Extensions of Digital-Analog Quantum Computation

ReAQCT 2024

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2. Digital-analog schedules
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1 - Introduction to DAQC

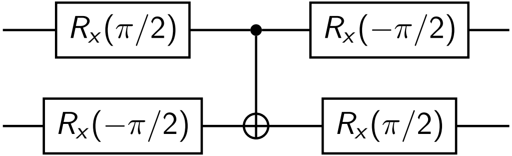
Analog quantum computing:

- Continuous evolution under a time-dependent Hamiltonian
- $U_{\text{AQC}} = \mathcal{T} \exp\left(-i \int_0^T dt\; H(t)\right)$
- Robust (quantum control theory)
- We can only do what the system is designed for

1 - Introduction to DAQC

Gate based/digital quantum computing (DQC):

- Discrete application of unitary operators
- $U_{\text{DQC}} = \prod_i U_i, \;\; U_i \in U(2^N)$



- Allows error correction
- Noisy two-qubit gates

1 - Introduction to DAQC

"Universal quantum computation can be performed using any entangling interaction and local unitary operations."

$$\{U_{\mathrm{SQG}}, e^{-itH_{\mathrm{TB}}}\} \text{ is universal}$$

J. L. Dodd, M. A. Nielsen, M. J. Bremner, and R. T. Thew, PRA 65, 040301(R) (2002)

Digital-analog quantum computing (DAQC):

- Combine digital single-qubit gates and analog blocks
- $U_{\text{DAQC}} = \prod_k U_{\text{SQG}} e^{-it_k H_S}$

This allows us to:

- Take advantage of the flexibility of digital quantum computing
- Maintain the robustness against noise of analog quantum computing

2 - Digital-analog schedules

2 - Digital-analog schedules

For simplicity, let's use ZZ Hamiltonians

$$H_S = \sum_{i<j}^{N} h_{i,j} \sigma_i^z \sigma_j^z,$$

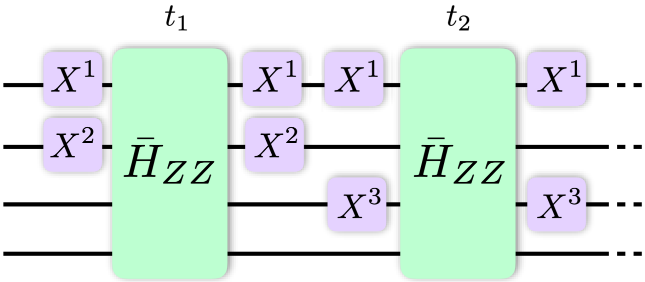
$$H_P = \sum_{i<j}^{N} g_{i,j} \sigma_i^z \sigma_j^z.$$

Objective,

$$e^{-iTH_P} = \prod_k U_k^\dagger e^{-it_k H_S} U_k.$$

A. Parra Rodriguez, P. Lougovski, L. Lamata, E. Solano, and M. Sanz, PRA 101, 022305 (2020)

Flip the effective sign of the couplings with Pauli gates



For ZZ Hamiltonians,

$$\sigma_i^x e^{-it\sigma_i^z\sigma_j^z}\sigma_i^x = e^{-it\sigma_i^x\sigma_i^z\sigma_j^z\sigma_i^x} = e^{it\sigma_i^z\sigma_j^z}.$$

2 - Digital-analog schedules

If $H_S \propto H_P$ the schedule is trivial. Else,

$$e^{-iTH_P} = \prod_k U_k^\dagger e^{-it_k H_S} U_k = \prod_k e^{-it_k H_S^{(k)}} = \exp\left(-i \sum_k t_k H_S^{(k)}\right),$$

which is equivalent to

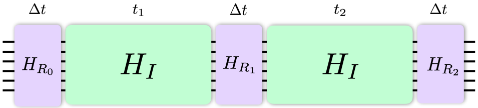
$$M \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_K \end{pmatrix} = T \begin{pmatrix} h_{1,2}/g_{1,2} \\ h_{1,3}/g_{1,3} \\ \vdots \\ h_{N-1,N}/g_{N-1,N} \end{pmatrix}, M \in \{\pm 1\}$$

M has the information about the effective signs of the couplings at each block.

