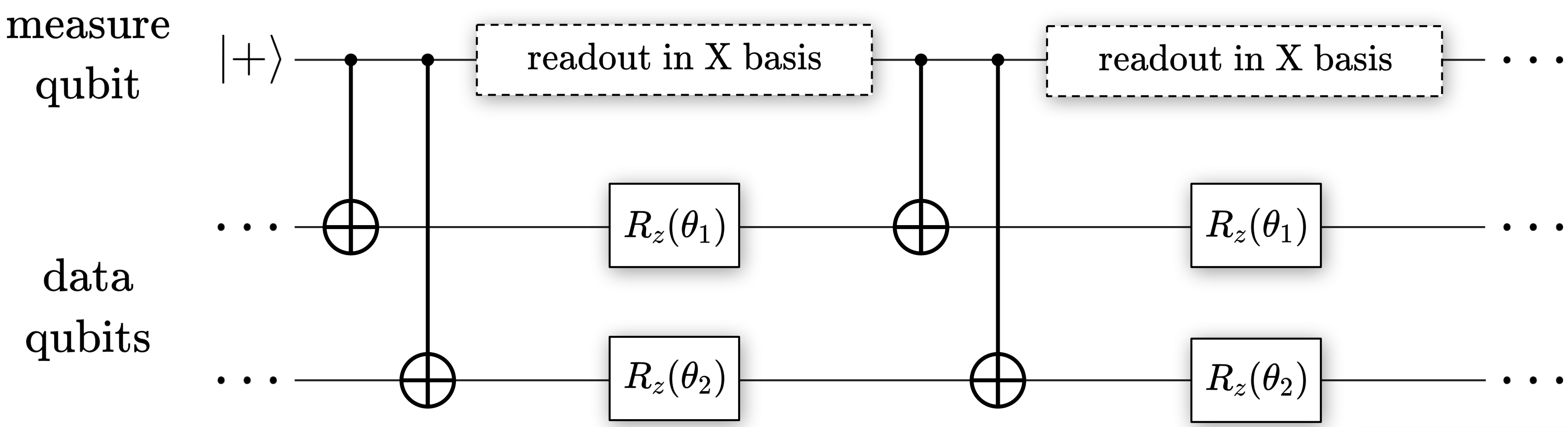
Coherent errors in stabilizer codes caused by quasi-static phase damping



arXiv:2401.04530

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$$\hat{U} = \prod_{j=1}^{n} e^{i\theta_j \hat{Z}_j} \quad , \quad \theta_j \sim \mathcal{N}(0, \sigma) \quad \text{and fixed for a single shot of experiment}$$

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1. How is this different from a simple phase-flip model?

$$\hat{U} = \prod_{j=1}^{n} e^{i\theta_j \hat{Z}_j} \quad , \quad \theta_j \sim \mathcal{N}(0, \sigma) \ \text{ and fixed for a single shot of experiment}$$

1. How is this different from a simple phase-flip model?

2. Are these errors more harmful, than phase-flip errors?

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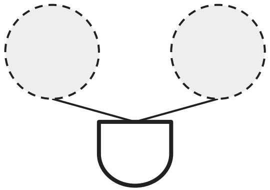
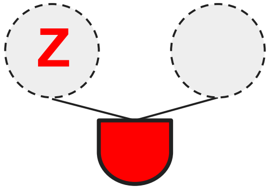
1. How is this different from a simple phase-flip model?
2. Are these errors more harmful, than phase-flip errors?
3. Surface code threshold?

Example: 2-qubit repetition code, single round

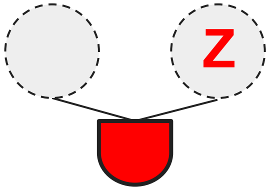
1)



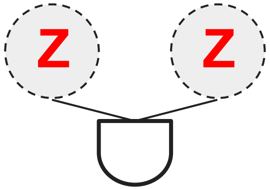
2)



3)

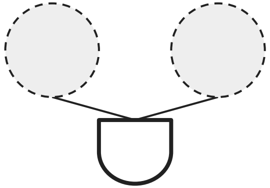


4)

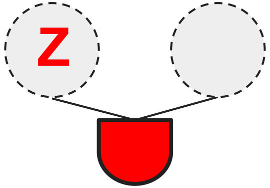


Example: 2-qubit repetition code, single round

1)



2)



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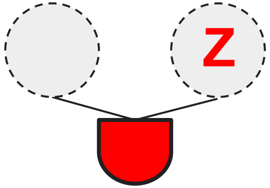
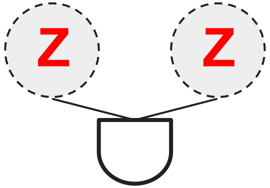


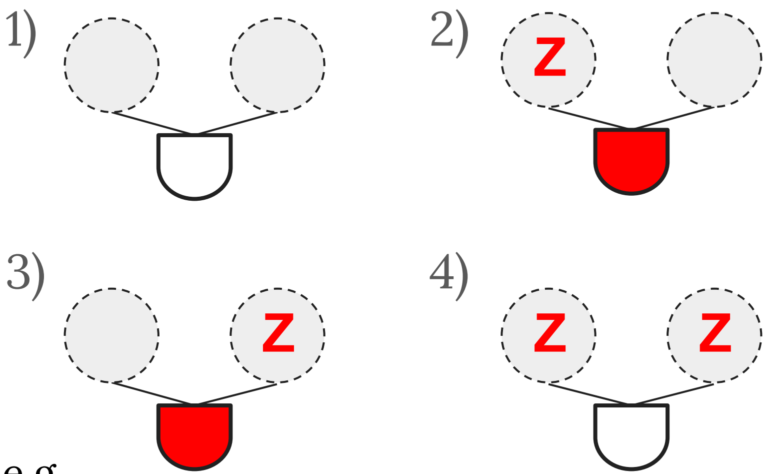
4)



e.g.

$$\rho_1 = \Pi_+ \mathcal{E}_p(\rho_0) \Pi_+ = (1-p)^2 \rho_0 + p^2 Z^L \rho_0 Z^L$$

Example: 2-qubit repetition code, single round



e.g.

