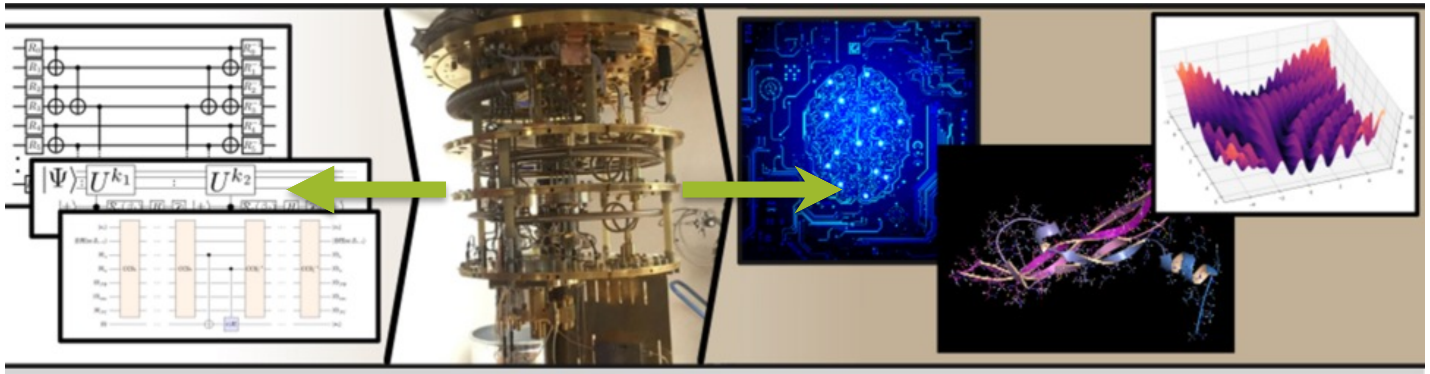
Reinforcement Learning in Bayesian Hamiltonian Tracking for Noise-Driven Coherent Rotation of a Spin Qubit

Evert van Nieuwenburg

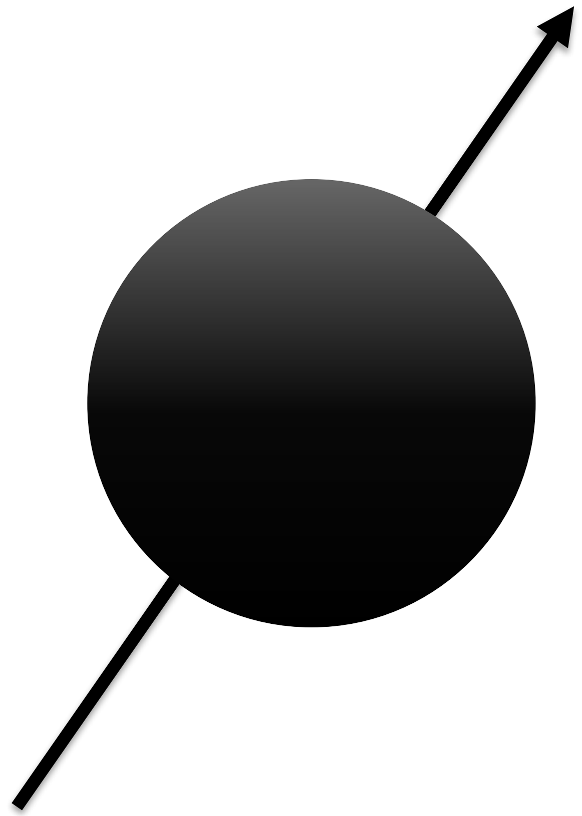
Jan A. Krzywda| Budapest 20.06.2024



Universiteit Leiden
The Netherlands

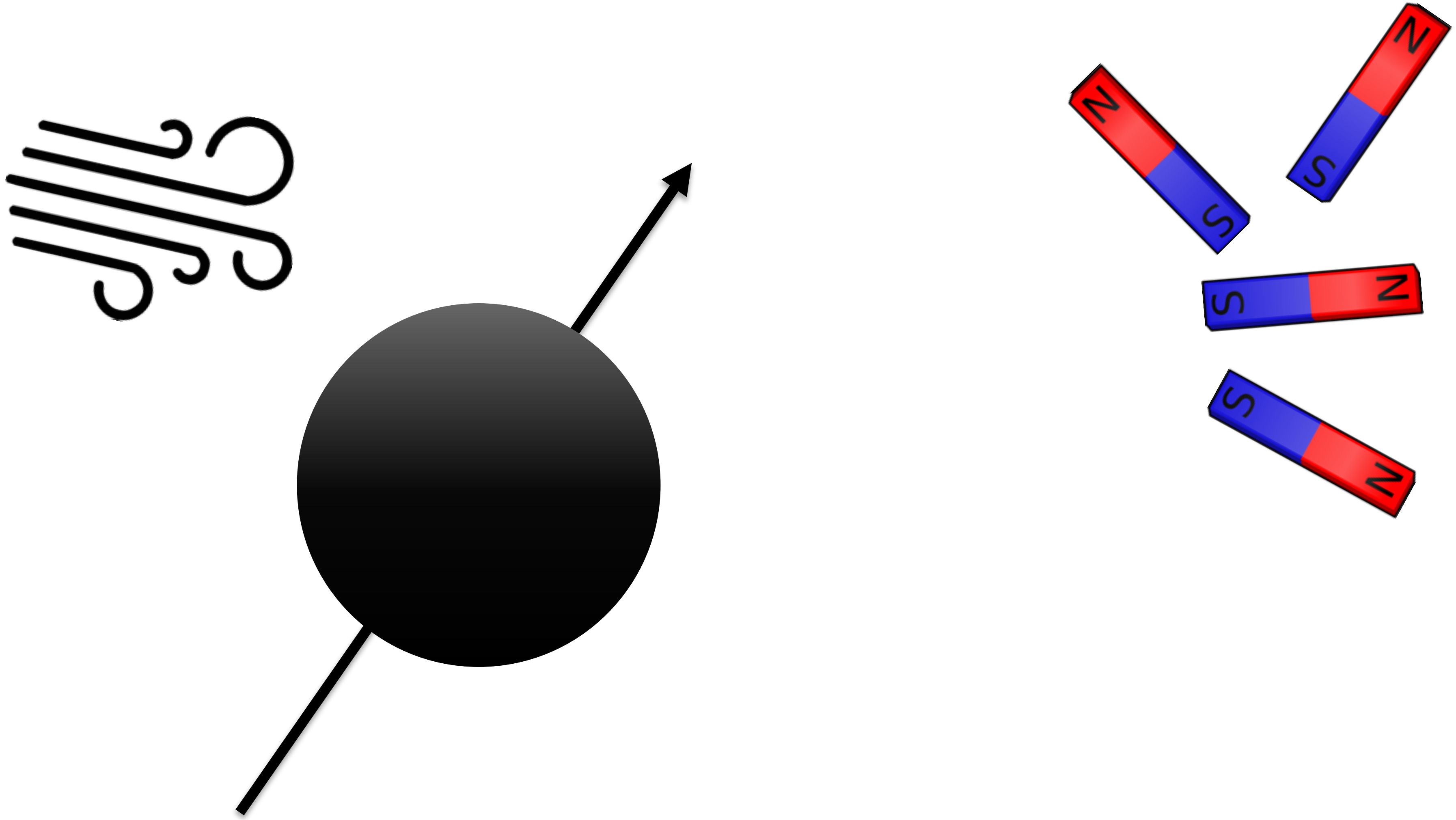

$S_z = -\frac{1}{2}\hbar$
$S_z = +\frac{1}{2}\hbar$
...... het draait rechtsom

Idea



$$H = \omega S_z$$

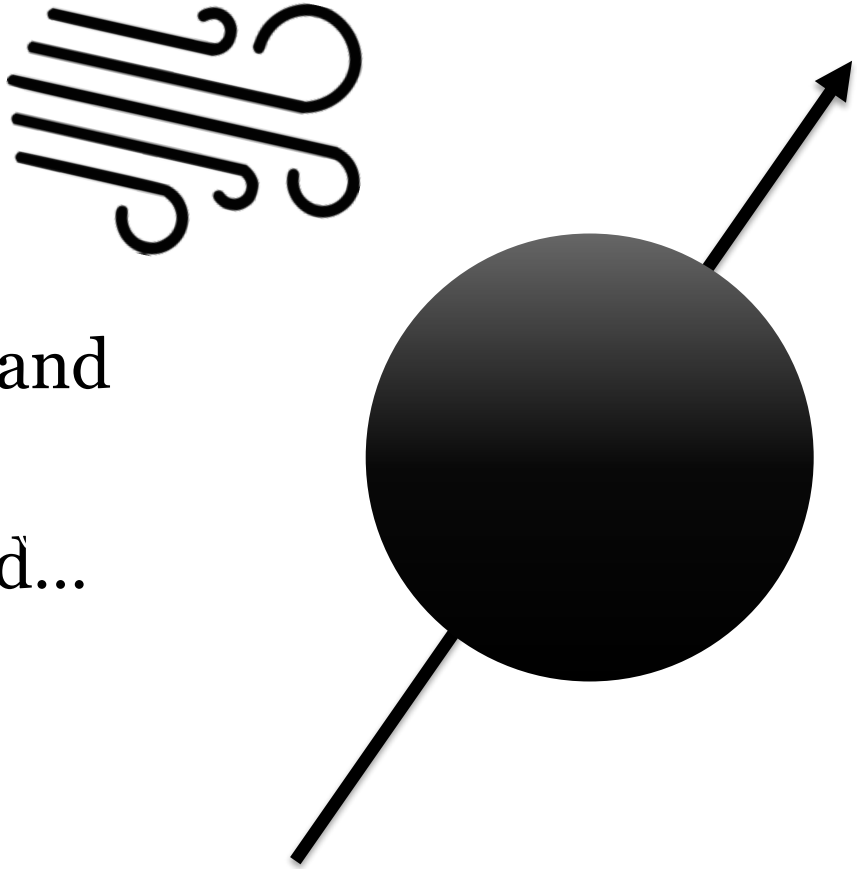
Idea

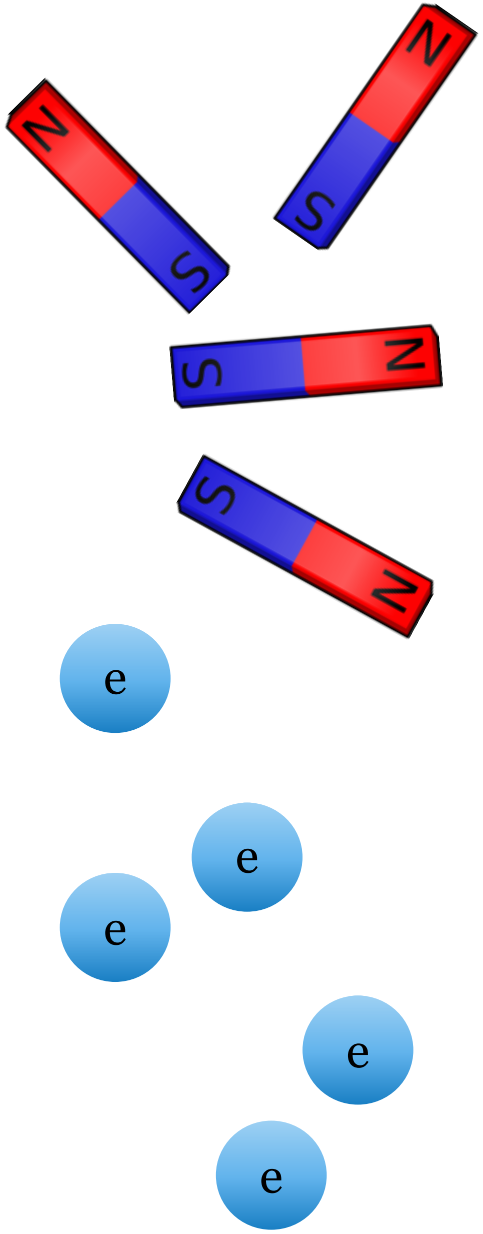


$$H = [\omega + \delta\omega(t)]S_z$$

Idea

Because noise and control are interconnected…





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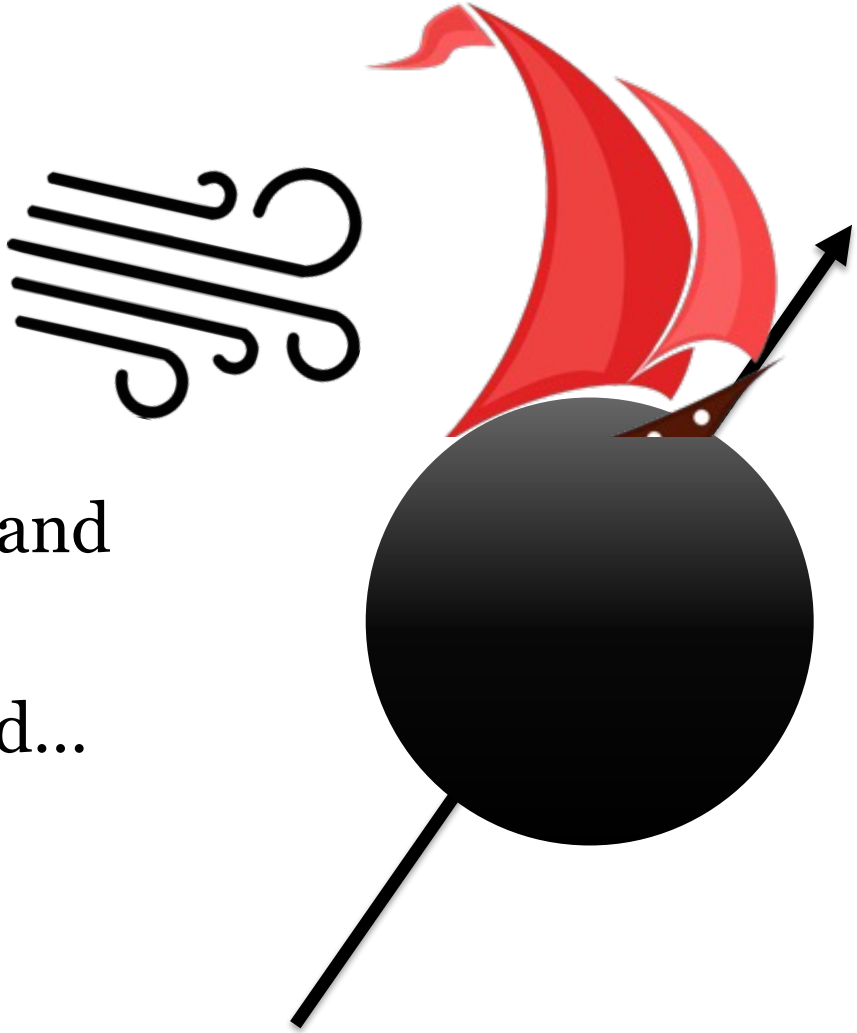
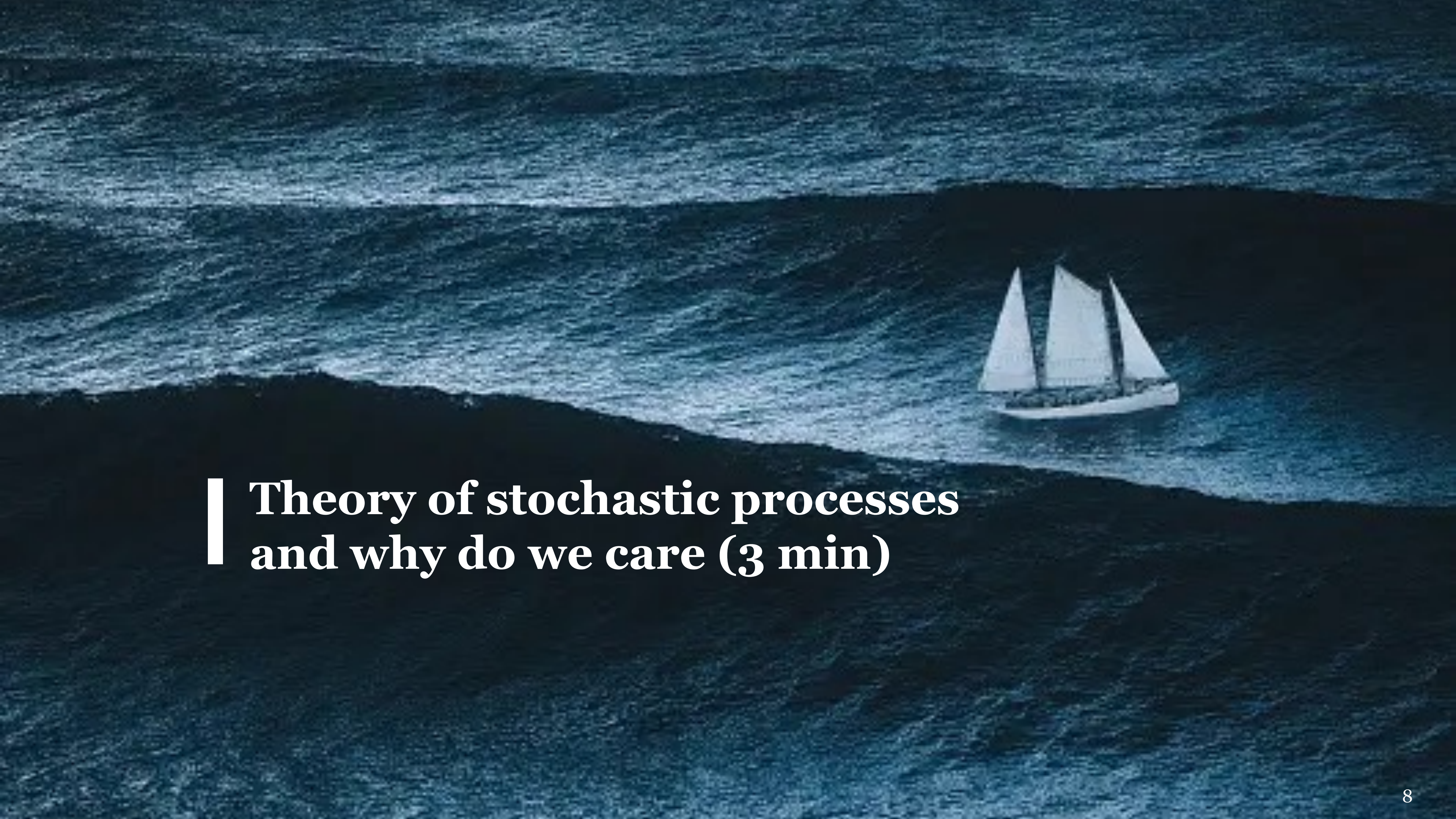


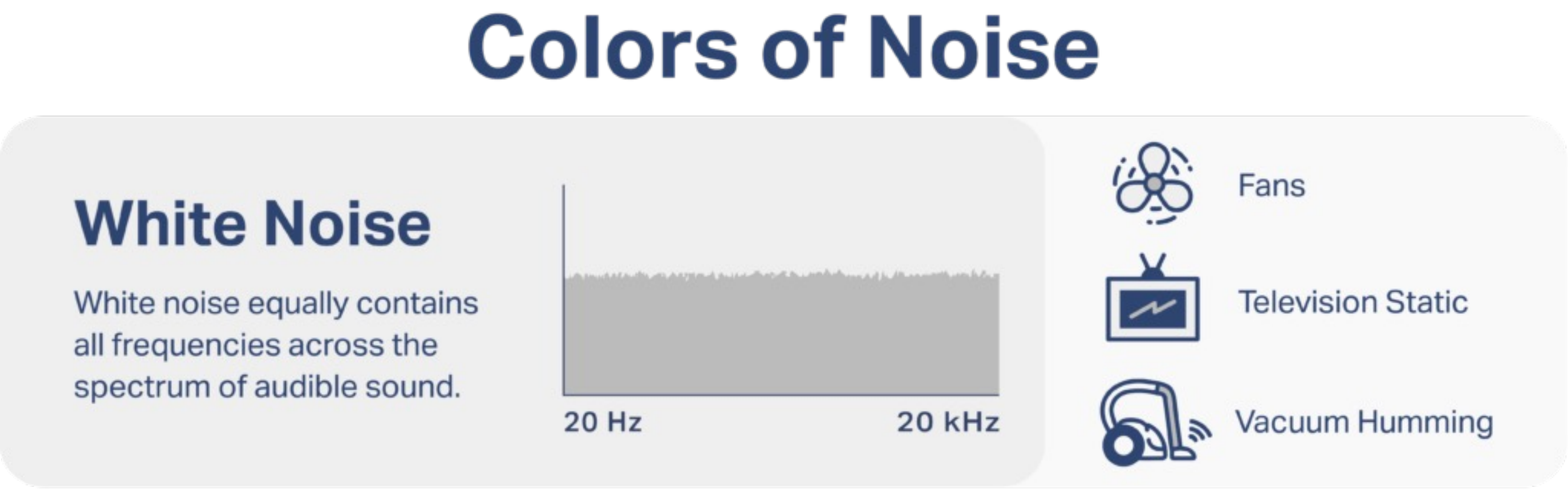
…why not use the noise for coherent control?

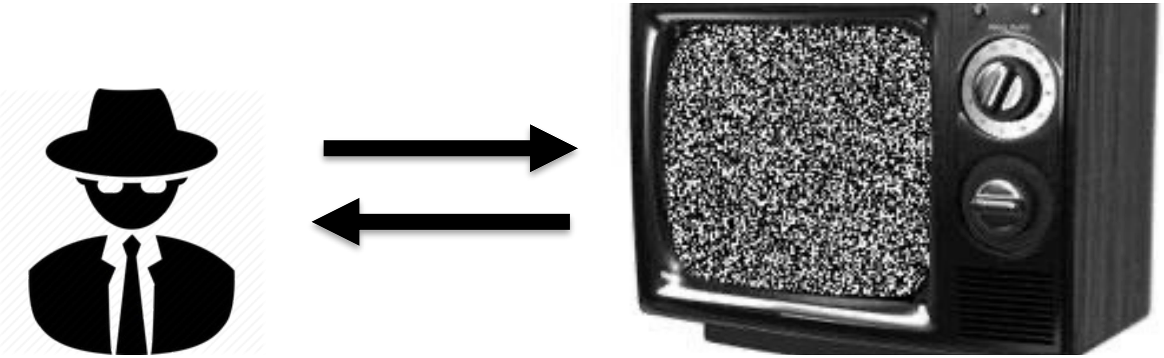
$$H = [\omega + \boldsymbol{\delta\omega(t)}]S_z$$



Theory of stochastic processes and why do we care (3 min)

Markovian
(Quantum optimal control,
Master equation, QEC, QEM)



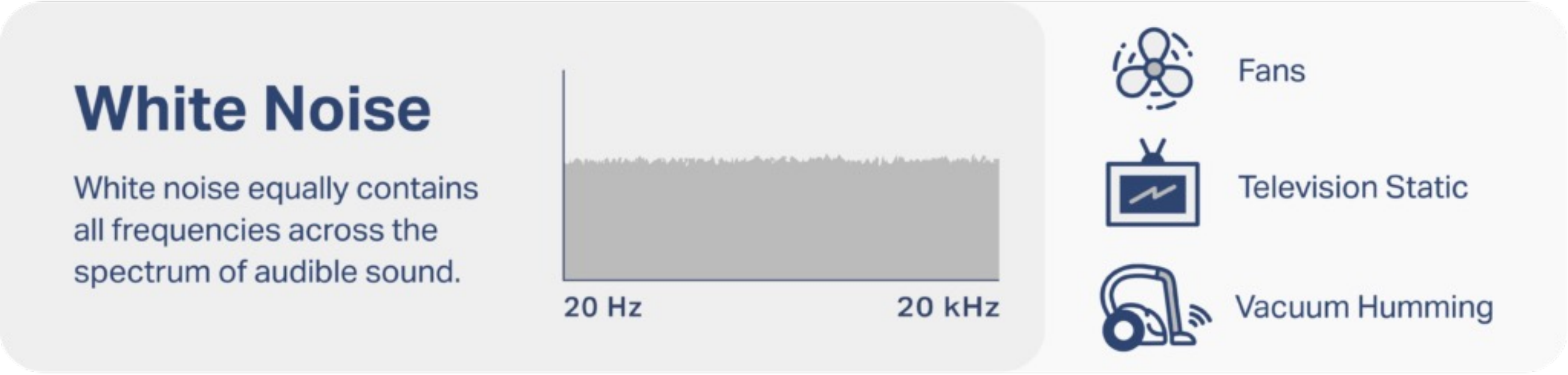


$$\rho(t+s) = e^{Ls}\rho(t)$$

https://www.sleepfoundation.org/noise-and-sleep/pink-noise-sleep

Markovian
(Quantum optimal control,
Master equation, QEC, QEM)

Colors of Noise



At the level of specific open challenges, the **question how reachability differs for non-Markovian compared to Markovian dynamics remains largely unexplored**

Koch, Christiane P., et al. "Quantum optimal control in quantum technologies. Strategic report on current status, visions and goals for research in Europe." *EPJ Quantum Technology* 9.1 (2022): 19.

Markovian
(Quantum optimal control,
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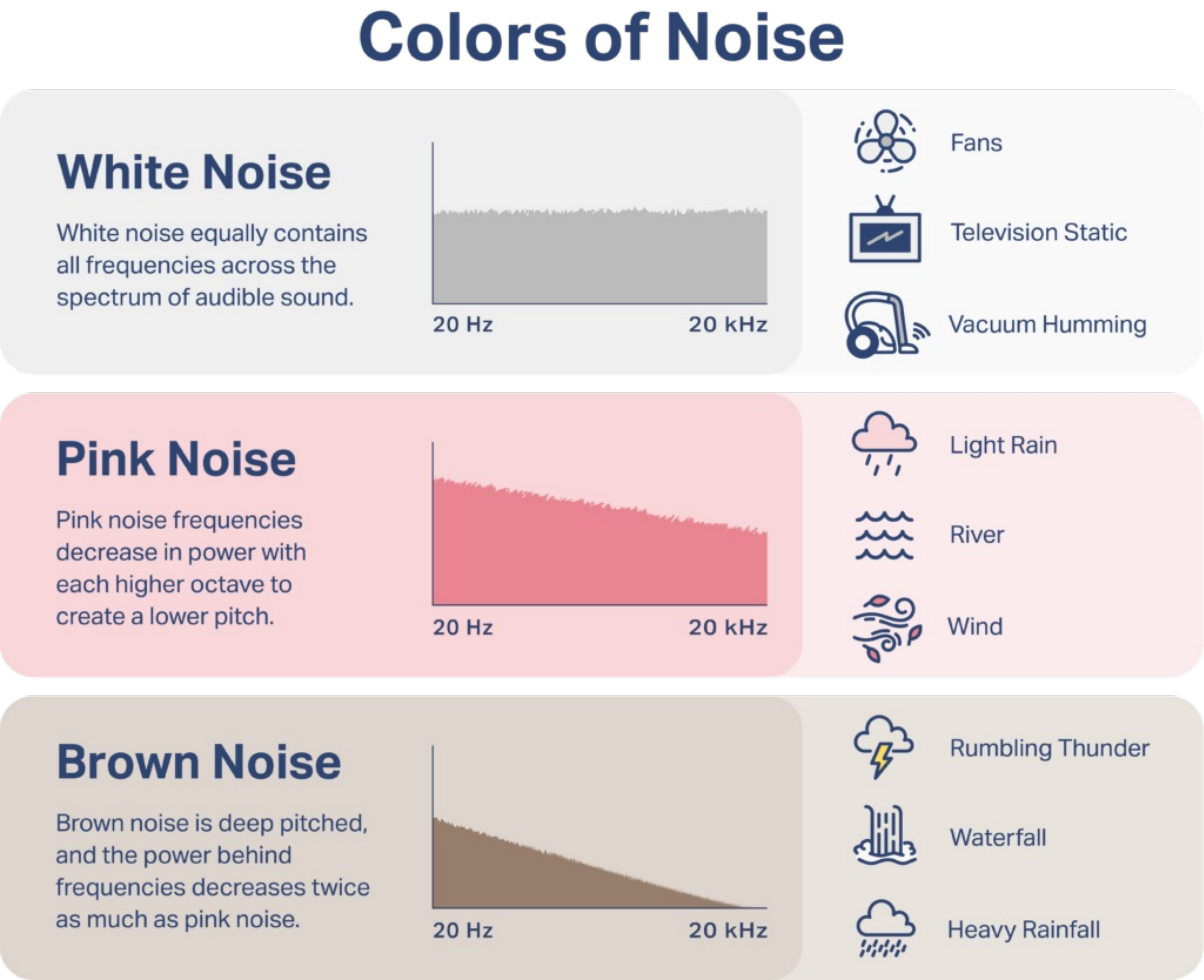
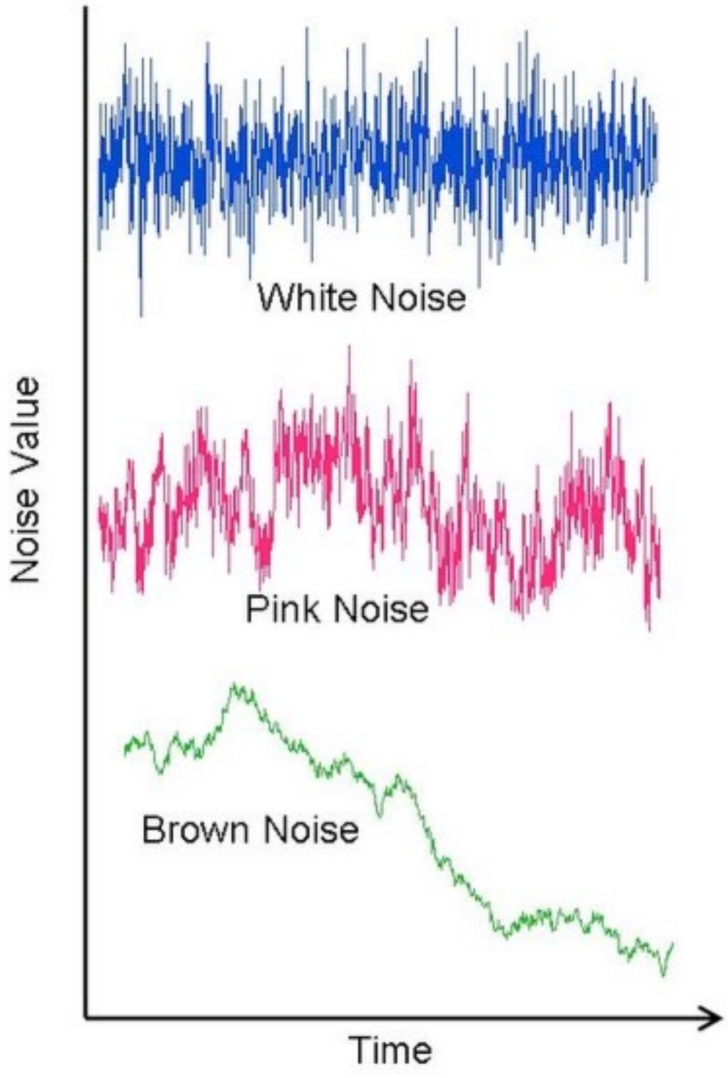


Charge noise

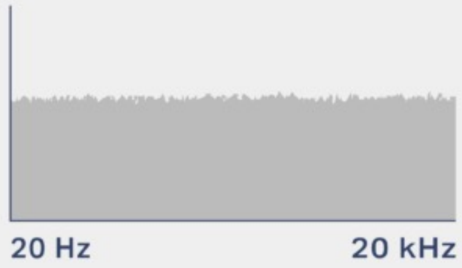
Magnetic noise

Colors of Noise



White Noise

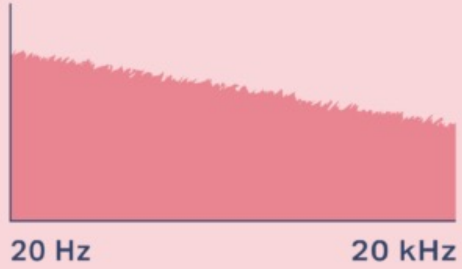
White noise equally contains all frequencies across the spectrum of audible sound.



- Fans
- Television Static
- Vacuum Humming

Pink Noise

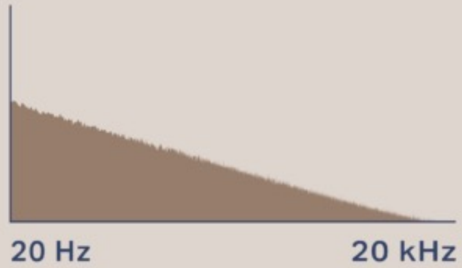
Pink noise frequencies decrease in power with each higher octave to create a lower pitch.



- Light Rain
- River
- Wind

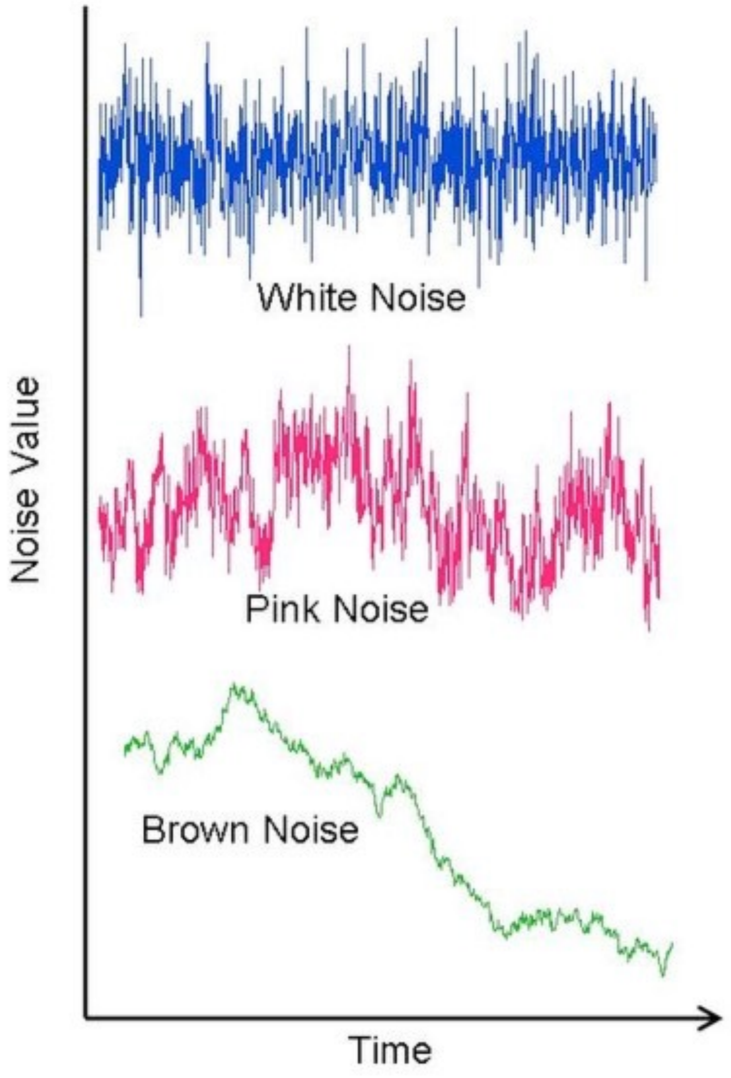
Brown Noise

Brown noise is deep pitched, and the power behind frequencies decreases twice as much as pink noise.