2 - Digital-analog schedules

We assumed a perfect setup $\begin{cases} \text{Instantaneous SQGs} \\ \text{OR} \\ \text{Ability to turn on and off the interactions on-demand} \end{cases}$

stepwise-DAQC (sDQAC)



We assumed a perfect setup { Instantaneous SQGs
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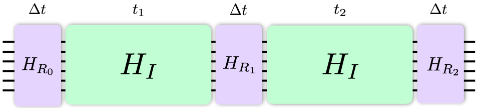
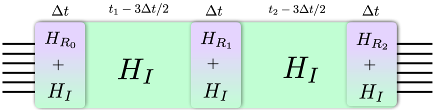
stepwise-DAQC (sDQAC)



banged-DAQC (bDAQC)



2 - Digital-analog schedules

Banging error:

$$U_{\text{Ideal}} = U_{\text{sDAQC}} = \prod_k e^{-it_k H_S} U_k = \prod_k e^{-it_k H_S} e^{-it_{SQG} H_{SQG}^{(k)}}$$

$$\approx U_{\text{bDAQC}} = \prod_k e^{-i(t_k - t_{SQG}) H_S} e^{-it_{SQG}\left(H_{SQG}^{(k)} + H_S\right)}$$

For the error to be negligible, $t_{\text{SQG}} \ll \min(t_k)$.
(usually is enough if $10^2 t_{\text{SQG}} \lesssim \min(t_k)$)

3 - Arbitrary two-body Hamiltonians (New!)

3 - Arbitrary two-body Hamiltonians

We wish to work with general two-body Hamiltonians

$$H = \sum_{i,j} \sum_{\mu,\nu \in \{x,y,z\}} h_{i,j}^{\mu,\nu} \sigma_i^{\mu} \sigma_j^{\nu} .$$

Before, we had to options to work with arbitrary two-body Hamiltonians:

- Use expensive term-by-term decompositions[1]
- Optimize specific cases by hand[2]

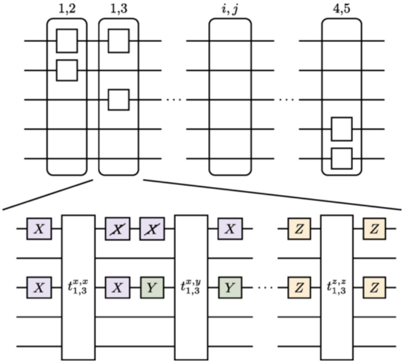
Can we develop a systematic procedure for solving this problem?

[1] A. Parra Rodriguez, P. Lougovski, L. Lamata, E. Solano, and M. Sanz, PRA 101, 022305 (2020)

[2] T. Gonzalez-Raya, R. Asensio-Perea, A. Martin, et al., PRX Quantum 2, 020328 (2021)

Digital-analog quantum computation with arbitrary two-body Hamiltonians

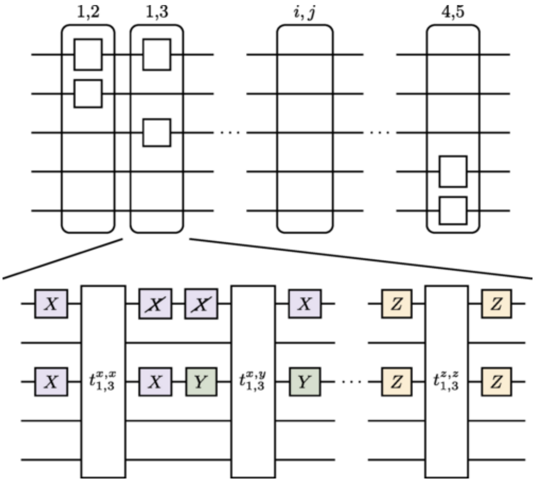
Mikel Garcia-de-Andoin, Álvaro Saiz, Pedro Pérez-Fernández, Lucas Lamata, Izaskun Oregi, and Mikel Sanz
Phys. Rev. Research **6**, 013280 – Published 14 March 2024