$$\rho_1 = \Pi_+ \mathcal{E}_p(\rho_0) \Pi_+ = (1-p)^2 \rho_0 + p^2 Z^L \rho_0 Z^L$$

$$\mathcal{E}_{\mathrm{coh}}(\rho_0) = U \rho_0 U^\dagger$$

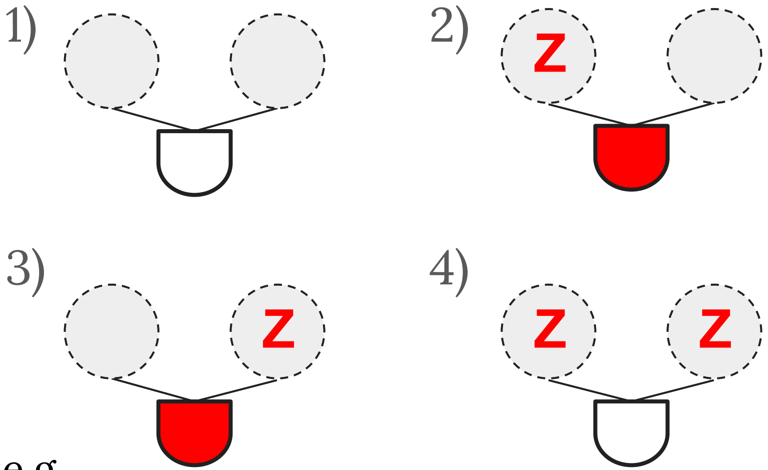
with $U = e^{i\theta_1 Z_1} e^{i\theta_2 Z_2}, \quad \theta_1,\, \theta_2 \sim \mathcal{N}(0, \sigma)$

$$\rho_0 = |\psi_L\rangle\langle\psi_L|$$

$$\tilde{\rho}_1 = \left\langle \Pi_+ \mathcal{E}_{\mathrm{coh}}(\rho_0) \Pi_+ \right\rangle_{\theta_1, \theta_2}$$

$$\tilde{\rho}_1 = \langle \cos^2 \theta_1 \rangle \langle \cos^2 \theta_2 \rangle \rho_0 + \langle \sin^2 \theta_1 \rangle \langle \sin^2 \theta_2 \rangle Z^L \rho_0 Z^L$$

Example: 2-qubit repetition code, single round



e.g.

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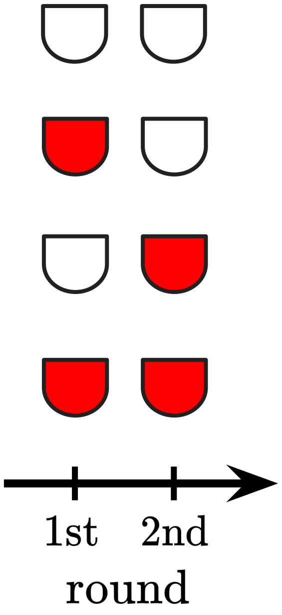
with $p = \langle \sin^2\theta_1 \rangle = \langle \sin^2\theta_2 \rangle = \frac{1}{2}\left(1 - e^{-2\sigma^2}\right) \quad \Rightarrow \quad \rho_1 = \tilde{\rho}_1$

for a single round of error detection **the two channels are the same**

Two rounds of error detection: distinct effect

$$\mathcal{E}_p \rightarrow \Pi_{s_1} \rightarrow \mathcal{E}_p \rightarrow \Pi_{s_2} \qquad \vdots \qquad \mathcal{E}_{\mathrm{coh}} \rightarrow \Pi_{s_1} \rightarrow \mathcal{E}_{\mathrm{coh}} \rightarrow \Pi_{s_2} \rightarrow \langle\, . \,\rangle$$

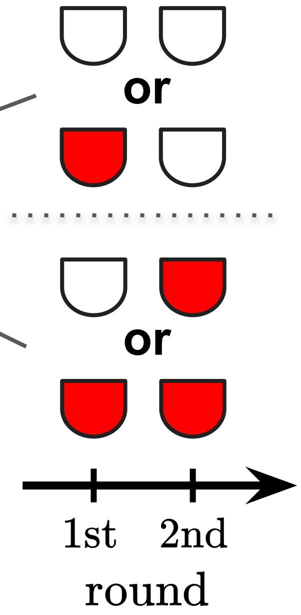


Two rounds of error detection: distinct effect

$$\mathcal{E}_p \rightarrow \Pi_{s_1} \rightarrow \mathcal{E}_p \rightarrow \Pi_{s_2}$$

$$\mathcal{E}_{\text{coh}} \rightarrow \Pi_{s_1} \rightarrow \mathcal{E}_{\text{coh}} \rightarrow \Pi_{s_2} \rightarrow \langle\, . \,\rangle$$

$$\rho_2(\boldsymbol{s}) = \begin{cases} c_{\boldsymbol{s}}\rho_0 + d_{\boldsymbol{s}} Z^L \rho_0 Z^L, & \text{for } s_2 = +1 \\ c_{\boldsymbol{s}} Z_1 \rho_0 Z_1 + d_{\boldsymbol{s}} Z_2 \rho_0 Z_2, & \text{for } s_2 = -1 \end{cases}$$



Two rounds of error detection: distinct effect

$$\mathcal{E}_p \;\to\; \Pi_{s_1} \;\to\; \mathcal{E}_p \;\to\; \Pi_{s_2}$$

$$\mathcal{E}_{\mathrm{coh}} \;\to\; \Pi_{s_1} \;\to\; \mathcal{E}_{\mathrm{coh}} \;\to\; \Pi_{s_2} \to \langle\,.\,\rangle$$

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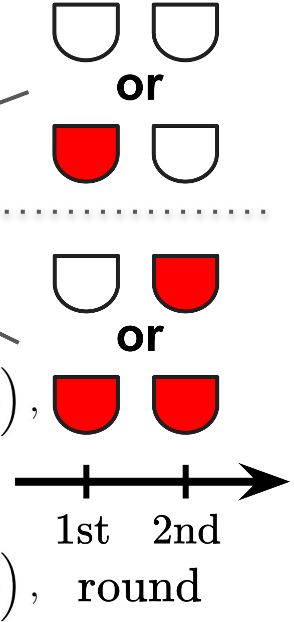


$$c_{++} = (1-p)^4 + p^4,$$
$$d_{++} = c_{-+} = d_{-+} = 2p^2(1-p)^2,$$
$$c_{+-} = d_{+-} = c_{--} = d_{--} = p(1-p)^3 + p^3(1-p),$$

$$\tilde{c}_{++} = \frac{1}{16}\left(e^{-16\sigma^2} + 2e^{-8\sigma^2} + 8e^{-4\sigma^2} + 5\right),$$
$$\tilde{d}_{++} = \tilde{d}_{-+} = \frac{1}{16}\left(1 - e^{-8\sigma^2}\right)^2,$$
$$\tilde{c}_{-+} = \frac{1}{16}\left(e^{-16\sigma^2} + 2e^{-8\sigma^2} - 8e^{-4\sigma^2} + 5\right),$$
$$\tilde{c}_{+-} = \tilde{d}_{+-} = \tilde{c}_{--} = \tilde{d}_{--} = \frac{1}{16}\left(1 - e^{-16\sigma^2}\right).$$

