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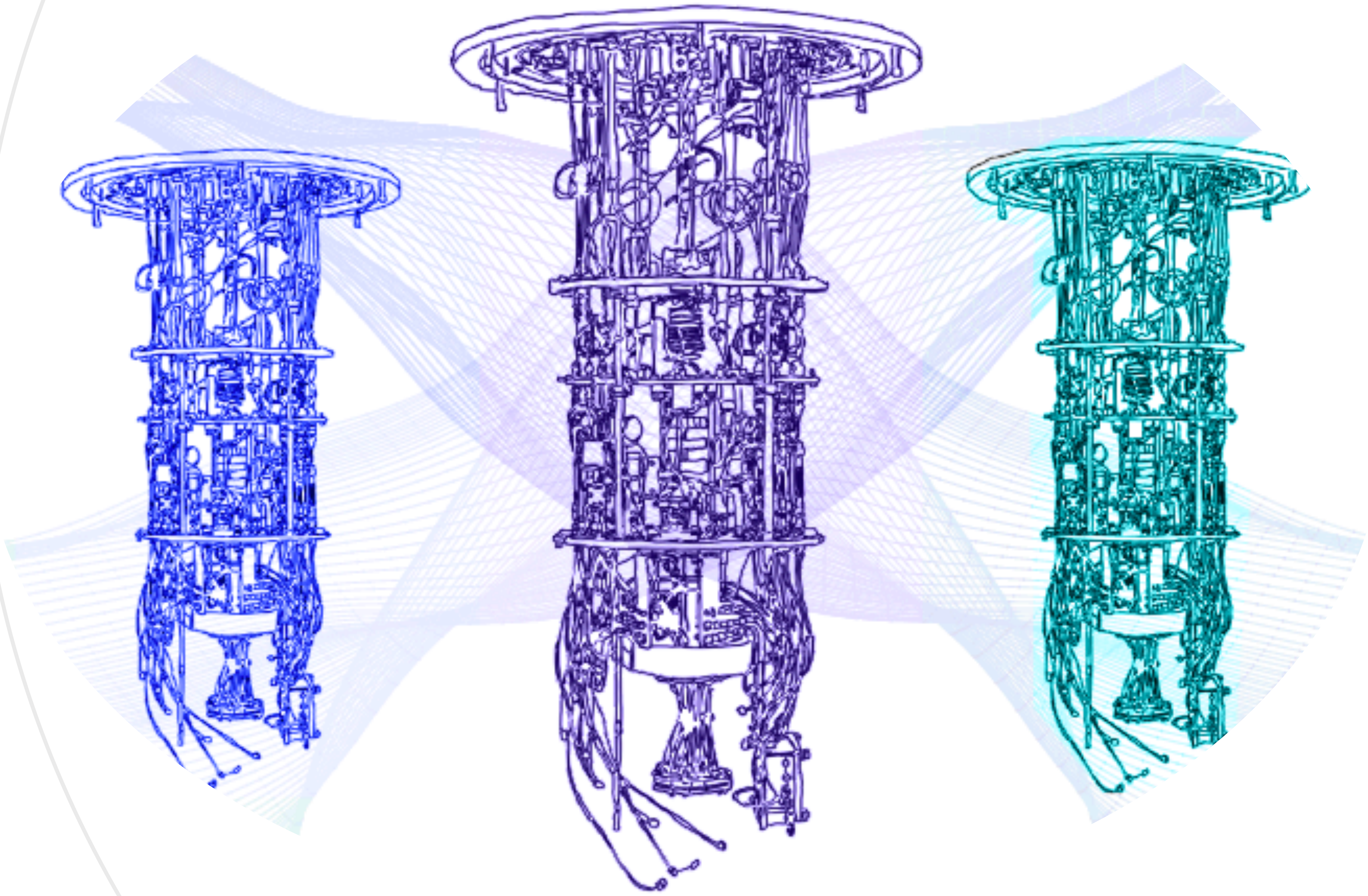

AN OVERVIEW ON QUANTUM ALGORITHMS FOR AMPLITUDE ENCODING OF CLASSICAL DATA INTO QUANTUM COMPUTERS

Javier Gonzalez-Conde

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EHU Quantum Center
Leioa, Spain

20th June 2024

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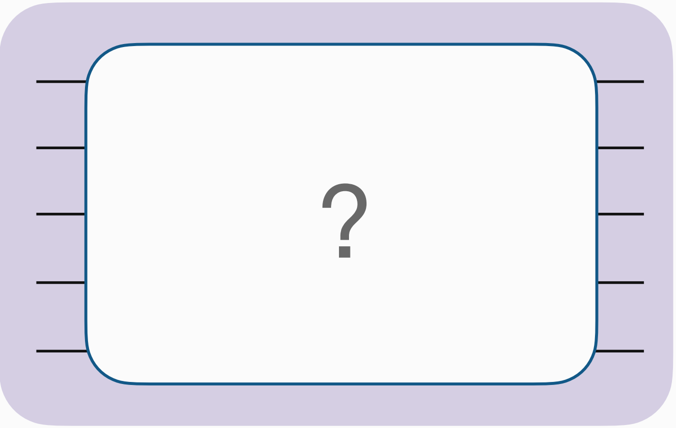
Amplitude Embedding

Given the normalized vector $\vec{x} = (x_1, \ldots x_{2^n})^T,\ x_i \in \mathbb{C}$

From $|0\rangle^{\otimes n}$ generate the state $|\Psi_{\text{data}}\rangle = \sum_{j=1}^{2^n} x_j |j\rangle$

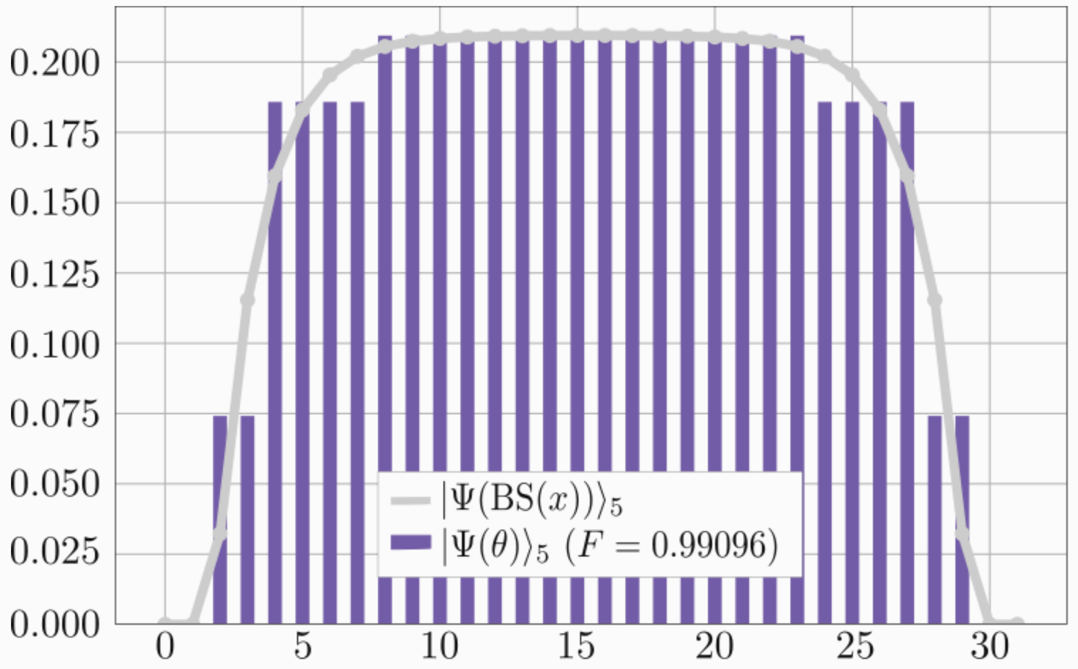
$|0\rangle^{\otimes n}$ 

$|\Psi_{\text{data}}\rangle$

Initial States for PDEs

Quantum Random Walk

Probability distributions



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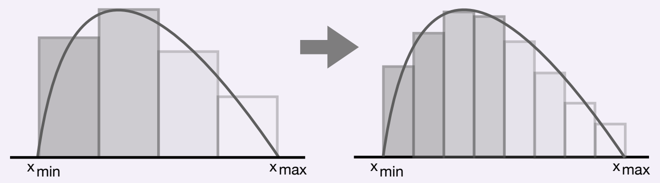
An overview on amplitude encoding

Brief historical review

Grover and Rudolph (2002)

- Iterative and deterministic
- Real valued functions



[Proc.Roy.Soc.Lond. A454 (1998)]
[arXiv preprint quant-ph/0208112 (2002)]
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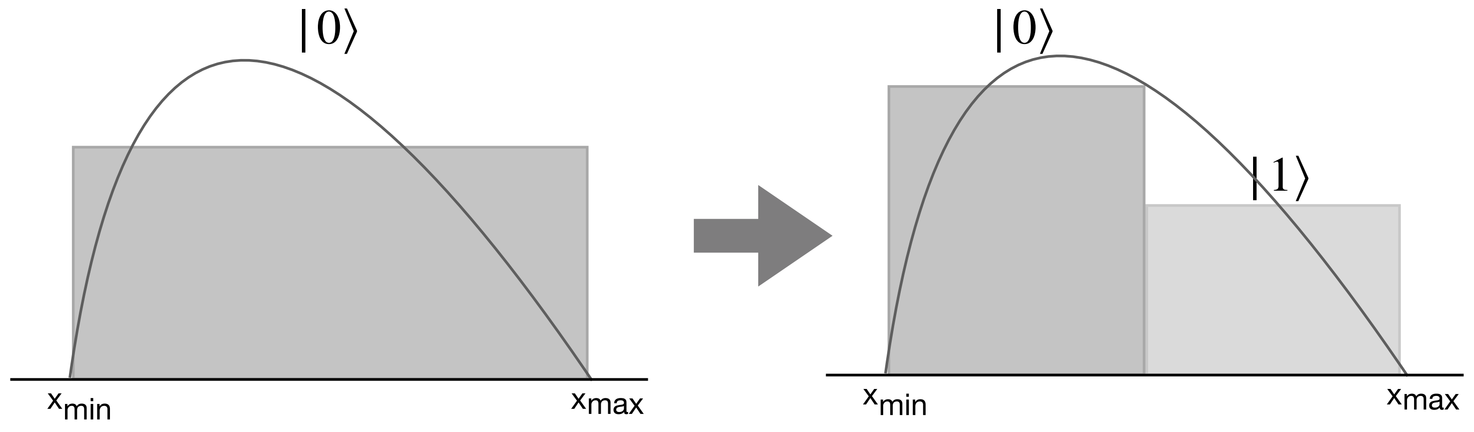
An overview on amplitude encoding

Grover and Rudolph

Grover and Rudolph (2002)

