Quantum approximated cloning-assisted density matrix exponentiation

arXiv:2311.11751

Pablo Rodriguez-Grasa, Rubén Ibarrondo, Javier González-Conde, Yue Ban, Patrick Rebentrost & Mikel Sanz

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June 19th, 2024 **ReAQCT, Budapest**

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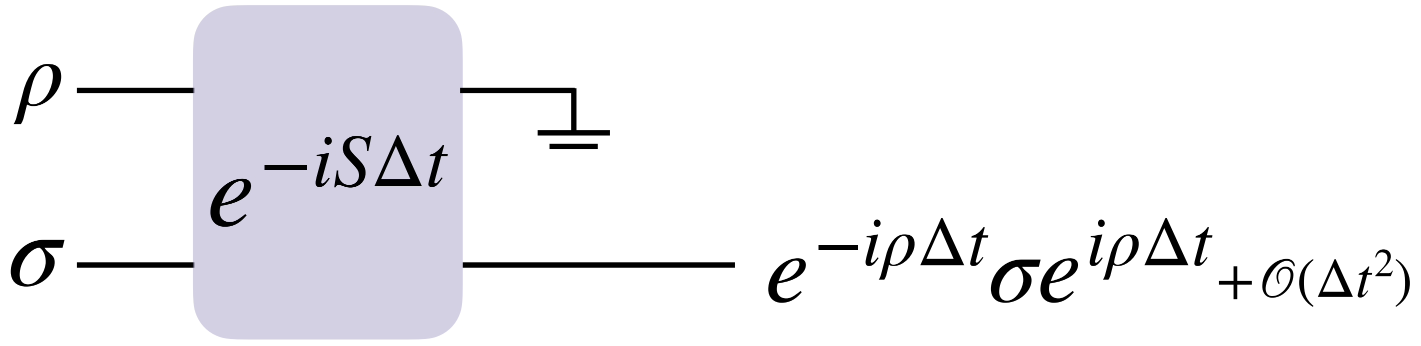
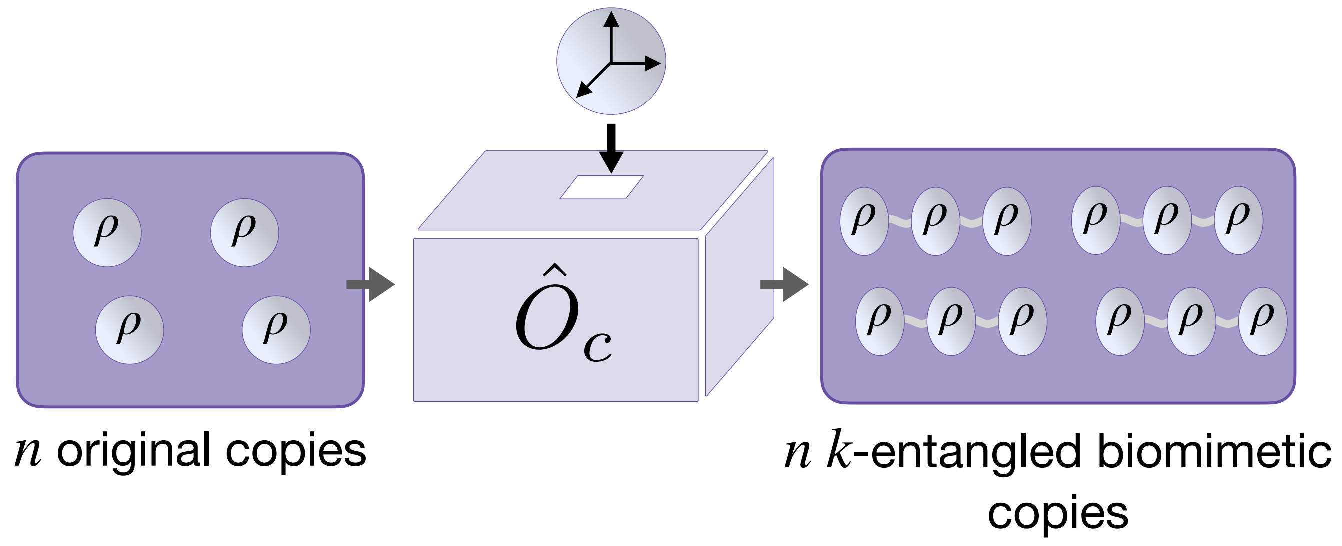
Density matrix exponentiation

LMR trick:

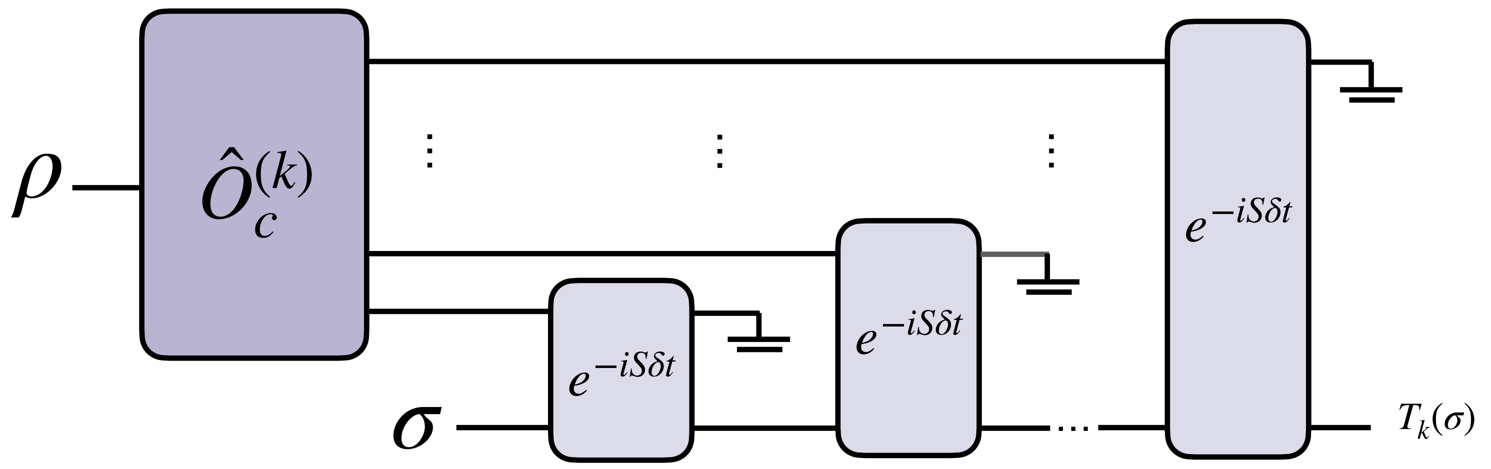


Imperfect cloning

Biomimetic cloning:



LMR assisted by biomimetic copies



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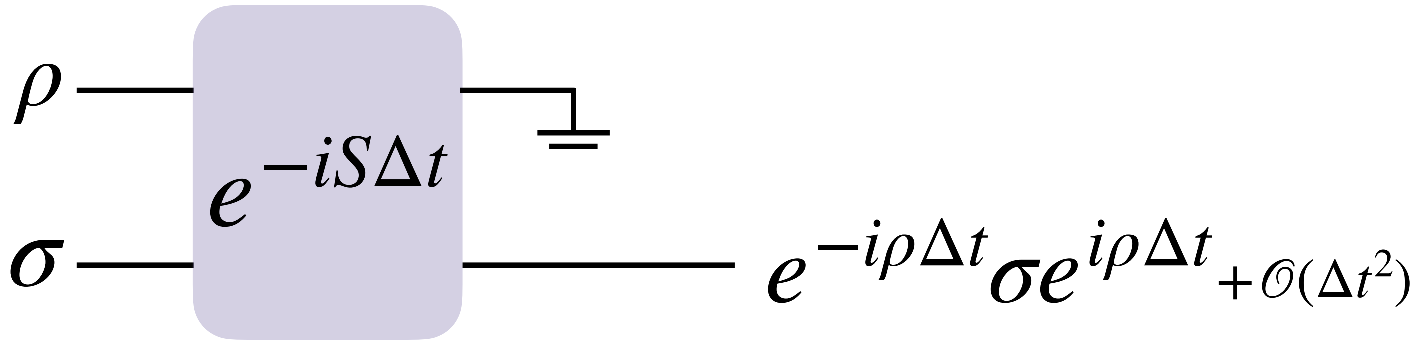