- Rumbling Thunder
- Waterfall
- Heavy Rainfall

We rely on the trajectory



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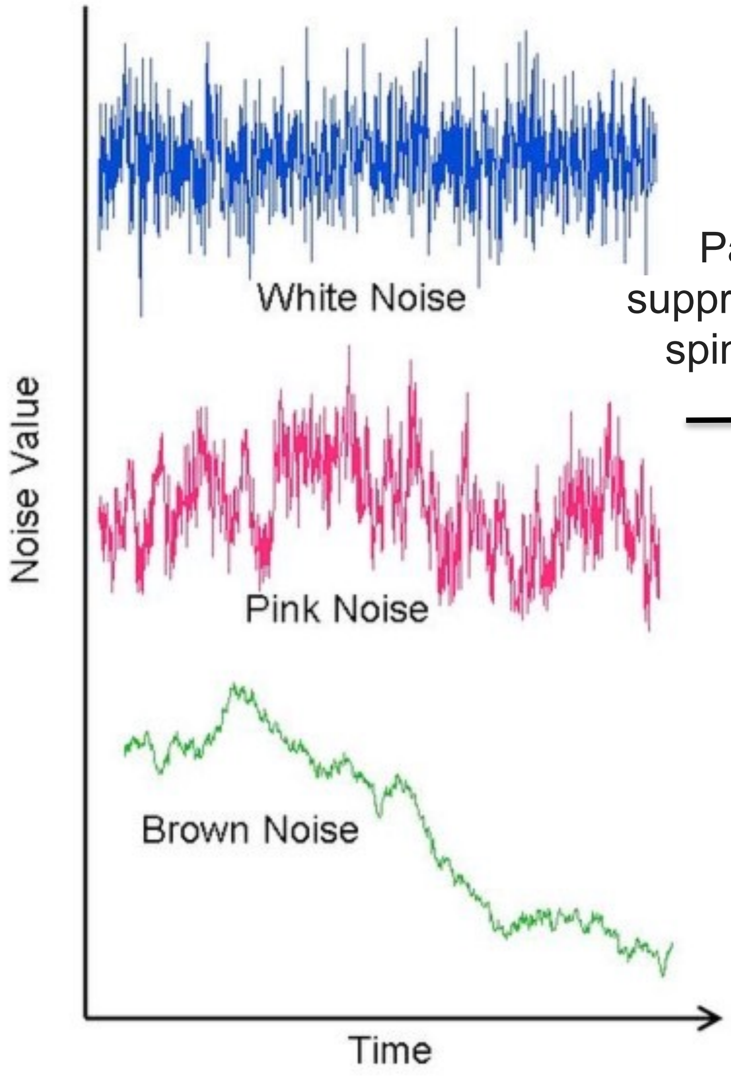
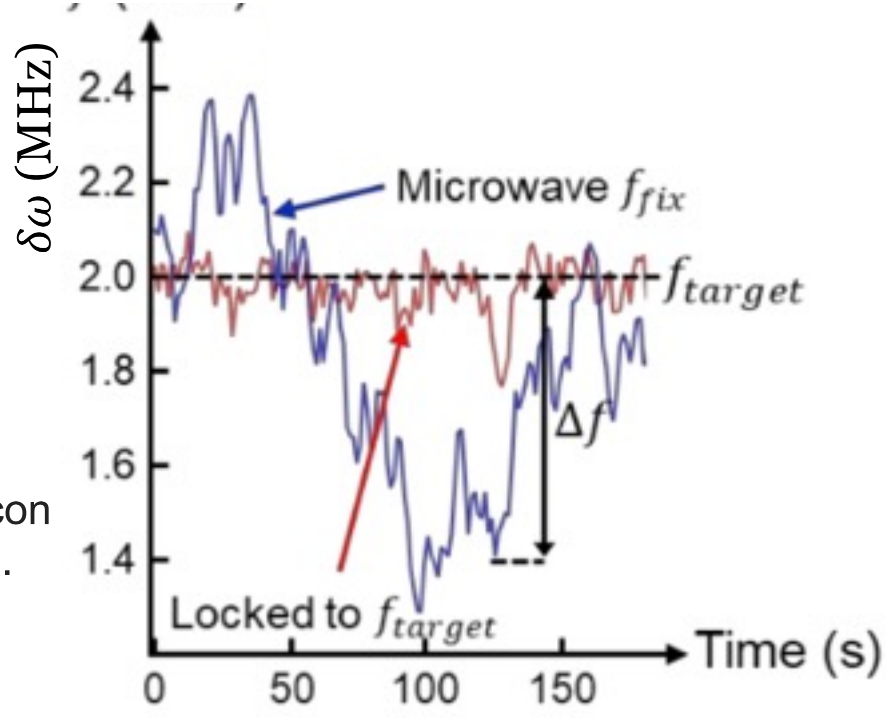


Park, J., et al. "Passive and active suppression of transduced noise in silicon spin qubits."*arXiv:2403.02666* (2024).



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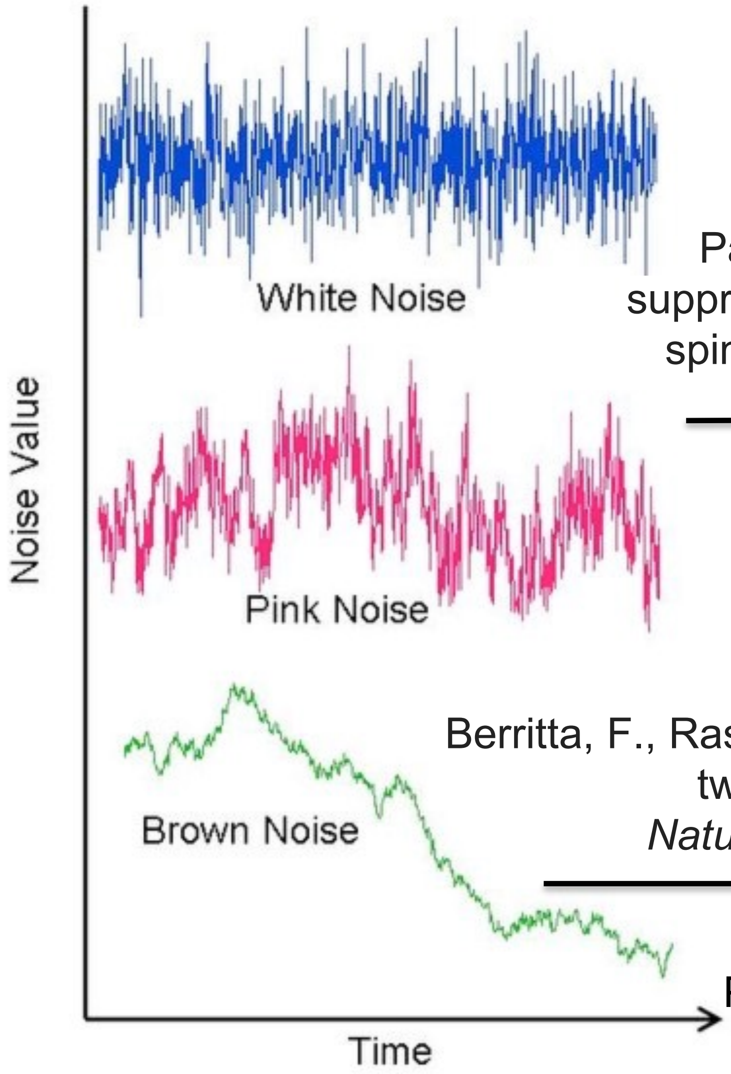

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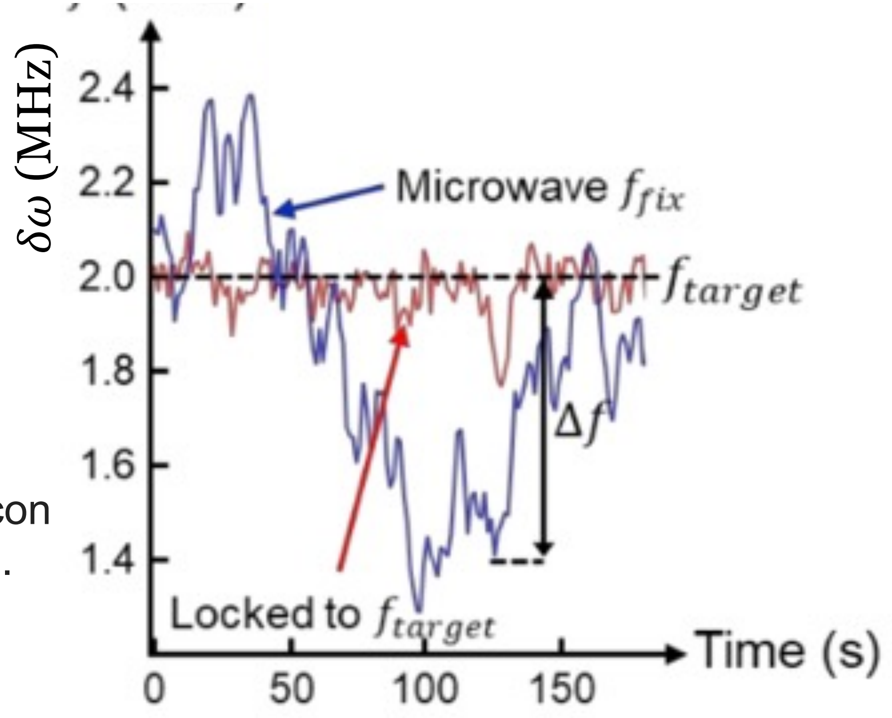
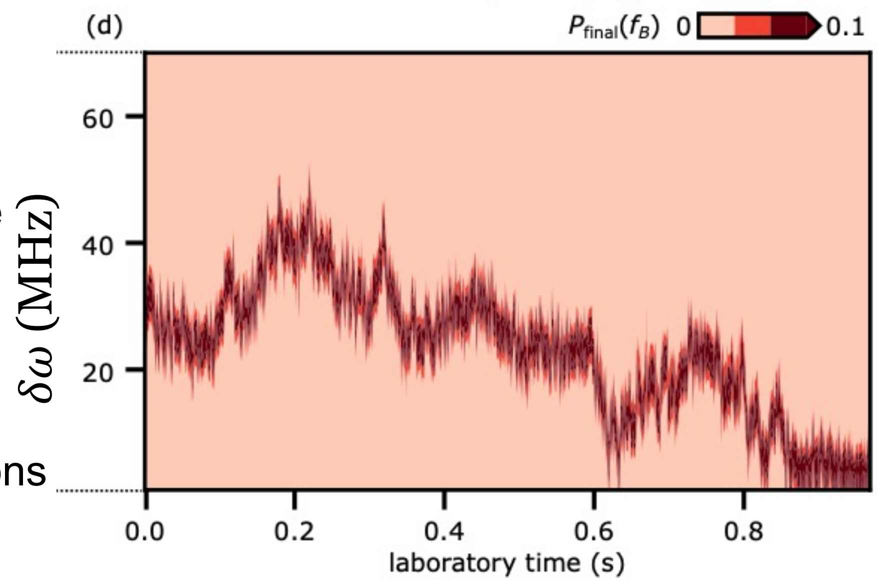
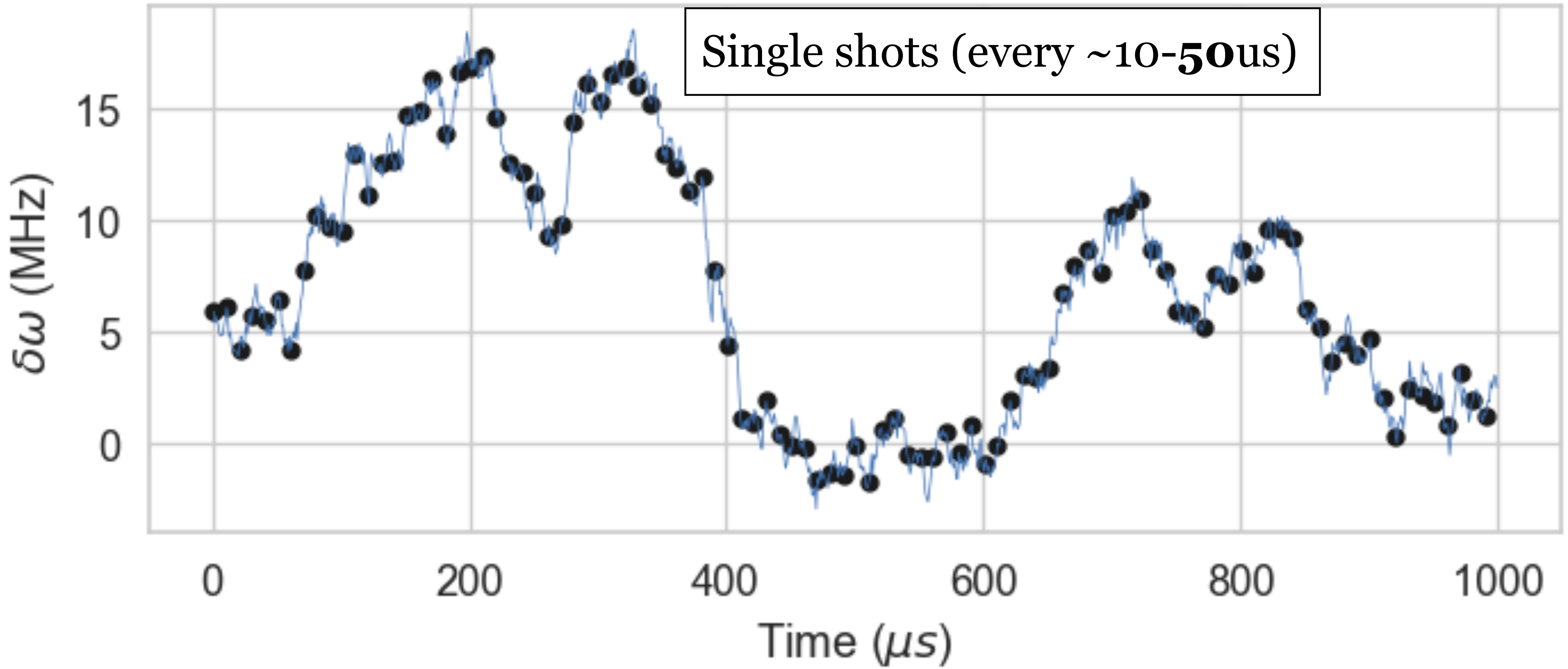


Berritta, F., Rasmussen, T., **J.A.K**. et al. (2024). Real-time two-axis control of a spin qubit. *Nature Communications*, *15*(1), 1676



Beritta F., **J.A.K** et al. (2024)
Physics-informed tracking of qubit fluctuations
arXiv:2404.09212

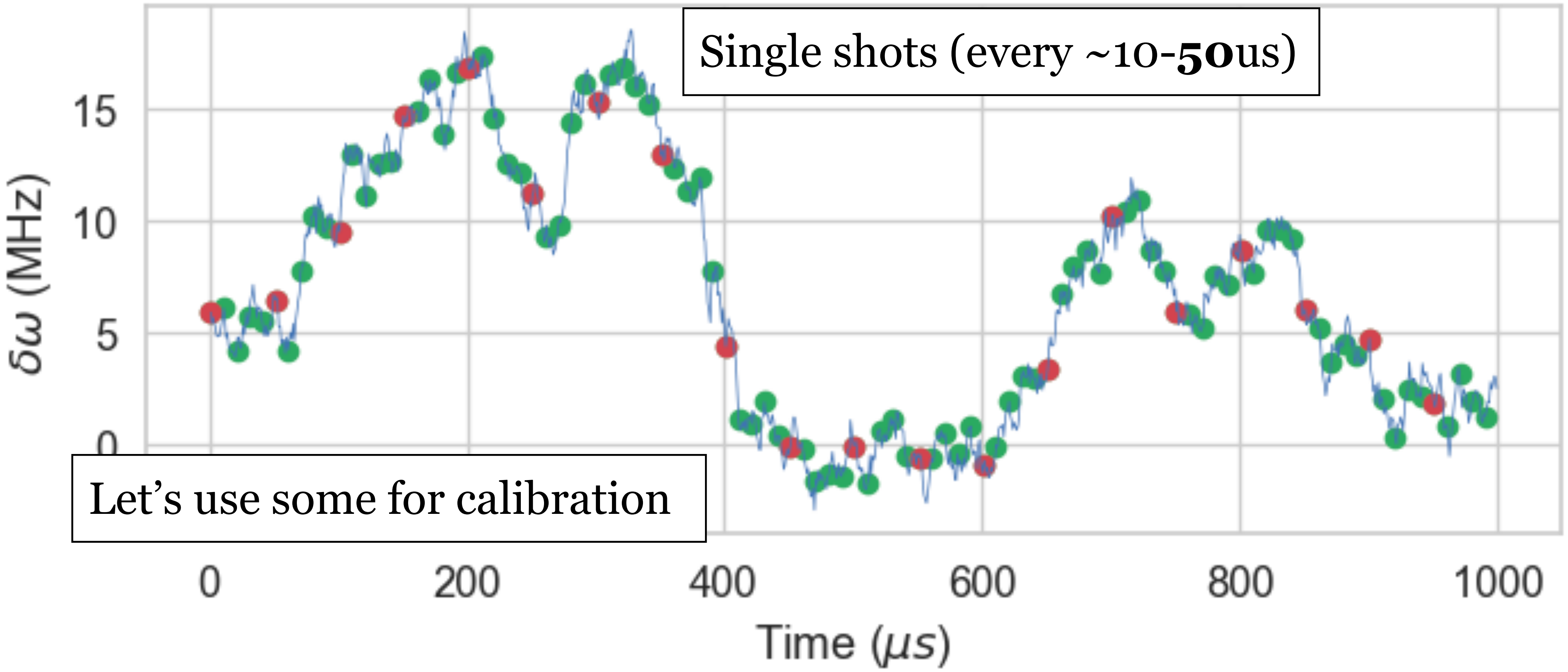
Research strategy – there is plenty of time



Quasistatic approximation

$$H_n = [\omega + \delta\omega_n] S_z$$

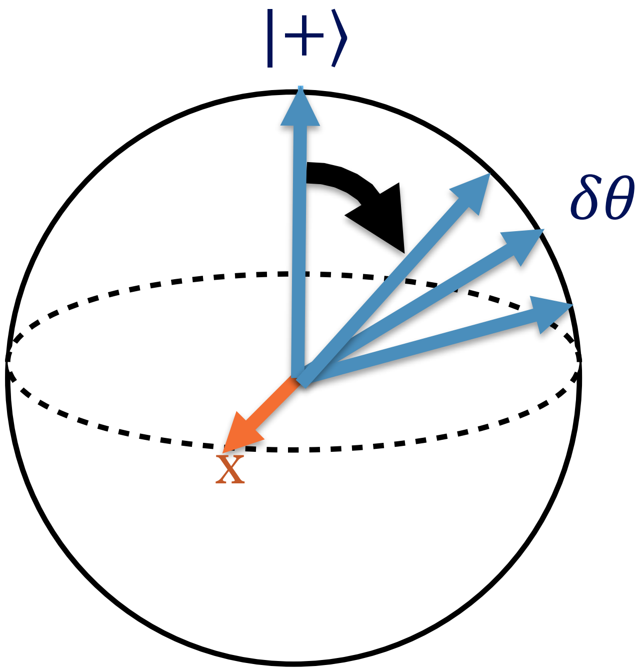
… to correct for the noise.

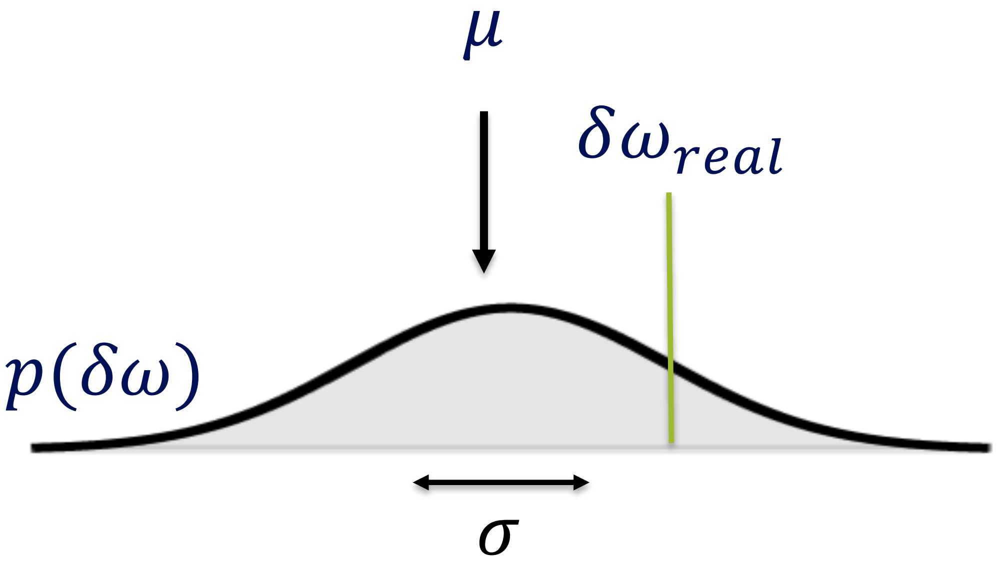


$$H_n = [\omega + \delta\omega_n] S_z$$

Bayesian estimation (2 min)

Phase gate

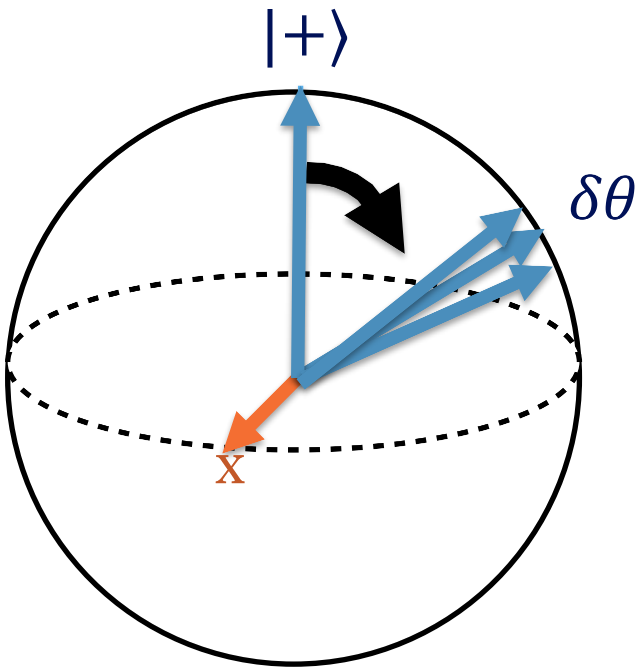


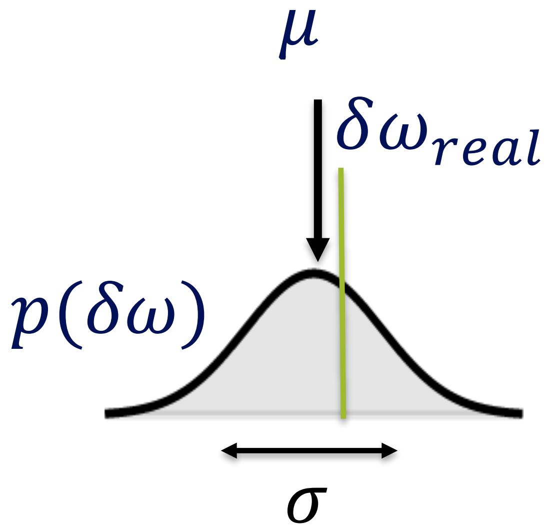


Gate infidelity

$$1 - F \propto \langle \delta\theta^2 \rangle \propto \sigma^2\theta^2/\mu^2$$

Phase gate





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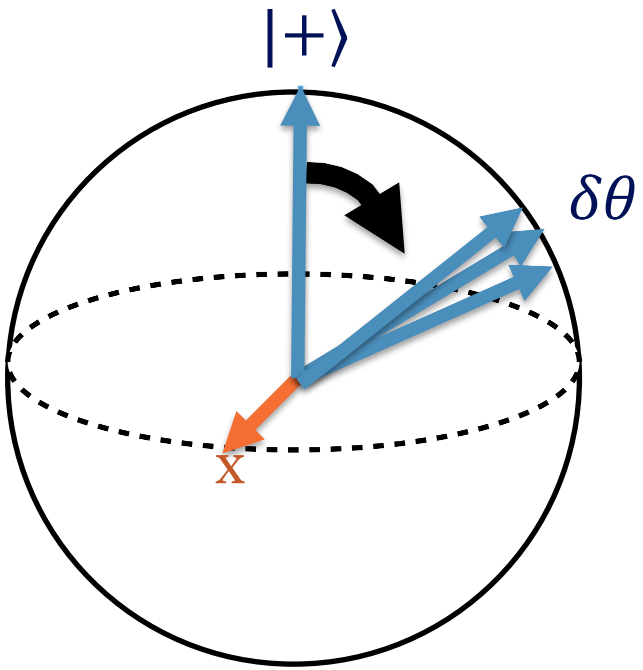

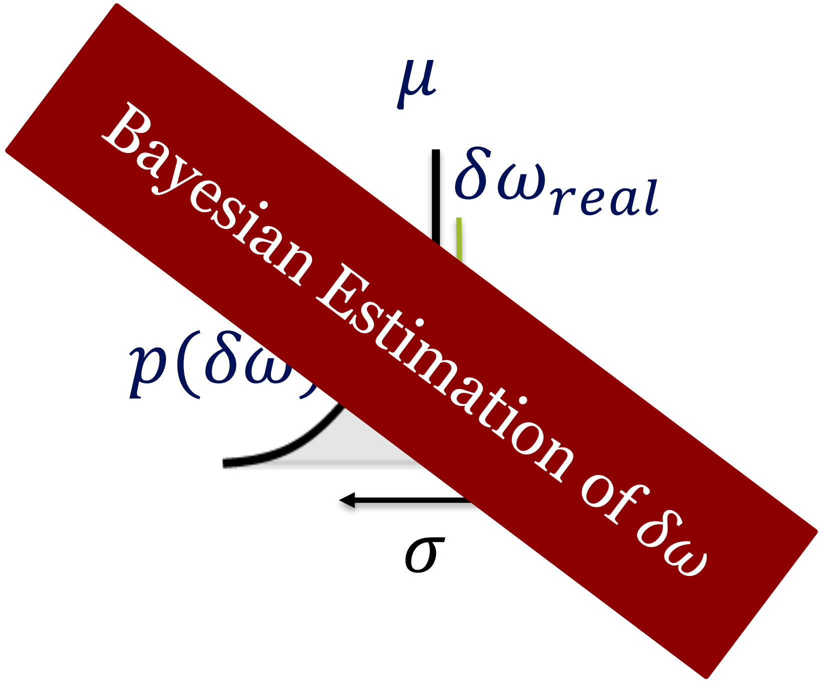

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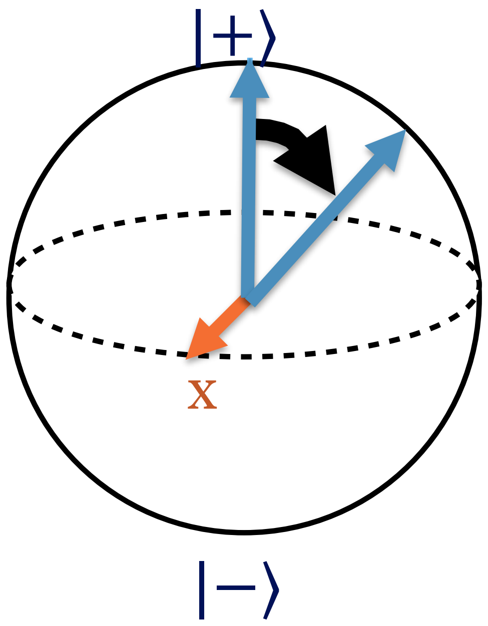
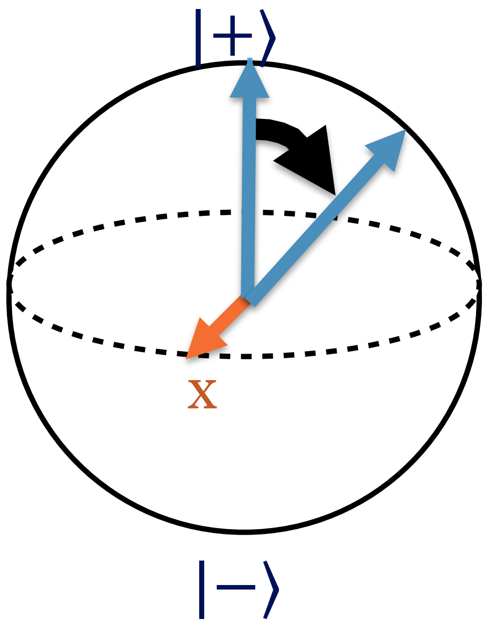


1. Find likelihood function

$$p(x|\delta\omega, \tau) \propto 1 + \mathrm{x}\cos(\delta\omega\, \tau)$$

Bayesian estimation



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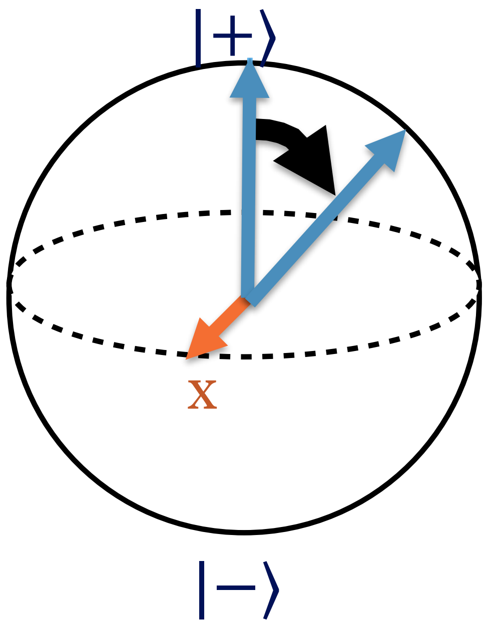
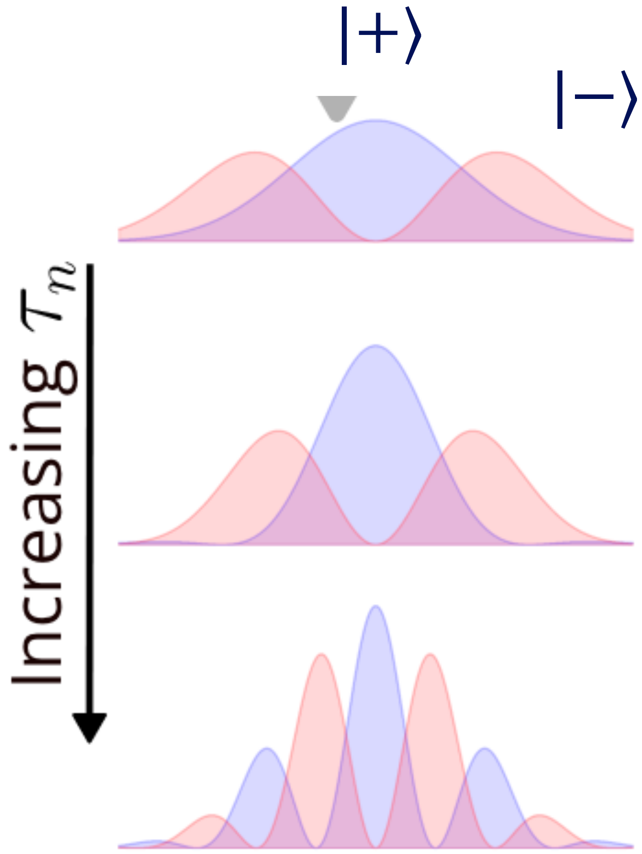
2. Use Bayesian formula

