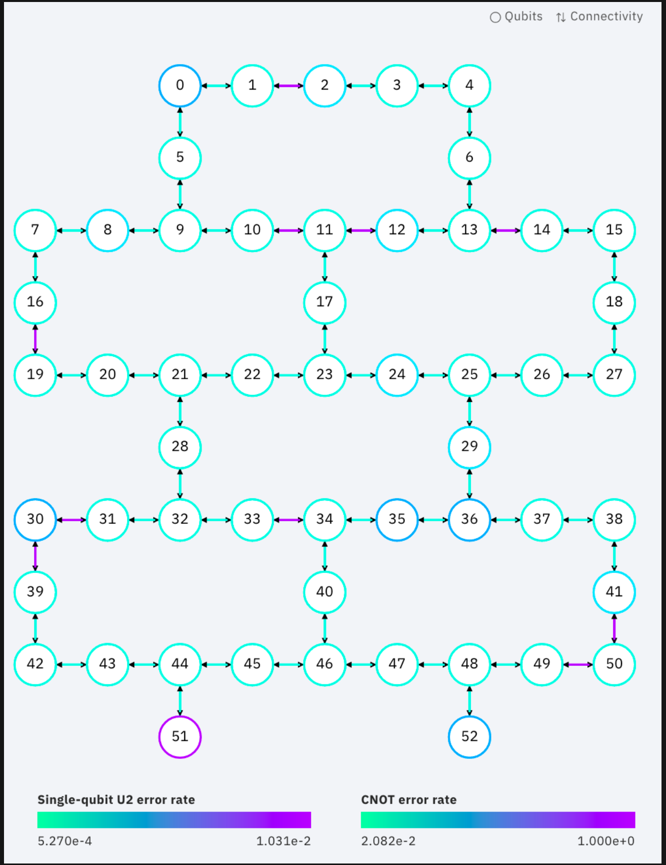
Introduction to Quantum Error Correction

James R. Wootton
IBM Quantum

Why do we need QEC?

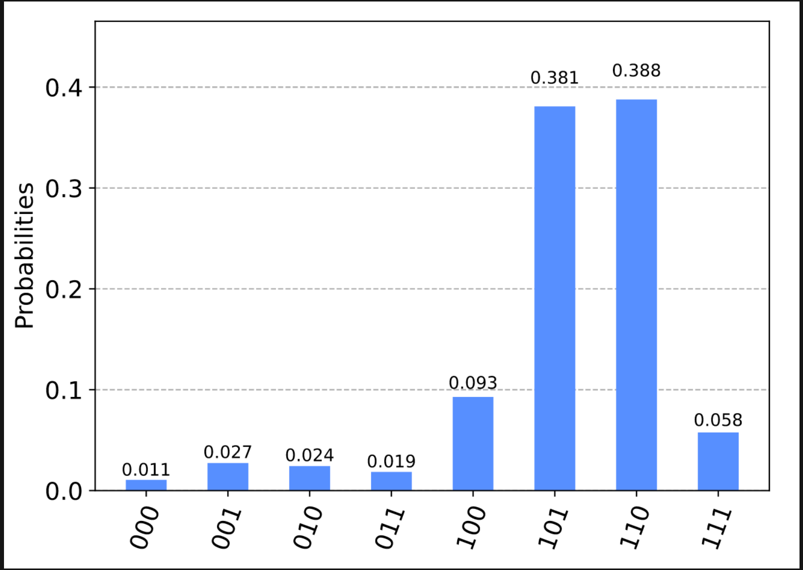
- Superconducting qubits don’t work exactly the way they should
  - Each gate is slightly wrong
  - Qubits get poked by (small) external forces
  - Measurements sometimes lie
- The same is true for spin qubits, topological qubits, photonic qubits, …
- **Any qubit made out of a real-life physical system will be at least a little bit rubbish**



Why do we need QEC?

IBM **Quantum**

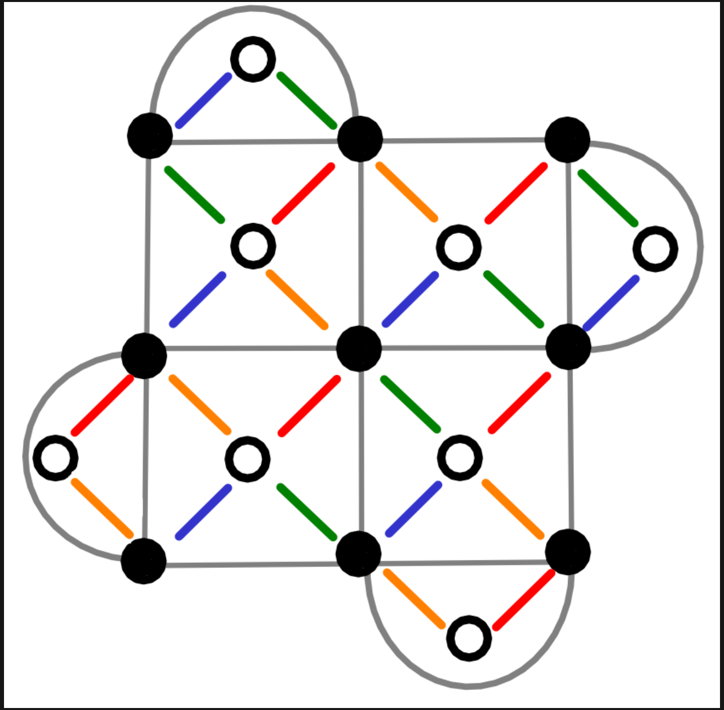
- Algorithms like Shor, Grover, etc assume that qubits are perfect
- Run them with imperfect *physical qubits*, and you'll get nonsense out
- Instead we need qubits that are flawless incarnations of the idea of quantum information
- Such ideal qubits are called *logical qubits*



- **We need to somehow make perfect logical qubits out of noisy physical qubits**

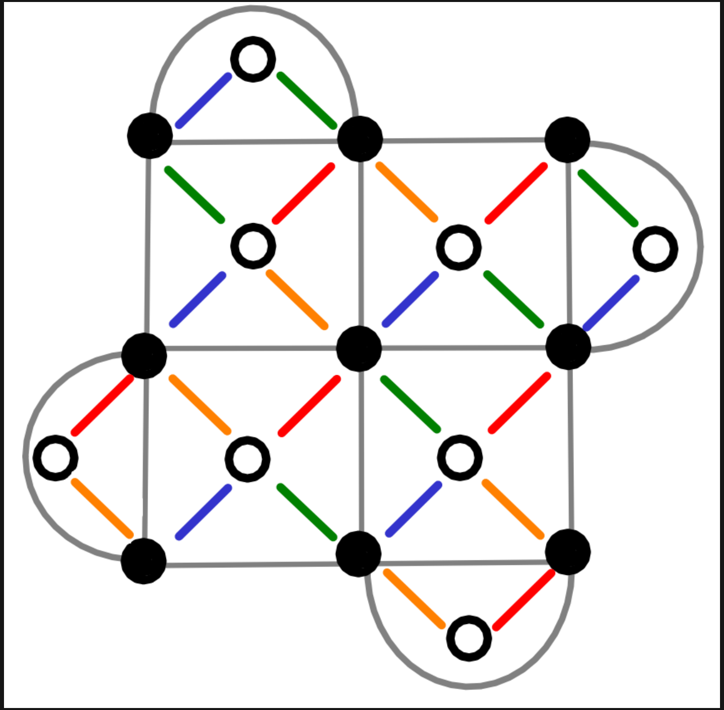
Why do we need QEC?

- QEC is how we do this!
- Typically, *many* physical qubits are needed per logical qubit
- Using QEC in algorithms is therefore a long-term goal
- As are the algorithms that require it
- In the near term, we'll try to make do and mitigate



In these lectures

- We'll look at the ideas behind error correction (classical and quantum)
- We'll go through specific examples
  - Repetition code
  - Surface code
- We'll see the most important techniques
  - Syndrome measurements
  - Decoding
  - Logical operations

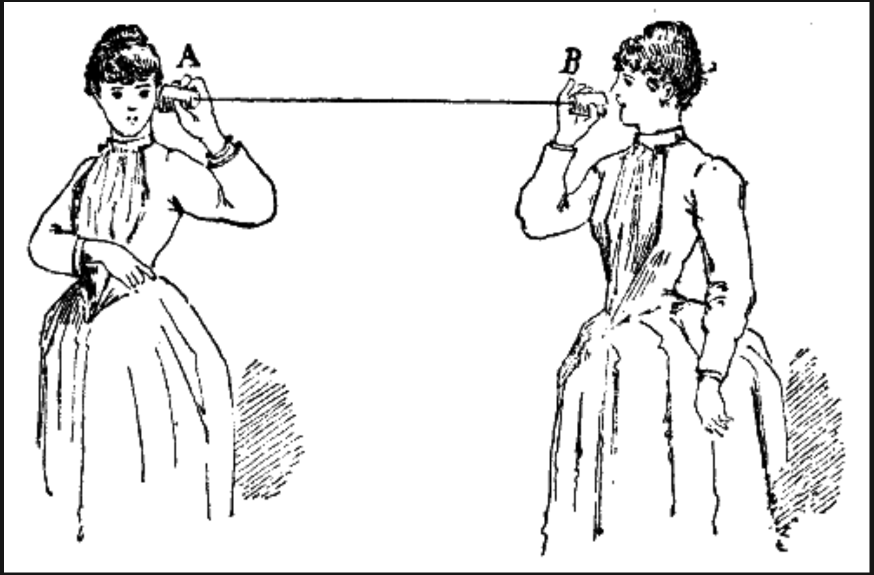


What is Error Correction?

- Before we explain *quantum* error correction, we should think about the general idea
- A simple example: you are talking on the phone, and need to answer a question with ‘yes’ or ‘no’.
- Two important things to consider:
  - How likely is it that you will be misheard?

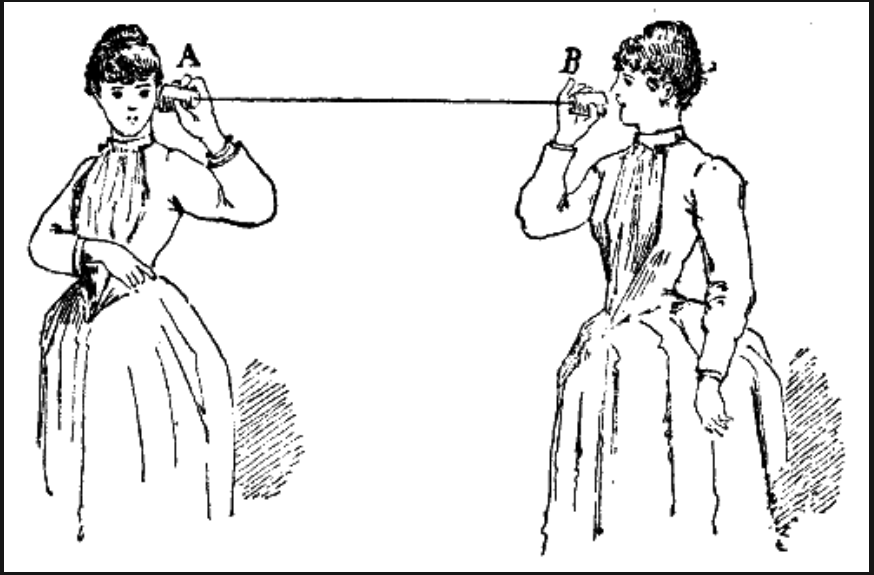
    p = probability that ‘no’ sounds like ‘yes’, etc
  - How much do you care about being misunderstood?

    P_a = maximum acceptable error probability



What is Error Correction?

- Usually $p \ll P_a$, so we don't need to worry.
- But what if we are being asked life-or-death questions over a noisy line?
- How can we make sure we are understood?



The Repetition Code

- We could repeat ourselves.
- With a lot of ‘no’s, it’s obvious we mean ‘no’.
- Same for mostly ‘no’s with a few random ‘yes’s thrown in.
- With this *encoding* of our message, it has become tolerant to small faults

The Repetition Code

- The receiver will need to *decode* the message.
- A sensible option is majority voting.
- A misunderstanding only happens when the majority of copies are flipped
- For d repetitions

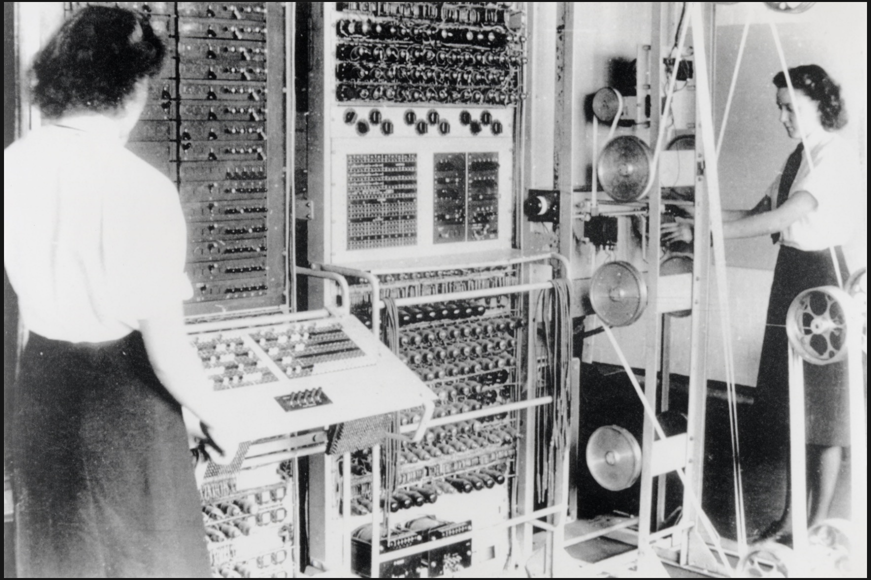
$$P = \sum_{n=0}^{\lceil d/2 \rceil} \binom{d}{n} p^n (1-p)^{d-n} \sim \left(\frac{p}{(1-p)}\right)^{\lceil d/2 \rceil}$$

- *P* decays exponentially with *d*
- With enough repetitions, we can make *P* as small as we like

Encoding and Decoding

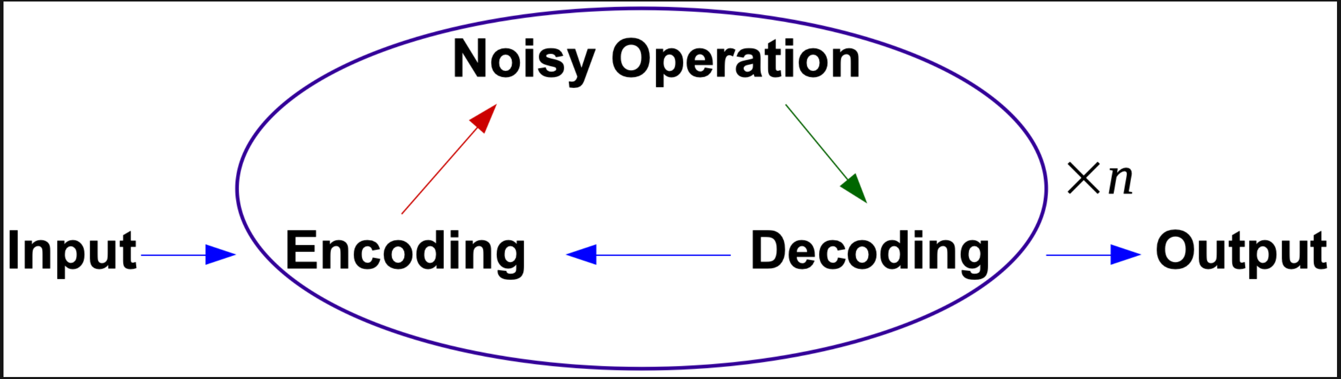
– This example contained all the basic features of any protocol for quantum error correction.

- **Input:** Some information to protect.
- **Encoding:** Transform the information to make it easier to protect.
- **Errors:** Random perturbations of the encoded message.
- **Decoding:** Trying to deduce the input from the perturbed message



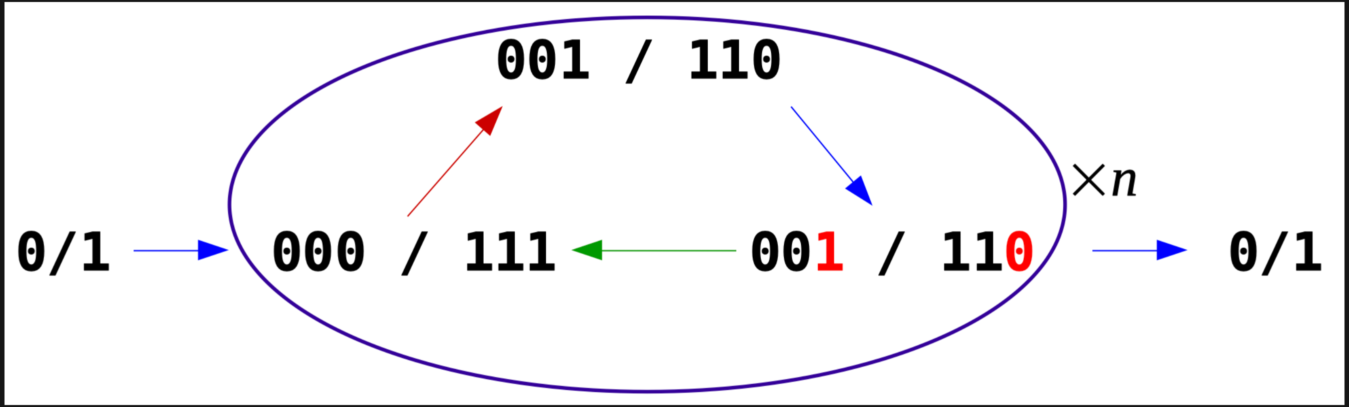
Computation

- The example we just considered was one of communication, with errors occurring during transmission.
- For computation, errors are introduced whenever we perform an operation.
- We need to correct errors as they are introduced.
- Can be done by constantly decoding and re-encoding



Computation

- For example, here’s a single bit using the Repetition code
- The ‘noisy operation’ here is just doing nothing (with some errors)
- Here we don’t completely decode and re-encode, but just do it enough to find and fix the errors.

