Tensor Core Computing: an example on Independent Component Analysis

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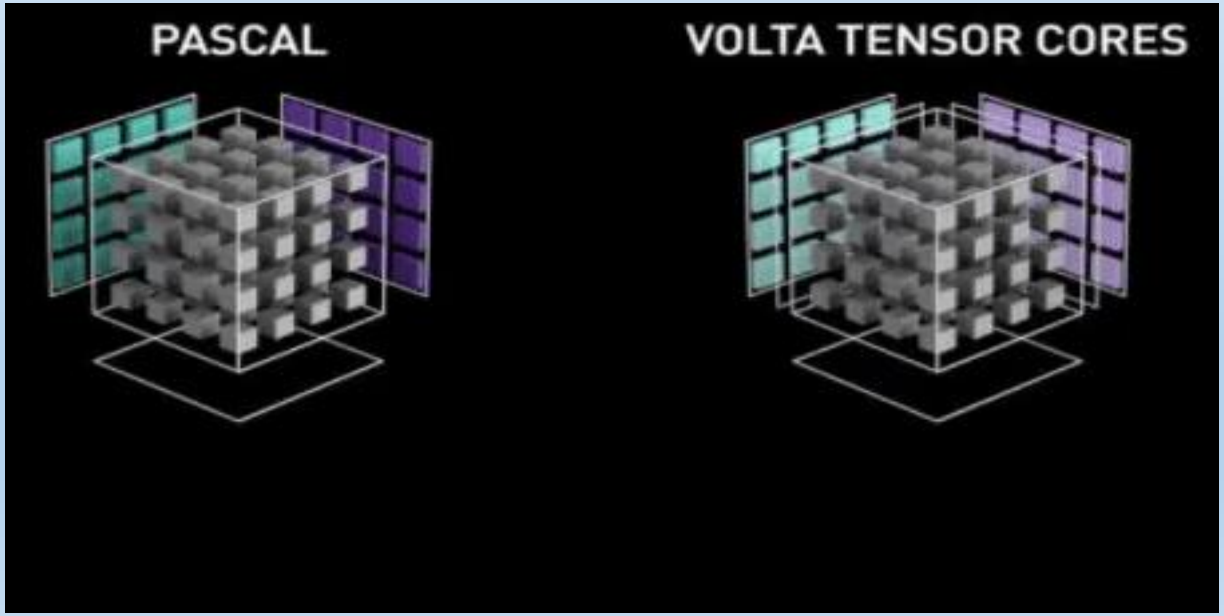
1.1 Abstract and hierarchy of the tensor core

The structure of GA100 SM

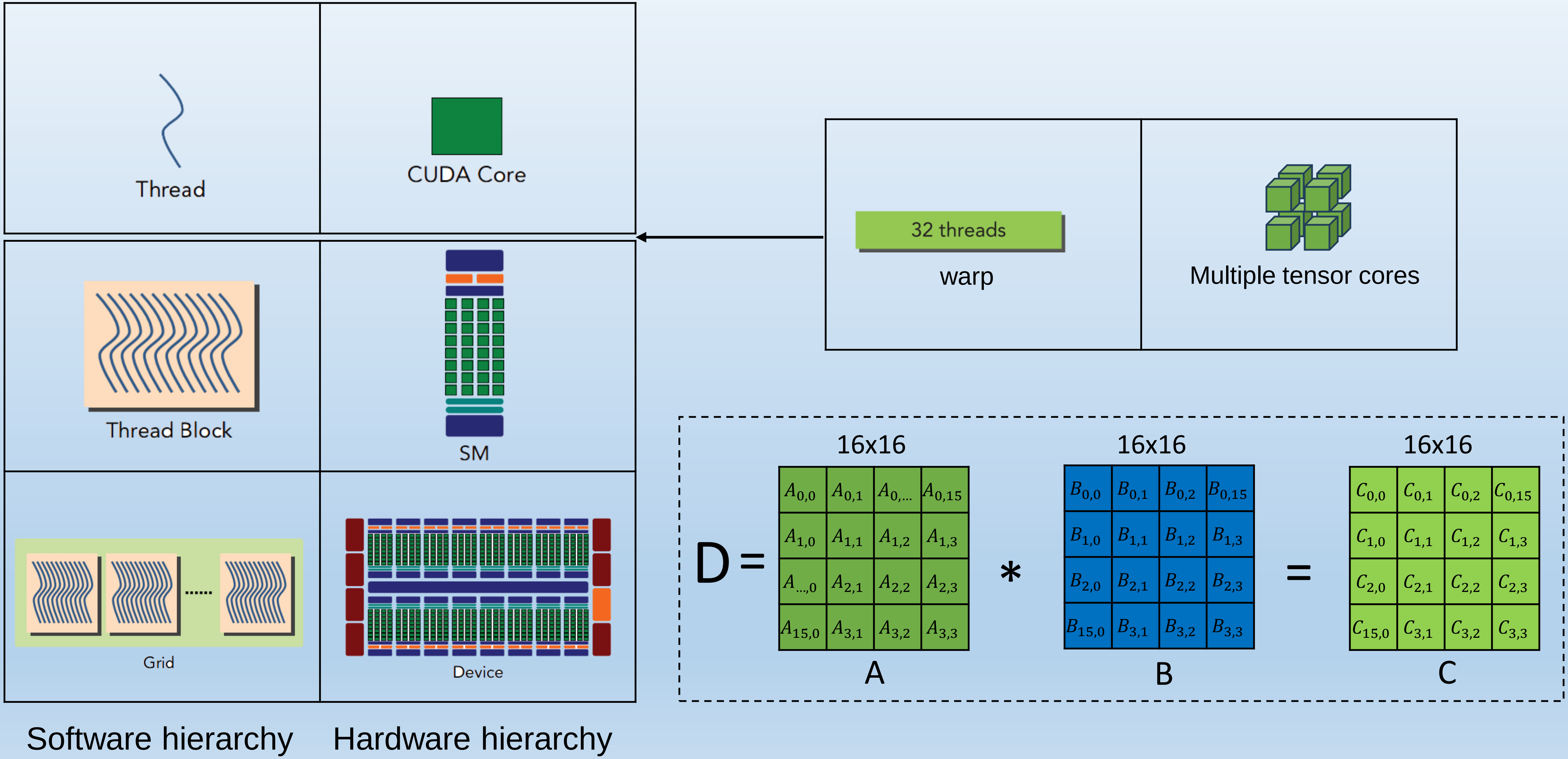
$$D = \underbrace{\begin{bmatrix} A_{0,0} & A_{0,1} & A_{0,2} & A_{0,3} \\ A_{1,0} & A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,0} & A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,0} & A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix}}_{A}^{4\times4} * \underbrace{\begin{bmatrix} B_{0,0} & B_{0,1} & B_{0,2} & B_{0,3} \\ B_{1,0} & B_{1,1} & B_{1,2} & B_{1,3} \\ B_{2,0} & B_{2,1} & B_{2,2} & B_{2,3} \\ B_{3,0} & B_{3,1} & B_{3,2} & B_{3,3} \end{bmatrix}}_{B}^{4\times4} = \underbrace{\begin{bmatrix} C_{0,0} & C_{0,1} & C_{0,2} & C_{0,3} \\ C_{1,0} & C_{1,1} & C_{1,2} & C_{1,3} \\ C_{2,0} & C_{2,1} & C_{2,2} & C_{2,3} \\ C_{3,0} & C_{3,1} & C_{3,2} & C_{3,3} \end{bmatrix}}_{C}^{4\times4}$$