$$p(\delta\omega|x_1, \tau) \propto p(x_1|\delta\omega, \tau)p_0(\delta\omega)$$

F. Berritta, T. Rasmussen, JAK, ... EVN et al., *Nature Communications*, *15*(1), 1676

Bayesian estimation





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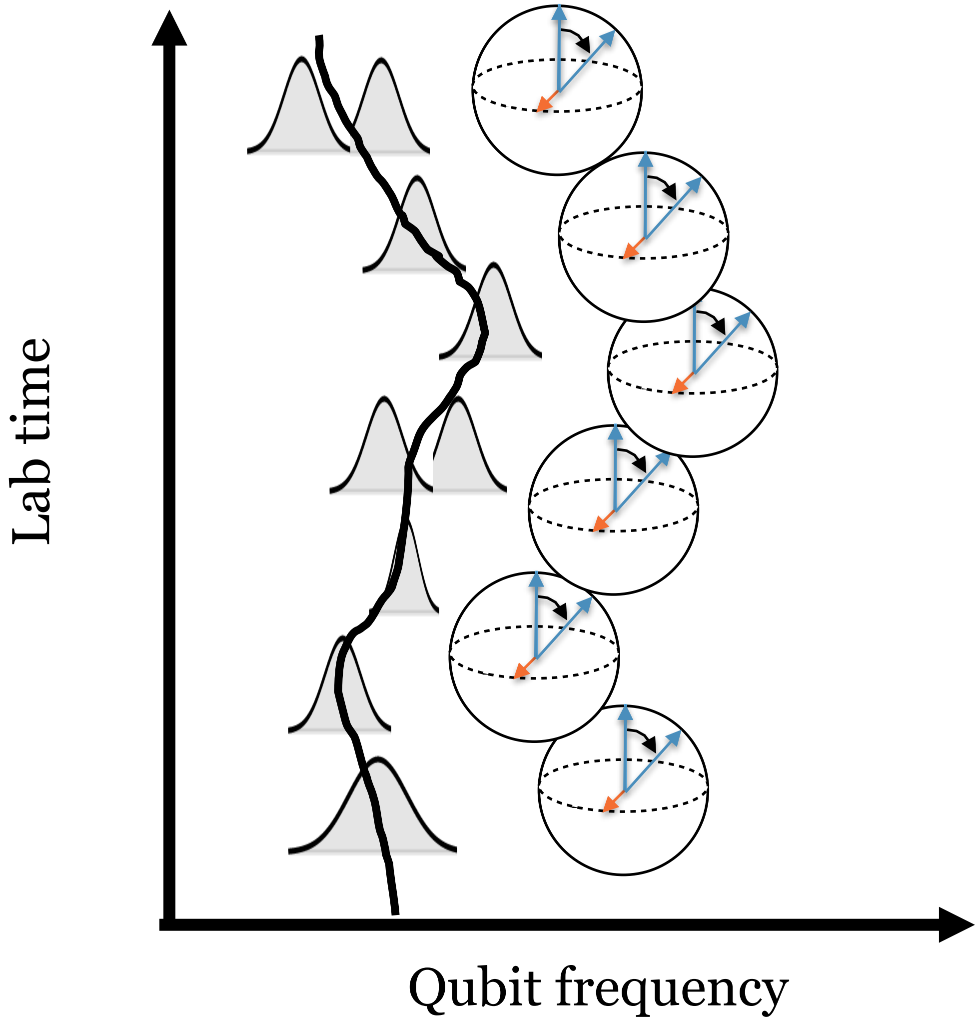
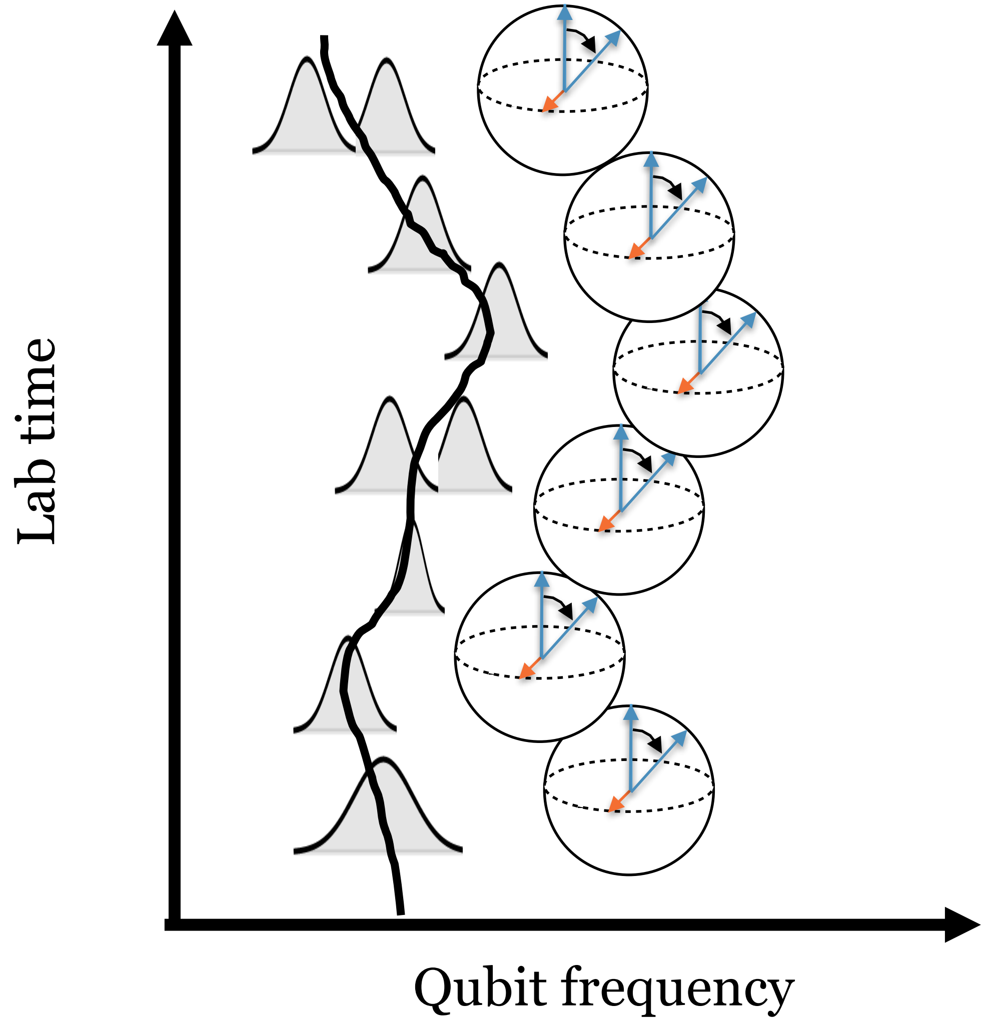
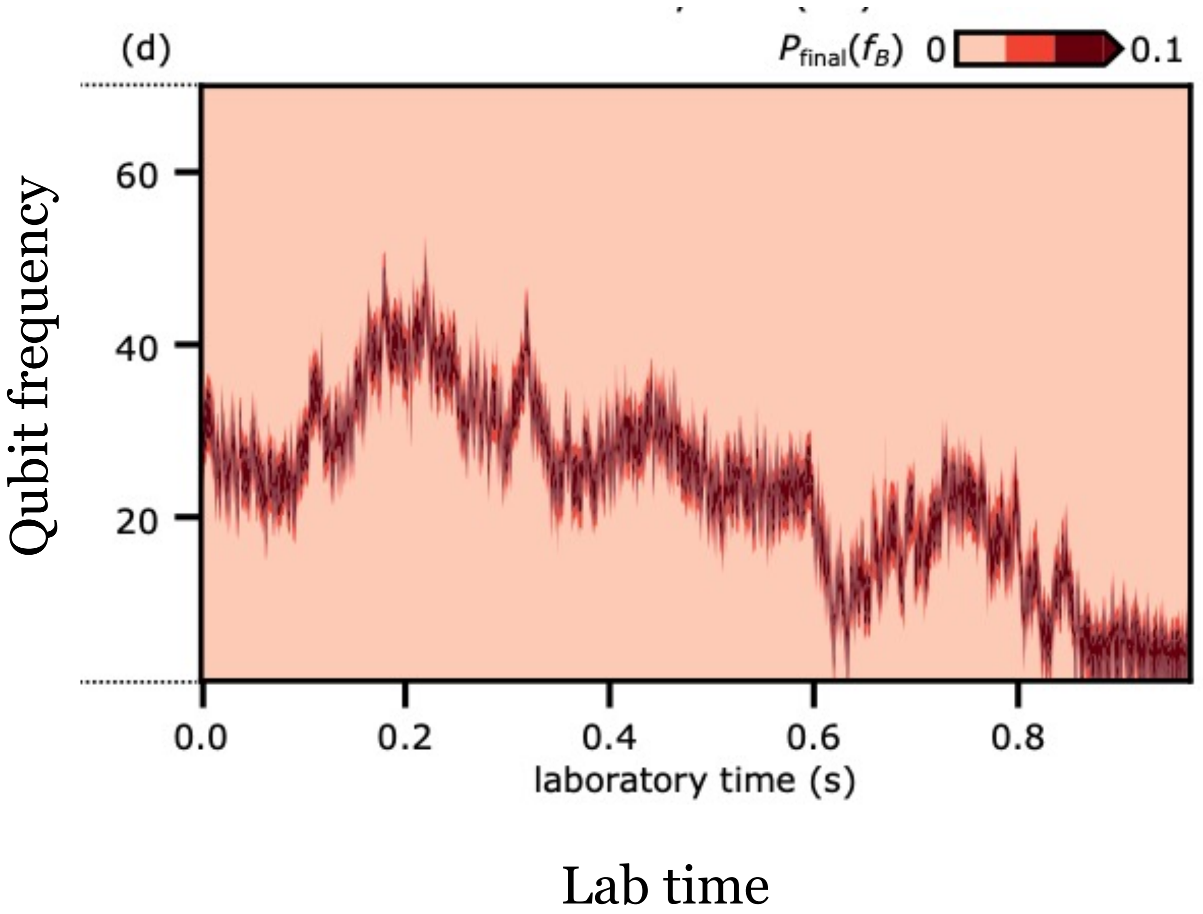
Bayesian tracking



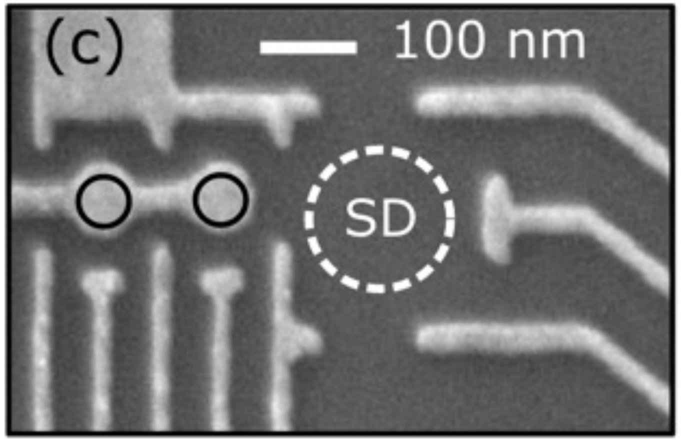
Bayesian tracking

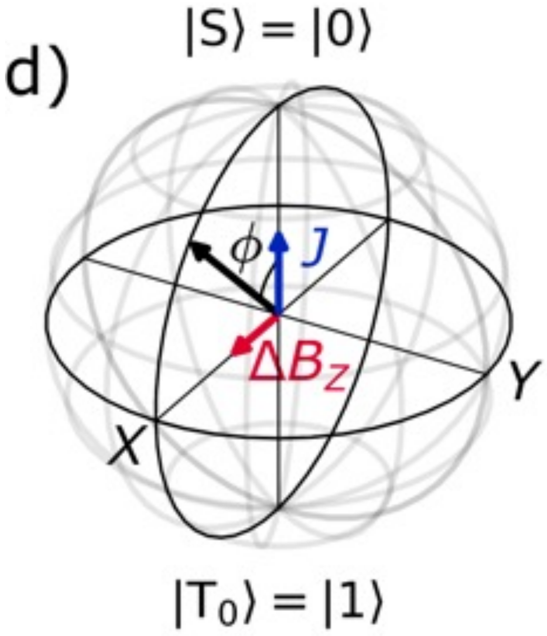




Coherent rotation around noise - experiment (3 min)

Experiment





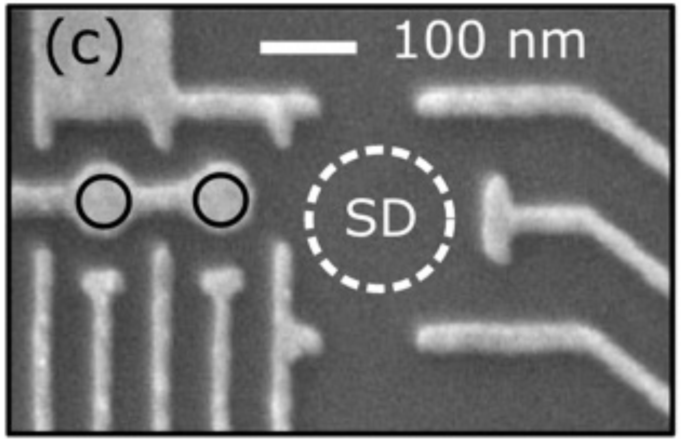
GaAs spin qubit

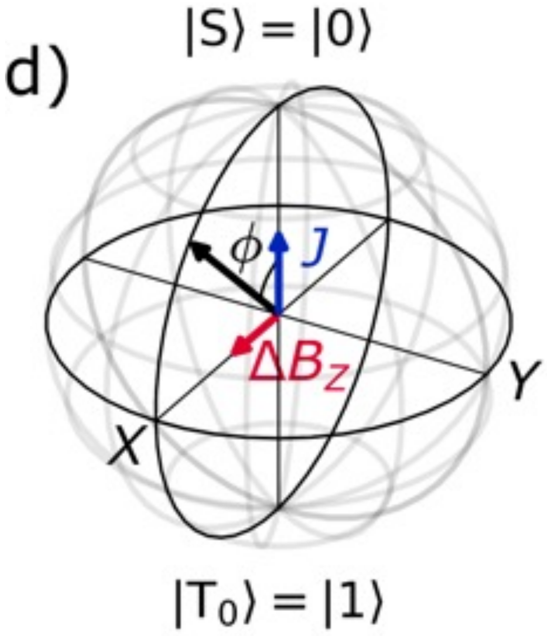
S-T subspace

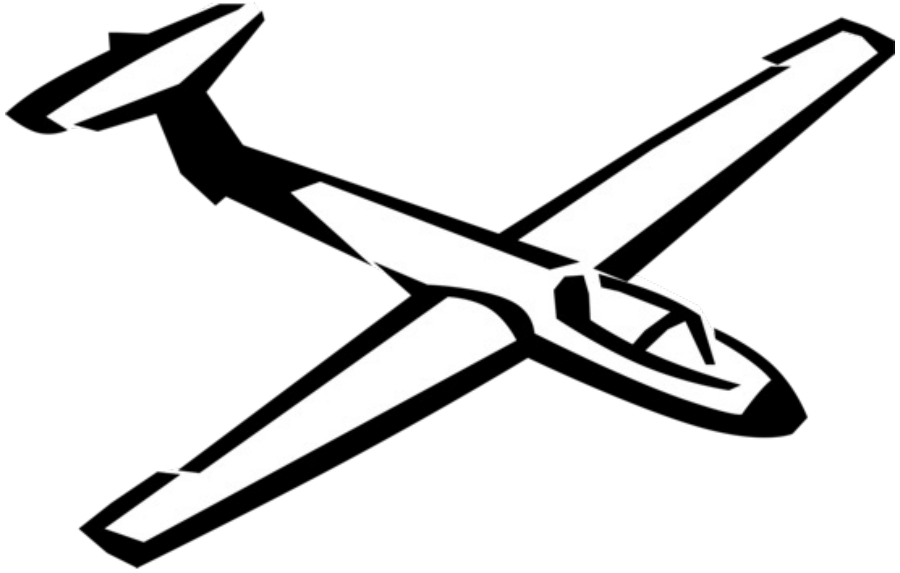
•

$$H(\epsilon) = \delta\omega(t)S_x + J(\epsilon)S_z$$

Experiment







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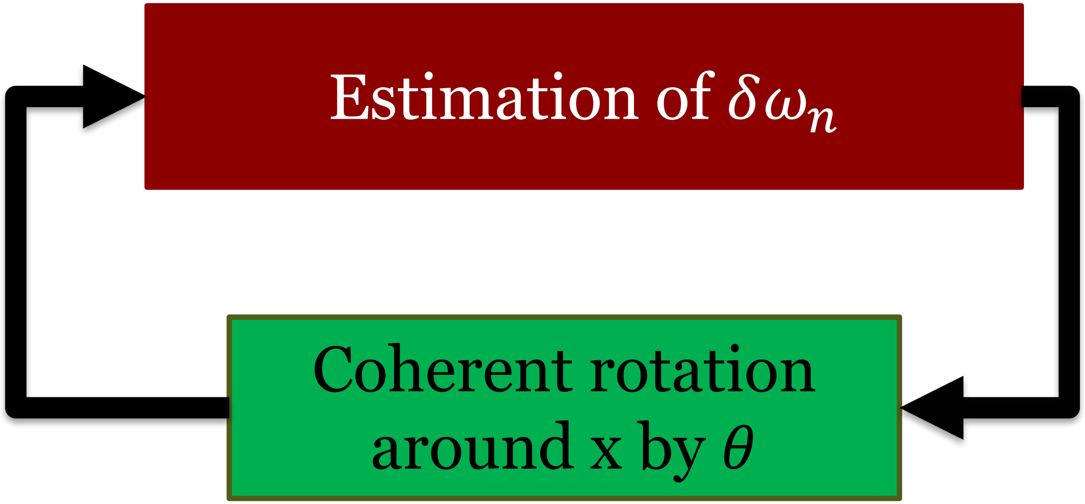


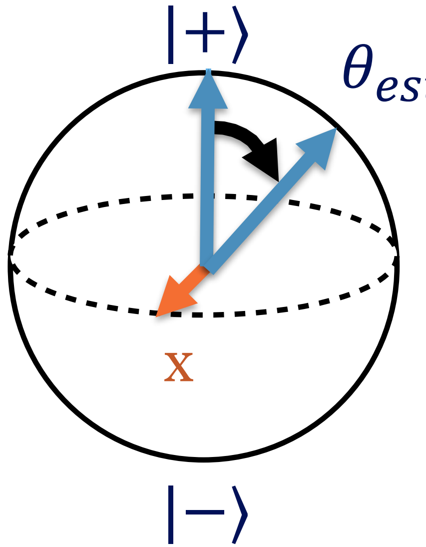
$$H(\epsilon_{11}) = \delta\omega(t) S_x$$

Zero-average

Experiment

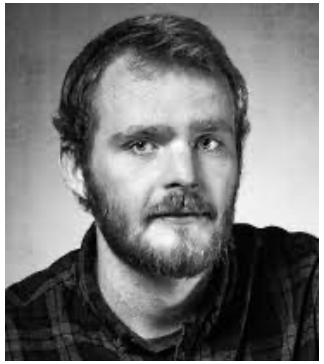
$$H(\epsilon) = \Omega_L S_x$$

$$\theta_{est} = \Omega_L \, t_i$$



Center for Quantum Devices
Niels Bohr Institute

Fabrizio Berritta
TALK PAS 2.4
(yesterday)



Torbjørn Raasø
Rasmussen

Experiment

$$H(\epsilon) = \Omega_L S_x$$

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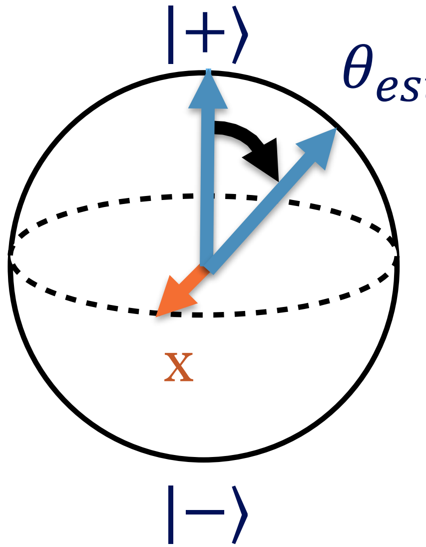

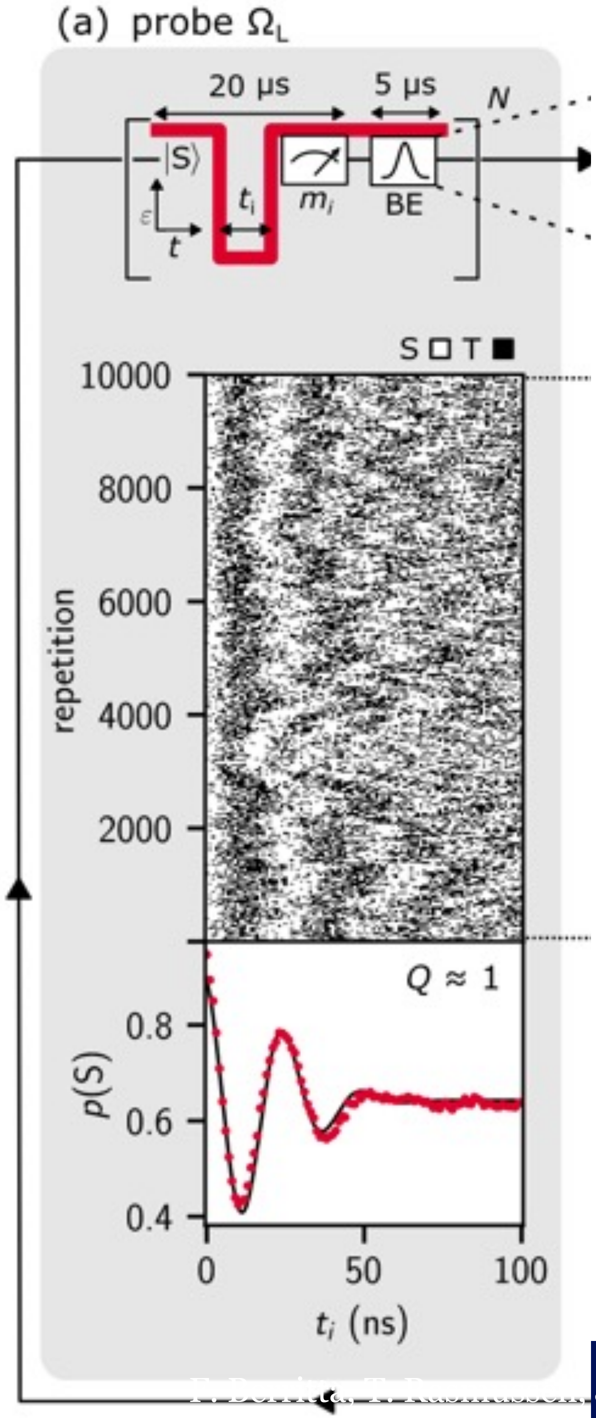

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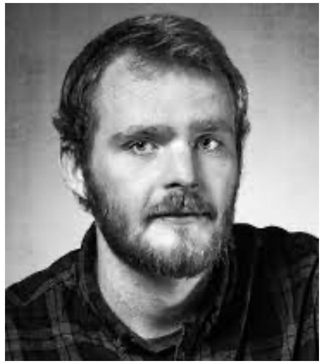

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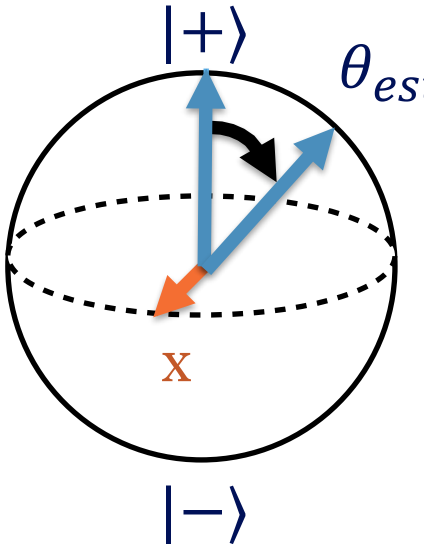

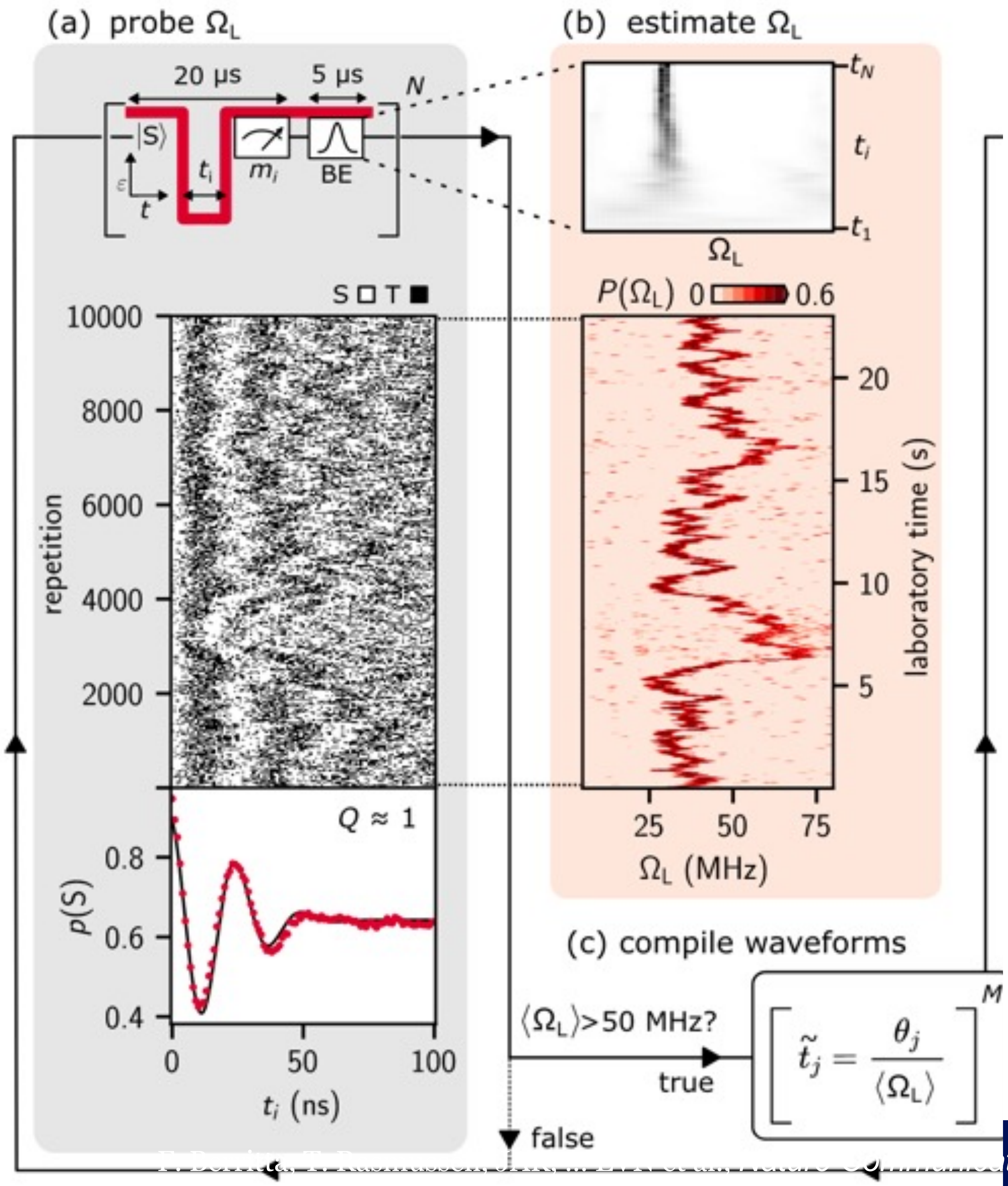

Experiment

$$H(\epsilon) = \Omega_L S_x$$

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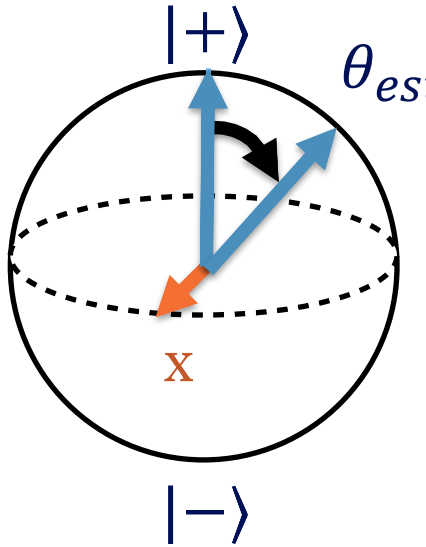
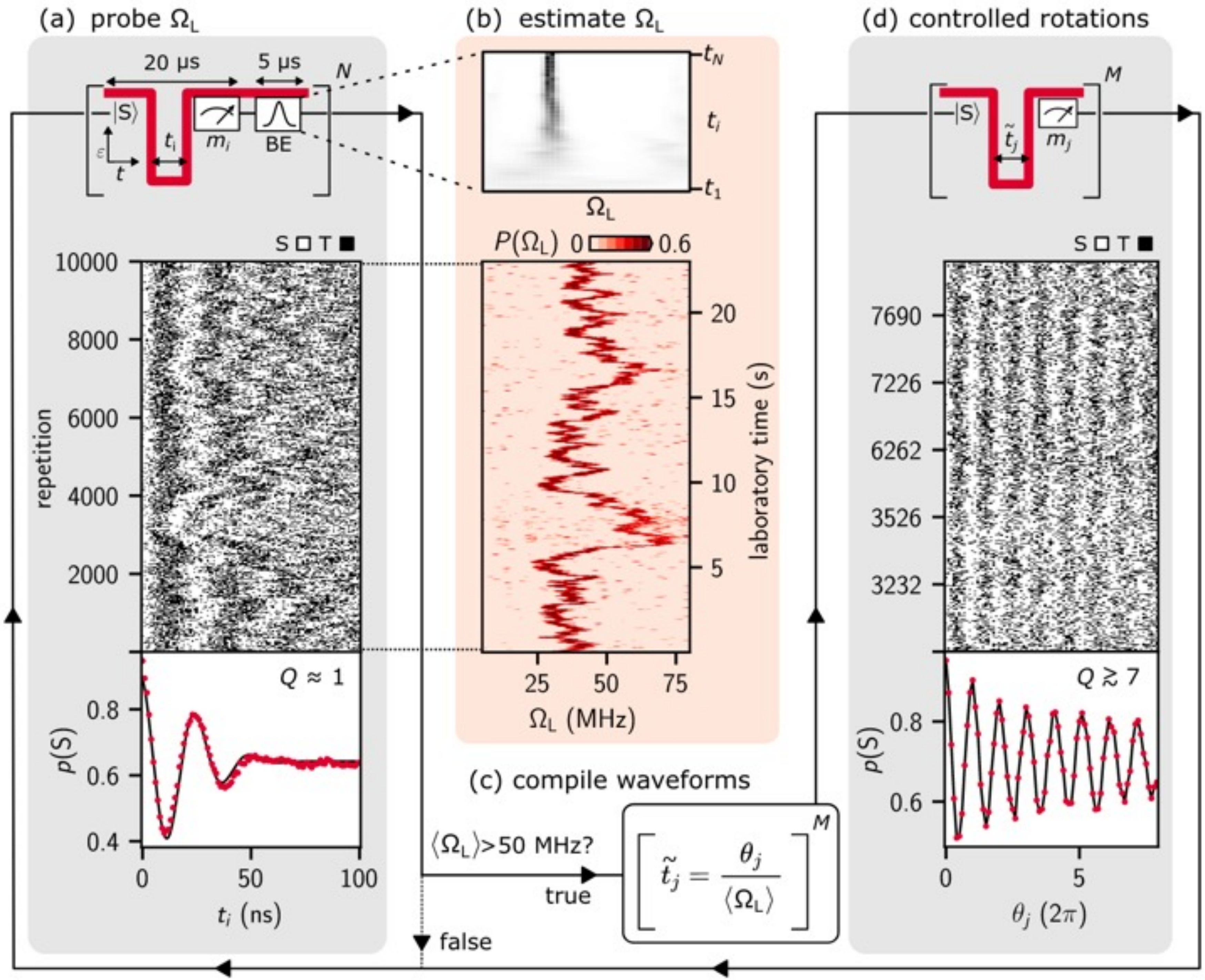