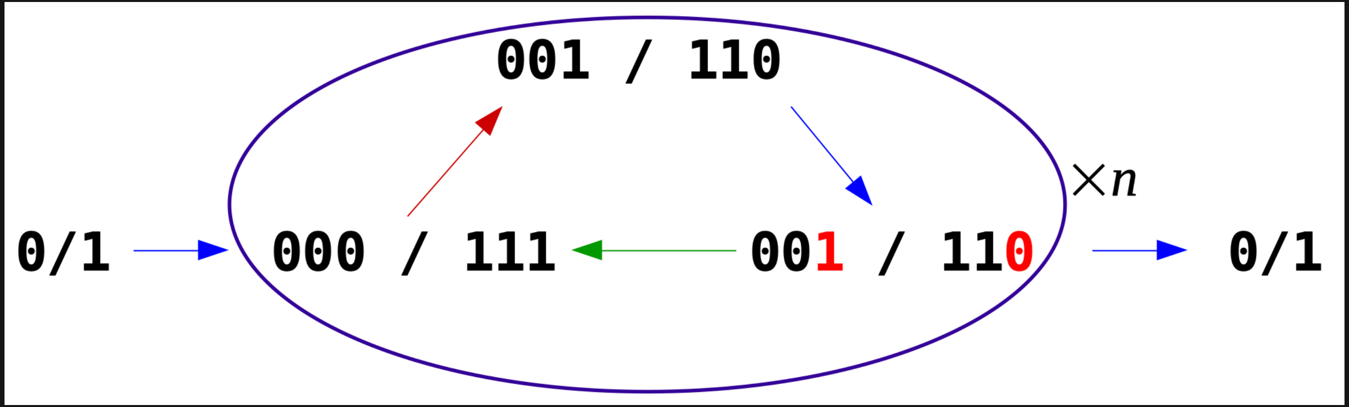


Quantum Computation

- This method works great for bits, but not for qubits
- Suppose we wanted to encode some superposition state

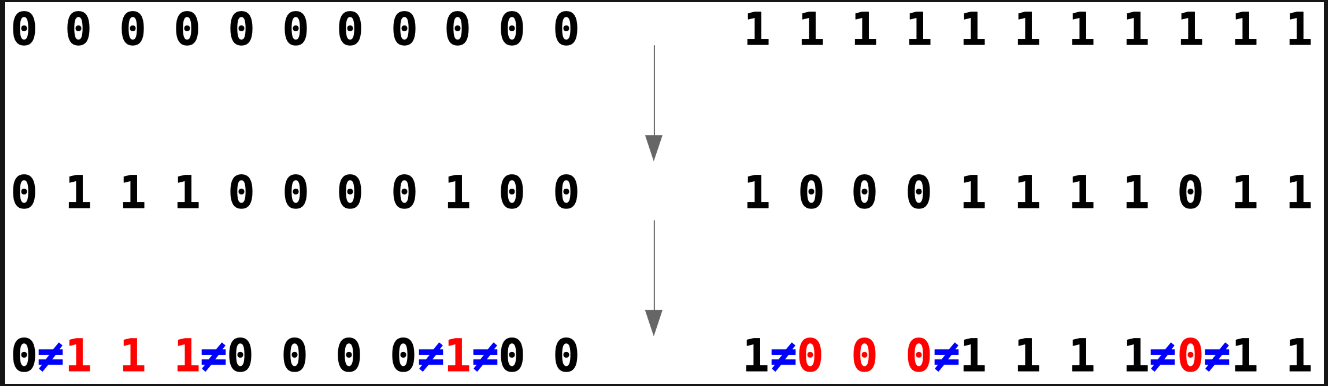
$$a|0\rangle + b|1\rangle \to a|000\rangle + b|111\rangle$$

- Decoding requires measurement, and that destroys the superposition.
- To protect against one bad thing, we caused another!



Quantum Computation

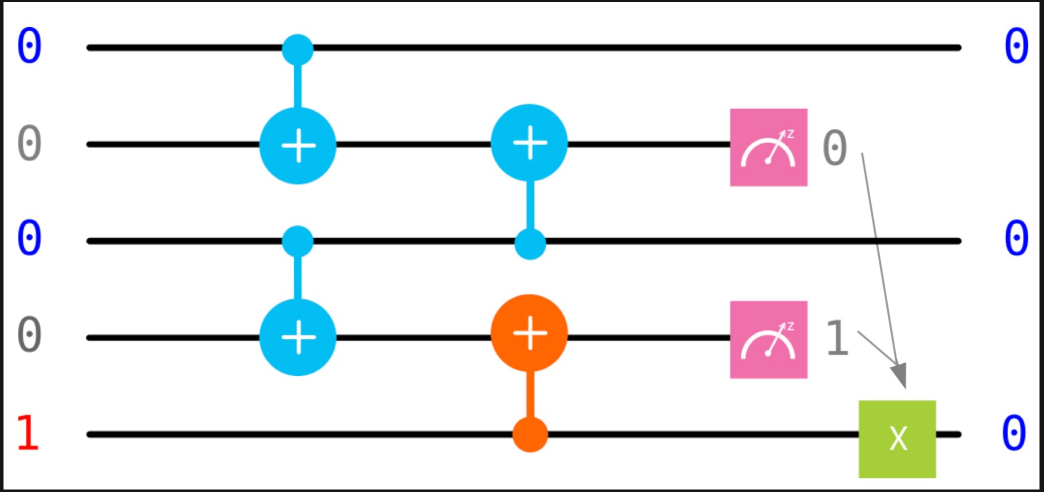
- To solve this, we need to be more careful with our measurements.
- We do need to measure, to get information about errors. But we must avoid learning about the encoded information
- For this, note that we don't actually need the bit values. We only need to know which ones have a different value to the rest.



Quantum Repetition Code

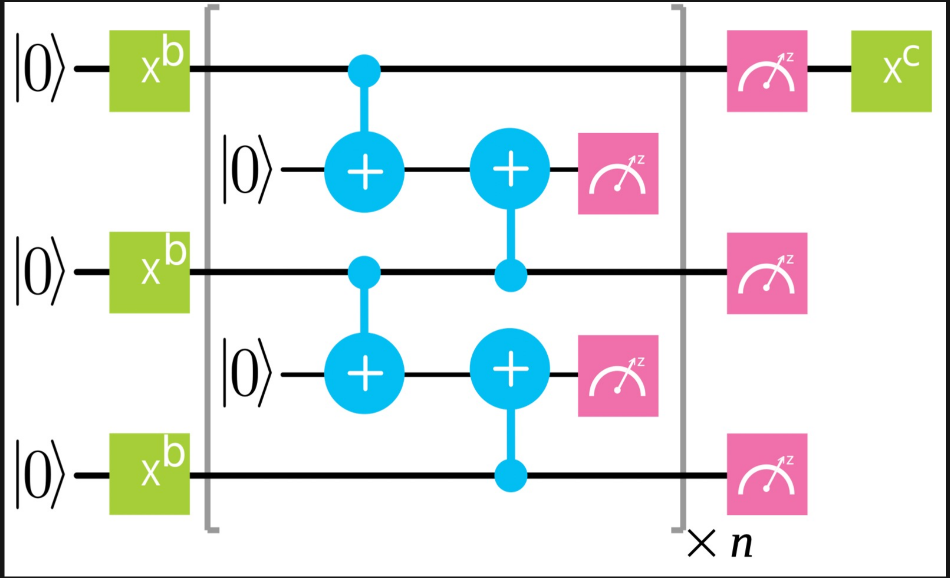
- Can be done with some extra qubits: one for each pair of code qubits.
- They are are initialized in state $|0\rangle$, and are used as the target for two CX gates
- The net effect is to measure the observable $Z_j Z_{j+1}$: the Z basis parity of the two qubits
- In short: whether they are the same or different

$$\text{cx}\,|00\rangle = |00\rangle$$
$$\text{cx}\,|01\rangle = |00\rangle$$
$$\text{cx}\,|10\rangle = |11\rangle$$
$$\text{cx}\,|11\rangle = |10\rangle$$



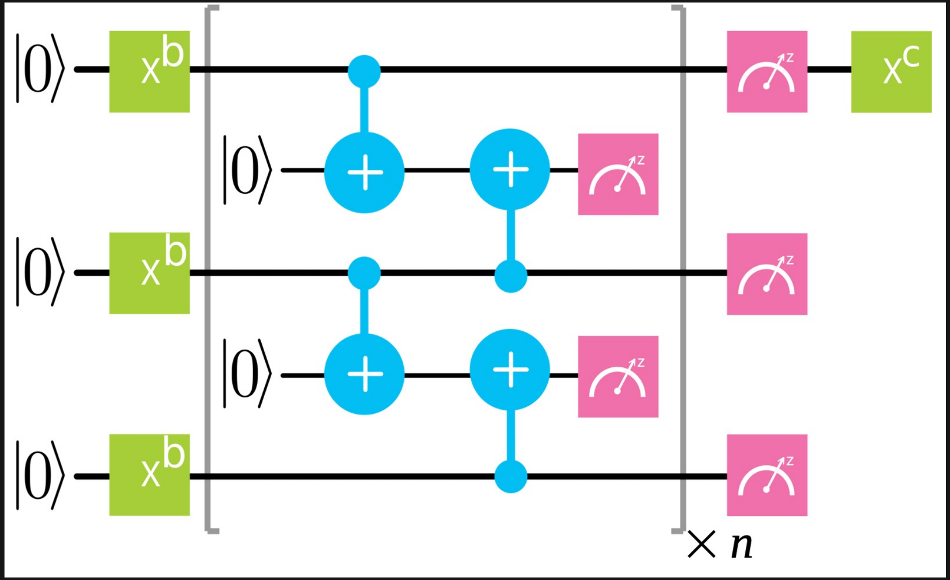
Quantum Repetition Code

- The parity measurements are repeated over the course of the computation
- The results are used to identify where errors likely occurred and how to remove their effects
- This is done by means of a classical algorithm: a decoding algorithm
- With this we can protect against bit flip errors for an arbitrarily long time



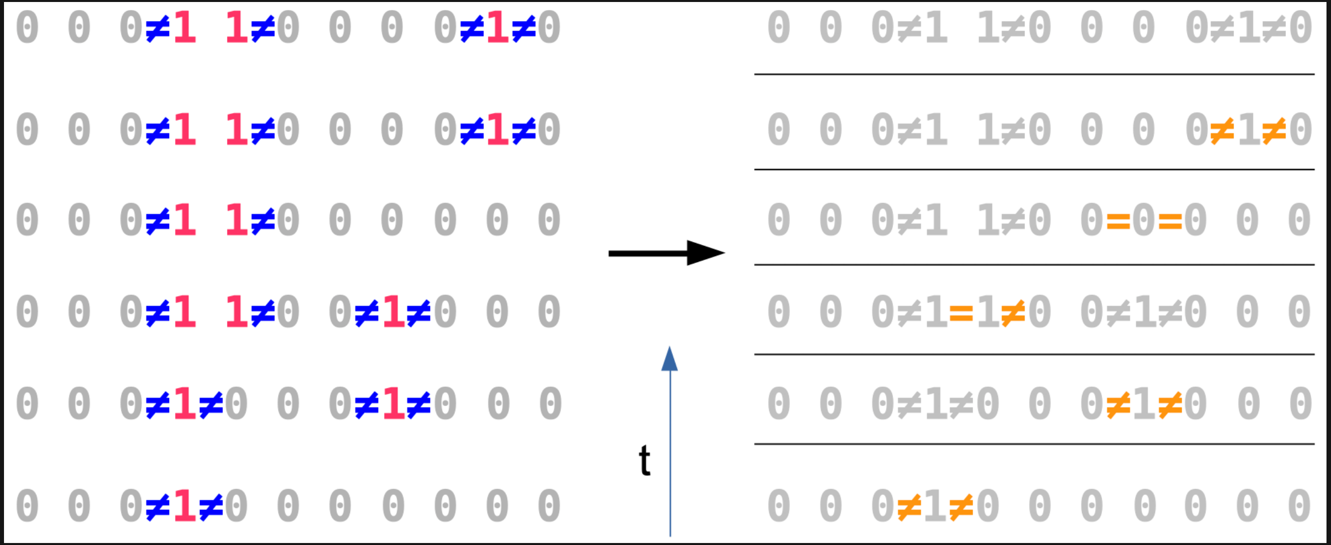
Syndrome Measurement

- This measurement is a *syndrome measurement*
- The result is known as the *syndrome*
- All codes have them
- They are ways to check that all the qubits are doing what they should be
- Their form changes from code to code, but their job is always the same
- They provide the clues that allow us to detect and correct errors



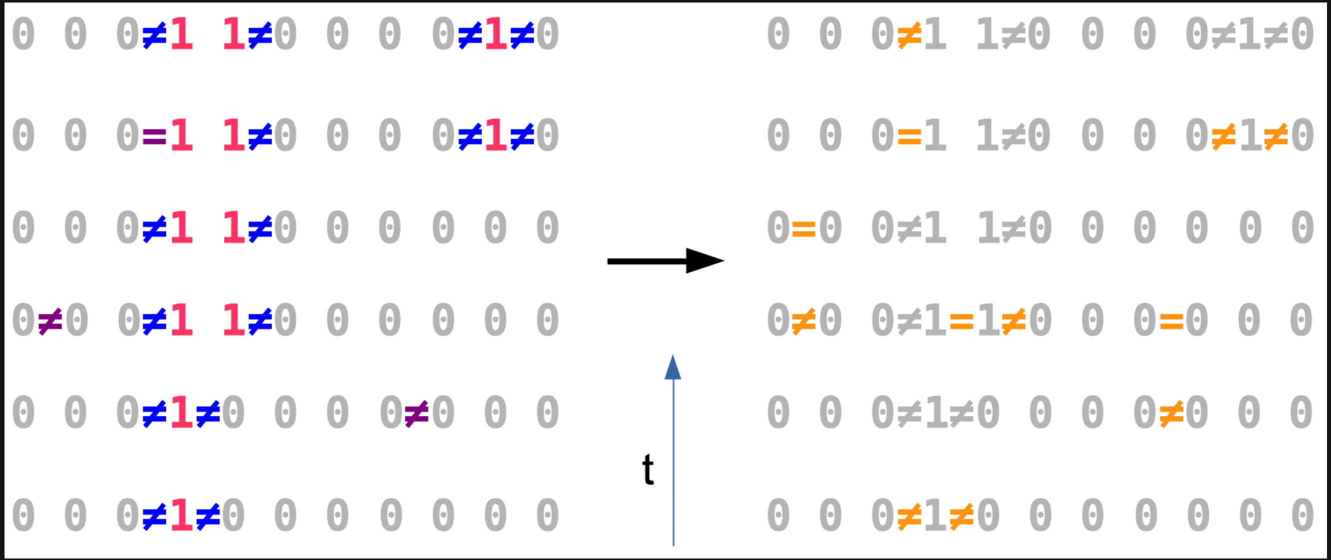
Decoding

- Start with an unrealistically simple case: errors between parity measurements only (not during)
- Focus on identifying errors for now (not correcting)
- Look for changes between rounds
- Errors create pairs of 'defects'. Majority voting can be used to find a minimal pairing.