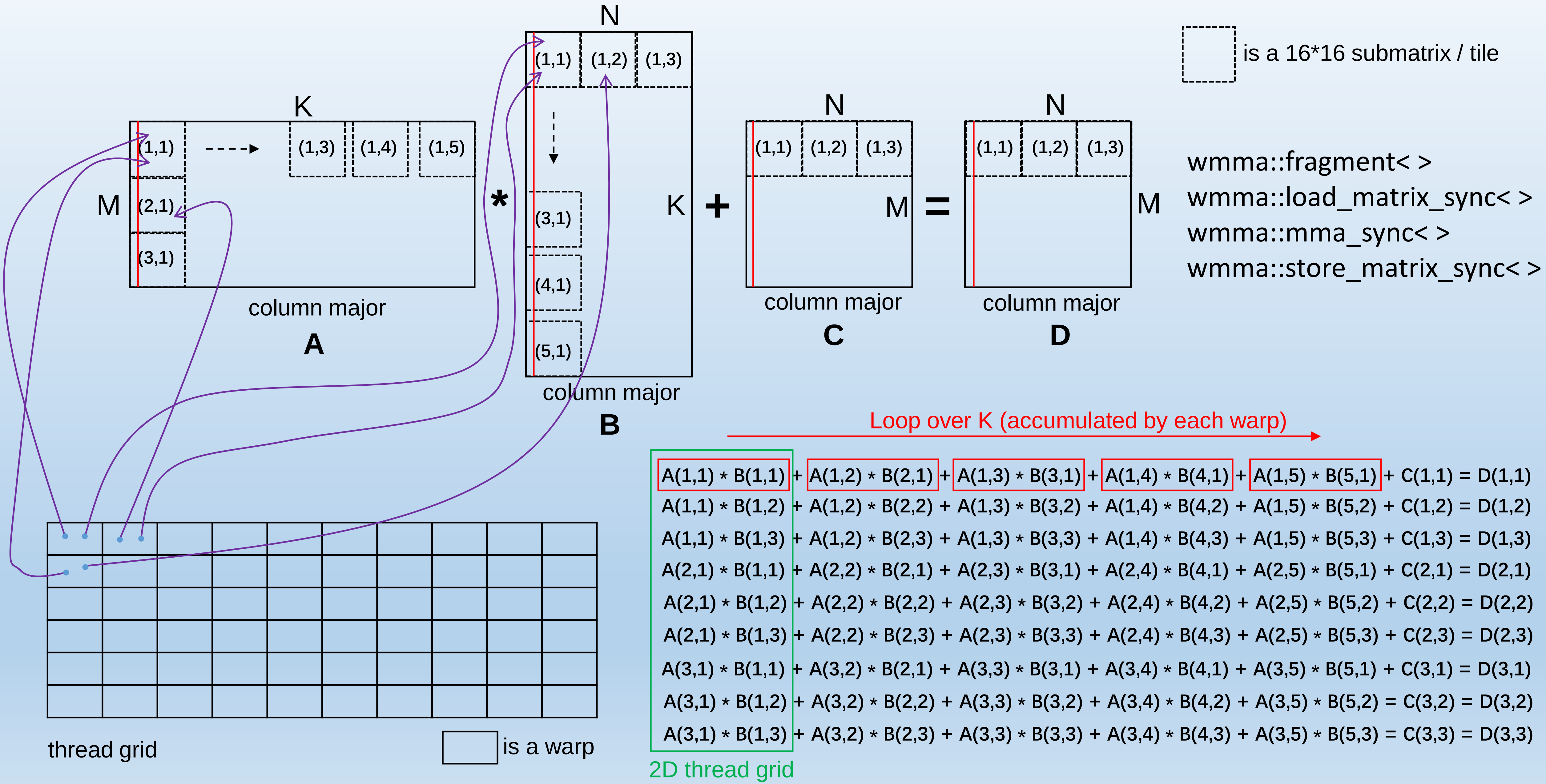
Some GPUs equipped with tensor cores

Model	Micro-architecture	Launch	CUDA cores	Memory size (GB)	Processing power (TFLOPS)		
					Tensor core FP32	Single precision	Double precision
V100	Volta	Jun 21, 2017	5120	16/32	112.2	14.0	7.0
T4	Turing	Sept 12, 2018	2560	16	64.8	8.1	0.2
A100	Ampere	May 14, 2020	6912	40/80	312.0	19.5	9.7
H100	Hopper	Mar 22, 2022	14592	80	756.4	51.2	25.6

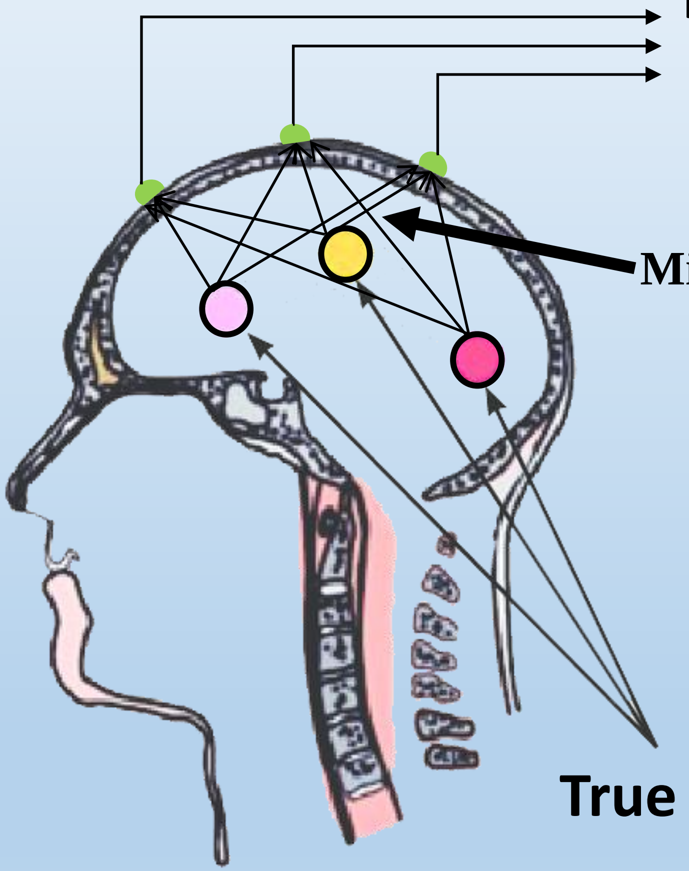
1.1 Abstract and hierarchy of the tensor core