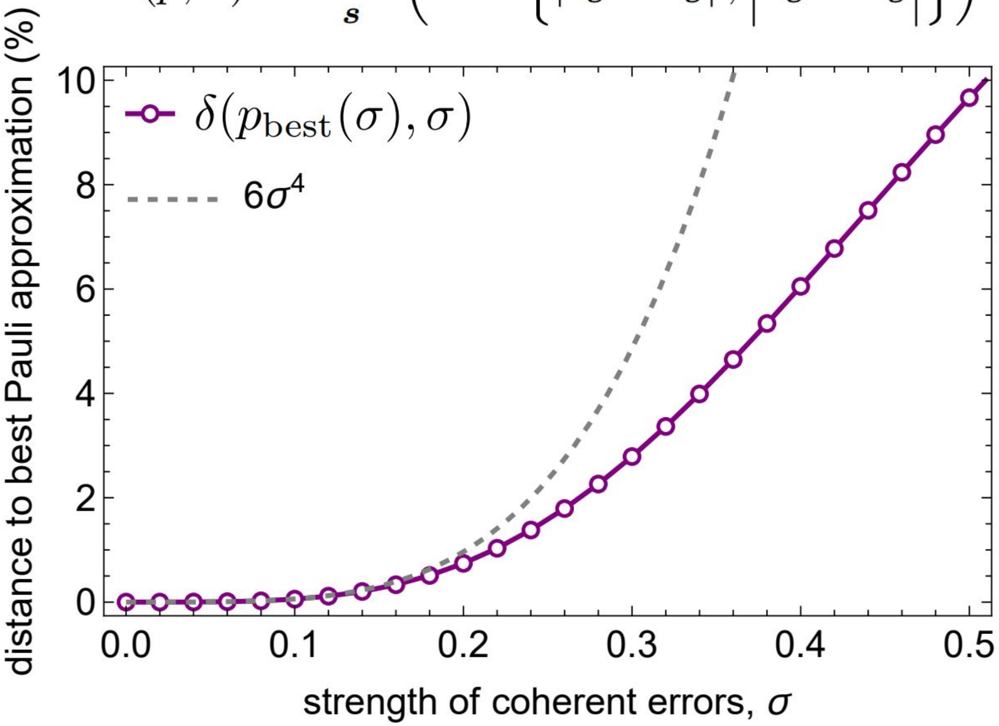
Two rounds: total variational distance is non-zero

- phase-flip error probability: p

- coherent errors:

  $\theta_1,\ \theta_2 \sim \mathcal{N}(0,\sigma)$

$$\delta(p,\sigma) = \max_{\boldsymbol{s}} \left(\max \left\{ |c_{\boldsymbol{s}} - \tilde{c}_{\boldsymbol{s}}|\,, \left|d_{\boldsymbol{s}} - \tilde{d}_{\boldsymbol{s}}\right| \right\} \right)$$



Quasistatic coherent errors **in stabilizer codes**

Single round of error detection/correction: equivalent to simple Pauli errors

$$\hat{\rho}_1(s) = \sum_{\hat{E}\in\mathcal{D}_s} P(\hat{E})\hat{E}\hat{\rho}_0\hat{E}, \qquad P(\hat{E}) = \prod_{j=1}^{n} \underbrace{\langle\cos^2\theta_j\rangle_{\boldsymbol{\theta}}}_{1-p}^{1-n_{\hat{E}}(j)} \underbrace{\langle\sin^2\theta_j\rangle_{\boldsymbol{\theta}}}_{p}^{n_{\hat{E}}(j)}$$

Multiple rounds: Pauli, but highly correlated/inhomogeneous

$$\hat{\rho}_2(\boldsymbol{s}) = \sum_{\boldsymbol{E}\in\mathcal{D}_{\boldsymbol{s}}} \tilde{P}(\boldsymbol{E})\hat{E}(\boldsymbol{E})\hat{\rho}_0\hat{E}(\boldsymbol{E})$$

e.g.

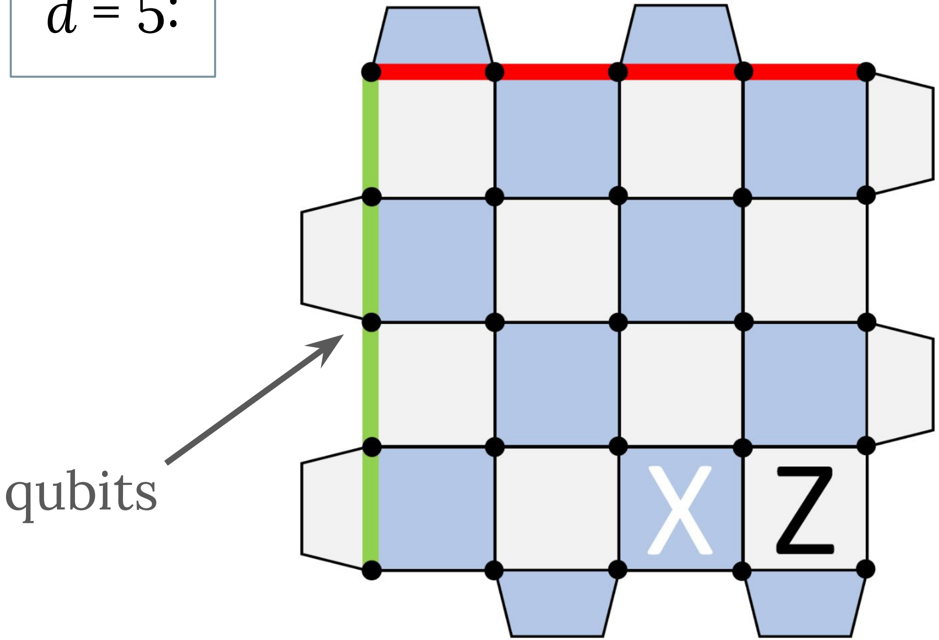
$$\tilde{P}(\hat{\mathbb{1}},\hat{Z}_1) = \Big\langle \cos^2(\theta_1)\sin^2(\theta_1)\cos^2(\theta_2)\cos(2\theta_2)\Big\rangle_{\boldsymbol{\theta}}$$

$$\tilde{P}(\hat{Z}_1,\hat{Z}_2) = \Big\langle \frac{1}{8}\sin^2(2\theta_1)\sin^2(2\theta_2)\Big\rangle_{\boldsymbol{\theta}}$$

$$\vdots$$

Error correction with the surface code [Phys. Rev. A **86**, 032324]

