Bayesian Quantum Amplitude Estimation

ReaQCT 2024

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This work is financed by National Funds through the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia, within project UIDB/50014/2020.

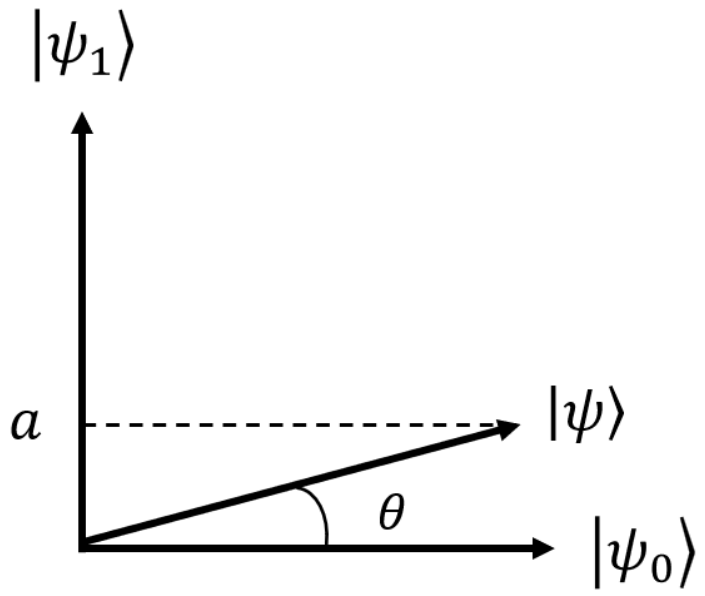
DOI 10.54499/UIDB/50014/2020 | https://doi.org/10.54499/uidb/50014/2020

Quantum Amplitude Estimation
(QAE)

We can prepare the quantum state

$$|\psi\rangle = \sqrt{\boldsymbol{a}}|\psi_1\rangle + \sqrt{1-\boldsymbol{a}}|\psi_0\rangle.$$

Find a.



Quantum Amplitude Estimation

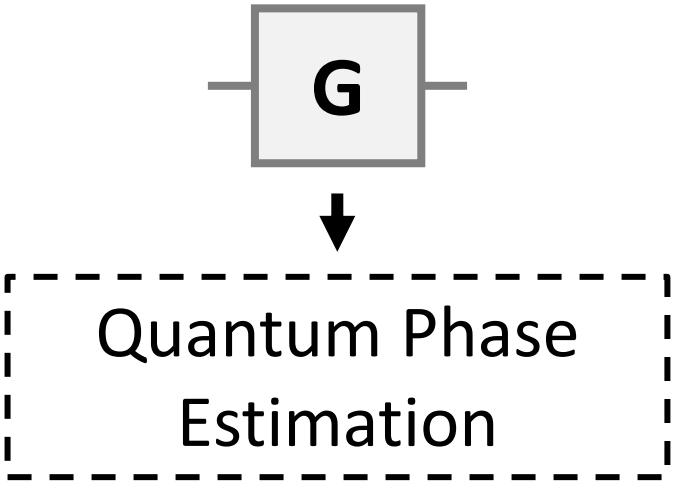
$$|\psi\rangle = \sqrt{a}|\psi_1\rangle + \sqrt{1-a}|\psi_0\rangle$$

Classical solution

Sample from $\mathrm{Binomial}\ (a)$ and take averages over N samples.

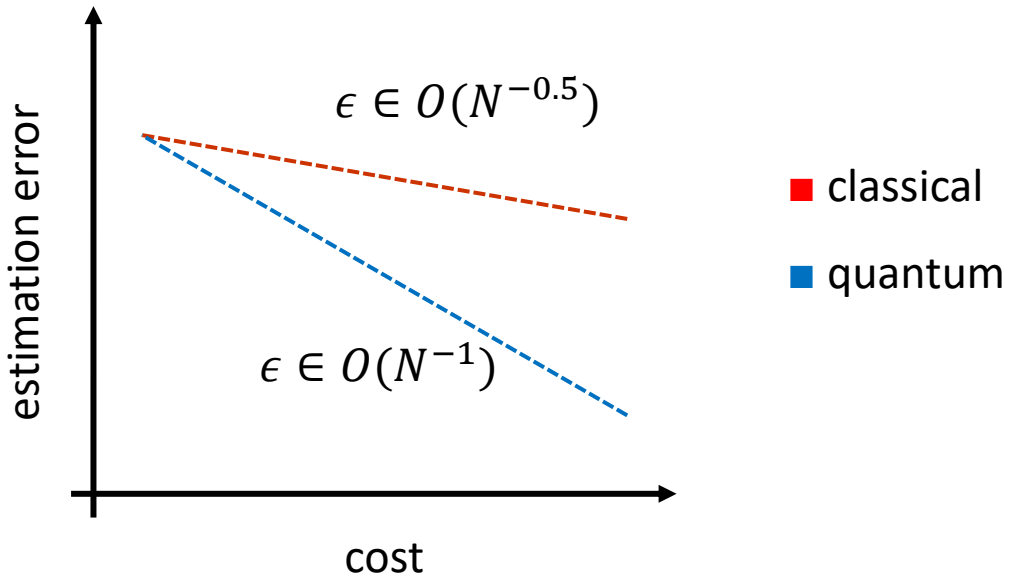
Quantum solution

Apply **phase estimation** to the Grover **quantum amplitude amplification** operator.