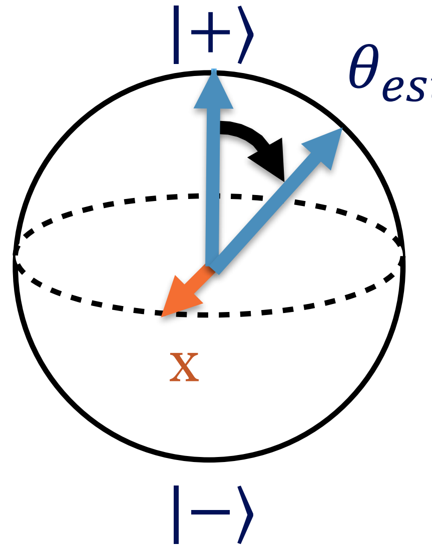


⟨aQa^L⟩

Experiment

$$H(\epsilon) = \Omega_L S_x$$

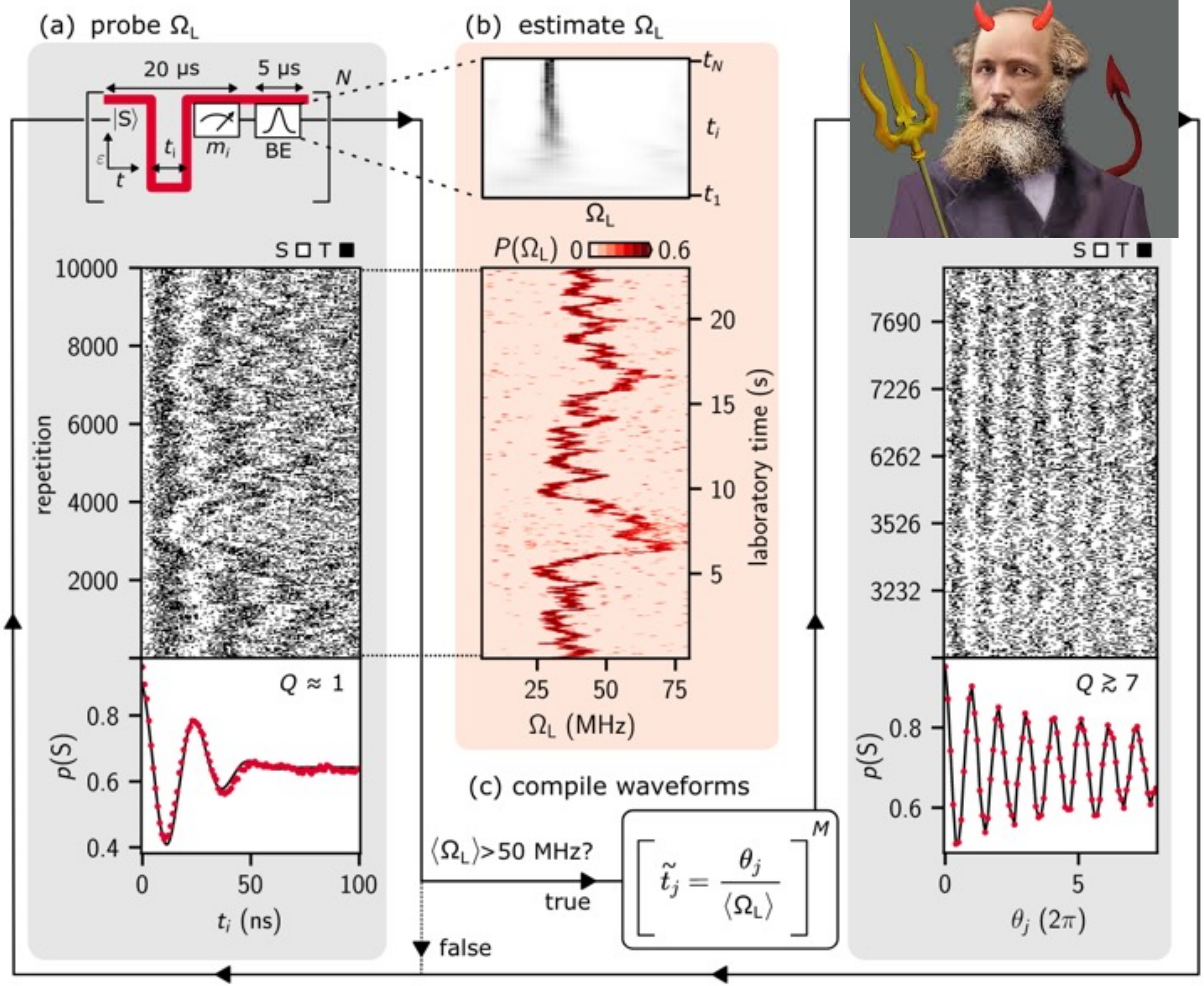
$\theta_{est} = \Omega_L\, t_i$



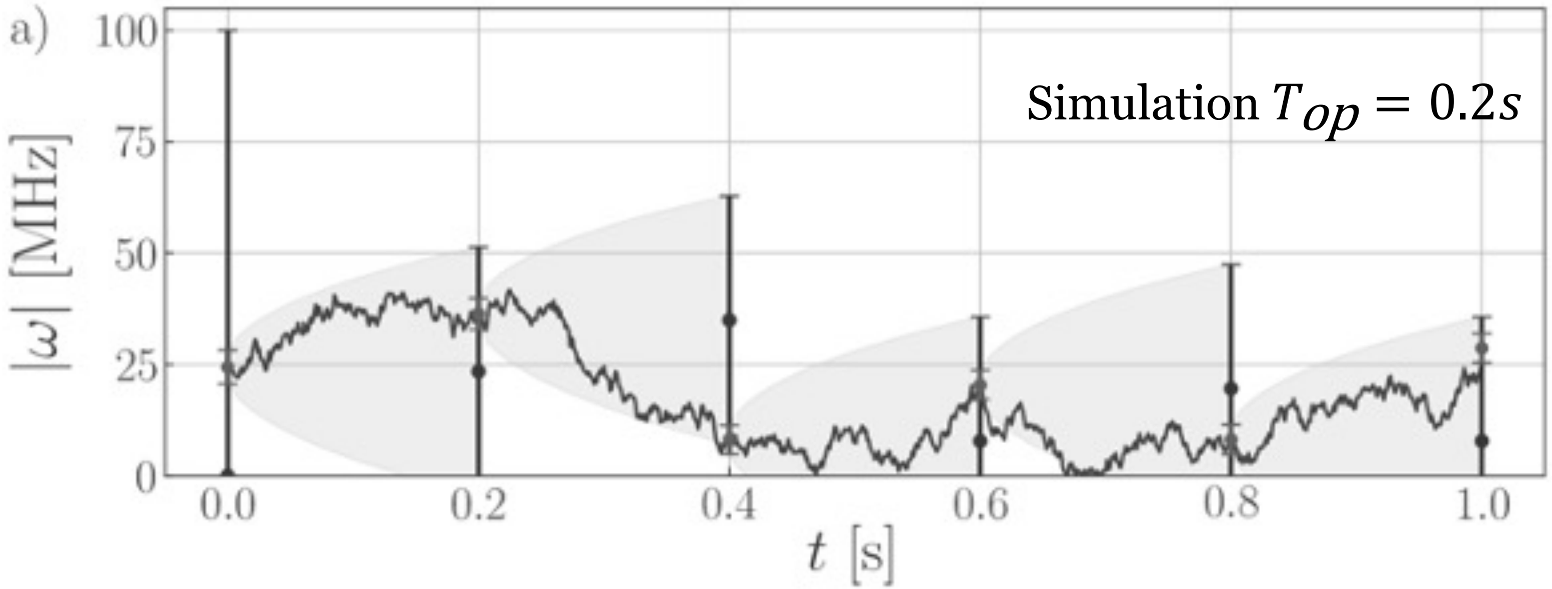
If Ω_L > 50 MHz

Rotate by θ, using time

$$\tau = \frac{\theta}{\widehat{\Omega}_L}$$



Our progress

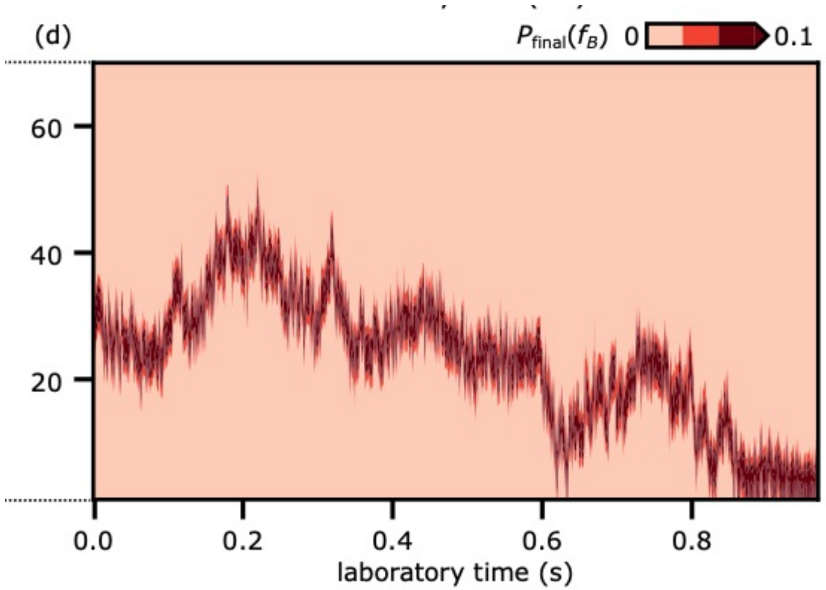


Fabrizio Berritta

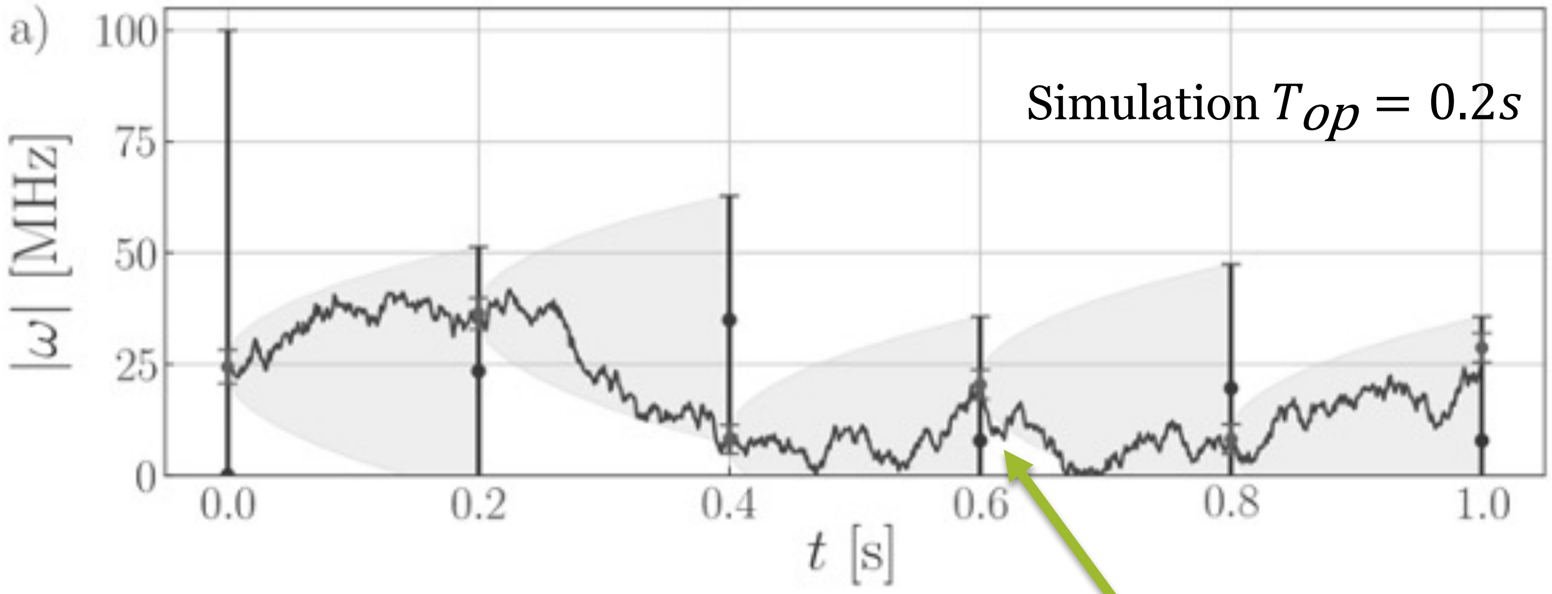
Jacob Benestad

1. Physics-informed estimation
   (No outliers)

$$P(\delta\omega'\, t|\delta\omega, 0)$$

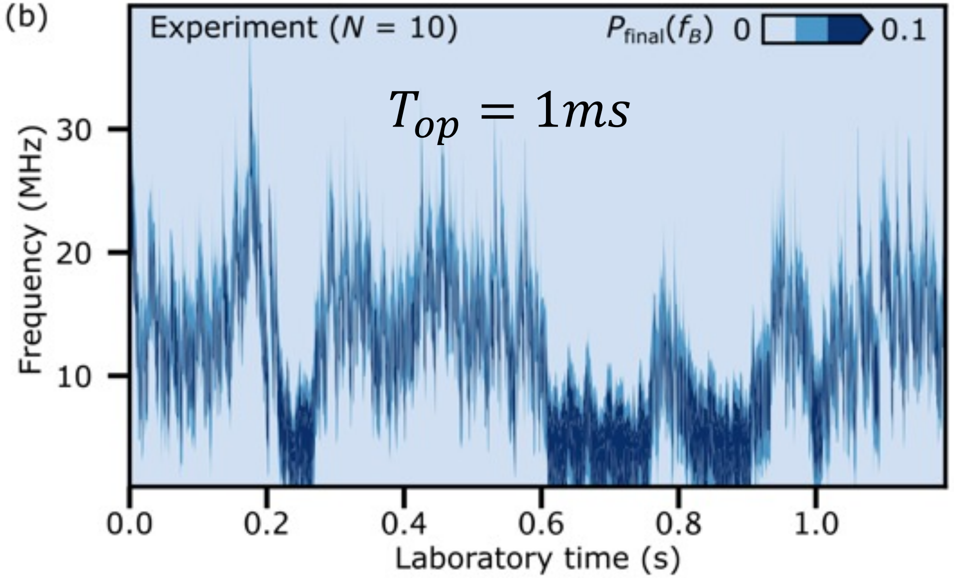


Our progress



Fabrizio Berritta

Jacob Benestad



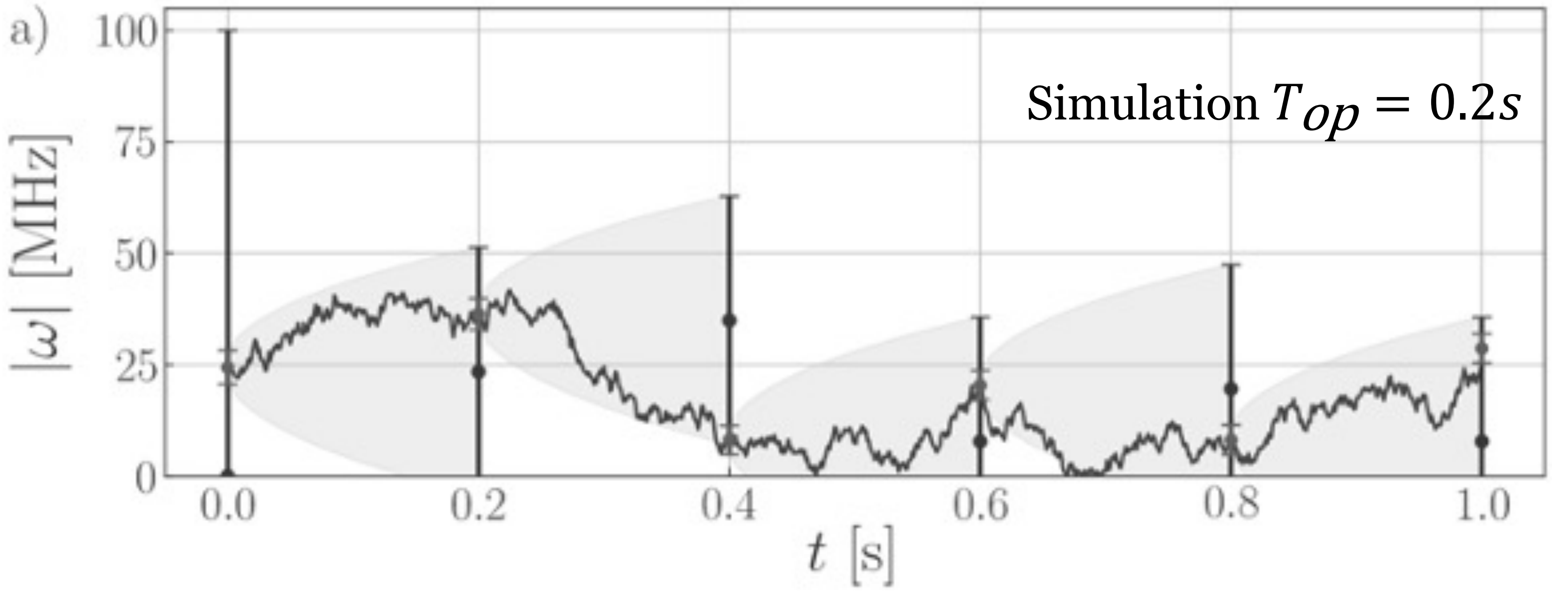
1. Physics-informed estimation
(No outliers)

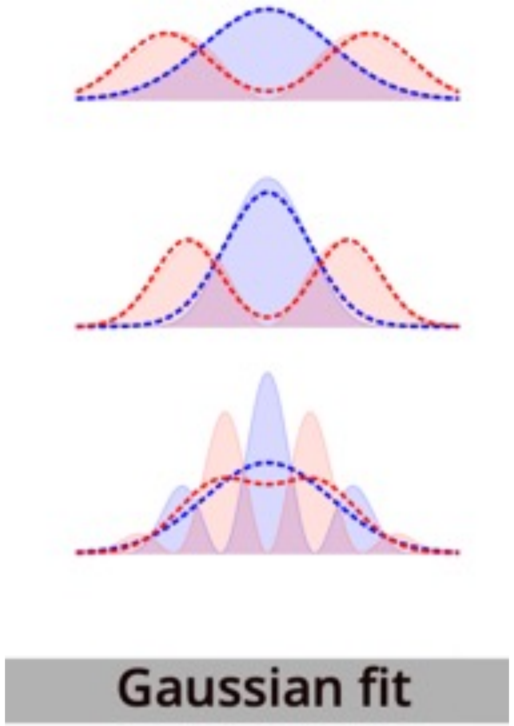
$$P(\delta\omega'\, t|\delta\omega, 0)$$

2. Adaptive time
(Few shots)

$$\tau_n = \frac{c}{\sigma_{n-1}}$$

Our progress





Jacob Benestad

1. Physics-informed estimation
(No outliers)

$$P(\delta\omega'\, t|\delta\omega, 0)$$

2. Adaptive time
(Few shots)

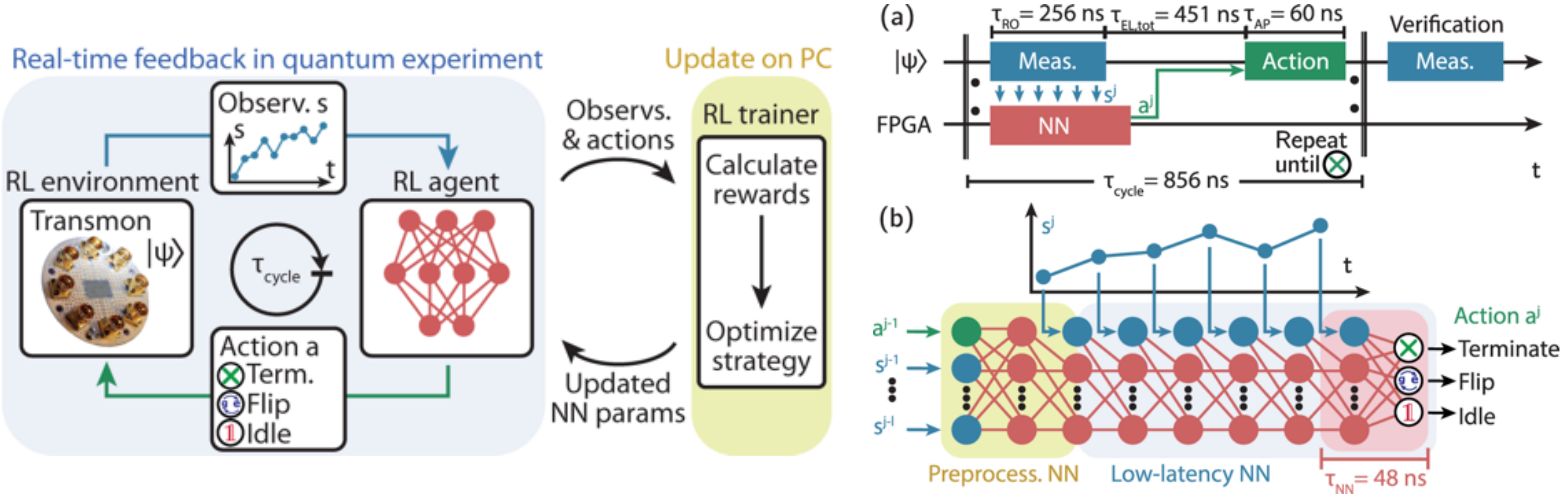
$$t_n = \frac{c}{\sigma_{n-1}}$$

3. Method of moments
(Low memory)

$$\mu_{n+1} = f(\mu_n, \sigma_n)$$
$$\sigma_{n+1} = g(\mu_n, \sigma_n)$$

The game and its heuristics (2 min)

Reinforcement learning on the FPGA



1. Hybrid optimisation method

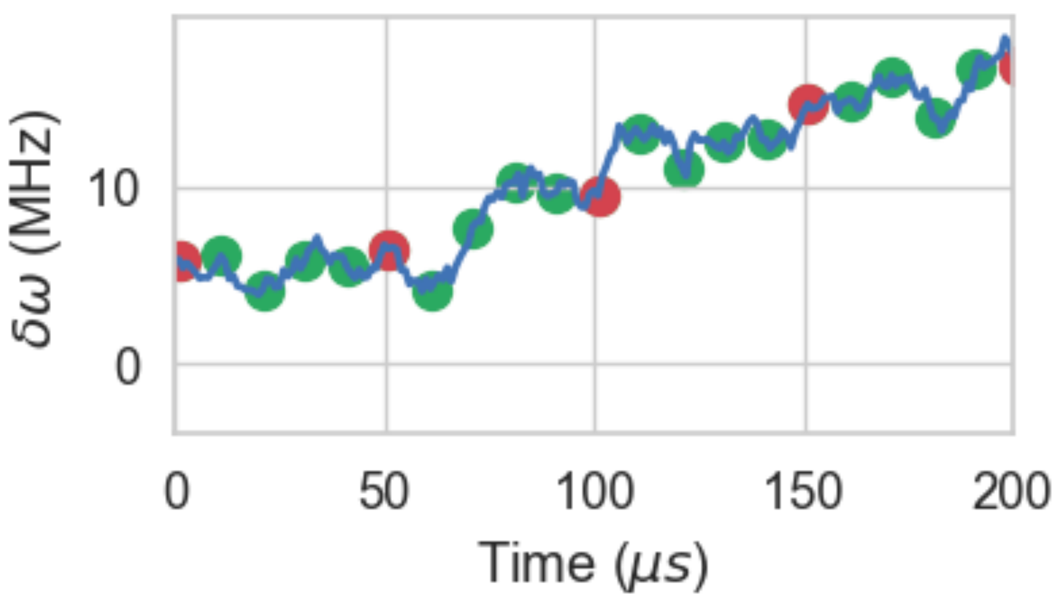
2. Improved initialisation

3. Markovian approach

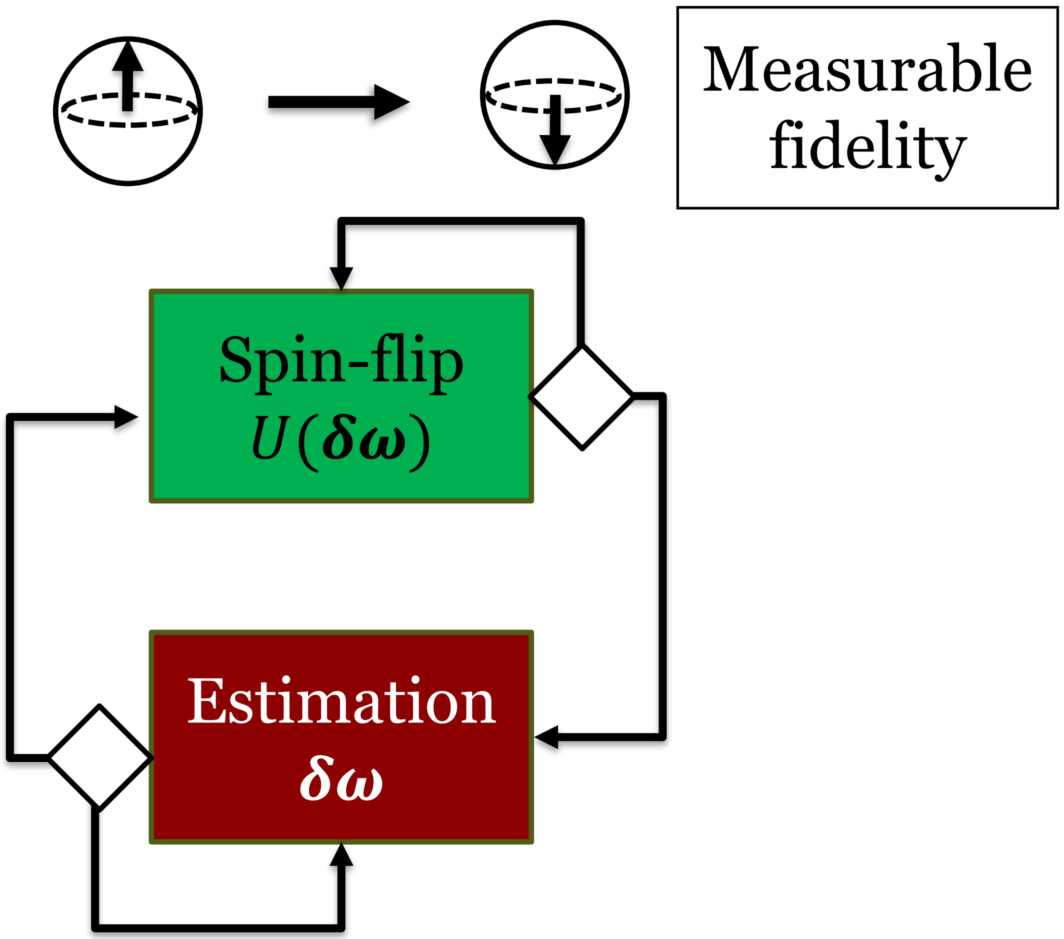
Reuer, Kevin, et al. "Realizing a deep reinforcement learning agent for real-time quantum feedback." *Nature Communications* 14.1 (2023): 7138.

⟨aQa⟩

The game



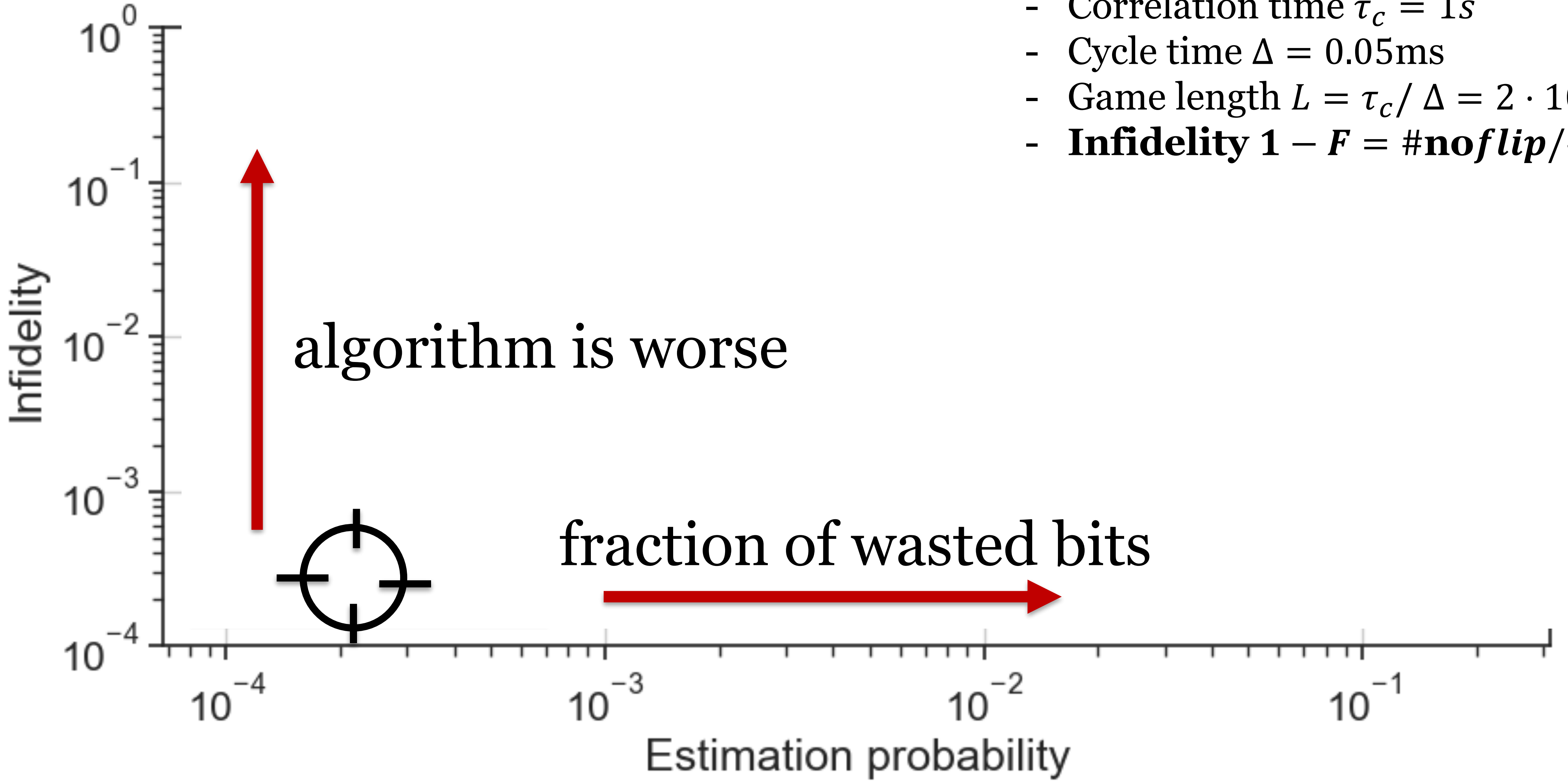
1. Resource allocation



2. Actions

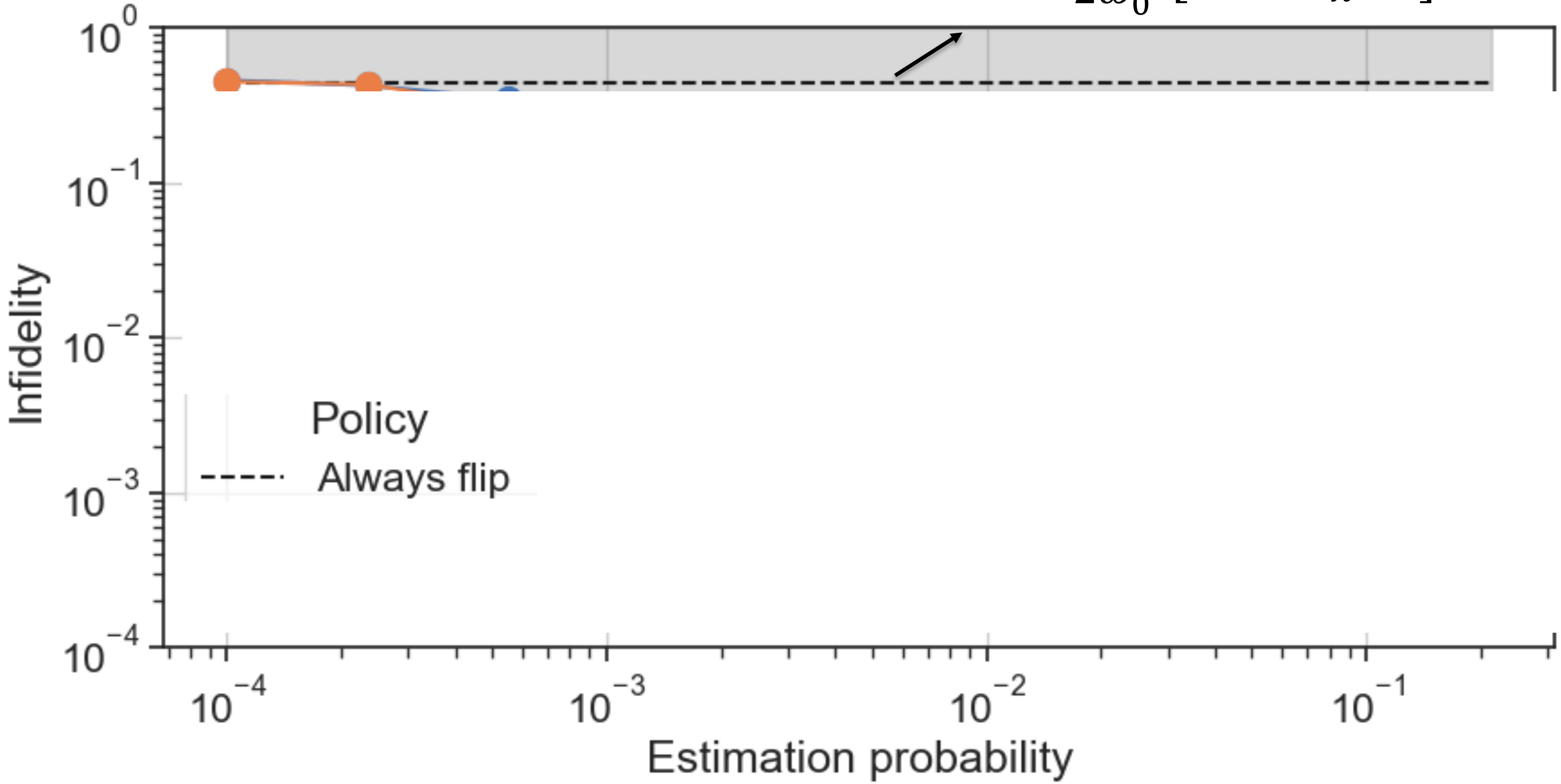
Heuristics to beat

- Noise amplitude 2MHz
- Target frequency 10MHz
- Initial error 0.5MHz
- Correlation time $\tau_c = 1s$
- Cycle time $\Delta = 0.05\text{ms}$
- Game length $L = \tau_c / \Delta = 2 \cdot 10^4$
- **Infidelity** $\boldsymbol{1 - F = \#noflip/\#x}$

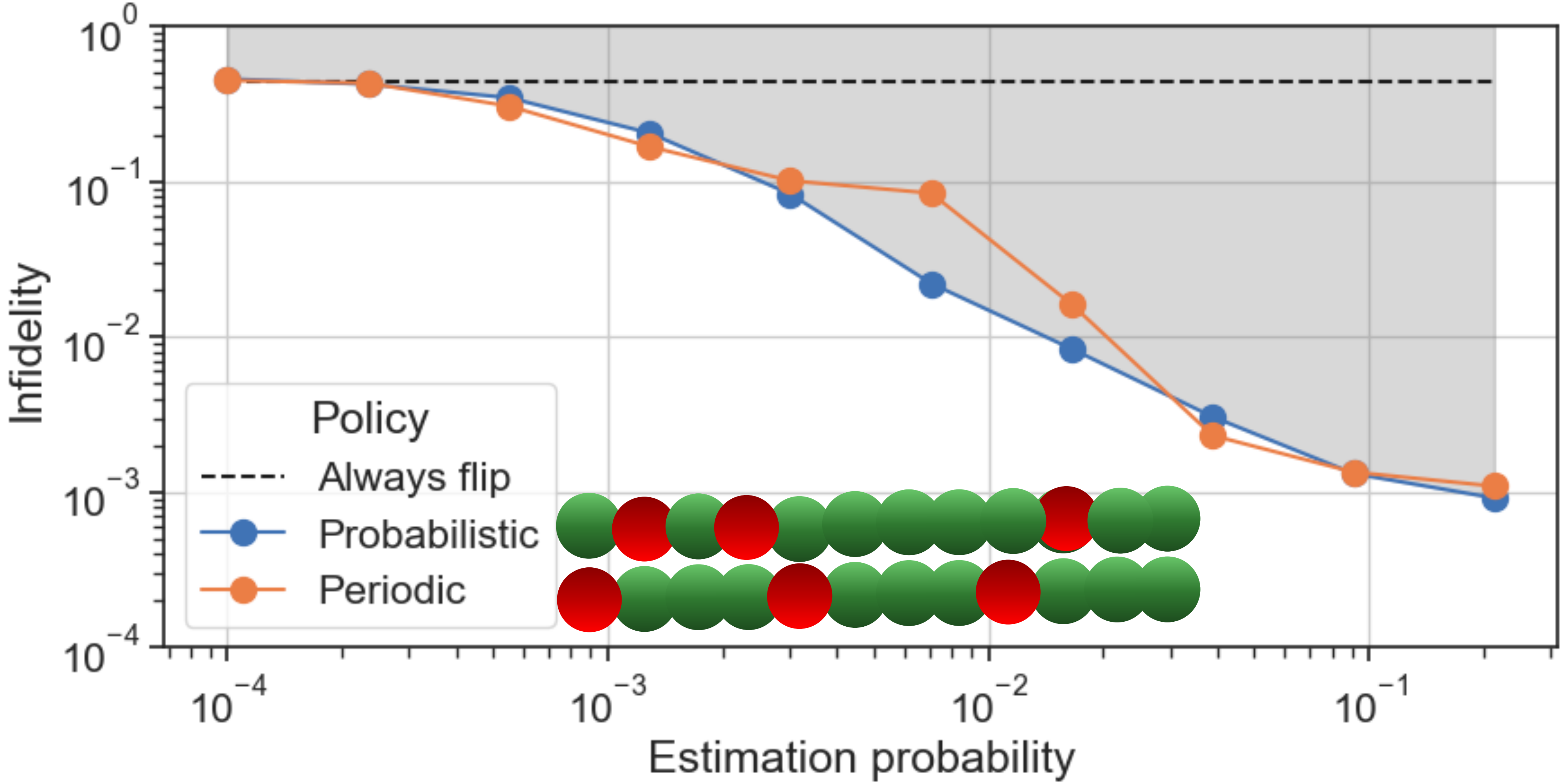


Heuristics to beat

$$\langle P(0)\rangle_{\delta\omega} \approx \frac{A^2\pi^2}{2\omega_0^2}\left[1 - \frac{1-e^{-x}}{x}\right] \quad x = \frac{N\Delta t}{\tau_c} = 1$$



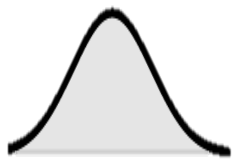
Heuristics to beat



Bringing the agent – Reinforcement learning (PPO) (5min)