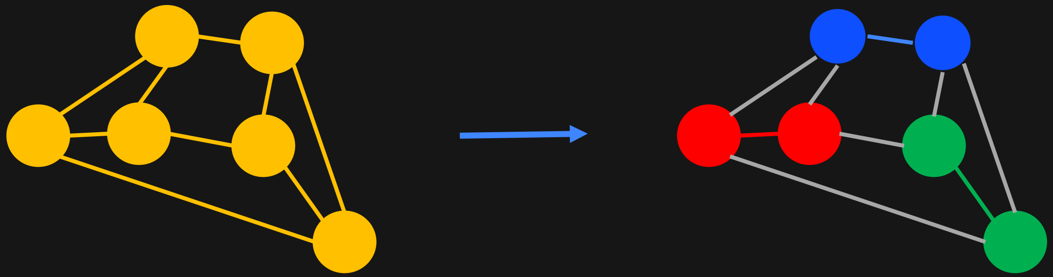
Decoding

- Next, a simple model of noise in the measurements: they randomly lie
- Again, look for changes between rounds
- Bit flips create defects with space-like separation, measurement errors with time-like
- Pairing is now a 2D problem, majority voting won't work



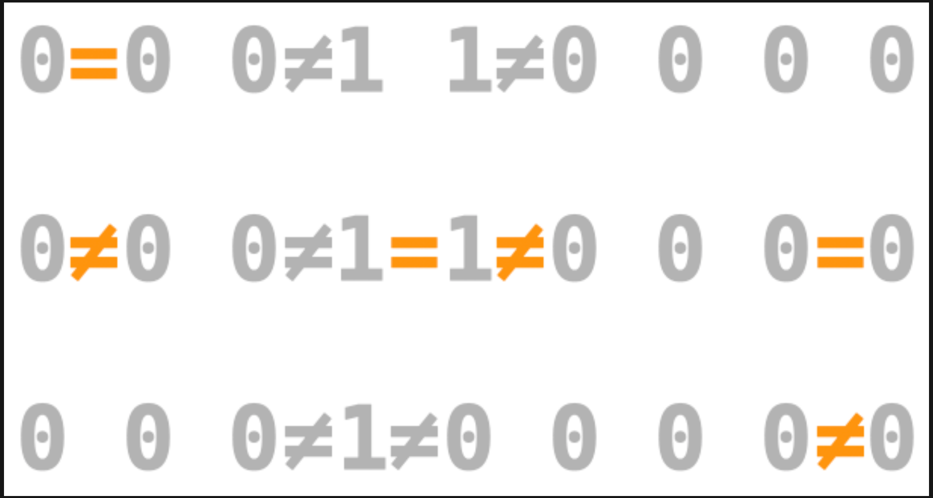
Decoding

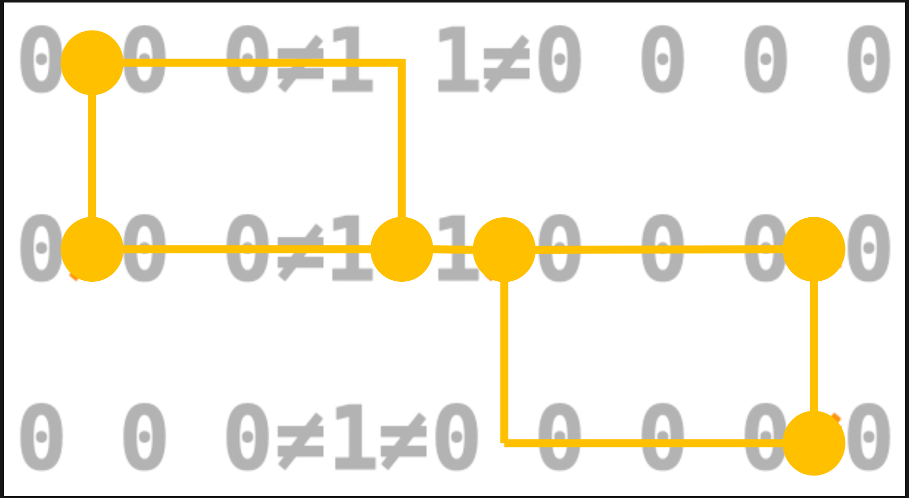
- There are many ways to do this, all with pros and cons
- One of the best is to think of the problem as a graph
    - Defects are nodes
    - The number of errors required to link them are weighted edges
- A likely set of errors corresponds to the 'minimum weight perfect matching' of the graph
- Efficiently computable using the 'Blossom' algorithm



Decoding

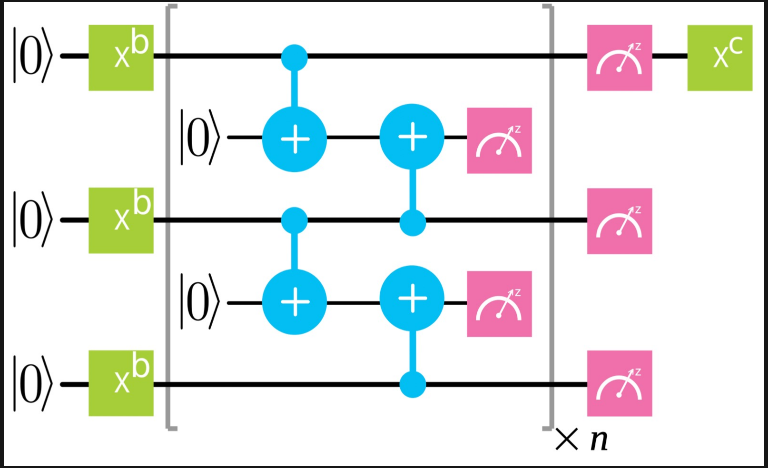
– Here it is in action for a portion of the example from earlier





Logical Operations

- If we wanted to manipulate the logical qubit, how could we do it?
- This requires a *logical operation*, made up of many physical operations
- The *logical* X gate is easy for the repetition code
  - Do *physical* X gates on code qubits
  - Code corrects imperfections
- An Rx gate essentially needs you to take the code apart and put it back together again
  - Not protected while unencoded

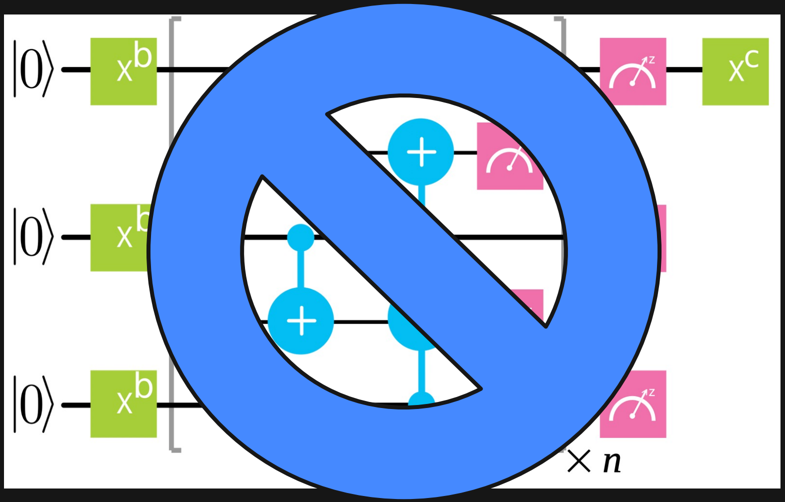


$$R_x(2\theta)|0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$

$$\rightarrow \cos\theta\,|000\rangle + \sin\theta|111\rangle$$

Quantum Repetition Code

- The limited set of fault-tolerant logical gates is a major problem with the repetition code
- But it is not the only one!
- The code only allows us to detect and correct bit fips, and only bit flips
- Though we made sure that it doesn't cause superpositions to collapse, it doesn't protect them either



- **The repetition code is a good first example of quantum error correction, but it cannot give us fault-tolerant quantum computation**

Towards a good quantum code

- The problem with the repetition code is that it treats z basis states very different to x and y basis states
- Example 1: z basis states are product states, the others are entangled

$$\begin{aligned} |0\rangle \quad &\rightarrow \quad |000\rangle \\ |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad &\rightarrow \quad \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \end{aligned}$$

- Example 2: Distinguishing encoded z basis requires a single qubit measurement, distinguishing x basis states requires *d*
- Example 3: Flipping between z basis states requires *d* gates, but the x basis only takes one

$$\begin{aligned} |0\rangle \rightarrow |1\rangle \;: \quad & X_0 X_1 X_2 |000\rangle = |111\rangle \\ |+\rangle \rightarrow |-\rangle \;: \quad & Z_j \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \end{aligned}$$

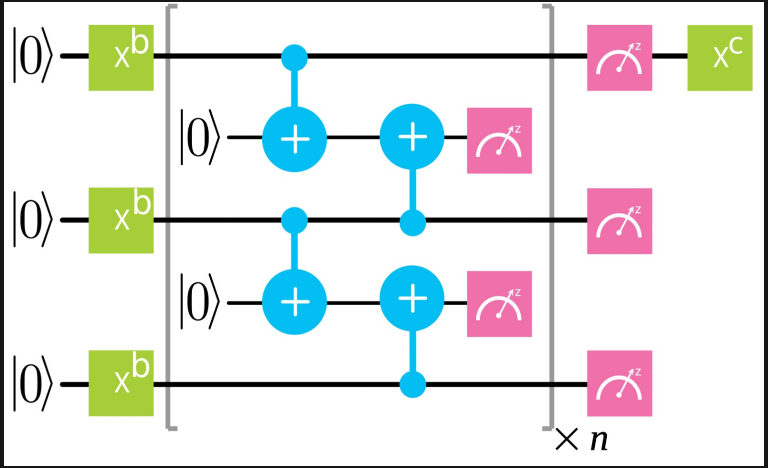
- It is not good when things are too easy, because they are easy for errors too!

Introduction to the Surface Code

James R. Wootton
IBM Quantum

Logical Operations

- If we wanted to manipulate the logical qubit, how could we do it?
- This requires a *logical operation*, made up of many physical operations
- The *logical* X gate is easy for the repetition code
  - Do *physical* X gates on code qubits
  - Code corrects imperfections
- An Rx gate essentially needs you to take the code apart and put it back together again
  - Not protected while unencoded

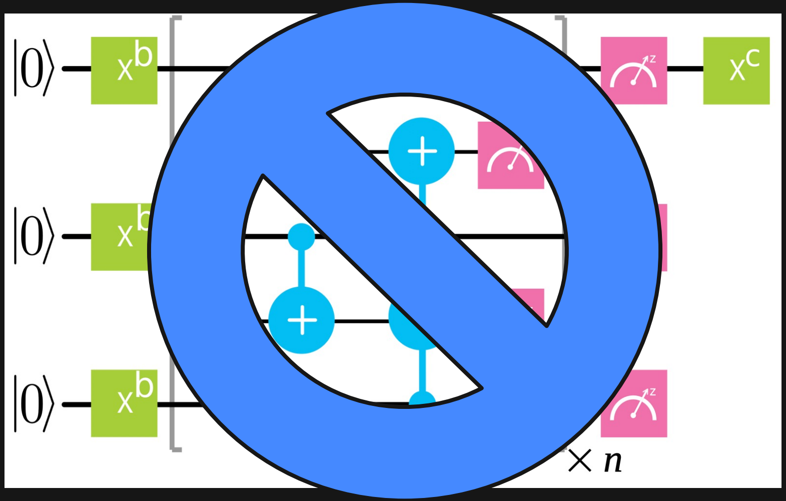


$$R_x(2\theta)|0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$

$$\rightarrow \cos\theta\,|000\rangle + \sin\theta|111\rangle$$

Quantum Repetition Code

- The limited set of fault-tolerant logical gates is a major problem with the repetition code
- But it is not the only one!
- The code only allows us to detect and correct bit fips, and only bit flips
- Though we made sure that it doesn't cause superpositions to collapse, it doesn't protect them either



- **The repetition code is a good first example of quantum error correction, but it cannot give us fault-tolerant quantum computation**

Towards a good quantum code

- The problem with the repetition code is that it treats z basis states very different to x and y basis states
- Example 1: z basis states are product states, the others are entangled

$$\begin{aligned} |0\rangle \quad &\rightarrow \quad |000\rangle \\ |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad &\rightarrow \quad \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \end{aligned}$$

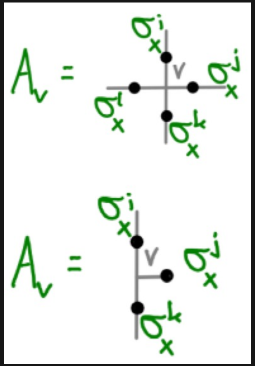
- Example 2: Distinguishing encoded z basis requires a single qubit measurement, distinguishing x basis states requires *d*
- Example 3: Flipping between z basis states requires *d* gates, but the x basis only takes one

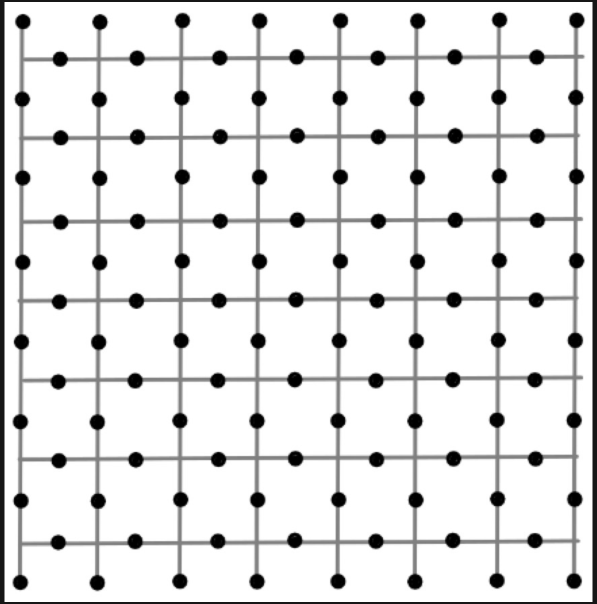
$$\begin{aligned} |0\rangle \rightarrow |1\rangle \ : \quad & X_0 X_1 X_2 |000\rangle = |111\rangle \\ |+\rangle \rightarrow |-\rangle \ : \quad & Z_j \tfrac{1}{\sqrt{2}}(|000\rangle + |111\rangle) = \tfrac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \end{aligned}$$

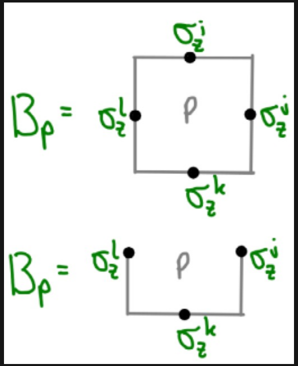
- It is not good when things are too easy, because they are easy for errors too!

The Surface Code

- Quantum error correcting codes are defined by the measurements we make
- Let's move beyond the simple $Z_j Z_{j+1}$ of the repetition code
- In the surface code we use a 2D lattice of code qubits, and define observables for plaquettes and vertices

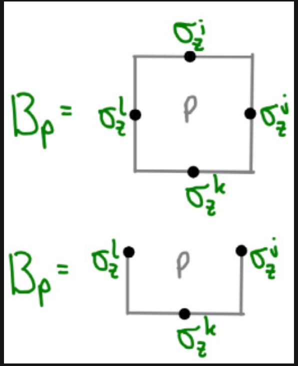

$A_v = \sigma_x^i \sigma_x^j \sigma_x^k \sigma_x^l$
v
$A_v = \sigma_x^i \sigma_x^j \sigma_x^k$
v

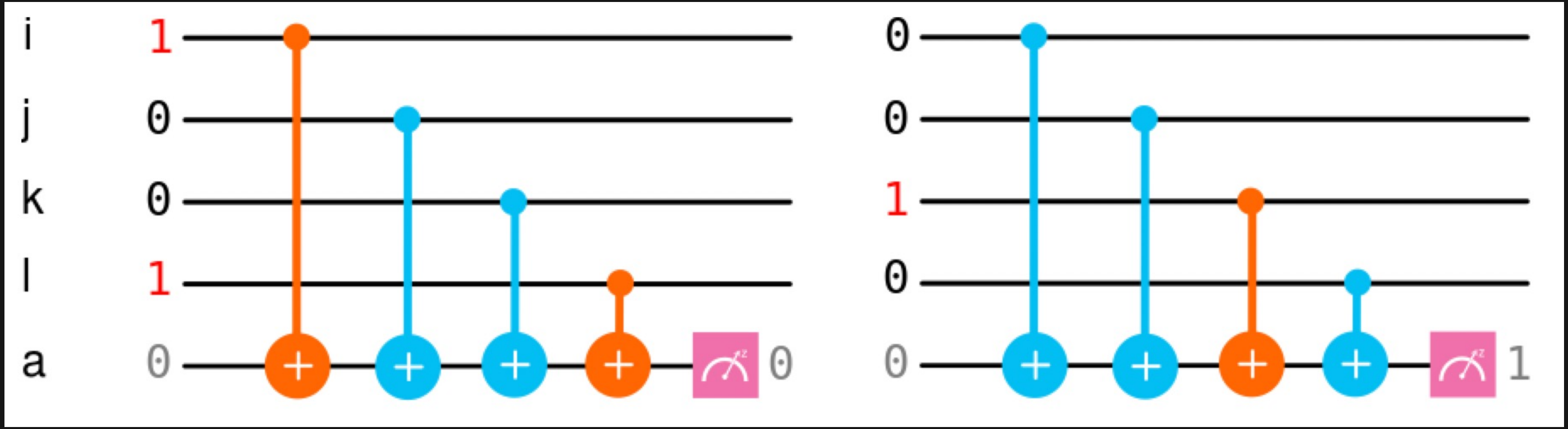



$B_p = \sigma_z^i \sigma_z^j \sigma_z^k \sigma_z^l$
p
$B_p = \sigma_z^l \sigma_z^j \sigma_z^k$
p

Plaquette Syndrome

- First let’s focus on the plaquette syndrome
- These are similar to the two qubit measurements in the repetition code
- Instead we measure the parity around plaquettes in the lattice
- Can again be done with CX gates and an extra qubit

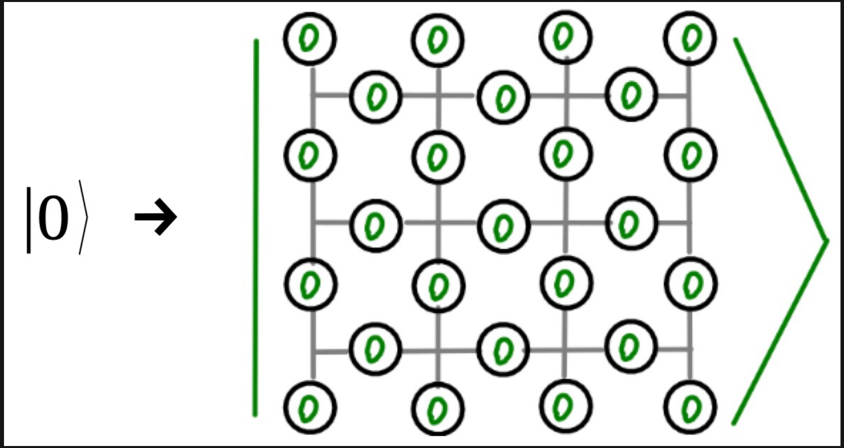

$B_p = \sigma_z^l \sigma_z^i \sigma_z^j \sigma_z^k$
p
$B_p = \sigma_z^l \sigma_z^j \sigma_z^k$
p



Plaquette Syndrome

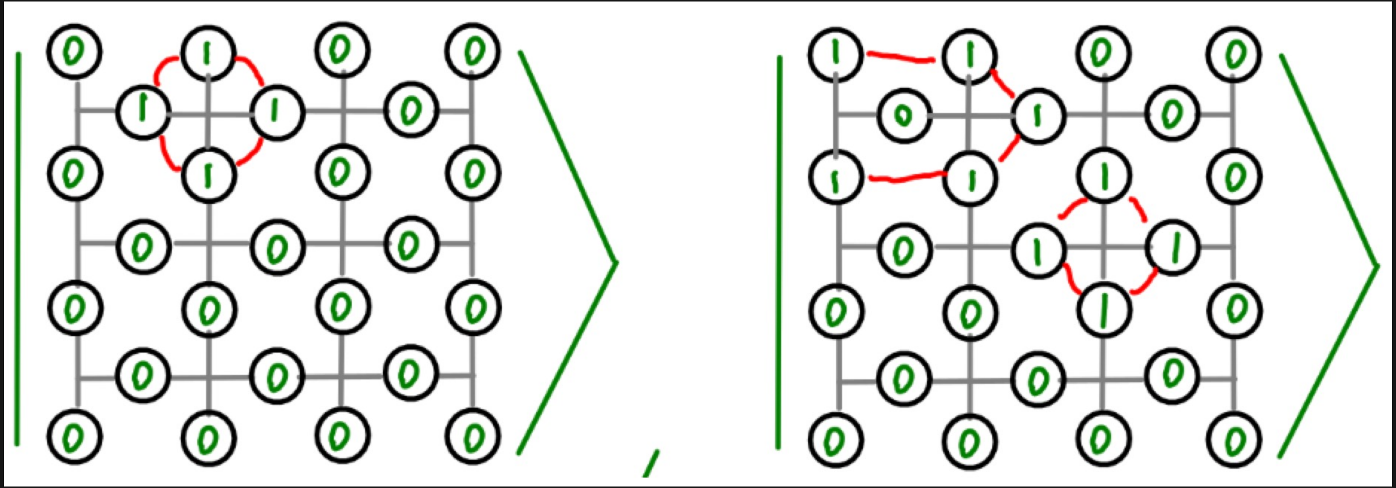
- We can define a classical code (storing only a bit) based on the plaquette syndrome alone
- Valid states are those with trivial outcome for all plaquette syndrome measurements:

  Even parity on all plaquettes
- How to store a 0 in this?
- How about the state where every code qubit is $|0\rangle$?



