

# Suppressing photon detection errors in nondeterministic state preparation

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## Quantum advantage using photons

The Quantum Information Group of USTC in Hefei (led by Jian-Wei Pan) demonstrated an advantage over classical computation in 2020, using the Gaussian Boson Sampling (GBS) scheme<sup>1</sup>.

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However, GBS only uses linear optical quantum gates  $\Rightarrow$  GBS is **non-universal!**

Difficult to implement non-linear (e.g., Kerr) gates in a photonic circuit  $\Rightarrow$  We try to **avoid non-linear gates**.

**Idea:** include ancilla detections and postselection!

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## Non-deterministic gates

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In our case, we use **particle number resolving detectors** (PNRDs).

## Example: Nonlinear phaseshift (NS) gate

**Goal:** Implement the following gate<sup>2</sup>:

$$\text{NS} : |0\rangle|0\rangle + |1\rangle|1\rangle \rightarrow |0\rangle|0\rangle + |1\rangle|1\rangle + |2\rangle|2\rangle \quad (1)$$

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## Example: Nonlinear phaseshift (NS) gate

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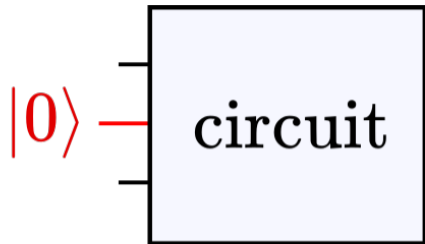
$$\text{NS} : |0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2| \not\equiv |0\rangle\langle 0| + |1\rangle\langle 1| - |2\rangle\langle 2| \quad (1)$$

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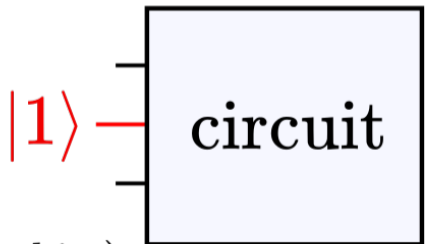
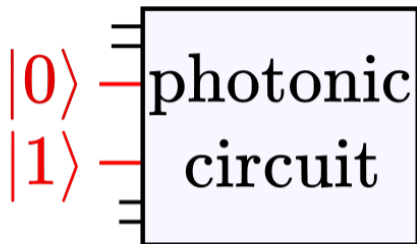
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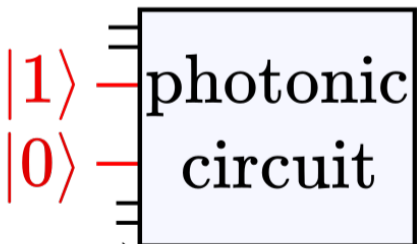
## Dual-rail encoding



$\equiv$



$\equiv$



(qubits)

(photons)

## Conditional sign flip gate

**Goal:** Implement the following transformation<sup>3</sup>:

$$\text{CZ} : |j++\rangle_{\text{qubit}} \mapsto |j-i\rangle = \frac{1}{2} (|0;0\rangle + |1;0\rangle + |0;1\rangle - |1;1\rangle)_{\text{qubit}} \quad (2)$$

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## Conditional sign flip gate

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We can implement this by using a specific interferometer denoted by  $U$ !

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## KLM protocol

The **KLM protocol** combines postselection and dual-rail encoding (+ gate teleportation).

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**Problem:** PNRDs are **biased** in practice  $\Rightarrow$  incorrect state may be postselected after faulty postselection!

## Detector efficiency matrix

The imperfections in the PNRD can be characterized by the **detector efficiency matrix**:

$$P_{m;n} := P_{\text{readout}}(m|n): \quad (3)$$

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<sup>4</sup>V. Resta et al., *Gigahertz Detection Rates and Dynamic Photon-Number Resolution with Superconducting Nanowire Arrays*. Nano Lett. (2023)

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An example<sup>4</sup>:

$$P = \begin{matrix} \text{O} & & & & & \text{1} \\ \text{C} & 1.0 & 0.1050 & 0.0110 & 0.0012 & 0.001 \\ \text{C} & 0.0 & 0.8950 & 0.2452 & 0.0513 & 0.0097 \\ \text{C} & 0.0 & 0.0 & 0.7438 & 0.3770 & 0.1304 \\ \text{C} & 0.0 & 0.0 & 0.0 & 0.5706 & 0.4585 \\ \text{A} & 0.0 & 0.0 & 0.0 & 0.0 & 0.4013 \end{matrix} : \quad (4)$$

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This will introduce an error in the nondeterministic gates!

