THE GAUGE SYMMETRY - BROKEN AND UNBROKEN!
- IN THE QCD GROUND STATE

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Lagrangian of QCD

H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B **47** (1973) 365

$$L_{QCD} = L_{YM} + L_{qg}$$

$$L_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + L_{g.f.} + L_{gh.}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \quad a = N_c^2 - 1, \quad N_c = 3$$

$$L_{qg} = i \bar{q}^j_\alpha D_{\alpha\beta} q^j_\beta + \bar{q}^j_\alpha m^j_0 q^j_\beta, \quad \alpha, \beta = 1, 2, 3, \quad j = 1, 2, 3, ... N_f$$

$$D_{\alpha\beta} q^j_\beta = (\delta_{\alpha\beta} \partial_\mu - ig(1/2) \lambda^a_{\alpha\beta} A^a_\mu) \gamma_\mu q^j_\beta$$

The λ^as are generators of $SU(3)$ color gauge group

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$$

$$q^A \quad \to \quad U(x)q^A(x), \quad U(x) = \exp(i\Theta^a(x)\lambda^a/2)$$

$$A_\mu(x) = A_\mu^a(x)\lambda^a/2 \;\to\; U(x)A_\mu(x)U^{-1}(x)+\frac{i}{g}U(x)\partial_\mu U^{-1}(x)$$

$$SU(N_f) \times SU(N_f) \times U_B(1) \times U_A(1), \quad q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$$

$$L_q = \bar{q}_\alpha^A \delta^{AB} m_0 q_\alpha^B$$

$$SU(N_f) \times SU(N_f) \to SU(N_f)$$

Properties of the QCD Lagrangian

QCD without quarks is Yang-Mills (YM)

1). Universal coupling constant is strong, $g \sim 1$.

2). In general, in QCD the PT does not work.

3). $g \ll 1$ only in the AF regime, where the PT works.

4). But there is a scale breaking, appears Λ^2_{QCD}.

5). The QCD Lagrangian by itself can explain neither scale breaking nor confinement of the coloured quarks and gluons.

Phase transitions in QCD

I. The confinement phase transition at the fundamental quark-gluon level in order to explain why all the physical states are colour-singlets.

II. The PCAC (partial conserved axial currents) phase transition at the hadronic level in order to explain the soft pion physics (current algebra results).

The both phase transitions were not explained up to present days though QCD has been formulated about five decades ago!

The confinement phase transition

The two conceptual problems of QCD

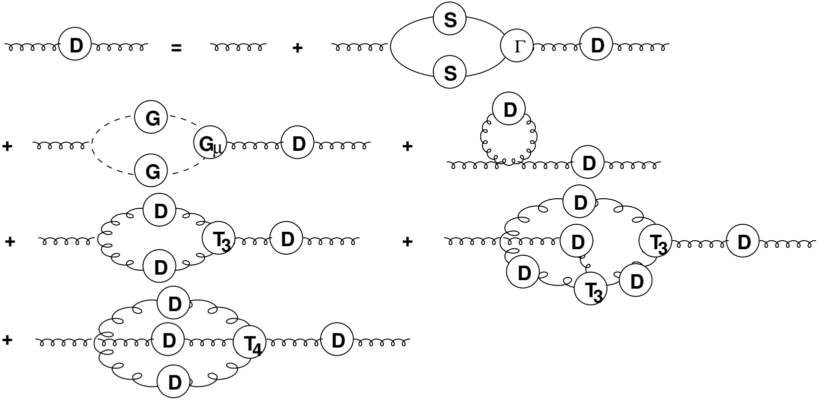
A. Whether the gauge symmetries of the QCD Lagrangian and its ground state coincide or not?

B. The dynamical generation of a mass squared in the vacuum of QCD, since the gauge invariance of its Lagrangian forbids such kind of terms, like the gluon mass term $M_g^2 A_\mu A_\mu$.

How does one get a mass out of massless theory?