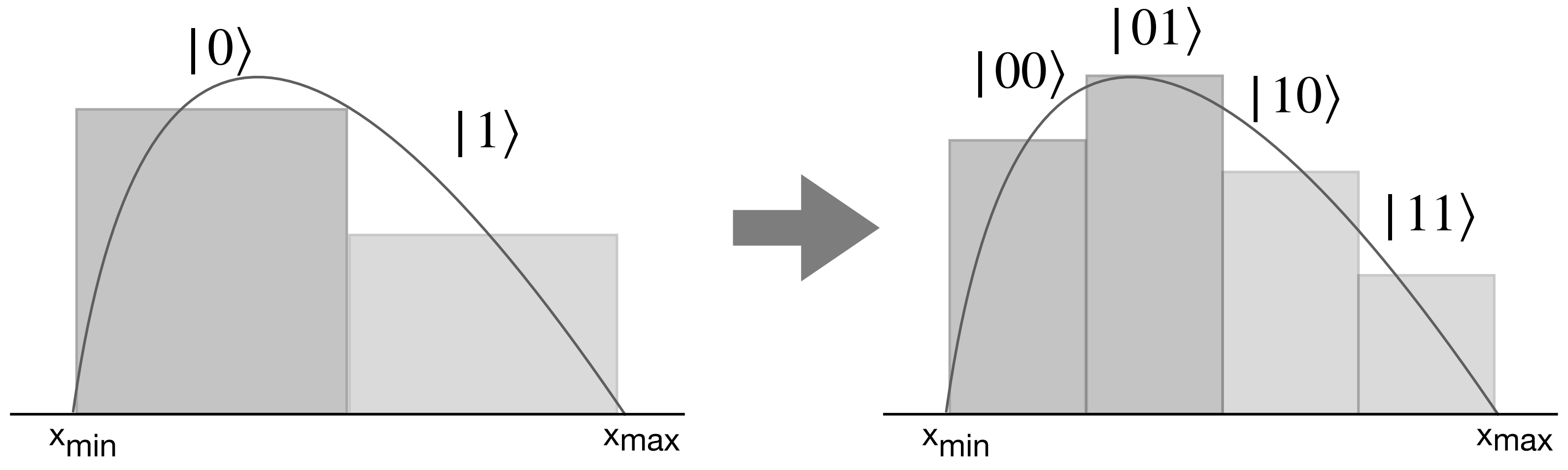
- Positive real valued function
- Iterative and deterministic method



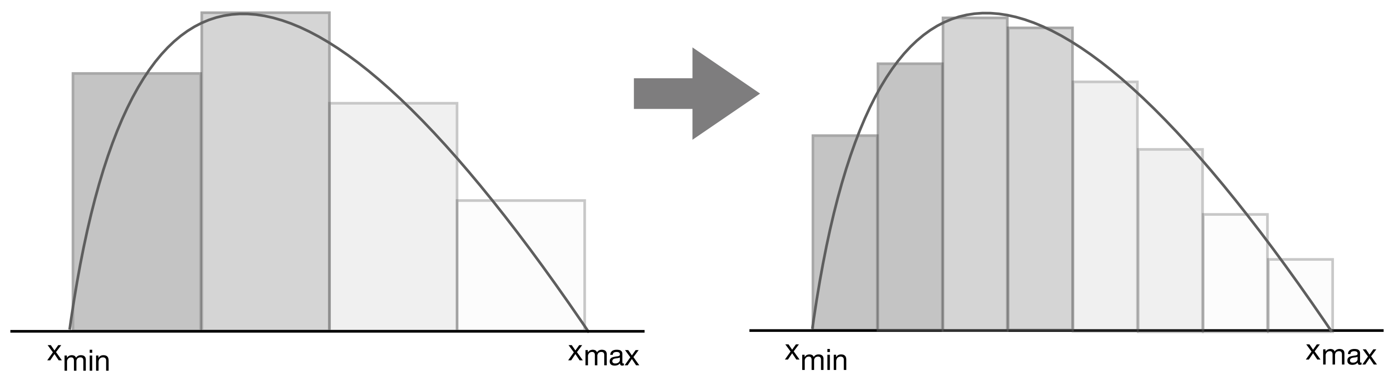
$$Ry(\theta^0)|0\rangle = \cos(\theta^0)|0\rangle + \sin(\theta^0)|0\rangle$$

How are the rotations implemented?

$$\theta^{(k-1)}_{bin(l)} = 2\arccos\left(\sqrt{\frac{\int_{x_{min}+(l-1)\delta_k}^{x_{min}+(l-1/2)\delta_k} f(x)dx}{\int_{x_{min}+(l-1)\delta_k}^{x_{min}+l\delta_k} f(x)dx}}\right)$$



$$P(A) = P(A|B)P(B)$$



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Encoding classical information

Overview

Grover and Rudolph (2002)

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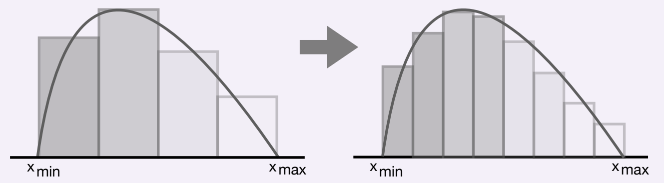

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Oracular Access

(ancillas for m bit precision)

Arithmetic

Rotation angles

$\mathcal{O}_{GR} : |l\rangle\,|0\rangle^{\otimes m} \to |l\rangle\,|\theta^{(k-1)}_{bin(l)}\rangle_m$

$\mathfrak{rot}\,|\theta\rangle\,|0\rangle := \sin(\theta)\,|\theta\rangle\,|0\rangle + \cos(\theta)\,|\theta\rangle\,|1\rangle$

Efficient ?
Quantum query complexity

[Phys. Rev. E 103, 063302 (2021)]
[Quantum 5, 431 (2021)]
[Quantum 5, 463 (2021)]

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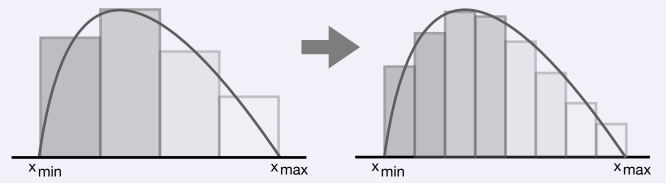

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Exact unity mappings
(needs classical pre-computation)

Exponential cost in ancillas
[Phys. Rev. Research 3, 043200 (2021)]
[Quantum 8, 1228 (2024)]

Sparse states (efficient)
[Phys. Rev. Lett. 129, 230504 (2022)]
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Exponential cost in depth
- Complex values
- Links two arbitrary states

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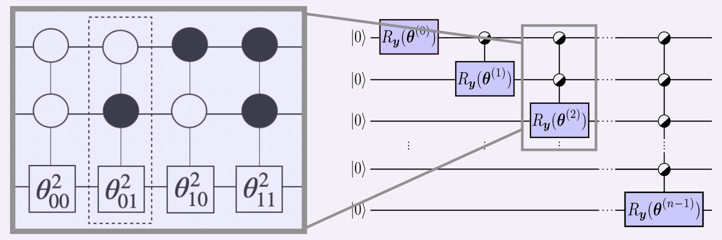

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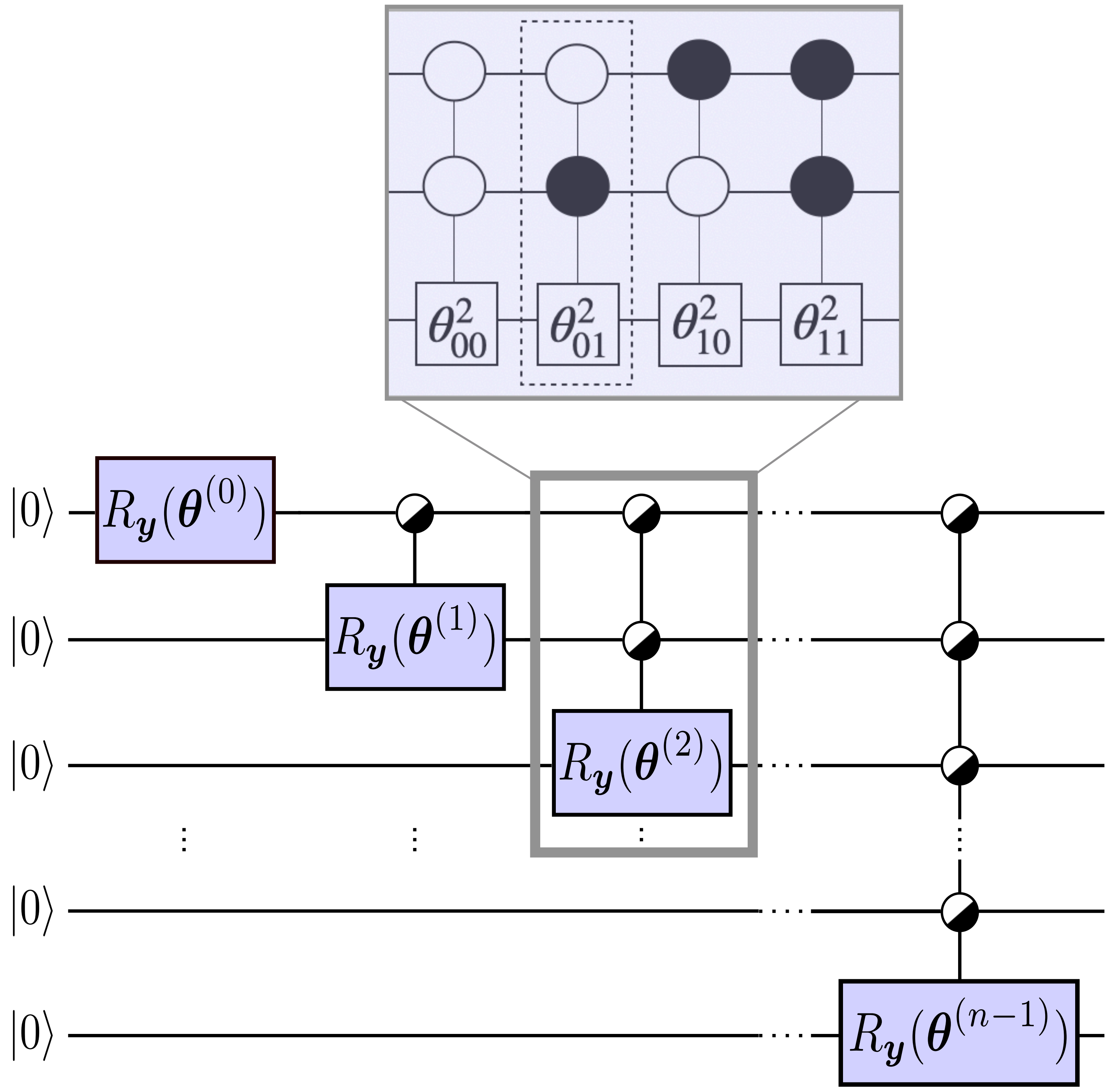

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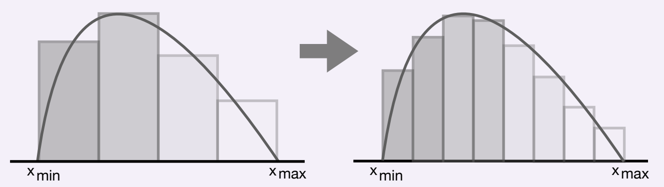

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~~Exact~~ **Approximated** unity mappings**?**
(needs classical pre-computation)

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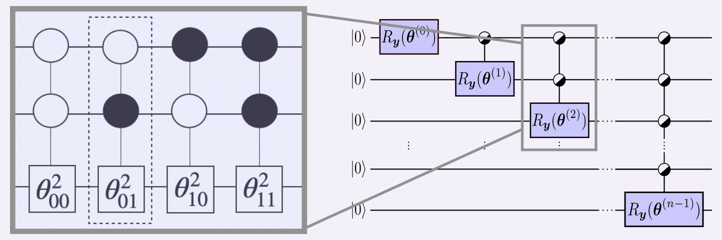

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Our first contribution!


