Full counting statistics and cumulant evolution in infinite temperature quantum spin chains

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BME PHYSICS

MŰEGYETEM 1782

Quantum Information National Laboratory HUNGARY

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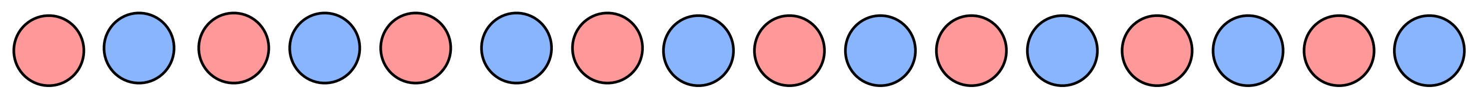
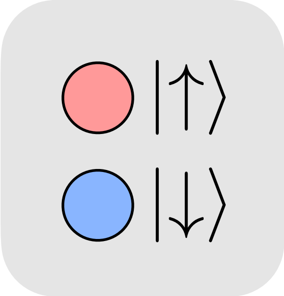
Spin transfer in S-1/2 anisotropic Heisenberg (XXZ) chain

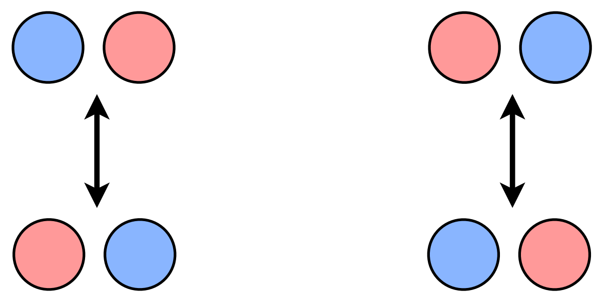




$$\mathcal{H}_{\mathrm{XXZ}} = J \sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$

drives dynamics **correlates spins**

$$\frac{1}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ \right)$$



symmetries

U(1) symmetry: conservation of S^z (charge)

SU(2) symmetry: conservation of S^2 — at Δ=1, isotropic point

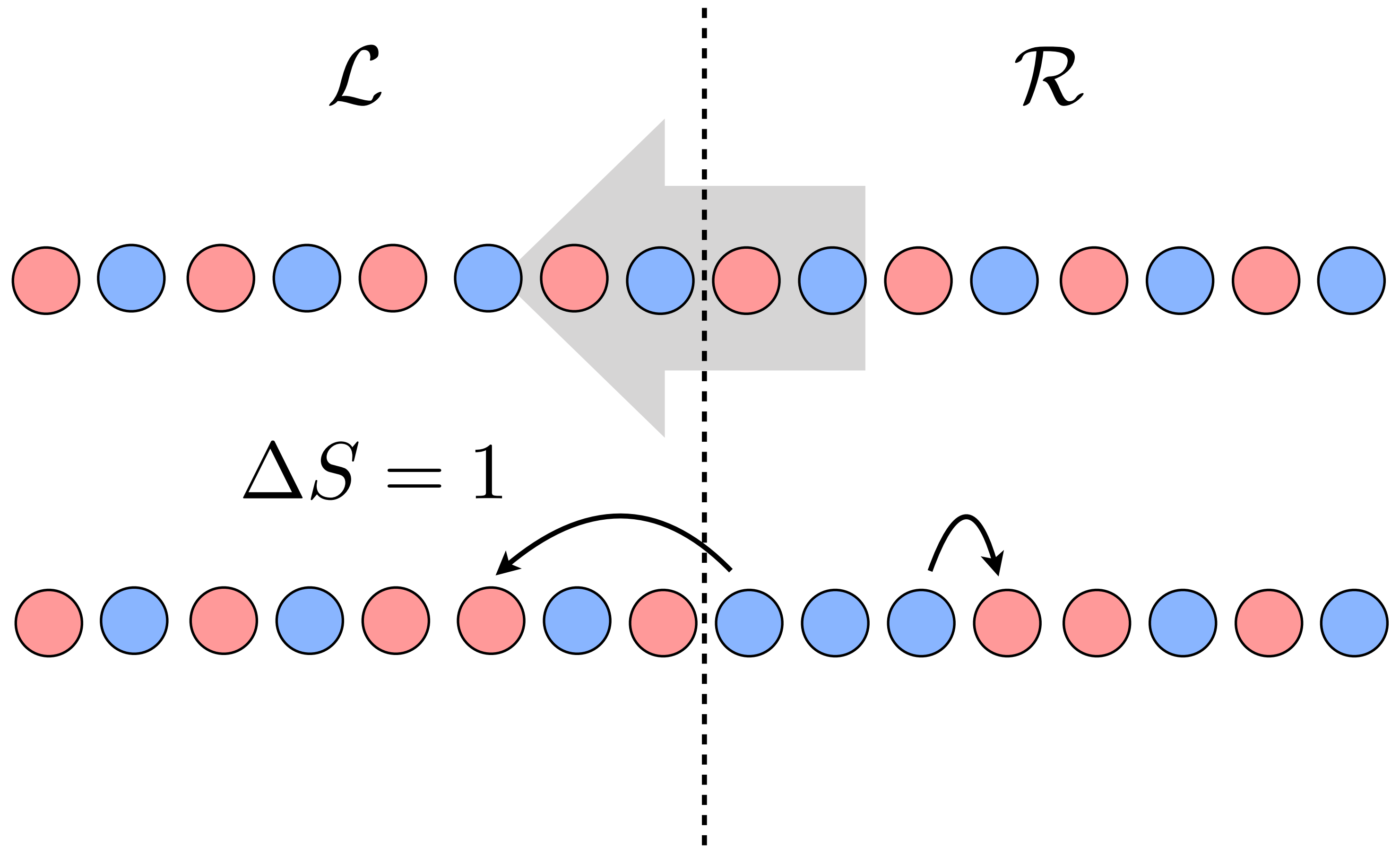
integrability

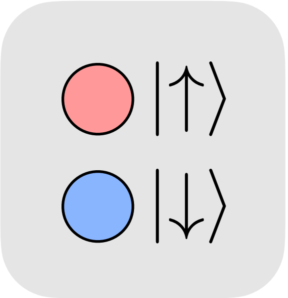
extensive set of conserved quantities: strongly impact dynamics

prototypical model (not exotic)

real-life realization e.g.: KCuF3, SrCuO2, …

Spin transfer in S-1/2 anisotropic Heisenberg (XXZ) chain





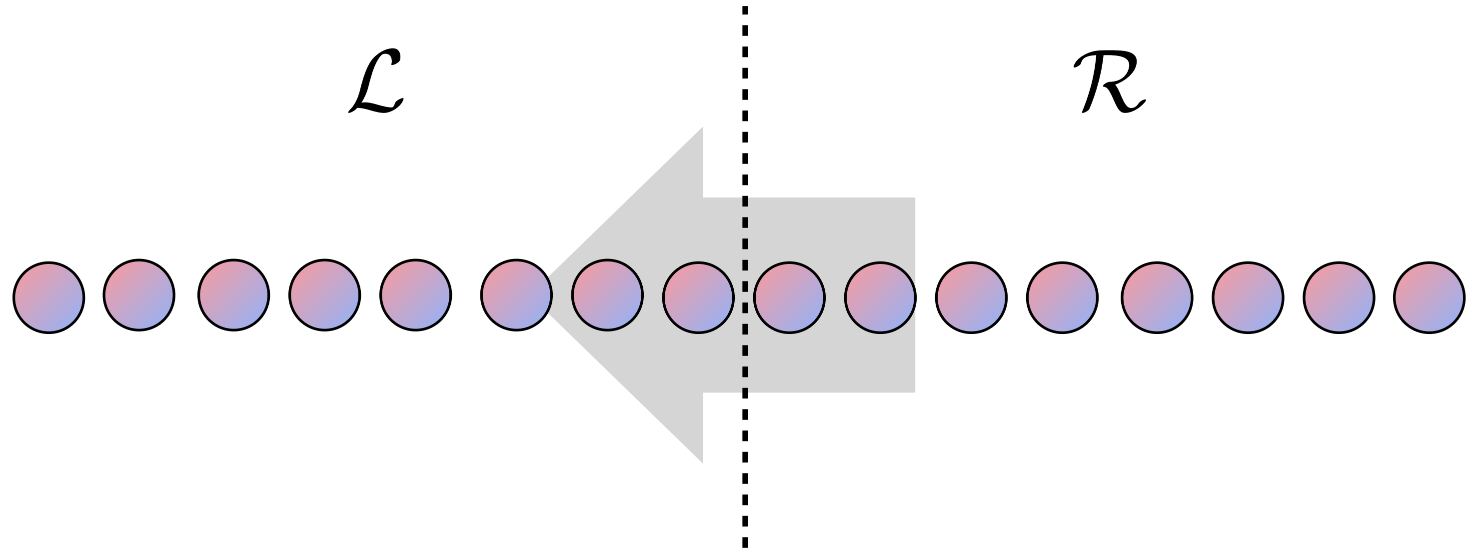
spin transfer across interface

$$\Gamma \longleftrightarrow \Delta S$$

full counting statistics

$$P(\Gamma)$$

probability distribution: characterizes the spin-transfer processes



$$\left.\begin{array}{c} \langle \Delta S \rangle = 0 \\ \langle (\Delta S)^2 \rangle \neq 0 \end{array}\right\}$$

infinite temperature state

$$\rho = \frac{1}{2^L} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{\otimes L}$$

Spin transport

naïve expectation for **conserved quantities**

$$\partial_t Q_n + \nabla_x j_n = 0$$

$$j_n = \underbrace{F_n[\{Q\}]}_{\text{ballistic}} + \sum_m \underbrace{D_{nm}\nabla Q_m}_{\text{diffusion}} + \dots$$

quantum quench protocol

$$\rho = \frac{1}{(1+\mu^2)^L} \bigotimes_{i=1}^{L/2} \begin{pmatrix} 1+\mu & 0 \\ 0 & 1-\mu \end{pmatrix} \bigotimes_{i=L/2+1}^{L} \begin{pmatrix} 1-\mu & 0 \\ 0 & 1+\mu \end{pmatrix}$$

probe **spin correlations** from spin profile

$$\langle S^z(x,t) S^z(0,0) \rangle = -\lim_{\mu \to 0} \frac{1}{\mu} \delta_x S^z$$

Anomalous diffusion

key observation: (numerical evidence) charge across interface

$$Q(t) = \int_0^t d\tau \; j(L/2, \tau) \propto t^{1/z}$$

superdiffusion with **dynamical exponent**:

$$z = 3/2$$

Ljubotina et al., Nat. Comm **8** (2017); PRL **122** (2019)