Quantum Amplitude Estimation

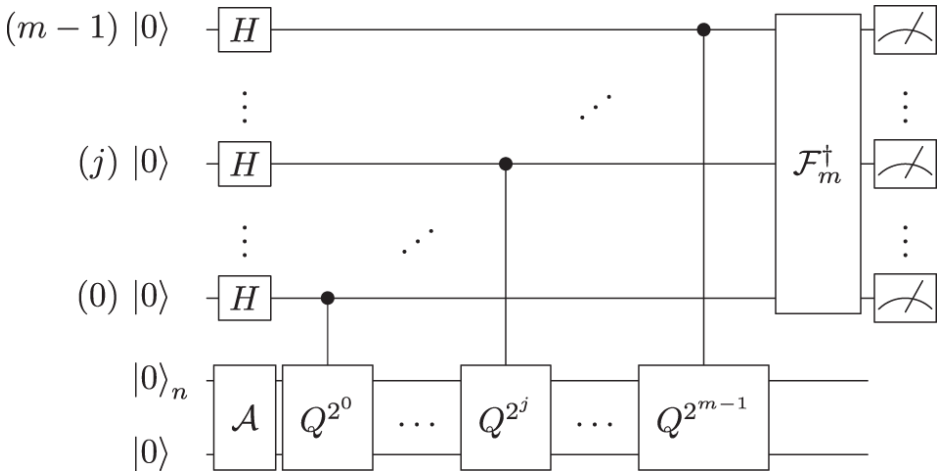
There is a **quadratic quantum speed-up** for this task.



QAE with Noisy Quantum Devices

The circuit is **not viable for today's quantum devices**.

Alternative strategies have been proposed.



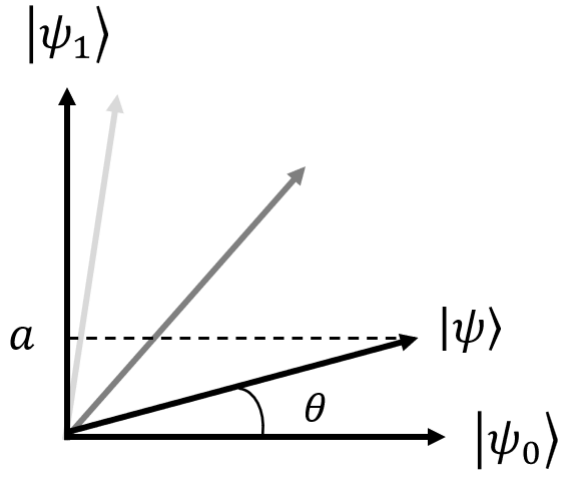
*in: Grinko, D., Gacon, J., Zoufal, C. et al. Iterative quantum amplitude estimation. npj Quantum Inf **7**, 52 (2021).*

QAE with Noisy Quantum Devices

We can sample from

$$p(\theta|k) \equiv \langle\psi_1|G^k|\psi\rangle = \sin^2(r_k\theta)$$

with cost in $\mathcal{O}(r_k)$.



Use this extra freedom to unlock otherwise unachievable learning rates.

QAE with Noisy Quantum Devices

Learn the hidden parameter of an observable probability distribution $\mathbf{Binomial}\ (\mathrm{p}(\theta), N_{shots})$.

Classical samples:

$$\mathbf{p}(\boldsymbol{\theta}) = \mathbf{sin}^{\mathbf{2}}\,\boldsymbol{\theta}$$

$$\epsilon \in O(N^{-0.5})$$

Quantum-enhanced samples:

$$\mathbf{p}(\boldsymbol{\theta}|\boldsymbol{r}) = \mathbf{sin}^{\mathbf{2}}(\boldsymbol{r\theta})$$