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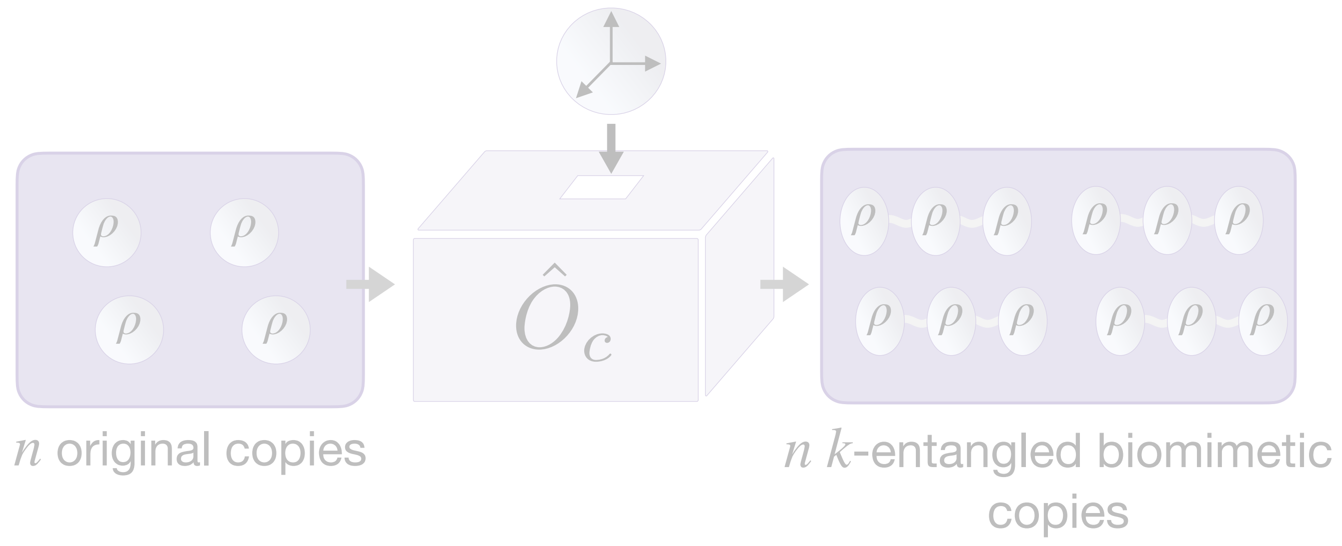

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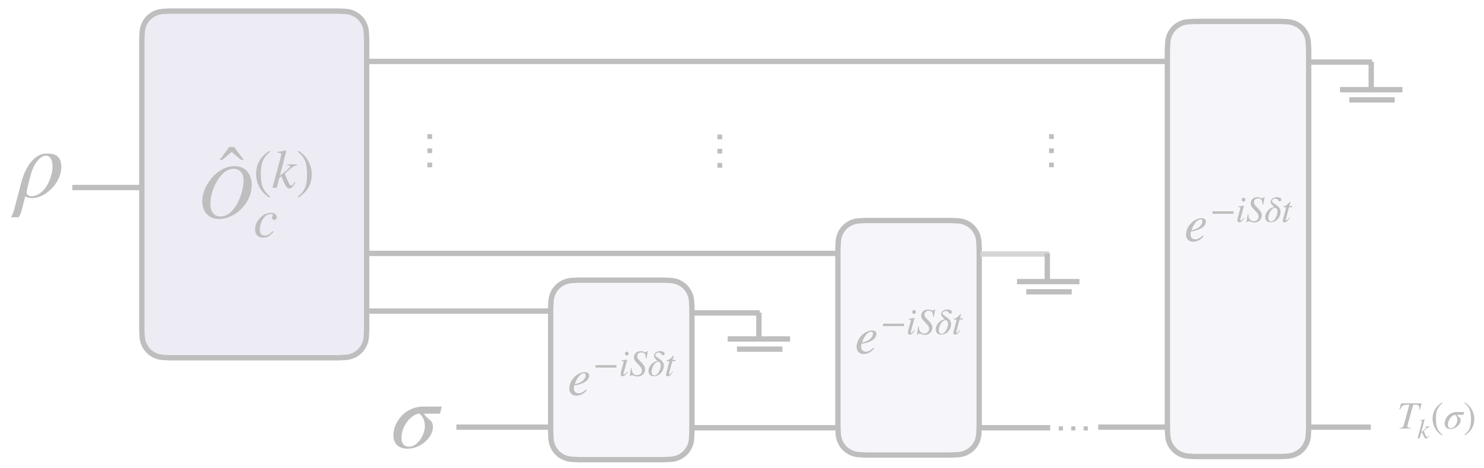
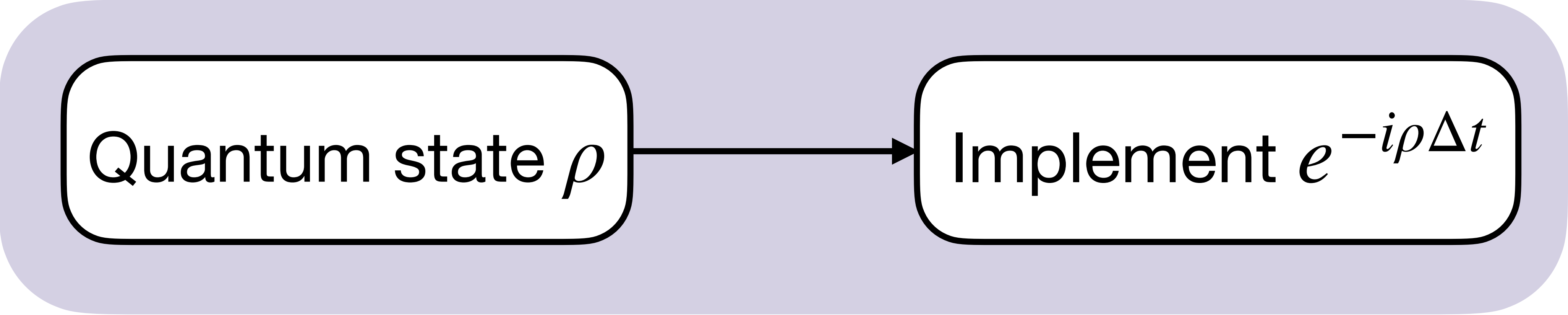
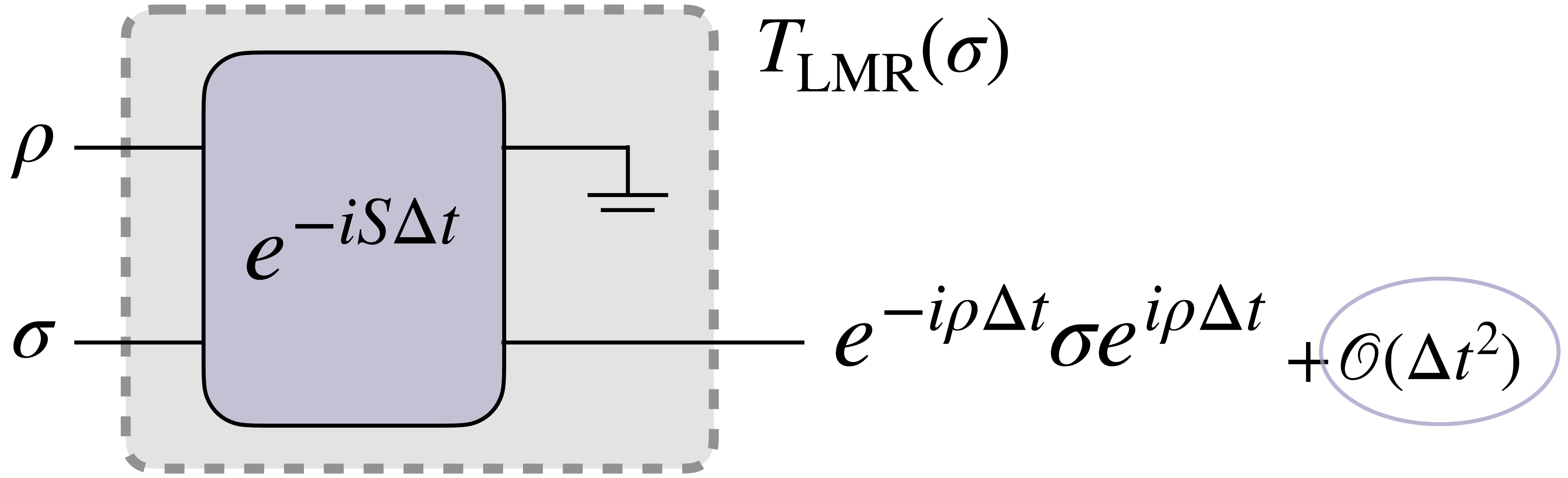


Density matrix exponentiation



LMR[1] trick:



Could we find a better operation? $\mathcal{O}(\Delta t^3)$?

LMR protocol is optimal[2]

[1] Lloyd, S., Mohseni, M., & Rebentrost, P. (2014). “Quantum principal component analysis”. *Nature Physics*, *10*(9), 631-633.
[2] Kimmel, S., Lin, C. Y. Y., Low, G. H., Ozols, M., & Yoder, T. J. (2017). “Hamiltonian simulation with optimal sample complexity”. *npj Quantum Information*, *3*(1), 13.