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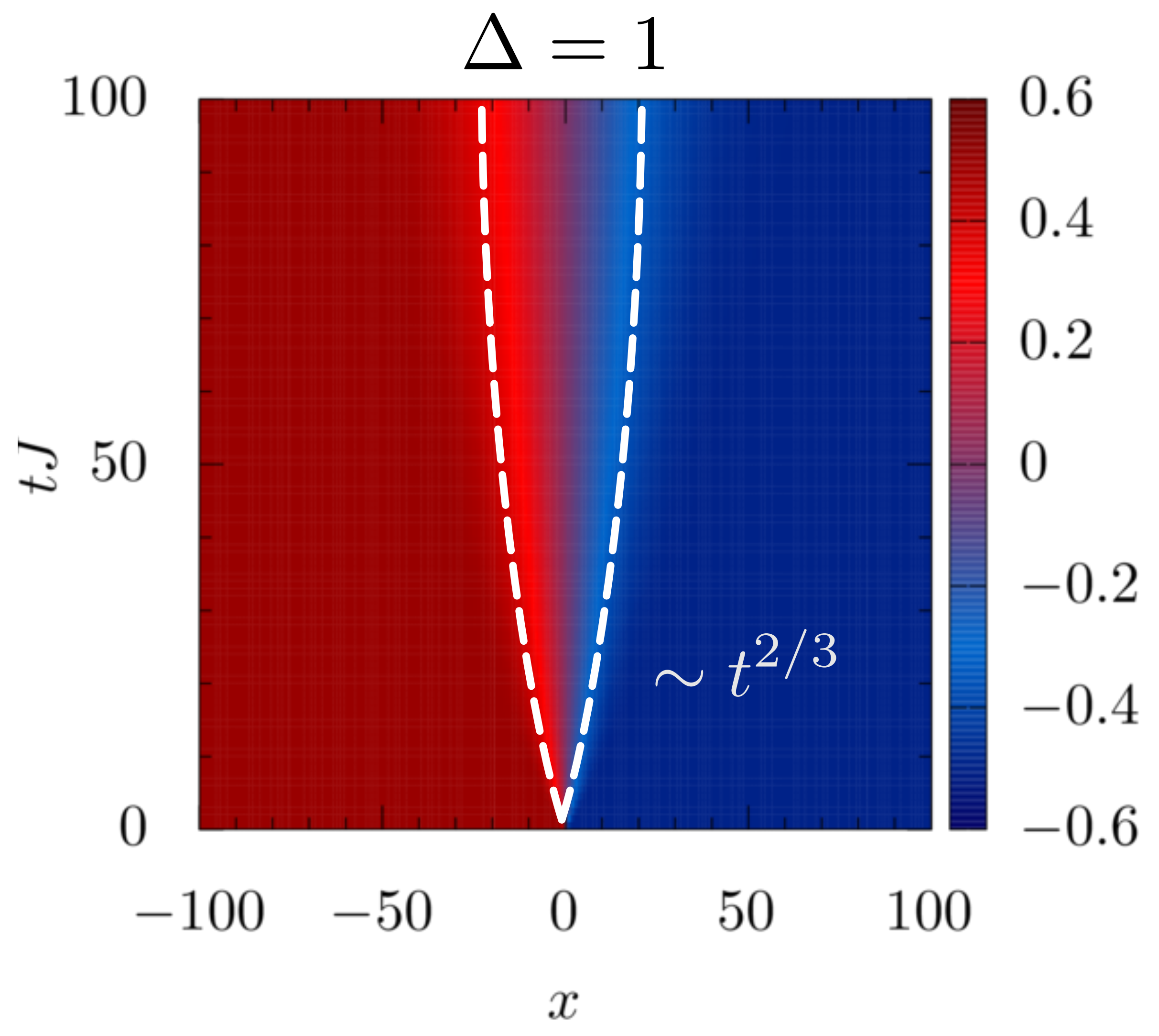
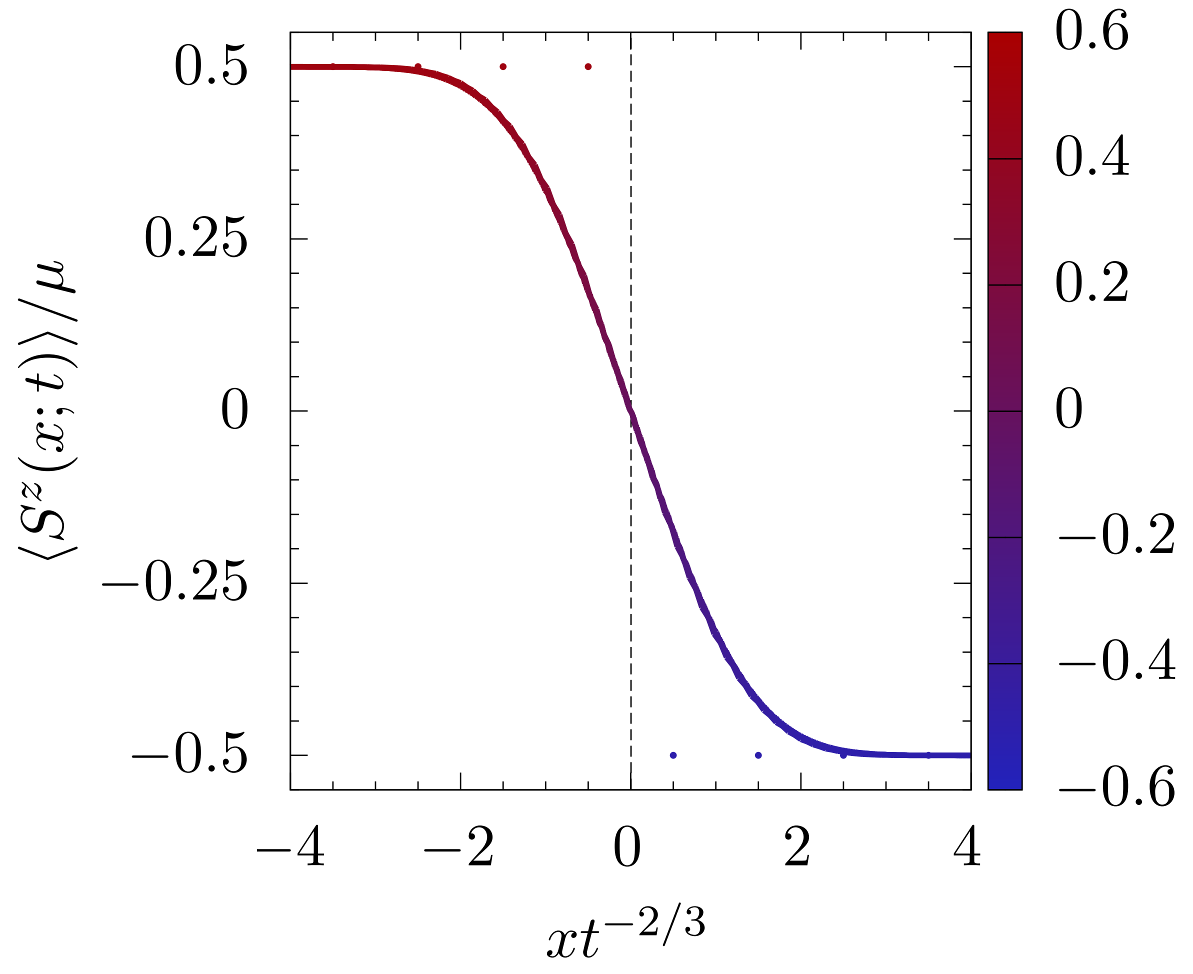
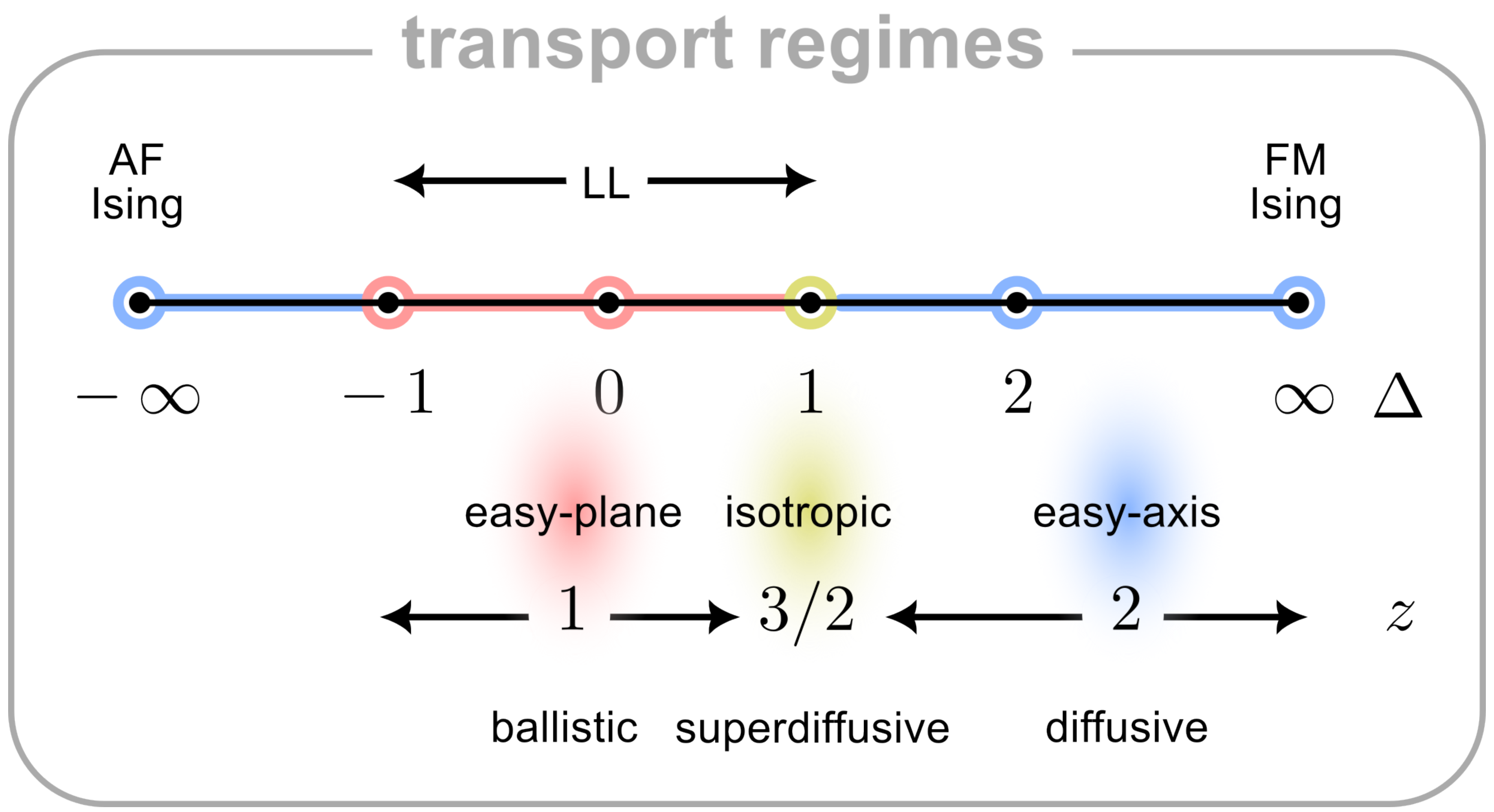
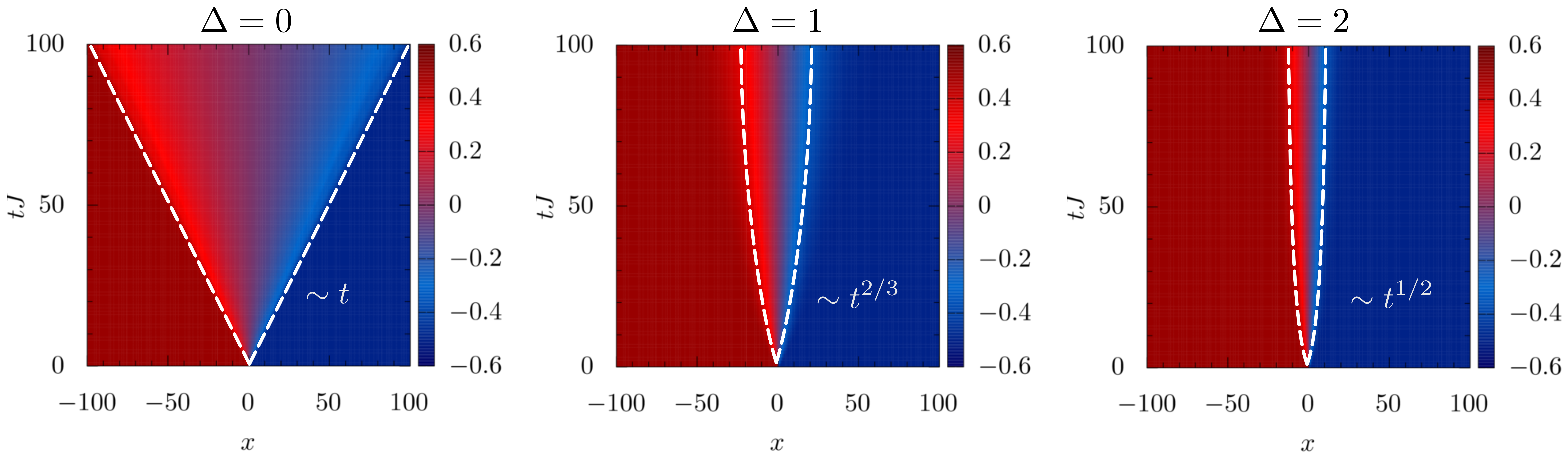