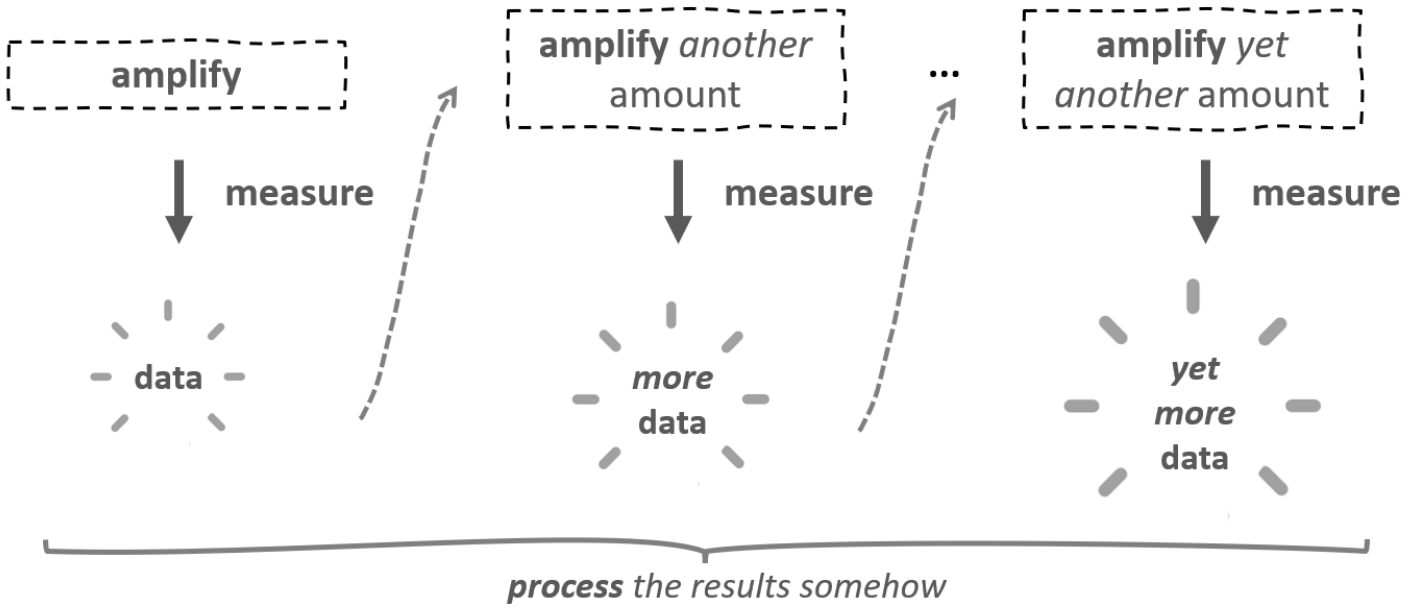
for $\boldsymbol{r}$ any odd integer

$$\epsilon \in O(N^{-1})$$

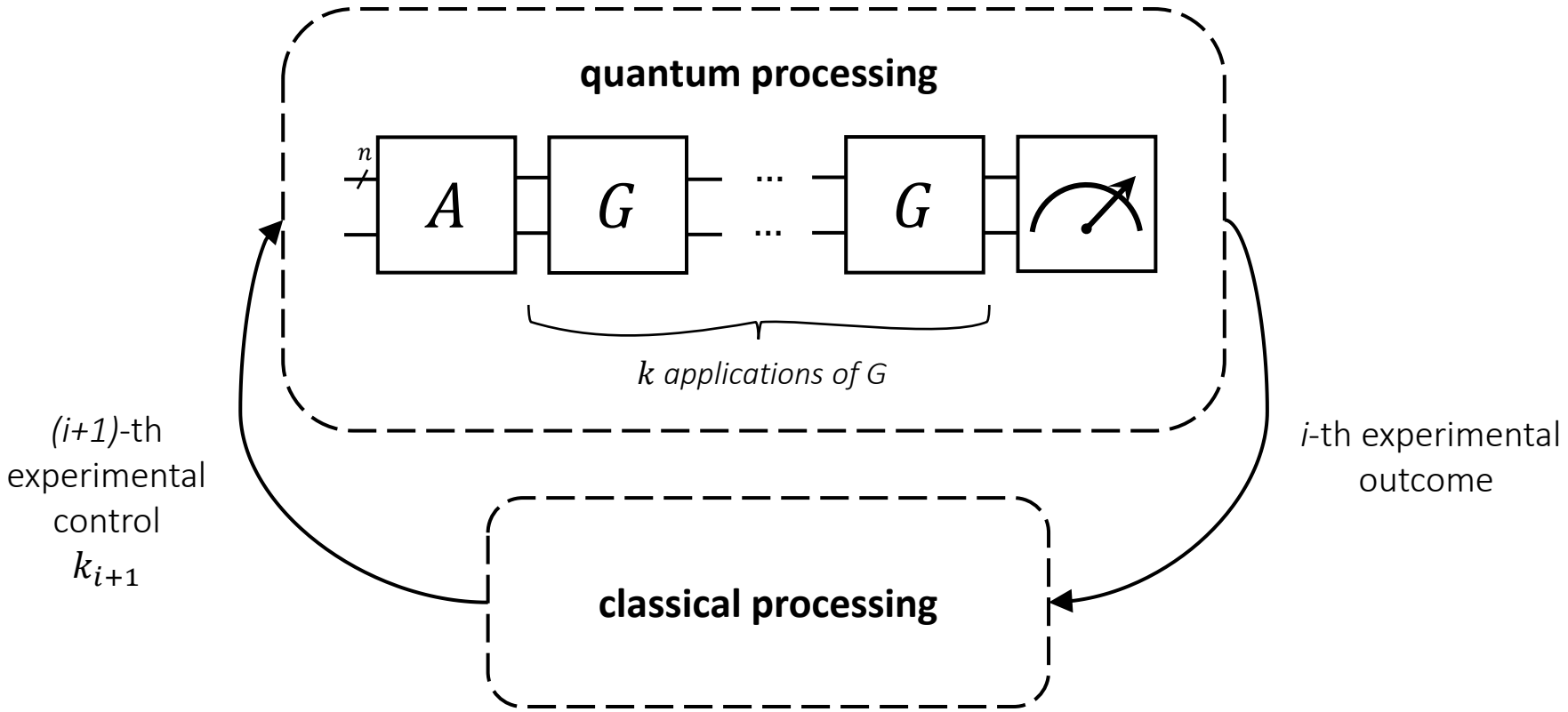
?