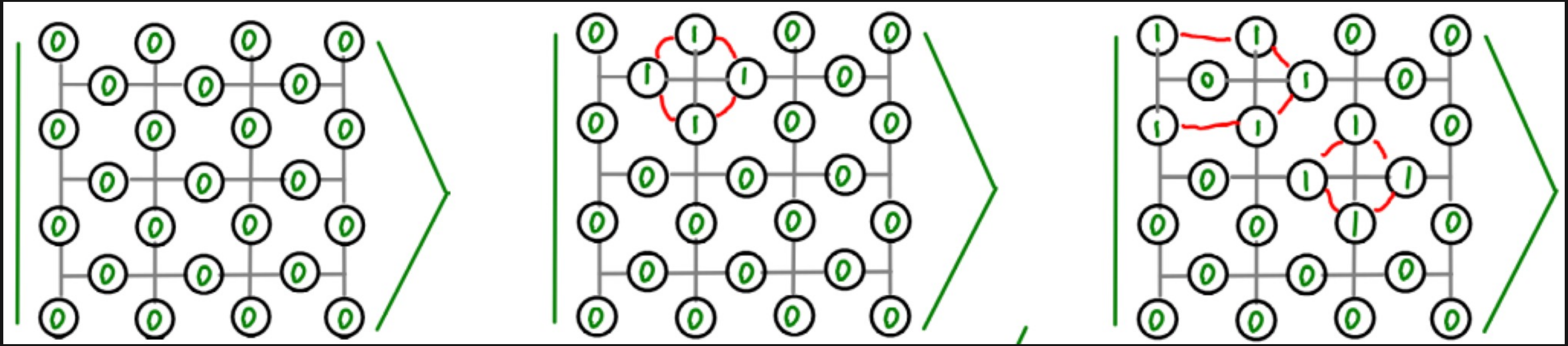
Plaquette Syndrome

- There are ‘nearby’ states that also have even parity on all plaquettes
- These can’t be a different encoded state: they are only a few bit flips away from our encoded 0 state
- We’ll treat them as alternative ways to store a 0



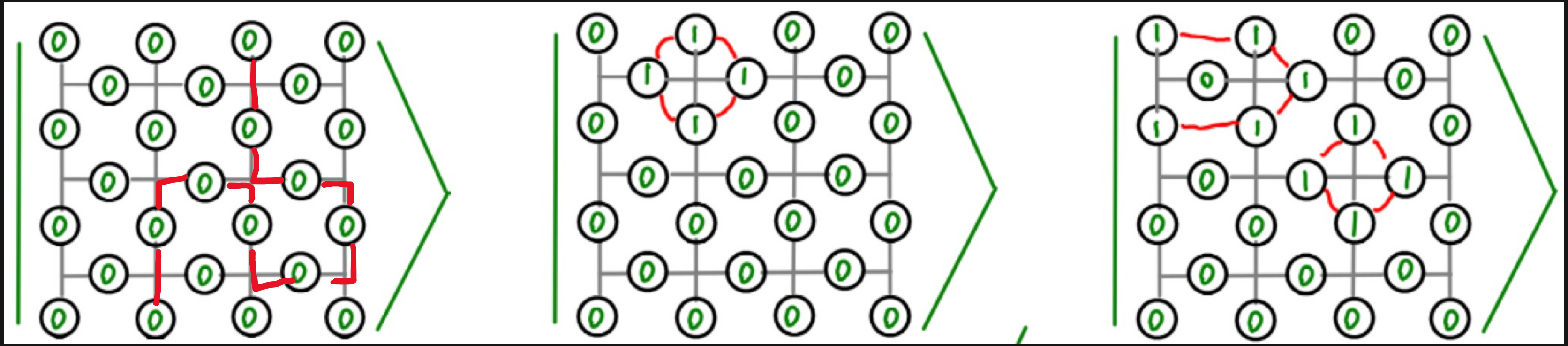
Plaquette Syndrome

- Given any state for an encoded 0
  - Pick a vertex
  - Apply bit flips around that vertex
- Now you have another valid state for 0
- This generates an exponentially large family



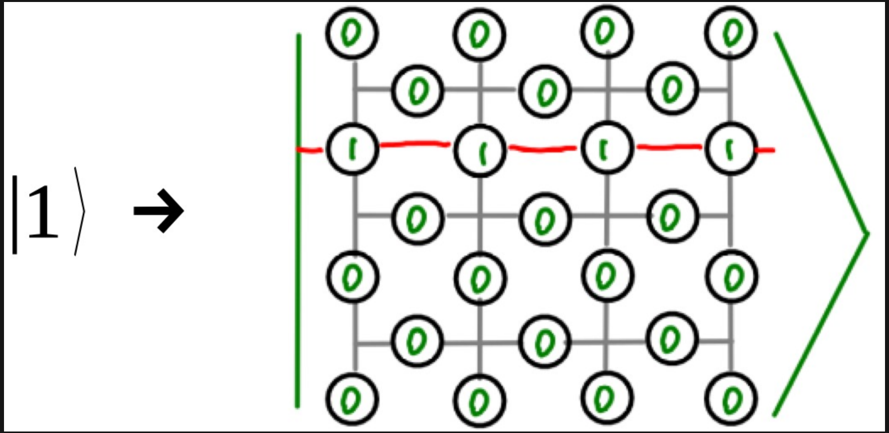
Plaquette Syndrome

- The states in this family can be very different
- But they all share a common feature
  - Any line from top to bottom (passing along edges) has even parity
- This is how we can identify an encoded 0
- And it gives us a clue about how to encode a 1



Plaquette Syndrome

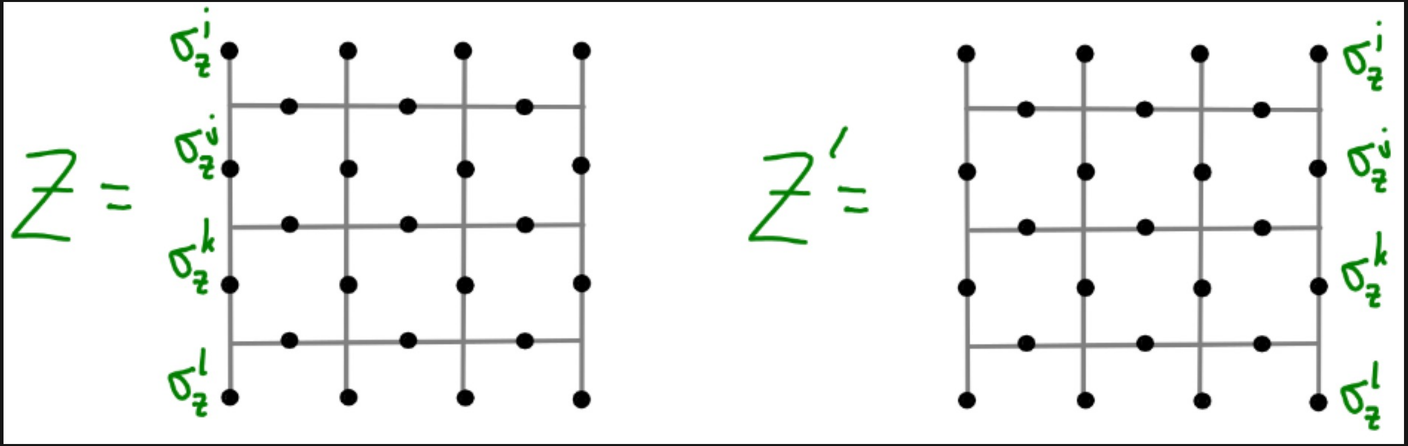
- For our basic encoded 1, we use a bunch of 0s with a line from left to right (passing through plaquettes)



- This also spawns an exponentially large family
- All have *odd* parity for a line from top to bottom
- Unlike the repetition code, distinguishing encoded 0 and 1 requires some effort (which is good!)

Logical X and Z

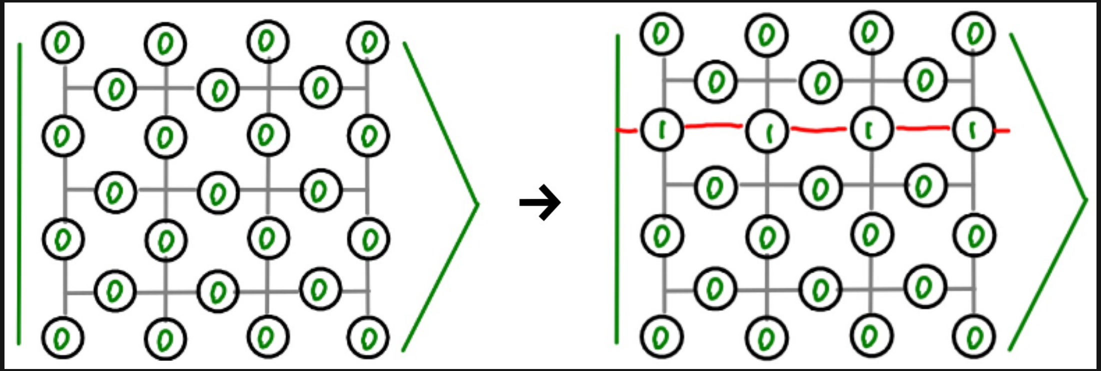
- Distinguishing 0 and 1 corresponds to measuring Z on the physical qubit
- The following observables detect what we need



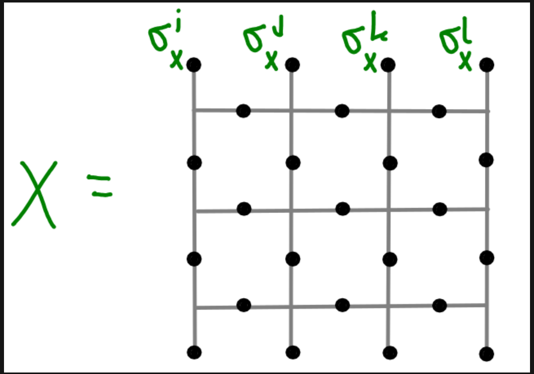
- Or the same on any line from top to bottom
- Uses the edges has a nice advantage: we can think of them as large (unenforced) plaquettes

Logical X and Z

– To flip between 0 and 1, we can flip a line of qubits

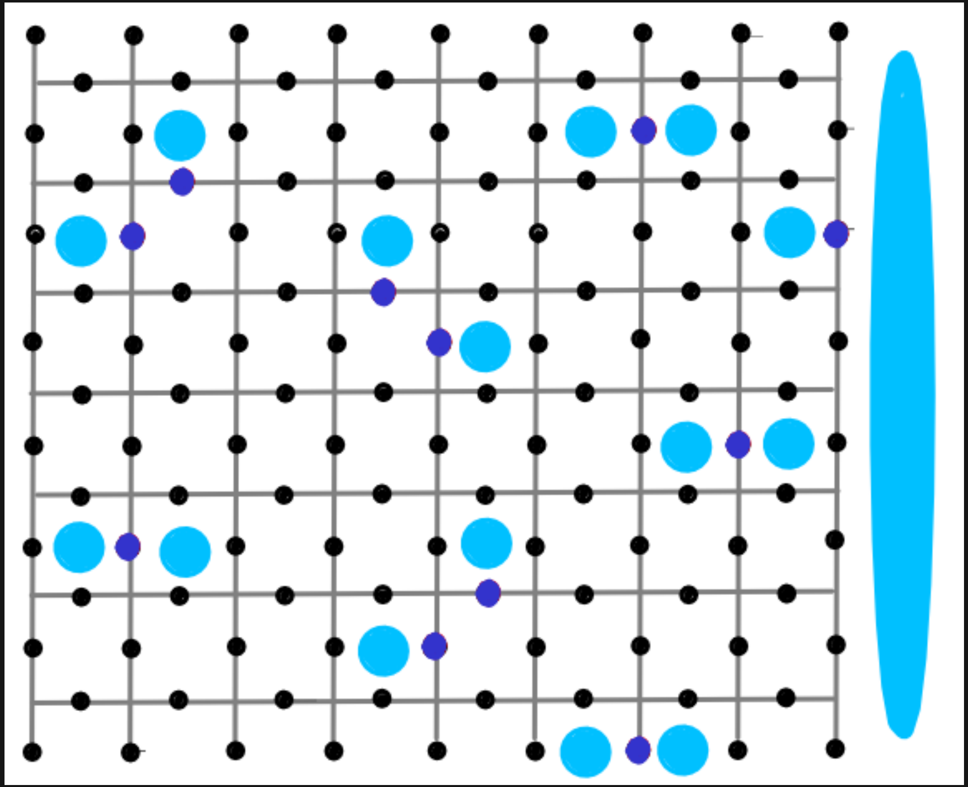


– Such lines of flips act as an X on the logical qubit



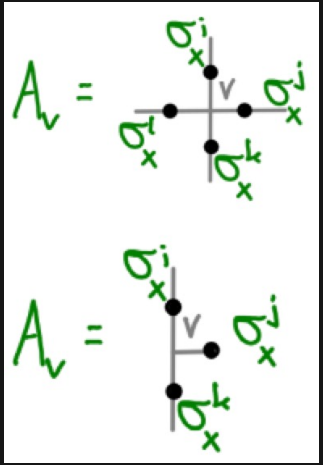
Effects of Errors

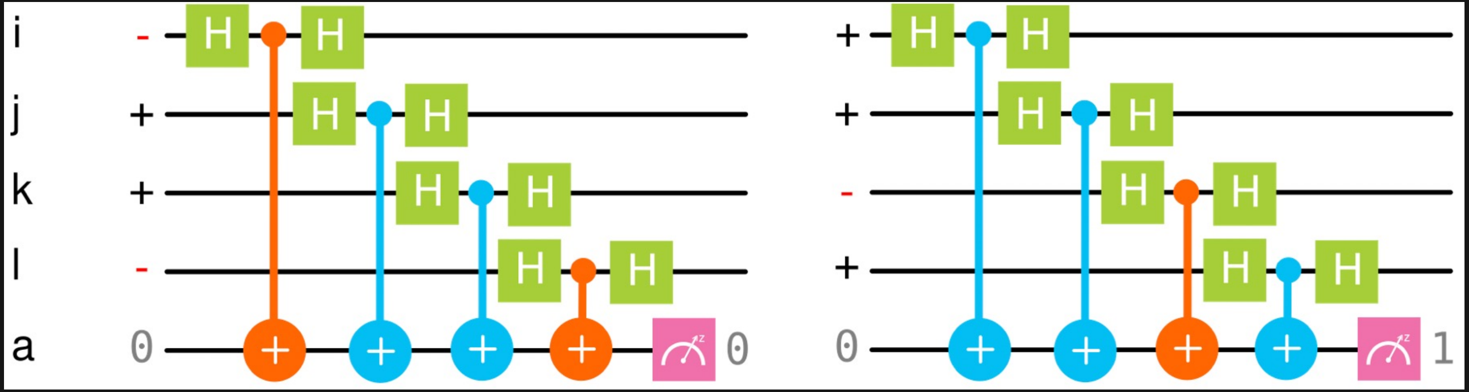
- Applying an X to any code qubit changes the parity of its two plaquettes
- An isolated X creates a pair of defects
- Further Xs can be move a defect, or annihilate pairs of them
- A logical X requires many errors to stretch across the lattice
- With the plaquette operators, we can encode and protect a *bit*