$d = 5$:



logical operators:

$$\hat{X}^L = \prod_{j \in |} \hat{X}_j$$

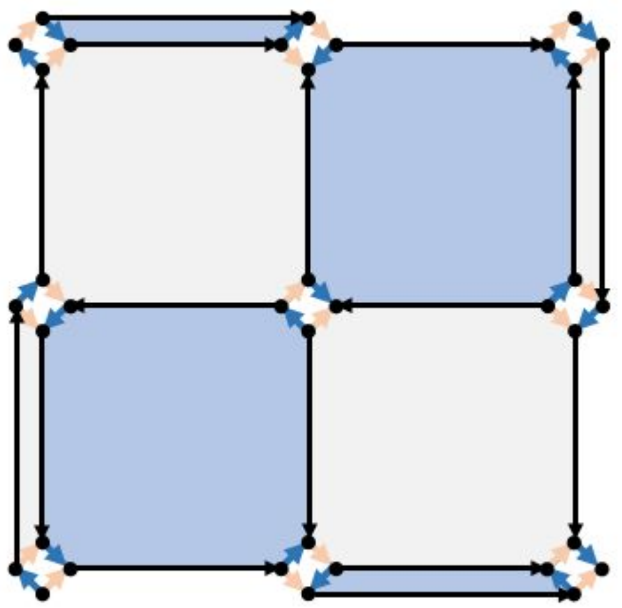
$$\hat{Z}^L = \prod_{j \in -} \hat{Z}_j$$

check operators:

$$\prod_{j \in \partial f} \hat{X}_j$$

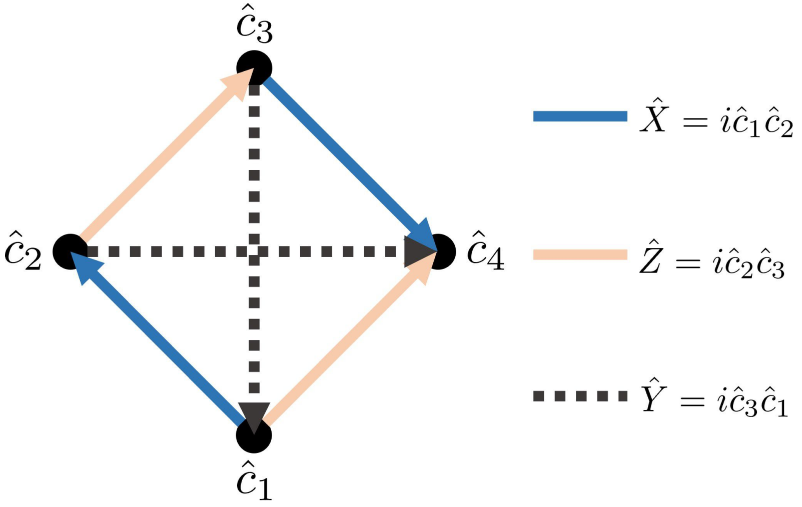
$$\prod_{j \in \partial f} \hat{Z}_j$$

Efficient surface code simulation using Fermionic Linear Optics



S. Bravyi *et al.* npj Quantum Inf **4**, 55 (2018)

Á. Márton and J. K. Asbóth, Quantum **7**, 1116 (2023)



$$\hat{c}_1 = i(\hat{a}_2 - \hat{a}_2^\dagger); \qquad \hat{c}_2 = \hat{a}_1 + \hat{a}_1^\dagger;$$
$$\hat{c}_3 = i(\hat{a}_1 - \hat{a}_1^\dagger); \qquad \hat{c}_4 = \hat{a}_2 + \hat{a}_2^\dagger.$$

$$|0\rangle = \hat{a}_2^\dagger |\emptyset\rangle ; \quad |1\rangle = \hat{a}_1^\dagger |\emptyset\rangle$$