QAE with Noisy Quantum Devices

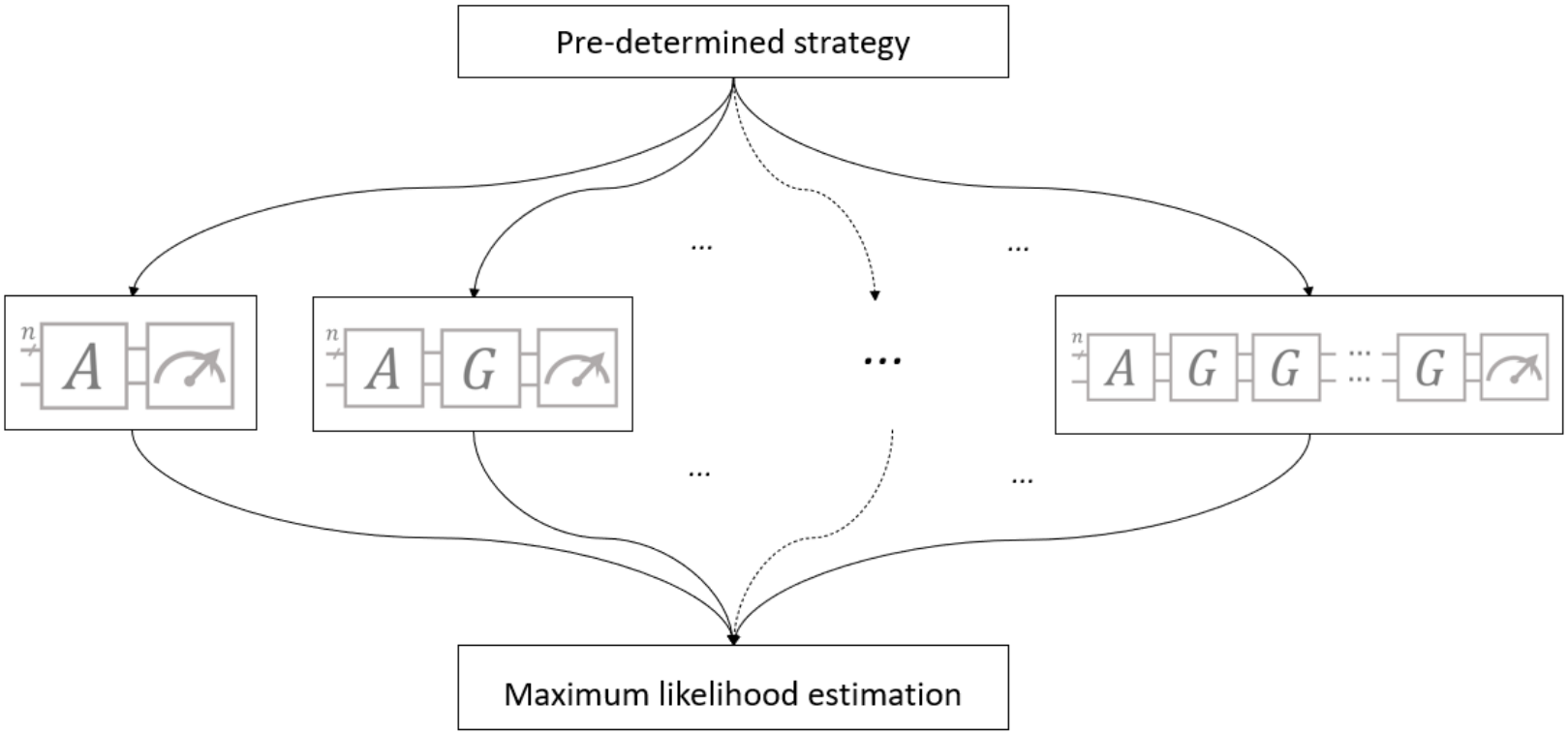
We can look at this as a **data science problem**.