Spin transport regimes: S-1/2 XXZ chain

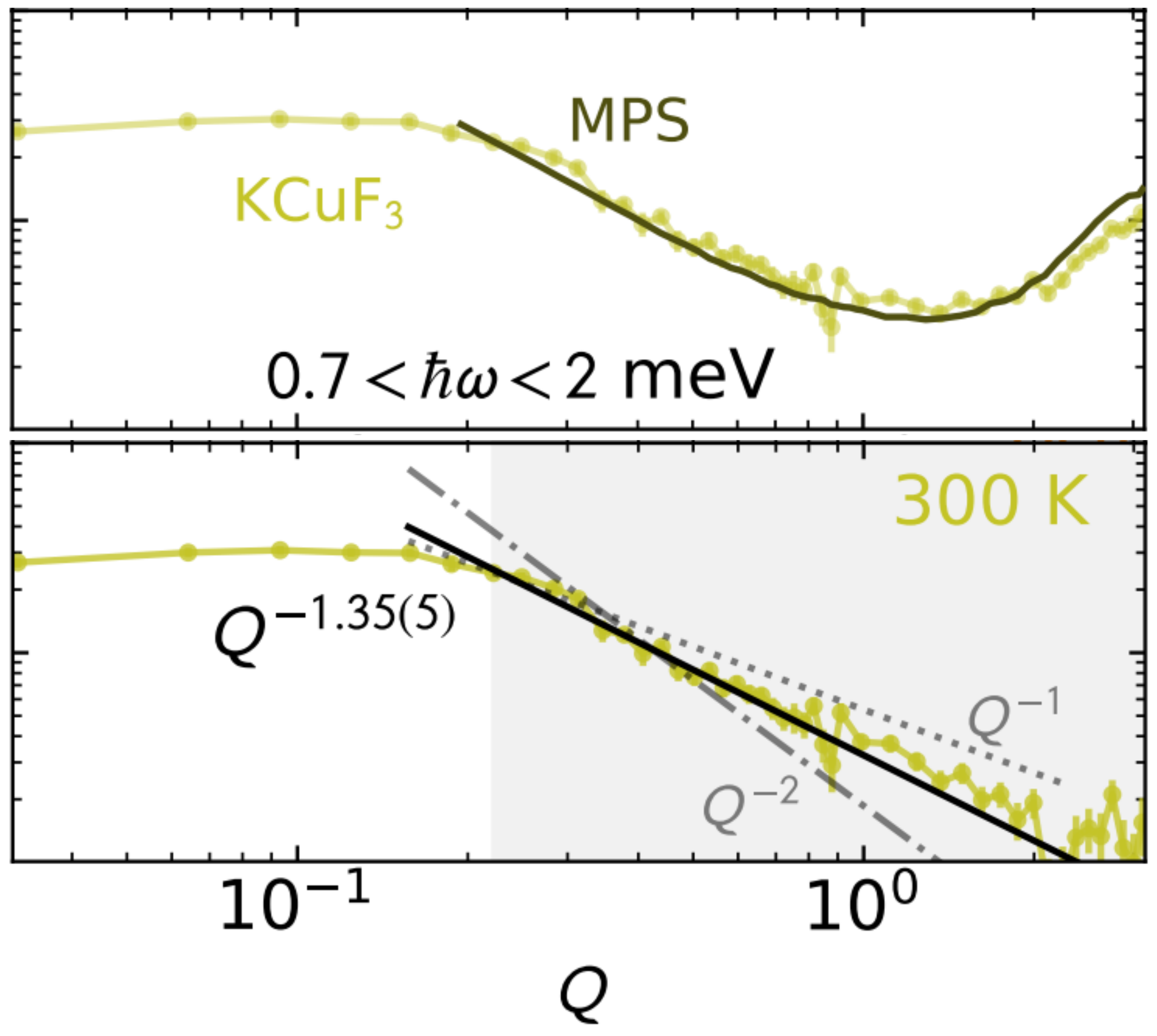




Experimental evidence

neutron scattering

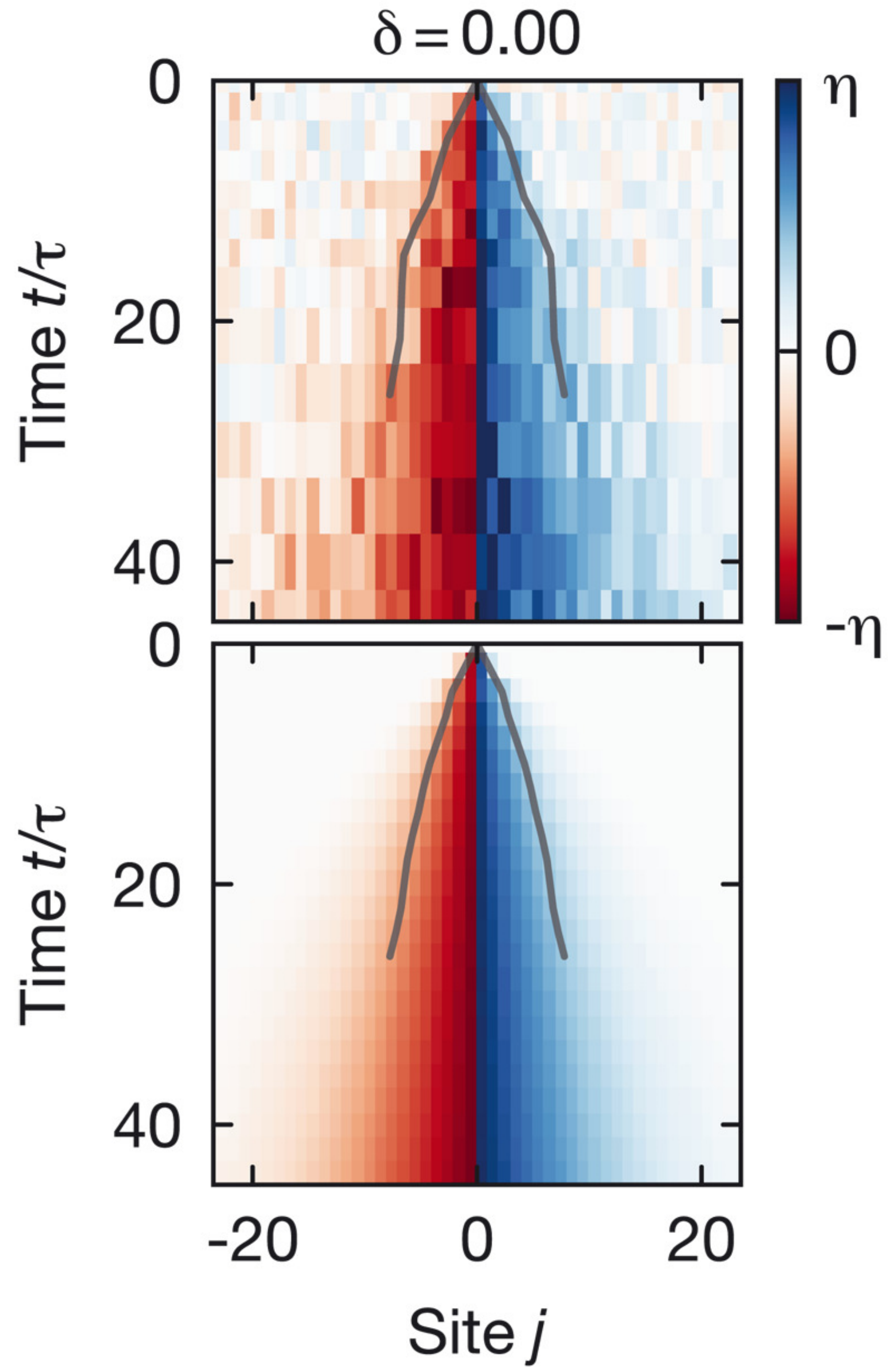
Tennant

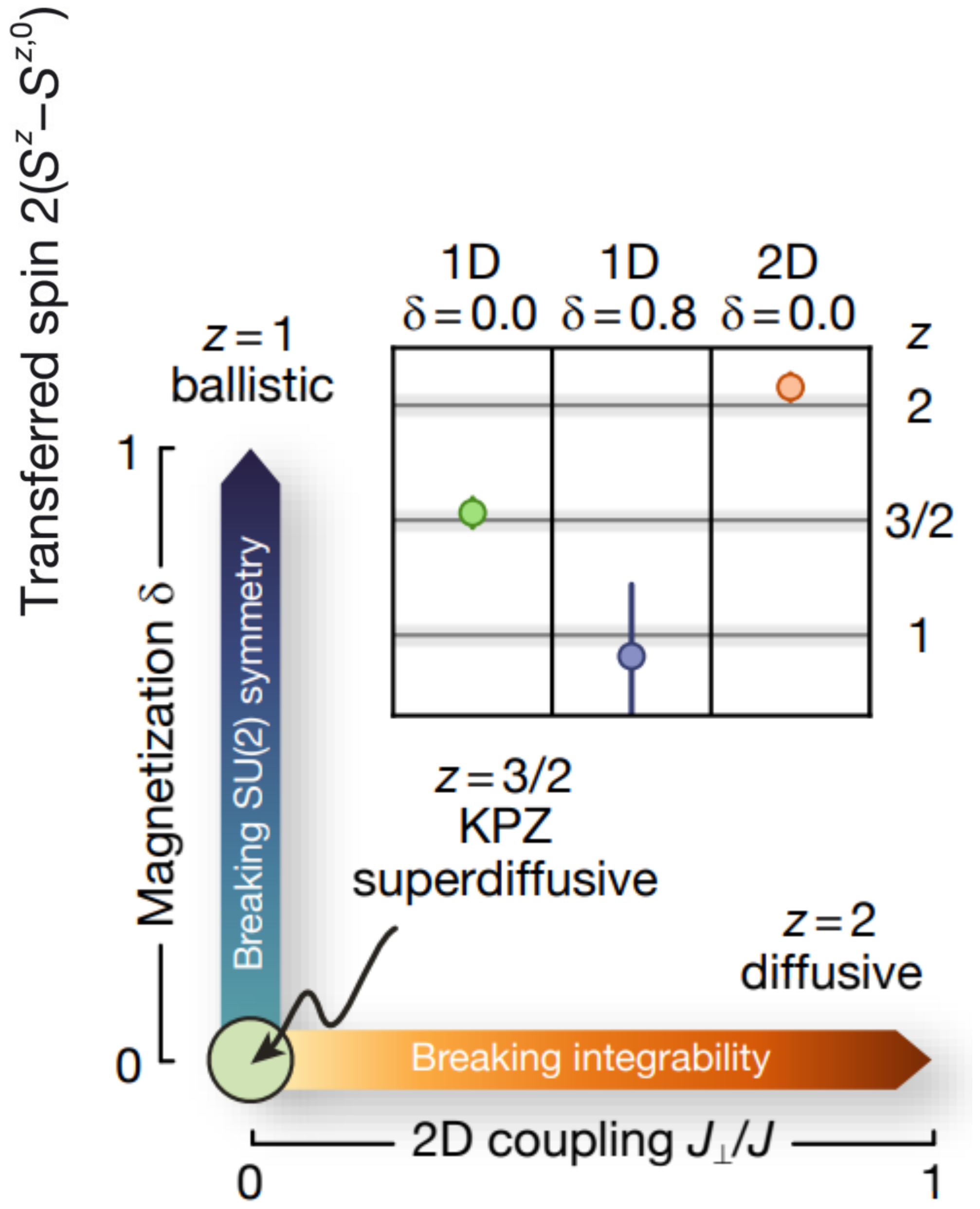


Schiele et al., Nat. Phys **17** (2021)

cold atoms setup

Bloch

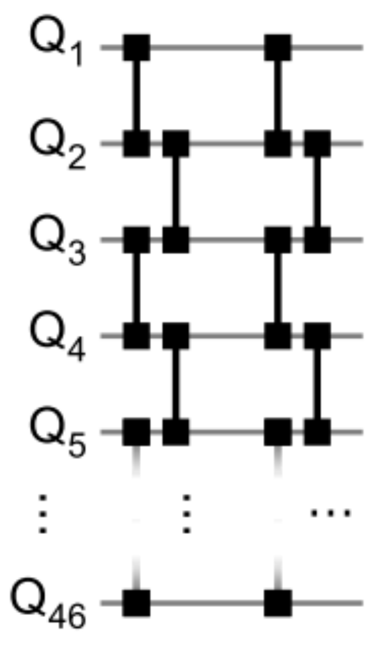


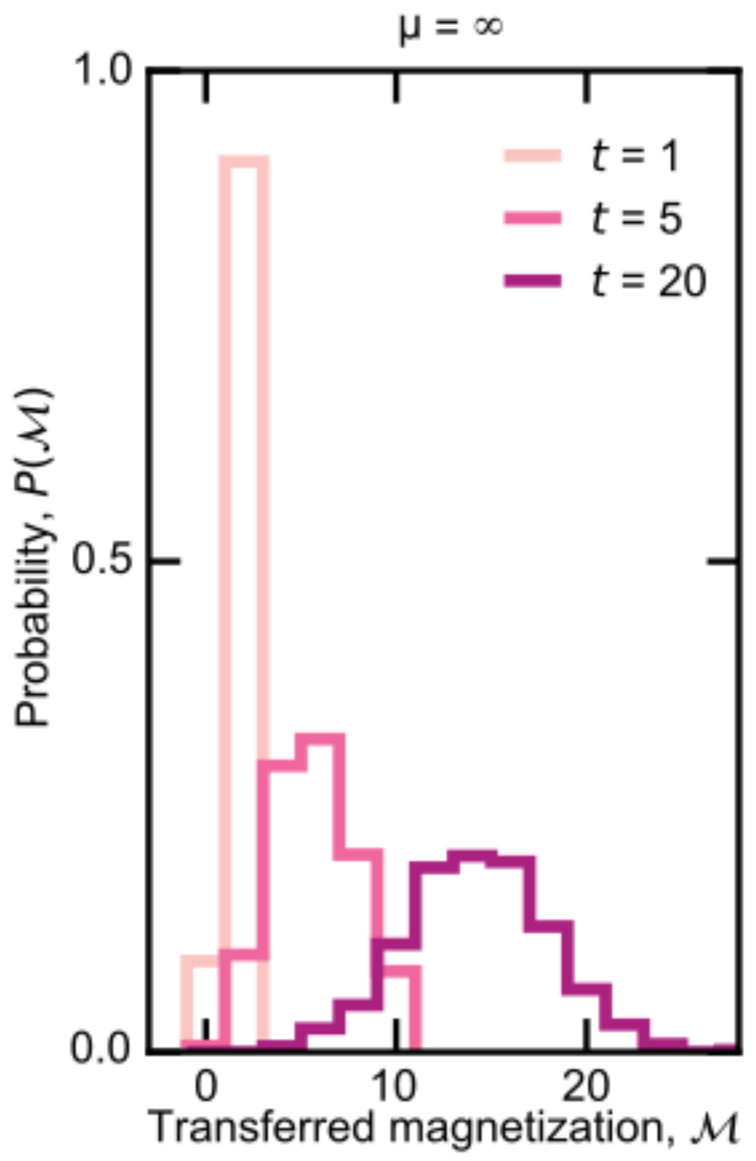


Wei et al., Science **376** (2022)

quantum simulators

Google Quantum AI / Prosen





Rosenberg et al., Science **384** (2024)

Kardar-Parisi-Zhang (KPZ) universality class

stochastic non-linear differential equation

$$\partial_t h = D\partial_x^2 h + \lambda(\partial_x h)^2 + \eta$$

diffusion non-linear δ-correlated noise

describe interface growth of **classical** processes

coffee stains

tumor cell

- burning paper,
- fire spread in a forest
- ice on a windscreen
- polymerization
- traffic
- …

i.i.d. **waiting times**

- linear growth speed
- lack of spatial correlations

Gaussian fluctuations

$$h(x,t) \sim t + \gamma t^{1/2}$$

i.i.d. **waiting times**

- linear growth speed
- height **correlated transversally** over long distances

Tracy-Widom fluctuations

$$h(x,t) \sim \gamma_0 t + \gamma_{1/3} t^{1/3} F_1$$

time : space : fluctuations

scaling like

3 : 2 : 1

Corwin NAMS **63** (2016)

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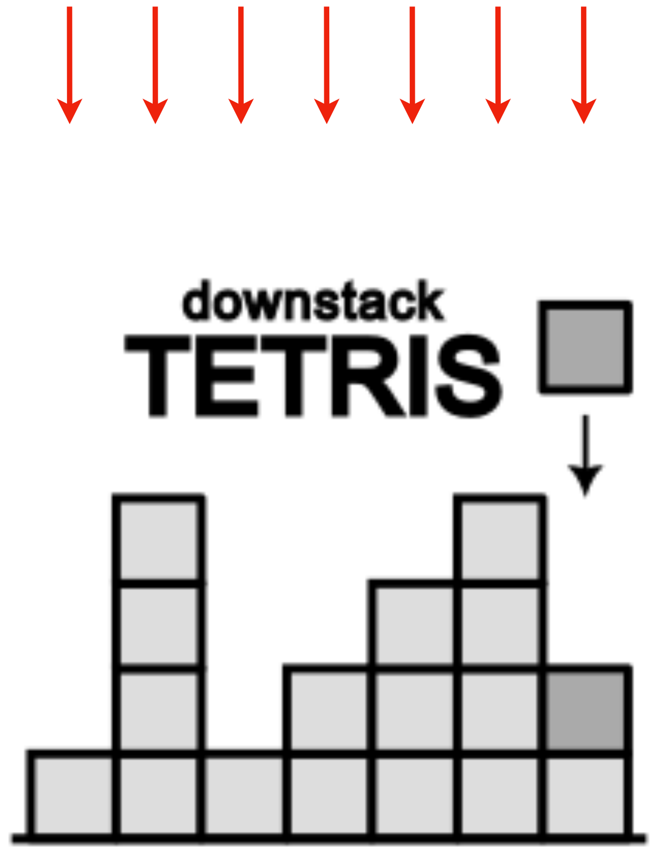

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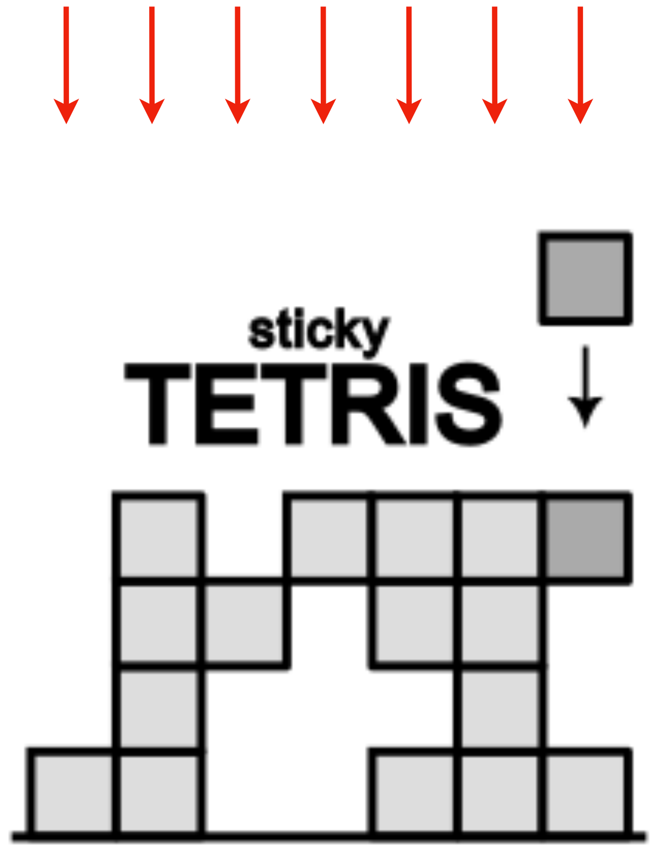

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Corwin NAMS **63** (2016)

Why is anomalous transport in quantum spin chains surprising?