Open Access
Quantum algorithms for approximate function loading
Gabriel Marin-Sanchez, Javier Gonzalez-Conde, and Mikel Sanz
Phys. Rev. Research 5, 033114 – Published 18 August 2023
Article | References | Citing Articles (2) | PDF | HTML | Export Citation

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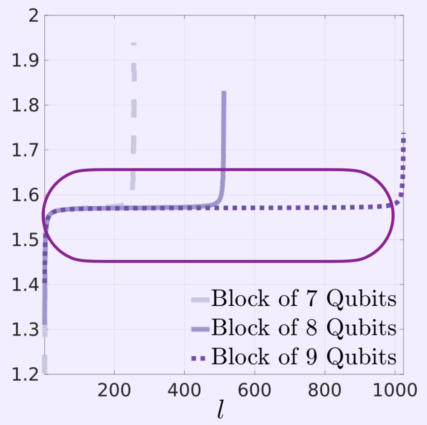
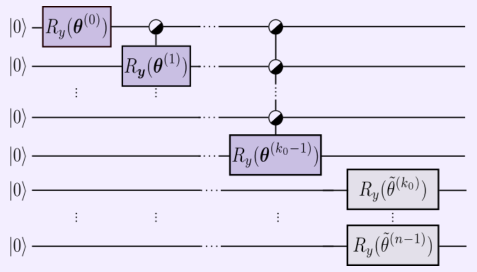


(c) Beta Dist. ($\alpha = 0.5$, $\beta = 0.5$)

Clustering angles!

$$\left|\theta^{(k-1)}_{l+1} - \theta^{(k-1)}_{l}\right| \le \frac{\delta_k^2}{4} \max_{y'\in[l\delta_k,(l+1)\delta_k]} \left|\left(\partial_y^2 \log f(y)\right)\Big|_{y=y'}\right|$$



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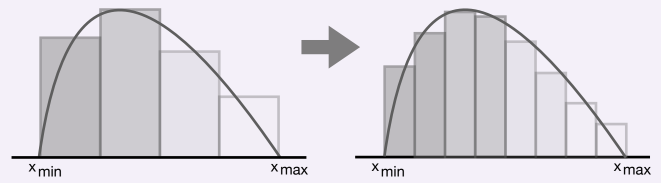

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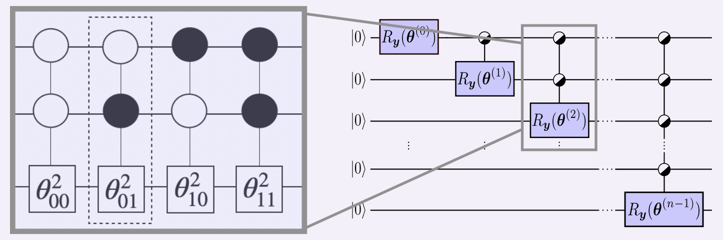

Oracular Access

(ancillas for m bit precision)

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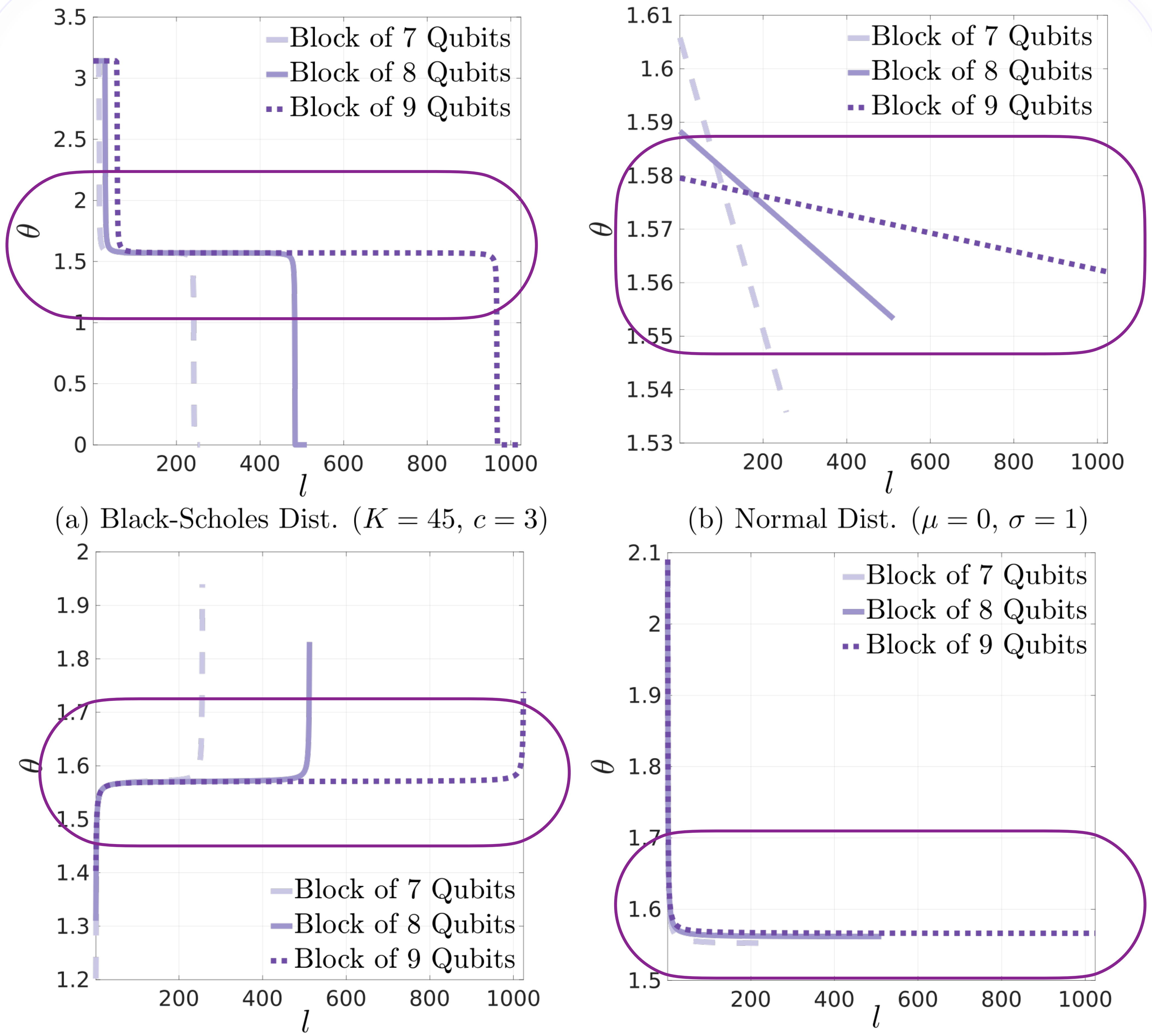
Our first protocol

Motivation

$$\theta_{bin(l)}^{(k-1)} = 2\arccos\left(\sqrt{\frac{\int_{x_{min}+(l-1)\delta_k}^{x_{min}+(l-1/2)\delta_k} f(x)dx}{\int_{x_{min}+(l-1)\delta_k}^{x_{min}+l\delta_k} f(x)dx}}\right)$$



(a) Black-Scholes Dist. ($K = 45$, $c = 3$)

(b) Normal Dist. ($\mu = 0$, $\sigma = 1$)

(c) Beta Dist. ($\alpha = 0.5$, $\beta = 0.5$)

(d) Gamma Dist. ($k = 1.5$, $\theta = 2$)

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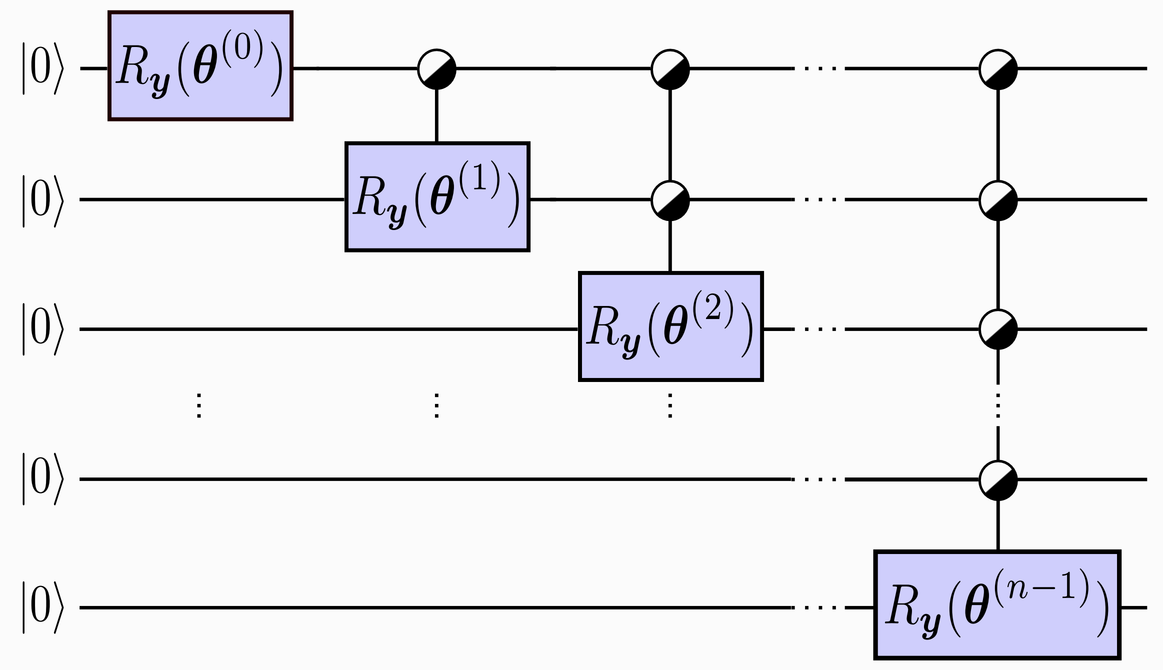
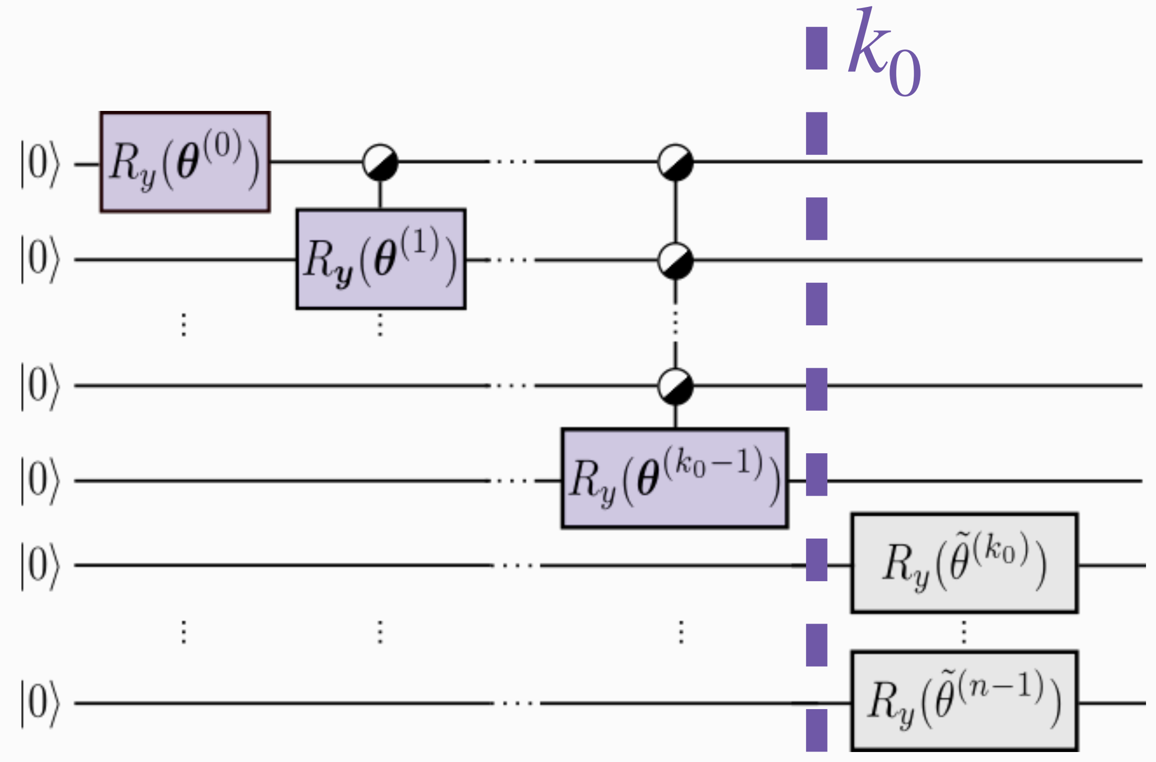
Our first protocol