QAE with Noisy Quantum Devices



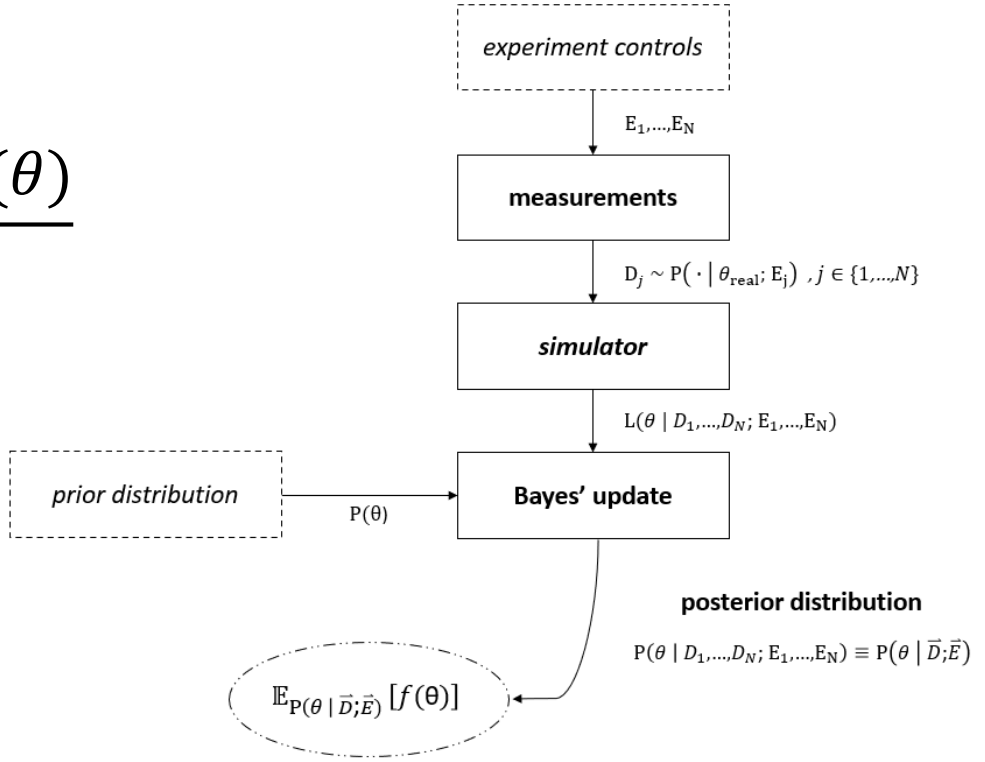
Maximum Likelihood QAE



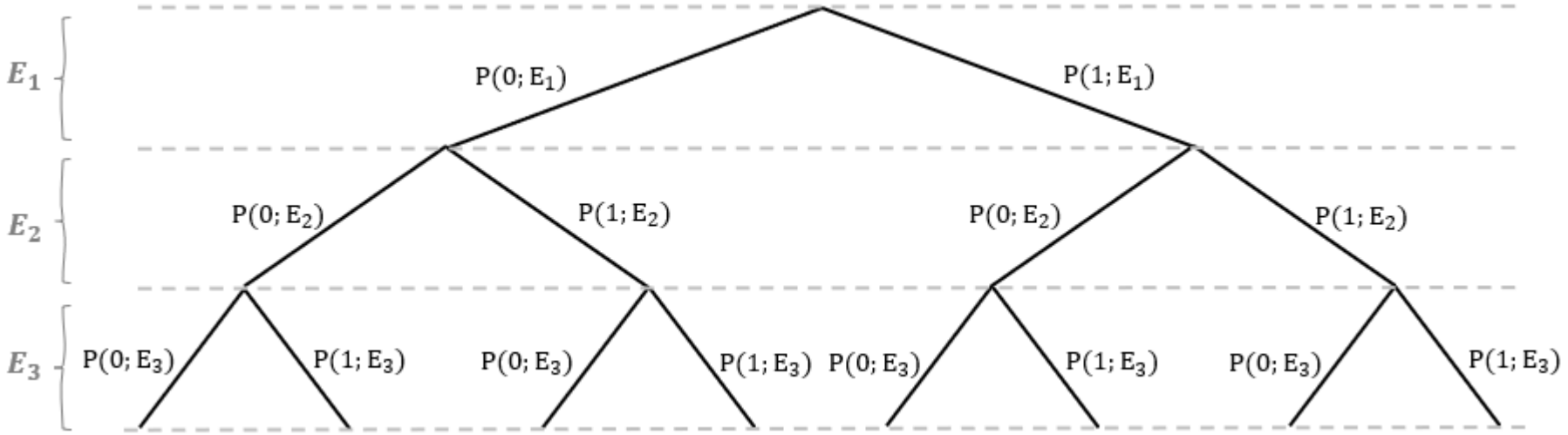
Bayesian Amplitude Estimation

Bayesian statistics offers a natural paradigm for QAE.

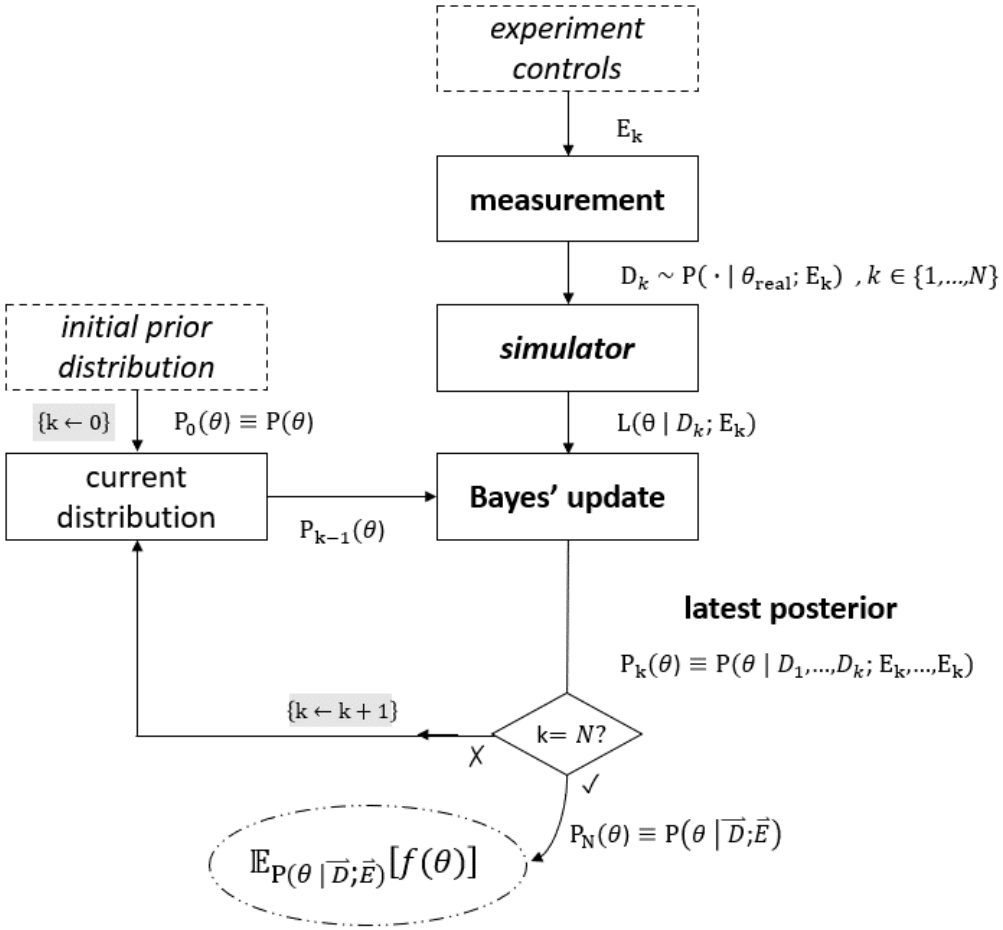
$$\boldsymbol{P}(\theta|D) = \frac{\boldsymbol{L}(\theta|D;E)\boldsymbol{P}(\theta)}{\boldsymbol{P}(D;E)}$$