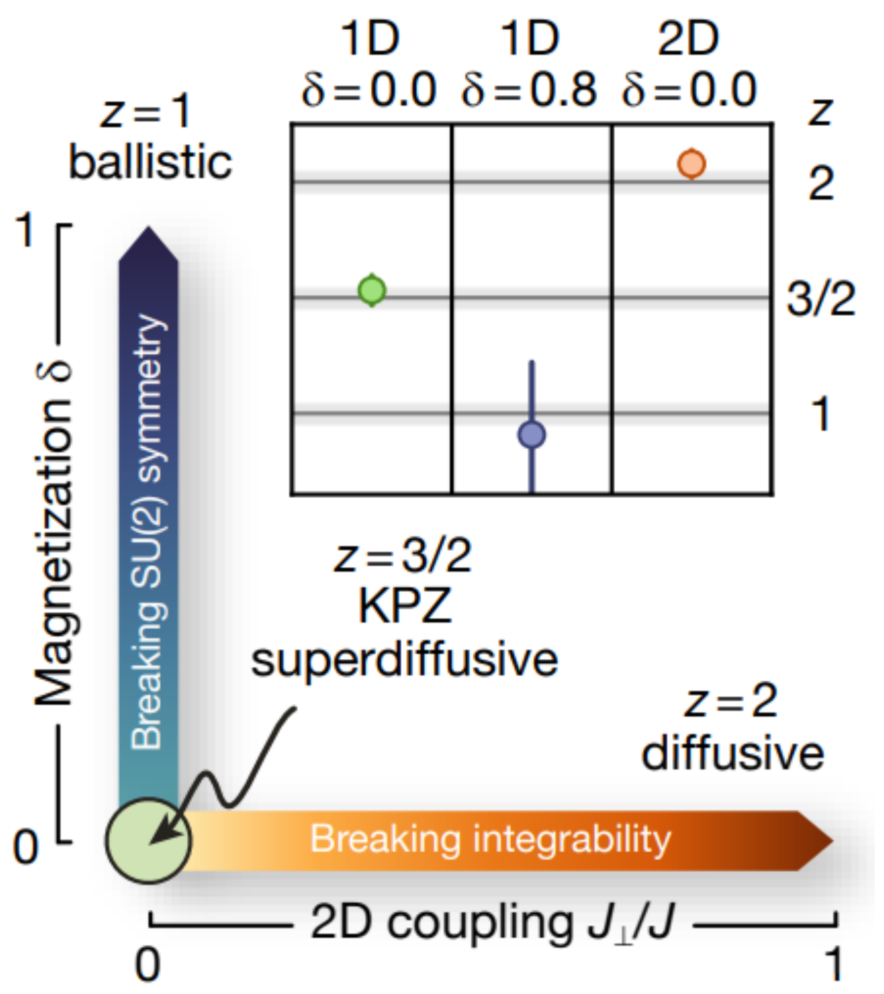
concept of **universality** in processes **far from equilibrium**

KPZ processes: **preferred direction in time**

classical systems ⟷ **quantum systems**

robust (universal) feature

“fragile” i.e., it depends on microscopic details:



integrability

non-abelian symmetry — e.g. SU(2)

Wei et al., Science **376** (2022)

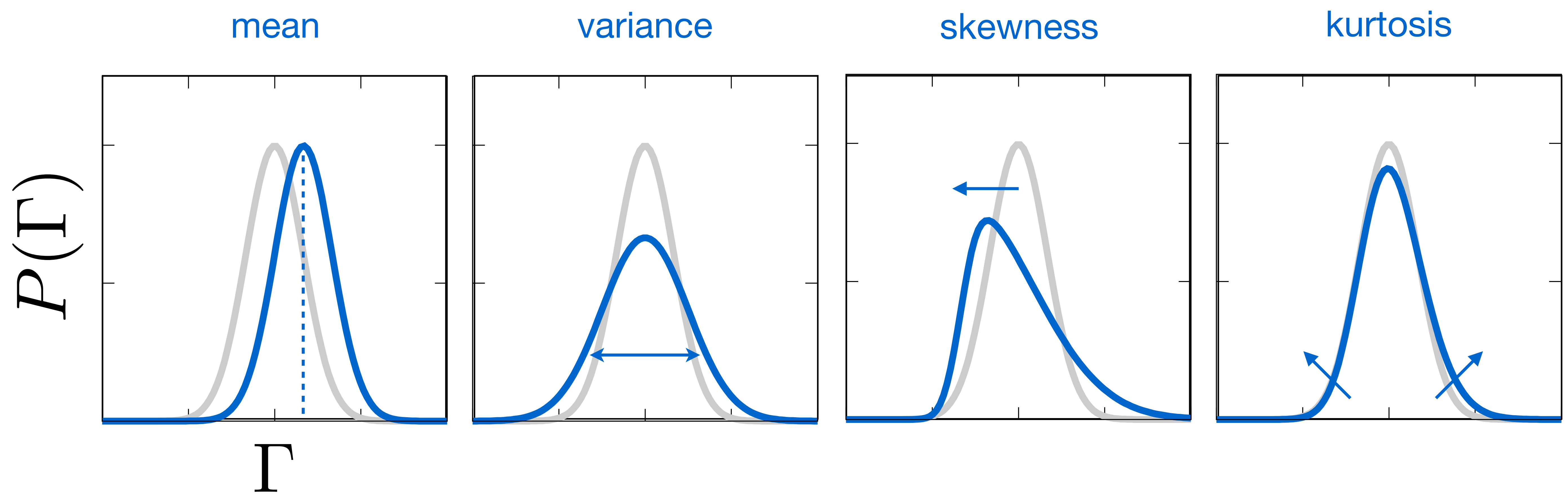
Higher-order correlation functions

full counting statistics

$$P(\Gamma) = \frac{1}{(2S+1)^L} \sum_{s,s'} \delta\left(\Sigma_{s'} - \Sigma_s - \Gamma\right) \left|\langle s'|e^{-\imath Ht}|s\rangle\right|^2$$

transferred spin

spin in config. s' and s



Quantum trajectories

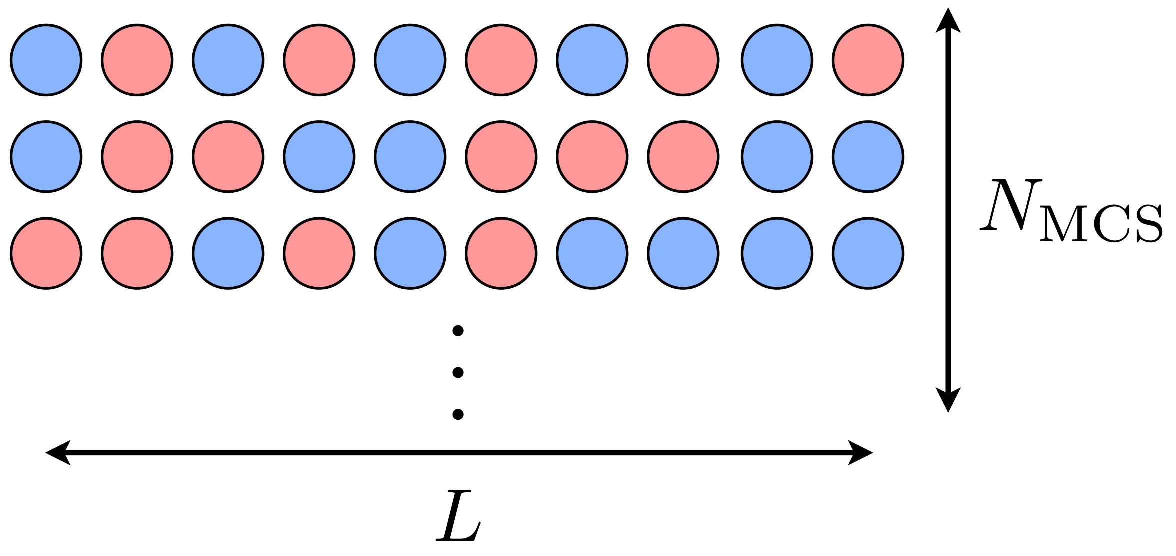
Schmidt decomposition of the time-evolution operator

$$U^t = \sum_\Gamma \sum_\alpha \Lambda_{\Gamma\alpha} O^A_{\Gamma\alpha} \otimes O^B_{\Gamma\alpha} = \sum_\Gamma O_\Gamma,$$

$$\boxed{P_j(\Gamma) = \sum_\alpha |\Lambda_{\Gamma\alpha}|^2}$$

Monte Carlo sampling:

$$|\psi(t)\rangle = \underbrace{U \cdots U}_{t} |\psi(0)\rangle$$



$$\mu_n = \sum_\Gamma \Gamma^n P(\Gamma)$$ (moments)

- access **directly** full counting statistics
- MPS bond dimension grows exponentially — **short timescales**

Generating function

MPO representation of spin on one side of the interface

$$R(\lambda) = e^{-\imath\lambda\Sigma} = \prod_{j<L/2} e^{-\imath\lambda S^z_j}$$

$$\boxed{G(\lambda, t) = \frac{1}{(2S+1)^L} \langle R(\lambda, t) R^\dagger(\lambda^*, 0)\rangle}$$

evaluate cumulants:

$$\kappa_n(t) = \frac{\partial^n}{\partial\lambda^n} \underbrace{\log G(\lambda, t)}_{F(\lambda,t)}\Big|_{\lambda=0}$$

truncated Taylor expansion

$$F_\phi(\lambda, t) = -\sum_{k=1}^{\infty} \frac{1}{2k!} \lambda^{2k} e^{\imath 2k\phi} \kappa_{2k}(t)$$

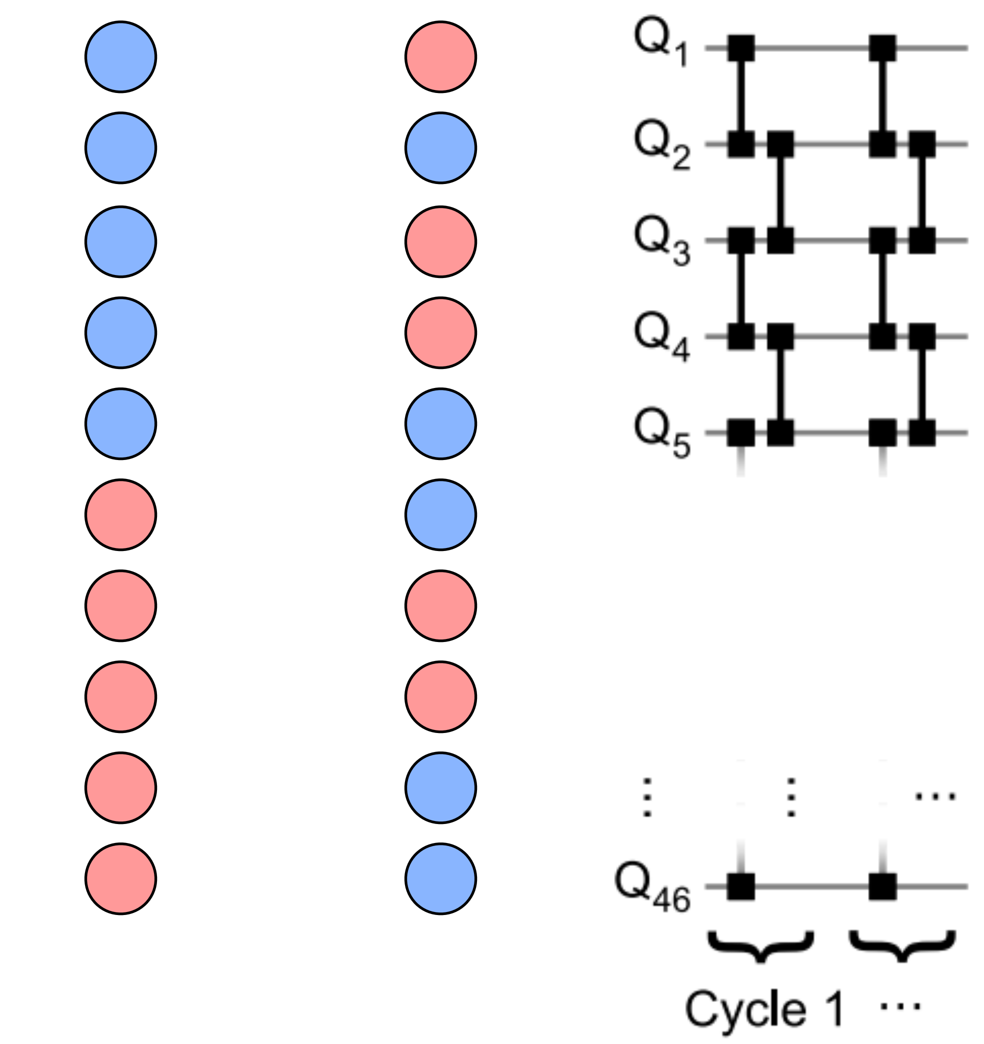
- MPO bond dimension grows slowly — **unprecedentedly-long timescales**
- access full counting statistics **indirectly** through moments/cumulants