Surface code simulation with readout errors

$\theta_j \sim \mathcal{N}(0, \sigma)$ and fixed for d rounds

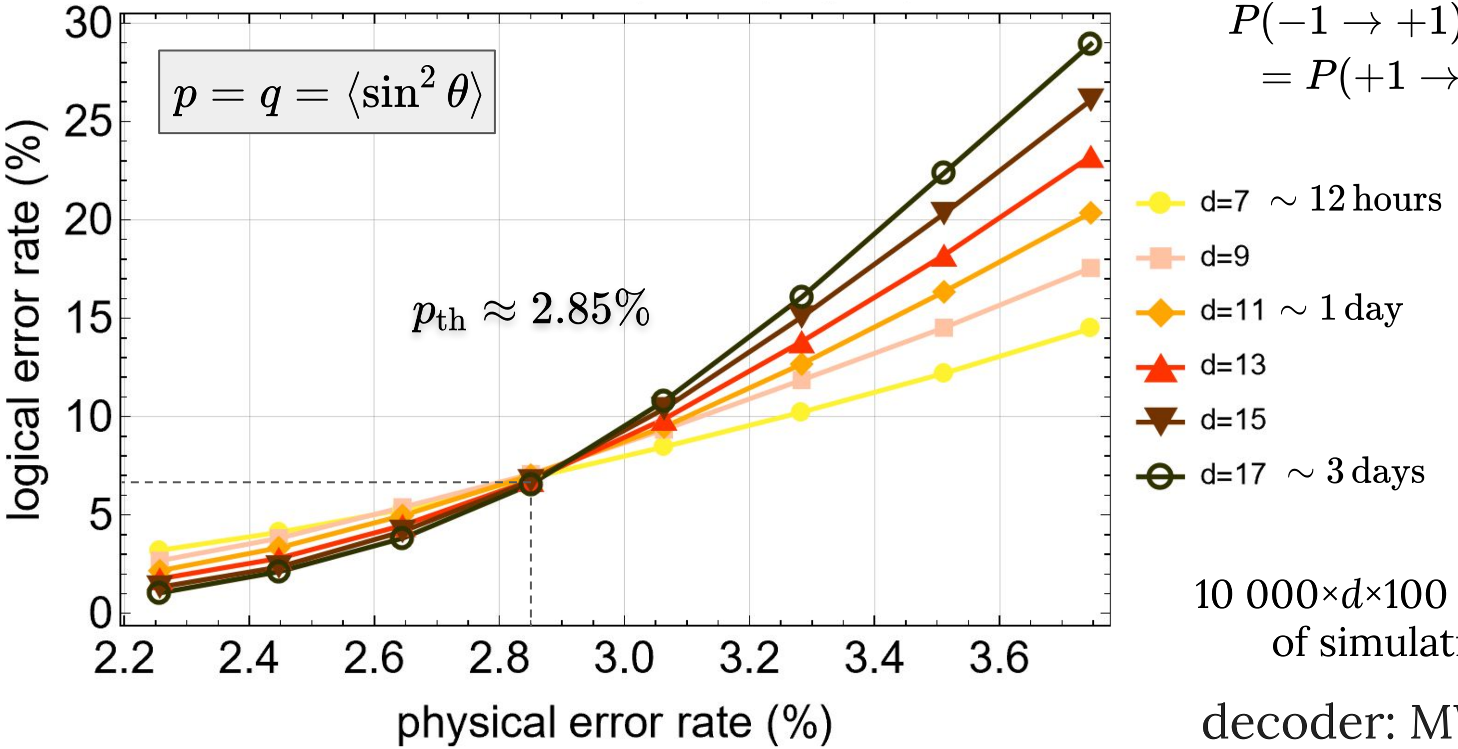
$$P(-1 \to +1) = P(+1 \to -1) = q$$

O. Higgott (2022): PyMatching

decoder: MWPM

Surface code simulation with readout errors

$\theta_j \sim \mathcal{N}(0,\sigma)$ and fixed for d rounds



$P(-1 \to +1)$
$= P(+1 \to -1) = q$

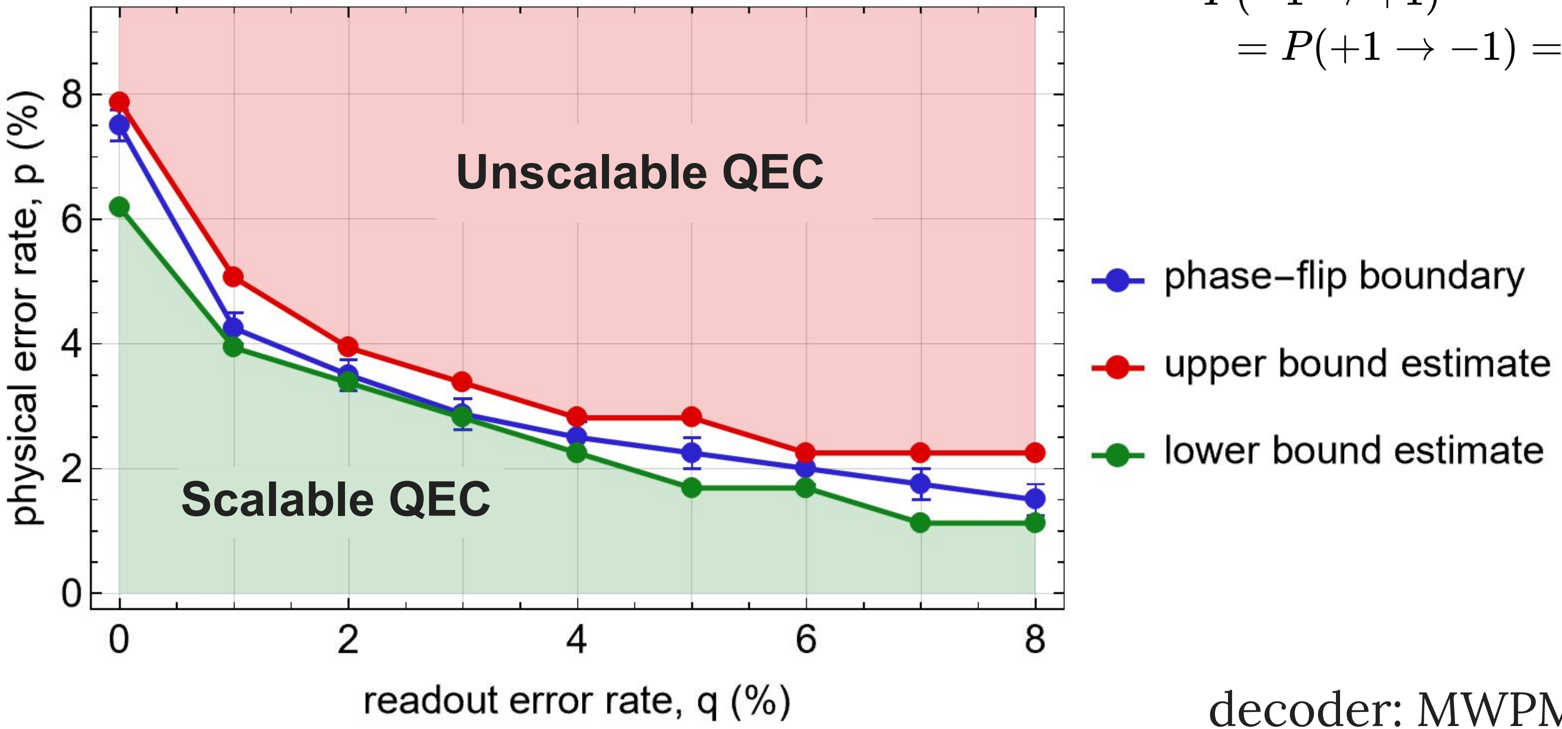
10 000×d×100 rounds of simulation

decoder: MWPM

Surface code simulation with readout errors

$$p = \langle \sin^2 \theta \rangle \neq q$$

$$P(-1 \to +1) = P(+1 \to -1) = q$$



decoder: MWPM

D. Pataki *et al.*
arXiv:2401.04530

Quasistatic coherent errors **in stabilizer codes**

$$U = \prod_{j=1}^{n} e^{i\theta_j Z_j}, \quad \theta_j \sim \mathcal{N}(0, \sigma) \text{ and fixed for a single shot of experiment}$$

- equivalent to simple Pauli errors for a **single-round** of error correction
- equivalent to a complicated Pauli model for **multiple rounds**
- **surface code threshold** is similar to Pauli-Z + readout errors, but the logical error rate is lower by 1.5% (at the threshold)