1.2 Tensor core programming paradigm



2.1 Independent Component Analysis (ICA)



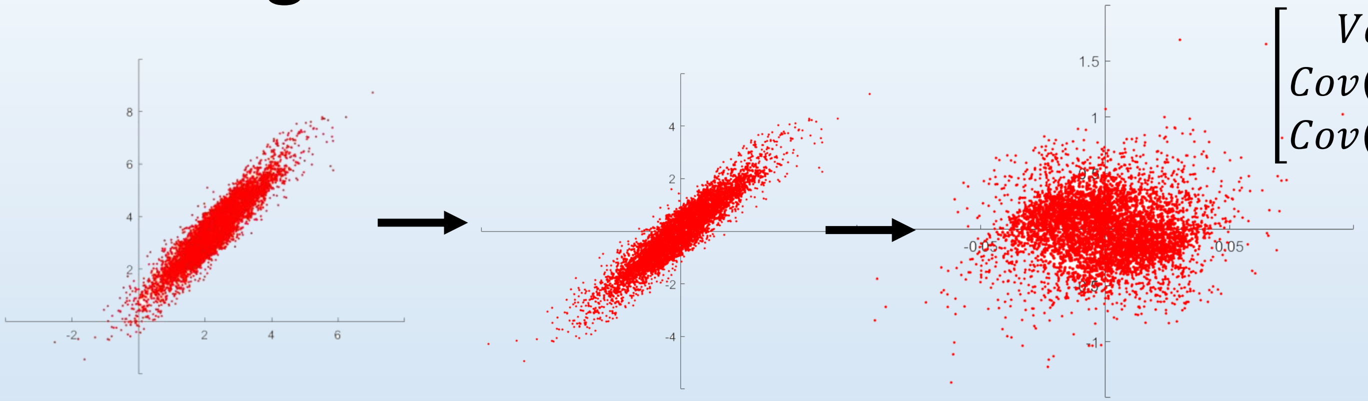
Unknown

$$\begin{bmatrix} x_{1(t)} \\ x_{2(t)} \\ x_{3(t)} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{32} \end{bmatrix} \cdot \begin{bmatrix} s_{1(t)} \\ s_{2(t)} \\ s_{3(t)} \end{bmatrix}$$

$$\boldsymbol{X} \qquad \boldsymbol{A} \qquad \boldsymbol{S}$$

$$\boldsymbol{X} = \boldsymbol{A} \cdot \boldsymbol{S} \Longrightarrow \begin{cases} \boldsymbol{S} = \boldsymbol{W} \cdot \boldsymbol{X} \\ \boldsymbol{W} = \boldsymbol{A}^{-1} \end{cases}$$

2.1 Algorithm flow



Raw data Centralized data Whitened data

$$Cov = XX^T$$

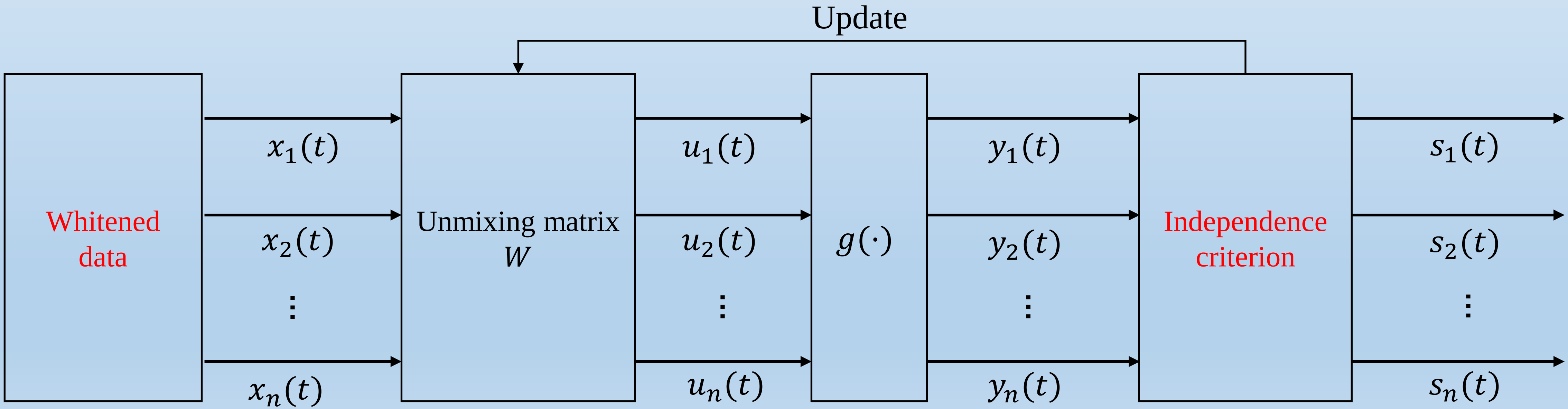
$$\begin{bmatrix} Var(CH1) & Cov(CH1,CH2) & Cov(CH1,CH3) \\ Cov(CH2,CH1) & Var(CH2) & Cov(CH2,CH3) \\ Cov(CH3,CH1) & Cov(CH3,CH2) & Var(CH3) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Singular Value Decomposition