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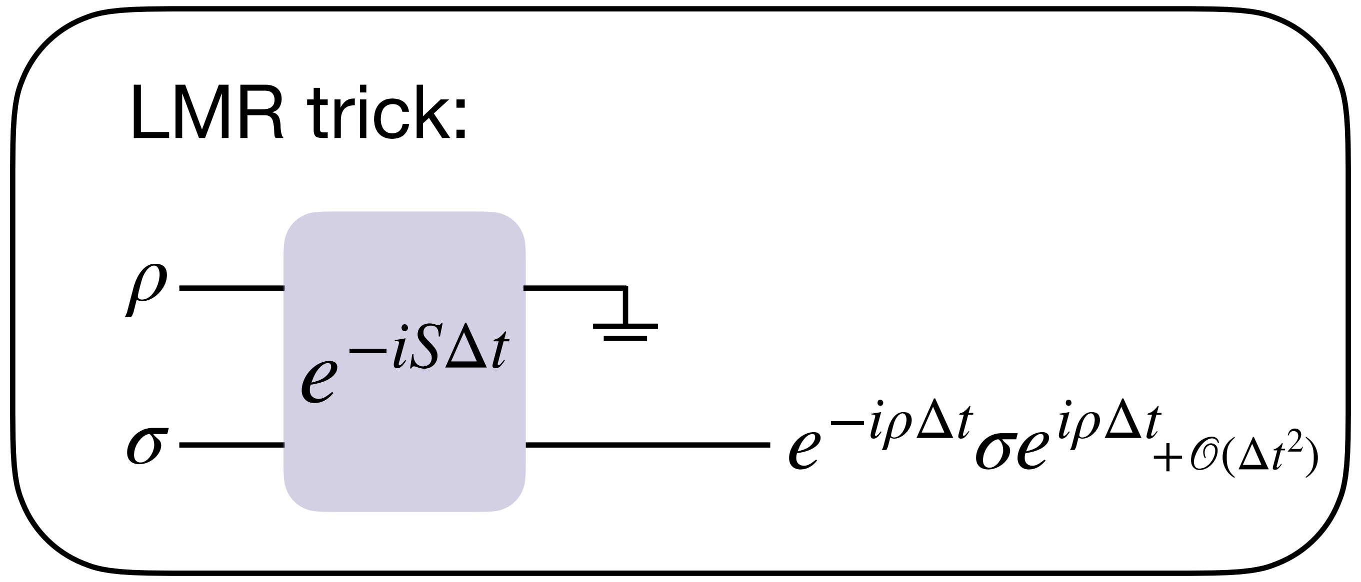
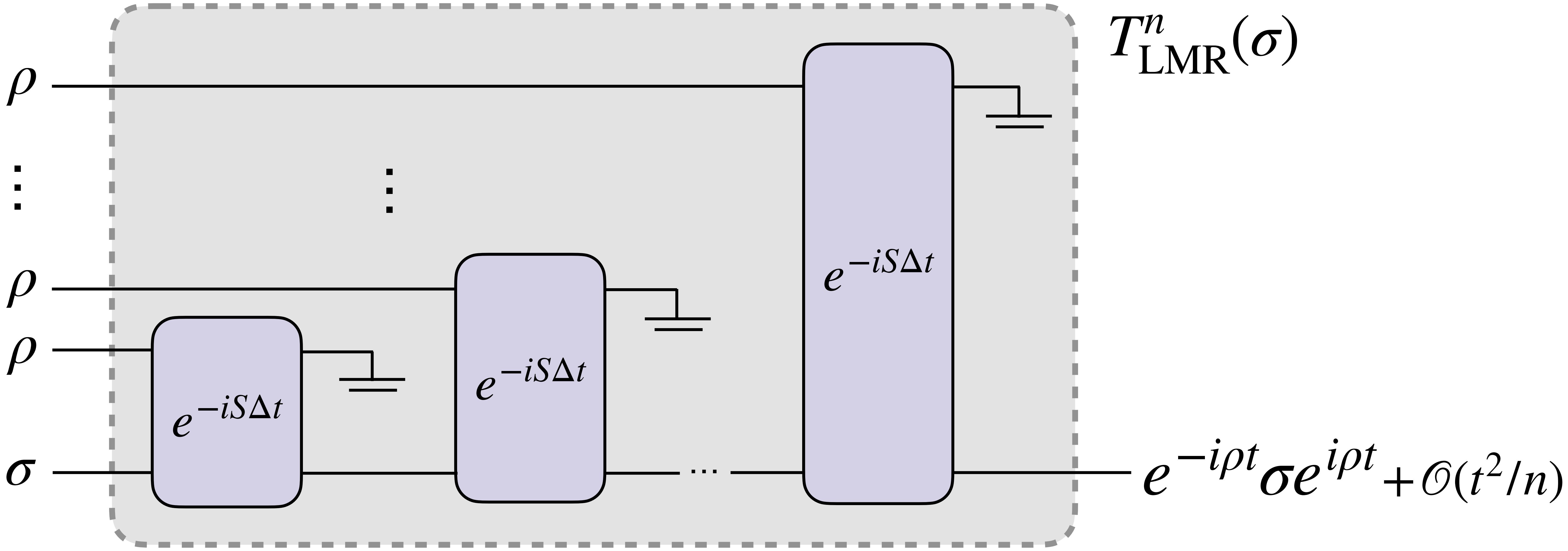
Density matrix exponentiation

What if we consider a time interval t ?





Access to n copies of ρ

$$\Delta t = t/n$$

$$n = \mathcal{O}(t^2/\varepsilon)$$

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Applicability

nature physics

LETTERS

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Quantum principal component analysis

Seth Lloyd[1,2★], Masoud Mohseni[3] and Patrick Rebentrost[2]

PRL 113, 130503 (2014) PHYSICAL REVIEW LETTERS week ending 26 SEPTEMBER 2014

Quantum Support Vector Machine for Big Data Classification

Patrick Rebentrost,[1,*] Masoud Mohseni,[2] and Seth Lloyd[1,3,†]

[1]*Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Google Research, Venice, California 90291, USA*
[3]*Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
(Received 12 February 2014; published 25 September 2014)

Quantum Machine Intelligence (2019) 1:41–51
https://doi.org/10.1007/s42484-019-00004-7

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Zhikuan Zhao[1,2,3] · Alejandro Pozas-Kerstjens[4] · Patrick Rebentrost[3] · Peter Wittek[5,6,7,8]

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Shelby Kimmel[1], Cedric Yen-Yu Lin[1], Guang Hao Low[2], Maris Ozols[3] and Theodore J. Yoder[2]

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Seth Lloyd,[1,2] Maria Schuld,[2] Aroosa Ijaz,[2] Josh Izaac,[2] and Nathan Killoran[2]

[1]*Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA*
[2]*Xanadu, Toronto, Canada*
(Dated: July 3, 2022)

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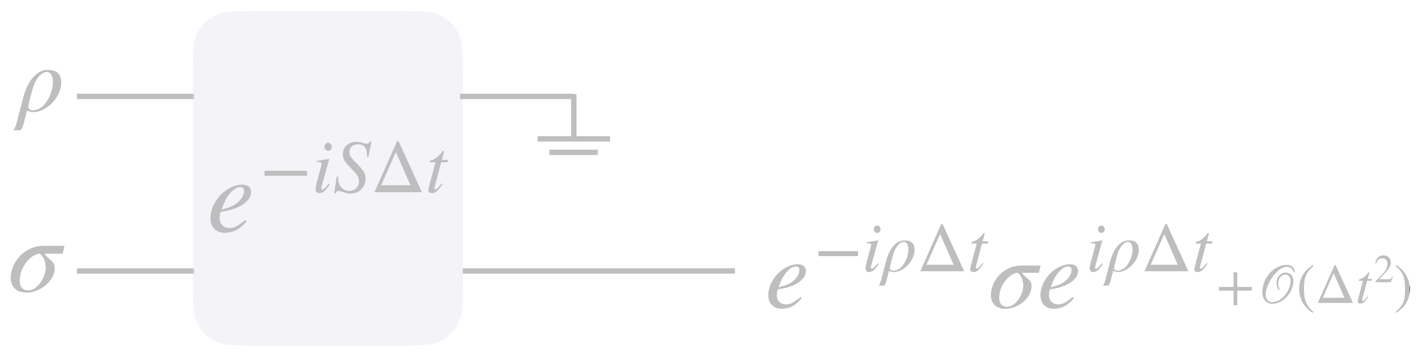

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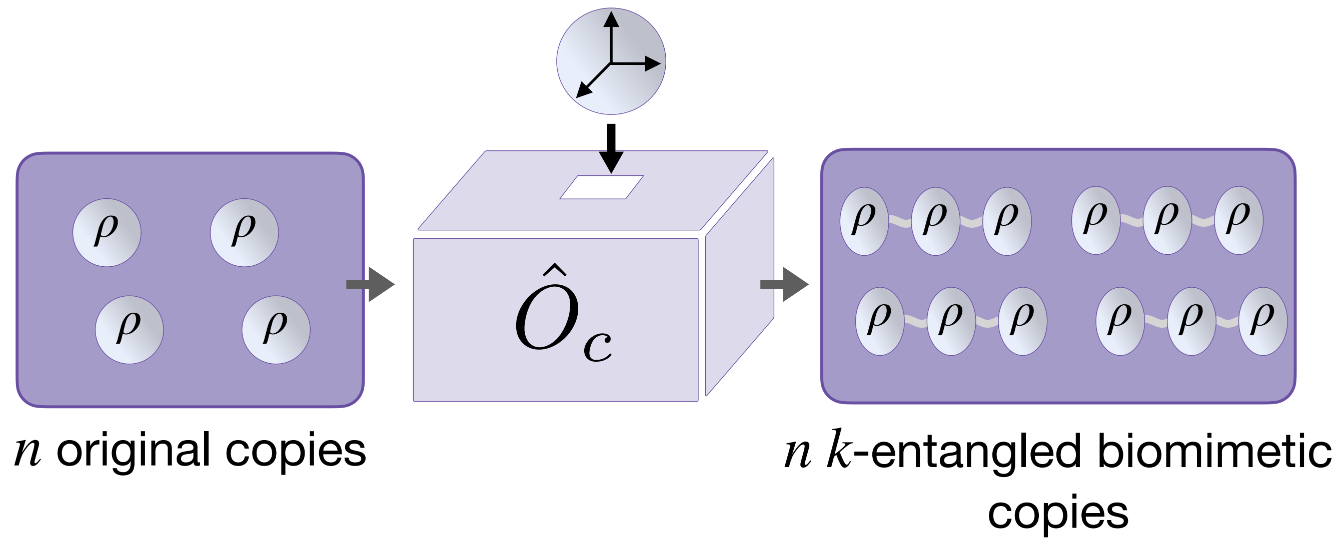