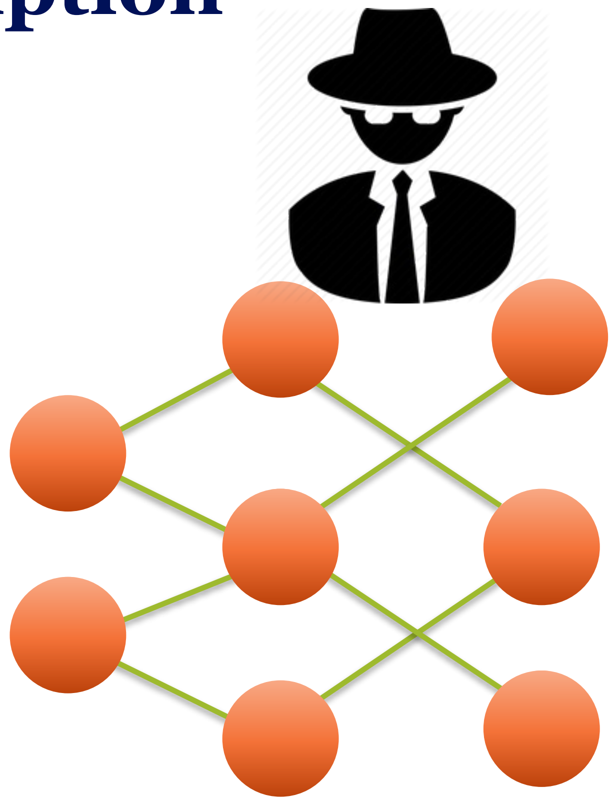
The agent description

$H(t) = \delta\omega\, S_x$



μ_n

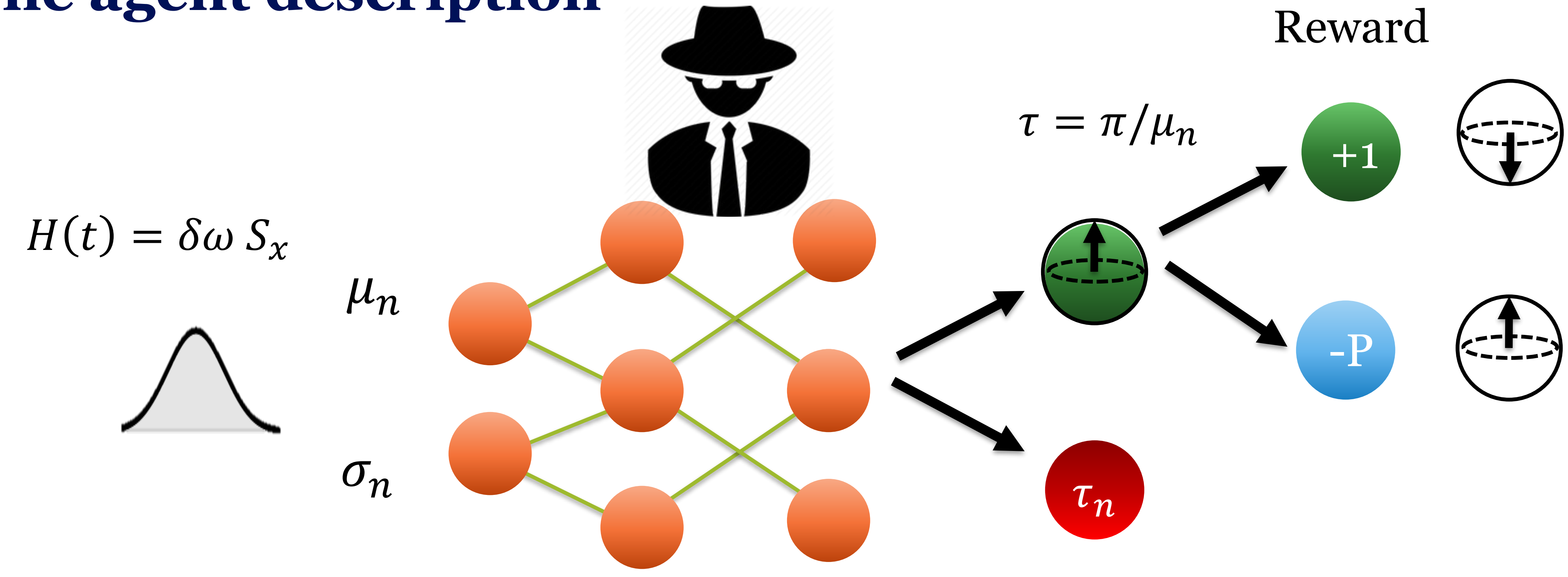
σ_n



1. Observations

μ_n, σ_n

The agent description



1. Observations

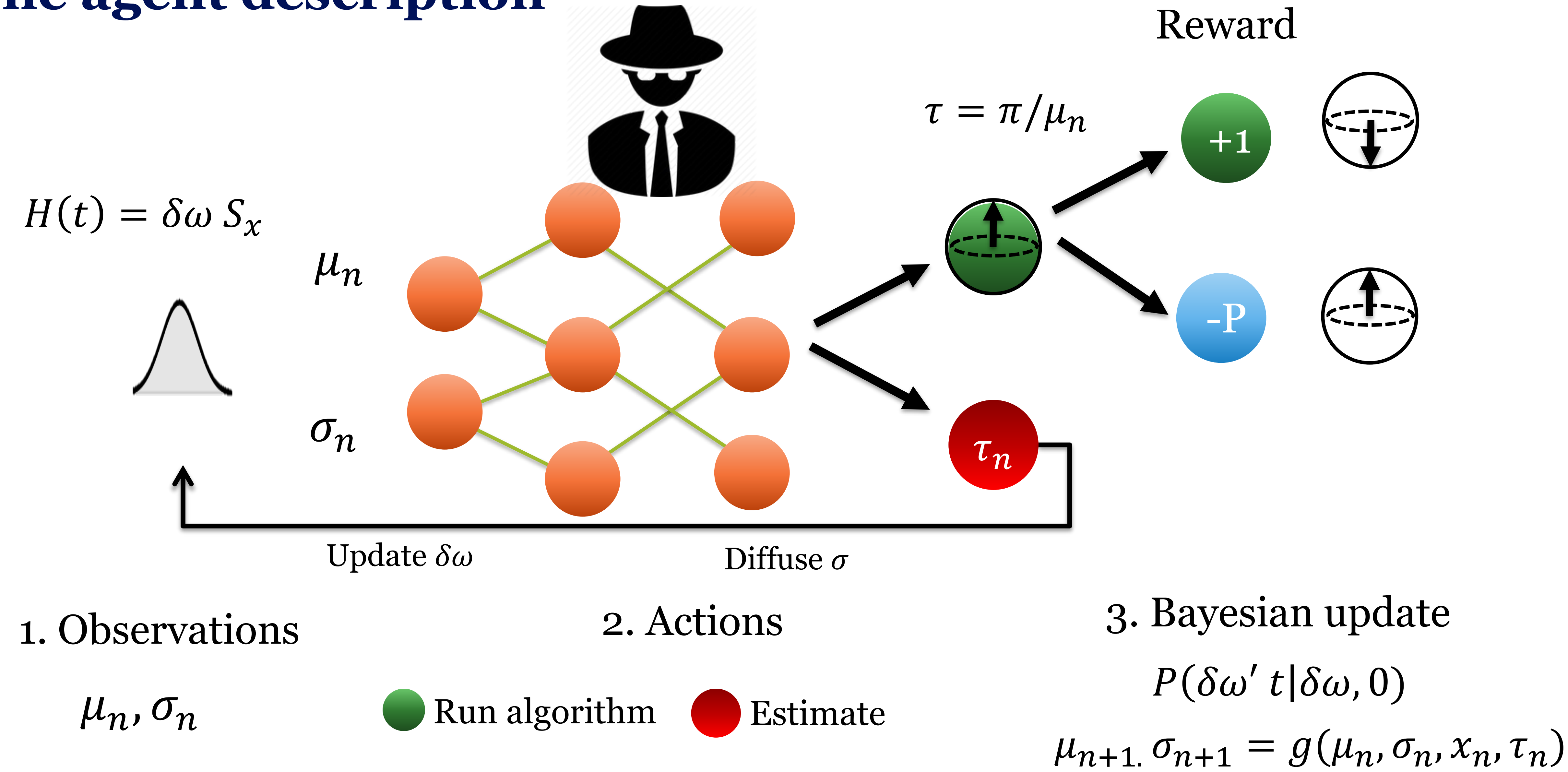
μ_n, σ_n

2. Actions

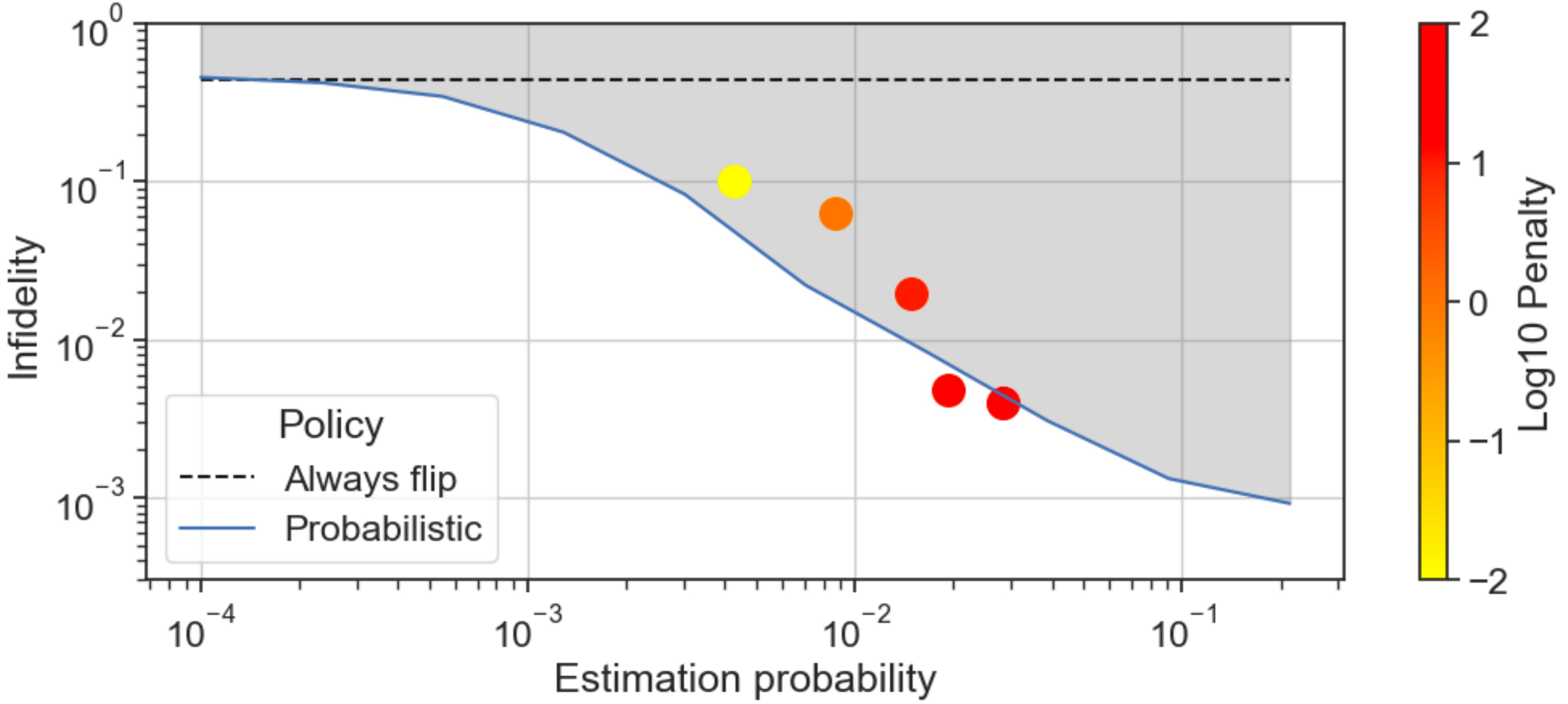
Run algorithm

Estimate

The agent description

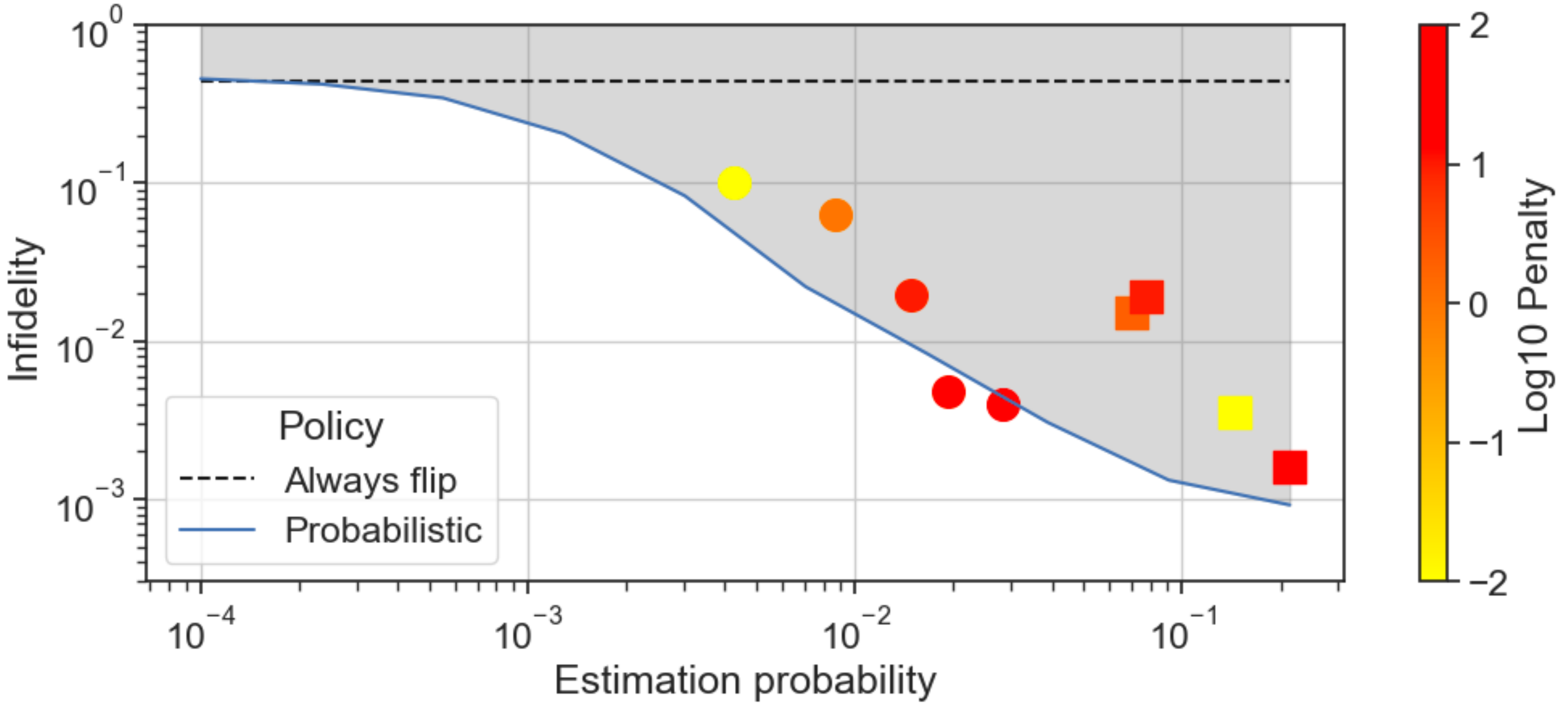


Agent cannot control time, $\tau_n = 1/c\sigma_{n-1}$



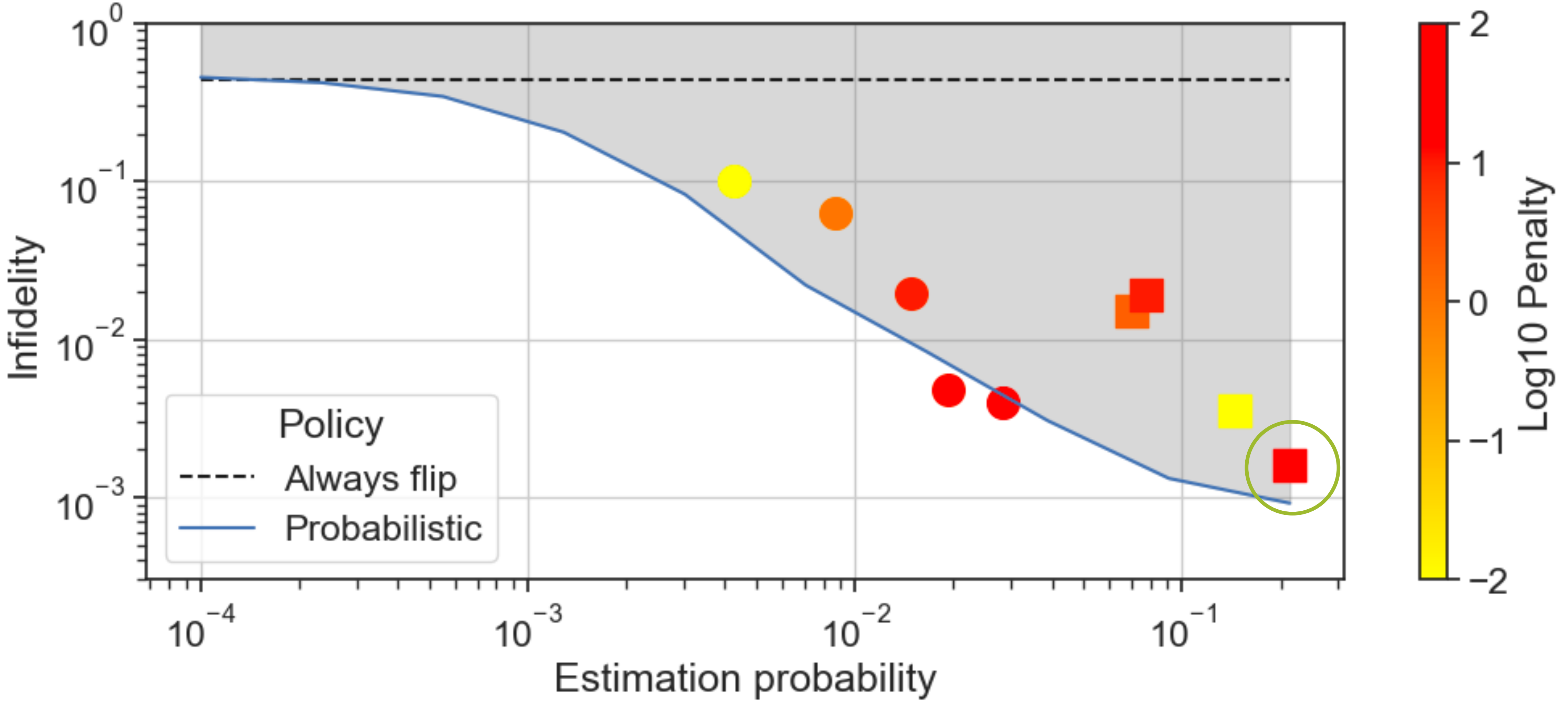
Agent can control time

-P

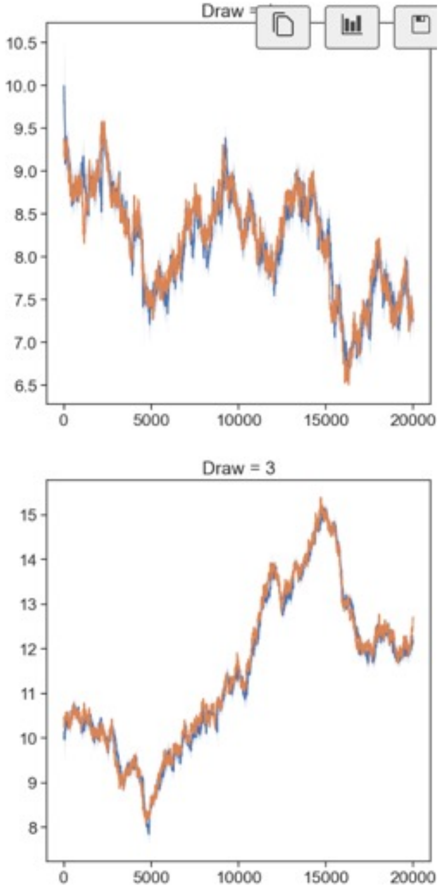


Agent can control time [redacted]

-P

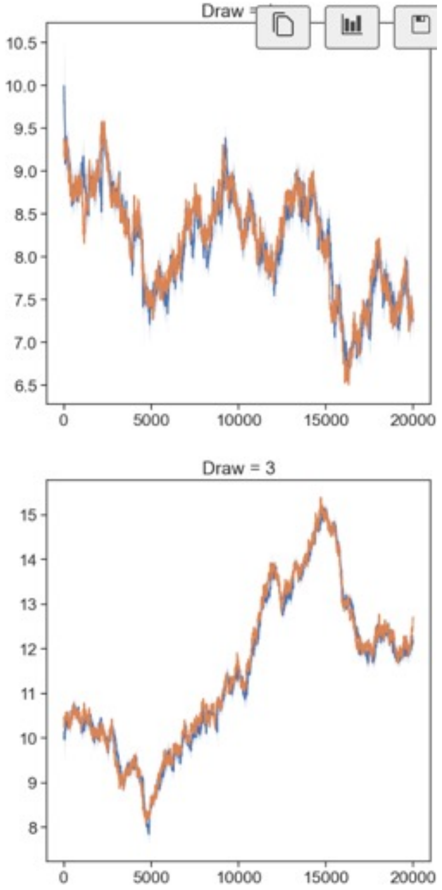


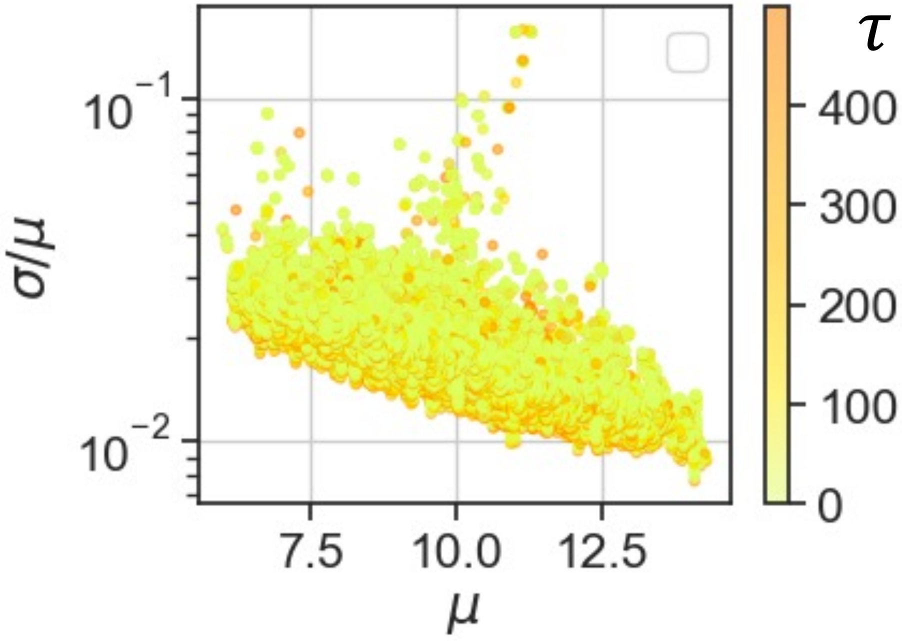
Understanding the best agent



1. Tracks trajectories

Understanding the best agent

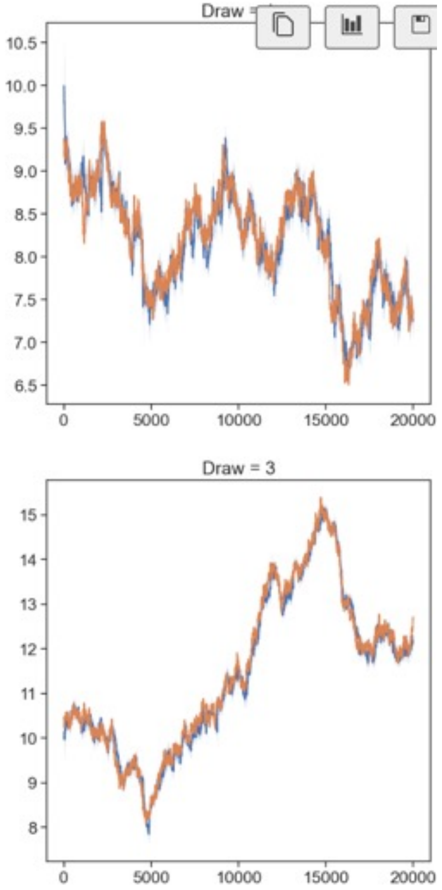




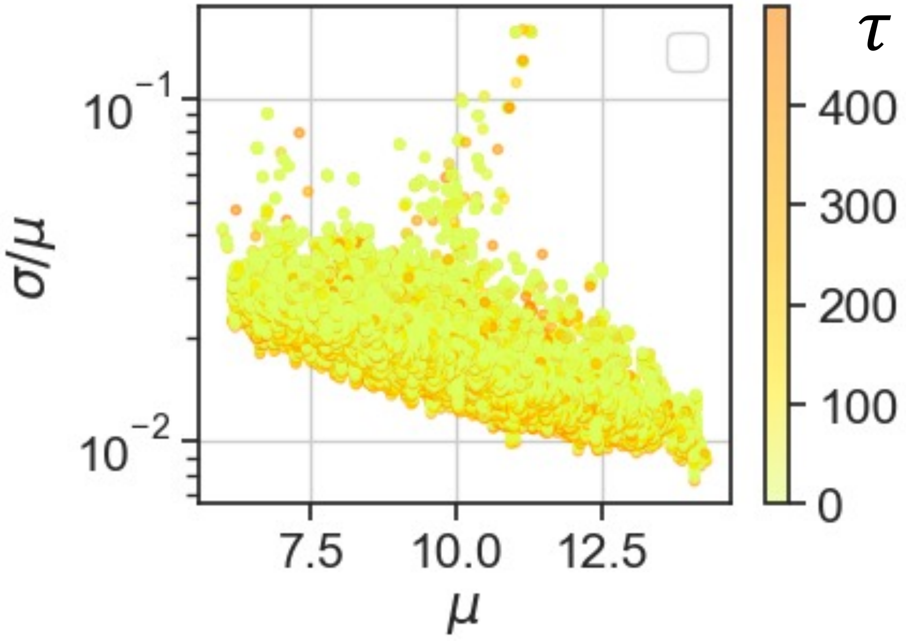
1. Tracks trajectories

2. Applies $\frac{1}{\sigma}$ strategy

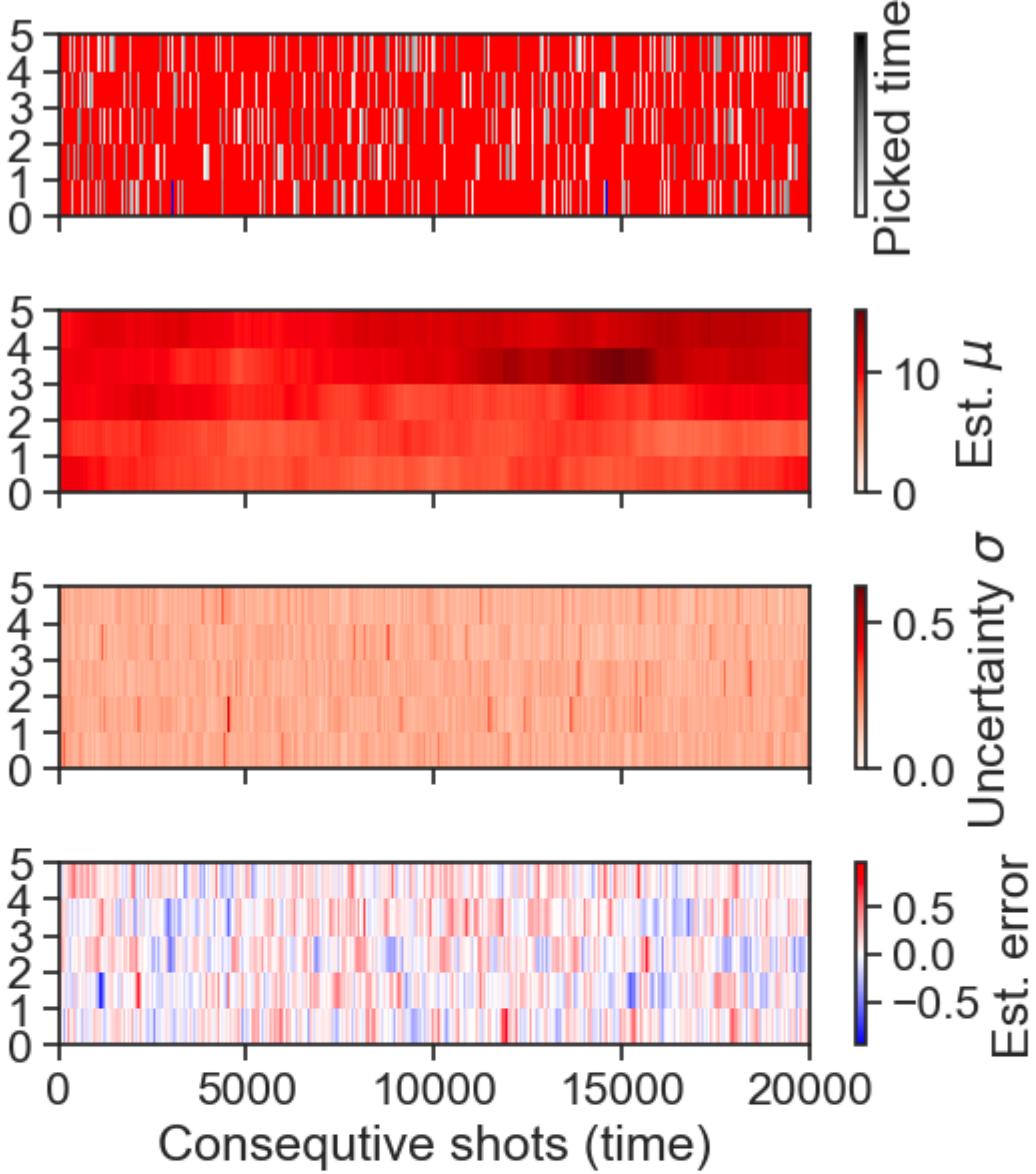
Understanding the best agent



1. Tracks trajectories

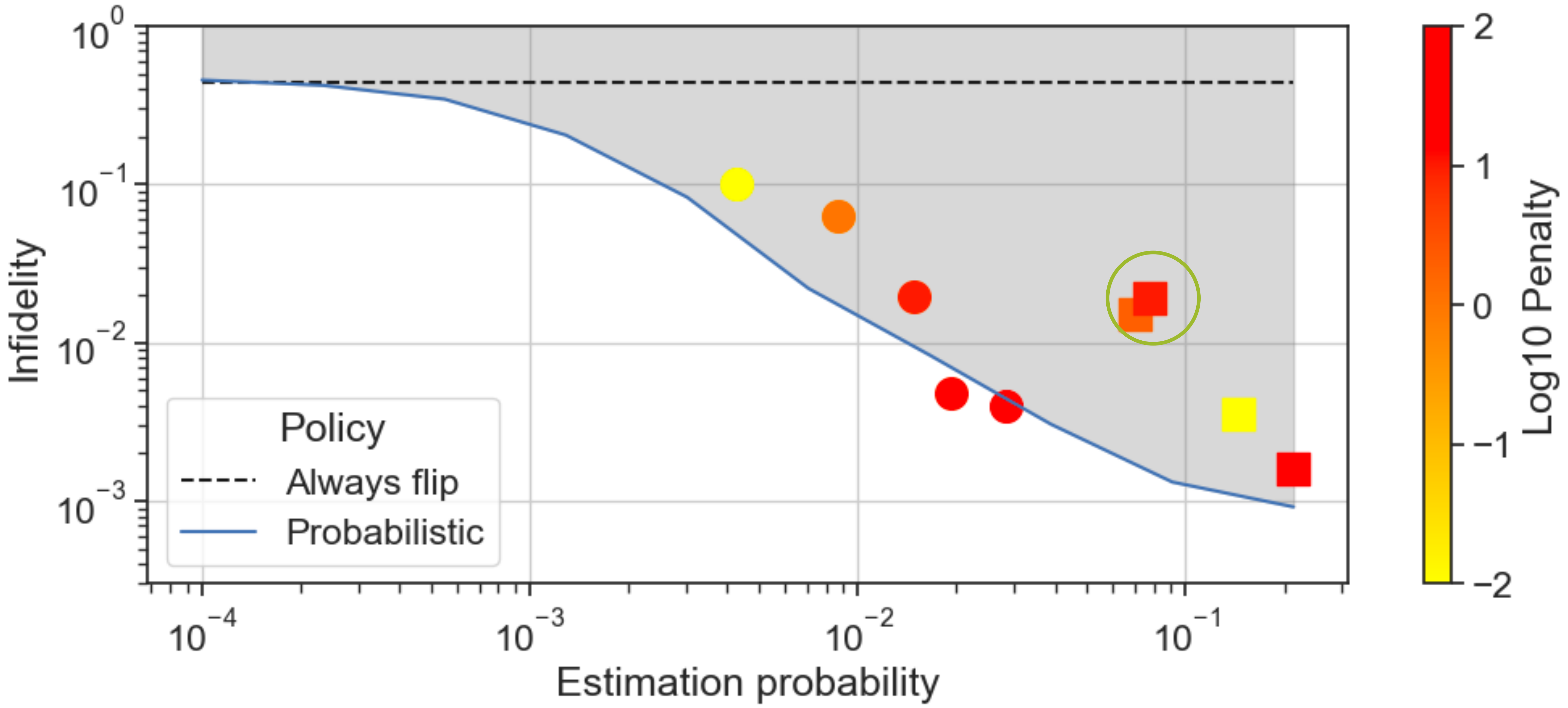


2. Applies $\frac{1}{\sigma}$ strategy

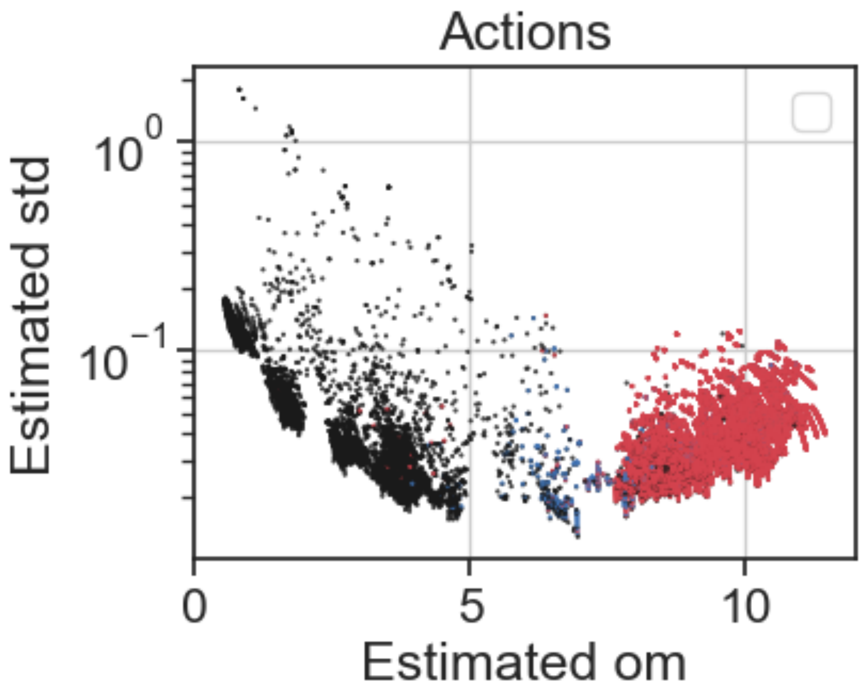


3. Behaves

But there is a more clever one

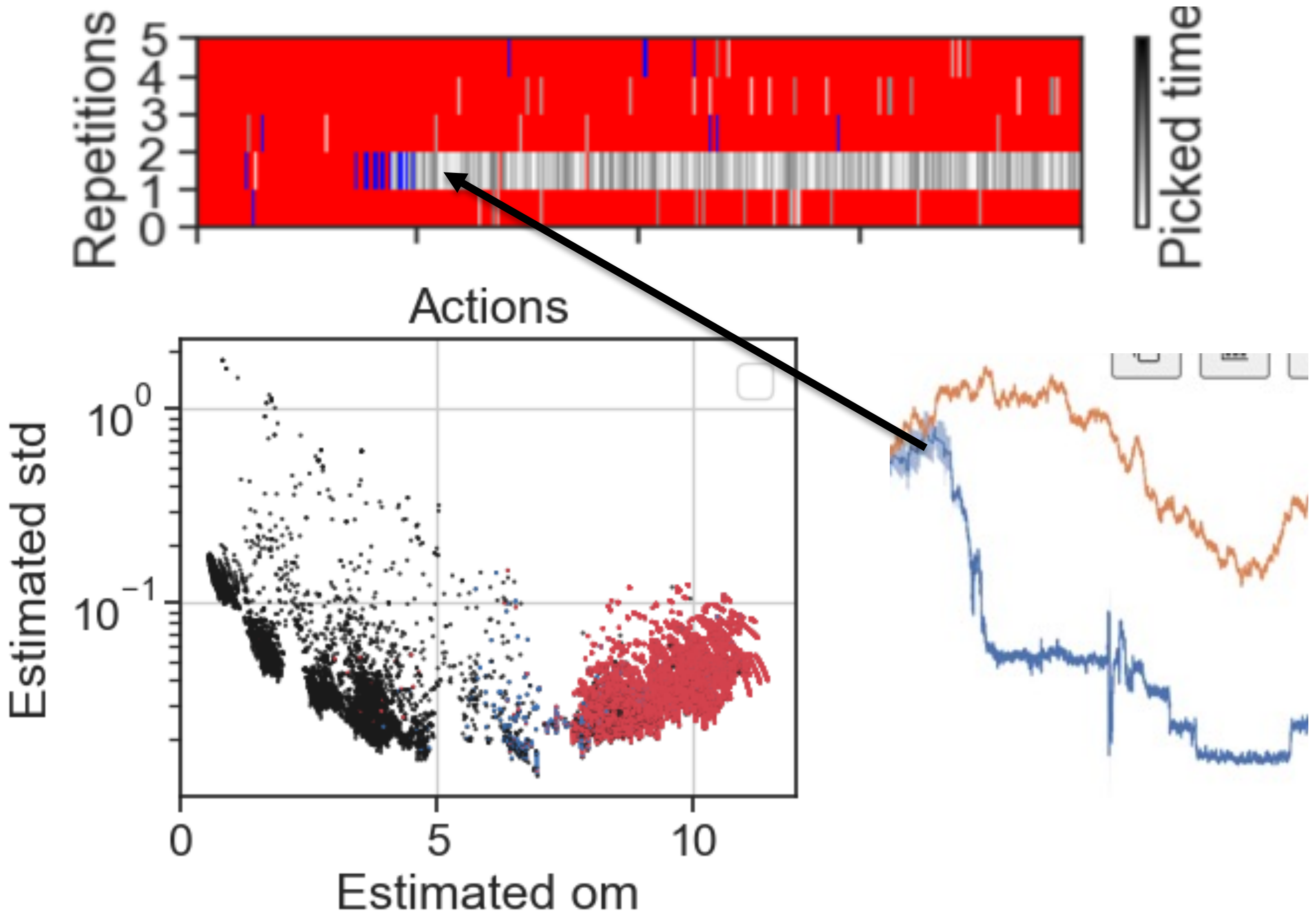


Smart agent (but not efficient)