$$X = U\Sigma V^T$$

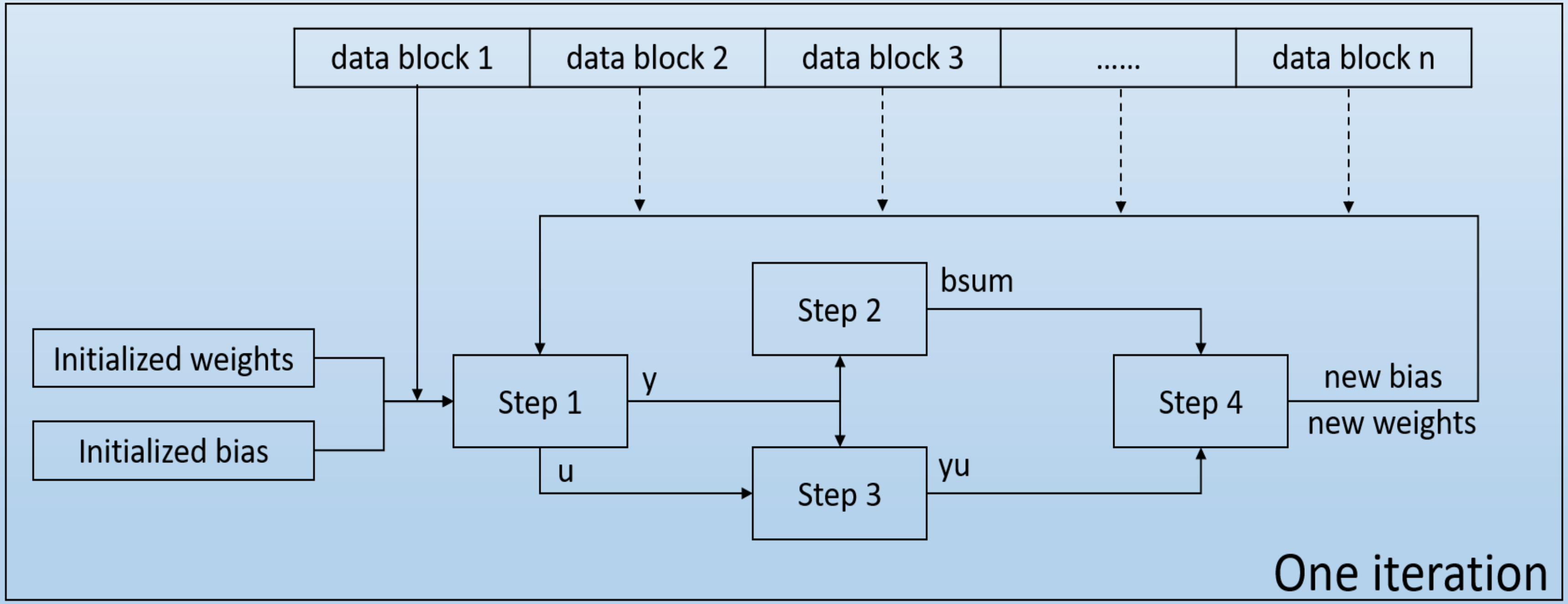
$$\Sigma = diag(\sigma_1, \sigma_2, \sigma_3, \dots)$$