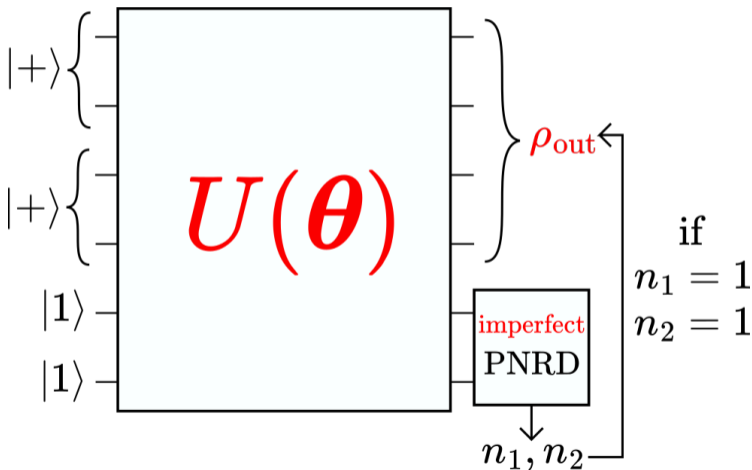
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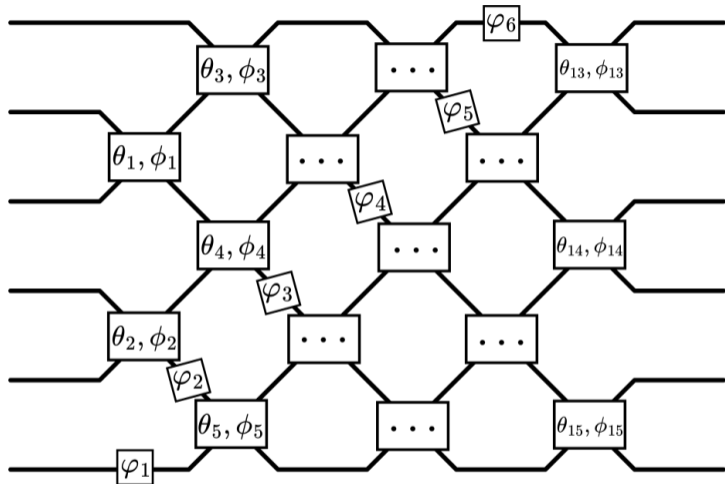
## Idea: adjust interferometer!

Changing the angles in the interferometer can increase the output state fidelity or the success probability of nondeterministic gates with imperfect PNRDs!



## Parametrizing the interferometer<sup>5</sup>

$$U(\boldsymbol{\theta}) \equiv$$



$$(\boldsymbol{\theta} = \{\theta_i, \phi_i, \varphi_i\}_i)$$

<sup>5</sup>W. R. Clements et al., *An Optimal Design for Universal Multiport Interferometers*, Optica (2016), [arXiv:1603.08788](https://arxiv.org/abs/1603.08788)

## Gradient-based optimization

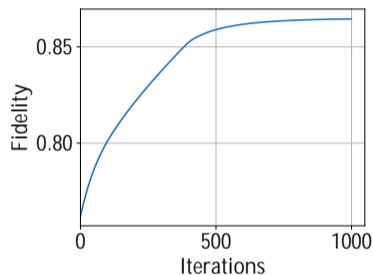
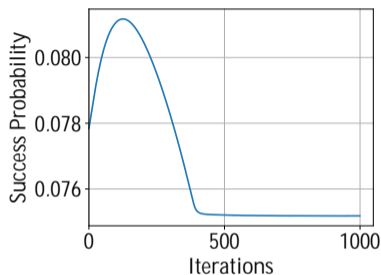
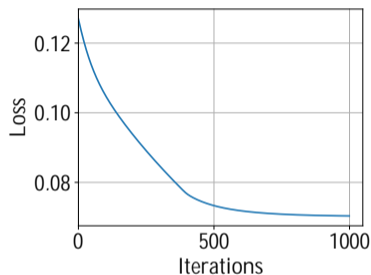
$$\text{out}(\theta) := \frac{1}{S(\theta)} \sum_n P(\mathbf{x}/\mathbf{n}) \text{tr}_2 \left[ U(\theta) U^y(\theta) (I - \mathbf{jn}i \mathbf{h} \mathbf{m} \mathbf{j})^{\circ} \right] \quad \text{(Final state)}$$
$$S(\theta) := \sum_n P(\mathbf{x}/\mathbf{n}) \text{tr} \left[ U(\theta) U^y(\theta) (I - \mathbf{jn}i \mathbf{h} \mathbf{m} \mathbf{j})^{\circ} \right] \quad \text{(Success rate)}$$

## Gradient-based optimization

$$L(\theta) := \frac{1}{n} \sum_{j=1}^n \text{softplus}(S - S_j(\theta)) \quad (S = \text{Target success rate})$$

$$\text{softplus}(x) := \frac{1}{2} \log(\exp(x) + 1) \quad (5)$$

# Optimization of conditional phase shift gate with imperfect PNRDs<sup>7</sup>



The simulations were executed using Piquasso<sup>6</sup>.

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<sup>6</sup>ZK et al., *Piquasso: A Photonic Quantum Computer Simulation Software Platform*, [arXiv:2403.04006](https://arxiv.org/abs/2403.04006) (2024)

<sup>7</sup>In this example, with target success rate  $S^* = 0.075$ , code available at <https://github.com/Budapest-Quantum-Computing-Group/supressing-loqc-pd-errors>

## Tradeoff between fidelity and success rate