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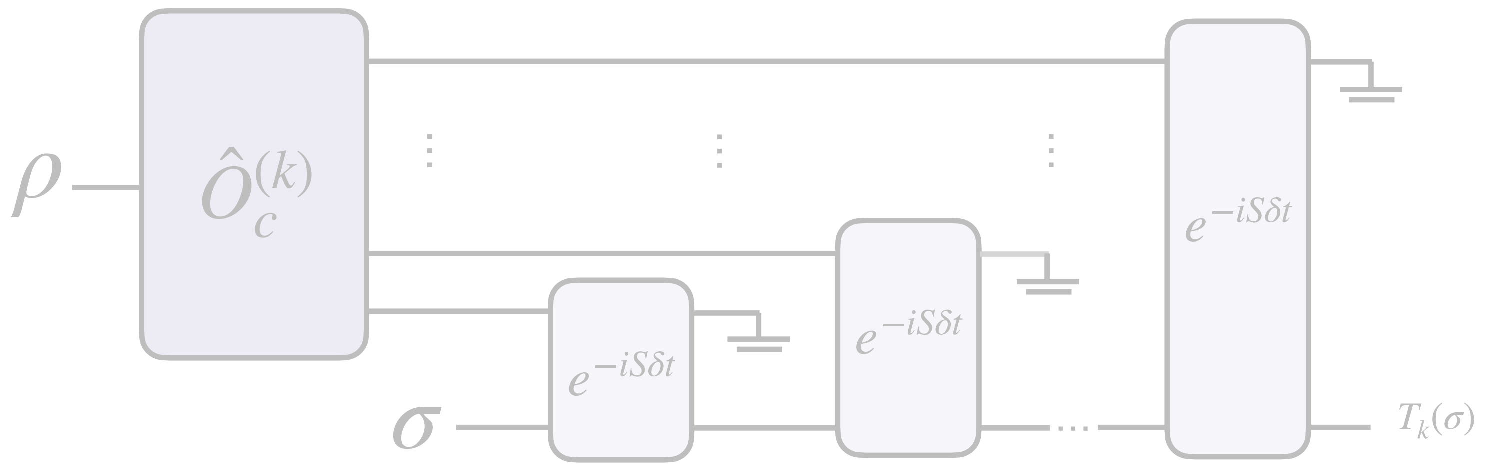
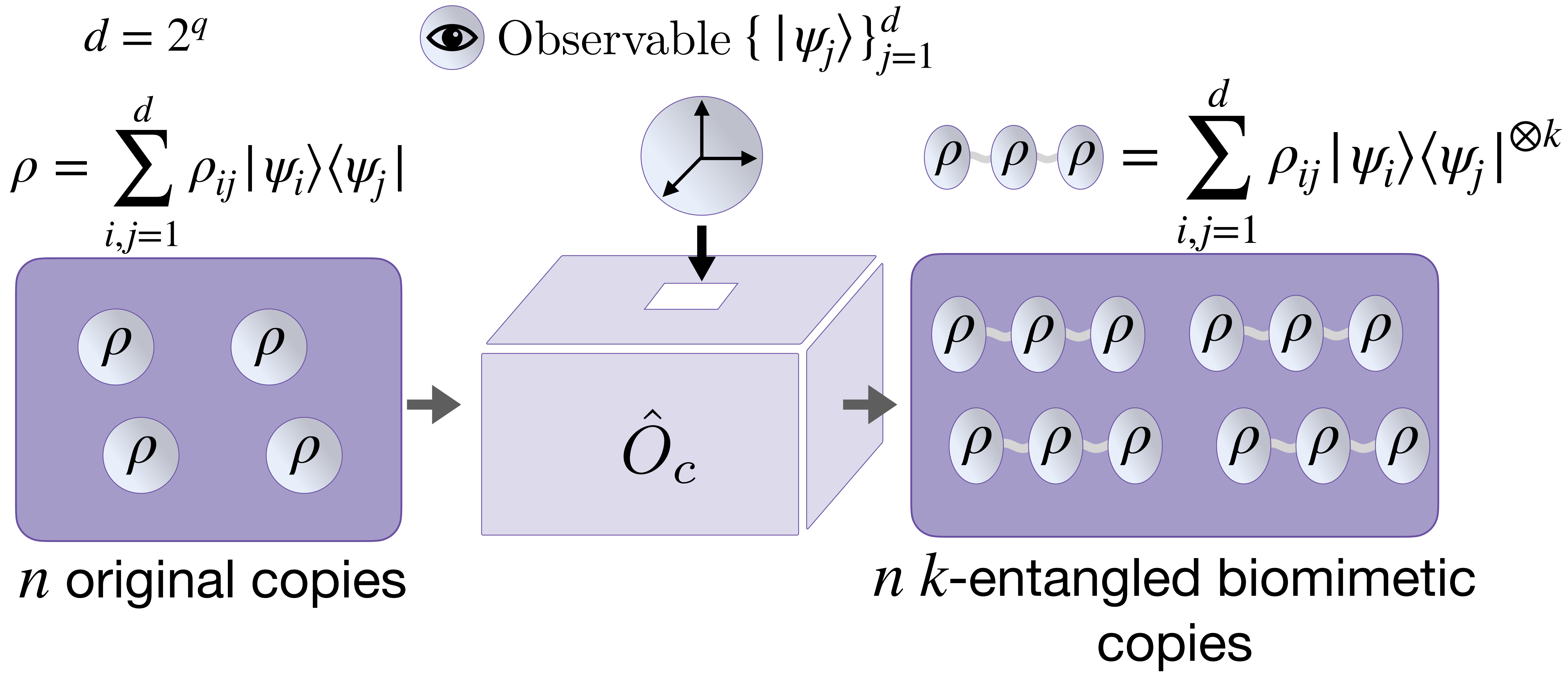


Biomimetic cloning [1]



The biomimetic cloning machine ($\hat{O}_C$) clones the statistics associated to an observable

[1] U. Alvarez-Rodriguez, M. Sanz, L. Lamata, and E. Solano. "Biomimetic Cloning of Quantum Observables". *Sci Rep 4, 4910 (2014).*

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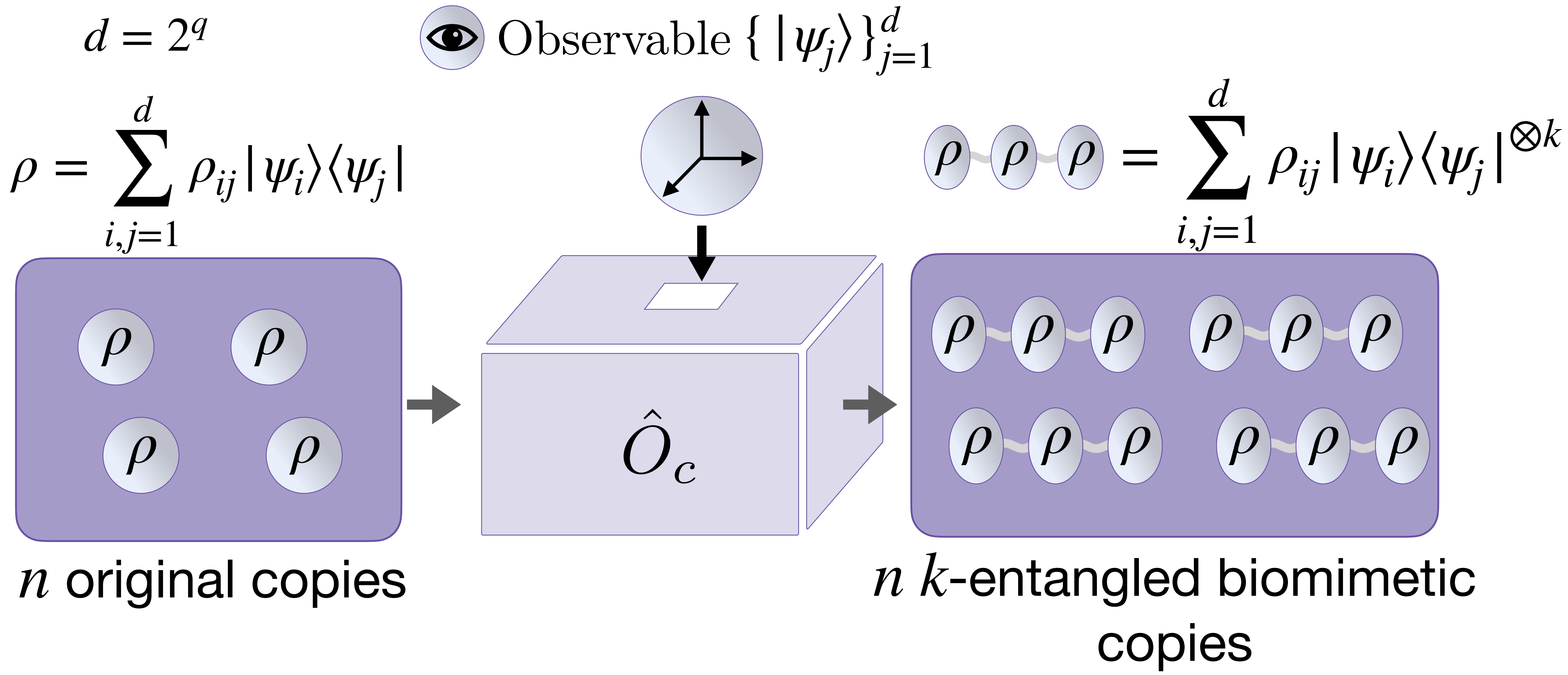
Biomimetic cloning [1]