$$X_{deco} = \sqrt{U^T X}$$



The process of solving the unmixing matrix

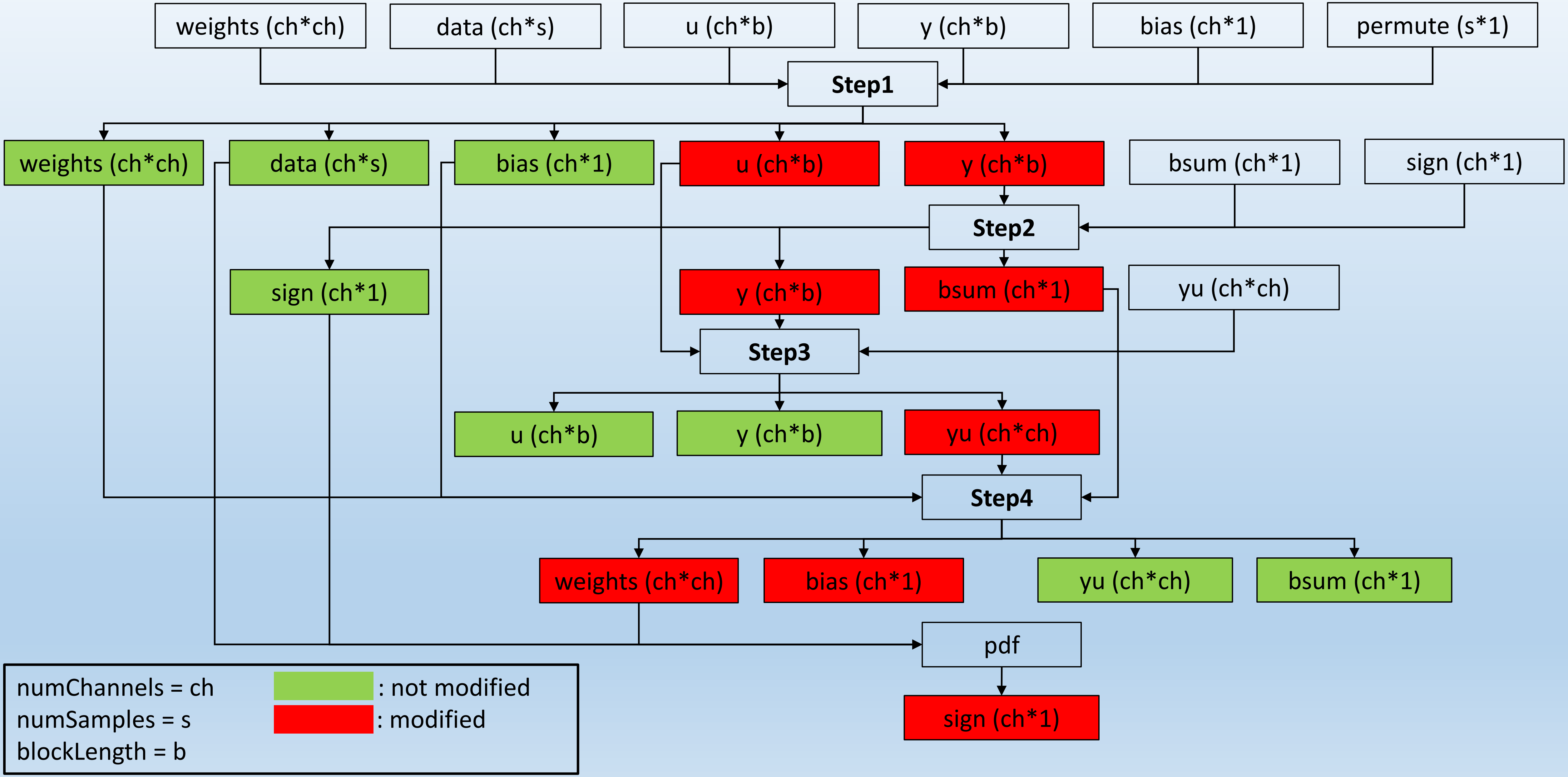
2.2 Parallel implementation



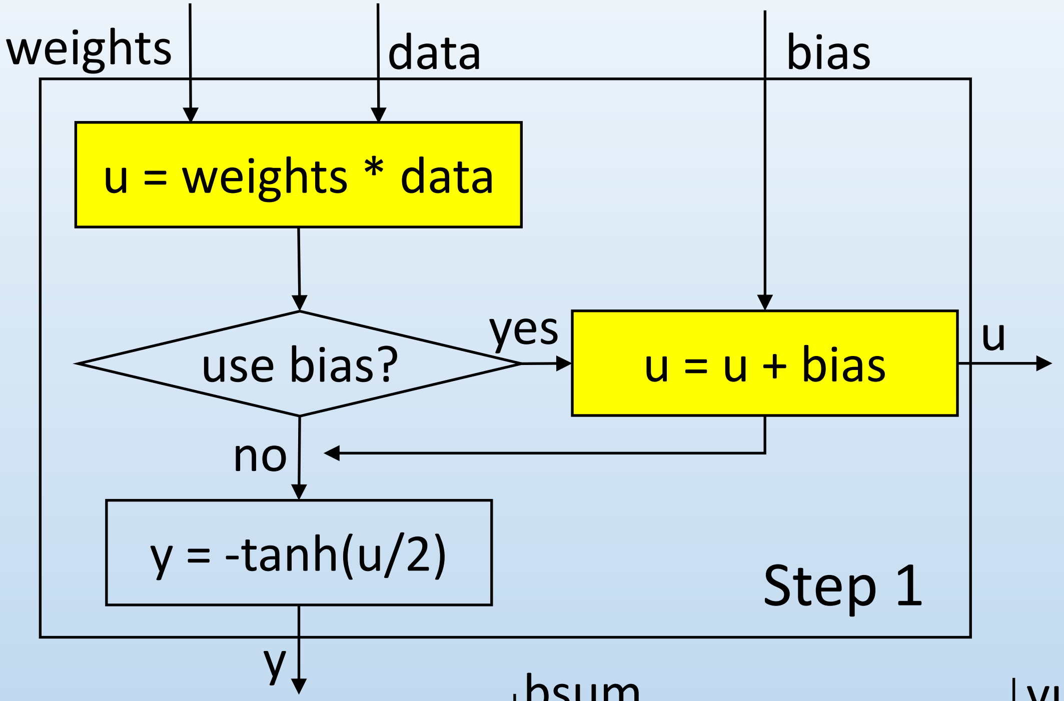
CUDA ICA

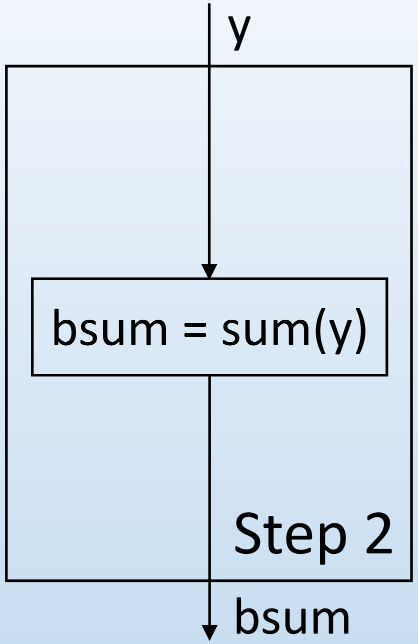
1. Initialize $W(0)$
2. **while** (not converged and t < limit)
    2.1 $\mathbf{Y}$ = permutate($\mathbf{X}$)
    2.2 **for** each block $\mathbf{B}$ in signal $\mathbf{Y}$
        2.2.1 $\mathbf{U} = \mathbf{WY} + bias$
        2.2.2 $\mathbf{Y} = tanh(\mathbf{U})$
        2.2.3 $\mathbf{W}^* =$
        $\mathbf{W} + \eta(\mathbf{I} - \mathbf{YU}^\mathbf{T} - \mathbf{UU}^\mathbf{T})\mathbf{W}$
        2.2.4 update bias
    end for
end while