By paying a Trotterization error, we treat each possible interaction in the Pauli basis individually.

$$\prod_k e^{-it_k H_S^{(k)}} \approx e^{-i\sum_k t_k H_S^{(k)}}$$

We build a valid DAQC schedule by using combinations of all Pauli gates.

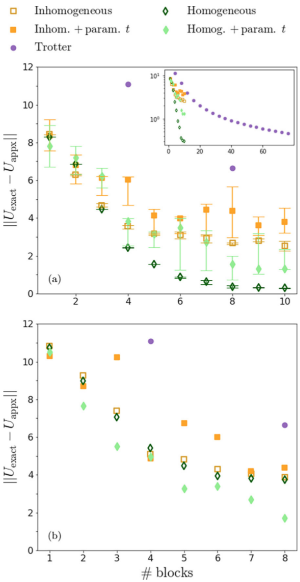


3 - Arbitrary two-body Hamiltonians

This was made assuming that all the interactions we want to simulate $\sigma_i^\mu \sigma_j^\nu$ appear on the source Hamiltonian.

Else,

- "Spray'n'pray", apply uniformly random single qubit gates.
- Optimize the rotation gates by using a tensor network proxy.

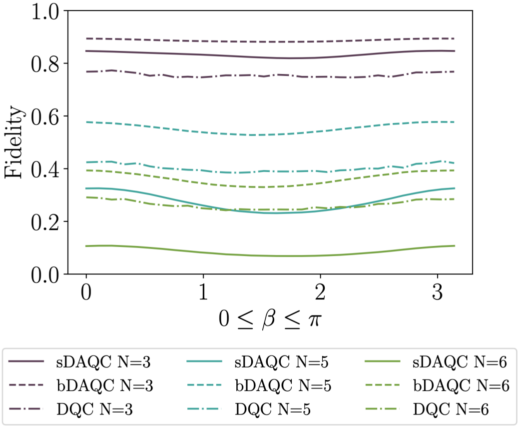


4 - Stability of DAQC (New!)

4 - Stability of DAQC

Noise sources:

- SQG errors
- Timing errors
- Bit-flip
- Decoherence
- Dephasing



QFT over a family of states
$|\Psi(\beta)\rangle = \sin\beta\,|W_N\rangle + \cos\beta\,|GHZ_N\rangle$

P. Garcia Molina et al. "Noise in Digital and Digital-Analog Quantum Computation", arXiv:2107.12969(2021)

4 - Stability of DAQC

We are interested in studying how DAQC scales with the number of qubits.

Informal definition:

A quantum simulation task is stable if the error committed when measuring a local observable depends on the error parameter but doesn't grow with the number of particles.

$$\varepsilon = f(\delta), \quad \varepsilon \neq f(\delta, N)$$

An analogue quantum simulation task on spin systems is stable for local* Hamiltonians.

R. Trivedi, A. Franco Rubio, J. Ignacio Cirac, arXiv:2212.04924 (2022)

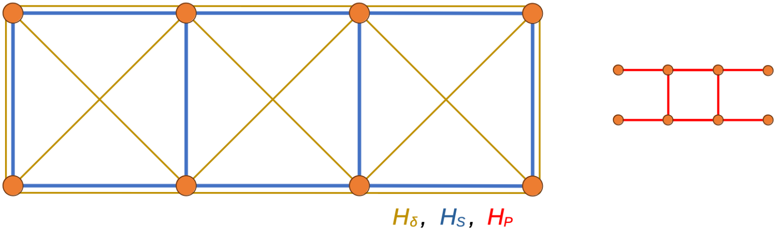
4 - Stability of DAQC

Since SQGs have a very high fidelity, we focus on errors on the source Hamiltonian

$$H_{S,\text{meas}} \longrightarrow H_{S,\text{real}} = H_{S,\text{meas}} + H_\delta.$$