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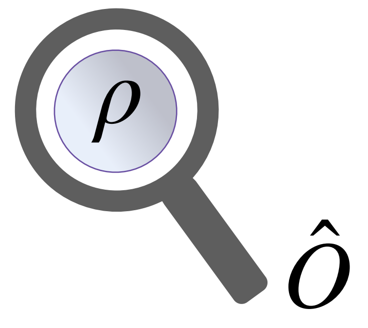

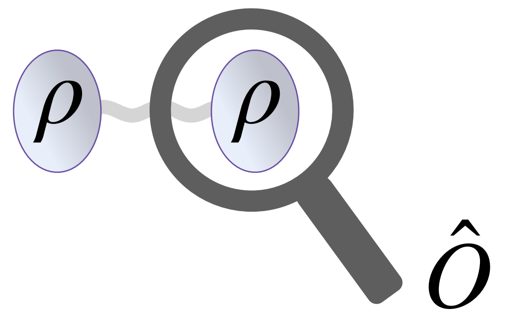

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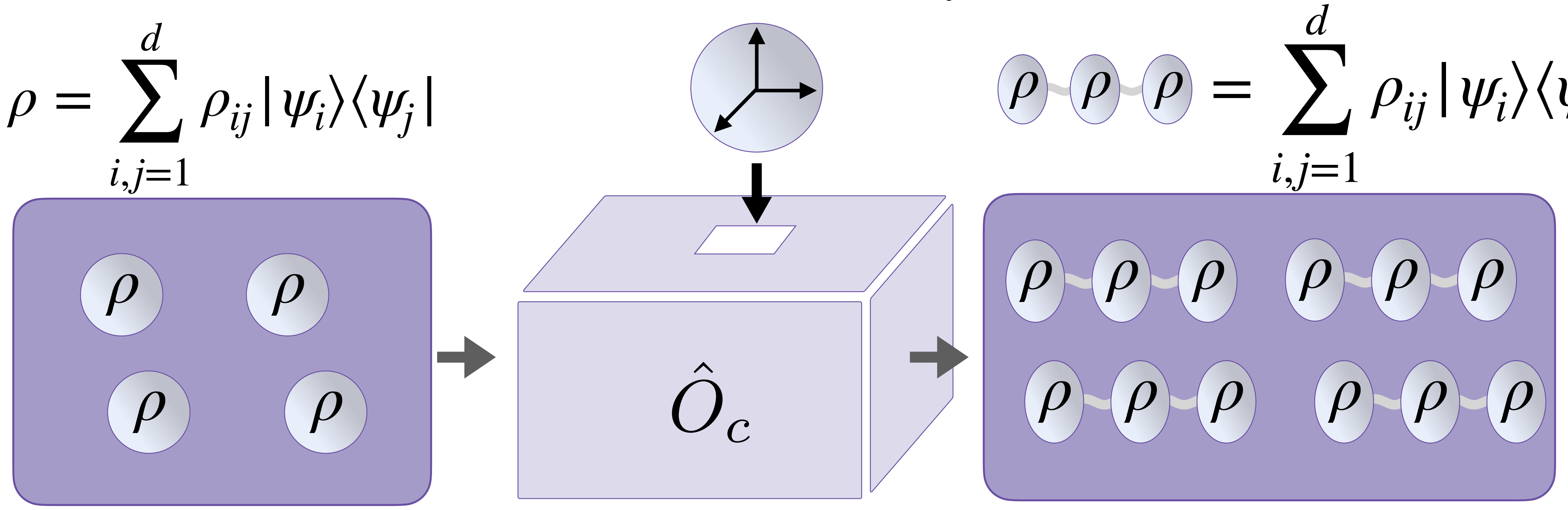
Biomimetic cloning [1]

$d = 2^q$

Observable $\{|\psi_j\rangle\}_{j=1}^d$

$$\rho = \sum_{i,j=1}^{d} \rho_{ij} |\psi_i\rangle\langle\psi_j|$$

$$\rho\text{-}\rho\text{-}\rho = \sum_{i,j=1}^{d} \rho_{ij} |\psi_i\rangle\langle\psi_j|^{\otimes k}$$



n original copies

n k-entangled biomimetic copies

We consider ρ as the observable

$$\rho = \sum_{i=1}^{d} p_i |\psi_i\rangle\langle\psi_i| \qquad \rho^{(2)} = \sum_{i=1}^{d} p_i |\psi_i\rangle\langle\psi_i|^{\otimes 2}$$

Acting k times:

$$\hat{O}_c^{(k)}(\rho) = \sum_{i=1}^{d} p_i \left(|\psi_i\rangle\langle\psi_i| \right)^{\otimes k} \equiv \rho^{(k)}$$

[1] U. Alvarez-Rodriguez, M. Sanz, L. Lamata, and E. Solano. “Biomimetic Cloning of Quantum Observables”. *Sci Rep 4, 4910 (2014).*

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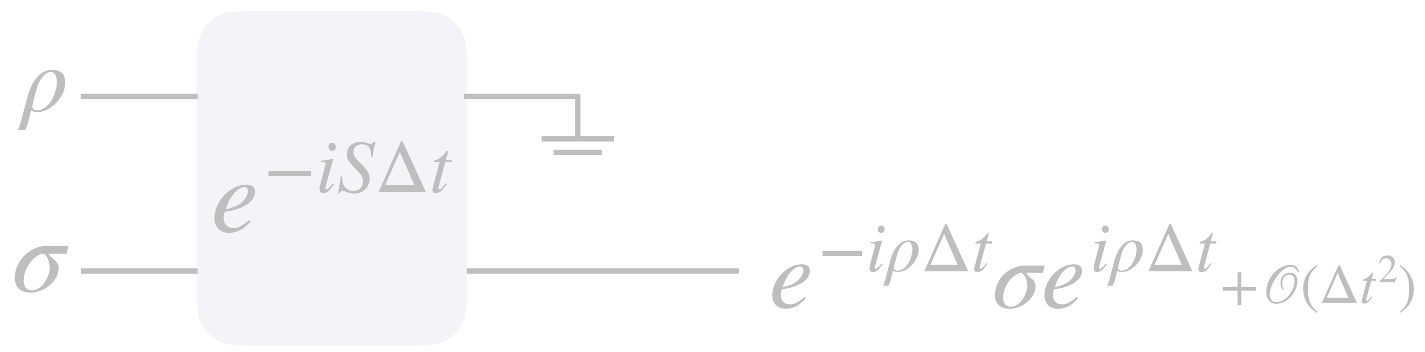

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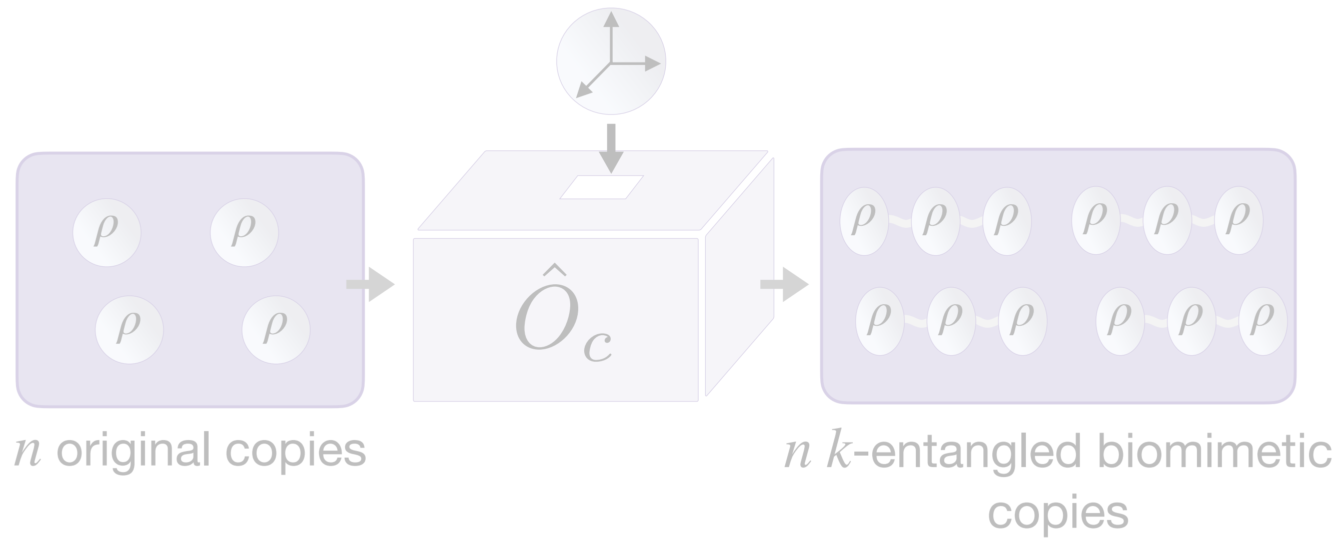