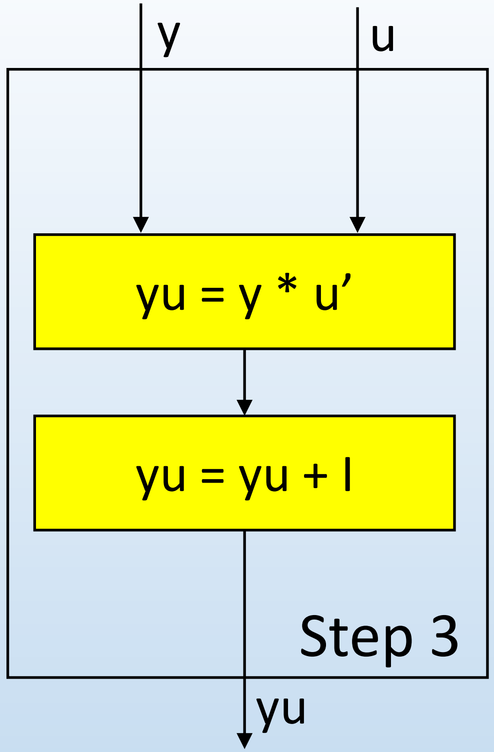
2.3 Variables analysis

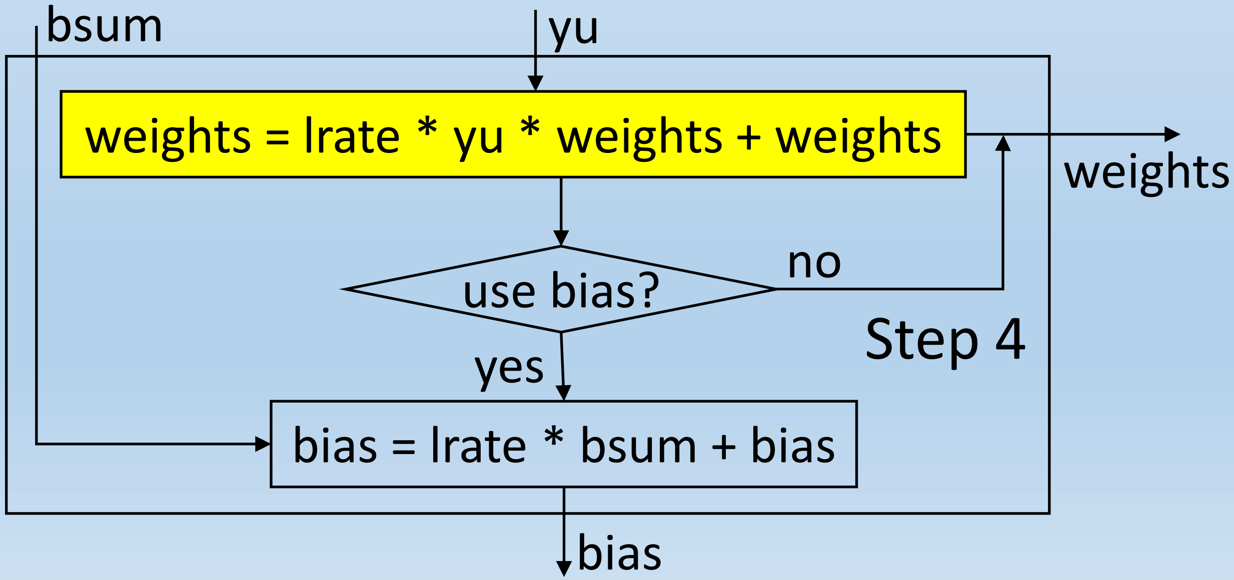


3.1 Locating matrix multiplication









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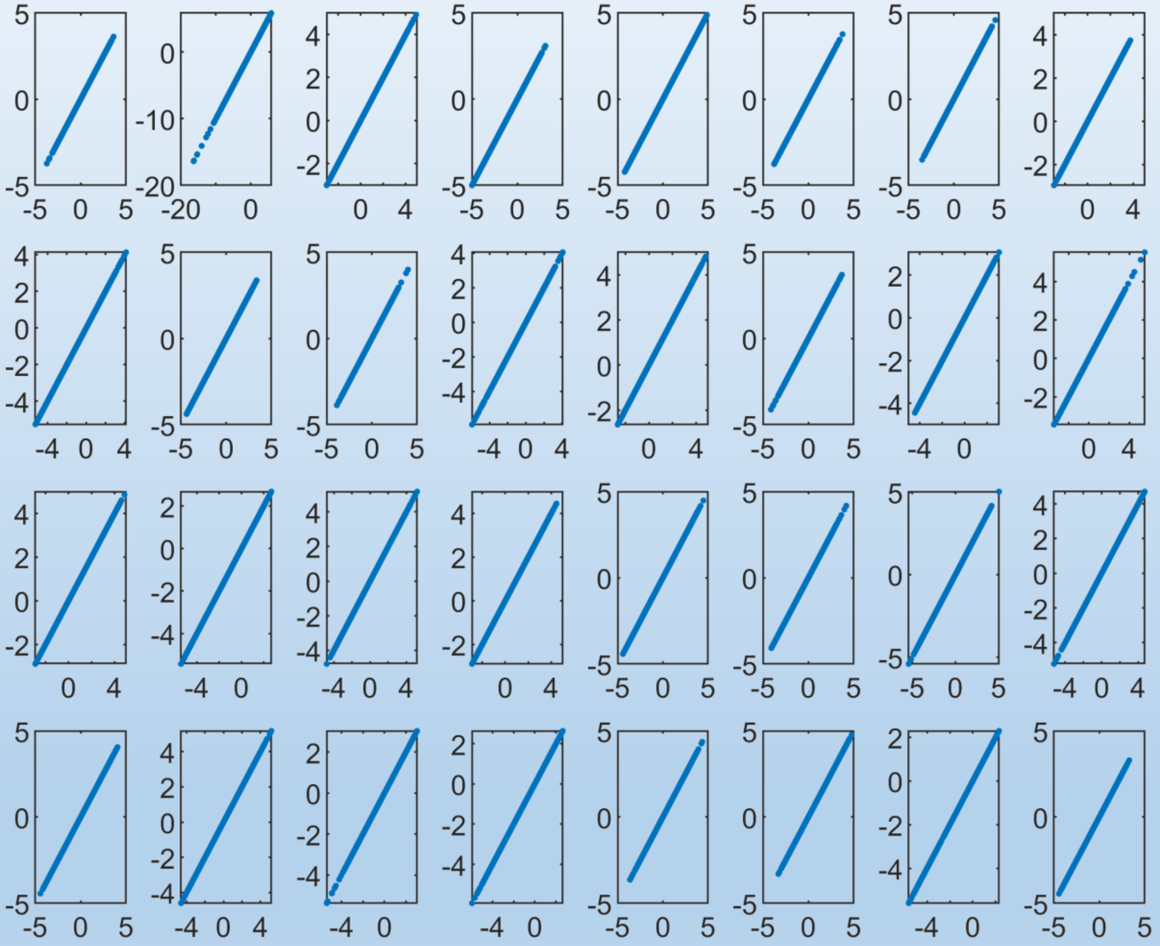
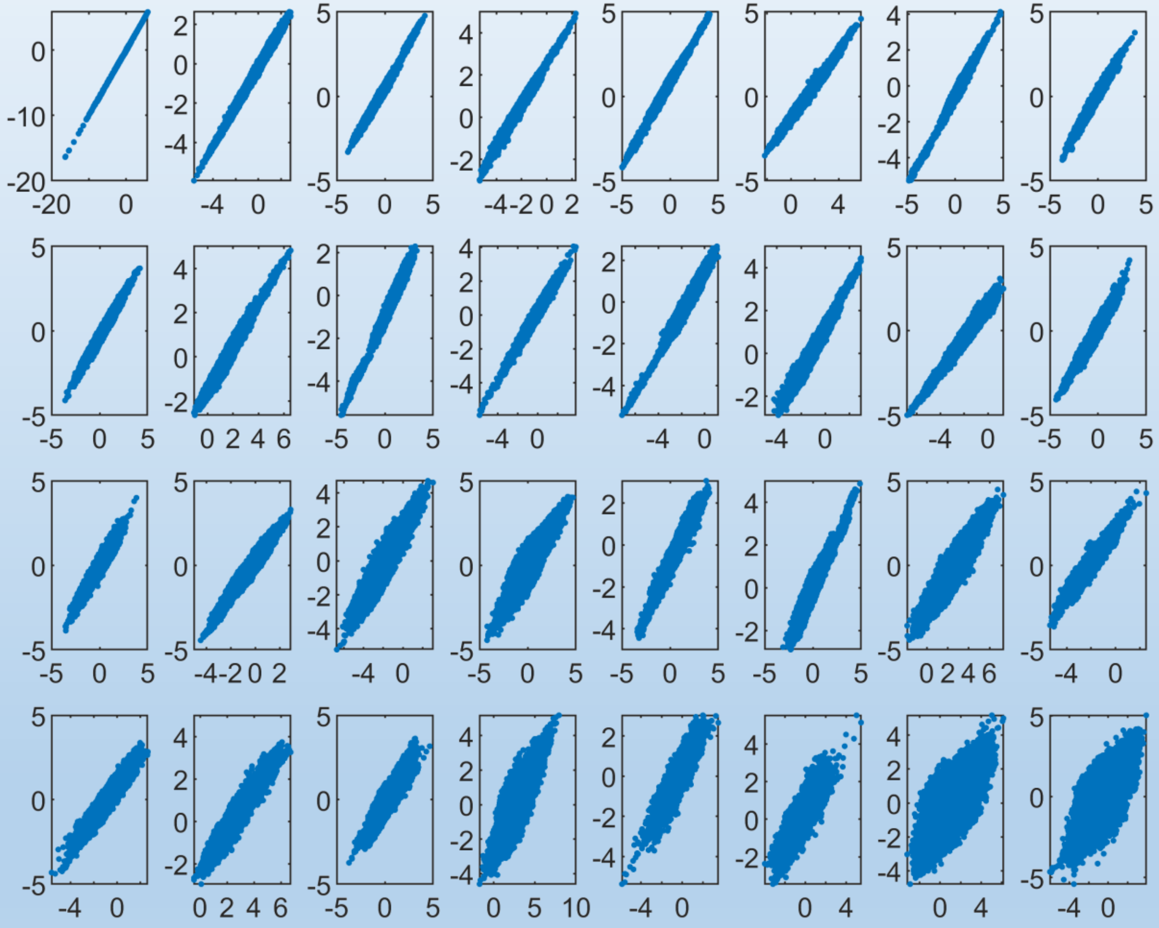
   end for
end while