This induces certain deviation to the simulated Hamiltonian

$$H_P \longrightarrow H_{P,\text{real}} = H_P + H_\varepsilon.$$



Since SQGs have a very high fidelity, we focus on errors on the source Hamiltonian, in particular calibration errors

$$H_{S,\text{meas}} \longrightarrow H_{S,\text{real}} = H_{S,\text{meas}} + H_{\delta}.$$

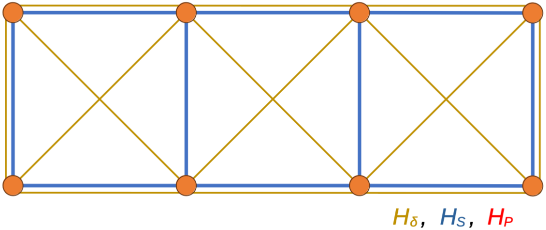
This induces certain deviation to the simulated Hamiltonian

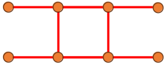
$$H_P \longrightarrow H_{P,\text{real}} = H_P + H_{\varepsilon}.$$

If the Hamiltonian is sufficiently local*, the simulation with DAQC is stable.

$$\varepsilon \lesssim 4mT\delta \mathrm{deg}(\mathcal{P}) \| h_P \oslash h_S \|_\infty + 4m\delta \mathrm{deg}(\mathcal{D} \backslash \mathcal{S}) t_A,$$

m number of measured qubits, T simulation time, t_A total time.

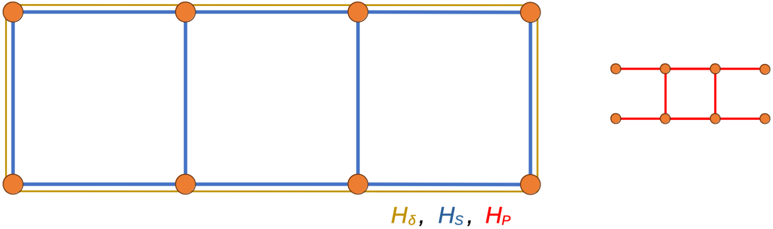




4 - Stability of DAQC

We can redesign our DAQC circuit to mitigate errors outside the problem (∼DD)

$$\varepsilon \lesssim 4mT\delta\deg(\mathcal{P})\|h_P \oslash h_S\|_\infty.$$



A. Ahedo, M. Garcia de Andoin, M. Sanz, in preparation

5 - Conclusions

Key points of digital-analog:

- Digital-analog is universal
- Using only SQGs makes it more noise resilient
- Can achieve faster circuit times

5 - Conclusions

Future works:

- Bounds for t_A
- Experimental implementations
- Qudit digital-analog

Affiliations:

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Thank you!

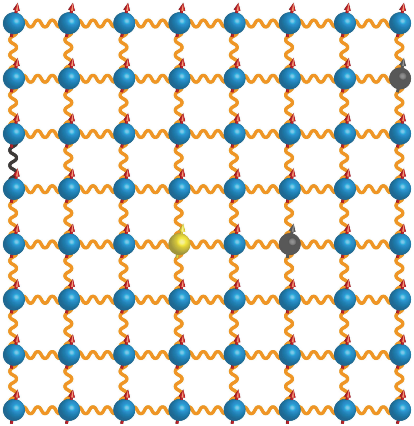
Try DAQC yourself! github.com/NQUIRE-Center/DAQC_simulator

Contact: mikelgda@gmail.com, @mgarciadeandoin

Practical experimentation

There are already successful experiments employing DAQC

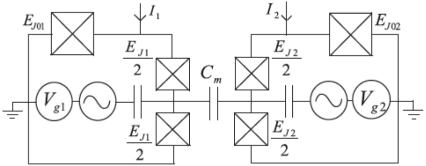
Quantum Neural Networks experiments on the Zuchongzhi hardware



M. Gong, H.-L. Huang, S. Wang, C. Guo, S. Li, . . . , and J.-W. Pan, arXiv:2201.05957 (2022)

DAQC platforms

- Superconducting circuits
- Trapped ions
- Rydberg atoms
- Liquid state NMR setups
- ...