Angles Clustering

We cluster the all the angles from the block $k_0 + 1$ onwards and find out how it affects on the fidelity



Complexity $\mathcal{O}(2^n)$



Complexity $\mathcal{O}(2^{k_0})$

Infidelity ϵ

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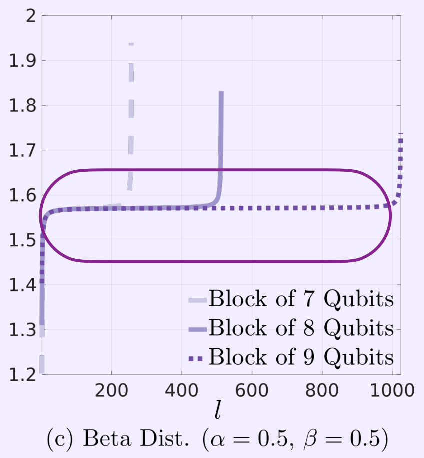
Encoding classical information

Overview

(Approximated) Unity mappings

Our first contribution!

[Phys. Rev. Research 5, 033114 (2023)]



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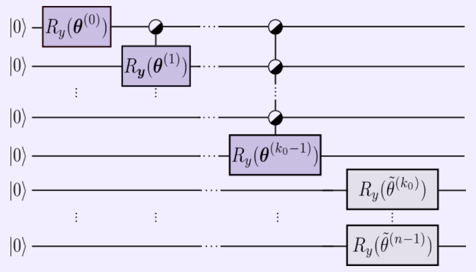

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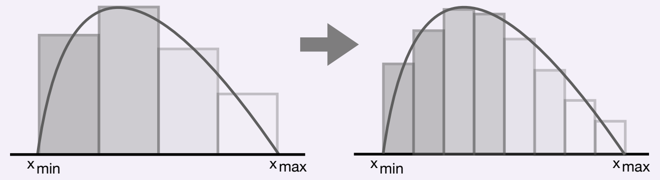

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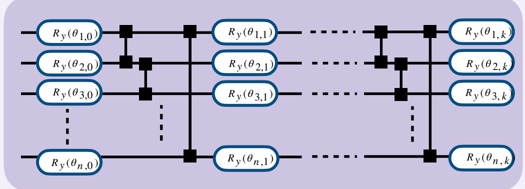
Generative models

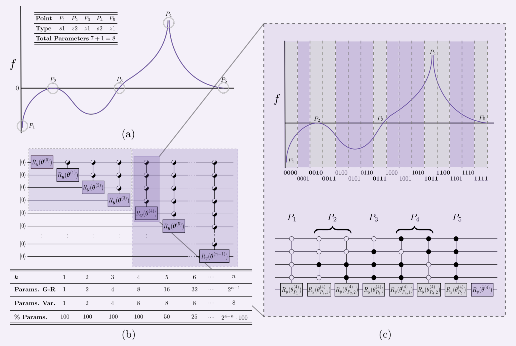


Training | Expressability

Quantum generative adversarial networks for learning and loading random distributions (IBM)

Our second contribution!

Adapted VQC!



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(needs classical pre-computation)

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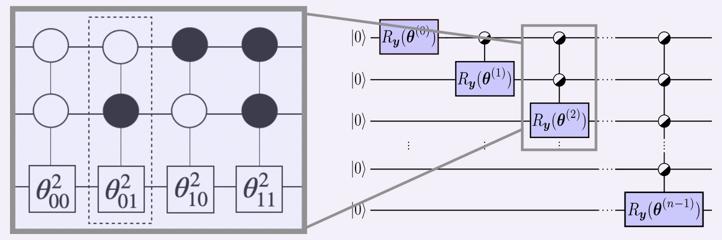

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Arithmetic

Rotation angles

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Efficient ? Quantum query complexity

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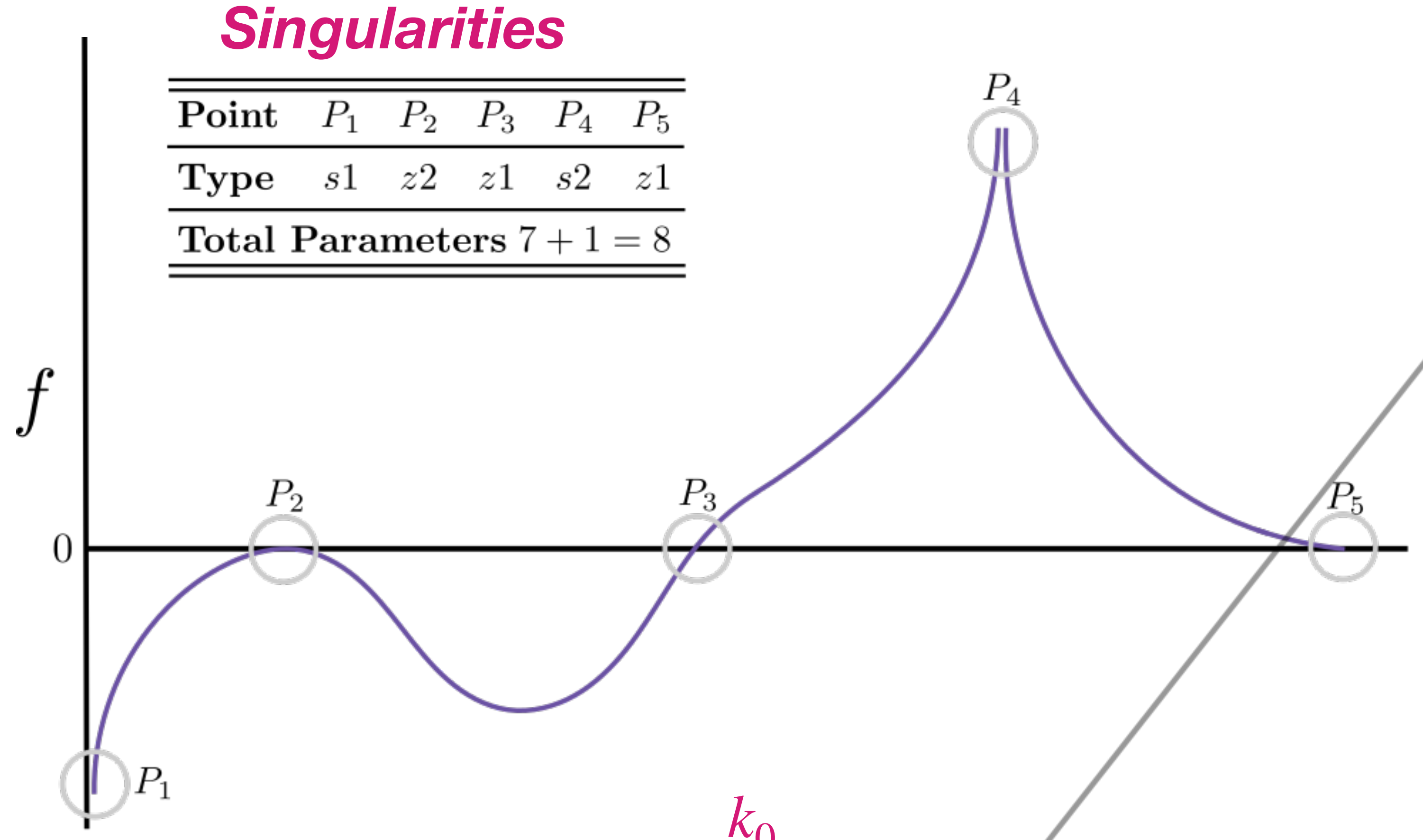
Variational Protocol