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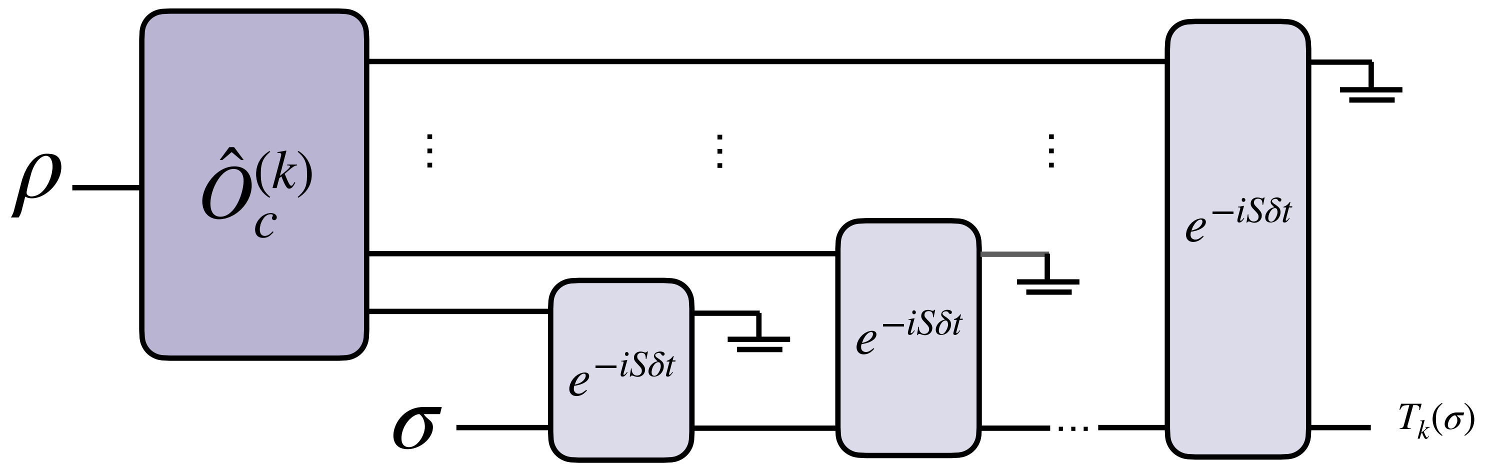
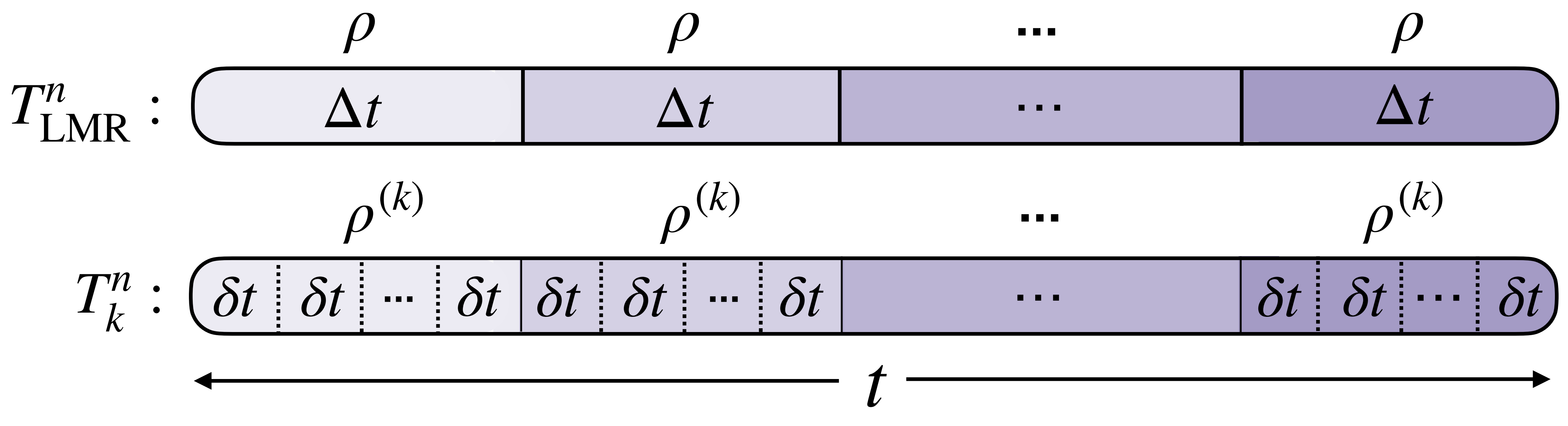