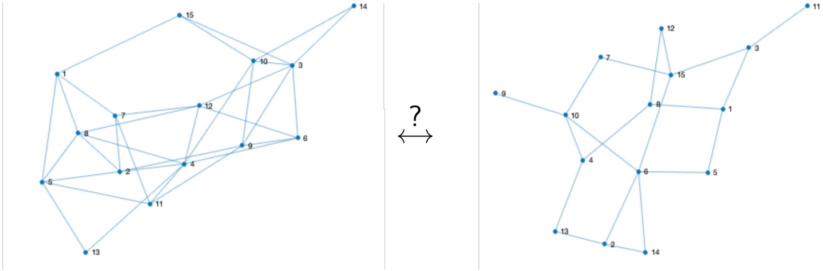
$$\begin{aligned} H = \quad & \frac{\omega_1}{2}\sigma_1^z + \Omega_1 \cos(\omega_1^{\text{rf}} t + \phi_1)\sigma_1^x \\ & + \frac{\omega_2}{2}\sigma_2^z + \Omega_2 \cos(\omega_2^{\text{rf}} t + \phi_2)\sigma_2^x \\ & + \frac{\omega_{xx}}{2}\sigma_1^x\sigma_2^x. \\ \rightarrow \cdots \rightarrow & \; H_{QF}^{\text{eff}} \sim \omega_{xx}\sigma_1^x\sigma_2^x \end{aligned}$$

G.S. Paraoanu, PRB 74, 140504(R) (2006). C. Rigetti, and M. Devoret, PRB 81, 134507 (2010).

Arbitrary connectivity

Simulation of dynamics of arbitrary Hamiltonians on an arbitrary system:

- If the connectivity graph of H_P doesn't match the one in H_S we need to implement SWAP strategies (really hard).



Other works

- A Scheme to Implement a Universal Two-Qubit Quantum Circuit using Cross-Resonance Hamiltonian.pdf
- Benchmarking Digital-Analog Quantum Computation.pdf
- Digital-analog co-design of the Harrow-Hassidim-Lloyd algorithm.pdf
- Digital-Analog Counterdiabatic Quantum Optimization with Trapped Ions.pdf
- Digital-analog for the Fourier quantum transform.pdf
- Digital-Analog Quantum Computation.pdf
- Digital-analog quantum convolutional neural networks for image classification.pdf
- Digital-analog quantum learning on Rydberg atom arrays.pdf
- Digital-Analog Quantum Simulations Using the Cross-Resonance.pdf
- Enhanced connectivity of quantum hardware with digital-analog control.pdf
- Enhancing Quantum Annealing in Digital-Analog Quantum Computing.pdf
- Fixed Depth Hamiltonian Simulation via Cartan Decomposition.pdf
- Hybrid DA simulation of many-body dynamics with SC qubits.pdf
- Noise in Digital and Digital-Analog Quantum Computation.pdf
- Qadence- a differentiable interface for digital-analog programs.pdf
- Quantum Computing with an Always-On Heisenberg Interaction.pdf
- Realization of programmable Ising models in a trapped-ion quantum simulator.pdf
- Stochastic error cancellation in analog quantum simulation.pdf

2 - Digital-analog schedules (example)

We want to simulate a ZZ-Ising Hamiltonian for a time T=1, with another ZZ-Ising Hamiltonian.