Motivation

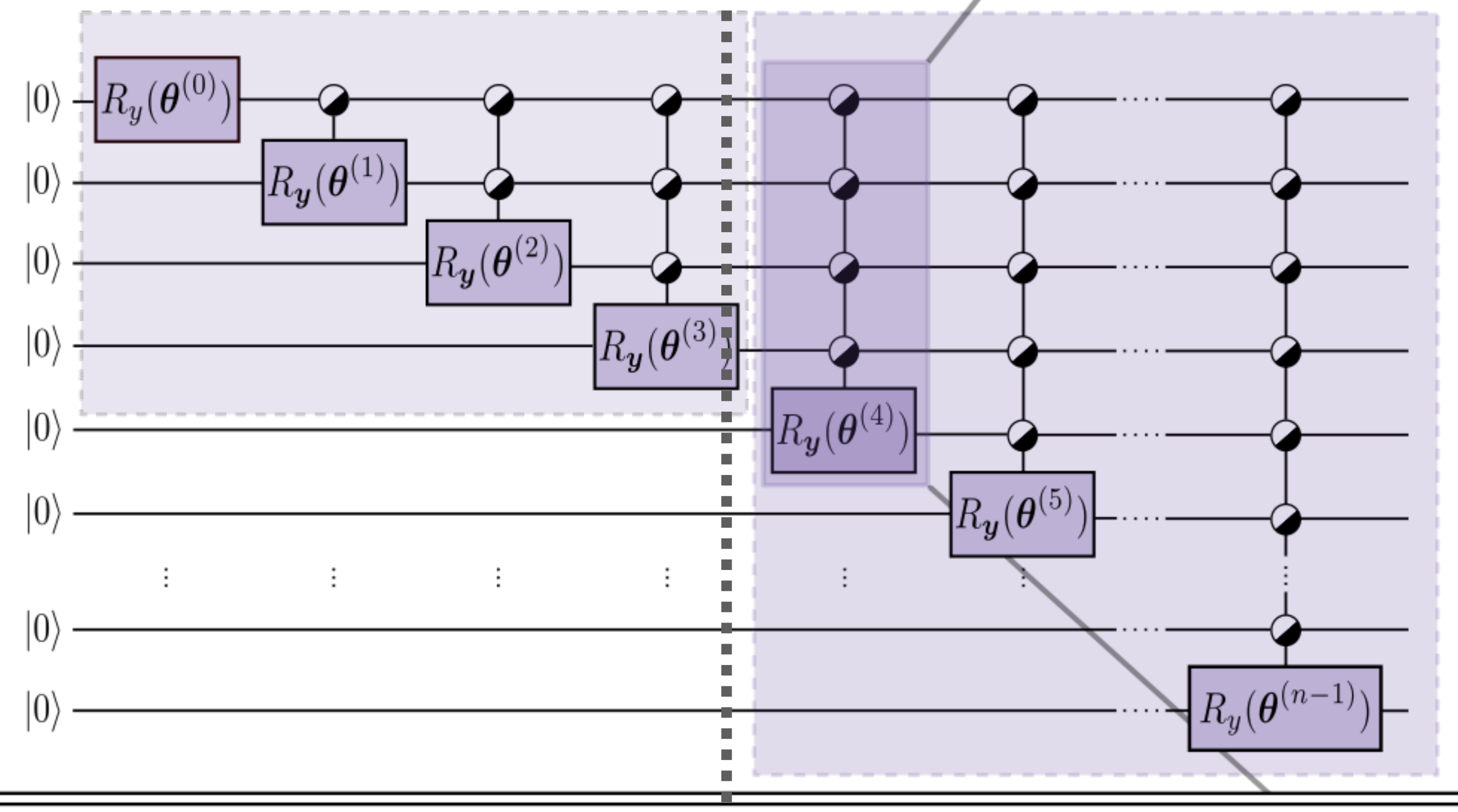
Singularities

Point	P_1	P_2	P_3	P_4	P_5
Type	$s1$	$z2$	$z1$	$s2$	$z1$
Total Parameters	$7+1=8$				



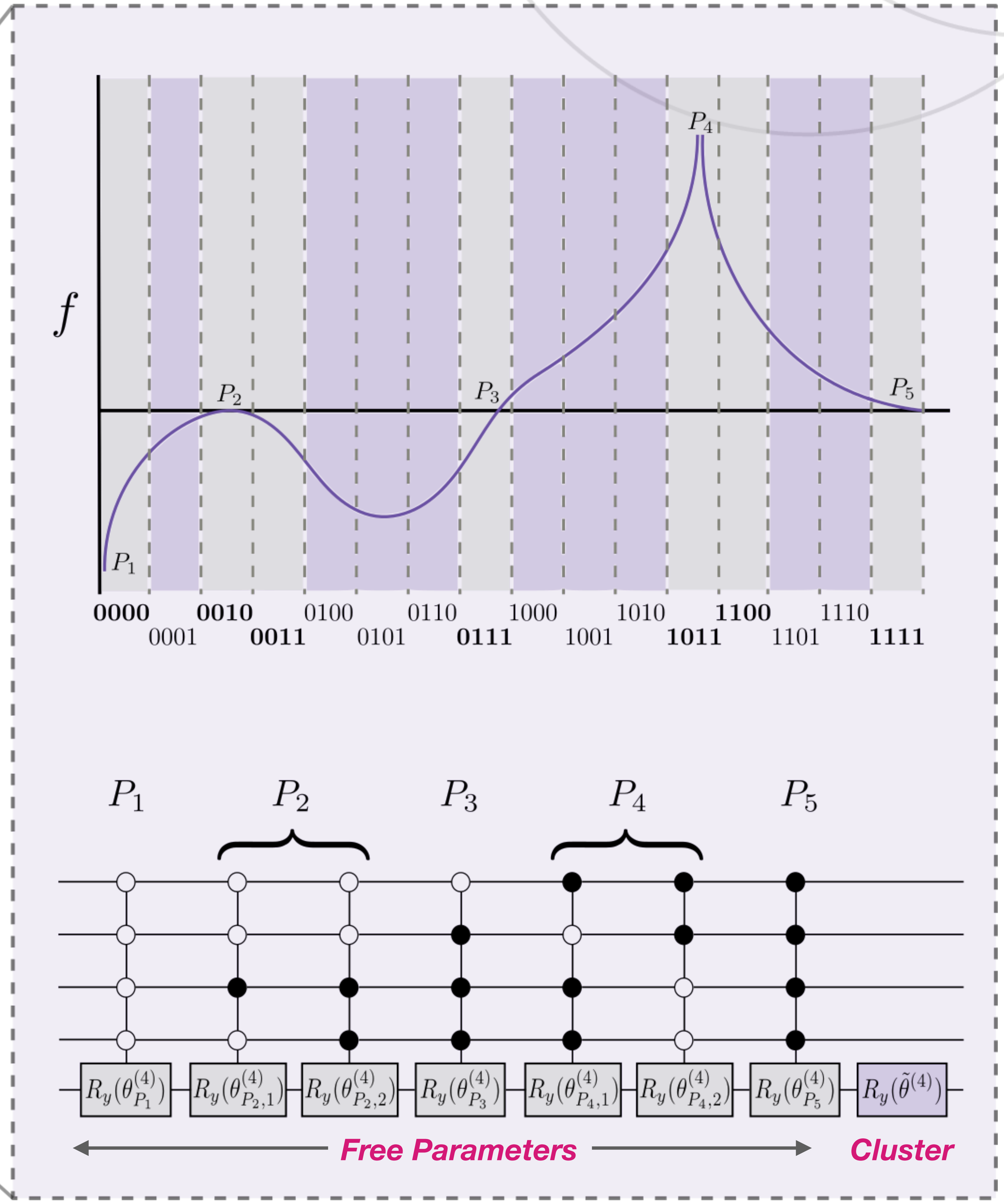
Unclustered

k_0



k	1	2	3	4	5	6	....	n
Params. G-R	1	2	4	8	16	32	....	2^{n-1}
Params. Var.	1	2	4	8	8	8	....	8
% Params.	100	100	100	100	50	25	....	$2^{4-n} \cdot 100$

Partially clustered



<u>How to proceed</u>

(i) Identify and classify singularities

(ii) Uncluster till k0 (hyperparameter)

(iii) Partially clustering from k0

(iv) Train

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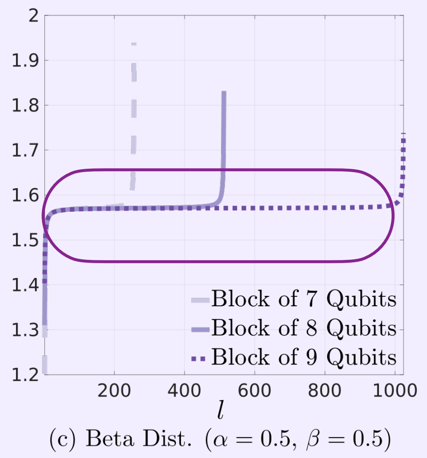

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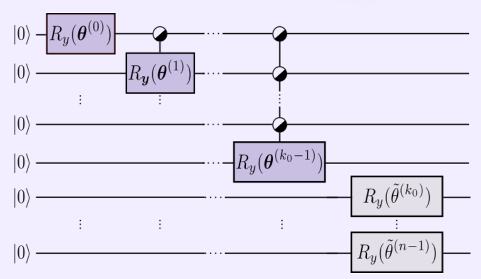

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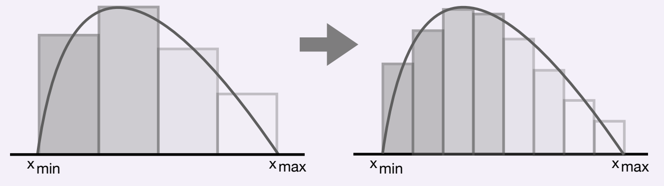

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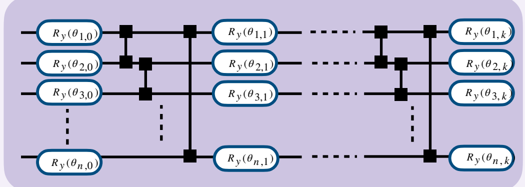

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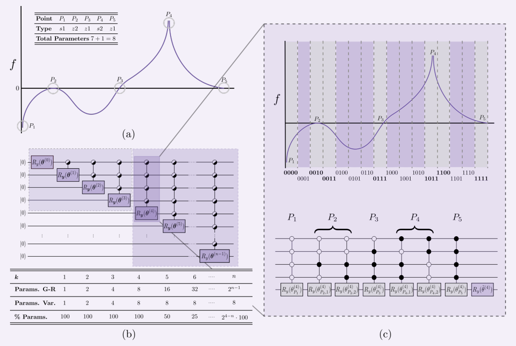

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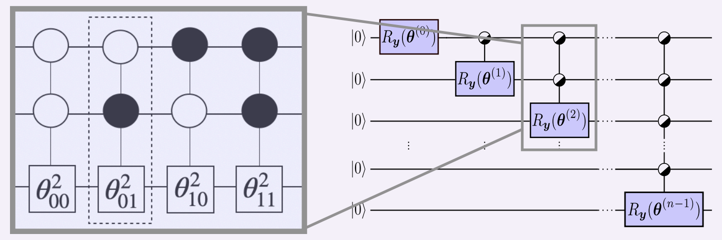

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Amplitude Oracle

$$\mathfrak{amp} : |l\rangle|0\rangle^{\otimes m} \rightarrow |l\rangle|x_l\rangle_m$$

Rotation Oracle

$$\mathfrak{rot}\,|\psi\rangle|0\rangle := \sin(\theta)|\psi\rangle|0\rangle + \cos(\theta)|\psi\rangle|1\rangle$$

$$\mathfrak{comp}\,|a\rangle|b\rangle|0\rangle := \begin{cases} |a\rangle|b\rangle|0\rangle & \text{if } a < b \\ |a\rangle|b\rangle|1\rangle, & \text{if } a \geq b \end{cases}$$

Grover (2000)

(Probabilistic, requires AA)
+ arithmetic

[Phys. Rev. Lett. 85, 1334 (2000)]

Probabilistic Grover

Grover 2 (2019)
[Phys. Rev. Lett. 122, 020502 (2019)]
Does not need arithmetics, inequality test instead

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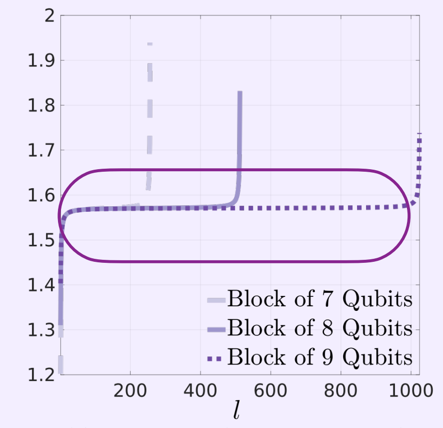

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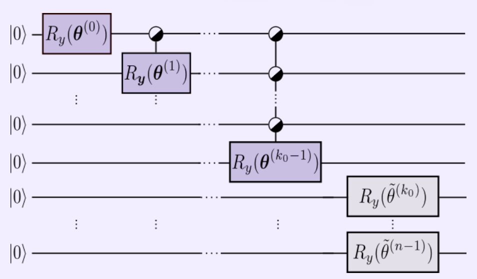