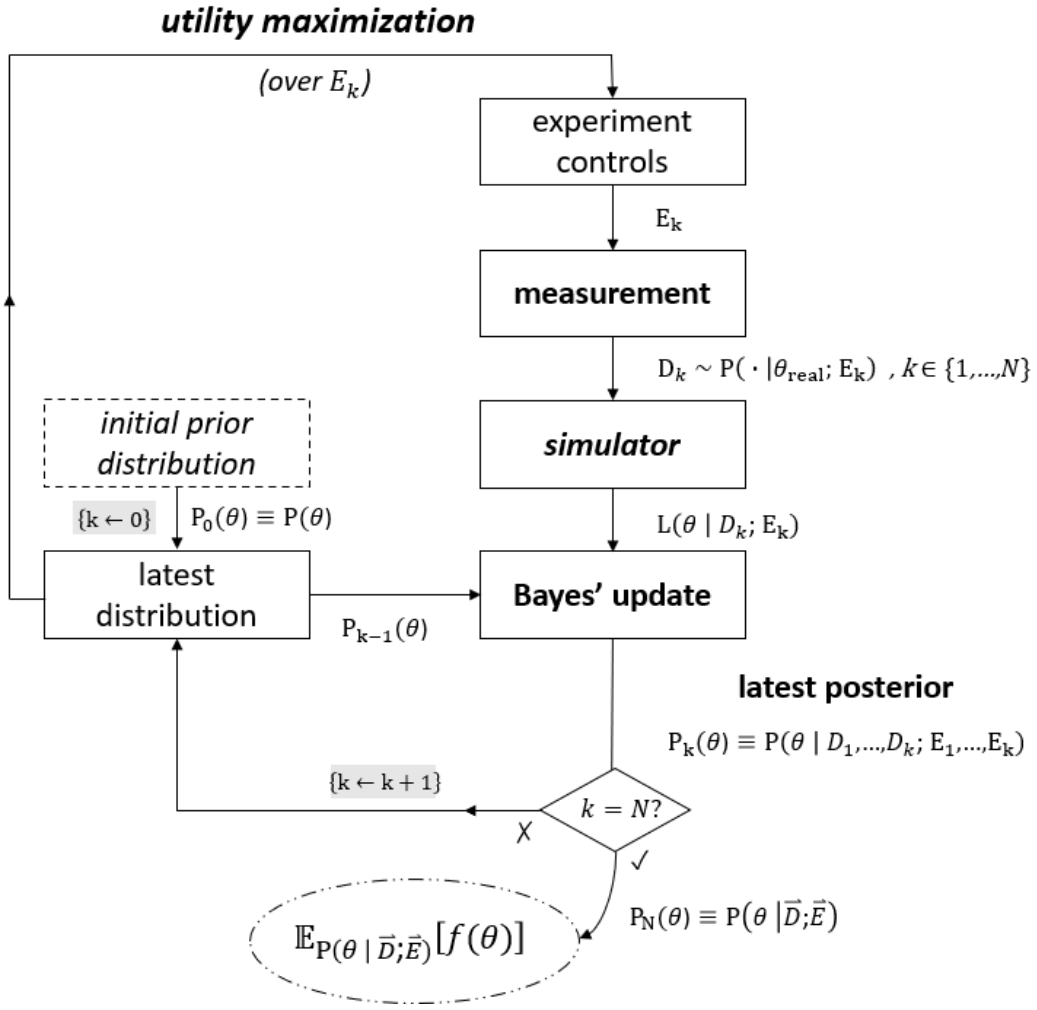
Bayesian Amplitude Estimation



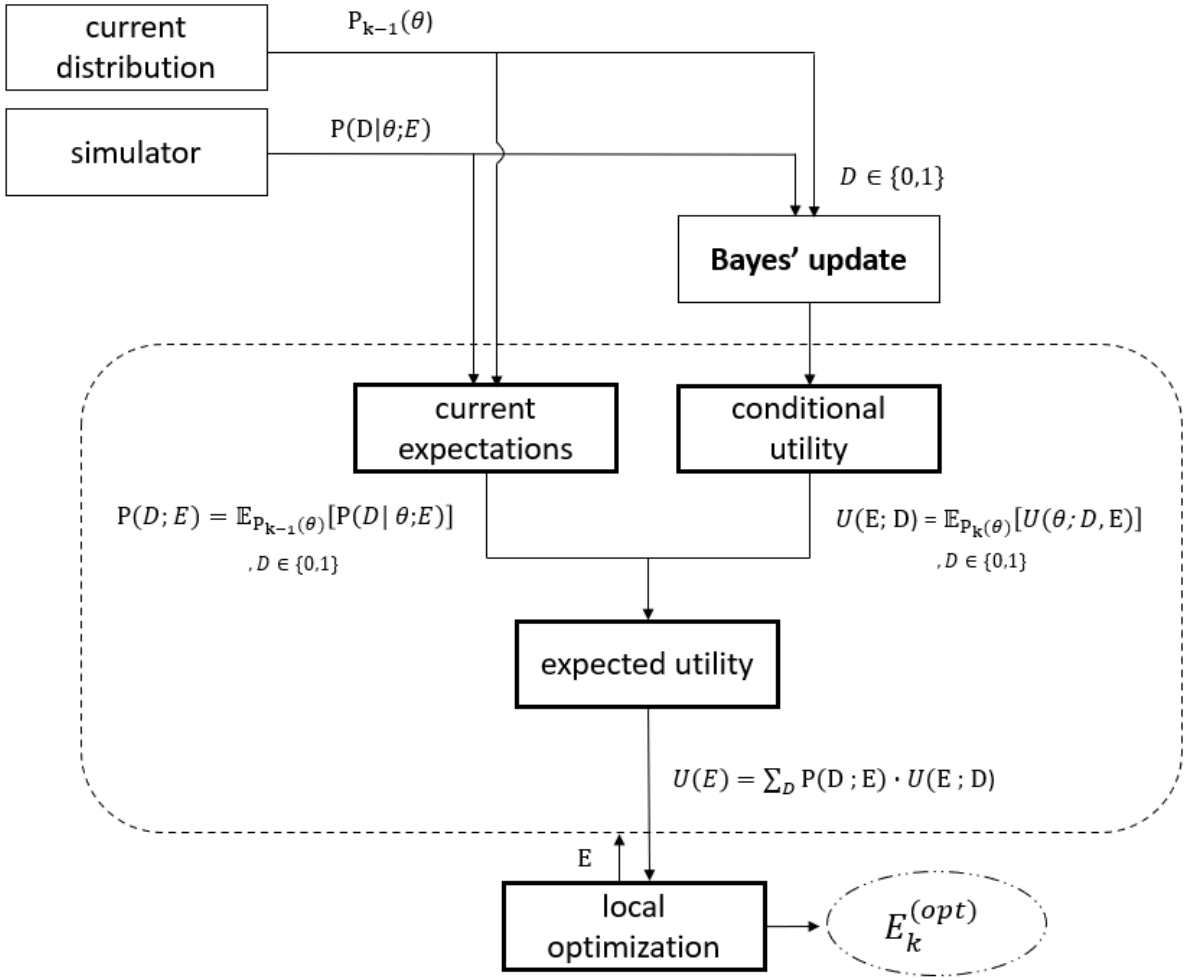
Bayesian Amplitude Estimation



Bayesian Amplitude Estimation



Bayesian Amplitude Estimation



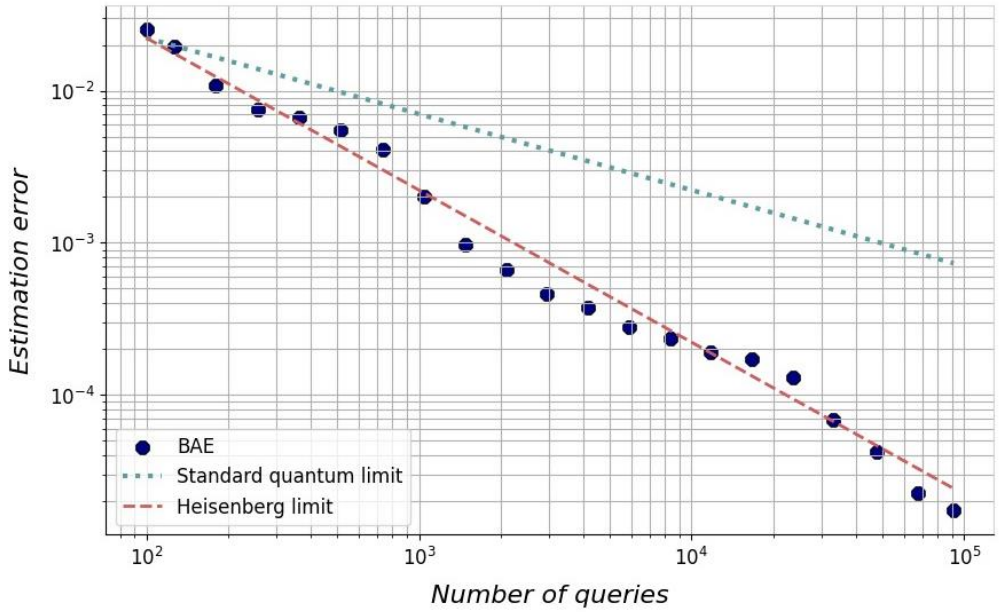
Bayesian Amplitude Estimation