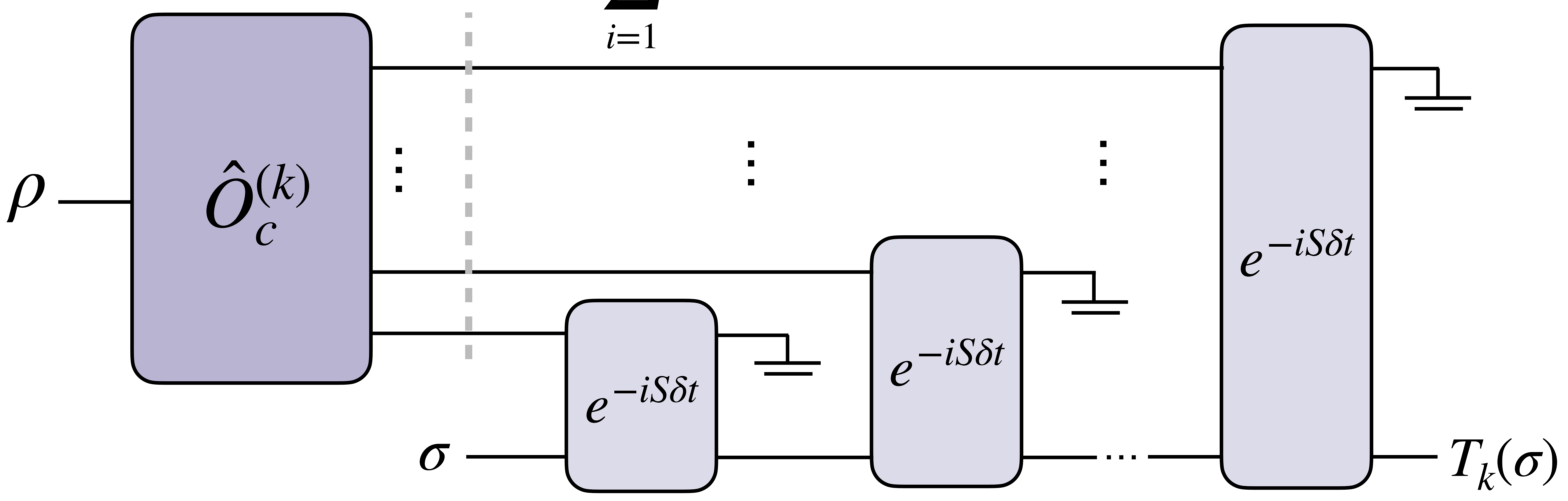
LMR assisted by biomimetic copies

Given n copies of ρ

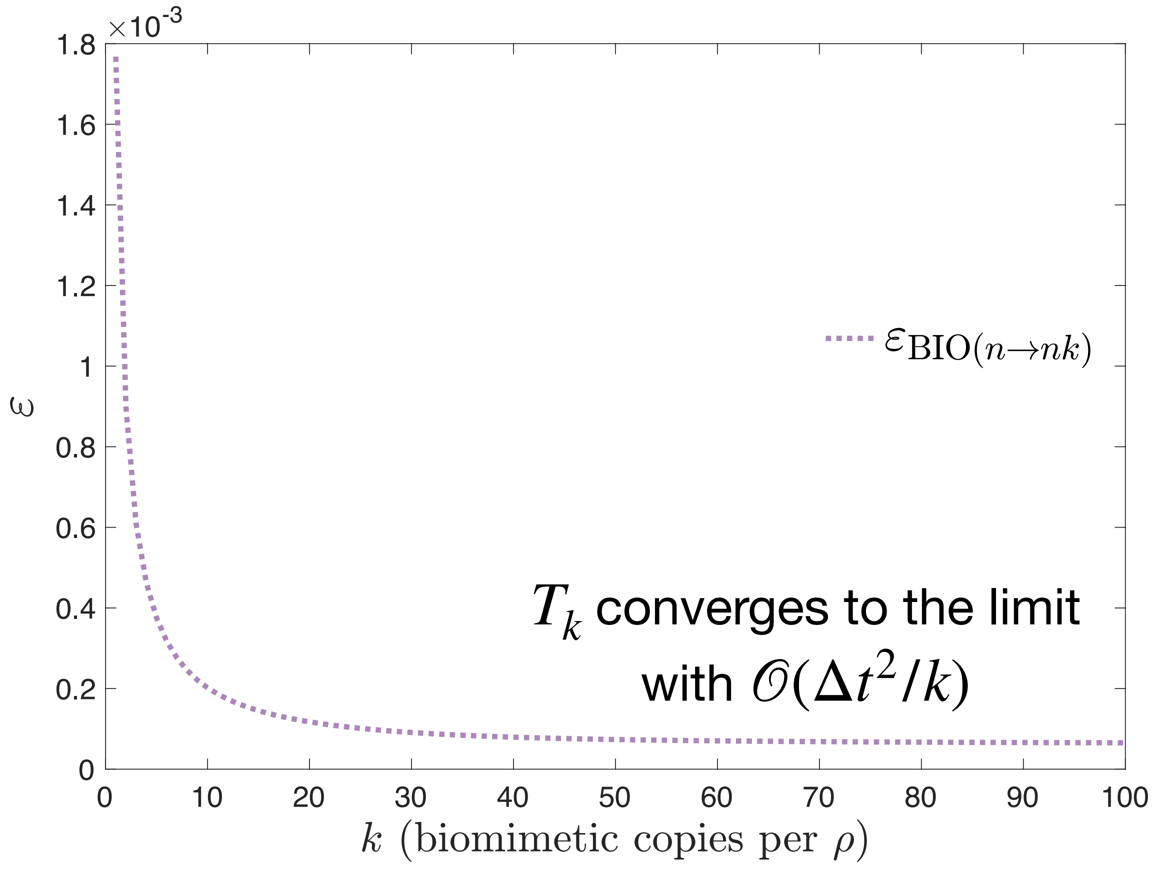


From each original copy

$$\rho^{(k)} = \sum_{i=1}^{d} p_i |\psi_i\rangle\langle\psi_i|^{\otimes k}$$



We consider the limit $k \to \infty$



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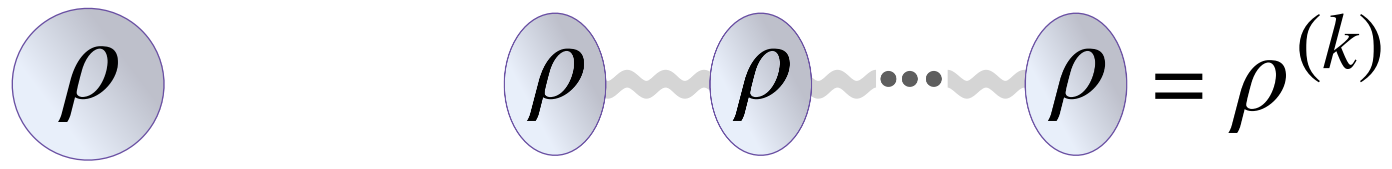
Error analysis

Is it worth disturbing the original copies of ρ to create k biomimetic copies?



Original protocol: $\varepsilon_{\mathrm{LMR}(n)} \approx \left\| [\rho, \sigma]_2 + 2(\rho - \sigma) \right\|_1 t^2/2n$

Does not violate optimality

Our protocol: $\varepsilon_{\mathrm{BIO}(n \to nk)} \approx \left\| [\rho, \sigma]_2 + 2\, \rho \circ \sigma - \{\rho, \sigma\} \right\|_1 t^2/2n$

$$\frac{\varepsilon_{\mathrm{LMR}(n)}}{\varepsilon_{\mathrm{BIO}(n \to nk)}} \approx \frac{\left\| [\rho, \sigma]_2 + 2(\rho - \sigma) \right\|_1}{\left\| [\rho, \sigma]_2 + 2\, \rho \circ \sigma - \{\rho, \sigma\} \right\|_1} \equiv Q_1$$

First order in ρ, σ

Second order in ρ, σ

Statistical case:

ρ and σ random density matrices

$$Q_1 \geq \frac{d}{8} \left(\frac{\left\| \rho - \sigma \right\|_1 - 32/d^2}{1 + 4/d} \right)$$

$\left\| \rho - \sigma \right\|_1 \to \mathrm{const.}$ [1]

Average $Q_1 \propto d$

[1] Puchała, Zbigniew, Łukasz Pawela, and Karol Życzkowski. "Distinguishability of generic quantum states." *Physical Review A* 93.6 (2016): 062112.

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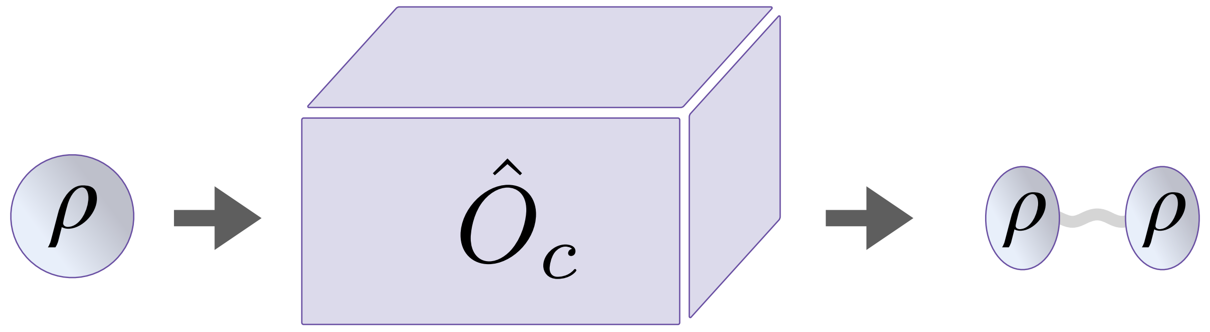
Cost analysis

Instead of cloning I could generate more original copies of ρ

$$l \cdot C_s \geq n \cdot (k-1) \cdot C_c$$

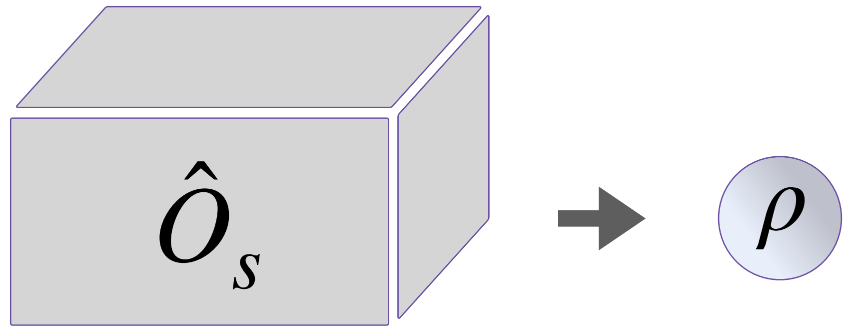
l to guarantee an error smaller or equal than the one with biomimetics copies

"Cloning":



Cost C_c

Generating ρ:



Access?

Cost C_s

$$C_c \leq \frac{C_s}{k-1}\left(Q_1 - 1\right)$$

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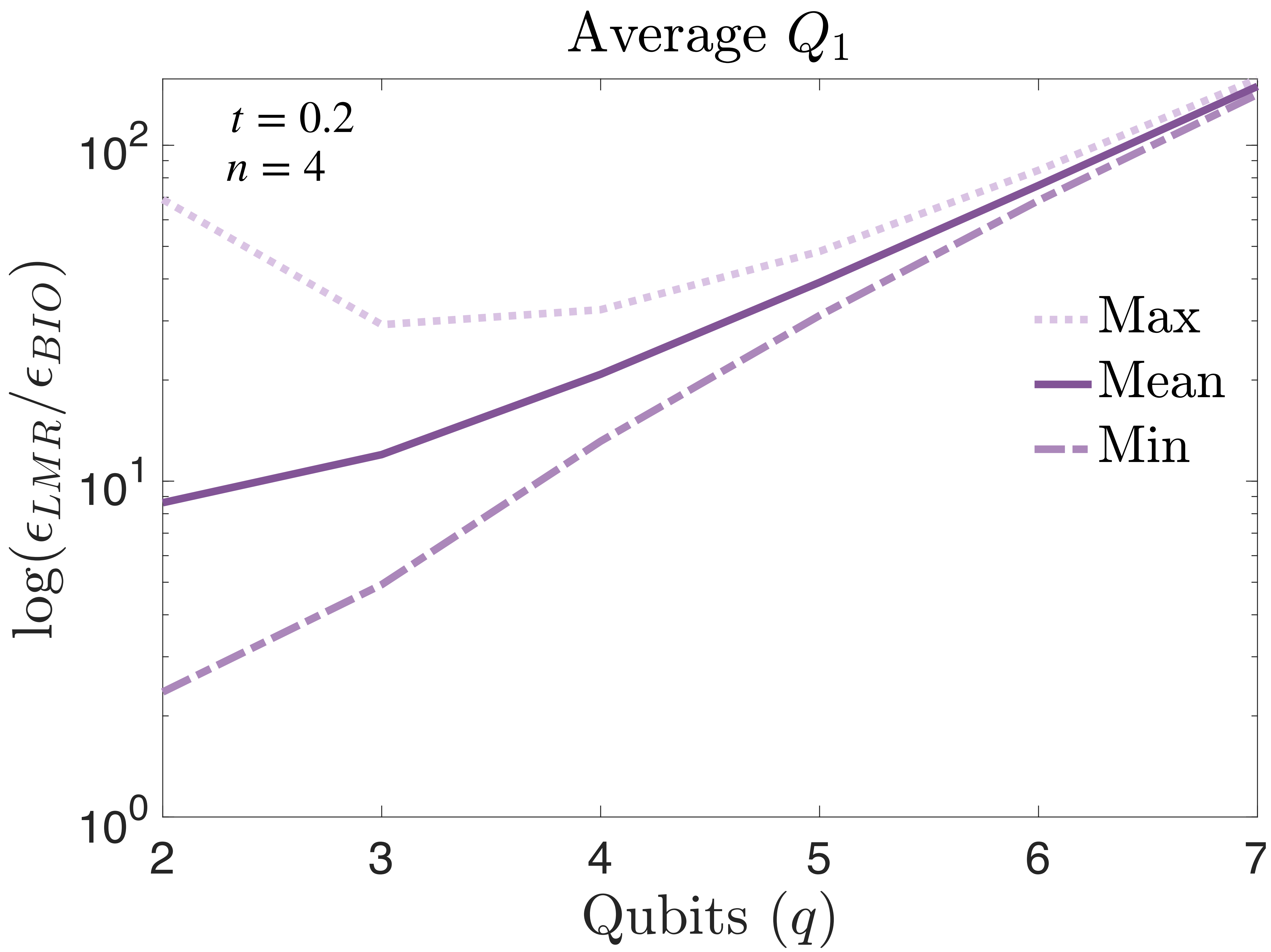
Performance analysis