Valli et al., in preparation

Google experiment

Sycamore is a **transmon** superconducting quantum processor

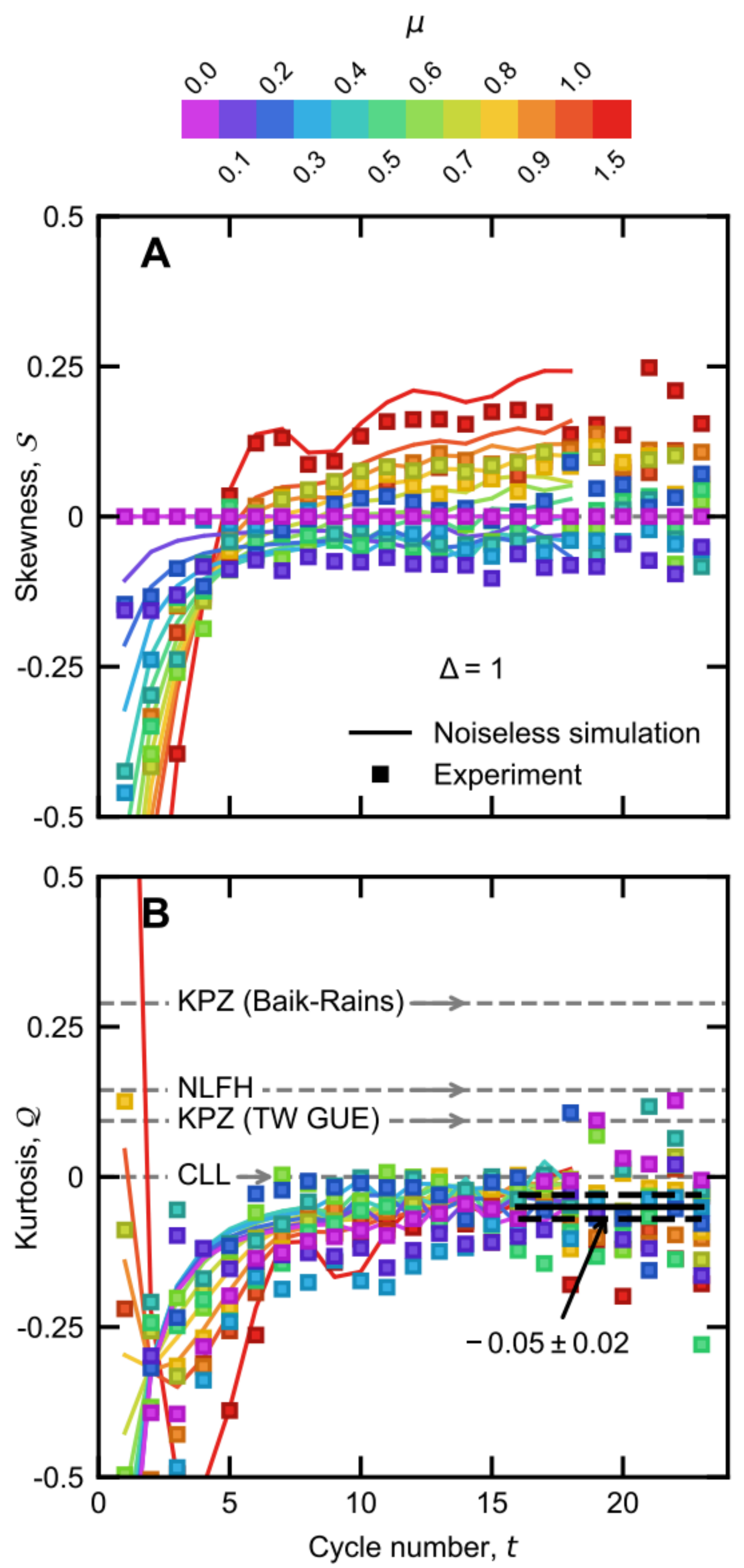
$\mu = \infty \qquad \mu = 0$



$$U = \prod_{j \in \text{even}} \text{fSim}_j(\theta,\phi) \prod_{j \in \text{odd}} \text{fSim}_j(\theta,\phi)$$

$$\text{fSim}(\theta,\phi) = \begin{pmatrix} e^{-\imath\phi/2} & 0 & 0 & 0 \\ 0 & \cos(\theta) & \imath\sin(\theta) & 0 \\ 0 & \imath\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & e^{-\imath\phi/2} \end{pmatrix}$$

higher-order correlation functions incompatible with KPZ



KPZ universality expected for $\mu \to 0$

$\mu = 0$ zero **skewness**
(symmetric distribution)

$\mu = 0$ weakly negative **kurtosis**

Rosenberg et al., Science **384** (2024)

Valli et al., in preparation

Transport regimes: integrable S-1/2 XXZ chain

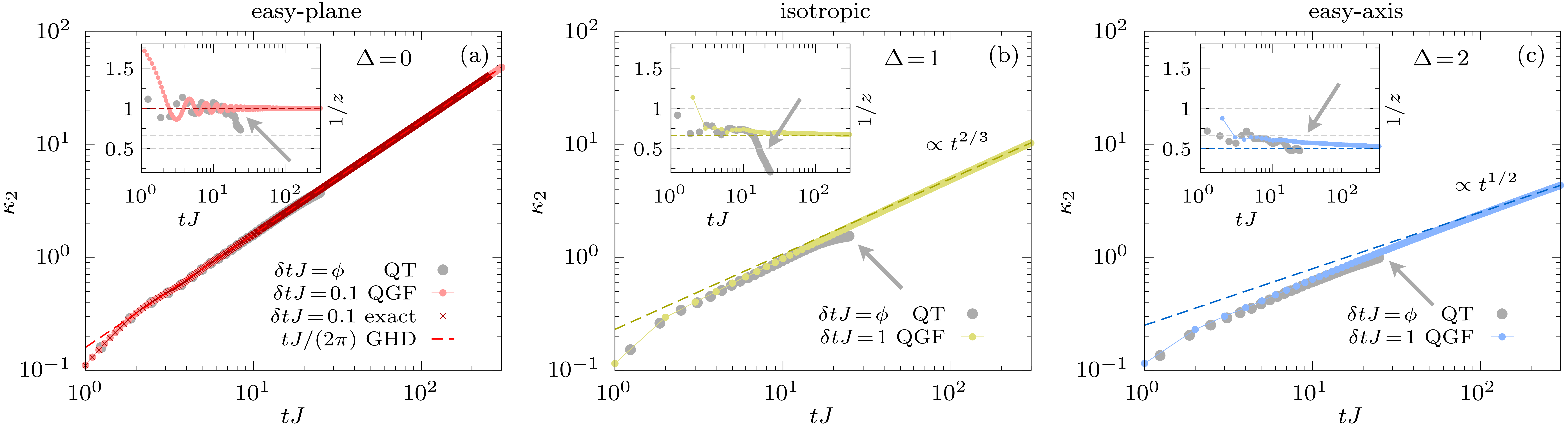
2. cumulant (variance)

$$\left\langle \left(\Gamma - \langle\Gamma\rangle\right)^2 \right\rangle = \kappa_2(t) \sim t^{1/z}$$

$$z = \begin{cases} 1 & \Delta < 1 \text{ (easy-plane)} \\ 3/2 & \Delta = 1 \text{ (isotropic)} \\ 2 & \Delta > 1 \text{ (easy-axis)} \end{cases}$$

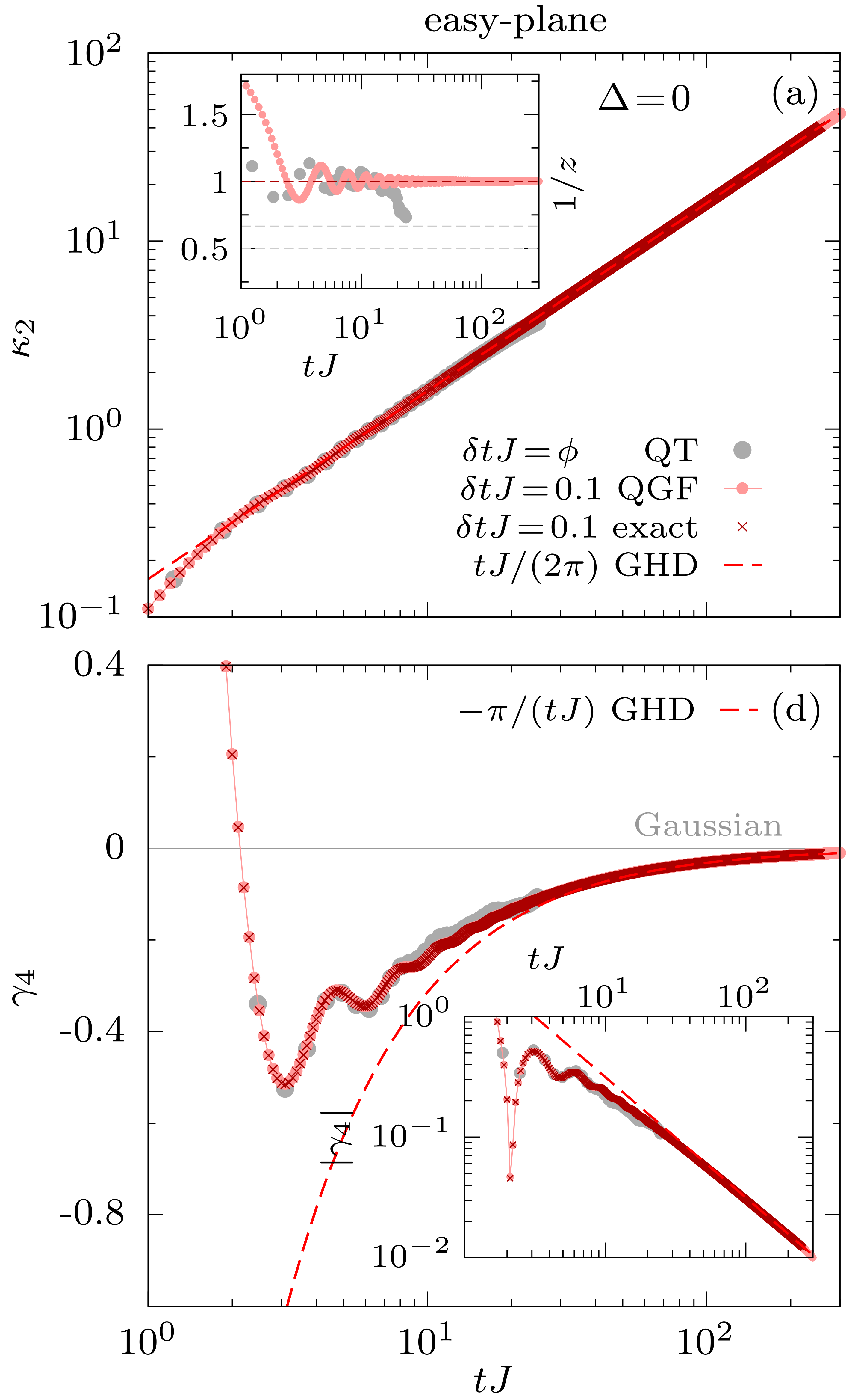
dynamic exponent

$$z^{-1} = \frac{d}{d\log t} \log \kappa_2(t)$$



Valli et al., in preparation

easy-plane — XX limit



cumulants

$\kappa_2 \sim t$

$\kappa_4 \sim t$

standardized moments

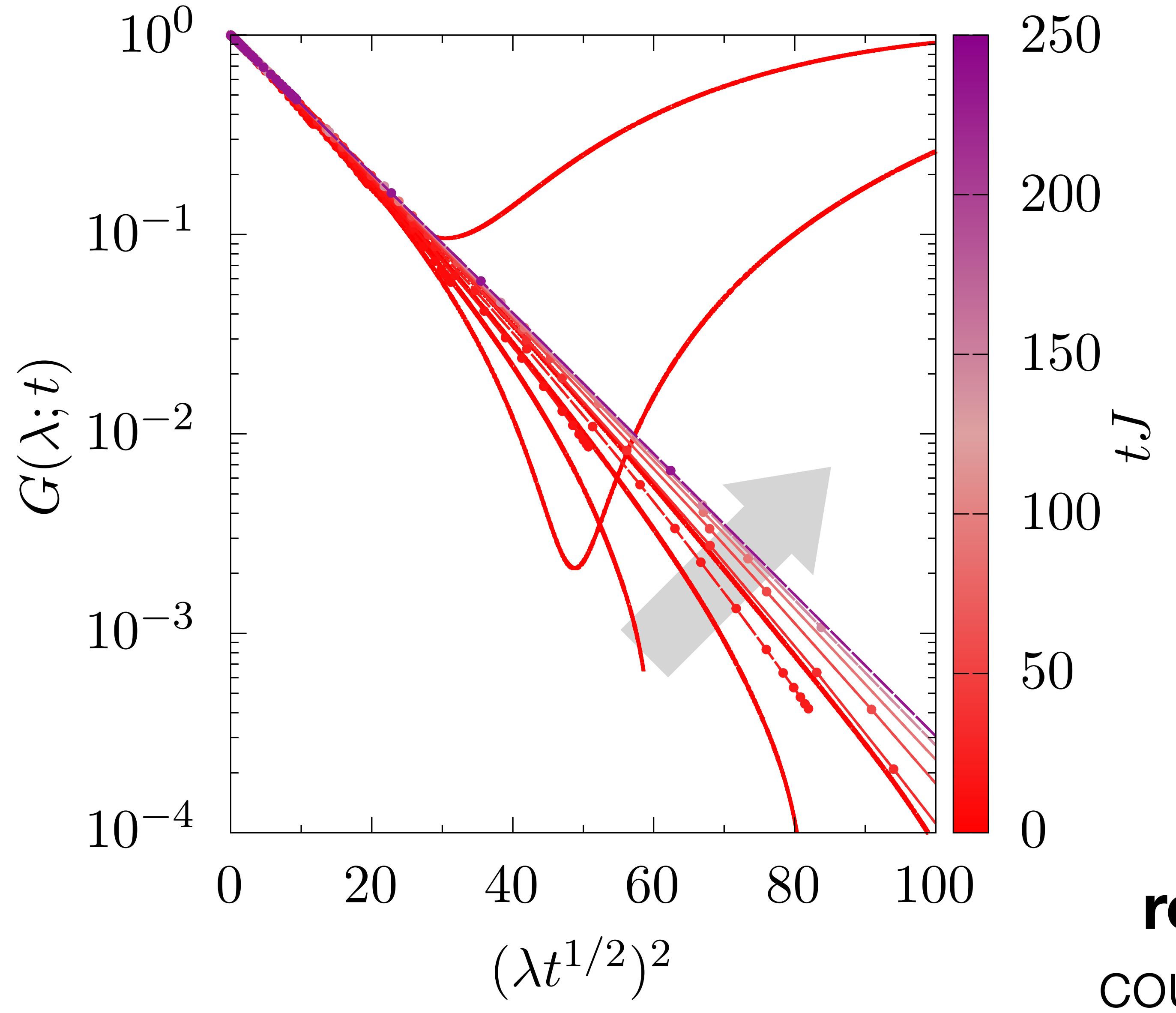
$\gamma_4 = \kappa_4/\kappa_2^2 \sim t^{-1}$

exact solution / GHD

$\kappa_{2n+1} = 0$	$\gamma_{2n+1} = 0$	**ODD**
$\kappa_{2n} \sim t$	$\gamma_{2n} \sim t^{1-n}$	**EVEN**