Vertex Syndrome

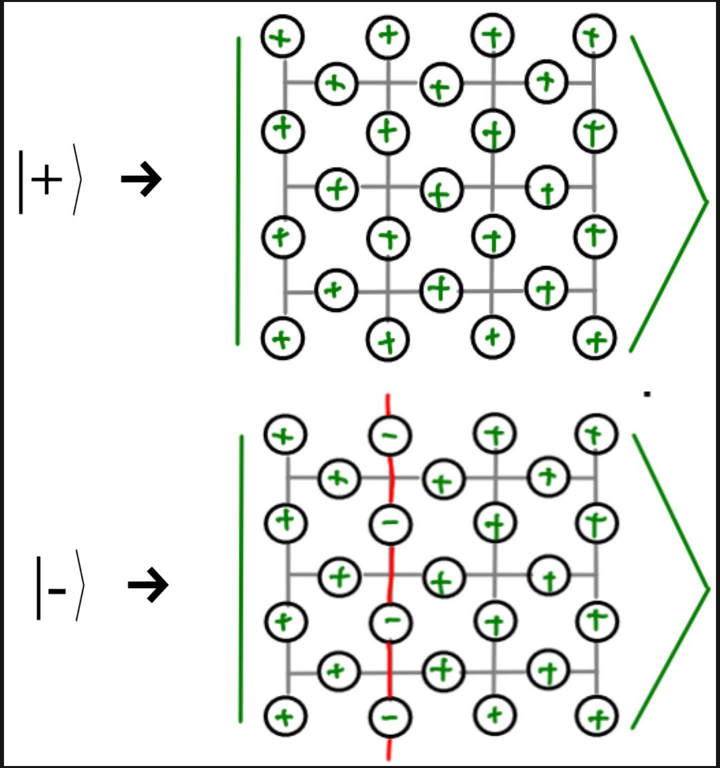
- Now forget the plaquettes and focus on vertices
- These observables can also be measured using CX gates an an ancilla
- In this case they look at the |+⟩ and |–⟩ states, and count the parity of the number of |–⟩s





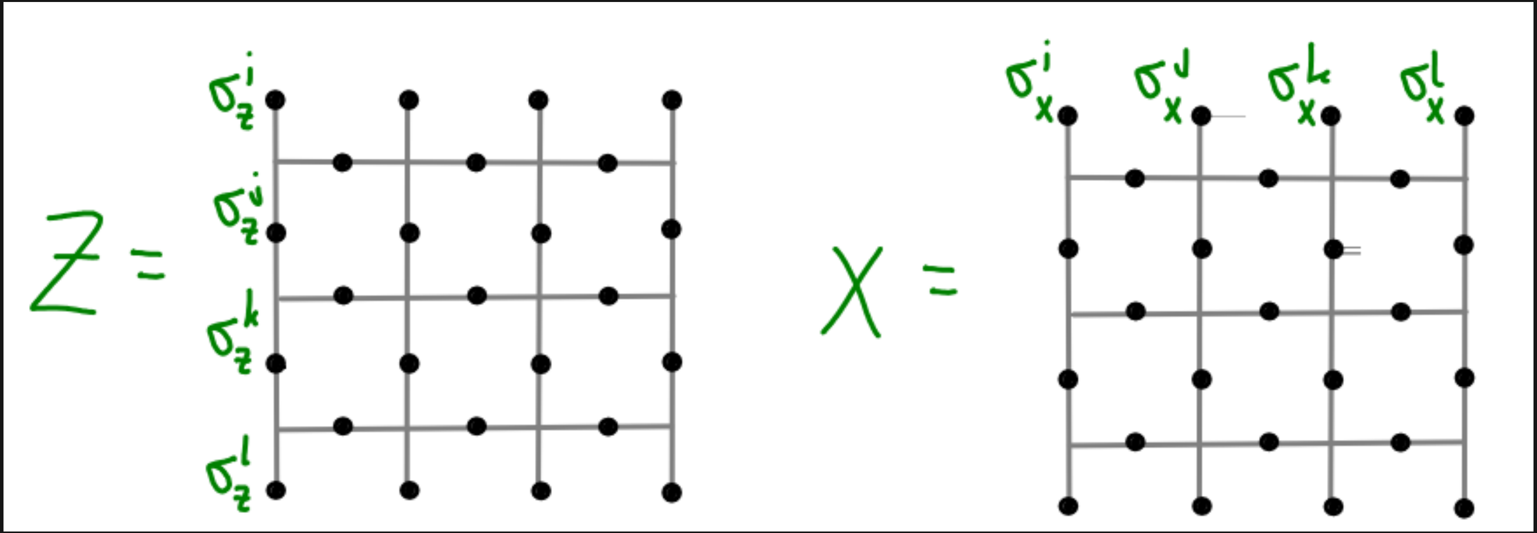
Vertex Syndrome

- These operators also allow is to encode and protect a bit value
- In this case, let’s use + and - to label the two states
- They are encoded using suitable patterns of |+⟩ and |−⟩ states for the code qubits
- As with the plaquettes, these also correspond to exponentially large families of states



Logical X and Z

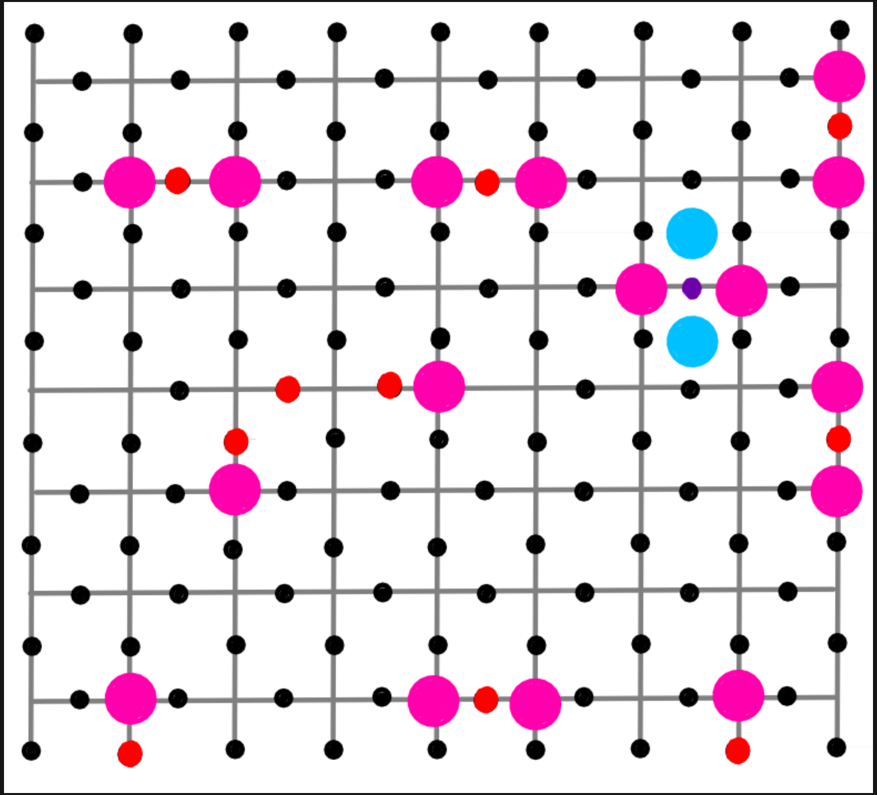
- What is the X operator (distinguish between $|+\rangle$ and $|-\rangle$)?
- What is the Z operator (flip between $|+\rangle$ and $|-\rangle$)?
- Turns out they are exactly the same as before!



Effects of Errors

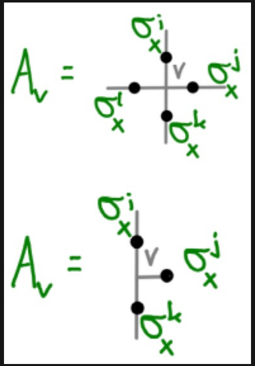
IBM **Quantum**

- Applying a Z to any code qubit changes the X parity of its two vertices
- An isolated Z creates a pair of defects
- Further Zs can be move a defect, or annihilate pairs of them
- A logical Z requires many errors to stretch across the lattice
- With the vertex operators, we can encode and protect a *bit*

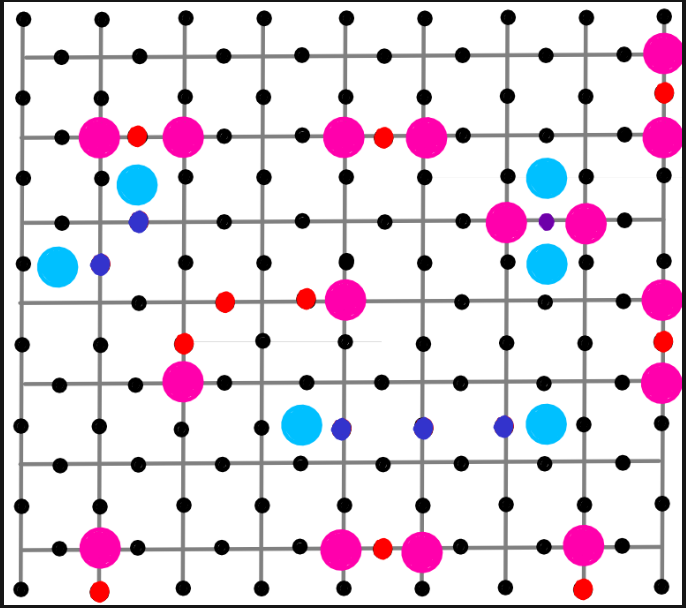


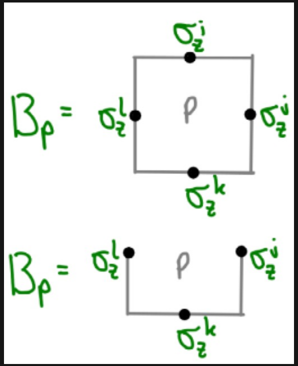
Putting it all Together

- The plaquette and vertex operators commute
- This allows us to detect both X and Z errors
- Since Y~XZ, we can detect Y errors too



$A_v = \sigma_x^i \sigma_x^j \sigma_x^k \sigma_x^l$
v
$A_v = \sigma_x^i \sigma_x^j \sigma_x^k$

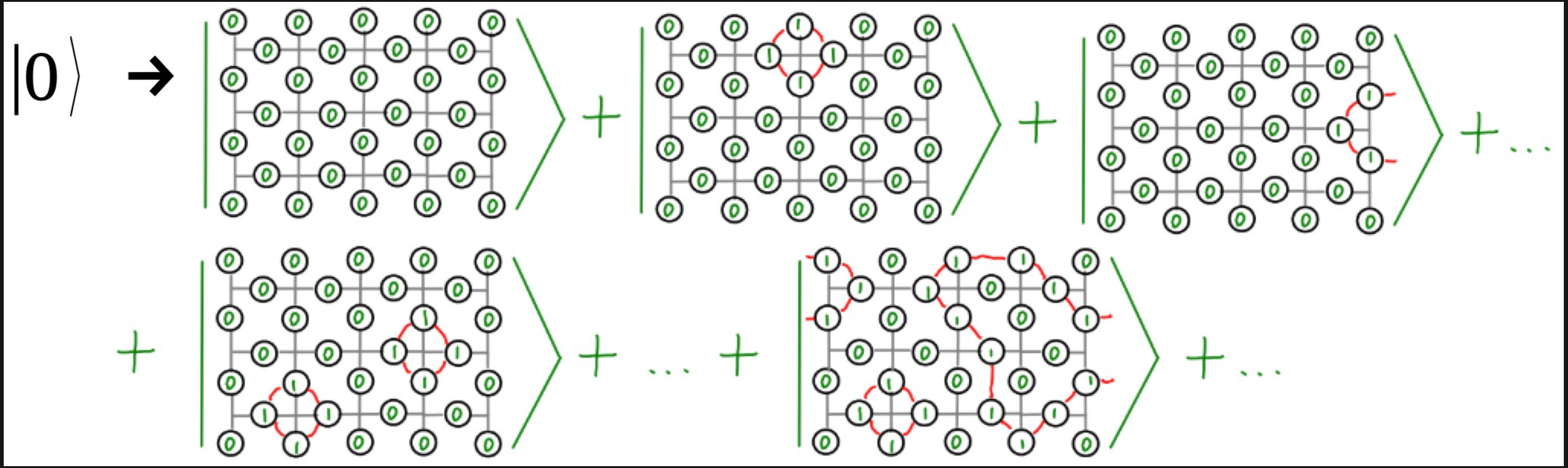




$B_p = \sigma_z^i \sigma_z^j \sigma_z^k \sigma_z^l$
p
$B_p = \sigma_z^i \sigma_z^j \sigma_z^k \sigma_z^l$

Putting it all Together

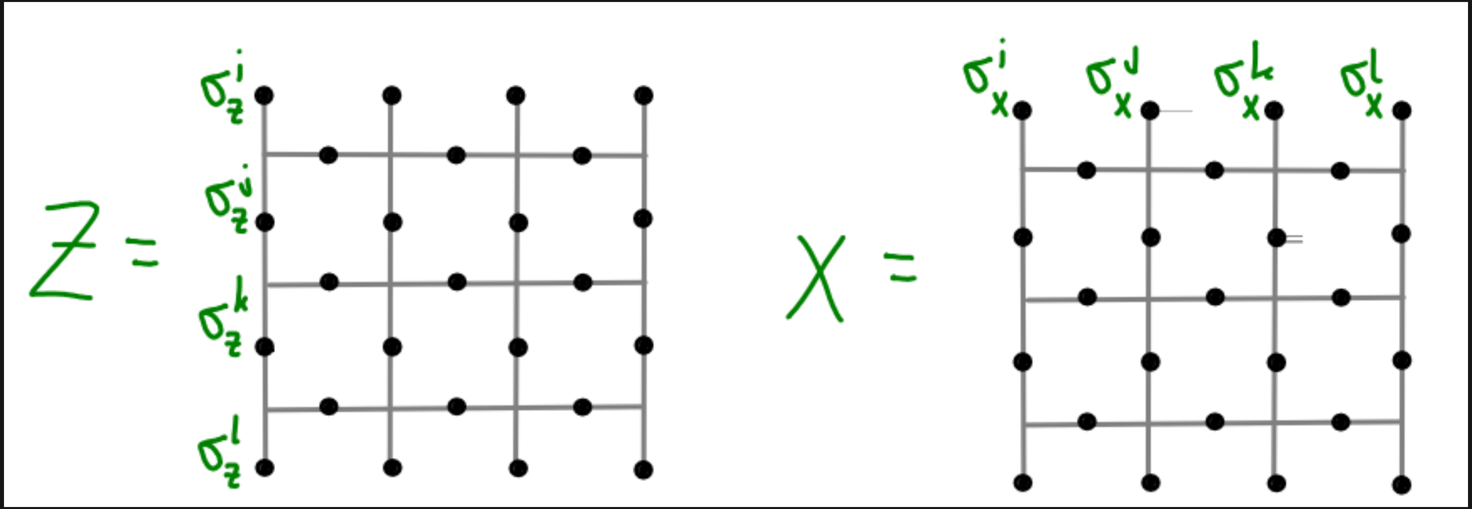
- Encoded states now unique: superposition of all previous solutions
- For example, the encoded 0



- Satisfies $A_v|\psi\rangle = |\psi\rangle$ and $B_v|\psi\rangle = |\psi\rangle$, so will give the 0 outcome for all stabilizer measurements

Putting it all Together

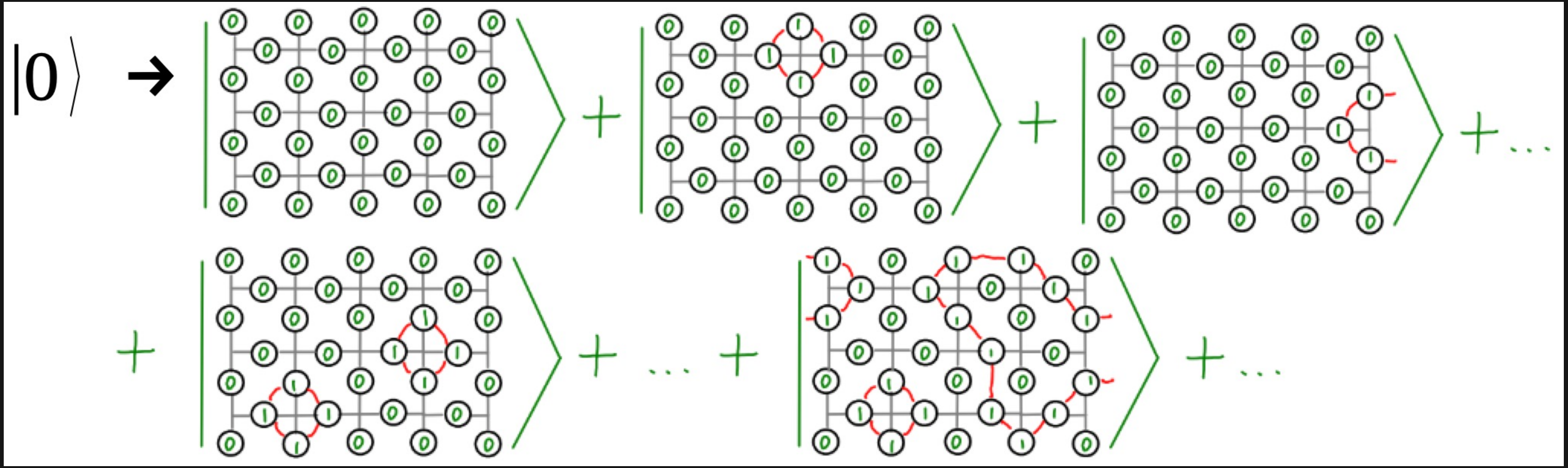
– The Z and X operators on the encoded qubit are exactly the same as before



– These, and the Hadamard, can be performed fault-tolerantly

Putting it all Together

- The states we need are highly entangled quantum states
- They are examples of topologically ordered states