'Mass without mass'! by F. Wilczek

Gluon SD equation



$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i \Pi_{\rho\sigma}(q; D) D_{\sigma\nu}(q)$$

$$\Pi_{\rho\sigma}(q; D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q; D) + \Pi^t_{\rho\sigma}(D)$$

$$\Pi^g_{\rho\sigma}(q; D) = \Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q; D^2) + \Pi^{(2)}_{\rho\sigma}(q; D^4) + \Pi^{(2')}_{\rho\sigma}(q; D^3)$$

$$\Pi^t_{\rho\sigma}(D) \sim \int d^4k D_{\alpha\beta}(k) T^0_{\rho\sigma\alpha\beta} = g_{\rho\sigma} \Delta^2_t(D) = [T_{\rho\sigma}(q) + L_{\rho\sigma}(q)] \Delta^2_t(D)$$

$$\Pi_{\rho\sigma}(q; D) \equiv \Pi_{\rho\sigma}(q; \tilde{\lambda}, \alpha, D), \quad L_{\rho\sigma}(q) = \frac{q_\rho q_\sigma}{q^2}$$

$$q^2 \to 0, \;\; q_i \to 0, \;\; q^2 \to -q^2, \;\; g_{\rho\sigma} \to \delta_{\rho\sigma}$$

Dilemma

The tadpole term breaks the QCD gauge symmetry! Why is it present in the vacuum of QCD at all if it makes theory un-renormalizable from the very beginning?

The standard solution was to remove this and all other such kind of QD constants in accordance with the QCD Lagrangian gauge symmetry! No mass scale parameter in this case!

However, here we are going to solve this dilemma by asking the question is it possible to retain the tadpole term in the QCD vacuum, but without affecting the PT renormalization properties of the theory?

Unlike to all other QD terms, it is only one which generate a mass squared scale parameter!

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i\Pi_{\rho\sigma}(q; D) D_{\sigma\nu}(q)$$

$$\Pi_{\rho\sigma}(q; D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q; D) + \delta_{\rho\sigma}\Delta^2_t(D)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i[\Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q; D)] D_{\sigma\nu}(q)$$

$$+ D^0_{\mu\rho}(q) i\delta_{\rho\sigma}\Delta^2_t(D) D_{\sigma\nu}(q)$$

Slavnov-Taylor (ST) identities

they are exact constraints on any solution to QCD

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi, \quad q_\mu q_\nu D^0_{\mu\nu}(q) = i\xi_0$$

$$D_{\mu\nu}(q) = i\left\{ T_{\mu\nu}(q) d(q^2) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}$$

$$D^0_{\mu\nu}(q) = i\left\{ T_{\mu\nu}(q) + \xi_0 L_{\mu\nu}(q) \right\} \frac{1}{q^2}$$

$$T_{\mu\nu}(q) = \delta_{\mu\nu} - (q_\mu q_\nu / q^2) = \delta_{\mu\nu} - L_{\mu\nu}(q)$$

$\xi = f(q^2; \xi_0)$? *the so-called gauge-fixing function in QCD*

The transverse relations

$$q_\rho q_\sigma \Pi_{\rho\sigma}(q;D) = \frac{(\xi_0 - \xi)}{\xi\xi_0}(q^2)^2$$

$$\Pi_{\rho\sigma}(q;D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q;D) + \delta_{\rho\sigma}\Delta^2_t(D)$$

$$q_\rho q_\sigma[\Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q;D)] = \frac{(\xi_0 - \xi)}{\xi\xi_0}(q^2)^2 - q^2\Delta^2_t(D)$$

By themselves they cannot remove the QD constants from the theory, as well as to fix the function $\xi = f(q^2;\xi_0)$

The satisfied transverse relations

$$q_\rho q_\sigma[\Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q;D)] = \frac{(\xi_0 - \xi)}{\xi\xi_0}(q^2)^2 - q^2\Delta_t^2(D)$$

$$q_\rho q_\sigma \Pi^q_{\rho\sigma}(q) = 0, \quad q_\rho q_\sigma \Pi^g_{\rho\sigma}(q;D) = 0$$

$$q_\rho q_\sigma[\Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q;D^2) + \Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3)] = 0$$

$$\frac{(\xi_0 - \xi)}{\xi\xi_0} = \frac{\Delta_t^2(D)}{q^2} \quad \rightarrow \quad \xi = f(q^2;\xi_0) = \frac{\xi_0 q^2}{q^2 + \xi_0\Delta_t^2(D)}$$

The proper subtraction scheme

$$\Pi^{s(q)}_{\rho\sigma}(q) = \Pi^{q}_{\rho\sigma}(q) - \Pi^{q}_{\rho\sigma}(0) = \Pi^{q}_{\rho\sigma}(q) - \delta_{\rho\sigma}\Delta^2_{\boldsymbol{q}},$$

$$\Pi^{s(g)}_{\rho\sigma}(q; D) = \Pi^{g}_{\rho\sigma}(q; D) - \Pi^{g}_{\rho\sigma}(0; d) = \Pi^{g}_{\rho\sigma}(q; D) - \delta_{\rho\sigma}\Delta^2_{g}(D),$$

so that $\Pi^{s(q)}_{\rho\sigma}(0) = \Pi^{s(g)}_{\rho\sigma}(0; D) = 0, \qquad q^2 = -\mu^2 \to 0$