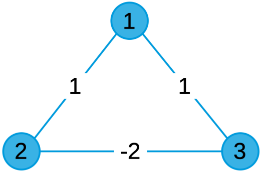


Figure: Source Hamiltonian (H_S)

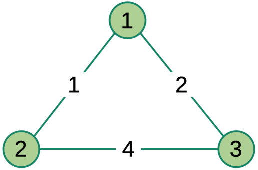


Figure: Problem Hamiltonian (H_P)

$$H_S = 1\sigma_1^z\sigma_2^z + 2\sigma_1^z\sigma_3^z - 2\sigma_2^z\sigma_3^z,$$

$$H_P = 1\sigma_1^z\sigma_2^z + 2\sigma_1^z\sigma_3^z + 4\sigma_2^z\sigma_3^z,$$

2 - Digital-analog schedules (example)

By sandwiching an X gate to a single qubit, we can change the effective sign of the couplings connected to it.

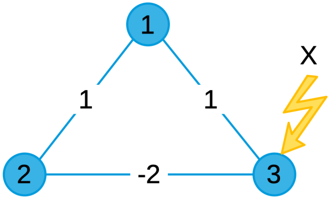


Figure: Sandwich block with X gate on qubit 3

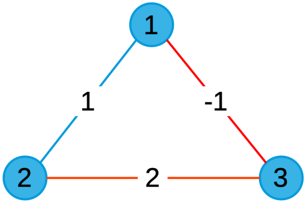


Figure: Effective Hamiltonian ($H_S^{(k)}$)

Run all possible combinations of SQGs to obtain

$$M = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

Find the times for the analog blocks.

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} h_{1,2}/g_{1,2} \\ h_{1,3}/g_{1,3} \\ h_{2,3}/g_{2,3} \end{pmatrix} = \begin{pmatrix} 1/1 = 1 \\ 2/2 = 1 \\ 4/-2 = -2 \end{pmatrix}$$

We want to minimize the total time of the circuit, and the times to be positive

$$\min_{\vec{t}} \sum_k t_k \quad \oplus \quad M \cdot \vec{t} = T\vec{h/g} \quad \& \quad t_k \geq 0 \ \forall k$$

The solution to this problem is

$$t_1 = 1,\ \ t_2 = 0,\ \ t_3 = 1.5,\ \ t_4 = 1.5.$$

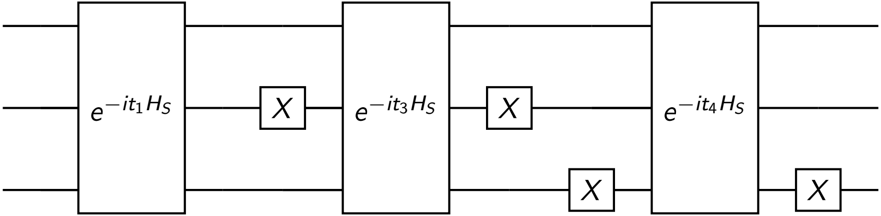
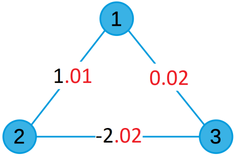
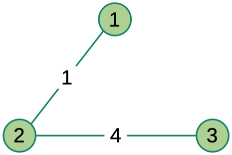


Figure: DAQC circuit for simulating H_P for T=1 with H_S.

4 - Stability of DAQC





Time optimal: eliminate non existent couplings

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ \cancel{1} & \cancel{1} & \cancel{-1} & \cancel{-1} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = T \begin{pmatrix} h_{1,2}/g_{1,2} \\ h_{1,3}/g_{1,3} \\ \cancel{0/0} \end{pmatrix}$$

Error mitigated: set non existent couplings to 0 (cost extra circuit time)

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = T \begin{pmatrix} h_{1,2}/g_{1,2} \\ h_{1,3}/g_{1,3} \\ 0/0 = 0 \end{pmatrix}$$