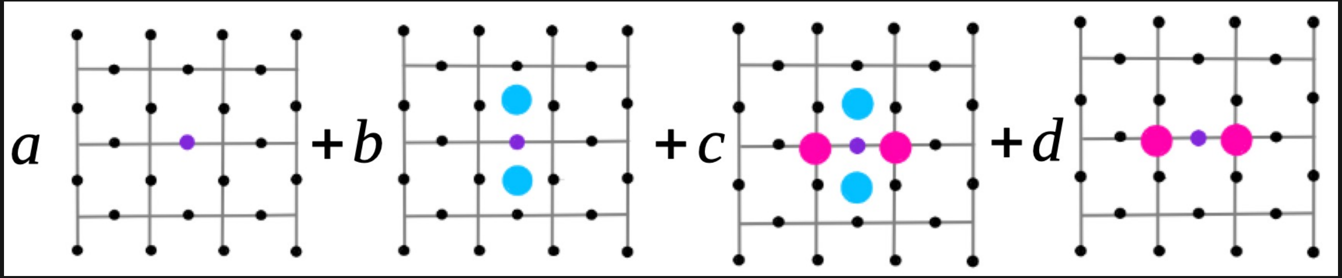
- Though such things can be hard to make, we create and protect them with the syndrome measurements

Putting it all Together

- We are not just protected against X and Z, but all local errors
- As mentioned earlier, Y~XZ
- Everything else can be expressed

$$E = a\,I + b\,X + c\,Y + d\,Z$$

- This creates a superposition of different types of error on the surface code



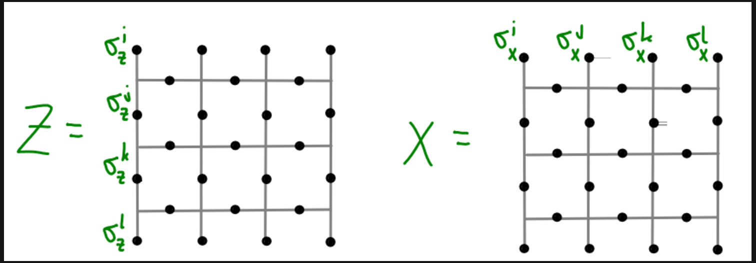
- Measuring the stabilizers collapses this to a simple X, Y or Z

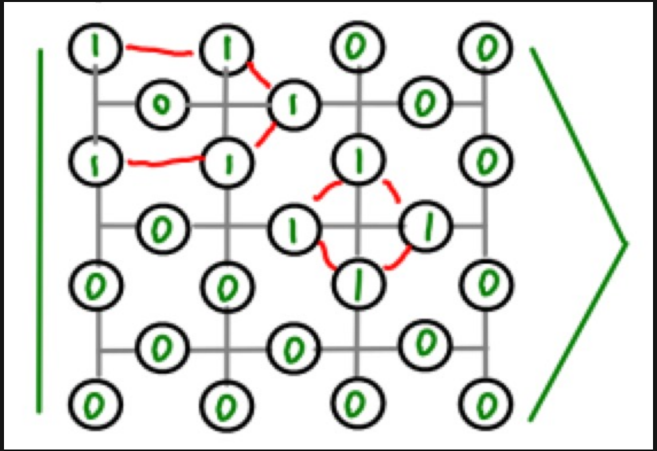
Final Readout

- The logical operators are many-body observables
- So how do we read them out fault-tolerantly

- When you decide on a basis for final measurement, you stop caring about some errors
- You can then measurement in a product basis
- Final readout and final stabilizer measurement can be constructed from the result

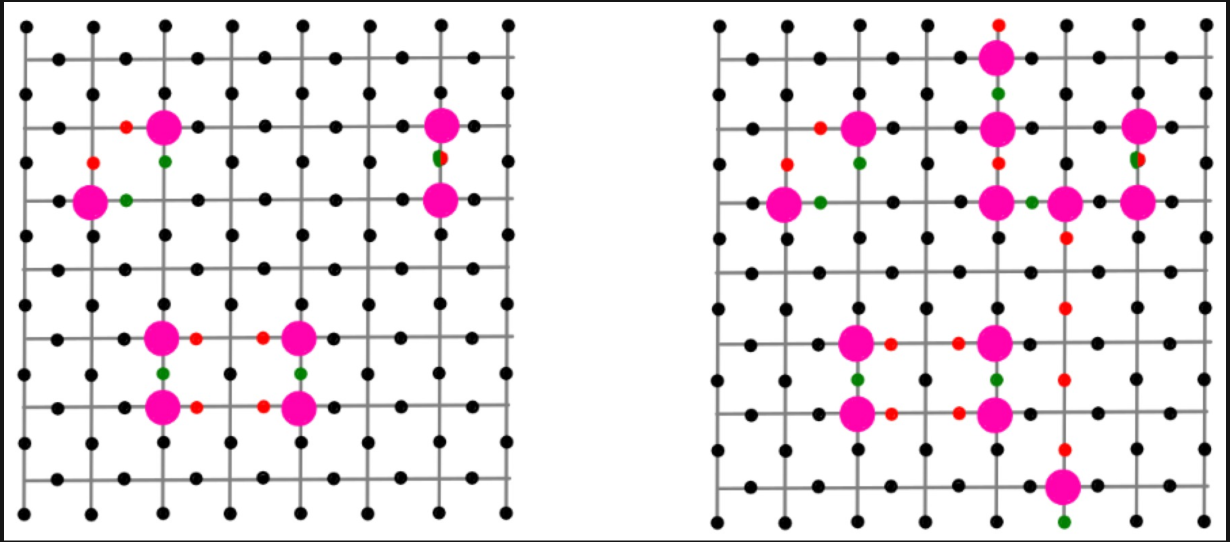
- Measurement errors are effectively the same as errors before measurement





Decoding

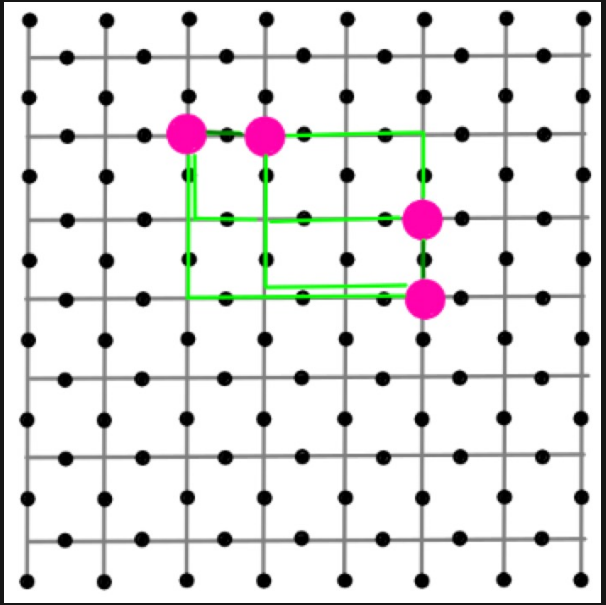
- Given the measurement results, we need to work out what errors happened
- More specifically, the 'equivalence class' of errors
- This is the job of the decoding algorithm

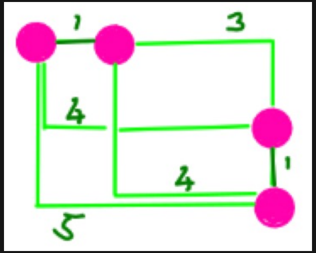


Decoding with MWPM

- A good option is MWPM
- Just as we considered for the repetition code

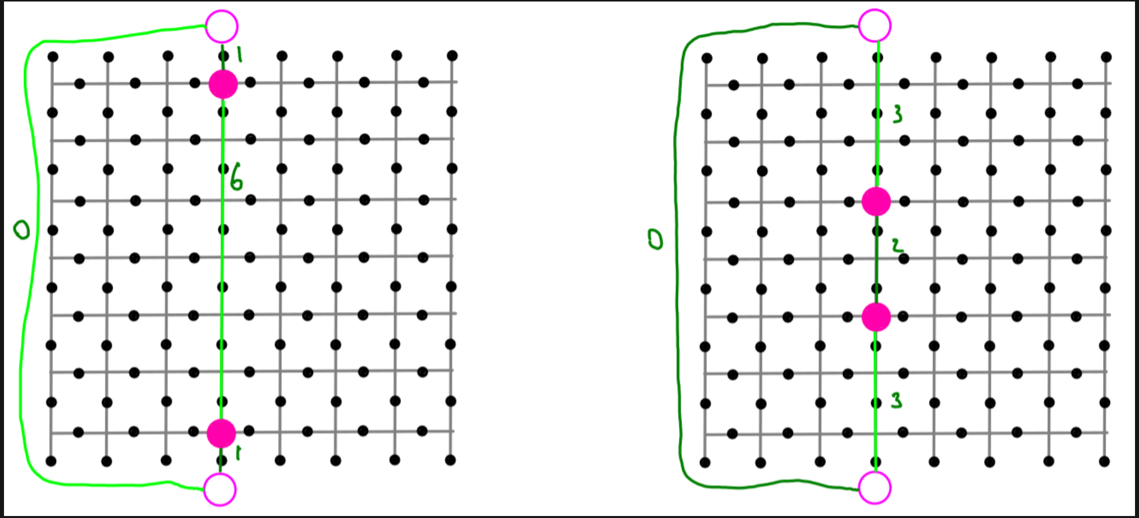
- Again we start with the simple and unrealistic case: errors only between measurement
- Each round can be decoded separately, corresponding to MWPM on a 2D graph
- Decoding for X and Z errors can also be done independently





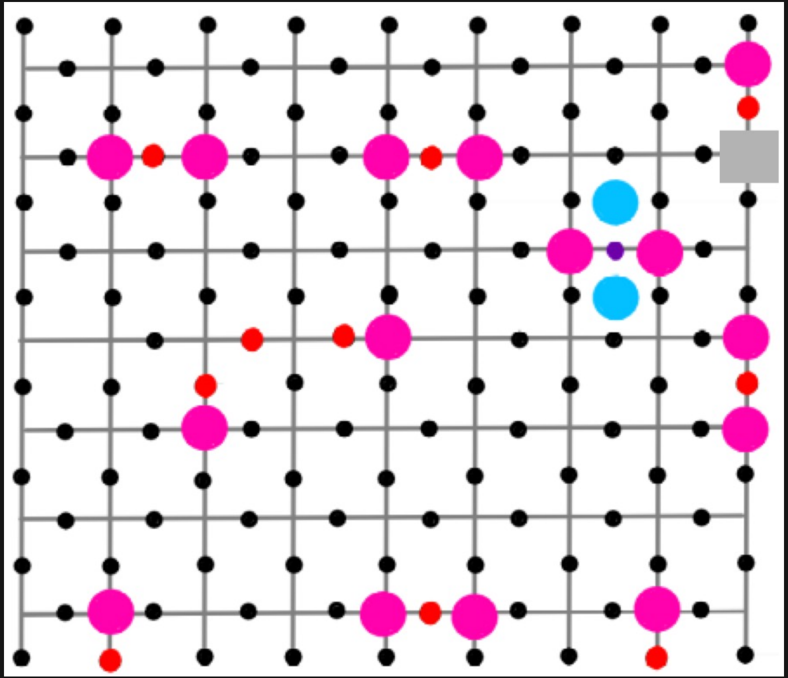
Decoding

- We need to be careful to account for the effects of the edges
- This is done by introducing extra ‘virtual nodes’
  (also required for the repetition code, but we ignored it earlier)



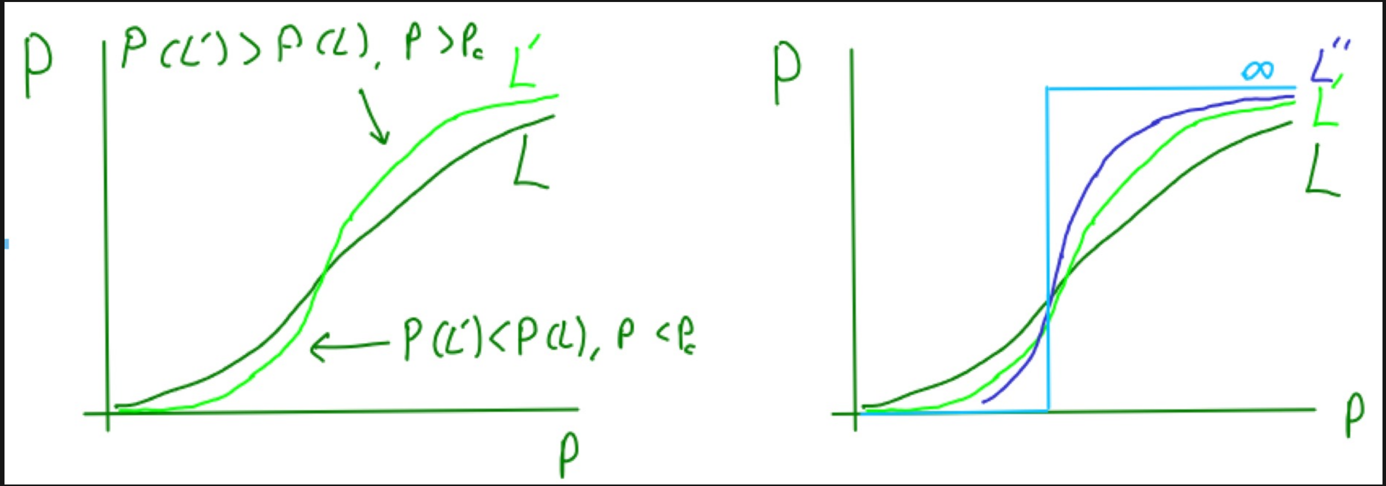
Imperfect Measurements

- Again, we have the problem of imperfect measurements
  - The measurements might lie
  - Errors on the additional qubit
  - Errors in the CX gates
- We base the decoding using syndrome changes
- This leads to a 3D MWPM problem (2D space + time)



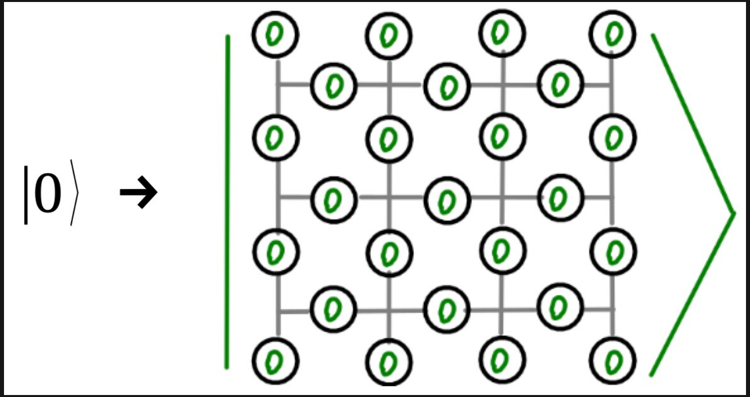
Threshold

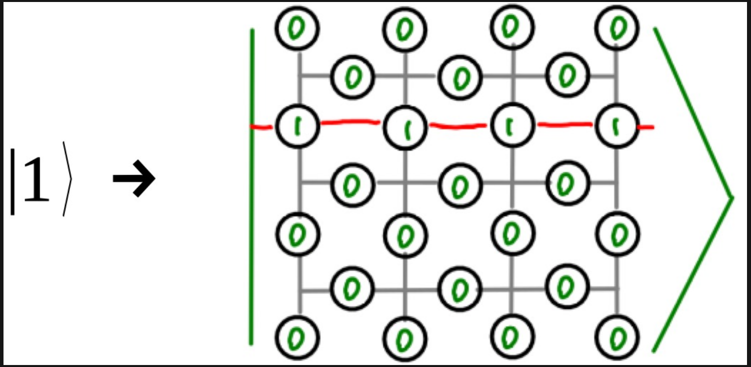
- Correcting according to the right class removes the effects of errors
- Correcting according to the wrong class causes an operation on the encoded qubit (without our knowing)
- What is the probability of such an error, *P*, given the probability on the qubits of the code, *p*?
- We find a phase transition as *L* is increased (for an *LxL* grid)



More Logical Gates

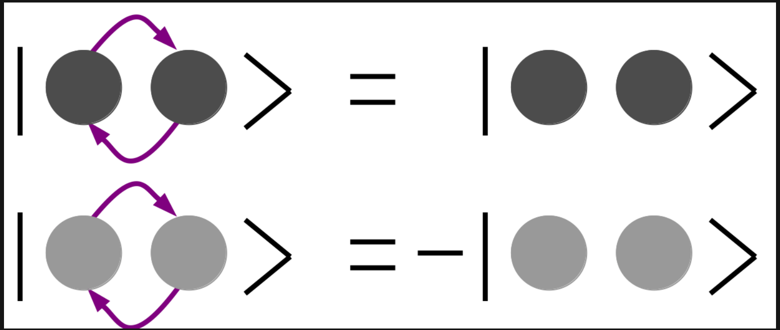
- We've seen how to do logical X and Z
- A logical CX can be done without much trouble
- A logical H requires the lattice to be rotated, but that can be done
- Other logical Clifford gates can be done with some crazy tricks
- But that's all! No other logical operations can be done fault-tolerantly.
- A solution is *magic state distillation*, using the logical gates we have to clean up the one we don't
- But that's beyond today's lecture...