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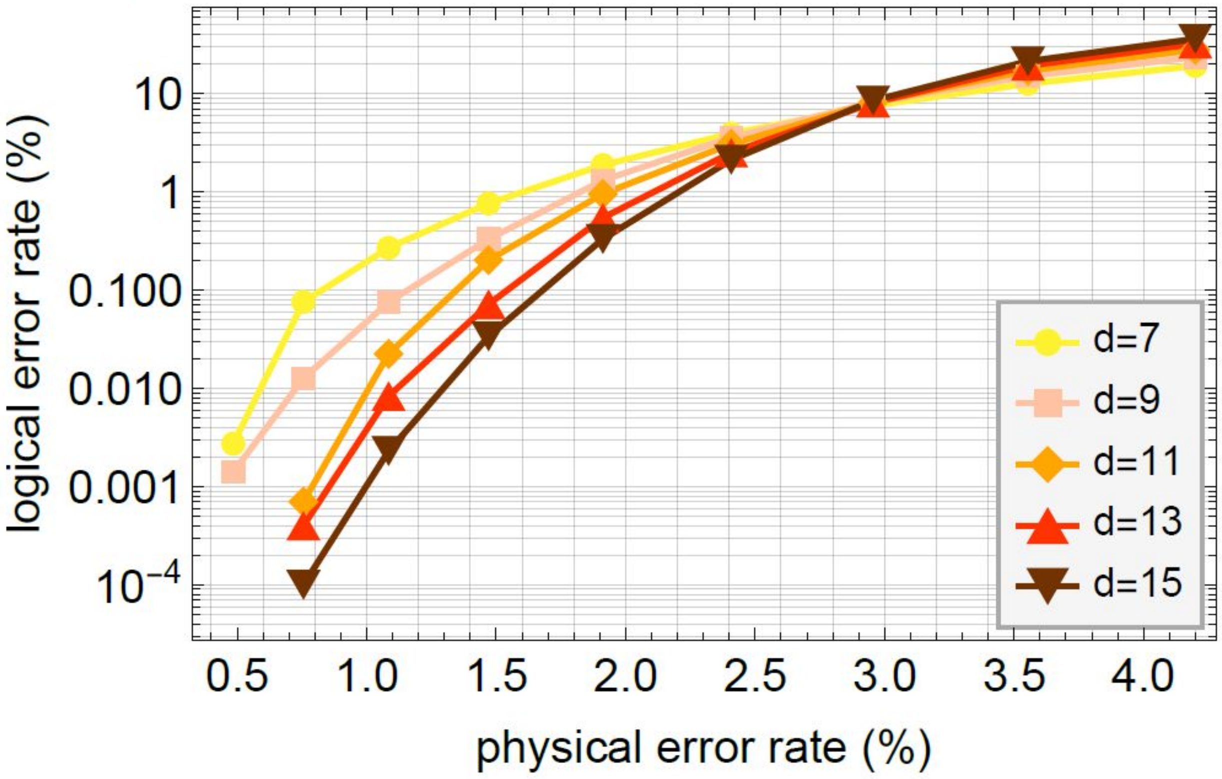
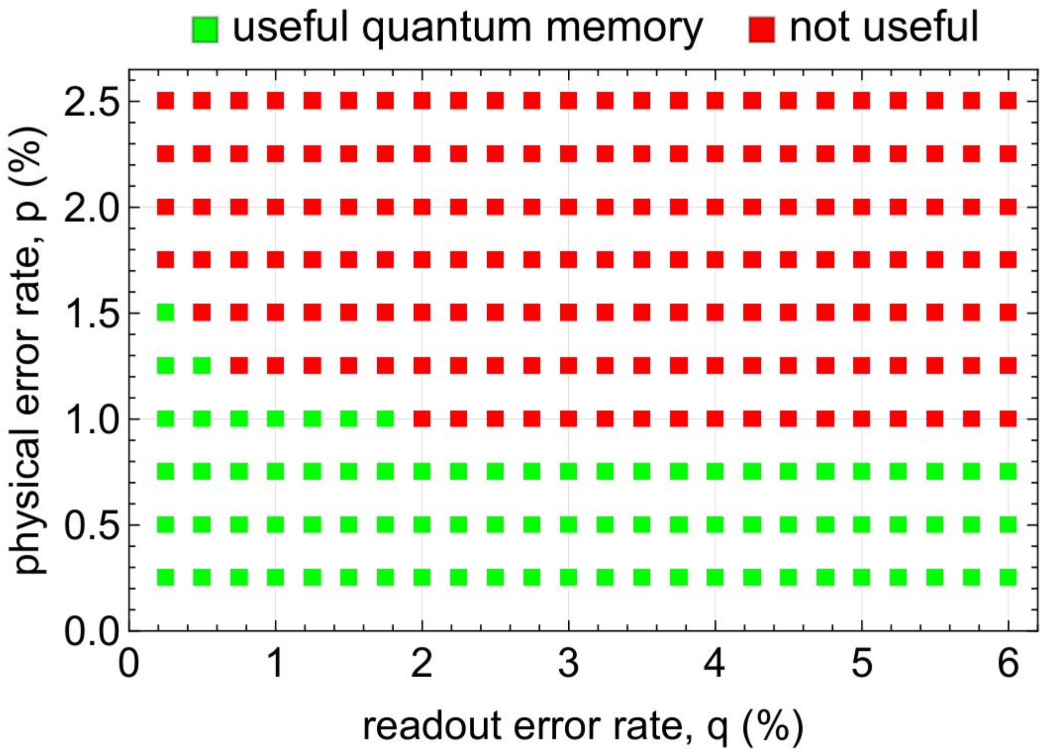

logical error rate (%)
10
1
0.100
0.010
0.001
10^{-4}
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
physical error rate (%)
d=7
d=9
d=11
d=13
d=15

For the surface-9 (d=3):