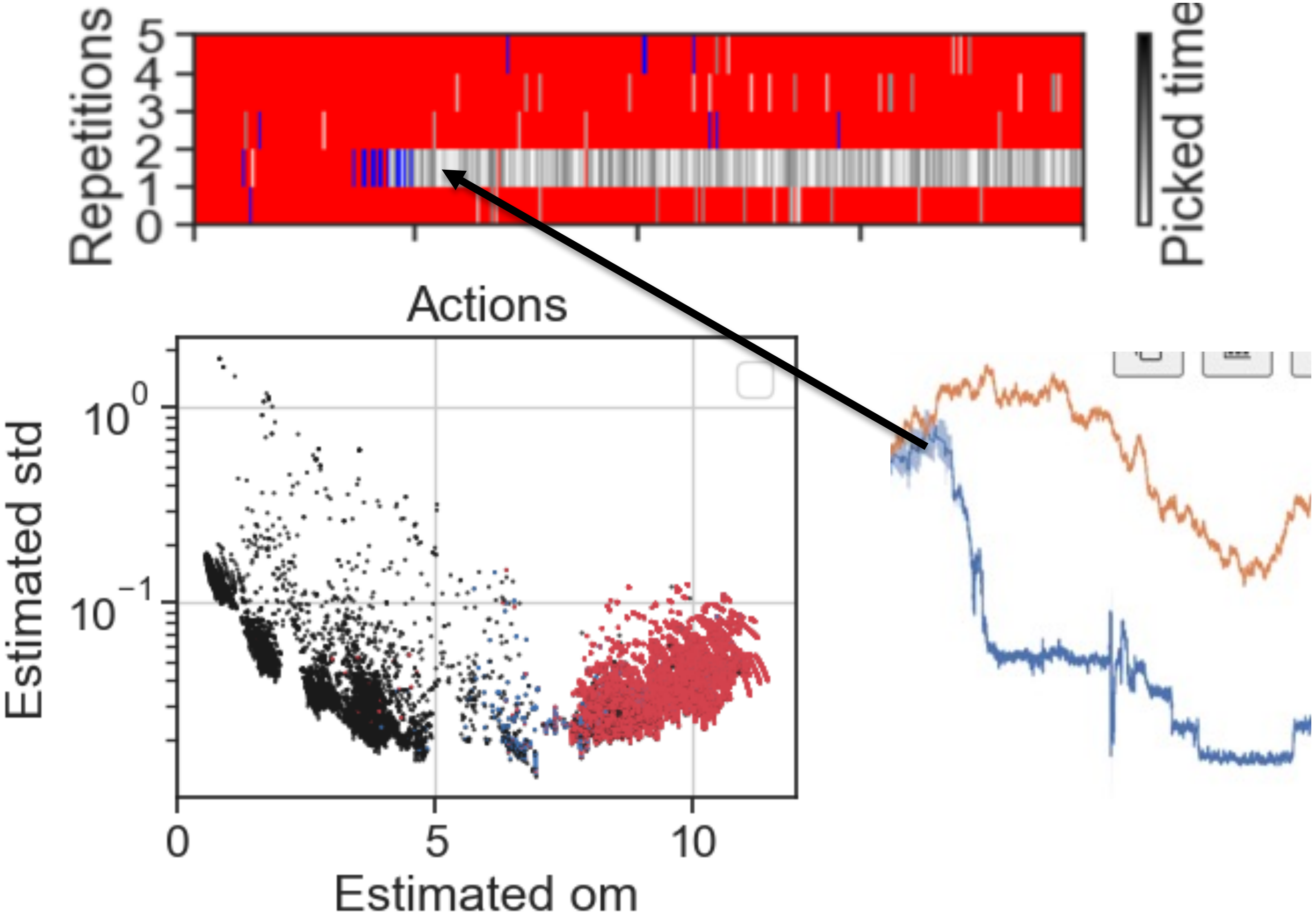
1. Waits for large field

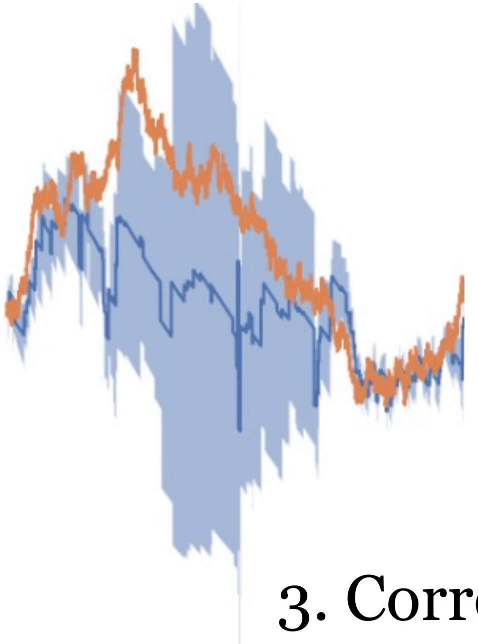
Smart agent (but not efficient)



1. Waits for large field 2. Knows when its lost

Smart agent (but not efficient)



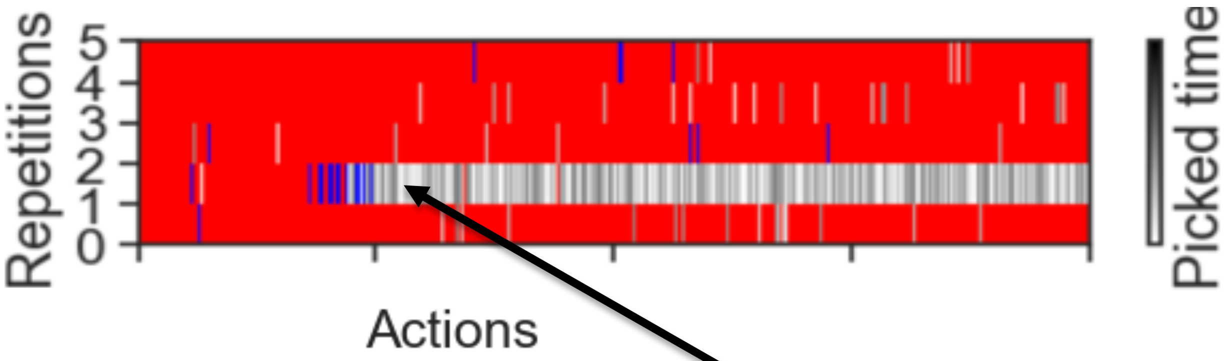


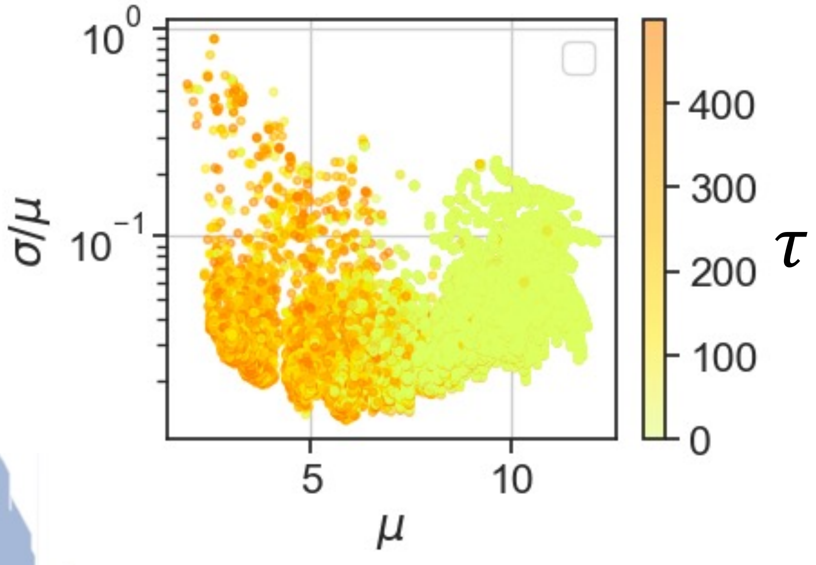
3. Correlates σ with error

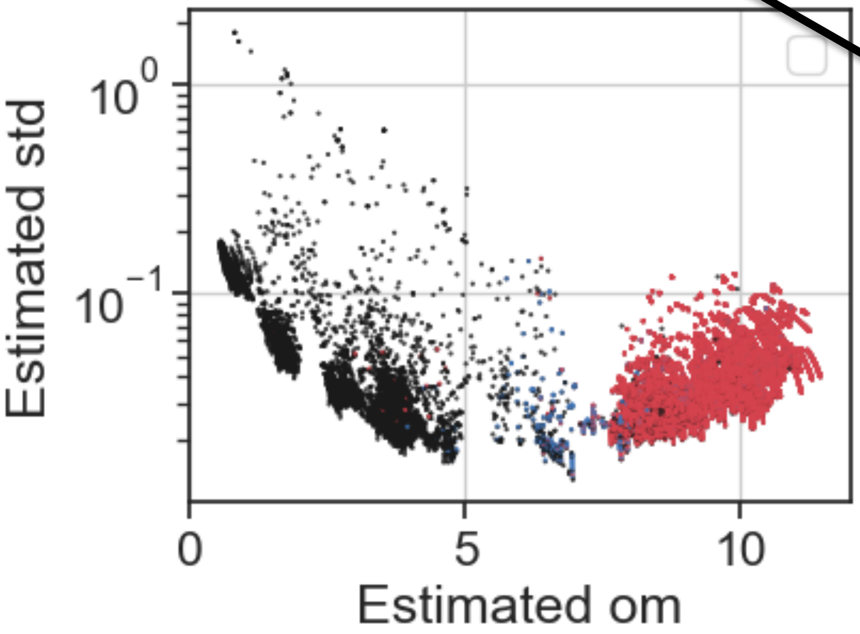
1. Waits for large field

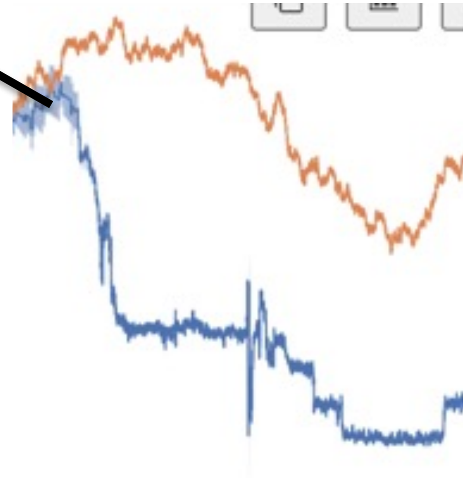
2. Knows when its lost

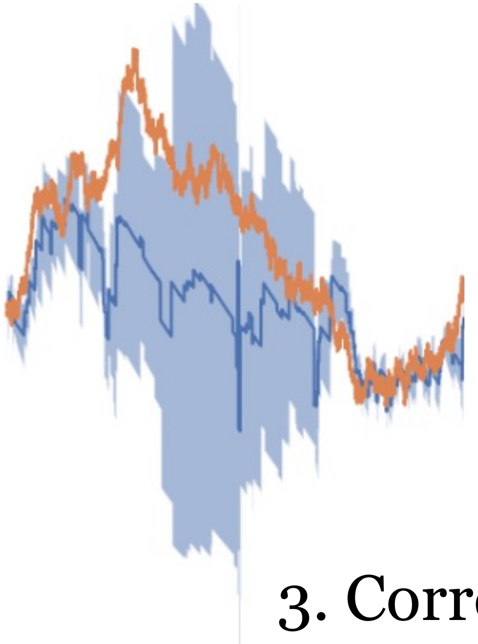
Smart agent (but not efficient)











4. Uses large τ
For small fields

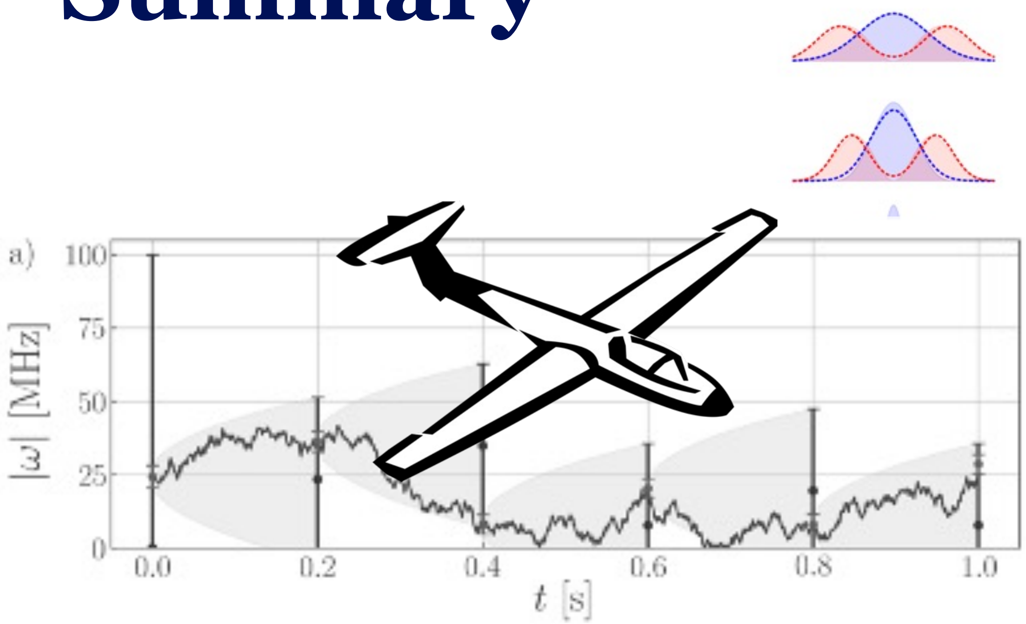
3. Correlates
σ with error

1. Waits for large field

2. Knows when its lost

Summary

Summary



Summary

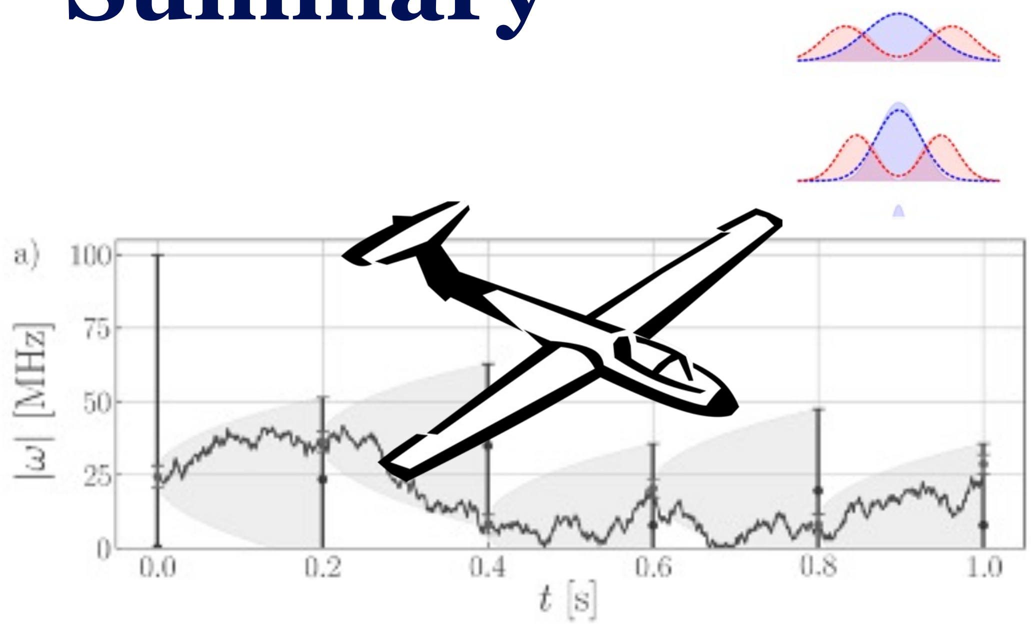
We developed methods
of fast, resource-efficient
field esitmation

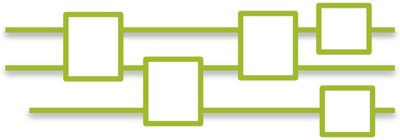
Outlook

Can we generalize to
arbitrary noise source?

j.a.krzywda@liacs.leidenuniv.nl

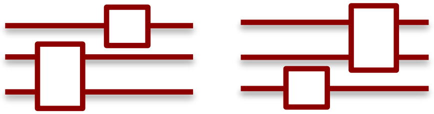
Summary





Operation
$U(\delta\omega)$

Estimation
$\delta\omega$



Summary

We developed methods of fast, resource-efficient field esitmation

We employed a RL agent to dynamically change between estimation and operation modes

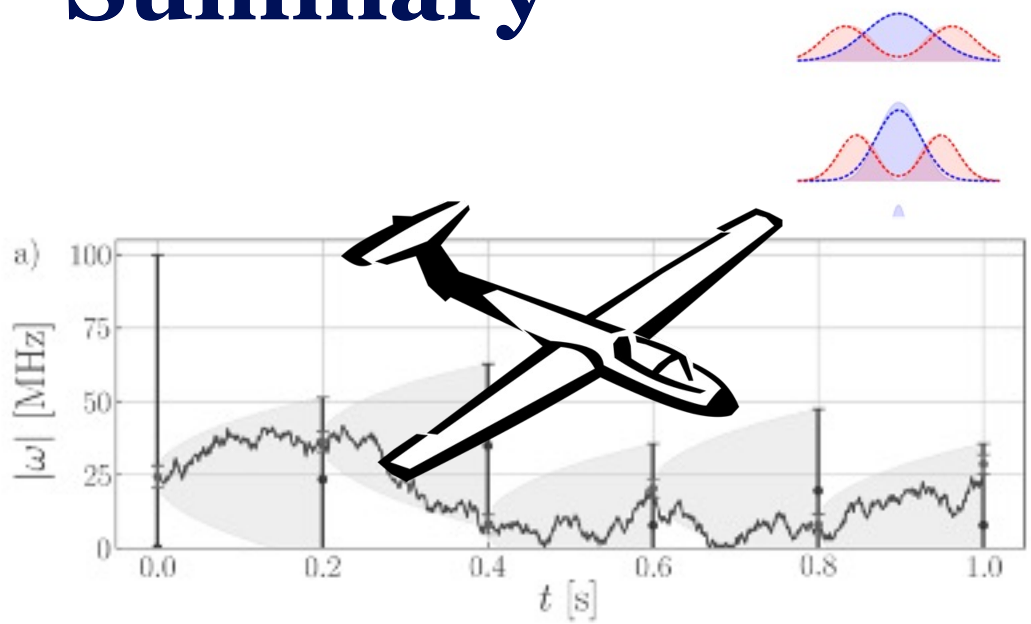
Outlook

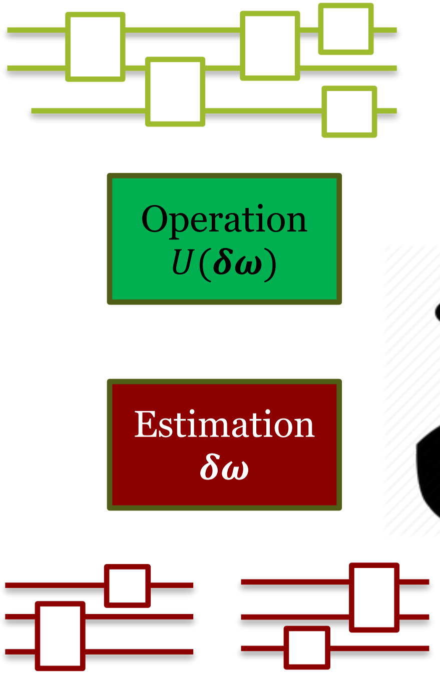
Can we generalize to arbitrary noise source?

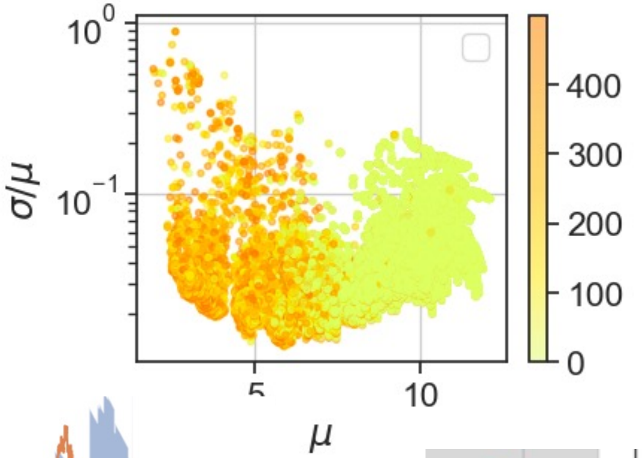
Can we do online learning (FPGA?)
Can we use it for two-qubit gates?

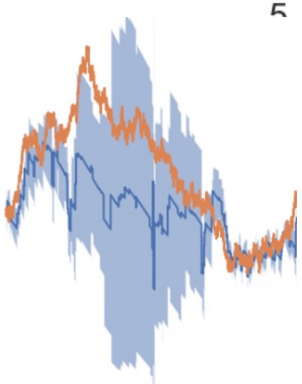
 j.a.krzywda@liacs.leidenuniv.nl

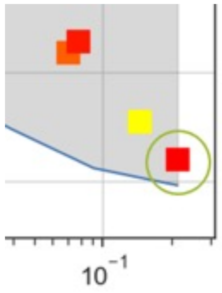
Summary











Summary

We developed methods of fast, resource-efficient field esitmation

We employed a RL agent to dynamically change between estimation and operation modes

The agent develped non-trivial strategies and matched best-known heuristics

Outlook

Can we generalize to arbitrary noise source?

Can we do online learning (FPGA?)
Can we use it for two-qubit gates?

Can we beat the heuristics?
Can we use the learned strategy to improve heuristics?

j.a.krzywda@liacs.leidenuniv.nl

Hadamard gate

$$J(\epsilon^*) = \Delta\hat{B}_z$$

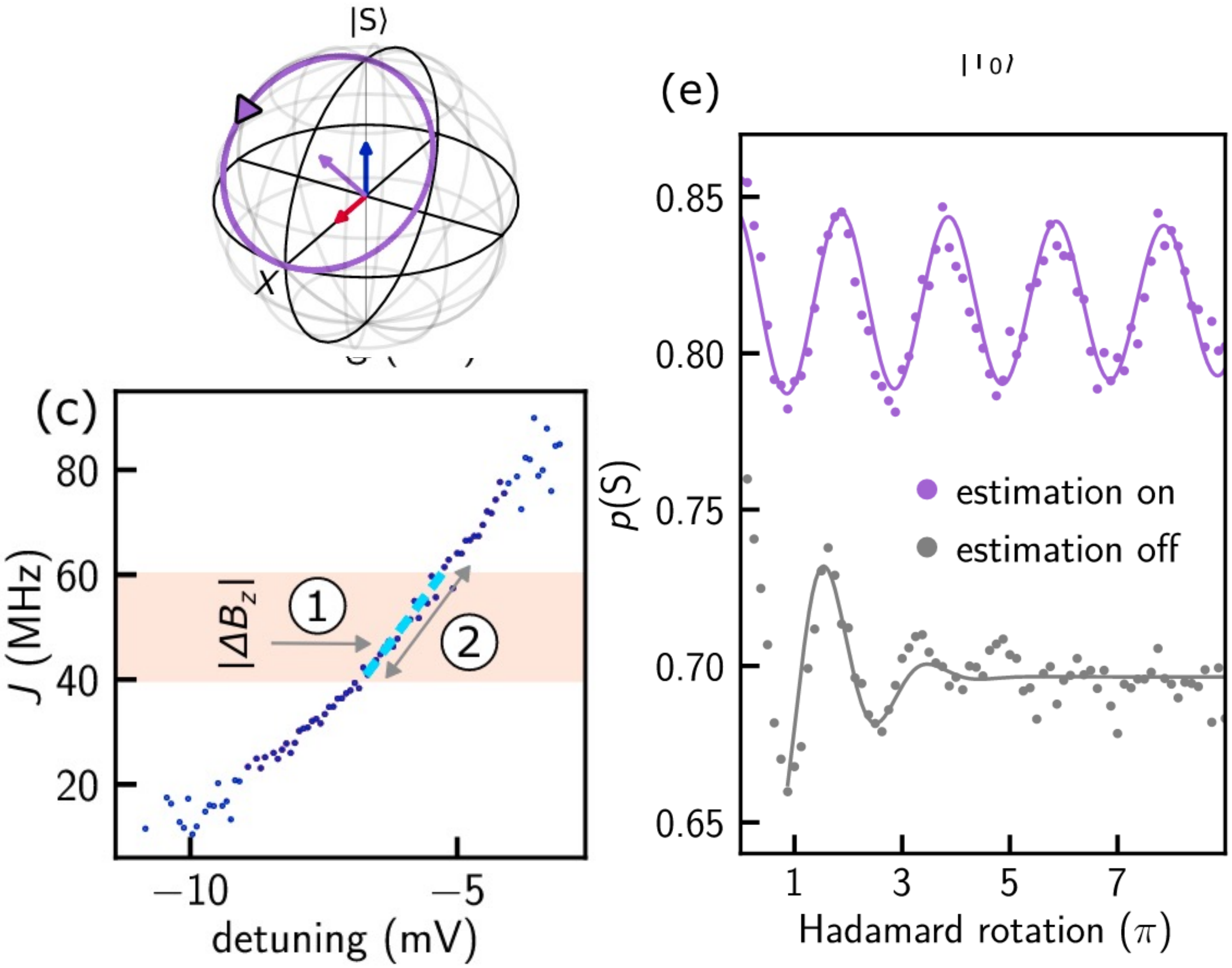
- For each shot, we performed two estimations:

  1. Estimation of $\Delta\hat{B}_z$
  2. Estimation of $\Omega(\hat{\epsilon})$, at $\hat{\epsilon}$

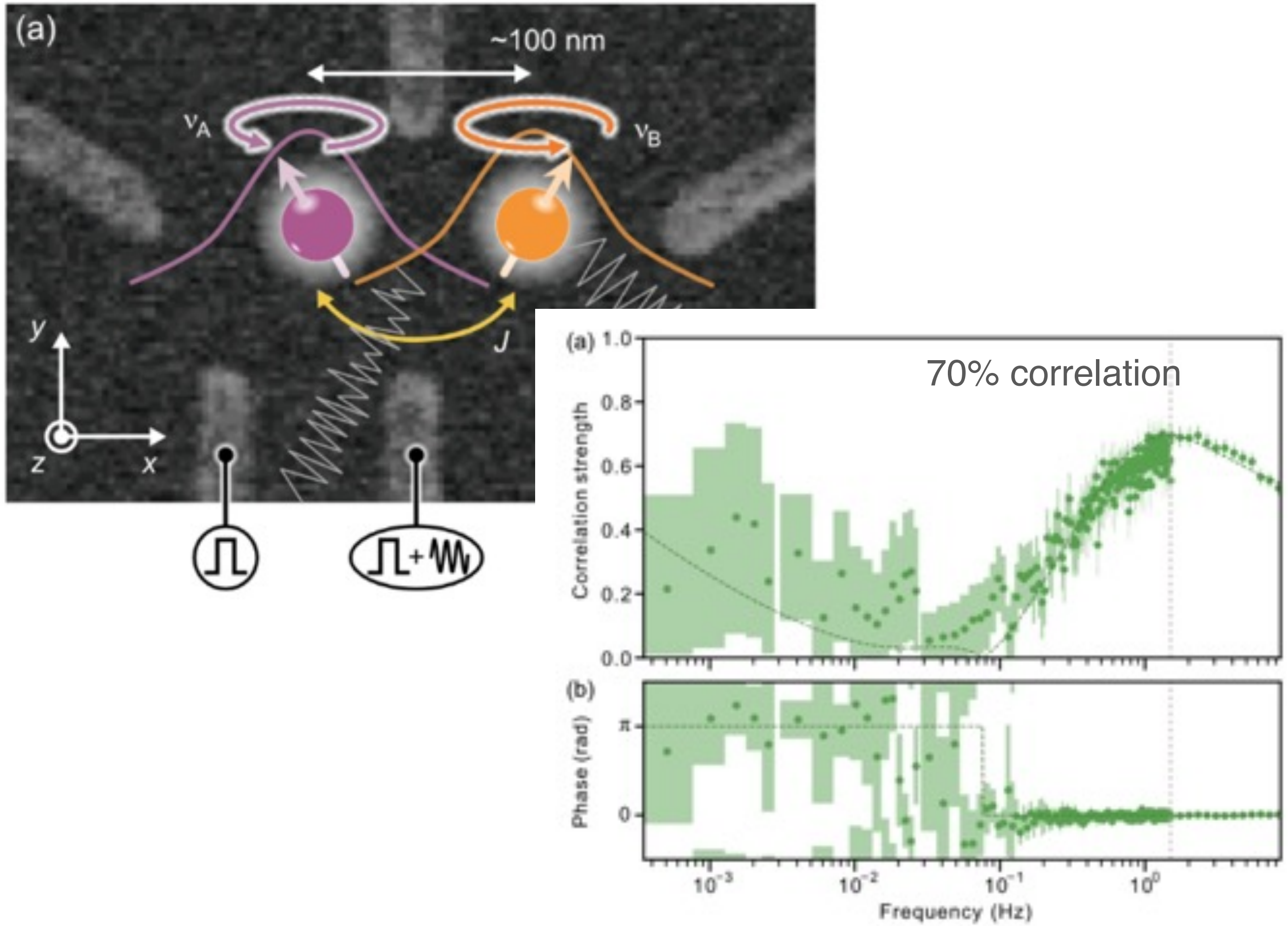
     where $J(\hat{\epsilon}) = \Delta\hat{B}_z$

- Based on $\Delta\Omega$:

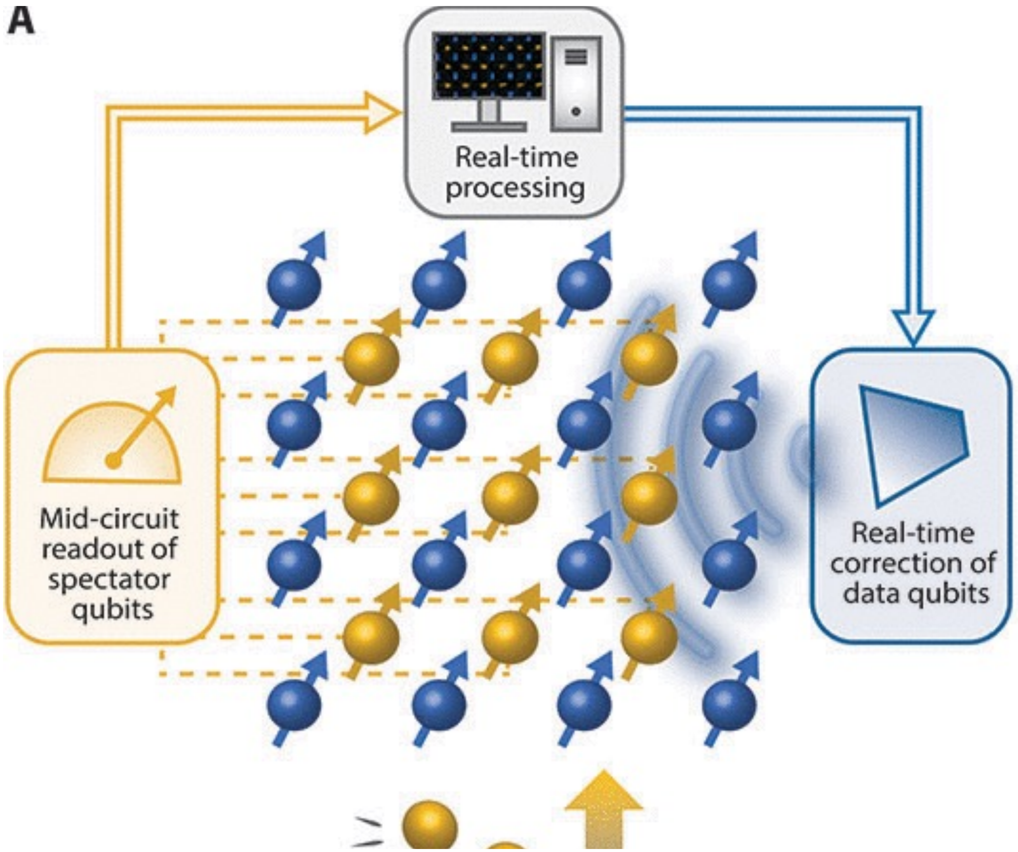
$$\Delta\epsilon = \hat{\epsilon} - \epsilon^*$$



Investigate utility of spatial correlations



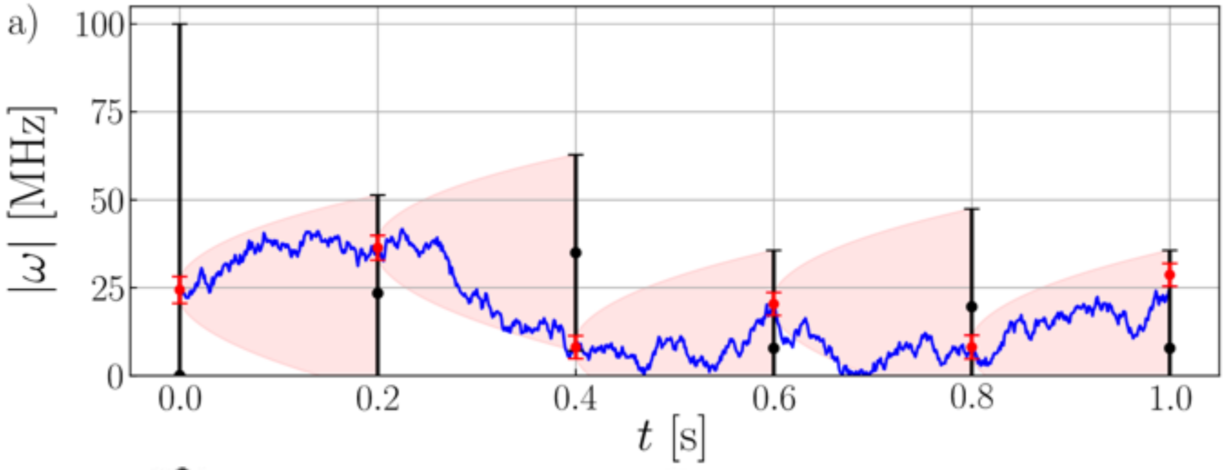
Y. Yoneda et al., arXiv:2208.14150 (2023)

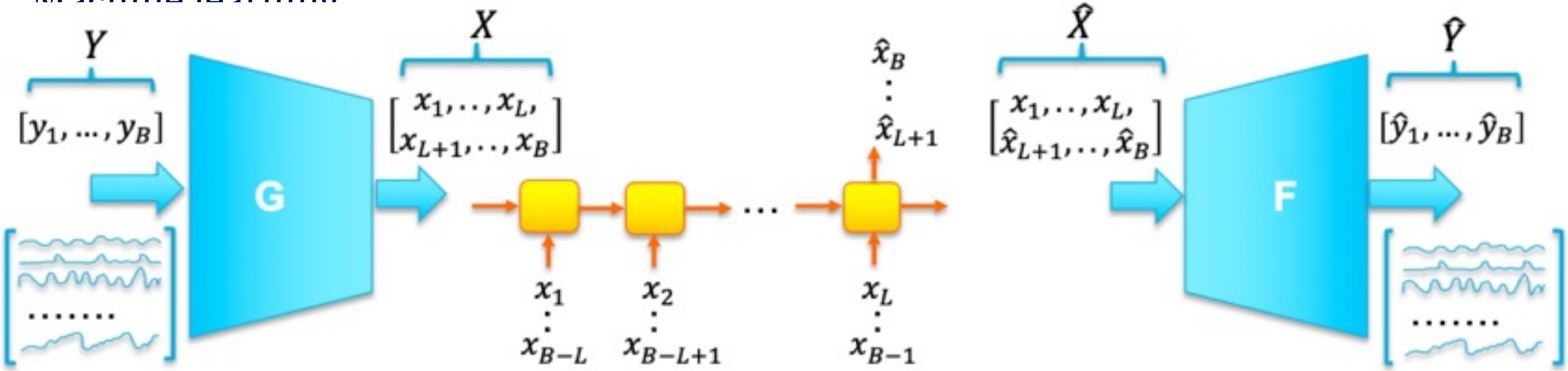


K. Singh *et al.* ,Mid-circuit correction of correlated phase errors using an array of spectator qubits *Science* **380**,1265-1269 (2023).