100,000 random cases uniformly distributed according to the Hilbert-Schmidt measure



$\mathbb{E}_{\rho,\sigma}[Q_1] = \mathcal{O}(\exp(q))$

Given n ⟶ Exponential reduction in ϵ

Given ϵ ⟶ Exponential reduction in n

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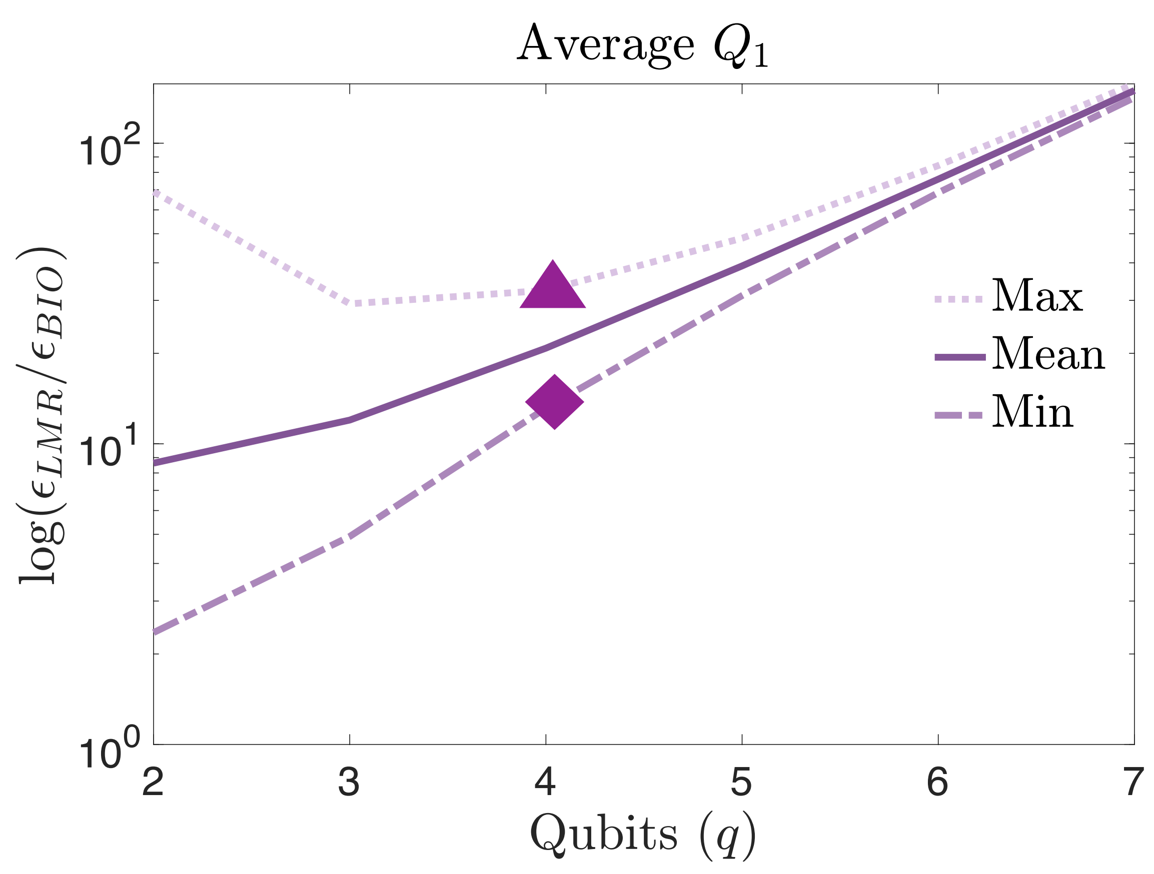
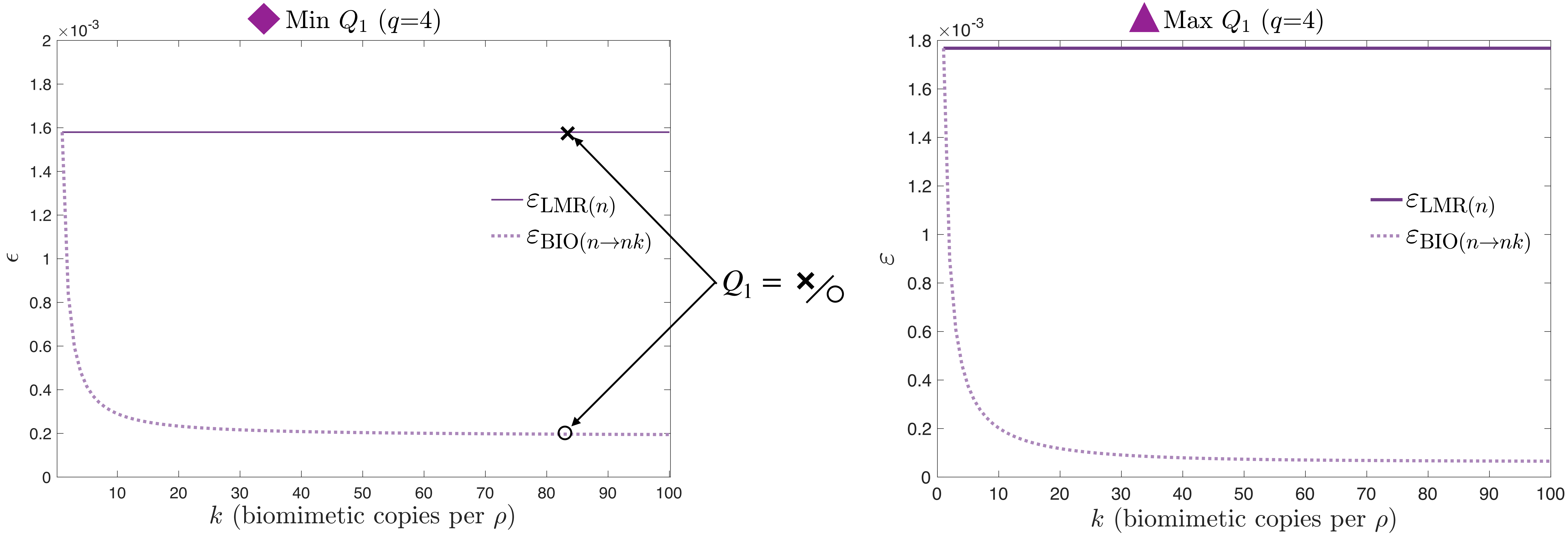
Performance analysis





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Outlook and conclusions

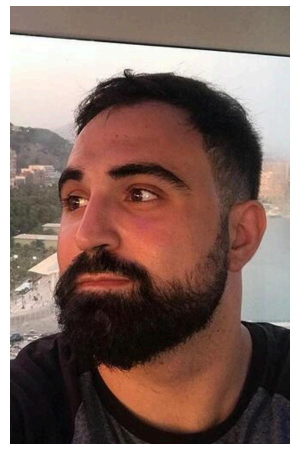
- Density matrix exponentiation can be enhanced
- On average, enhancement scales with the dimension of the system
- Using imperfect cloning could enhance other protocols requiring copies

Killing application ?

Block-diagonalization ?

Collaborators:


Ruben Ibarrondo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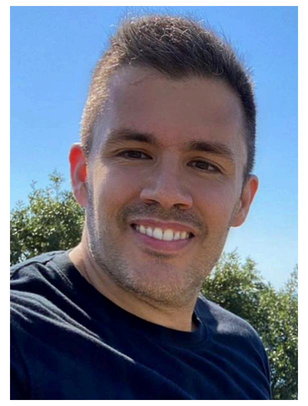
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Thank you for your attention!