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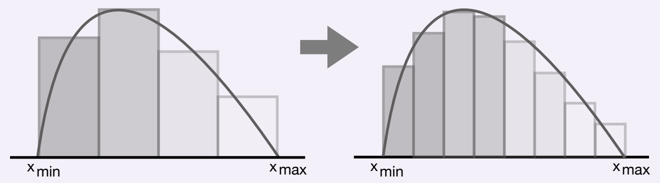

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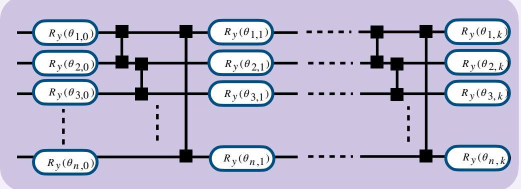

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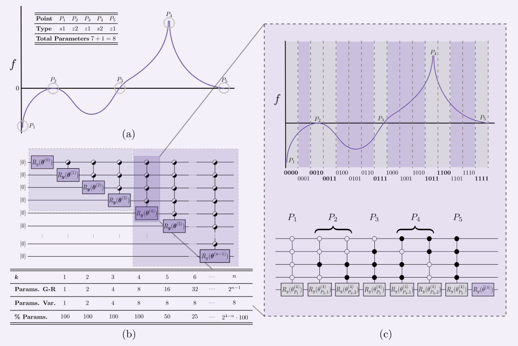

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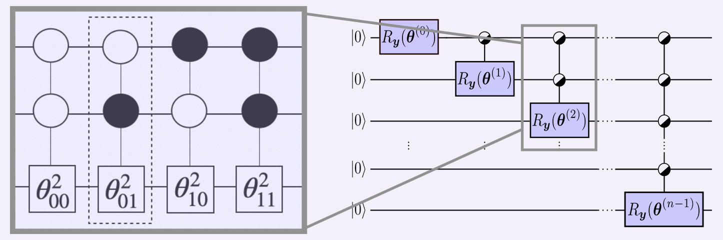

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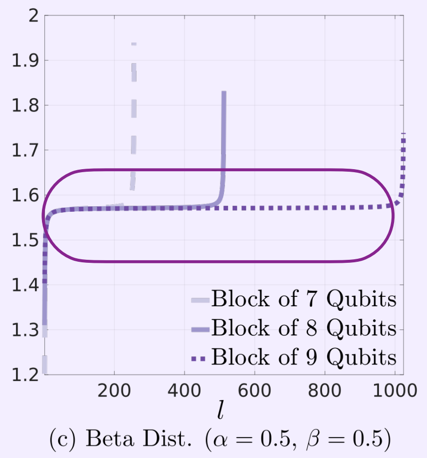

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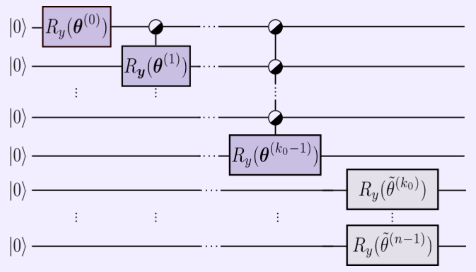

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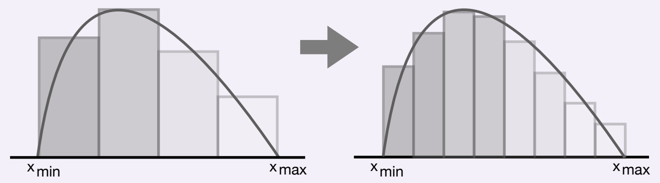

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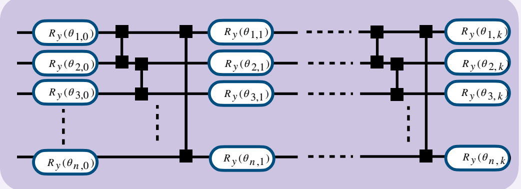

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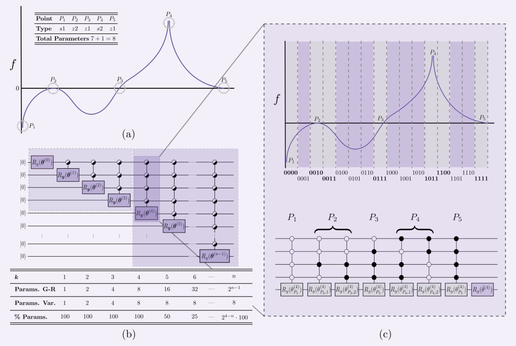

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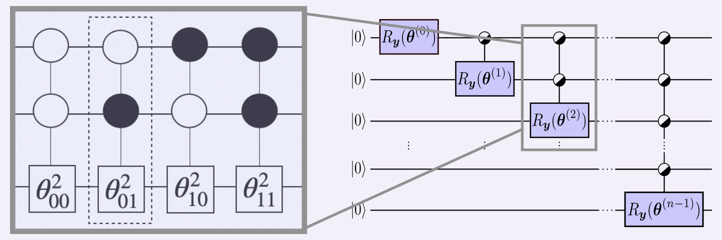

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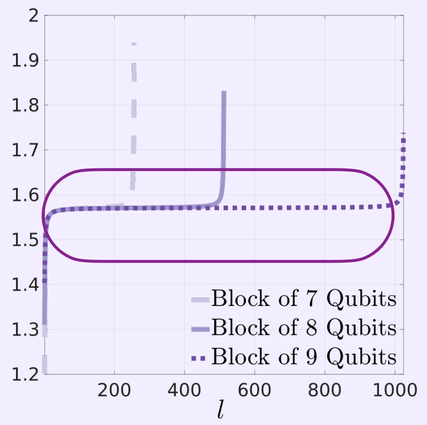

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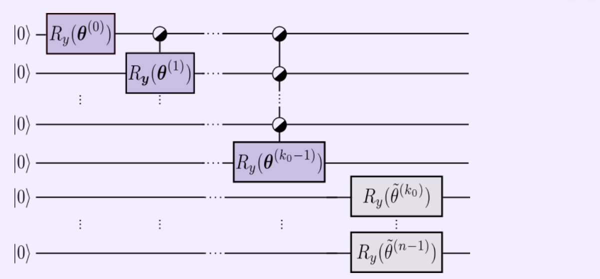

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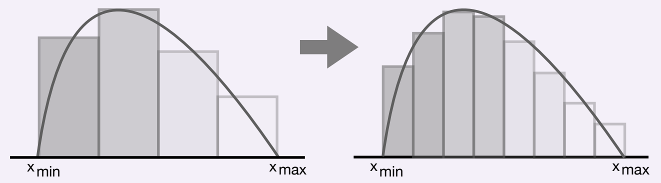

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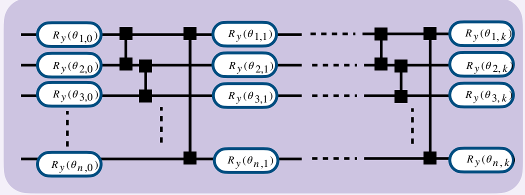

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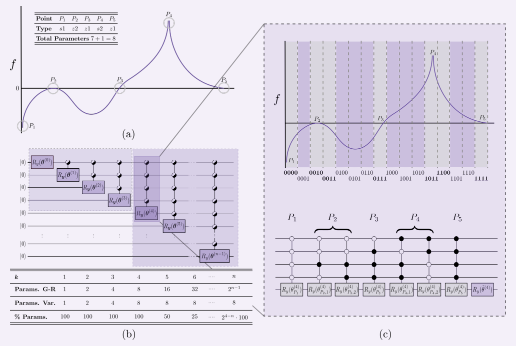

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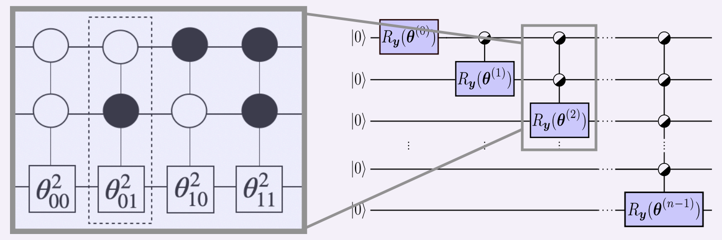

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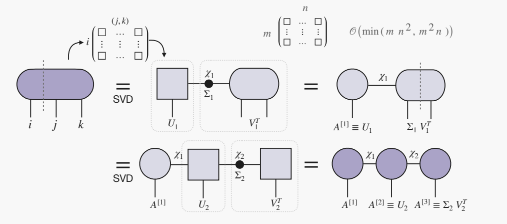

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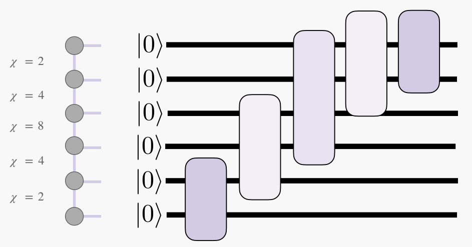

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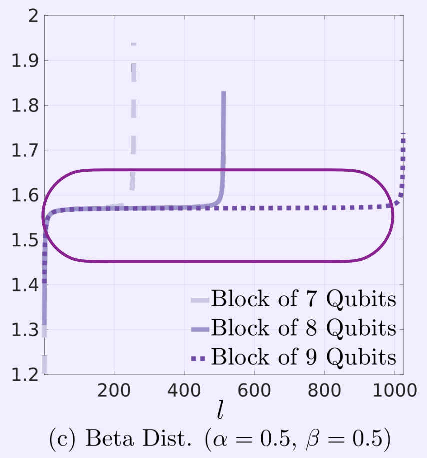

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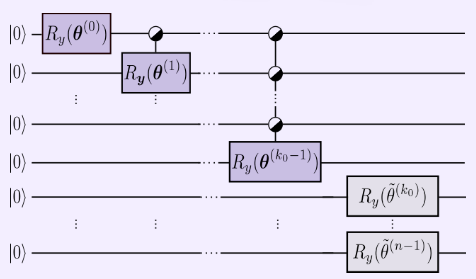

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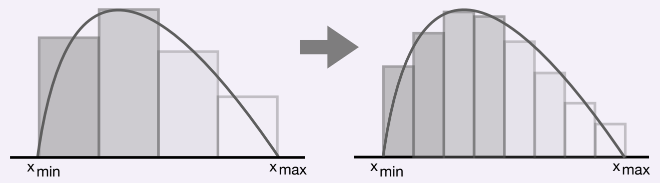

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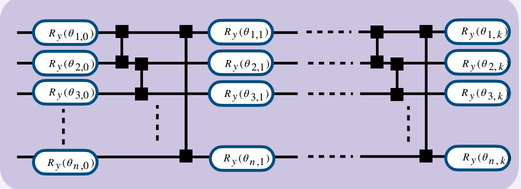

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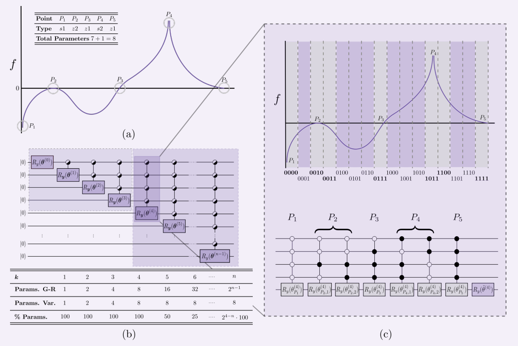