Trajectory analysis and prediction

- Representing the sequential estimation and field broadening in the language of functional integral
- Process identification and prediction enabled by Machine learning

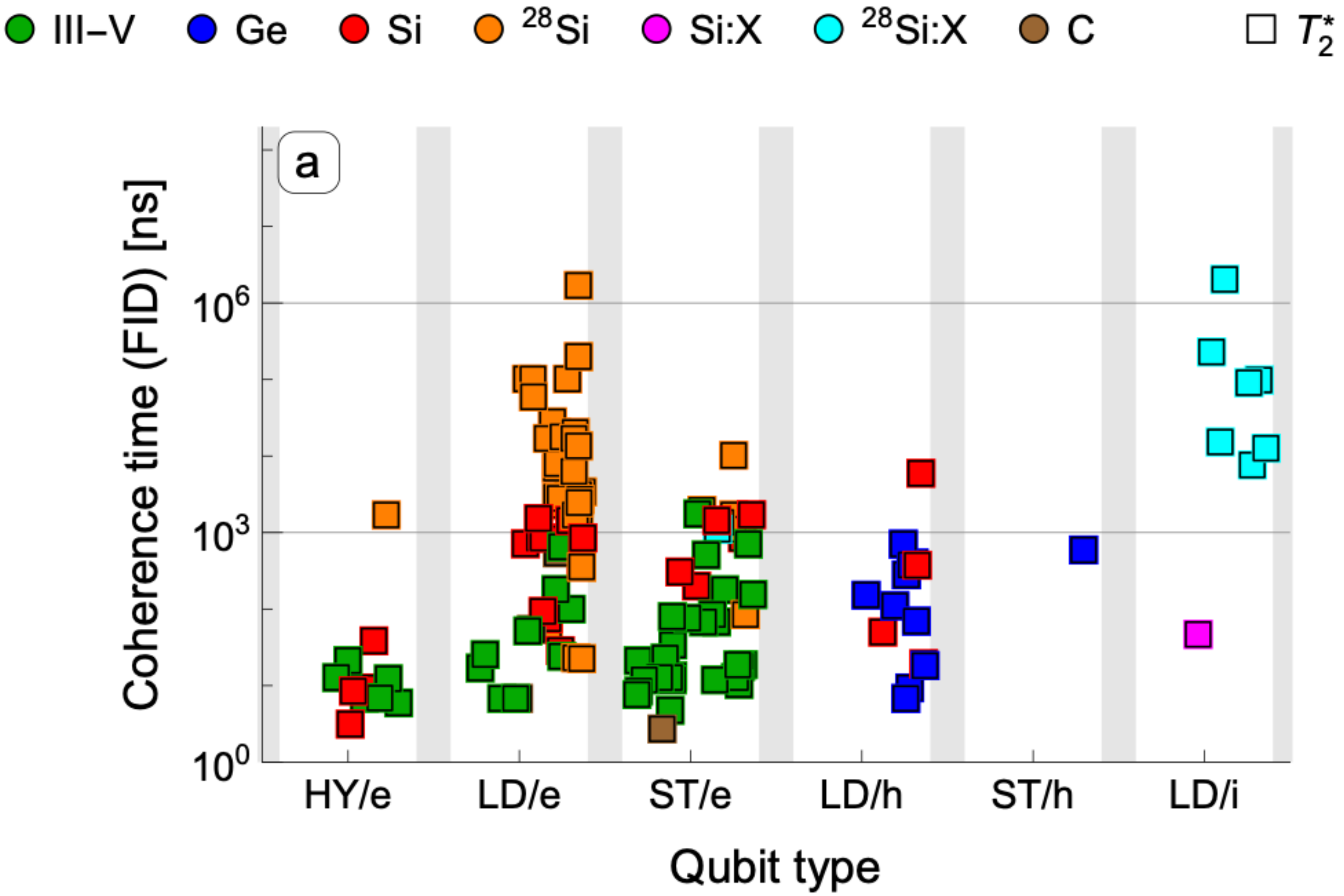




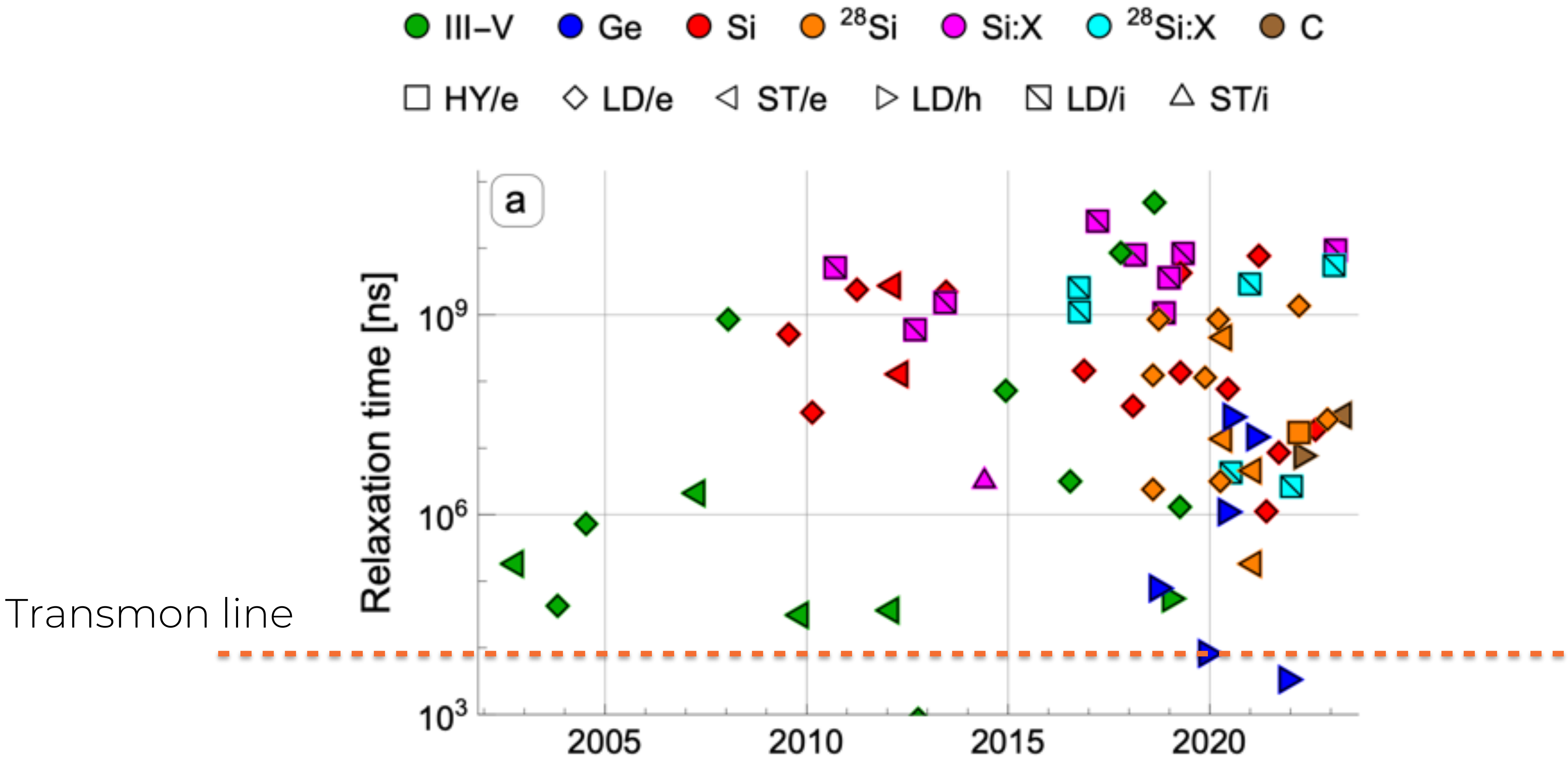
Felix Frohnert

N. Nguyen, B. Quanz, Temporal Latent Auto-Encoder: A Method for Probabilistic Multivariate Time Series Forecasting, arXiv:2101.10460 (2021)

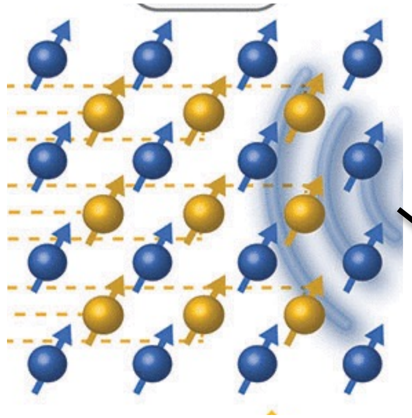
Spin qubits are limited by inhomogeneous broadening…



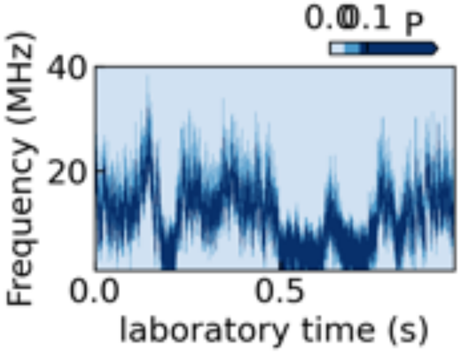
… and we can forget about relaxation times T_1

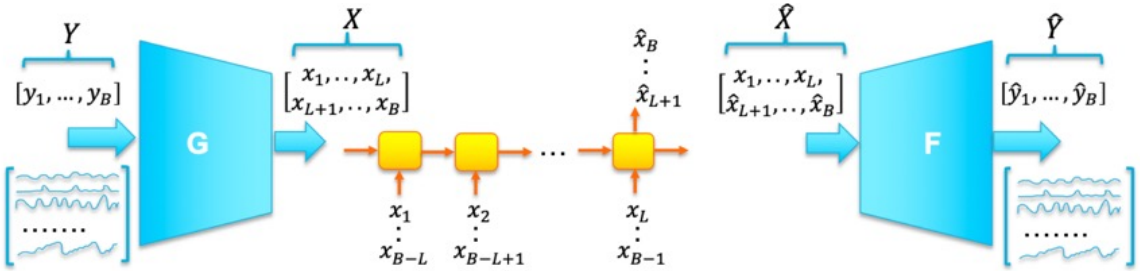


Anyway the Wind Blows: Bayesian Estimation and Feedback for Correlated

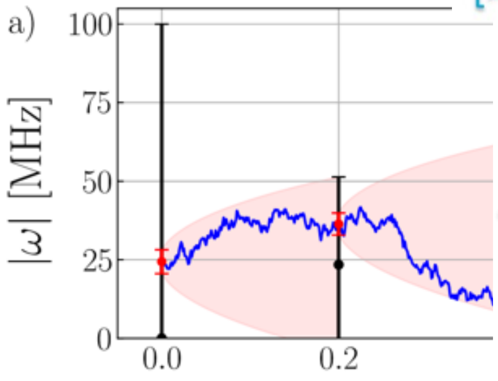


Future quantum computer

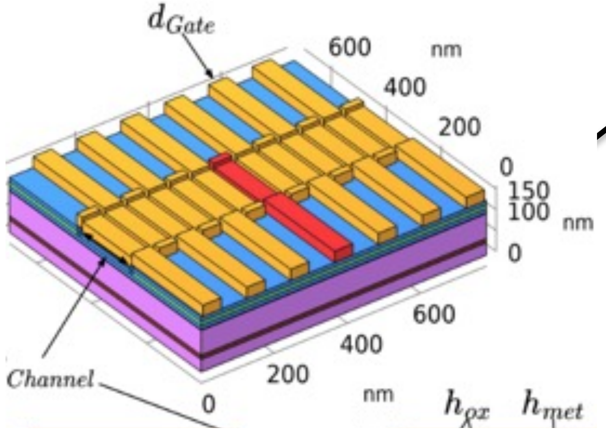




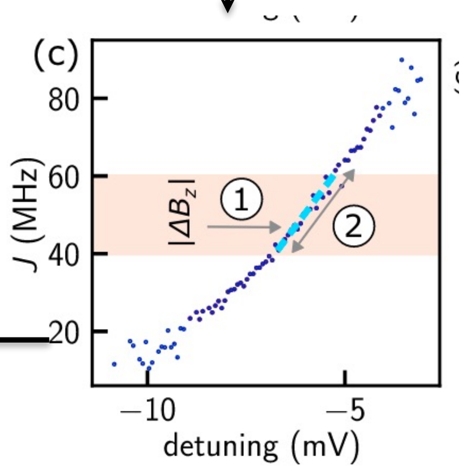
PLAN: Use ML, filtering and Bayesian tools



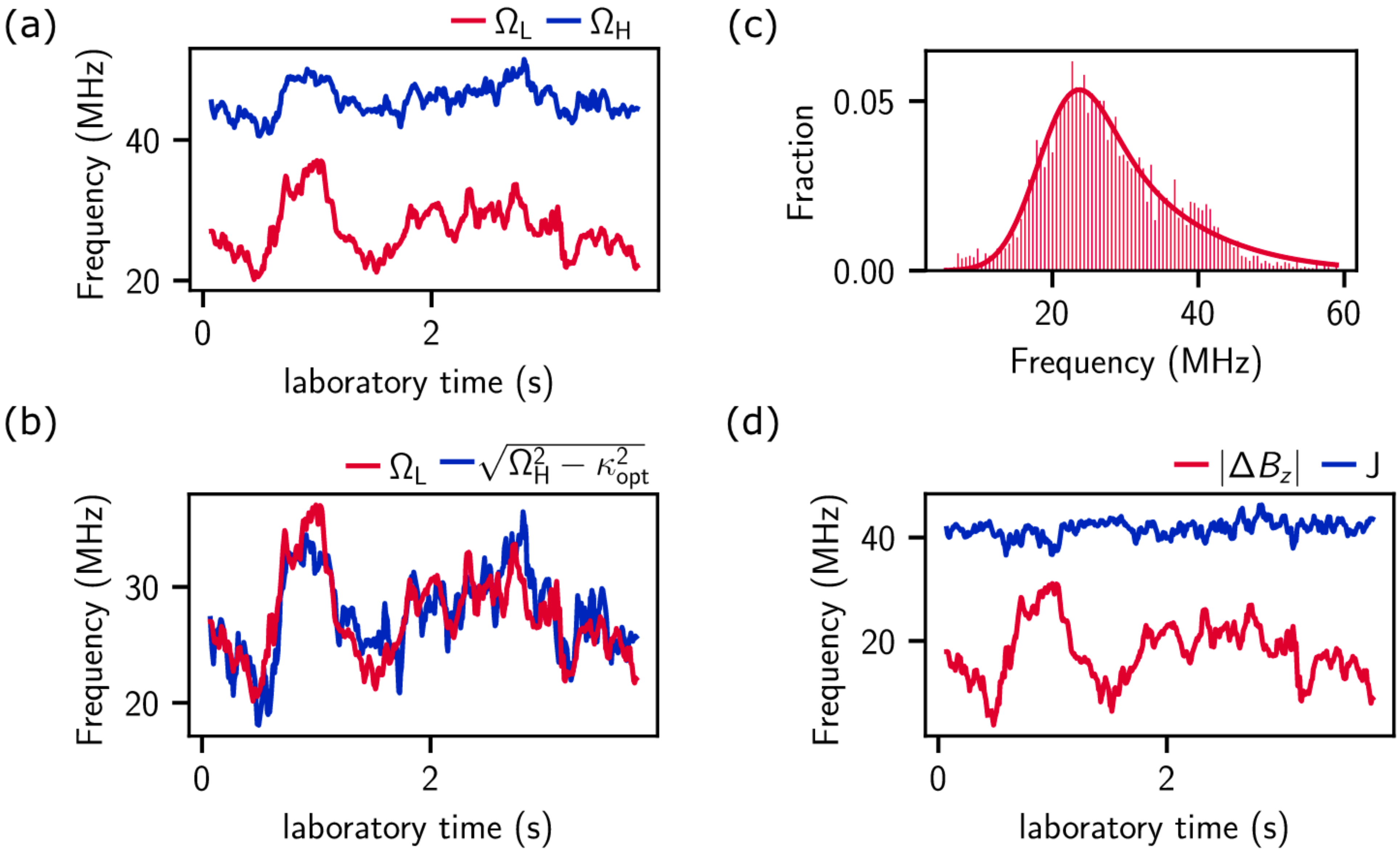
Tracking Hamiltonian Parameters



Model of charge noise
And spatial correlations



Feedback for Hadamard gate



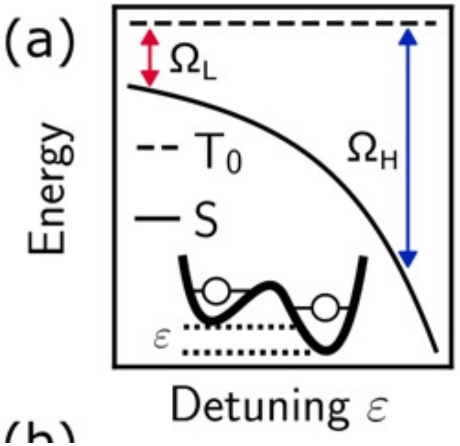
(a)
Ω_L Ω_H
Frequency (MHz)
40
20
0
2
laboratory time (s)
(b)
Ω_L $\sqrt{\Omega_H^2 - \kappa_{opt}^2}$
Frequency (MHz)
30
20
0
2
laboratory time (s)
(c)
Fraction
0.05
0.00
20
40
60
Frequency (MHz)
(d)
$|\Delta B_z|$ J
Frequency (MHz)
40
20
0
2
laboratory time (s)

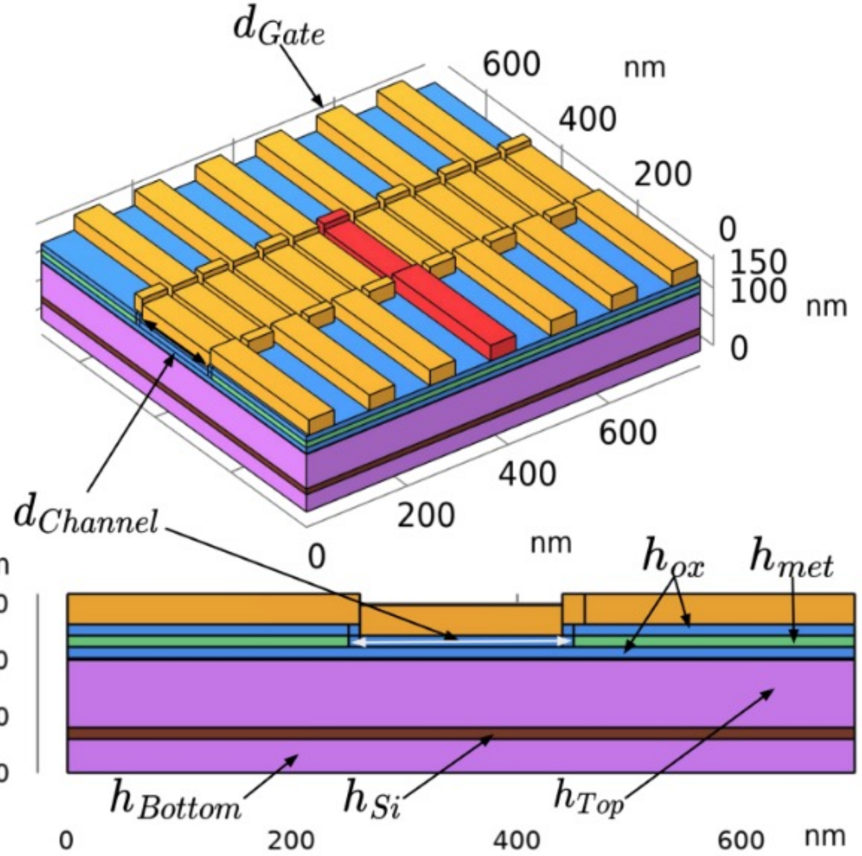
Charge noise

- Charge noise limits almost all spin qubits. For S-T we have

$$J(\epsilon + \delta\epsilon) = J(\epsilon) + \partial_\epsilon J(\epsilon)\delta\epsilon$$

- It is believed to originate at the isolated chargers at the oxide-semiconductor interface





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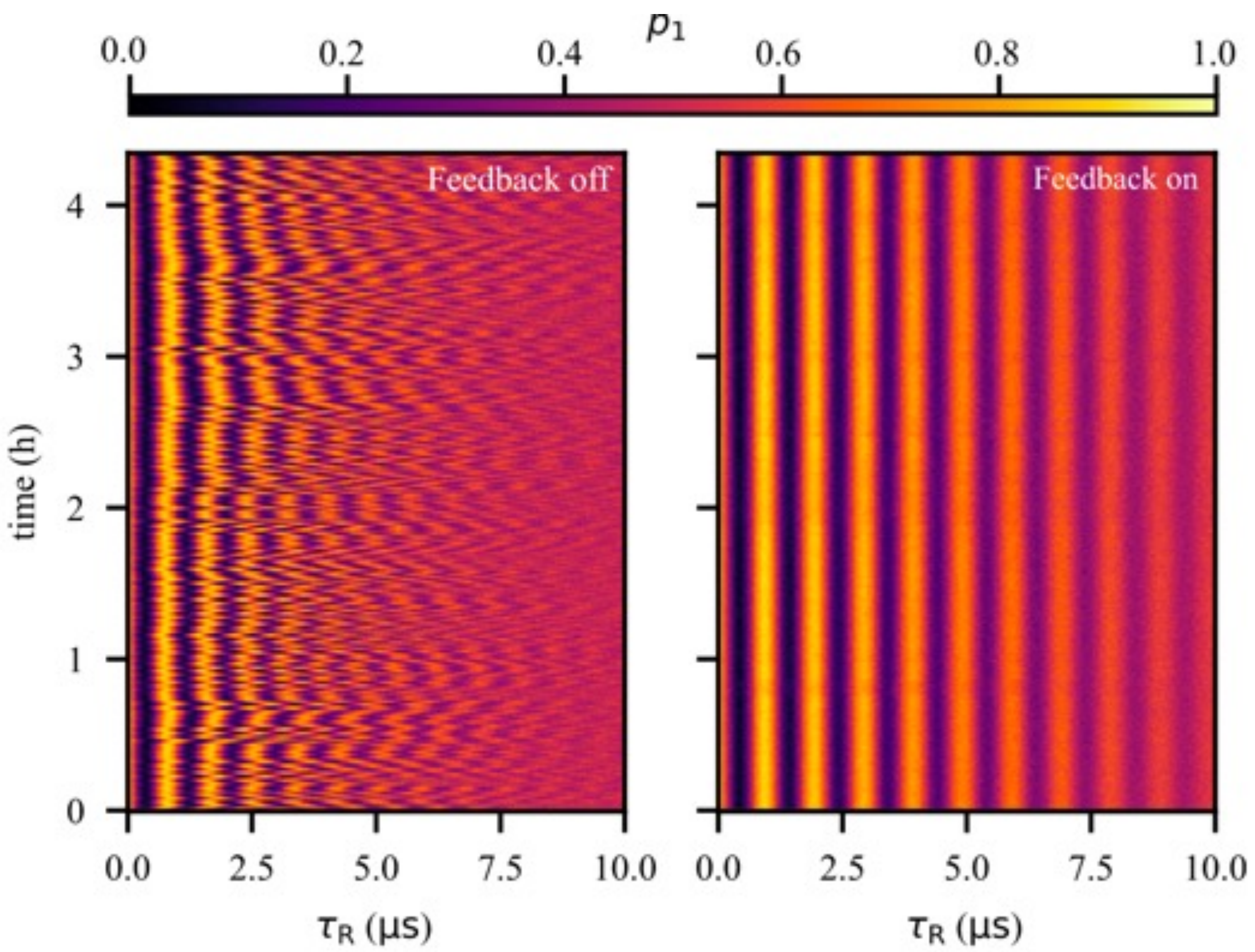
- Each defect is a source of telegraph noise, which result in 1/f spectral density
- In comparison to ΔB_z charge noise have mor weight at finite frequency.



Vepsäläinen, *et al*. Improving qubit coherence using closed-loop feedback. *Nat Commun* **13**, 1932 (2022).

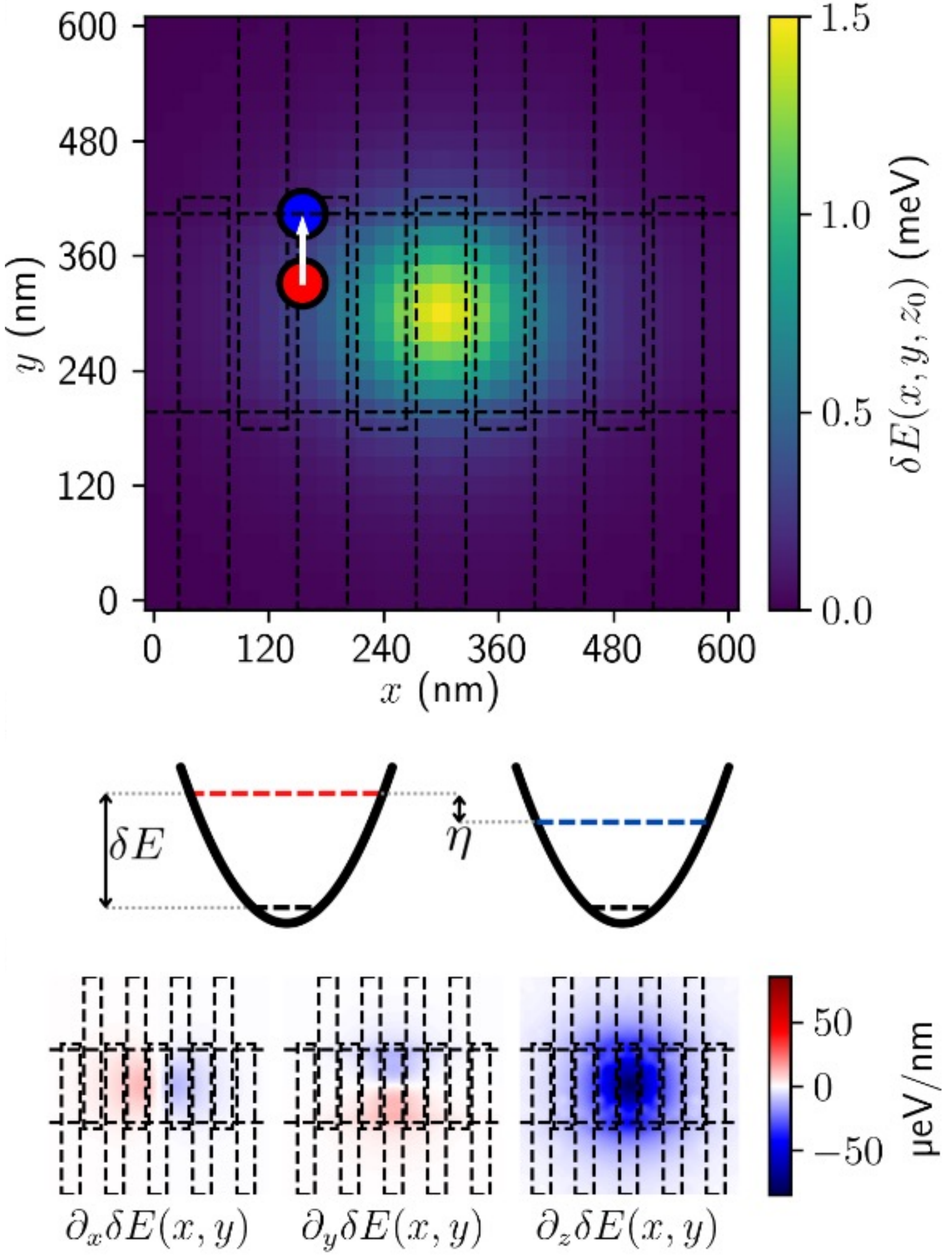
Model of charge noise

- Noise come from the sum of TLF

$$\sigma^2 = \sum_n \eta_n^2$$

- Each defect charge can physically move by random vector $\delta\mathbf{r}_n$
- Use finite-element method (COMSOL) to compute shift of dots energy $\delta E(x, y, z_0)$ due to defect at (x, y, z_0)
- We compute the contribution from charge movement using spatial derivatives:

$$\eta_n = \nabla\delta E \cdot \delta\mathbf{r}_n$$

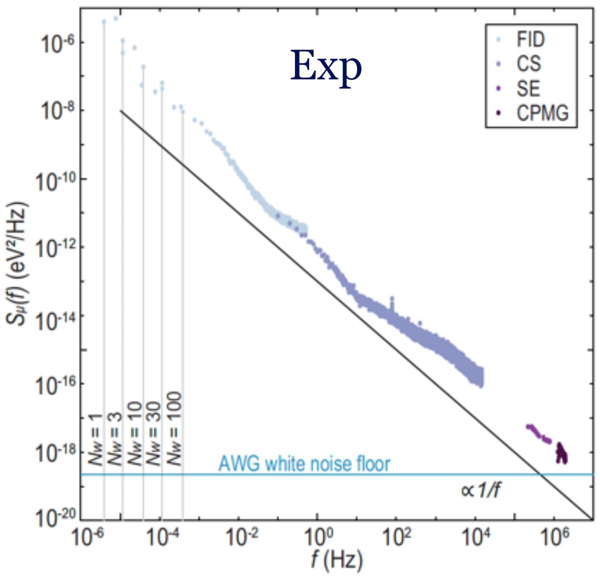


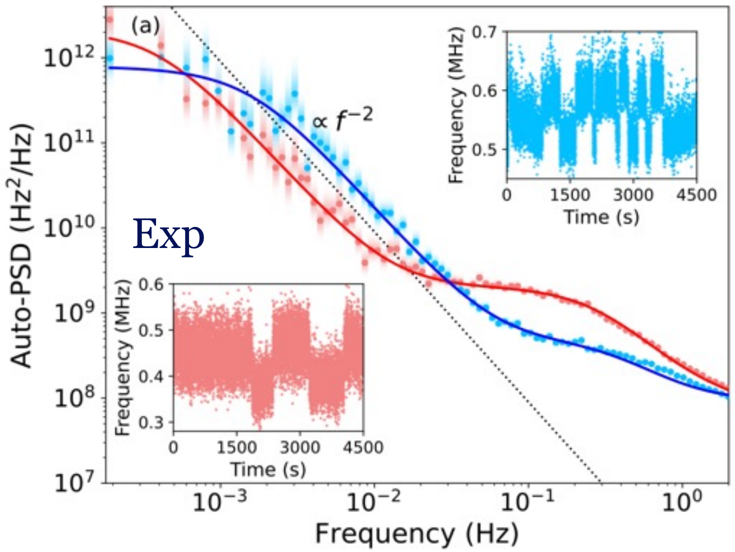
M. Kępa, N. Focke, Ł. Cywiński, J. A. Krzywda; Simulation of 1/*f* charge noise affecting a quantum dot in a Si/SiGe structure. *Appl. Phys. Lett.* 17 July 2023; 123 (3): 034005.