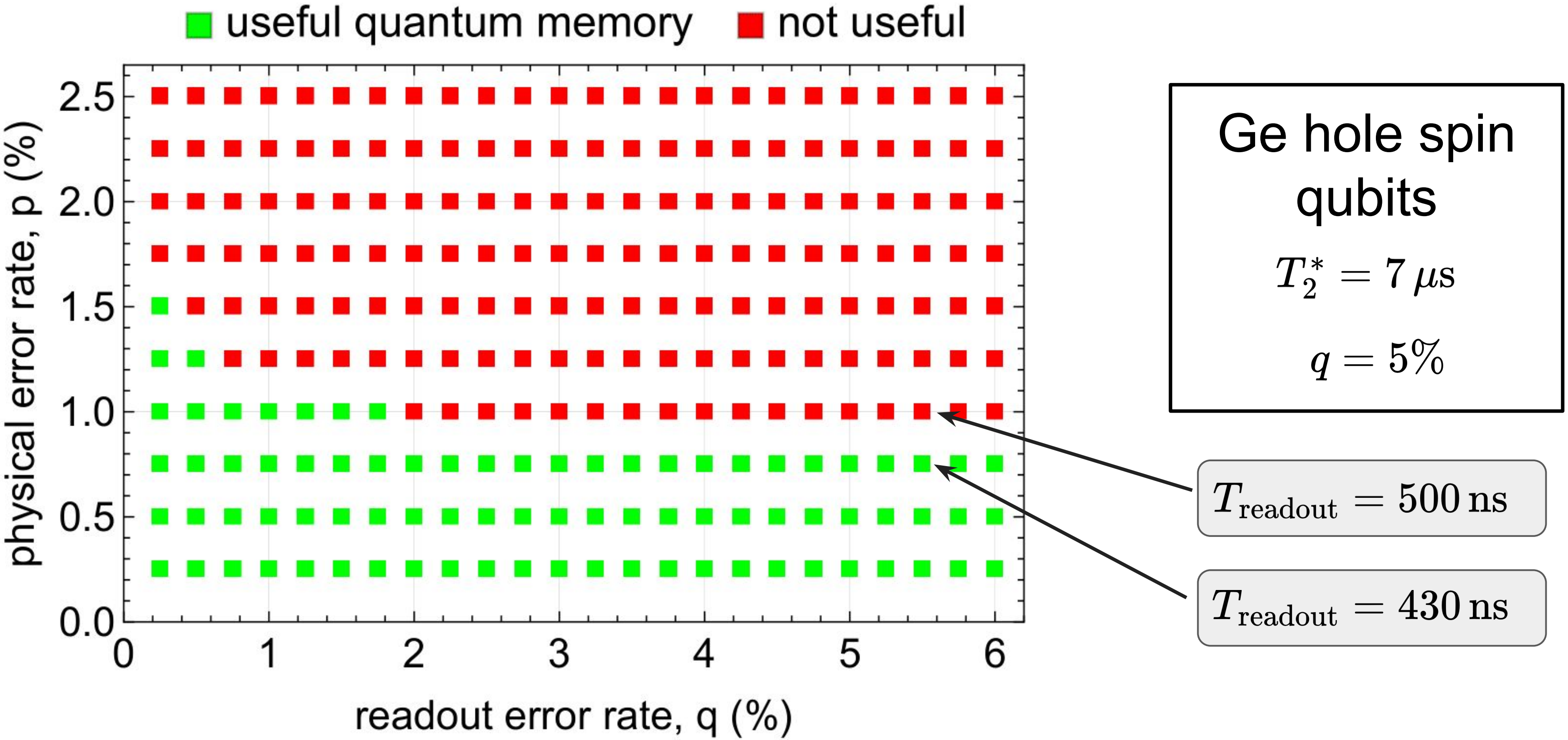
$$\sigma = \sqrt{2}\,\frac{T_{\text{meas}}}{T_2^*},$$

$$p = \langle \sin^2\theta \rangle = \frac{1}{2}\left(1 - e^{-2\sigma^2}\right)$$

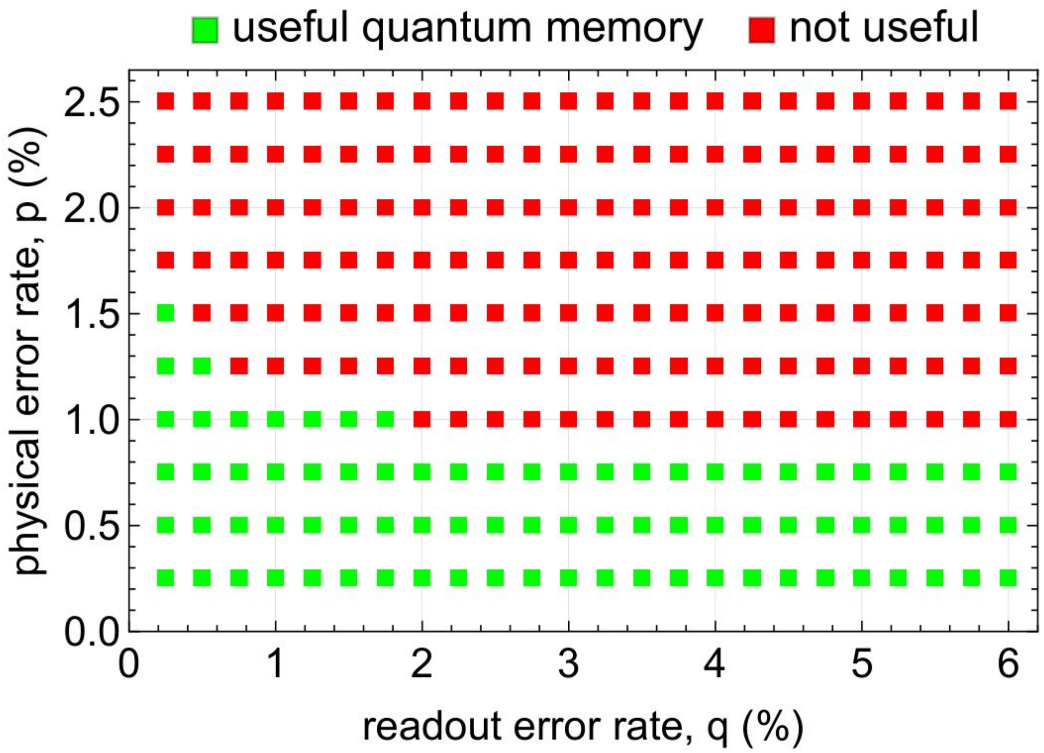
e.g. $T_{\text{meas}} = 500\ \text{ns}$

$T_2^* = 10\,\mu\text{s}$

$\Rightarrow p \approx 0.5\%$


useful quantum memory
not useful
physical error rate, p (%)
readout error rate, q (%)
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6
Ge hole spin qubits
$T_2^* = 7\,\mu s$
$q = 5\%$
$T_{\mathrm{readout}} = 500\,\mathrm{ns}$
$T_{\mathrm{readout}} = 430\,\mathrm{ns}$

For the surface-9 (d=3):



$$\sigma = \sqrt{2}\frac{T_{\text{meas}}}{T_2^*},$$

$$p = \langle \sin^2\theta \rangle = \frac{1}{2}\left(1 - e^{-2\sigma^2}\right)$$

e.g. $T_{\text{meas}} = 500$ ns

$T_2^* = 10\,\mu$s

$\Rightarrow p \approx 0.5\%$

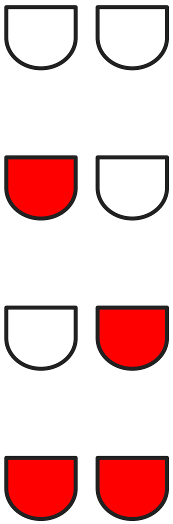
suppressed by *dynamical decoupling*, but e.g. $1/f^2$ noise still remains quasistatic