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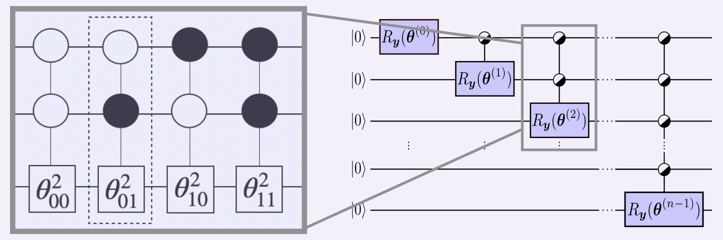

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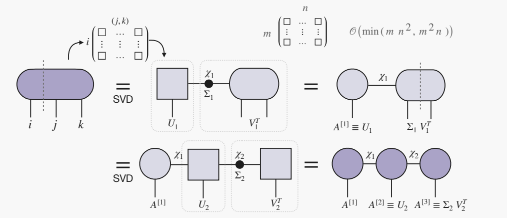

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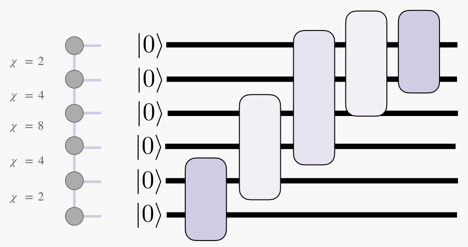
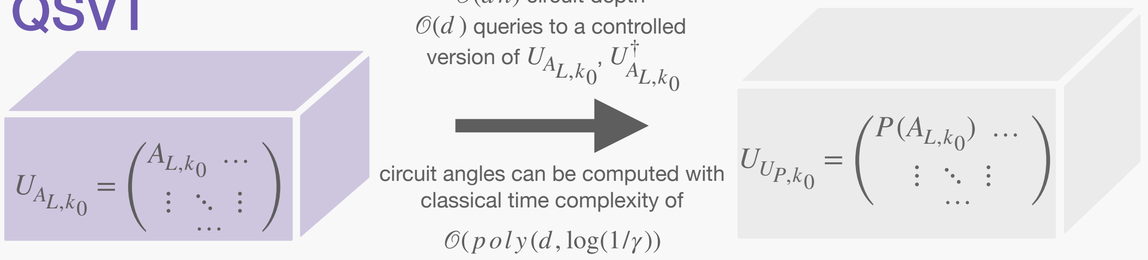


QSVT

$\mathcal{O}(dn)$ circuit depth
$\mathcal{O}(d)$ queries to a controlled version of $U_{A_{L,k_0}}$, $U^{\dagger}_{A_{L,k_0}}$



circuit angles can be computed with classical time complexity of $\mathcal{O}(poly(d, \log(1/\gamma)))$

[Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing (STOC 2019)]

Amplitude encoding

Linear function

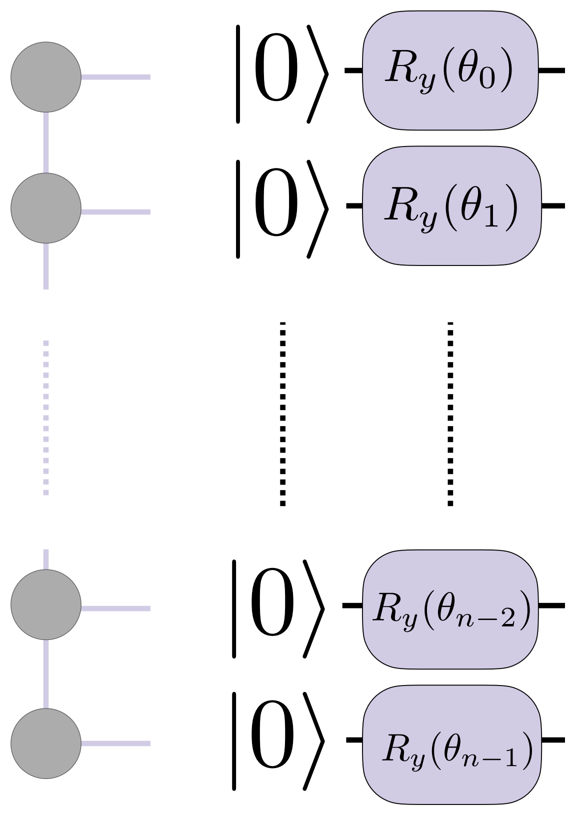
Linear Function

Exact MPS $\chi = 2$

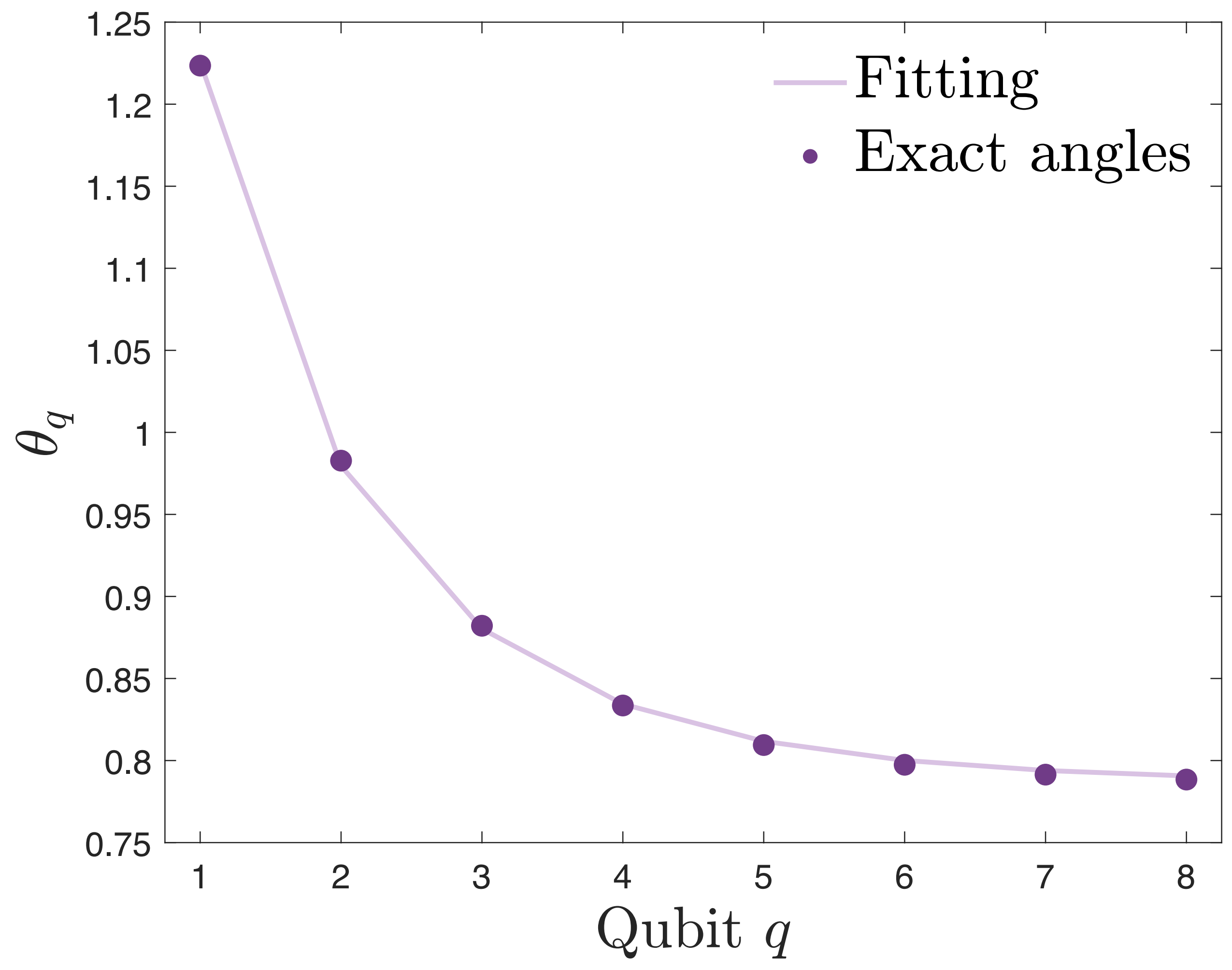
$$|\Phi_L\rangle = \sum_{j_0\cdots j_{n-1}} \begin{pmatrix} j_0/C & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2j_1/C & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 \\ 2^{n-1}j_n/C \end{pmatrix} |j_{n-1}\cdots j_0\rangle\,.$$

Approximated MPS

$\chi = 1$

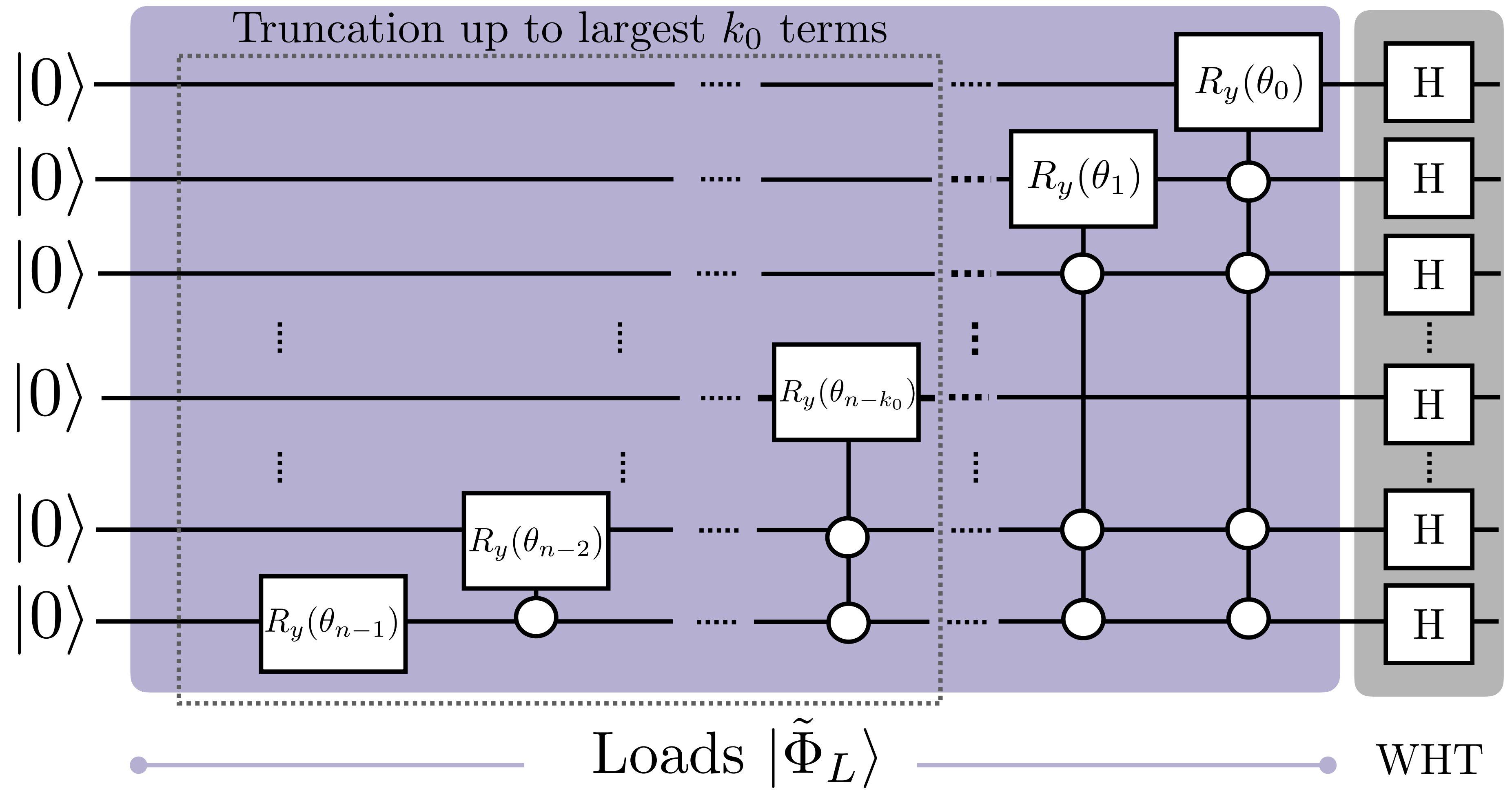


Variational QC



Walsh Hadamard Series

$$(x_0^{(n)} \ldots x_{N-1}^{(n)}) \qquad x_k^{(n)} = \begin{cases} 2^{n/2}(2^n - 1)/2 & \text{if } k = 0 \\ -2^{n/2}k/2 & \text{if } |k|_b = 1 \\ 0 & \text{otherwise} \end{cases}$$