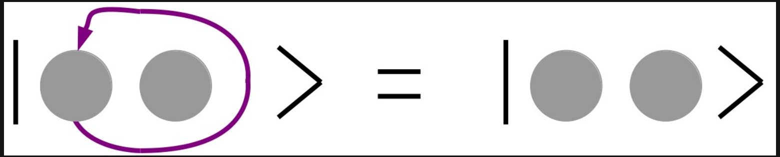


Anyons in the Surface Code

- There are many variants on how qubits can be encoded and manipulated in the surface code
- They all depend on the unique topological nature
- The 'defects' created by errors in the surface code behave like particles

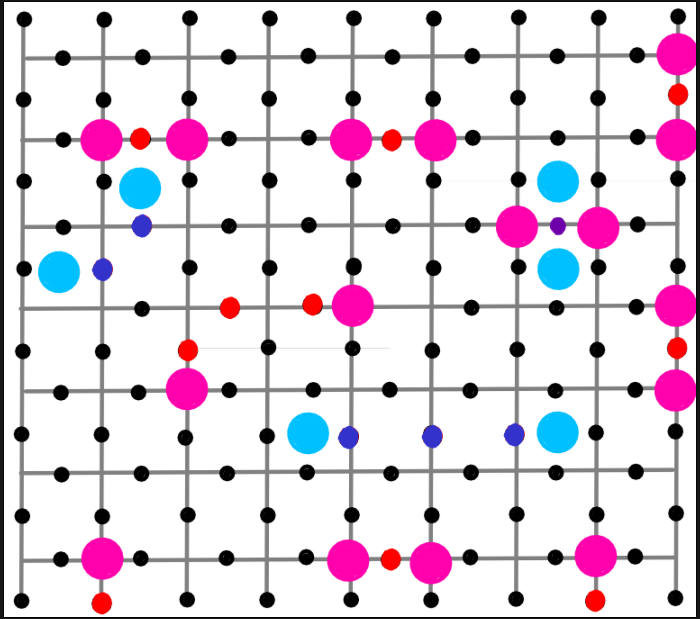


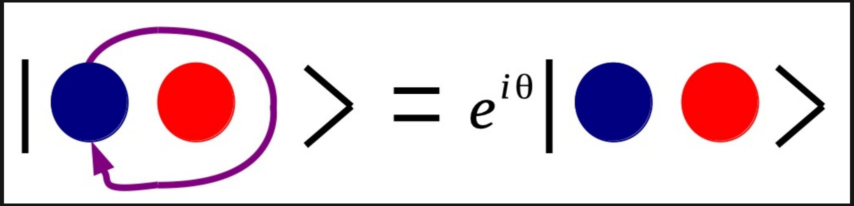
- All particles in our universe are either bosons or fermions
- This due to topological restrictions in a 3D universe



Anyons in the Surface Code

- The surface code is only a 2D ‘universe’, so doesn’t have these restrictions
- How do these particles behave?



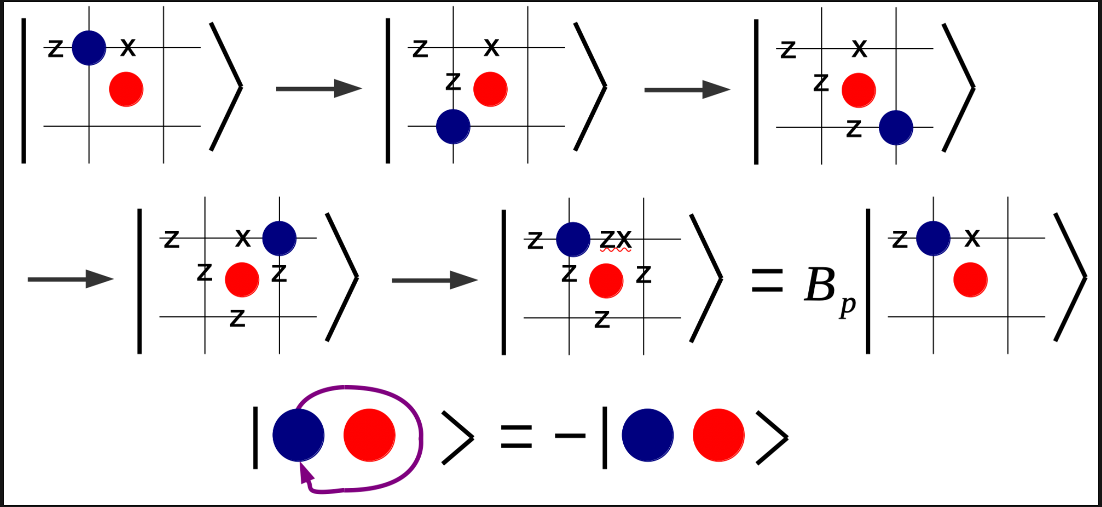

$| \bullet \; \bullet \rangle = e^{i\theta} | \bullet \; \bullet \rangle$

Anyons in the Surface Code

– Braiding a particle corresponds to applying a stabilizer

– Their eigenstates defines the braiding phase

– Neither bosons nor fermions, but anyons!



Introduction to LDPC Codes

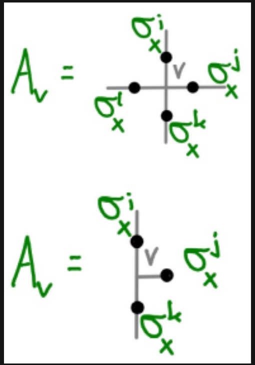
James R. Wootton
IBM Quantum

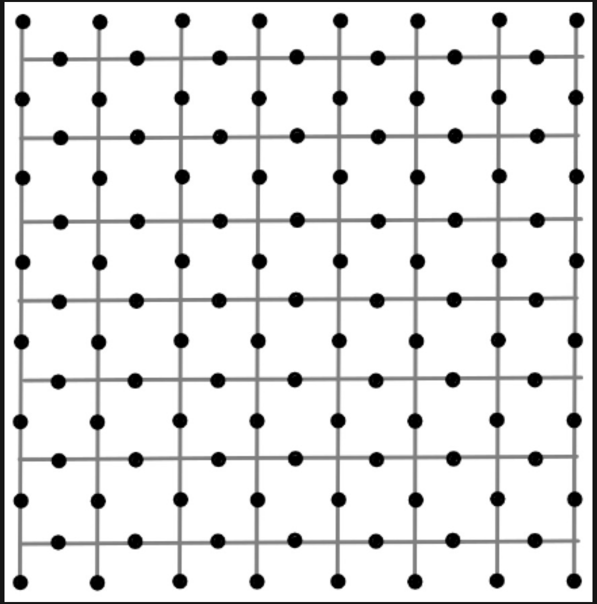
IBM **Quantum**

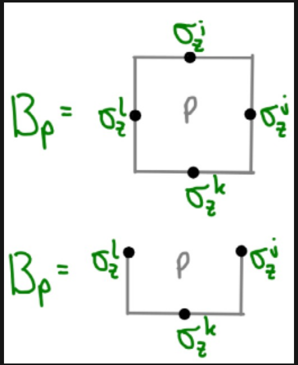
Why we like the surface code

IBM **Quantum**

- Each qubit is involved in only a finite number of syndrome measurements
- Each syndrome measurement requires only a finite number of qubits
- Qubits can be restricted to a 2D lattice with nearest neighbour entangling gates







Why we don't like the surface code

IBM **Quantum**

- We refer to codes using the parameters [[n,k,d]]
  - n: the number of physical qubits
  - k: the number of logical qubits
  - d: the code distance
- For a surface code

$$n \sim d^2, \qquad k = 1, \qquad d = d$$

- Logical qubits made with the surface code are very expensive

$$R = \lim_{n\to\infty} \frac{k}{n} = 0, \qquad d \sim n^{1/2}$$

- Can we find codes with better scaling, while keeping the nice features?

LDPC codes

- ”Low density parity check” codes are classical EC codes for which
  - Each bit is involved in only a finite number of checks
  - Each check involves only a finite number of bits

- qLDPC codes are the same, but quantum

- Good qLDPC codes are those with good sets of parameters, such as

$$R = \lim_{n\to\infty} \frac{k}{n} = O(1), \qquad d \sim n$$

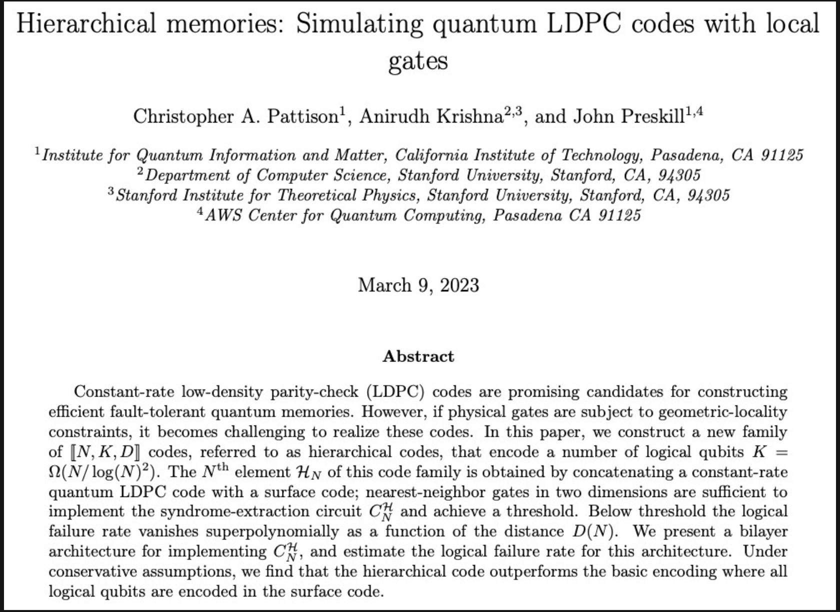
- But how much do they deviate from a 2D lattice?

qLDPC codes

- We know a few bounds for purely 2D layouts, e.g.
  - $kd^2 \lesssim n$ [Bravyi, Poulin, Terhal 2010]
  - At least $\sqrt{\frac{k}{n}}d$ interactions of range $\sqrt{\frac{k}{\sqrt{d}}}$ are required [Baspin, Krishna 2022]
- These can also be violated, at a price
  - For example

$$k \sim \frac{n}{\log^2 n}, \quad d \sim n^{1/2}$$

but P decays only superpolynomially



Hierarchical memories: Simulating quantum LDPC codes with local gates

Christopher A. Pattison[1], Anirudh Krishna[2,3], and John Preskill[1,4]

[1] *Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, CA 91125*
[2] *Department of Computer Science, Stanford University, Stanford, CA, 94305*
[3] *Stanford Institute for Theoretical Physics, Stanford University, Stanford, CA, 94305*
[4] *AWS Center for Quantum Computing, Pasadena CA 91125*

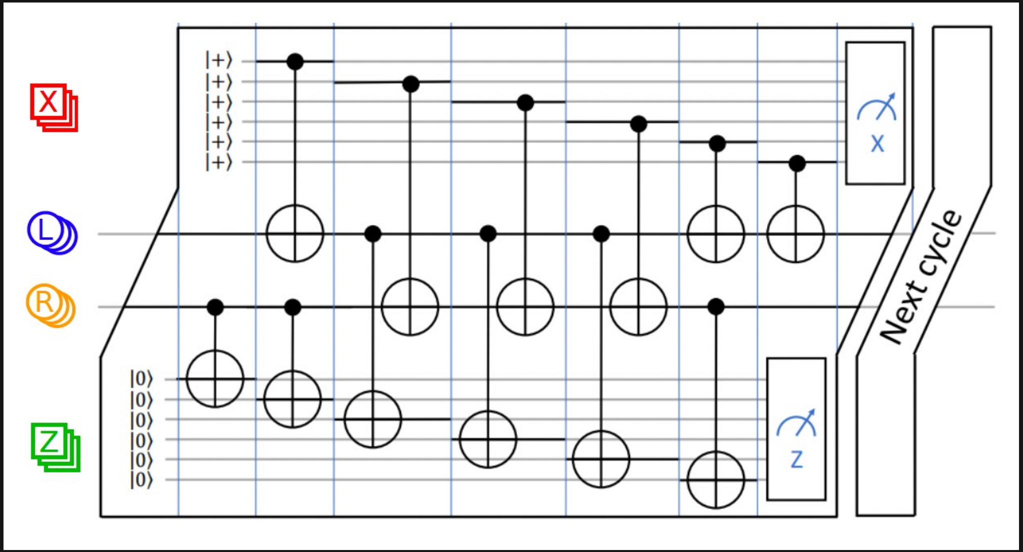
March 9, 2023

Abstract

Constant-rate low-density parity-check (LDPC) codes are promising candidates for constructing efficient fault-tolerant quantum memories. However, if physical gates are subject to geometric-locality constraints, it becomes challenging to realize these codes. In this paper, we construct a new family of $[\![N, K, D]\!]$ codes, referred to as hierarchical codes, that encode a number of logical qubits $K = \Omega(N/\log(N)^2)$. The N^{th} element $\mathcal{H}_N$ of this code family is obtained by concatenating a constant-rate quantum LDPC code with a surface code; nearest-neighbor gates in two dimensions are sufficient to implement the syndrome-extraction circuit $C_N^{\mathcal{H}}$ and achieve a threshold. Below threshold the logical failure rate vanishes superpolynomially as a function of the distance $D(N)$. We present a bilayer architecture for implementing $C_N^{\mathcal{H}}$, and estimate the logical failure rate for this architecture. Under conservative assumptions, we find that the hierarchical code outperforms the basic encoding where all logical qubits are encoded in the surface code.

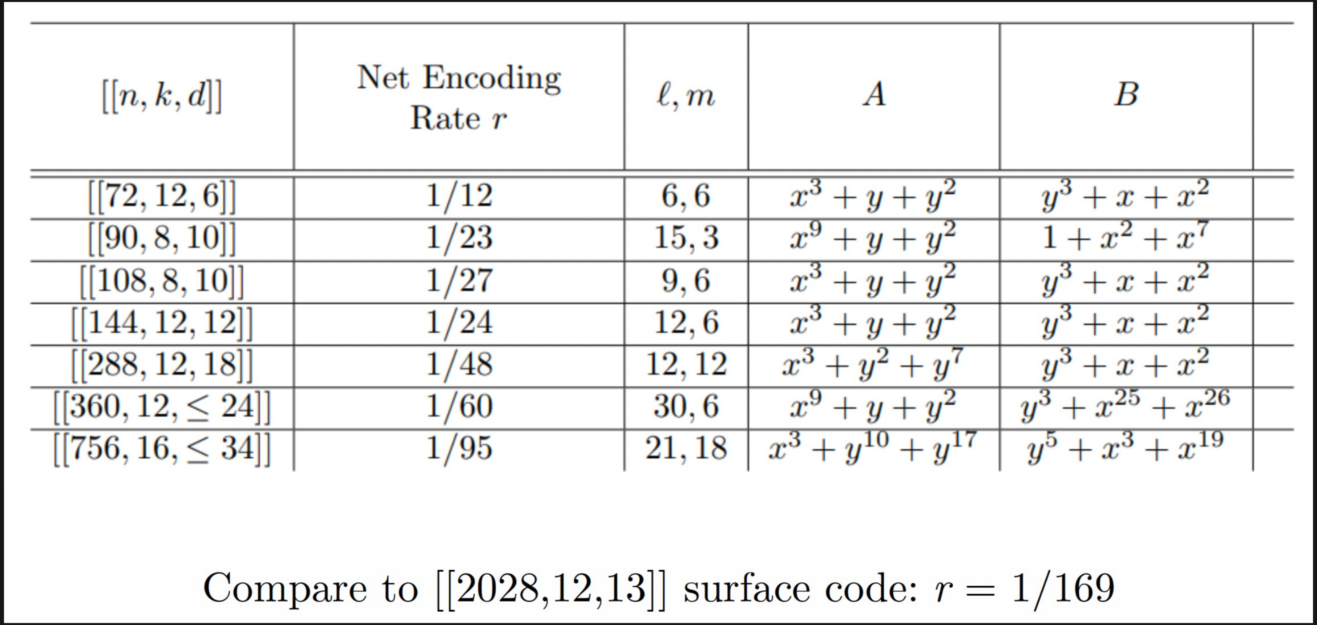
qLDPC codes at IBM

- At IBM we want codes with
  - High distance and encoding rate
  - A high threshold (or pseudothreshold) for circuit noise
  - Superconducting qubit implementation
  - A short-depth syndrome extraction circuit



qLDPC codes at IBM

- Answer comes from ”bivariant bicycle codes”
  - Variant of quasi-cyclic codes [Kovalev, Pryadko 2013]

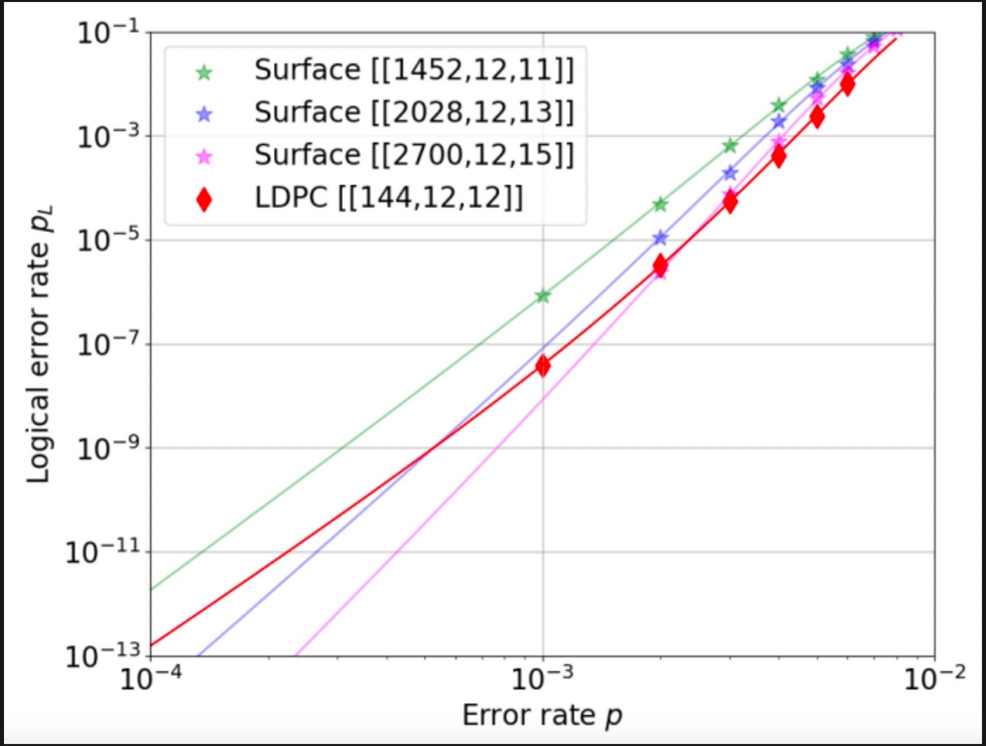


$[[n,k,d]]$	Net Encoding Rate r	ℓ, m	A	B
$[[72,12,6]]$	$1/12$	$6,6$	x^3+y+y^2	y^3+x+x^2
$[[90,8,10]]$	$1/23$	$15,3$	x^9+y+y^2	$1+x^2+x^7$
$[[108,8,10]]$	$1/27$	$9,6$	x^3+y+y^2	y^3+x+x^2
$[[144,12,12]]$	$1/24$	$12,6$	x^3+y+y^2	y^3+x+x^2
$[[288,12,18]]$	$1/48$	$12,12$	$x^3+y^2+y^7$	y^3+x+x^2
$[[360,12,\leq 24]]$	$1/60$	$30,6$	x^9+y+y^2	$y^3+x^{25}+x^{26}$
$[[756,16,\leq 34]]$	$1/95$	$21,18$	$x^3+y^{10}+y^{17}$	$y^5+x^3+x^{19}$