[Ł. Cywiński *et al.*, Phys. Rev. B 77, 174509]

2-qubit repetition code, two rounds of error detection

$$\mathcal{E}_{\text{coh}} \rightarrow \Pi_{s_1} \rightarrow \mathcal{E}_{\text{coh}} \rightarrow \Pi_{s_2} \rightarrow \langle\, . \,\rangle$$

$$\mathcal{E}_p \rightarrow \Pi_{s_1} \rightarrow \mathcal{E}_p \rightarrow \Pi_{s_2}$$



$$p_{++} \approx 1 - 4\sigma^2 + 20\sigma^4 \qquad p_{++} \approx 1 - 4p + 8p^2$$

$$p_{-+} \approx 12\sigma^4 \qquad p_{-+} \approx 4p^2$$

$$p_{+-} \approx 2\sigma^2 - 16\sigma^4 \qquad p_{+-} \approx 2p - 6p^2$$

$$p_{--} \approx 2\sigma^2 - 16\sigma^4 \qquad p_{--} \approx 2p - 6p^2$$

2-qubit repetition code, two rounds of error detection

For example, choosing $\sigma = \pi/10$:

$$p_{++} \approx 0.738 \quad = \quad 1 - 4p + 8p^2 \quad \rightarrow \quad p = 0.076$$

$$p_{-+} \approx 0.064 \quad = \quad 4p^2 \quad \rightarrow \quad p = 0.148$$

$$p_{+-} \approx 0.099 \quad = \quad 2p - 6p^2 \quad \rightarrow \quad p = 0.059$$

$$p_{--} \approx 0.099 \quad = \quad 2p - 6p^2 \quad \rightarrow \quad p = 0.059$$

#	$\boldsymbol{E}$	$\hat{E}(\boldsymbol{E})$	$\boldsymbol{s}$	$P(\boldsymbol{E})$	$\tilde{P}(\boldsymbol{E})$
1	$(\hat{\mathbb{1}}, \hat{\mathbb{1}})$	$\hat{\mathbb{1}}$	$(+1, +1)$	$(1-p)^4$	$1 - 4p + 11p^2 - 26p^3 + 46p^4 - 60p^5 + 56p^6 - 32p^7 + 8p^8$
2	$(\hat{Z}_A\hat{Z}_B, \hat{Z}_A\hat{Z}_B)$	$\hat{\mathbb{1}}$	$(+1, +1)$	p^4	$p^2 - 6p^3 + 26p^4 - 52p^5 + 56p^6 - 32p^7 + 8p^8$
3	$(\hat{\mathbb{1}}, \hat{Z}_A\hat{Z}_B)$	$\hat{Z}_A\hat{Z}_B$	$(+1, +1)$	$(1-p)^2p^2$	$2p^2 - 12p^3 + 34p^4 - 56p^5 + 56p^6 - 32p^7 + 8p^8$
4	$(\hat{Z}_A\hat{Z}_B, \hat{\mathbb{1}})$	$\hat{Z}_A\hat{Z}_B$	$(+1, +1)$	$(1-p)^2p^2$	$2p^2 - 12p^3 + 34p^4 - 56p^5 + 56p^6 - 32p^7 + 8p^8$
5	$(\hat{Z}_A, \hat{Z}_A)$	$\hat{\mathbb{1}}$	$(-1, +1)$	$(1-p)^2p^2$	$4p^2 - 16p^3 + 36p^4 - 56p^5 + 56p^6 - 32p^7 + 8p^8$
6	$(\hat{Z}_B, \hat{Z}_B)$	$\hat{\mathbb{1}}$	$(-1, +1)$	$(1-p)^2p^2$	$4p^2 - 16p^3 + 36p^4 - 56p^5 + 56p^6 - 32p^7 + 8p^8$
7	$(\hat{Z}_A, \hat{Z}_B)$	$\hat{Z}_A\hat{Z}_B$	$(-1, +1)$	$(1-p)^2p^2$	$2p^2 - 12p^3 + 34p^4 - 56p^5 + 56p^6 - 32p^7 + 8p^8$
8	$(\hat{Z}_B, \hat{Z}_A)$	$\hat{Z}_A\hat{Z}_B$	$(-1, +1)$	$(1-p)^2p^2$	$2p^2 - 12p^3 + 34p^4 - 56p^5 + 56p^6 - 32p^7 + 8p^8$
9	$(\hat{\mathbb{1}}, \hat{Z}_A)$	$\hat{Z}_A$	$(+1, -1)$	$(1-p)^3p$	$p - 6p^2 + 19p^3 - 40p^4 + 58p^5 - 56p^6 + 32p^7 - 8p^8$
10	$(\hat{Z}_A\hat{Z}_B, \hat{Z}_B)$	$\hat{Z}_A$	$(+1, -1)$	$(1-p)p^3$	$-p^2 + 9p^3 - 30p^4 + 54p^5 - 56p^6 + 32p^7 - 8p^8$
11	$(\hat{\mathbb{1}}, \hat{Z}_B)$	$\hat{Z}_B$	$(+1, -1)$	$(1-p)^3p$	$p - 6p^2 + 19p^3 - 40p^4 + 58p^5 - 56p^6 + 32p^7 - 8p^8$
12	$(\hat{Z}_A\hat{Z}_B, \hat{Z}_A)$	$\hat{Z}_B$	$(+1, -1)$	$(1-p)p^3$	$-p^2 + 9p^3 - 30p^4 + 54p^5 - 56p^6 + 32p^7 - 8p^8$
13	$(\hat{Z}_A, \hat{\mathbb{1}})$	$\hat{Z}_A$	$(-1, -1)$	$(1-p)^3p$	$p - 6p^2 + 19p^3 - 40p^4 + 58p^5 - 56p^6 + 32p^7 - 8p^8$
14	$(\hat{Z}_B, \hat{Z}_A\hat{Z}_B)$	$\hat{Z}_A$	$(-1, -1)$	$(1-p)p^3$	$-p^2 + 9p^3 - 30p^4 + 54p^5 - 56p^6 + 32p^7 - 8p^8$
15	$(\hat{Z}_A, \hat{Z}_A\hat{Z}_B)$	$\hat{Z}_B$	$(-1, -1)$	$(1-p)p^3$	$-p^2 + 9p^3 - 30p^4 + 54p^5 - 56p^6 + 32p^7 - 8p^8$
16	$(\hat{Z}_B, \hat{\mathbb{1}})$	$\hat{Z}_B$	$(-1, -1)$	$(1-p)^3p$	$p - 6p^2 + 19p^3 - 40p^4 + 58p^5 - 56p^6 + 32p^7 - 8p^8$