$$\Delta^2_g(D) = \Delta^2_{gh} + \Delta^2_1(D^2) + \Delta^2_2(D^4) + \Delta^2_{2'}(D^3)$$

$$\Pi^{q}_{\rho\sigma}(q) = \Pi^{q}_{\rho\sigma}(q) - \Pi^{q}_{\rho\sigma}(0) + \Pi^{q}_{\rho\sigma}(q)$$

$$\Pi^{q}_{\rho\sigma}(q) = \Pi^{s(q)}_{\rho\sigma}(q) + \Pi^{q}_{\rho\sigma}(0), \quad \Pi^{g}_{\rho\sigma}(q; D) = \Pi^{s(g)}_{\rho\sigma}(q; D) + \Pi^{q}_{\rho\sigma}(0; D)$$

Independent tensor decomposition

$$\Pi^{s(q)}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{s(q)}_t(q^2) - q_\rho q_\sigma \Pi^{s(q)}_l(q^2)$$

$$\Pi^{q}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{q}_t(q^2) - q_\rho q_\sigma \Pi^{q}_l(q^2)$$

$$\Pi^{s(g)}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{s(g)}_t(q^2) - q_\rho q_\sigma \Pi^{s(g)}_l(q^2)$$

$$\Pi^{g}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{g}_t(q^2) - q_\rho q_\sigma \Pi^{g}_l(q^2)$$

$$\Pi^{s(q)}_t(q^2),\ \Pi^{s(q)}_l(q^2),\ \Pi^{s(g)}_t(q^2),\ \Pi^{s(q)}_l(q^2)$$

$$\Pi_t^{s(q)}(q^2) = \Pi_t^q(q^2) - \frac{\Delta_q^2}{q^2}, \quad \Pi_l^{s(q)}(q^2) = \Pi_l^q(q^2) + \frac{\Delta_q^2}{q^2}$$

$$\Pi_t^{s(g)}(q^2) = \Pi_t^g(q^2) - \frac{\Delta_g^2(D)}{q^2}, \quad \Pi_l^{s(g)}(q^2) = \Pi_l^g(q^2) + \frac{\Delta_g^2(D)}{q^2}$$

$$\Pi_t^{s(g)}(q^2) + \Pi_l^{s(g)}(q^2) = \Pi_t^g(q^2) + \Pi_l^g(q^2)$$

$$\Pi_t^{s(q)}(q^2) + \Pi_l^{s(q)}(q^2) = \Pi_t^q(q^2) + \Pi_l^q(q^2)$$

$$q_\rho q_\sigma \Pi_{\rho\sigma}(q; D) = q^2 \Delta_t^2(D)$$

$$q_\rho q_\sigma [\Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q; D)] = 0$$

$$q_\rho q_\sigma \Pi^q_{\rho\sigma}(q) = q_\rho q_\sigma \Pi^g_{\rho\sigma}(q; D) = 0$$

$$\Pi^q_l(q^2) = \Pi^g_l(q^2; D) = 0$$

$$\Pi_l^{s(q)}(q^2) = \frac{\Delta_q^2}{q^2}, \quad \Pi_l^{s(g)}(q^2) = \frac{\Delta_g^2(D)}{q^2}$$

$$\Delta_q^2 = \Delta_g^2(D) = 0, \quad \Delta_t^2(D) \neq 0$$

$$\Delta_{gh}^2 = \Delta_1^2(D^2) = \Delta_2^2(D^4) = \Delta_{2'}^2(D^3) = 0$$

$$\Pi_{\rho\sigma}^q(q) + \Pi_{\rho\sigma}^g(q; D) = T_{\rho\sigma}(q) q^2 \Pi_t^s(q^2; D)$$

$$\Pi_t^s(q^2; D) = \Pi_t^{s(q)}(q^2) + \Pi_t^{s(g)}(q^2; D) = \Pi(q^2; D)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i\Pi_{\rho\sigma}(q; D) D_{\sigma\nu}(q)$$

$$\Pi_{\rho\sigma}(q; D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q; D) + \delta_{\rho\sigma}\Delta^2_t(D) = T_{\rho\sigma}(q) q^2 \Pi(q^2; D)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2 \Pi(q^2; D) D_{\sigma\nu}(q)$$

$$+ D^0_{\mu\rho}(q) i \delta_{\rho\sigma} \Delta^2_t(D) D_{\sigma\nu}(q)$$

THE GAUGE SYMMETRY IS BROKEN

The gluon SDE in the generalized gauge

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) \left[q^2 \Pi(q^2; D) + \Delta_t^2(D) \right] D_{\sigma\nu}(q)$$

