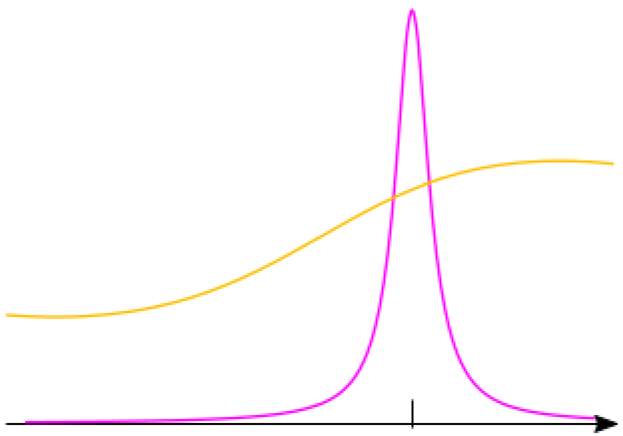

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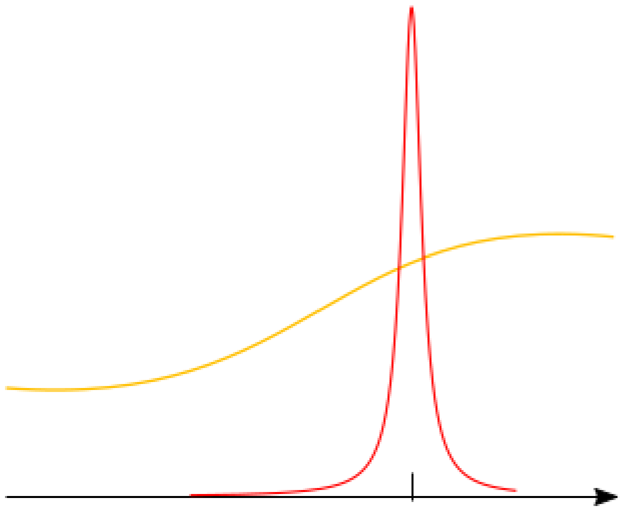

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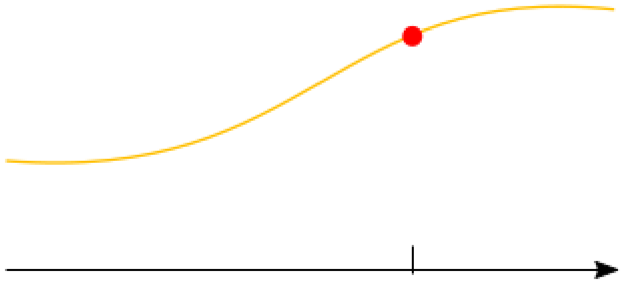
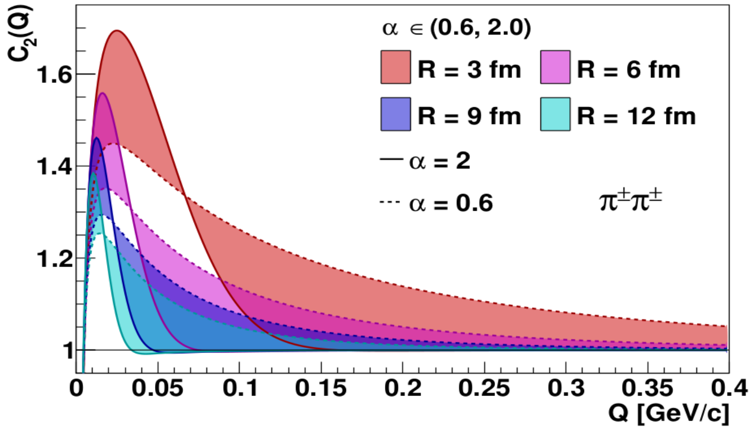


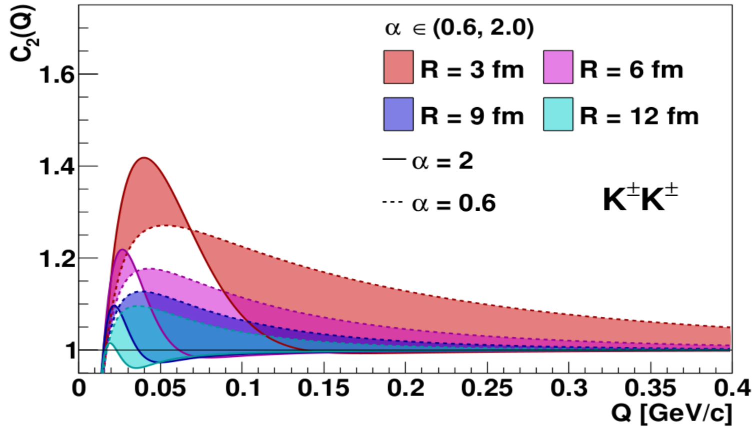
New method for Coulomb effect

Numerically implemented (Gauss-Krohnrod algorithm from boost C++ library)

→ real time calculation becomes possible! github.com/csanadm/CoulCorrLevyIntegral

Examples: ππ, KK pairs





Next steps:

→ non-spherically symmetric case (math: ready)

3D Lévy measurement becomes possible in a self-consistent way

→ Treatment of (s-wave) strong interaction

Summary and outlook

Phenomenology of Bose-Einstein correlations:

Femtoscopy: introduction, uses, ingredients (Core-halo picture, Coulomb effect)

→ versatile tool to map out particle emitting source geometry

→ results a

Methods of self-consistent description; *new mathematical method introduced*

Ready to use (in fact, used) in experimental analyses

(*Some*) open topics *(under investigation)*:

- 3D calculation of C(**Q**); Coulomb effect included
- Inclusion of strong interaction into new framework (interesting for investigation of strong interaction itself for exotic particles... recent direction)

See next talk for various recent experimental results

Thanks for your attention (&sorry...) !!!