Spectral density

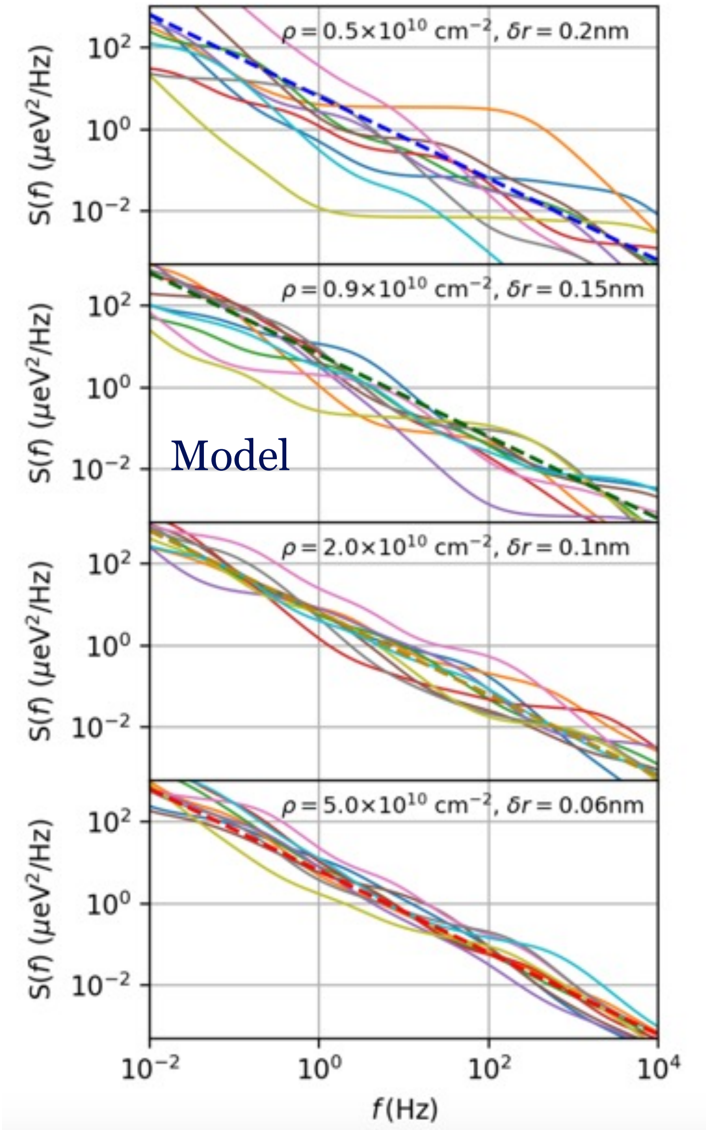
- Compared against experimentally measured spectra
- Adjusted two free parameters
  - Defect density ρ
  - Typical size of the displacement δr

Nature Communications **13**, 940 (2022)





arXiv 2302.11717 (2023)



Planar motion of the charges?

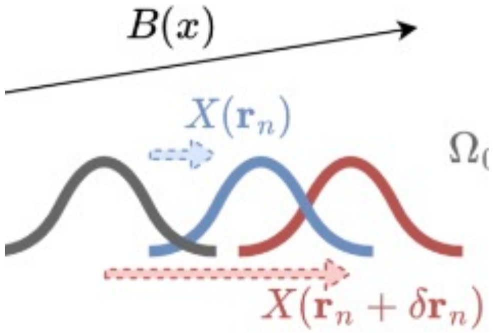
- Charge noise induces spin splitting noise(SOI)

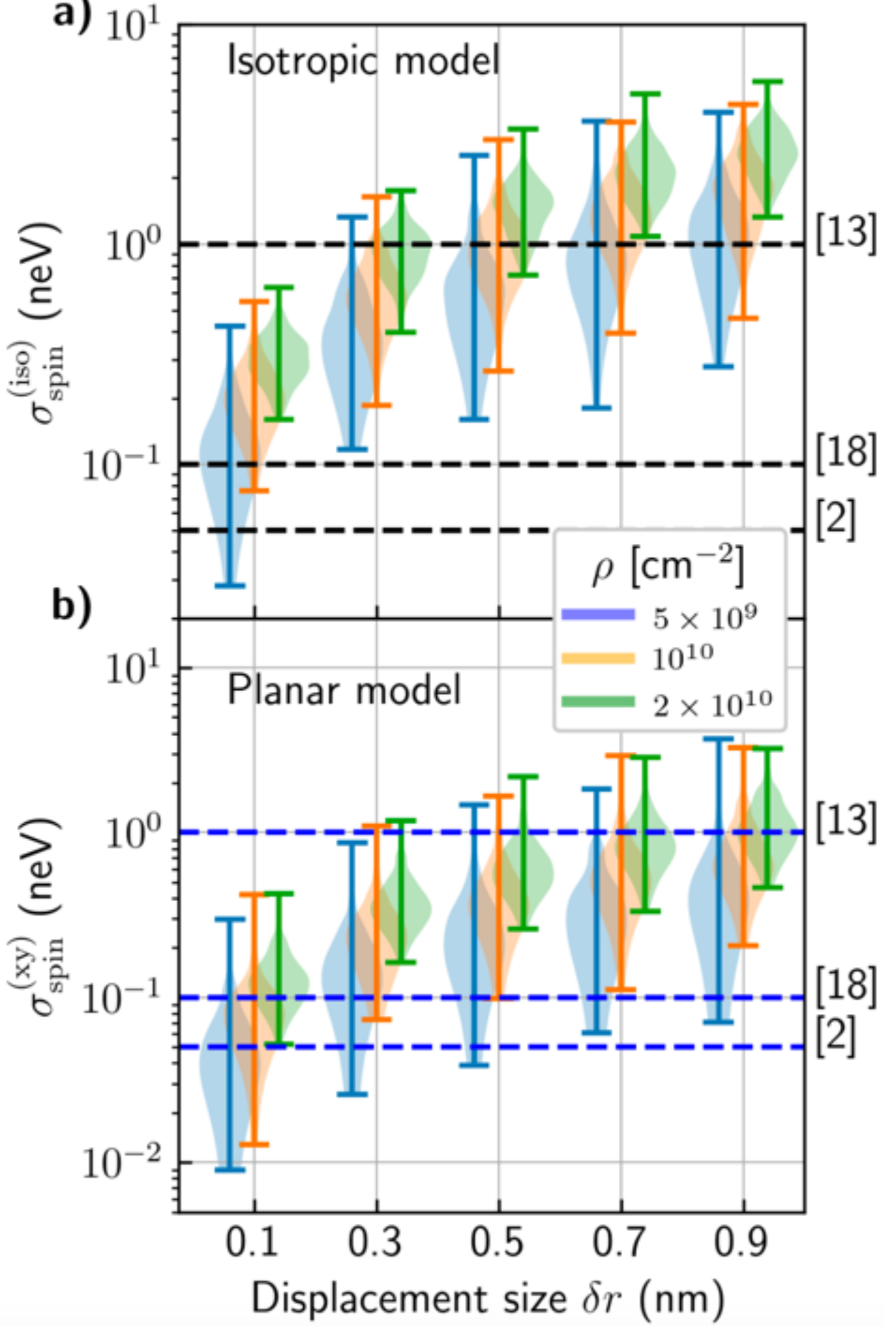
$$H = \mathbf{B}_{\parallel} \cdot \boldsymbol{\sigma} = \Omega \hat{\sigma}_z'$$

- It is caused by shaking of wavefunction in presence of mag. field gradient

$$\delta\Omega(\mathbf{r}_n) = g\mu_B \Delta B_{\parallel} \cdot \delta R(\mathbf{r_n})$$

- From this analysis we made a hypothesis that charges move in the planar direction
- We predict $\rho \approx 10^{10}\,cm^{-2}, \delta r = 0.5\,nm$



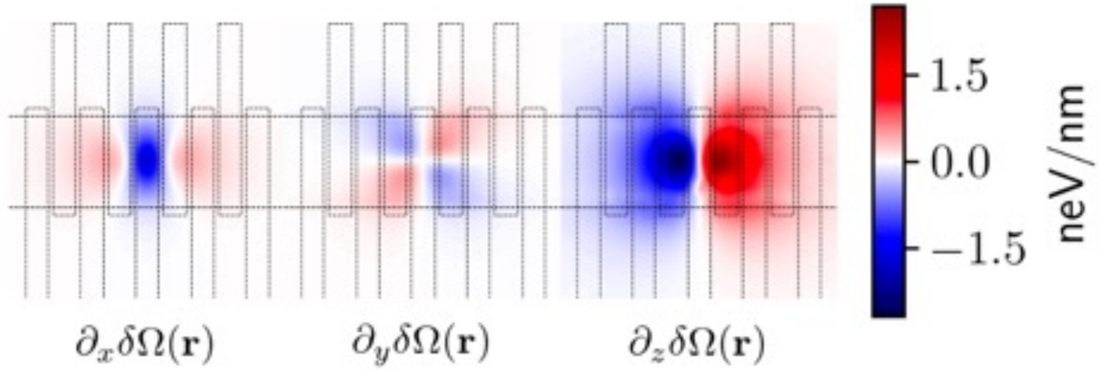


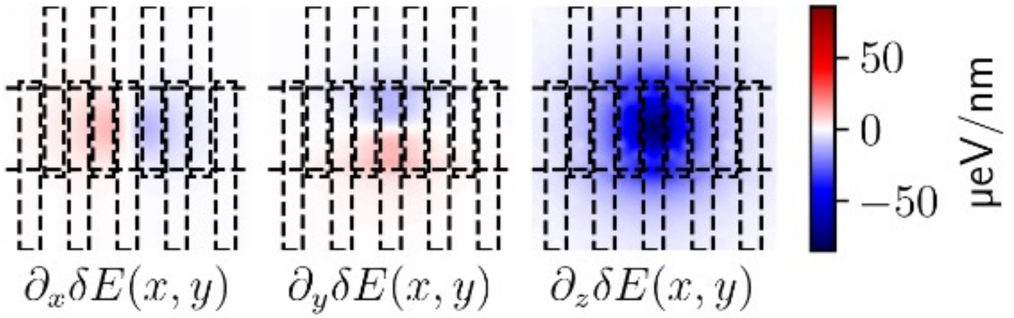
M. Kępa, Ł. Cywiński, J. A. Krzywda; Correlations of spin splitting and orbital fluctuations due to 1/f charge noise in the Si/SiGe quantum dot. *Appl. Phys. Lett.* 17 July 2023; 123 (3): 034003.

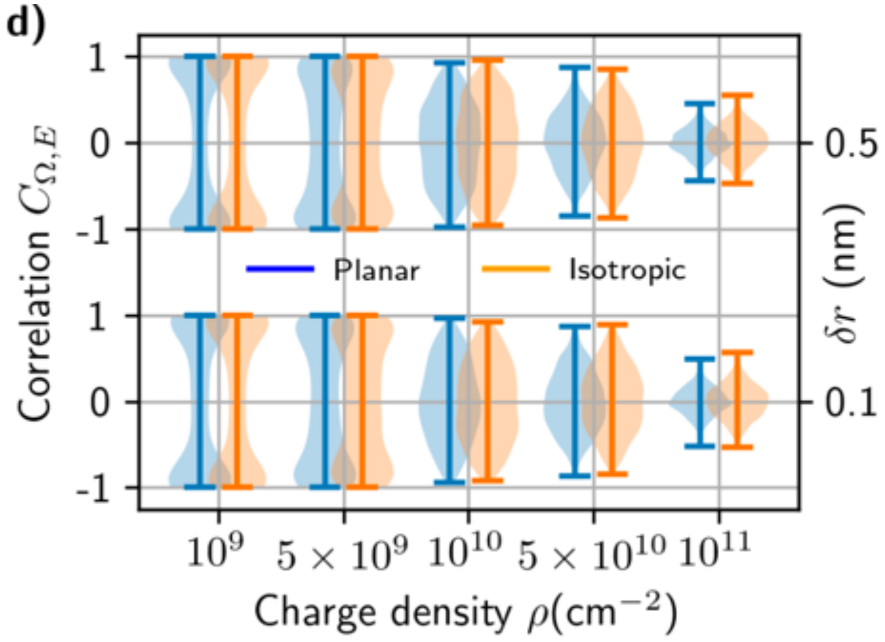
Orbital noise and spin splitting noise are (spatially) correlated

- We found that both orbital noise and spin-splitting noise can lead to spatial correlations

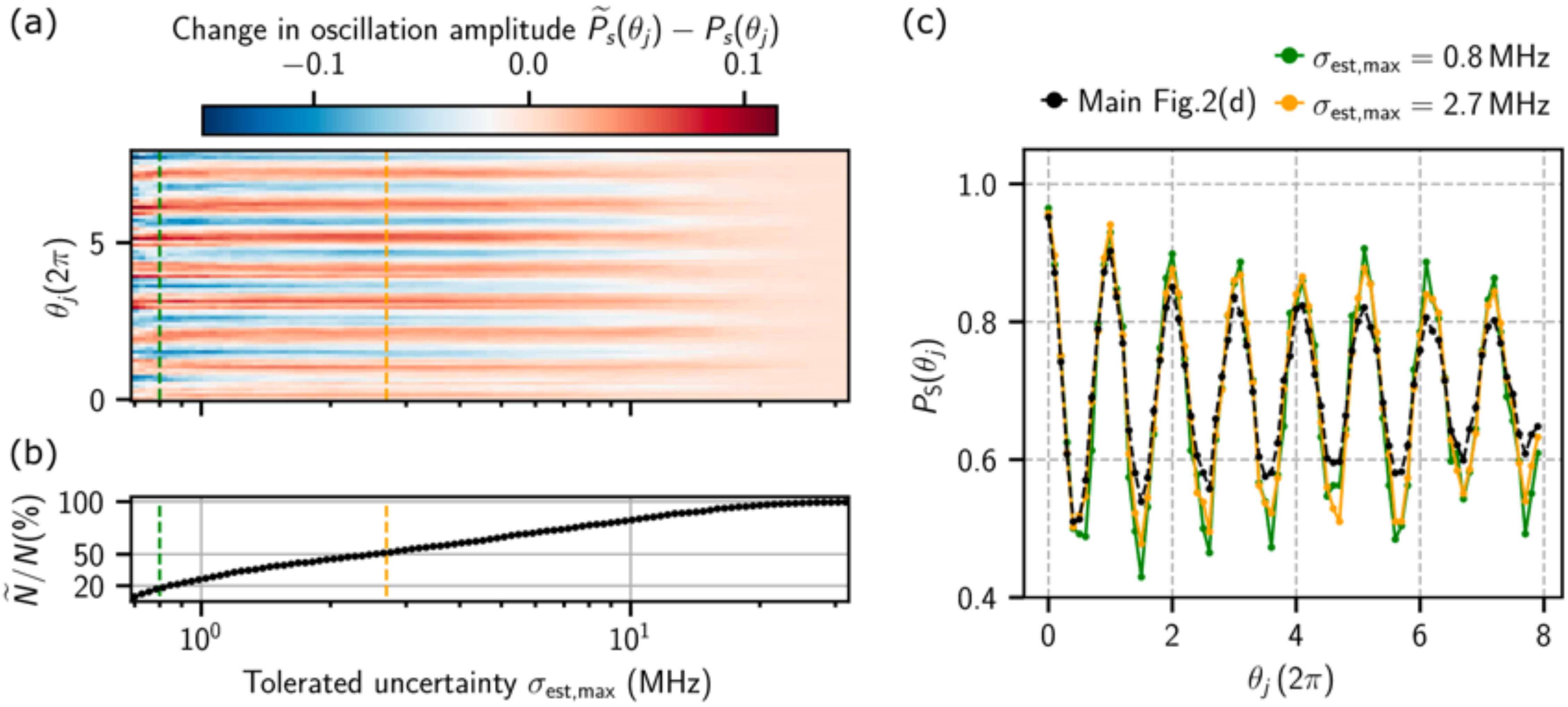
$$\langle \epsilon_L \epsilon_R \rangle \neq 0, \langle \Omega_L \Omega_R \rangle \neq 0$$







- Apart from hat non-zero correlations between them is also expected.
- This means that $\langle \delta\epsilon \delta\Delta B_z \rangle \neq 0$



(a)
Change in oscillation amplitude $\widetilde{P}_s(\theta_j) - P_s(\theta_j)$
−0.1
0.0
0.1
$\theta_j(2\pi)$
5
0
(b)
$\widetilde{N}/N(\%)$
100
50
20
10^0
10^1
Tolerated uncertainty $\sigma_{\mathrm{est,max}}$ (MHz)
(c)
Main Fig.2(d)
$\sigma_{\mathrm{est,max}} = 0.8\,\mathrm{MHz}$
$\sigma_{\mathrm{est,max}} = 2.7\,\mathrm{MHz}$
$P_S(\theta_j)$
1.0
0.8
0.6
0.4
0
2
4
6
8
$\theta_j\,(2\pi)$

Time scales

- Correlation time $T_c = 1s$
- Relaxation time $T_1 \approx 1ms$
- Estimation time $T_{est} = 100\mu s$
- Qubit cycle $T_{cycle} = 10\mu s$
- Effective dephasing time $T_2' = 1/MHz = 1\mu s$
- Dephasing time $T_2^* = 20ns$

$T_1 = 10^{-3}T_c$

$T_{est} = 10^{-4}T_c$

$T_{cycle} = 10^{-5}T_c$

$T_2' = 10^{-6}T_c$

$T_2^* = 10^{-8}T_c$

Time

- 10^4 single shots in 0.1T_c = 0.1s. During this time:
  - 0.1% estimation
  - 10% coherent evolution
  - 90% readout

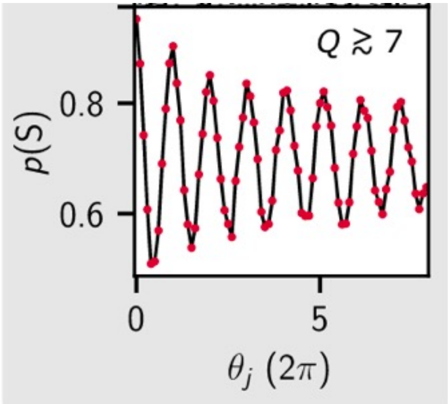
Hadamard gate synthetisation

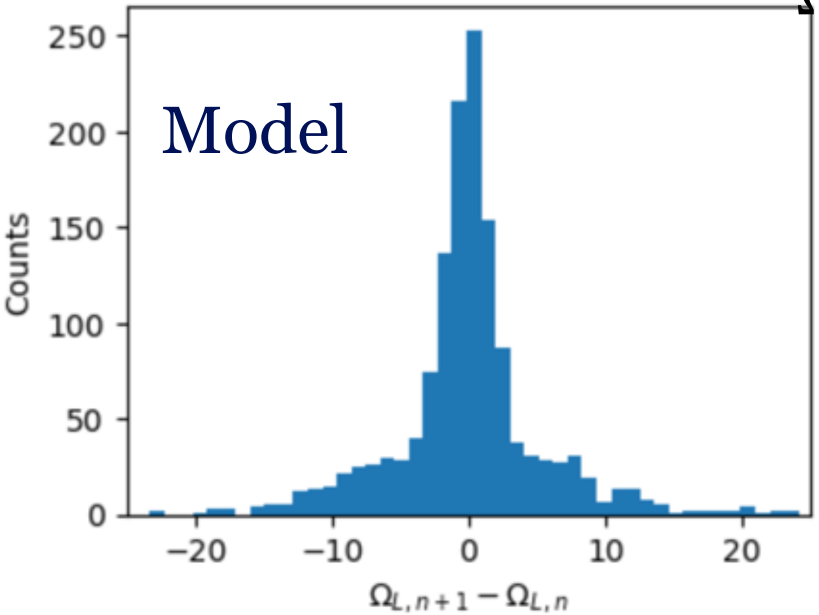
- Stable oscillations are achieved with feedback,
- Lack of contrast due to outliers:

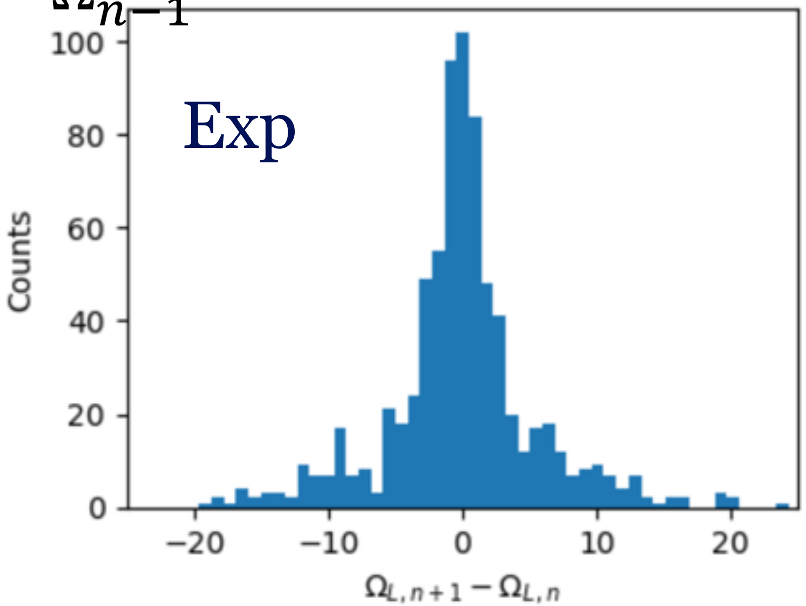
$$\delta\Omega \sim P_f \mathcal{N}(0, \sigma_{out}^2) + (1 - P_f)\mathcal{N}($$

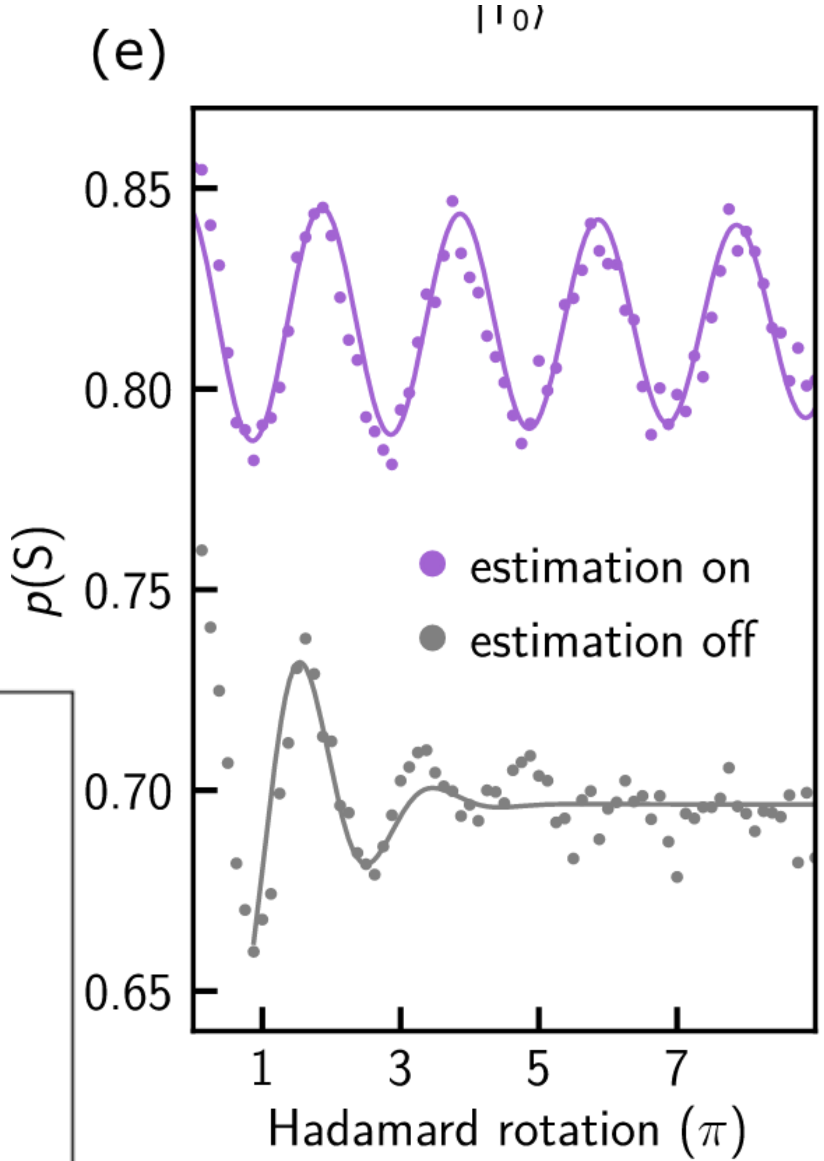
$$1 - P_f = 0.65$$

$$\hat{\Omega}_n - \hat{\Omega}_{n-1}$$









Outline (Different outline)

1. Spin qubit Quantum Computer
2. Correlated noise
3. Gates driven by noise
4. Perspective and outlook


Anasua Chattejee


Ferdinand Kuemmeth

Experiment (NBI Copenhagen)


Jeroen Danon

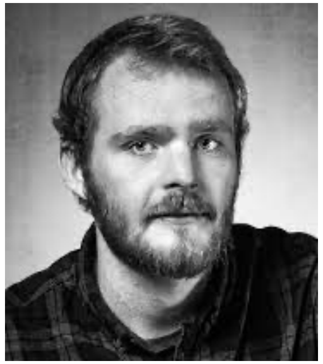
Theory (NTNU)


Evert van Nieuwenburg

Machine learning (Leiden)

ConSpiQuOS


Fabrizio Berritta


Torbjørn Raasø Rasmussen


Jacob Benestad

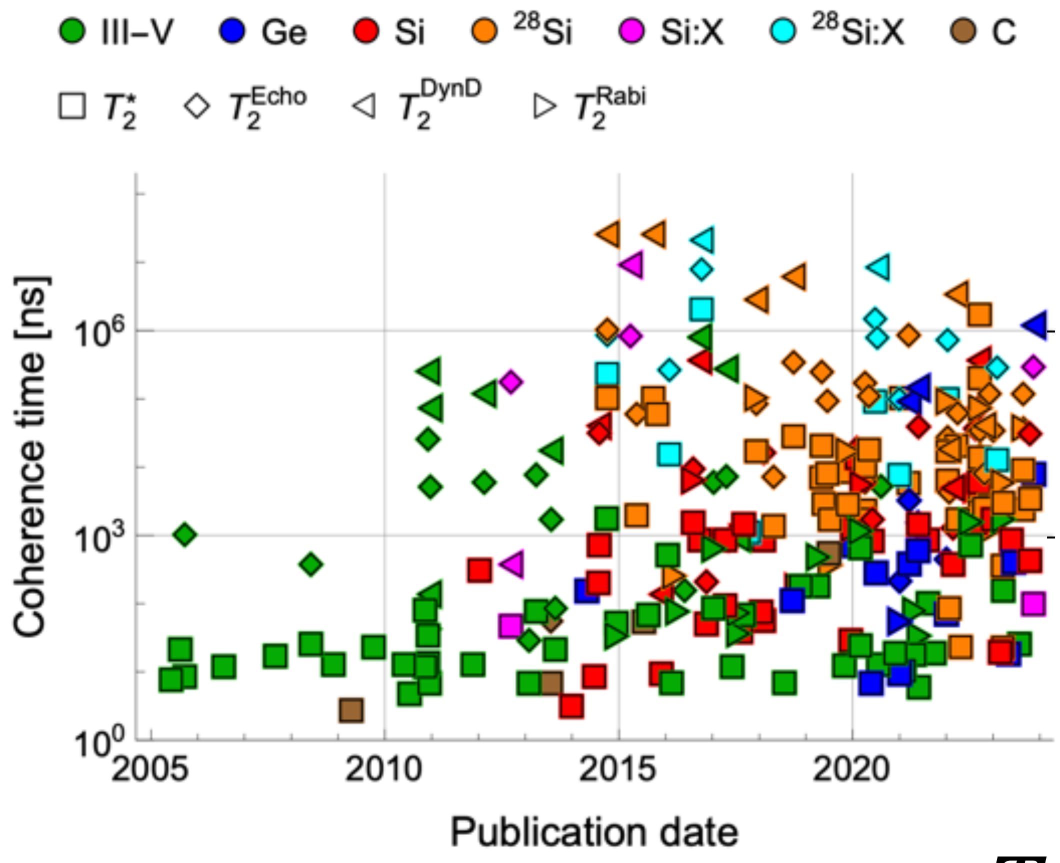
Etiuda Preludium

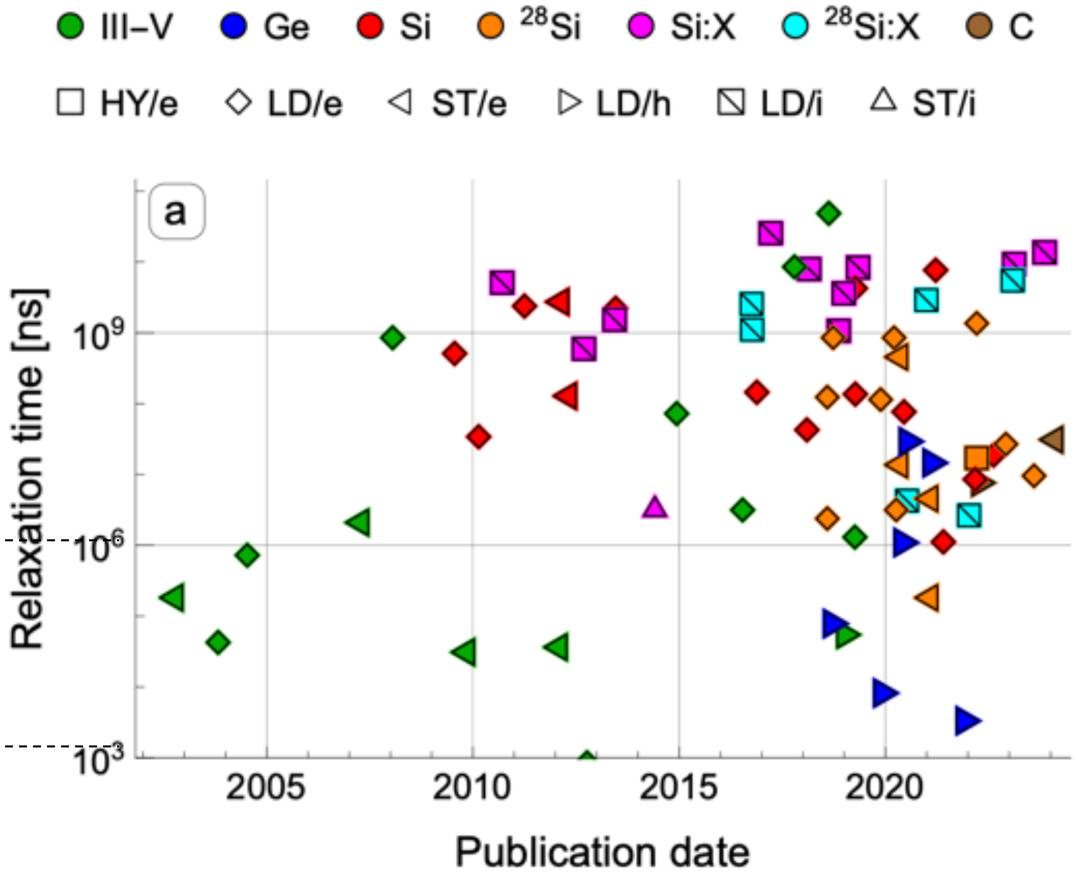

Łukasz Cywiński


Marcin Kępa

Because the noise is classical…

Qubits — No energy exchange — Environment





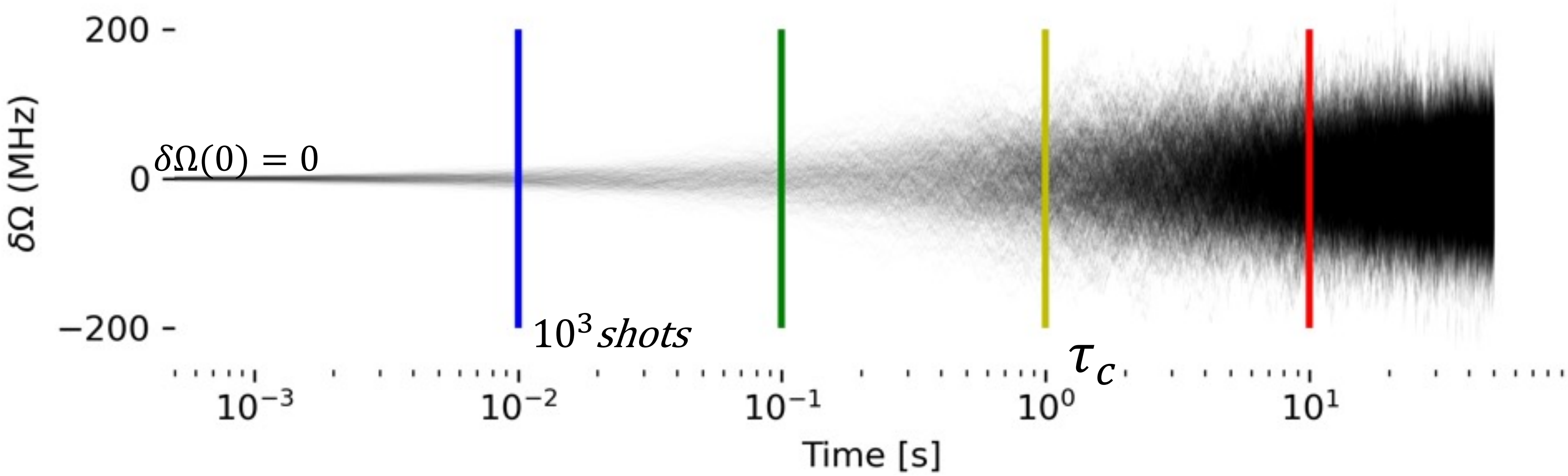
Qubits — Energy exchange — Environment

$$T_2 \gg T_1$$

Example: Nuclear noise

$$\sigma^2(t) = \sigma_f^2 - [\sigma_f^2 - \sigma^2(0)]e^{-2t/\tau_c}$$

$$\mu(t) = \delta\Omega(0)e^{-t/\tau_c}$$



$$\delta W = \frac{1}{2N}\langle\langle\delta\Omega_n^2\rangle\rangle\tau^2 = \frac{\sigma_f^2\tau^2}{2}\left(1 - \frac{1}{N}\sum_n e^{-n\Delta t/\tau_c}\right) = \frac{\sigma_f^2\tau^2}{2}\left(1 - \frac{1 - e^{-N\Delta t/\tau_c}}{e^{\Delta t/\tau_c} - 1}\right)$$