$$D^0_{\mu\rho}(q) i L_{\rho\sigma}(q) \Delta_t^2(D) D_{\sigma\nu}(q)$$

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi_0 \left(1 - \xi \frac{\Delta_t^2(D)}{q^2} \right) = i\xi$$

$$\xi = f(q^2; \xi_0) = \frac{\xi_0 q^2}{q^2 + \xi_0 \Delta_t^2(D)}$$

The QCD full gluon propagator

$$D_{\mu\nu}(q) = \frac{iT_{\mu\nu}(q)}{q^2 + q^2\Pi(q^2; D) + \Delta_t^2(D)} + iL_{\mu\nu}(q)\frac{\lambda^{-1}}{q^2 + \lambda^{-1}\Delta_t^2(D)}$$

$$\xi_0 = \lambda^{-1}, \quad d^{-1}(q^2; D) = [1 + \Pi(q^2; D) + (\Delta_t^2(D)/q^2]$$

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi(q^2; \lambda^{-1}) = i\frac{\lambda^{-1}q^2}{q^2 + \lambda^{-1}\Delta_t^2(D)}$$

$\lambda^{-1} = 0$, $\lambda^{-1} = 1$. The formal $\lambda^{-1} = \infty$.

This system of the regularized equations constitutes that the $SU(3)$ colour gauge symmetry of the QCD Lagrangian is not a symmetry of its ground state.

Why do we need the tadpole term 'alive'?

A. Summation of the severe IR singularities and suppression of the PT gluon states

$$D_{\mu\nu}(q) \sim i\frac{\delta_{\mu\nu}}{q^2} - i\frac{T_{\mu\nu}(q)}{q^2}\left[\Pi(q^2;D) + \frac{\Delta_t^2(D)}{q^2}\right] - iL_{\mu\nu}(q)\frac{\Delta_t^2(D)}{(q^2)^2} + ...,$$

$$D_{\mu\nu}(q) \sim T_{\mu\nu}\frac{\Delta_t^2(D)}{(q^2)^2}\sum_{k=0}^{\infty}\left(\frac{\Delta_t^2(D)}{q^2}\right)^k \Phi_k(g^2, \tilde{\lambda}...) + O_{\mu\nu}(1/q^2)$$

The confinement of the free gluons will be proven in the presence of the tadpole term. This is a singular part of the Laurent expansion but, in fact, this is a 'cluster' one.

B. The massive gluons (Minkowski signature)

$$D_{\mu\nu}(q) = \frac{-iT_{\mu\nu}(q)}{q^2 + q^2\Pi(q^2; D) - M^2} - iL_{\mu\nu}(q)\frac{\lambda^{-1}}{q^2 - \lambda^{-1}M^2}$$

If the denominator may have a pole at $q^2 = M_g^2$, then

$$D^0_{\mu\nu}(q; M_g^2) = \frac{-i}{(q^2 - M_g^2)}\left[g_{\mu\nu} - (1 - \lambda^{-1})\frac{q_\mu q_\nu}{(q^2 - \lambda^{-1}M_g^2)}\right]$$

It is expressed in the Stueckelberg gauge. So this gauge is a particular case of the generalized gauge derived above.

The self-consistency condition for the gauge choice

$$\frac{\lambda^{-1}q^2}{q^2-\lambda^{-1}M_g^2}=\frac{aq^2}{q^2-aM_g^2}, \quad a = \textit{finite number}, \quad \lambda^{-1}=a$$

$$\frac{\lambda^{-1}q^2}{q^2-\lambda^{-1}M_g^2}=-\frac{q^2}{M_g^2}, \quad a=\infty, \textit{ canonical gauge}, \quad \textit{no solution}!$$

Mass-shell $q^2=M_g^2$ at any finite gauge

$$D^0_{\mu\nu}(q;M_g^2)=\frac{-i}{(q^2-M_g^2)}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{M_g^2}\right]$$

Mass-shell $q^2=M_g^2$ at the canonical gauge $\lambda^{-1}=\infty$

Since the canonical gauge is not self-consistent in QCD, there is no mass-shell for the massive gluon fields within the generalized gauge-fixing function approach.

C. 'Dimensional Transmutation'

In the presence of the mass squared scale parameter - the tadpole term - the role of the QCD coupling constant g^2 becomes unimportant. This is also evidence of the 'dimensional transmutation', $g^2 \to \Delta_t^2(D)$, which occurs whenever a massless theory acquires mass dynamically. It is a general feature of spontaneously symmetry breaking in field theories.