Compare to [[2028,12,13]] surface code: $r = 1/169$

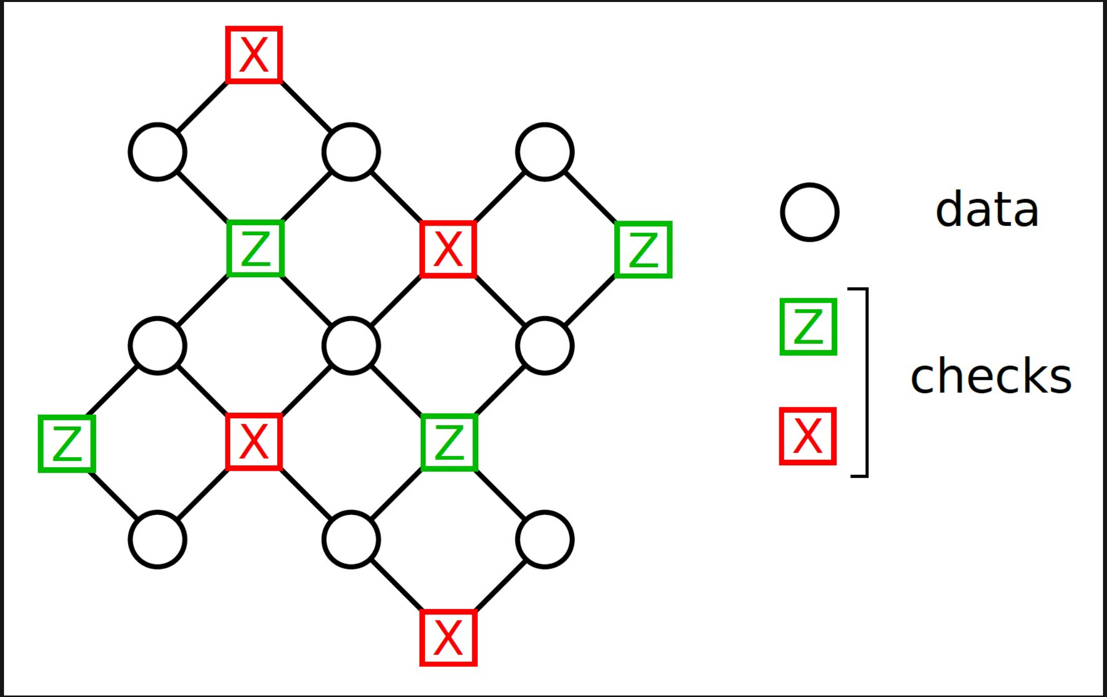
qLDPC codes at IBM

– Matches surface code performance, but with 10x fewer qubits!



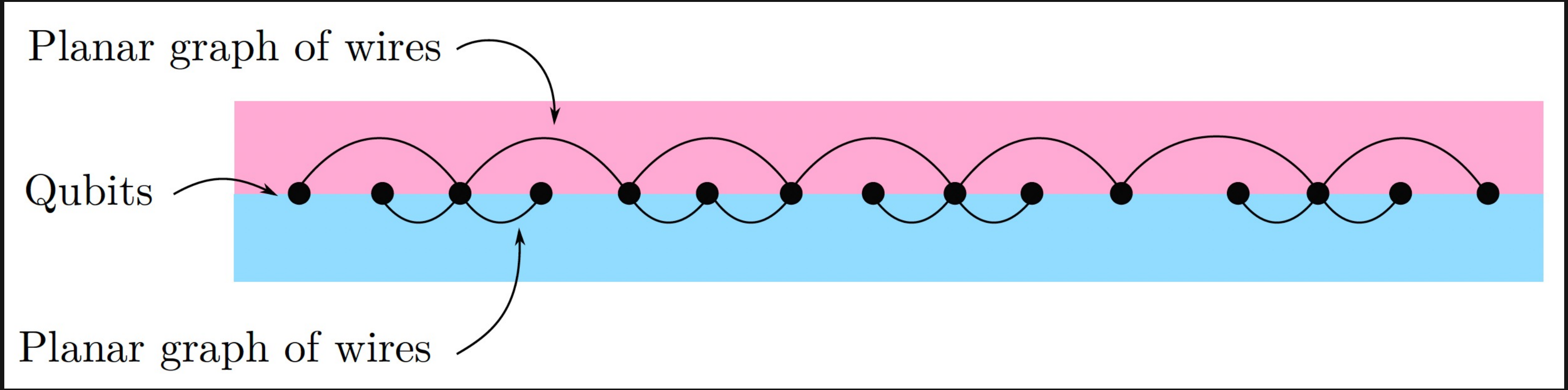
Bilayer representation

– First: Tanner graphs



Bilayer representation

- If planar graphs aren't good enough, we go for thickness-2
  - Union of two planar graphs

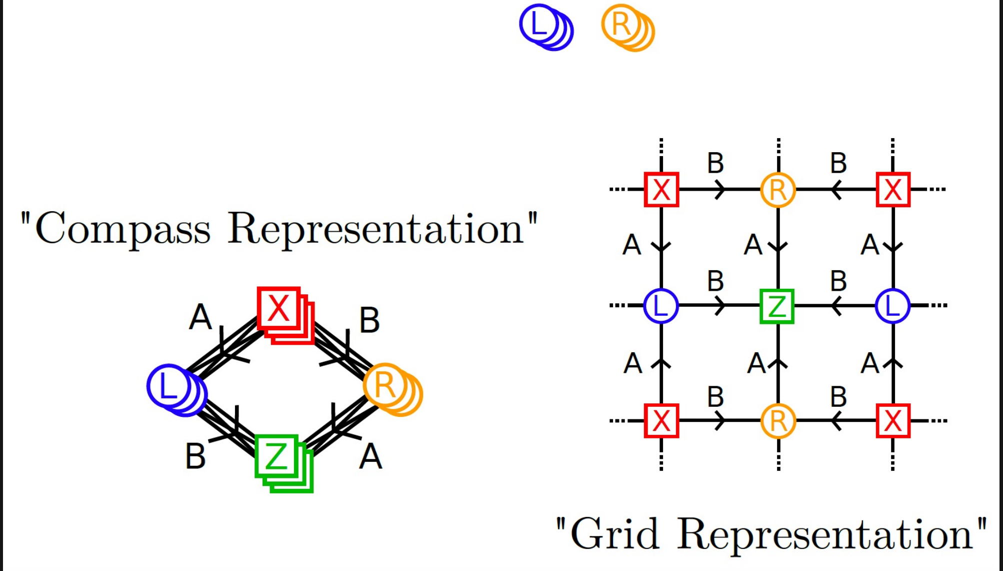


Bilayer representation

– Tanner graph for these codes

$$H_X = \begin{bmatrix} A & B \end{bmatrix}$$

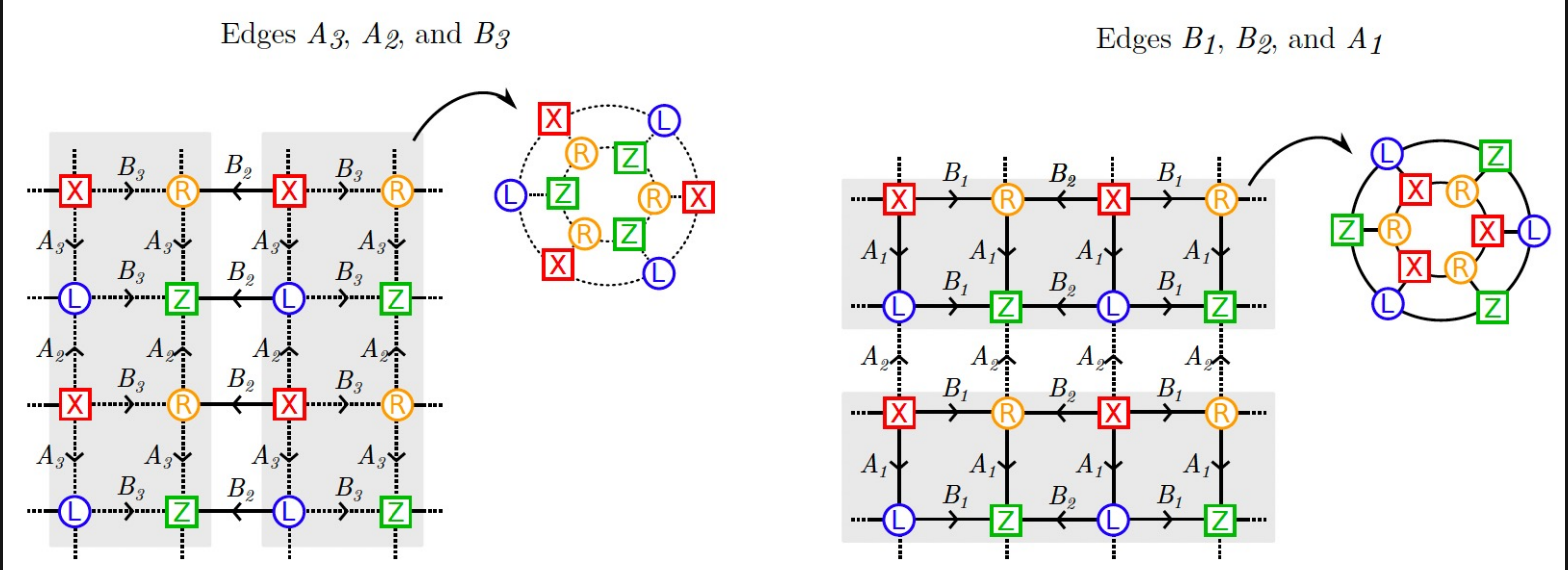
$$H_Z = \begin{bmatrix} B^T & A^T \end{bmatrix}$$



Bilayer representation

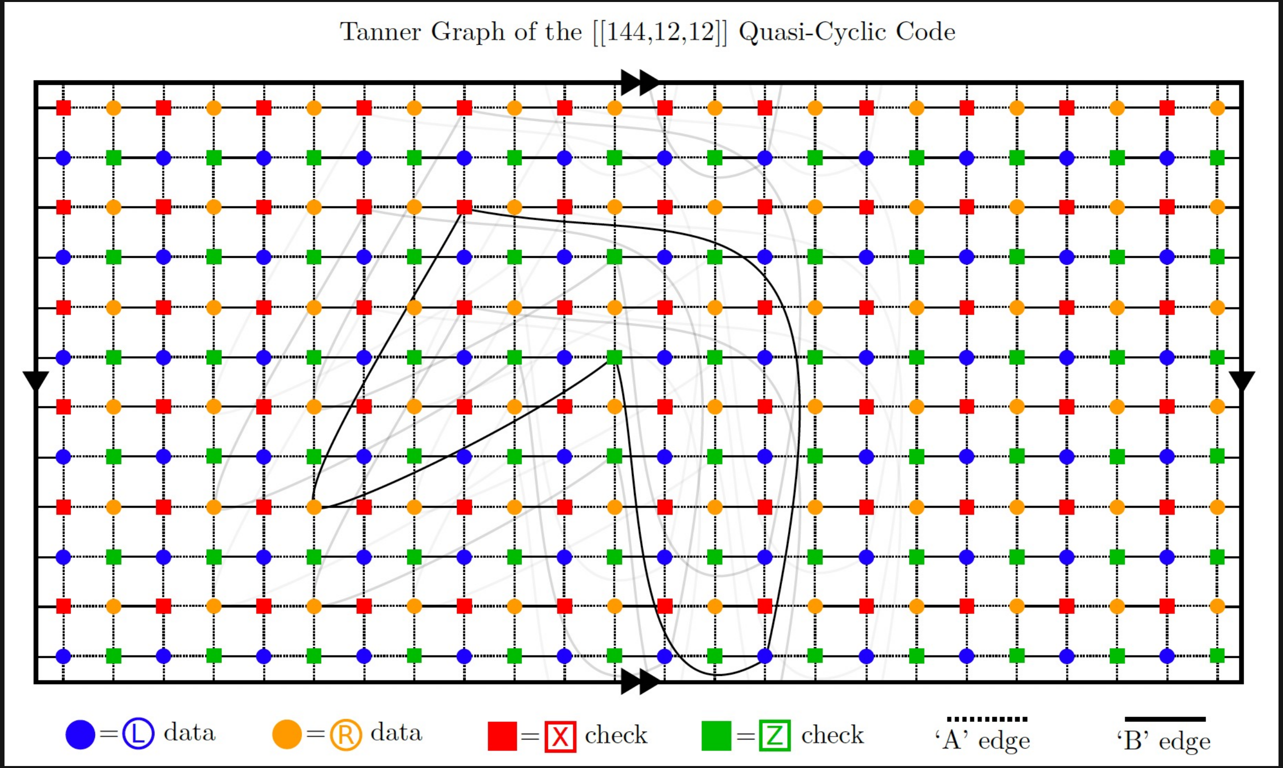
– Visual proof of thickness-2

$$A = A_1 + A_2 + A_3 \qquad B = B_1 + B_2 + B_3$$



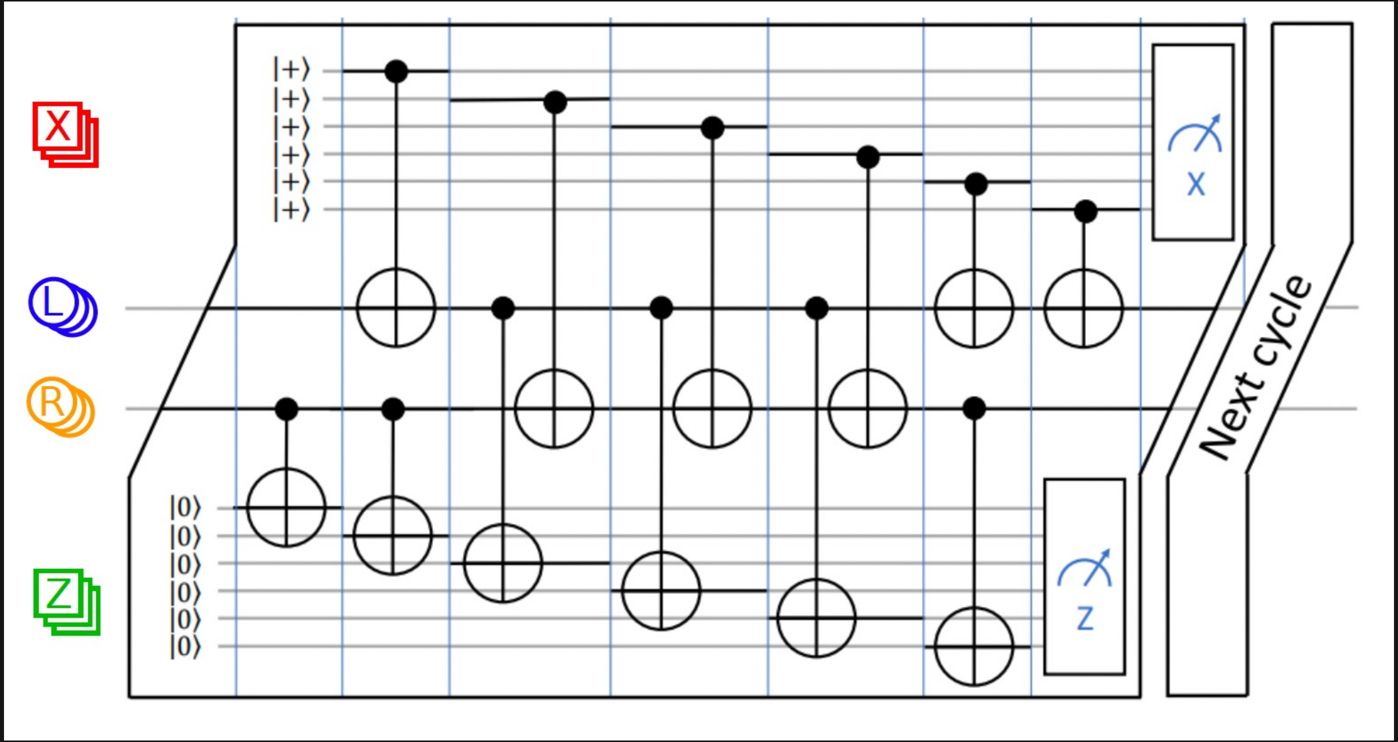
Bilayer representation

– Tanner graph of [[114,12,12]]



Syndrome measurement circuit

IBM **Quantum**



Pseudo-thresholds around 0.8%

Conclusions

- qLDPC codes that outperform the surface code
  - Better rate
  - Same error suppression
  - Similar pseudo-threshold
- The cost is a more complex Tanner graph
  - But bilayer architecture is something we can achieve!

Bravyi et al., arXiv:2308.07915 (2023)

Thanks for your attention

qisk.it/decoders