$$\boldsymbol{P}(\theta|D) = \frac{\boldsymbol{L}(\theta|D;E)\boldsymbol{P}(\theta)}{\boldsymbol{P}(D;E)}$$

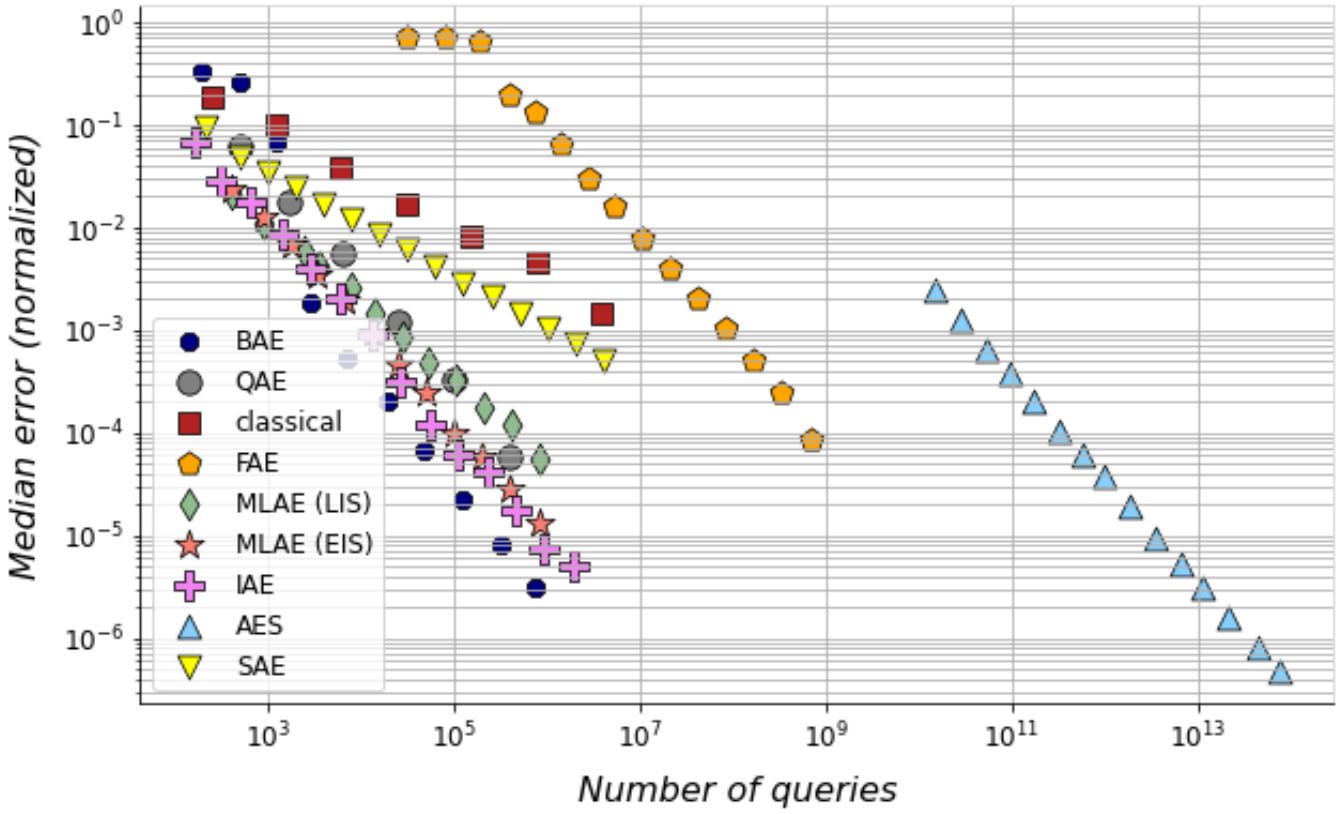
The use of Bayesian inference does not fully determine the protocol; the **optimization** and **representation** methods employed are determinant for performance.