We distinguish between the PT and full QCD by the explicit presence of the tadpole term in the latter one, and not by the magnitude of the coupling constant. In the both cases the gluon fields remain strongly interacted, apart from the asymptotic freedom (AF) regime.

D. The tadpole term and asymptotic freedom (AF)

$$D_{\mu\nu}(q) \sim g_{\mu\nu} \left[\frac{g^2}{1 + g^2 b_0 \ln(q^2/\Lambda^2_{QCD})} \right] (1/q^2), \quad q^2 \to \infty$$

Any mass to which can be assigned some physical meaning

$$M \sim \mu \exp(-1/b_0 g^2), \qquad g^2 \to 0$$

None a finite mass can survive in the PT weak coupling limit or, equivalently, in the PT $q^2 \to \infty$ regime. So the question where the finite mass comes from? cannot be answered by the PT! It has to come from the IR region, controlled precisely by the tadpole term. Its renormalized (finite) version should somehow related to Λ^2_{QCD}.

THE GAUGE SYMMETRY IS UNBROKEN

$$\Delta_t^2(D) = 0 \quad \to \quad \xi = f(\xi_0) = \xi_0, \quad \to \quad D \longrightarrow D^{PT}$$

$$D_{\mu\nu}^{PT}(q) = D_{\mu\nu}^0(q) + D_{\mu\rho}^0(q) i T_{\rho\sigma}(q) q^2 \Pi^s(q^2; D^{PT}) D_{\sigma\nu}^{PT}(q)$$

$$D_{\mu\nu}^{PT}(q) = i \left[\frac{1}{1 + \Pi(q^2; D^{PT})} T_{\mu\nu}(q) + \xi_0 L_{\mu\nu}(q) \right] \frac{1}{q^2}$$

This system of eqs. is free of all the types of the scale parameters having the dimensions of mass squared, forbidden by the exact gauge symmetry of the QCD Lagrangian.

Conclusions

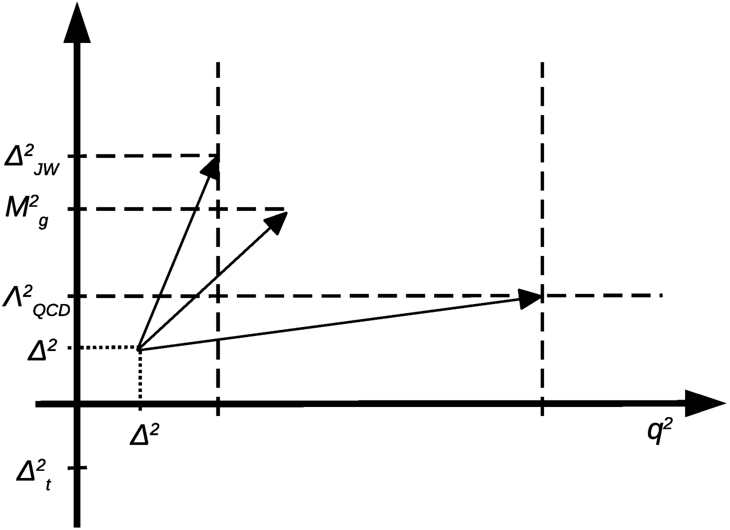
a. The two satisfied transverse relations exist in QCD making the theory renormalizable .
b. The subtraction scheme has been formulated, which makes it possible to remove from theory the quadratically UV divergent constants on the general basis.
c. The two different types of the gluon field configurations with broken and unbroken gauge symmetries in the QCD vacuum exist. The dynamical source of this effect is the tadpole term.
d. The system of the relations which removes all the QD constants apart from the tadpole term are exact mathematical results, i.e., they are not prescriptions.
e. The NL dependence ξ on ξ_0 in the generalized gauge in the PT $q^2 \to \infty$ or, equivalently, formal $\Delta_t^2(D) = 0$ limits becomes linear one $\xi = \xi_0$.

Summary

In the QCD ground state exist such gluon field configurations which break/violate the gauge symmetry of its Lagrangian, but the PT renormalizability of the theory is not affected. Existence of this effect includes establishing the true role of the tadpole term in the QCD ground state. The gluon field configurations with unbroken gauge symmetry are also exist.

To understand the phenomenon of the gauge symmetry violation is a key to the explanation of the confinement phase transition in QCD.

QCD is the self-consistent quantum field gauge theory. It does not need some extra degrees of freedom in order to generate a mass scale parameters.


Δ^2_{JW}
M^2_g
Λ^2_{QCD}
Δ^2
Δ^2
q^2
Δ^2_t