: FMA operation

3.2 Numerical correctness



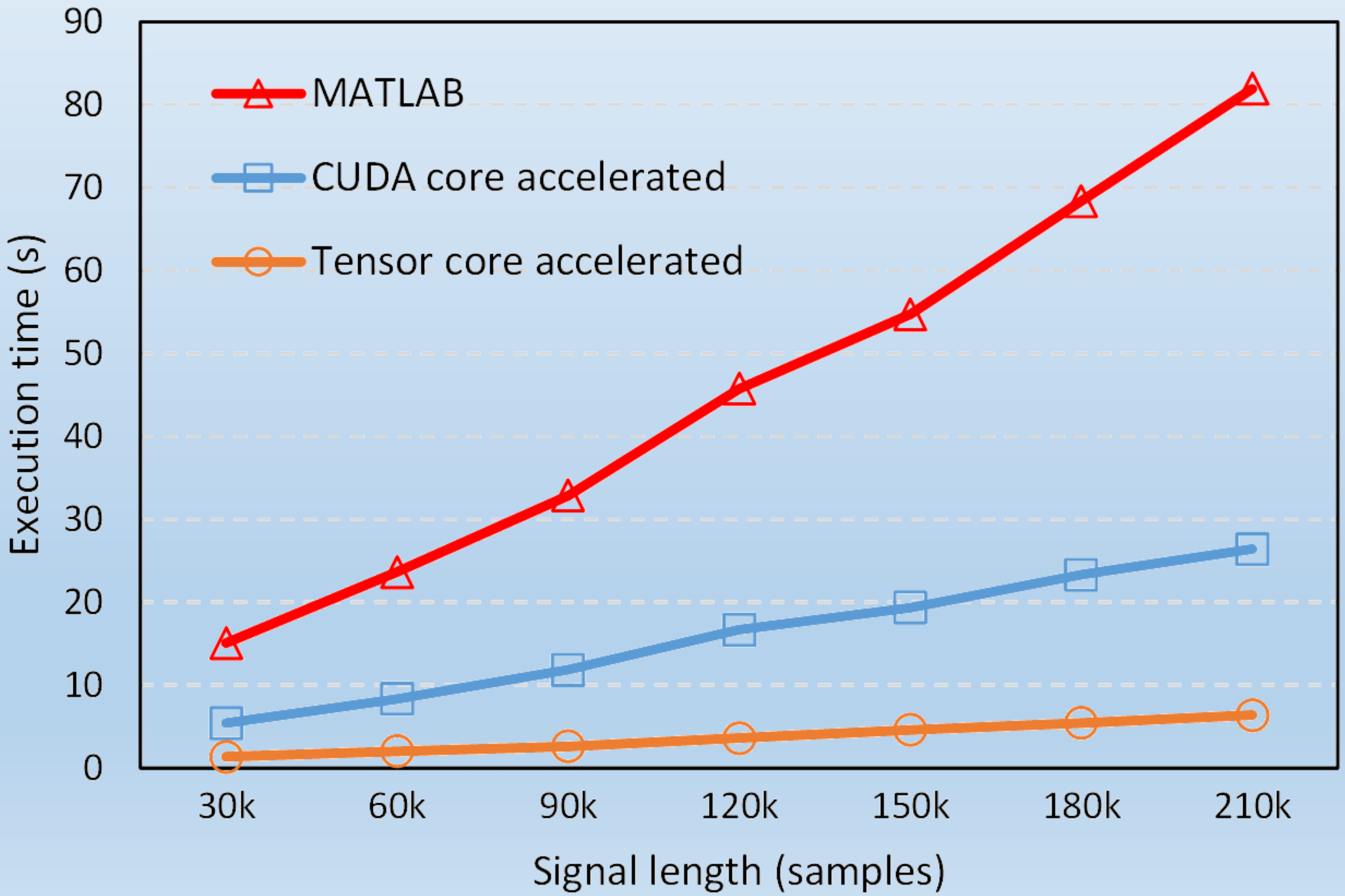
The diagonal lines produced by IC pairs from MATLAB-ICA and CUDAICA



The diagonal lines produced by IC pairs from tensor core ICA and CUDAICA

3.3 Performance preview

32-channel EEG data, up to 210k time points, CUDA core and tensor core execution time are produced by A100 GPU on Komondor supercomputer. The MATLAB execution time is produce by a 8-core CPU (i7-9700k)



3x speedup

4x speedup

4 Future works

- Numerical correctness issue
- Tensor core + shared memory strategy
- Further performance testing
  - New strategy and larger dataset
  - EEG processing pipeline integration
- Impact of mixed precision computing
  - FP32, TF32, FP16, BF16 ……
- **Other potential algorithms that tensor cores can optimize**
  - **Are there a lot of matrix multiplication operations in the algorithm? Why not consider using tensor cores to speed it up?**

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Decorrelation

Samples

Channels

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \end{bmatrix}$$

Data matrix

$$Cov(X,Y) = \frac{1}{n}\sum_{i=n}^{n}(x_i - E(x))(y_i - E(Y))$$

Covariance

$$\begin{bmatrix} Var(CH1) & Cov(CH1,CH2) & Cov(CH1,CH3) \\ Cov(CH2,CH1) & Var(CH2) & Cov(CH2,CH3) \\ Cov(CH3,CH1) & Cov(CH3,CH2) & Var(CH3) \end{bmatrix}$$