Del Vecchio[2] & Doyon, J. Stat. Mech. (2022)

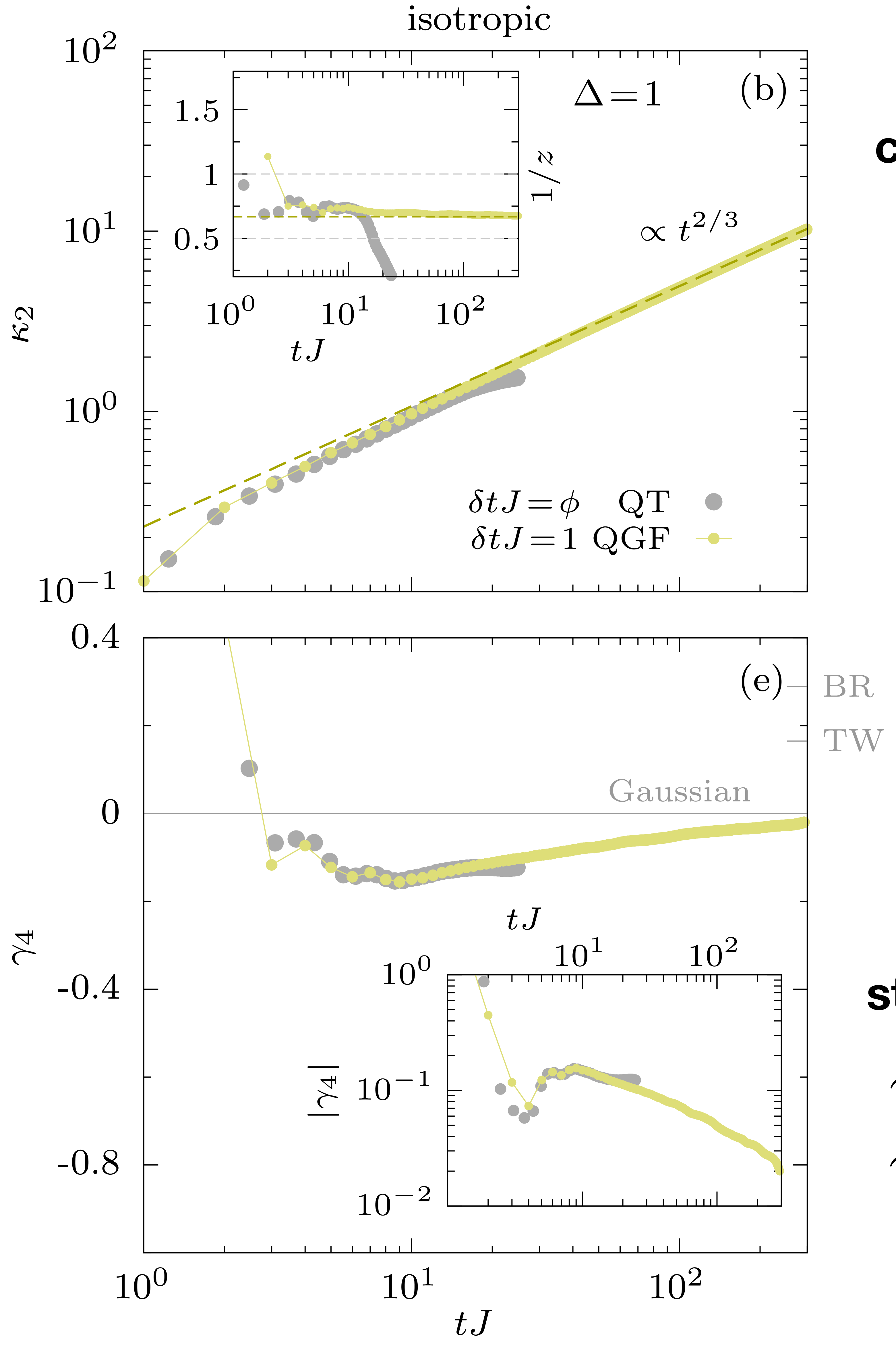
converges to **Gaussian** distribution



rescaling counting field

Valli et al., in preparation

SU(2) isotropic point



cumulants

$$\kappa_2 \sim t^{2/3}$$

$$\kappa_{2n+1} = 0$$

standardized moments

$$\gamma_4 \to 0? \quad \text{OR} \quad \to \text{const.?}$$

$$\gamma_{2n+1} = 0$$

Valli et al., in preparation

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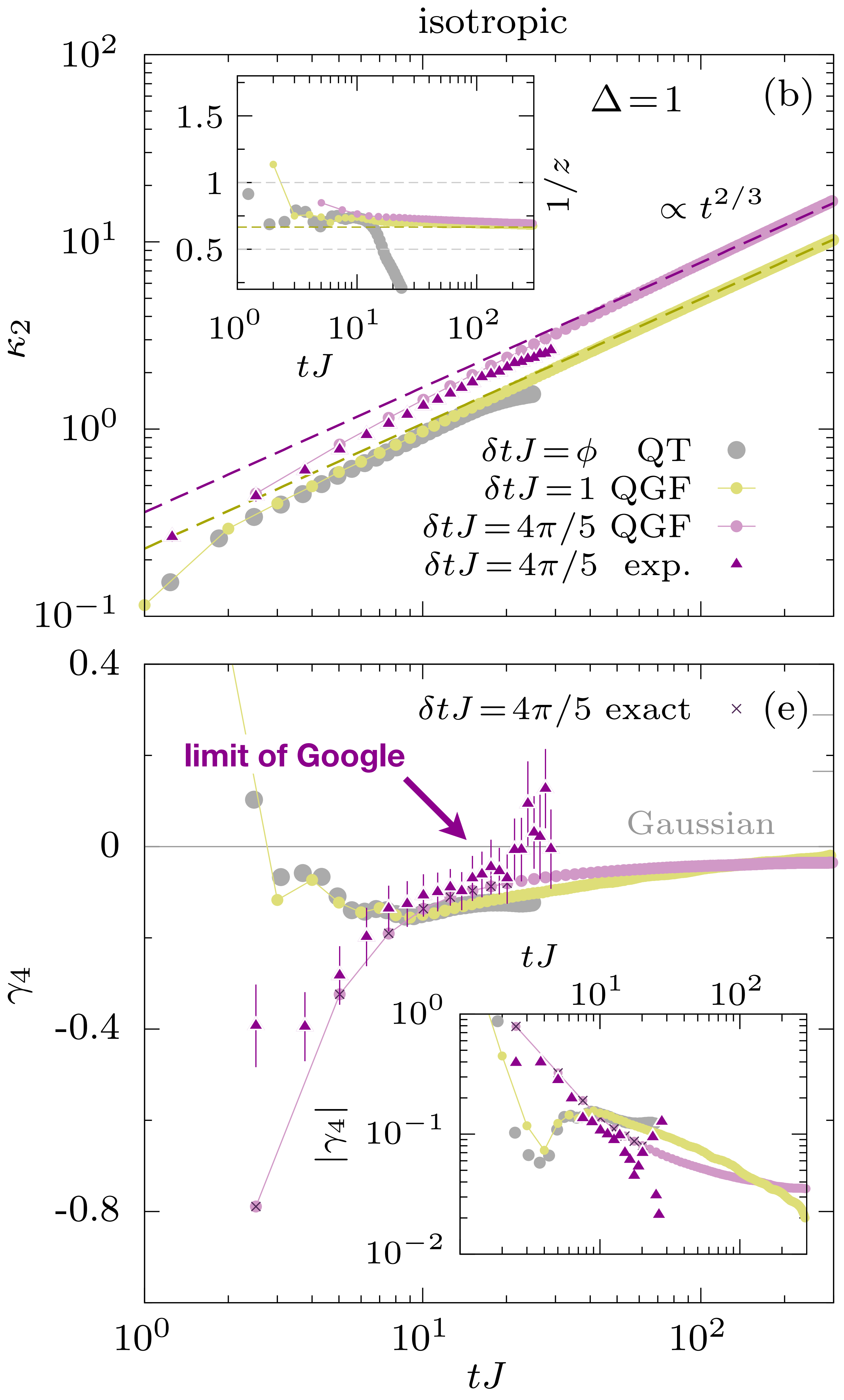

cumulants

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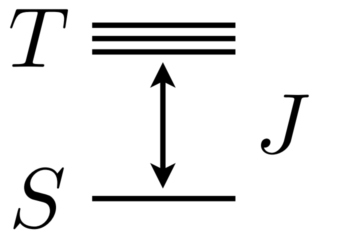
weakly negative **kurtosis**?

standardized moments

$$\gamma_4 \to 0? \quad \text{OR} \quad \to \text{const.}?$$
$$\gamma_{2n+1} = 0$$

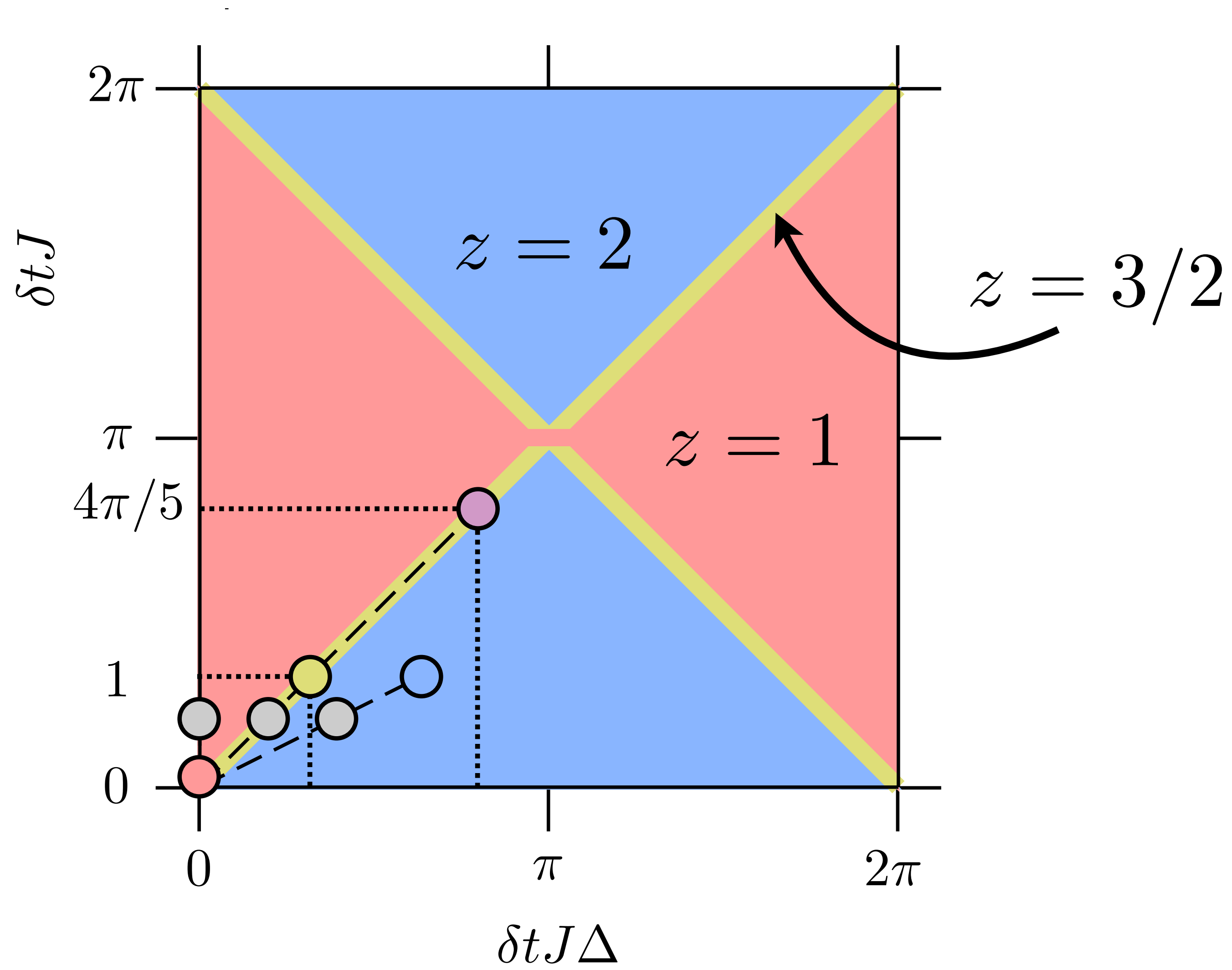
Floquet time evolution

$$\mathrm{fSim}(\theta,\phi) = \begin{pmatrix} e^{-\imath\phi/2} & 0 & 0 & 0 \\ 0 & \cos(\theta) & \imath\sin(\theta) & 0 \\ 0 & \imath\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & e^{-\imath\phi/2} \end{pmatrix}$$



$$\delta tJ = \phi$$

Floquet dynamical exponent



Valli et al., in preparation

Take-home

XXZ chain: **superdiffusion** with **KPZ-like dynamical exponent**:

- integrability
- non-abelian symmetry

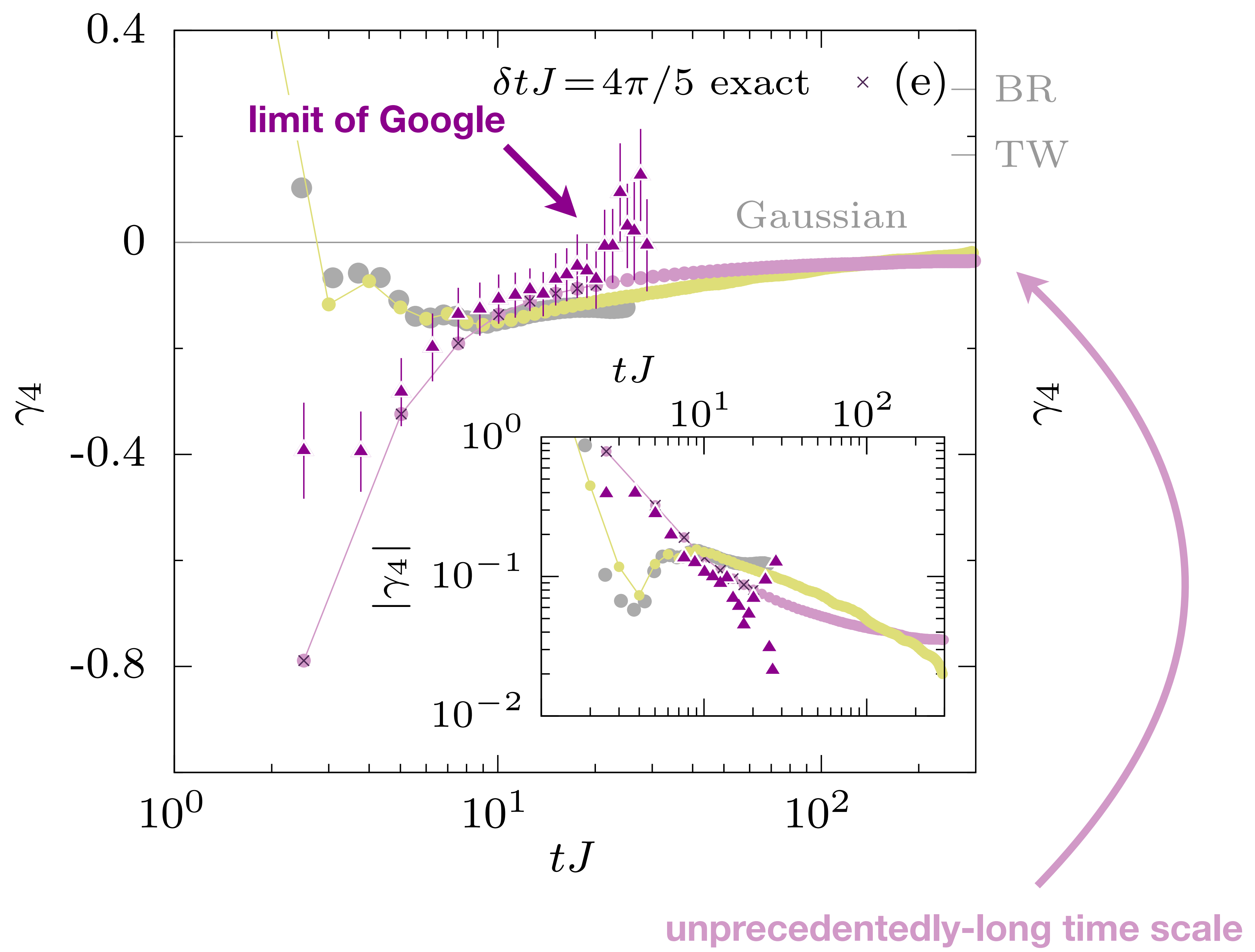
nature of **fluctuations unclear**

NEW MPO METHOD

full counting statistics through **cumulants**

skewness and **kurtosis** seem **incompatible with KPZ**

comparison vs. Google experiment

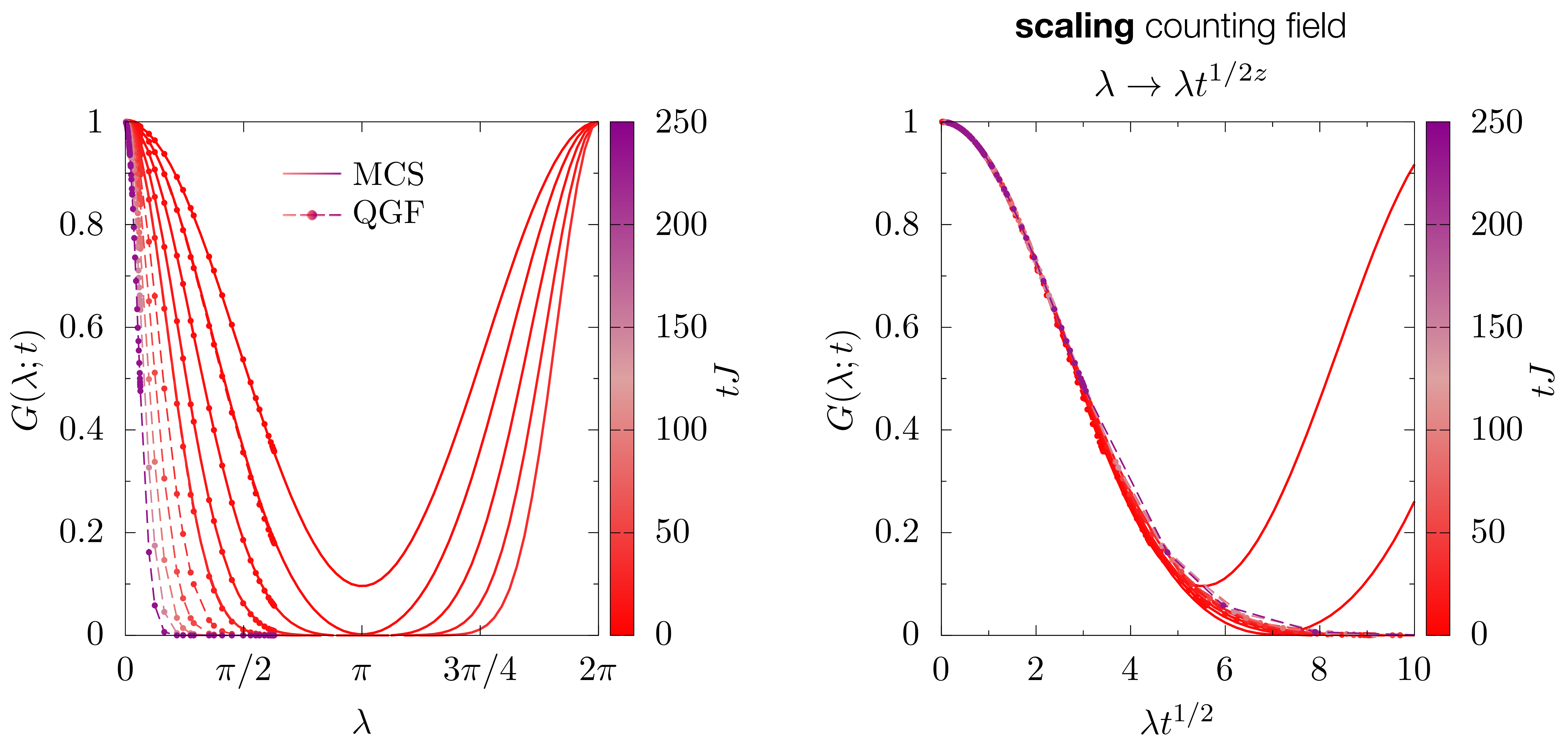


Thank you for your attention!

Backup

Integrable quantum spin chain $S = 1/2$

generating function — XX limit



QGF: low bond dimension sufficient whereas **MCS breaks down** at $t_{\max} = t(M, \delta t)$

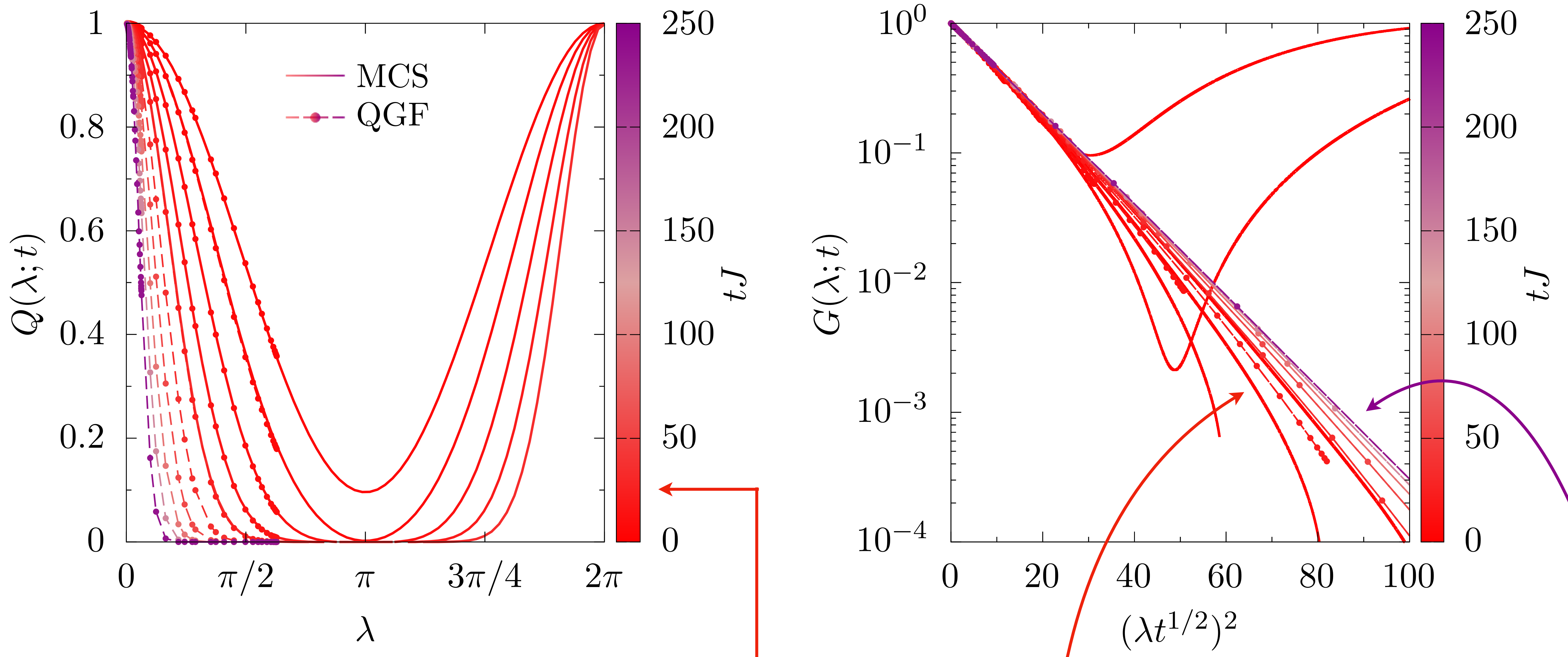
Valli et al., preprint (soon)

Integrable quantum spin chain $S = 1/2$

generating function — XX limit

scaling counting field

$\lambda \to \lambda t^{1/2z}$



quantum trajectories simulations
degrade at longer timescales

$t_{\max} J \approx 25$

converges to **Gaussian** distribution

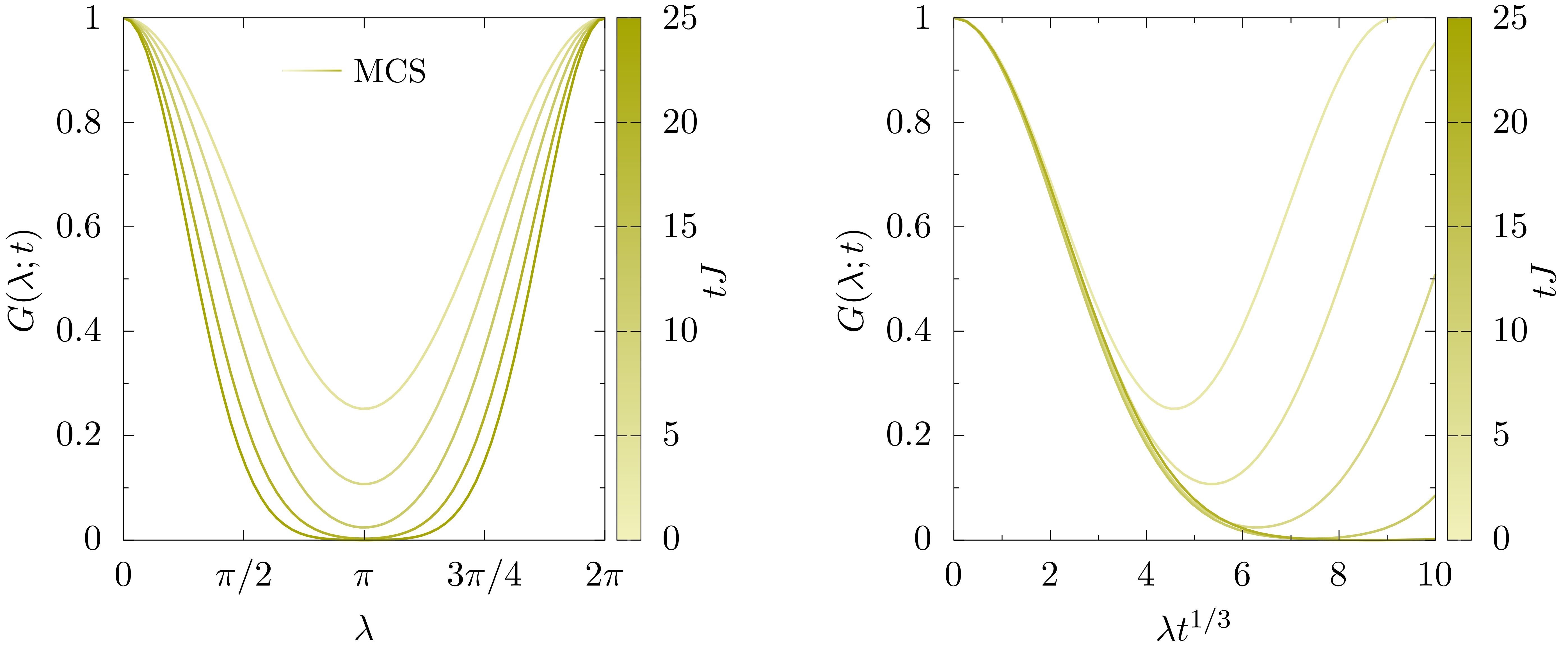
Valli et al., preprint (soon)