Bayesian Amplitude Estimation

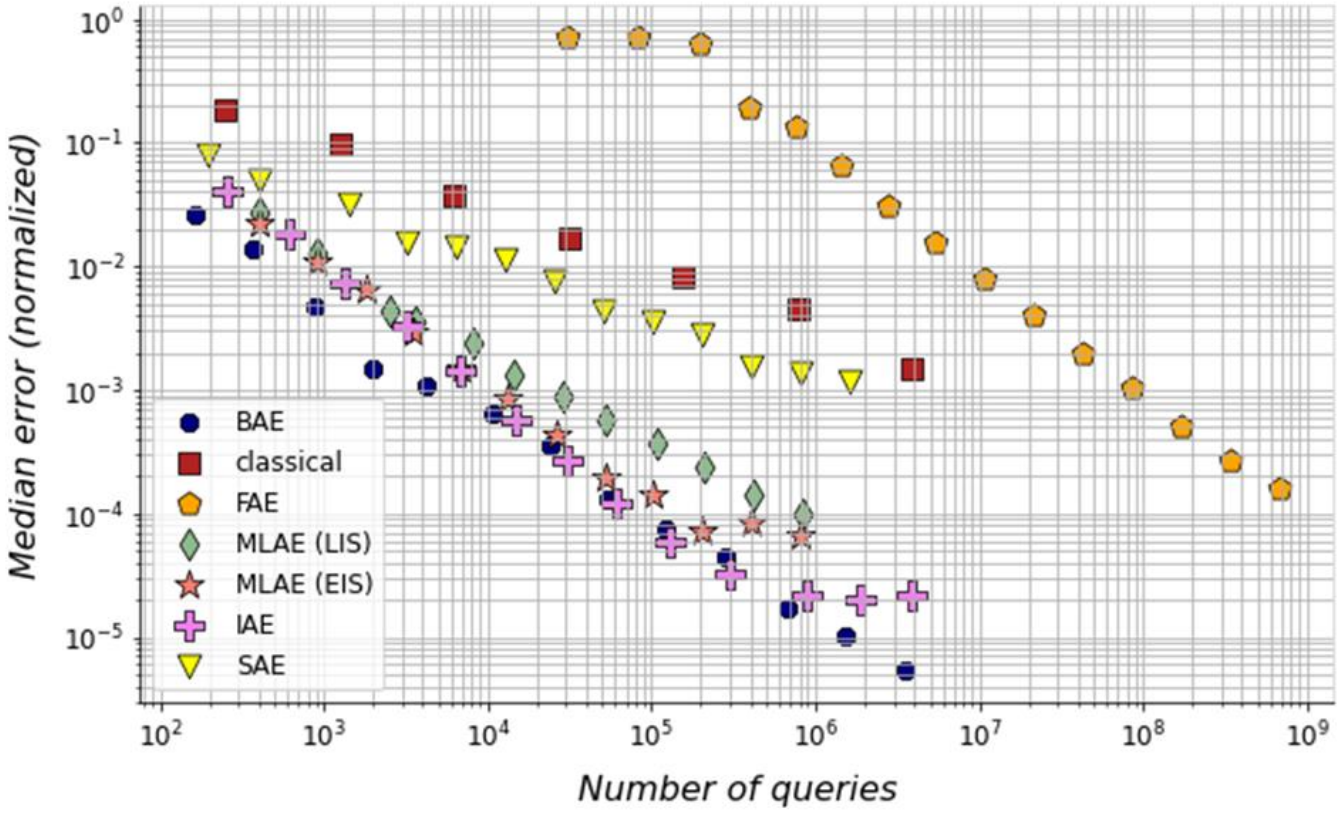
We combine general, scalable and adaptive-compatible **statistical methods** with **problem-tailored heuristics** to keep the problem tractable while retaining the quantum advantage.



Comparative Results
- ideal case



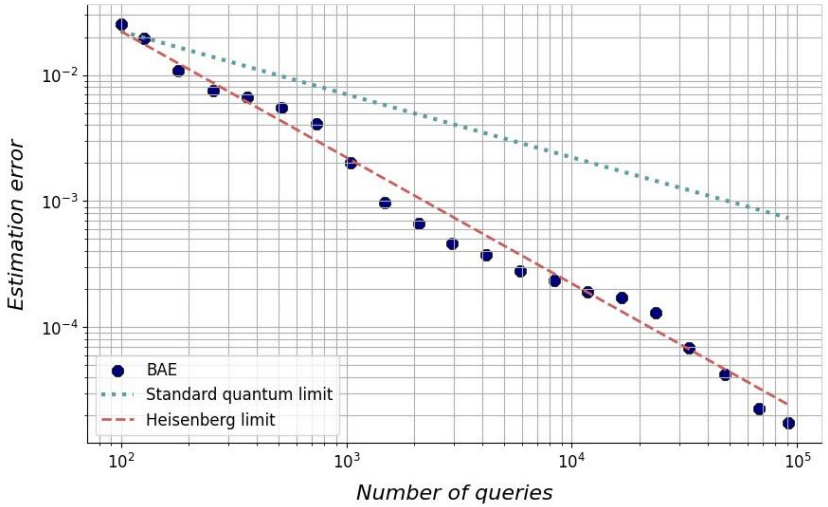
Comparative Results
- noisy case



Key Takeaways

Our Bayesian amplitude estimation algorithm:

- Achieves the Heisenberg limit;
- Is highly adaptable, parallelizable and scalable;
- Is capable of dealing with noise.



Thank you for your attention!

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