Covariance matrix

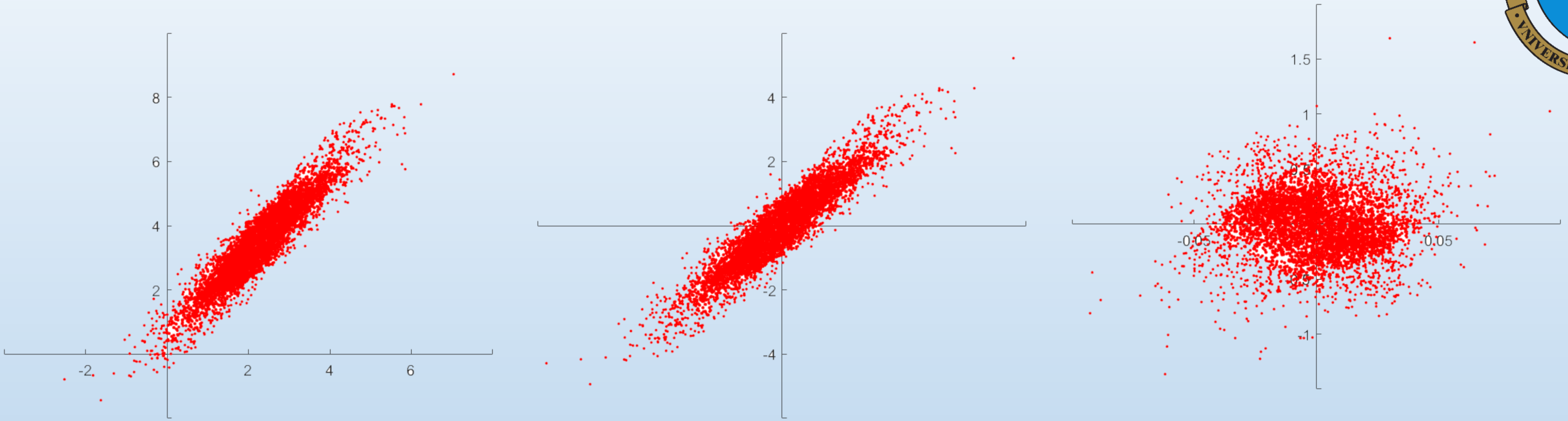
Diagonalization

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Diagonal matrix

$$Cov(X,Y)\begin{cases} > 0: & \textit{Positive correlation} \\ < 0: & \textit{Negative correlation} \\ = 0: & \textit{No correlation} \end{cases}$$

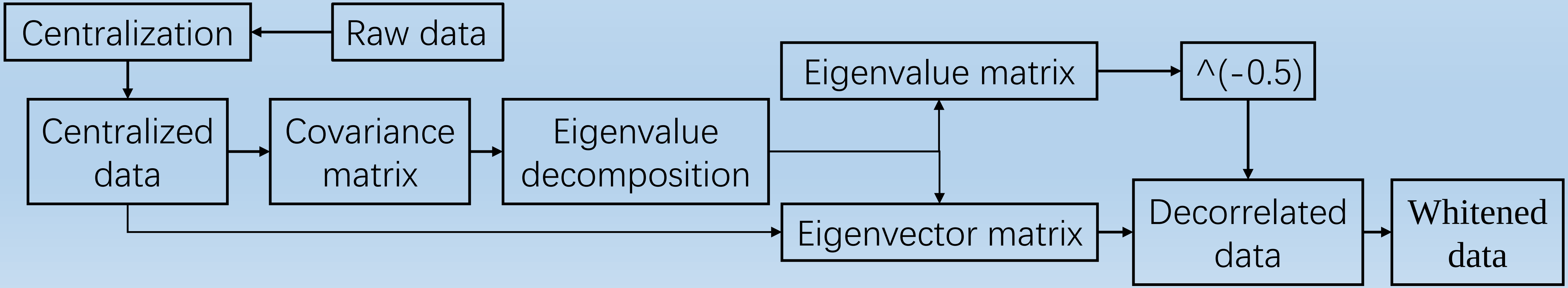
Decorrelation



Raw data

Centralized data

Whitened data



$$X = U\Sigma V^T$$

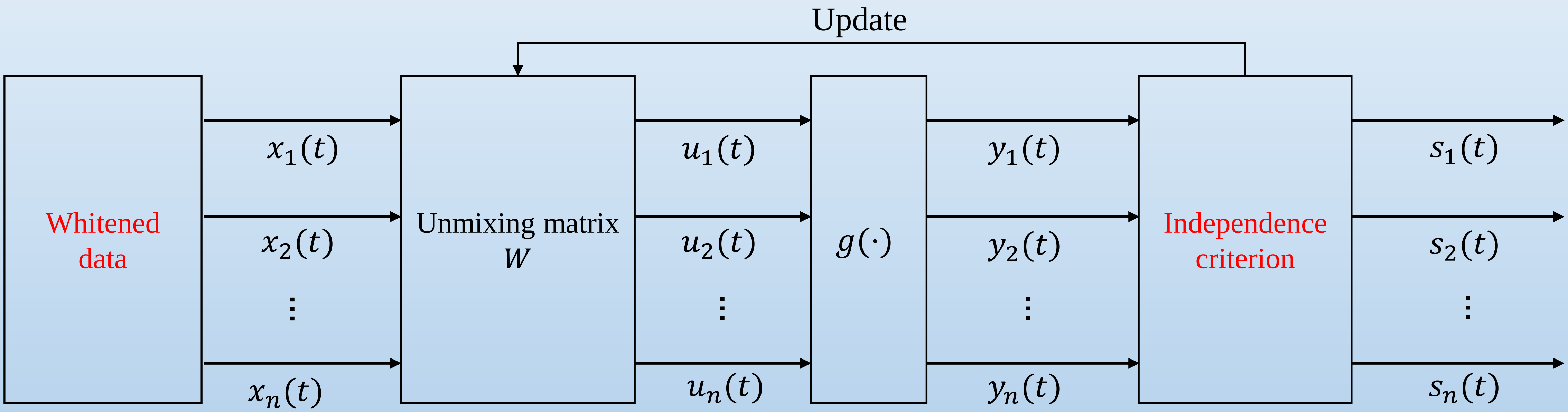
$$UU^T = I$$
$$V^T V = I$$

$$XX^T = U\Sigma V^T * (U\Sigma V^T)^T$$
$$XX^T = U\Sigma V^T * V\Sigma U^T$$
$$XX^T = U\Sigma U^T$$
$$XX^T U = U\Sigma^2$$
$$U^T XX^T U = U^T U\Sigma^2$$

Set $Y = U^T X$

$$YY^T = \Sigma^2$$

Remove dependency



The process of solving the unmixing matrix