Integrable quantum spin chain $S = 1/2$

generating function — XXZ isotropic point

scaling counting field

$\lambda \to \lambda t^{1/2z}$



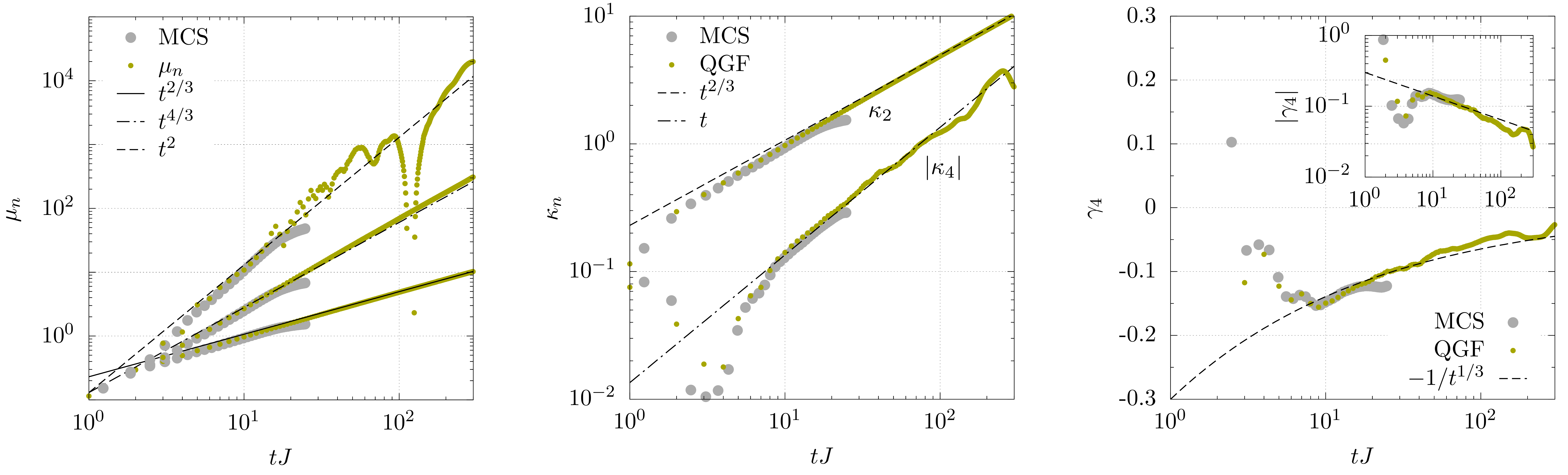
KPZ exponent

Valli et al., preprint (soon)

Integrable quantum spin chain $S = 1/2$

higher-order cumulants

central moments: $\mu_n \sim t^{n/2z}$

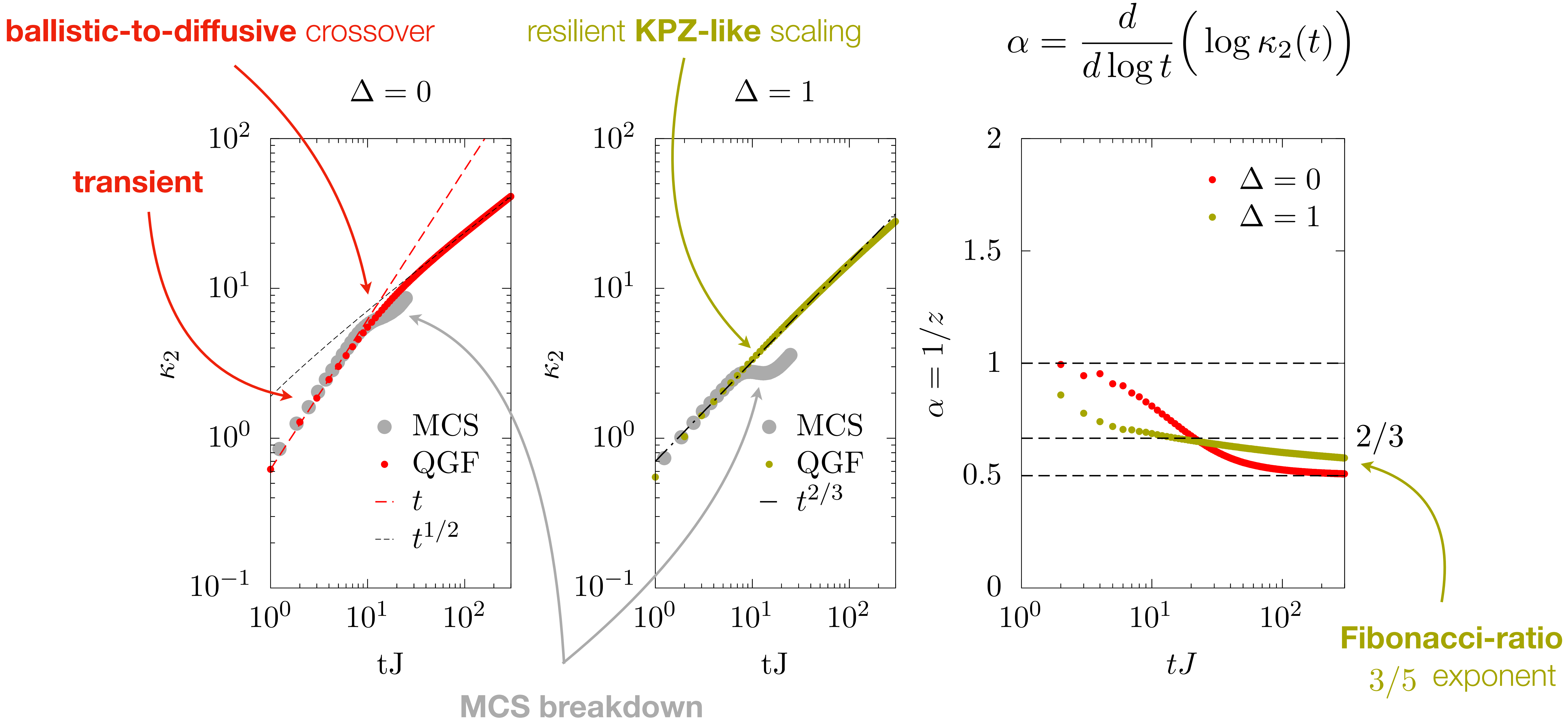


Non-integrable quantum spin chain $S = 1$

KPZ-like scaling from second cumulant κ_2

dynamical exponent

$$\alpha = \frac{d}{d\log t}\Big(\log \kappa_2(t)\Big)$$



Popkov et al., PNAS **112** (2015)

numerics suggests **near-integrability**

quantum quench protocol

$$\rho = \frac{1}{(1+\mu^2)^L} \bigotimes_{i=1}^{L/2} \begin{pmatrix} 1+\mu & 0 \\ 0 & 1-\mu \end{pmatrix} \bigotimes_{i=L/2+1}^{L} \begin{pmatrix} 1-\mu & 0 \\ 0 & 1+\mu \end{pmatrix}$$

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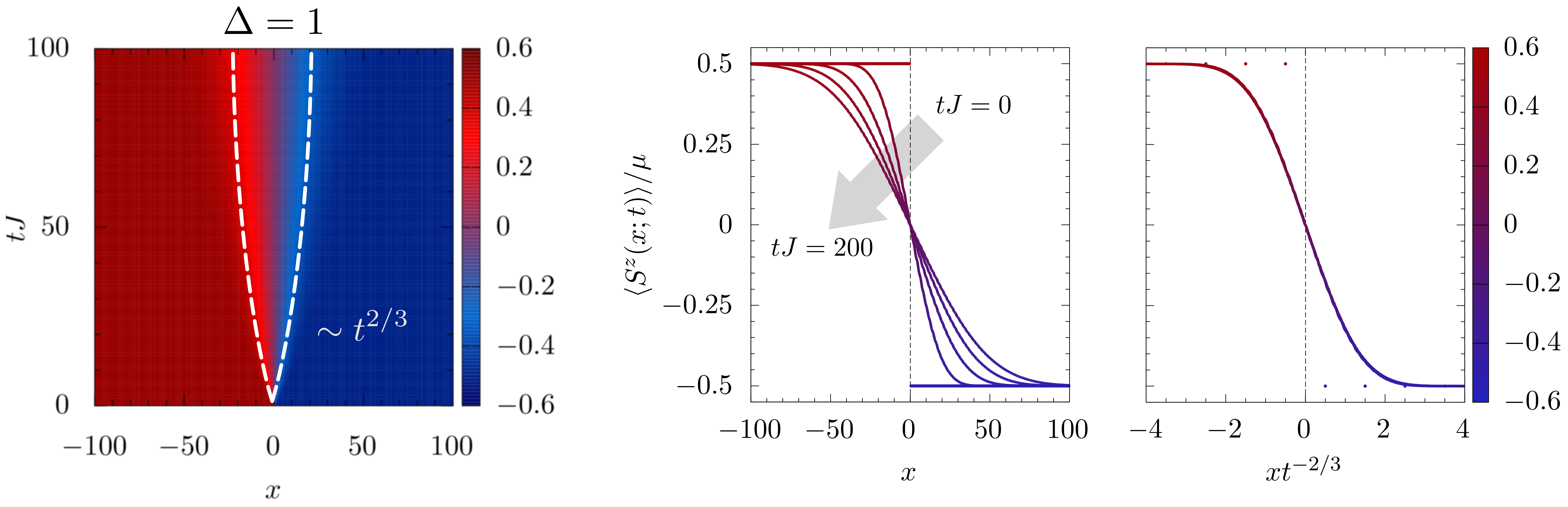

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diffusion

non-linear

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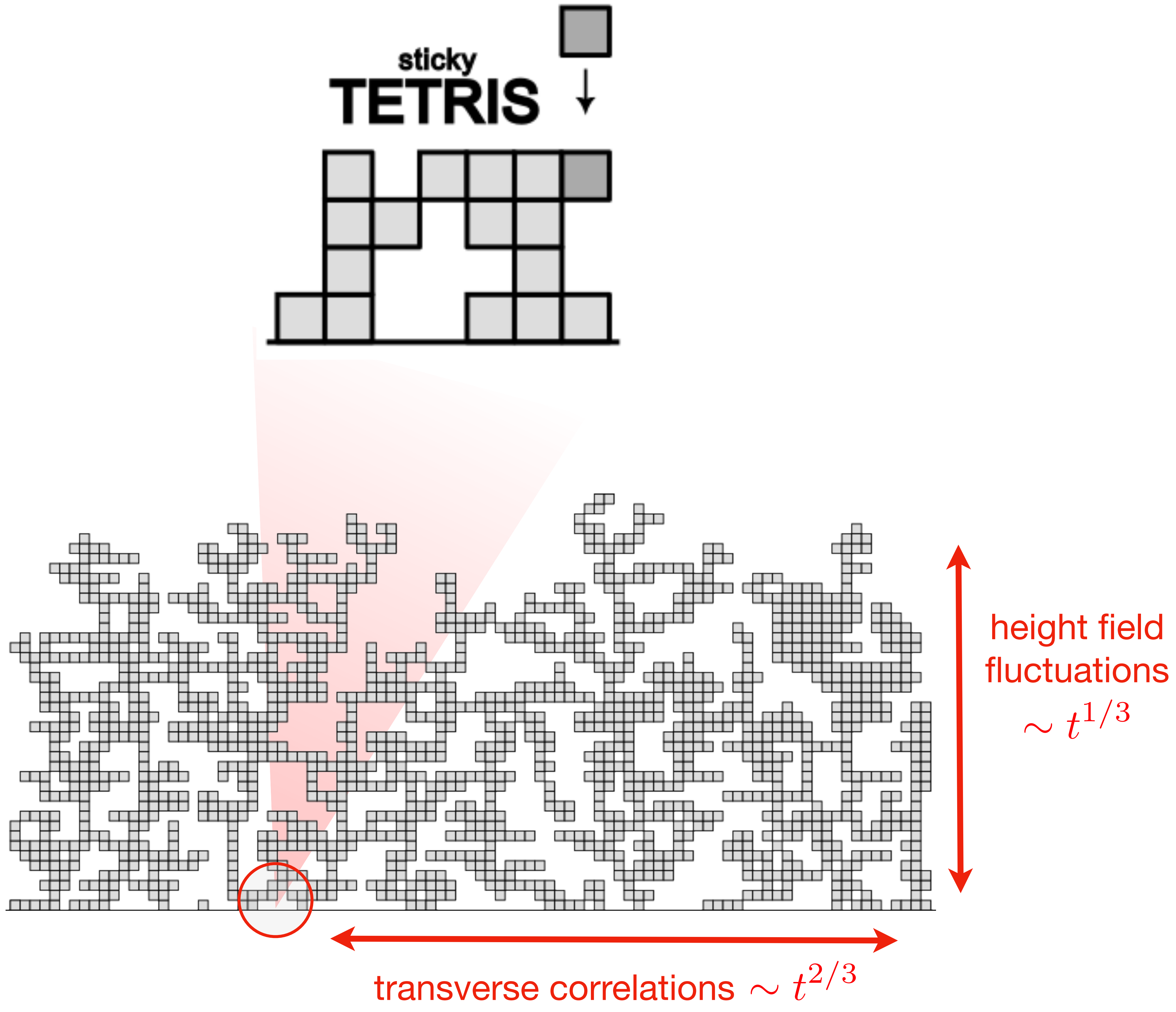

Corwin NAMS **63** (2016)