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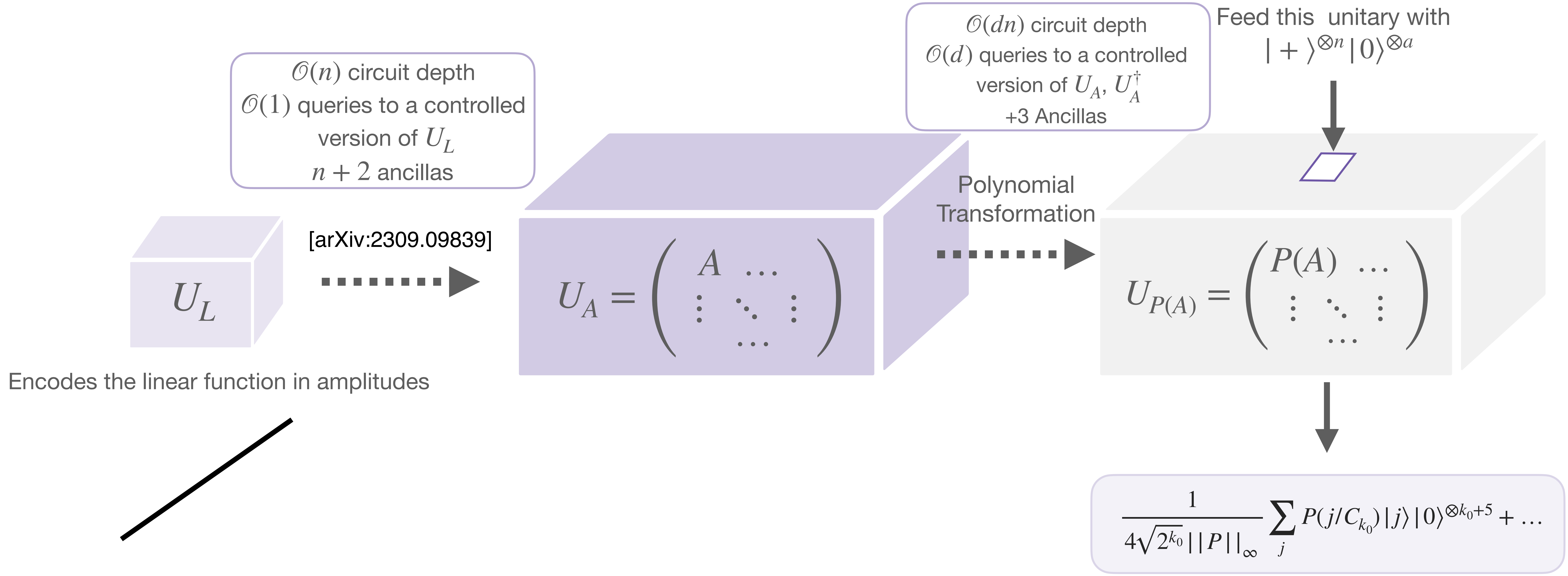
Amplitude encoding

QSVT



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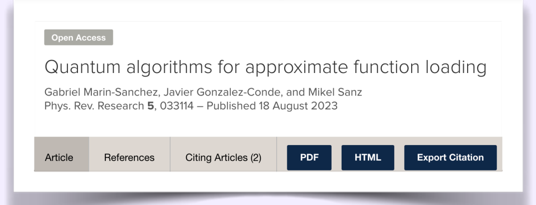
Encoding classical information

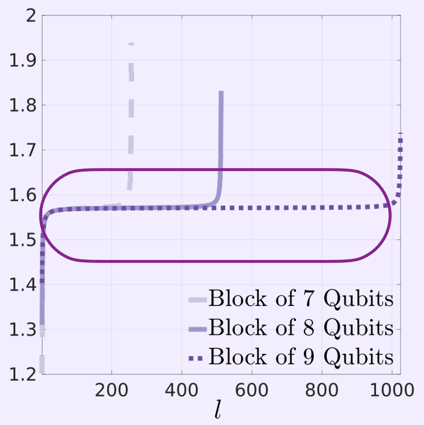
Different embeddings

(Approximated) Unity mappings

Our first contribution!


Quantum algorithms for approximate function loading
Gabriel Marin-Sanchez, Javier Gonzalez-Conde, and Mikel Sanz
Phys. Rev. Research 5, 033114 – Published 18 August 2023

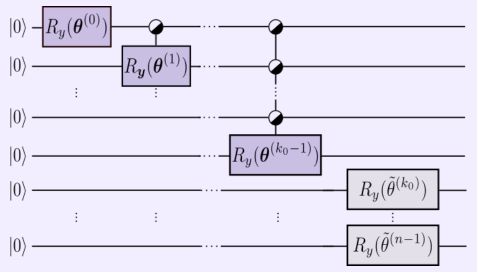
[Phys. Rev. Research 5, 033114 (2023)]



(c) Beta Dist. $(\alpha = 0.5, \beta = 0.5)$

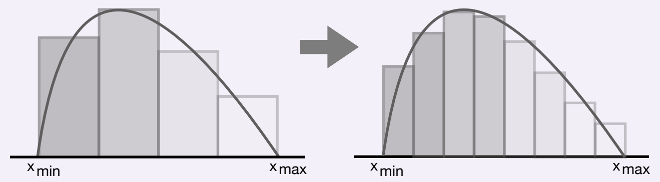
Clustering angles!

$$\left|\theta_{l+1}^{(k-1)} - \theta_l^{(k-1)}\right| \leq \frac{\delta_k^2}{4} \max_{y' \in [l\delta_k, (l+1)\delta_k]} \left| \left(\partial_y^2 \log f(y)\right)\right|_{y=y'}$$



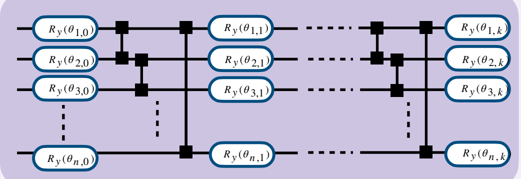
Grover and Rudolph (2002)

- Iterative and deterministic
- Real valued functions



[Proc.Roy.Soc.Lond. A454 (1998)]
[arXiv preprint quant-ph/0208112 (2002)]
[arXiv preprint quant-ph/0407102, (2004)]

Generative models



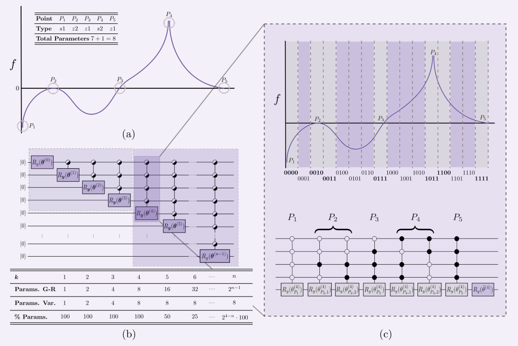
Training | Expressability

Quantum generative adversarial networks for learning and loading random distributions (IBM)

Our second contribution!

Adapted VQC!


Quantum algorithms for approximate function loading



[Phys. Rev. Research 5, 033114 (2023)]

Exact unity mappings
(needs classical pre-computation)

Exponential cost in ancillas
[Phys. Rev. Research 3, 043200 (2021)]
[Quantum 8, 1228 (2024)]

Sparse states (efficient)
[Phys. Rev. Lett. 129, 230504 (2022)]
[arXiv:2108.06150]

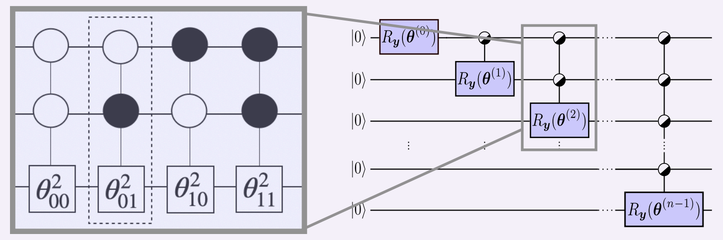
[arXiv:2310.19309]

Exponential cost in depth
- Complex values
- Links two arbitrary states

[Quantum Information & Computation Volume 5 Issue 6 (2005)]
https://arxiv.org/pdf/quant-ph/0407102.pdf

Schmidt decomposition
[Phys. Rev. A 83, 032302 (2011)]

CNOT synthesis
[arXiv:2401.01009]



Oracular Access
(ancillas for m bit precision)

Amplitude Oracle

$$\mathtt{amp} : |l\rangle|0\rangle^{\otimes m} \rightarrow |l\rangle|x_l\rangle_m$$

Rotation Oracle

$$\mathtt{rot}|\psi\rangle|0\rangle := \sin(\theta)|\psi\rangle|0\rangle + \cos(\theta)|\psi\rangle|1\rangle$$

$$\mathtt{comp}|a\rangle|b\rangle|0\rangle := \begin{cases} |a\rangle|b\rangle|0\rangle & \text{if } a < b \\ |a\rangle|b\rangle|1\rangle, & \text{if } a \geq b \end{cases}$$

[https://arxiv.org/abs/2405.20225]

Arithmetic

Grover (2000)
(Probabilistic, requires AA)
+ arithmetic

[Phys. Rev. Lett. 85, 1334 (2000)]

Grover 2 (2019)
[Phys. Rev. Lett. 122, 020502 (2019)]
Does not need arithmetics, inequality test instead

Grover 3 (2022)
[Quantum 6, 773 (2022)]
Improves AA

Grover 4 (2022)
[New J. Phys. 24 103004]
Inverse-coefficient

Rotation angles

$$\mathcal{O}_{GR} : |l\rangle|0\rangle^{\otimes m} \rightarrow |l\rangle|\theta_{bin(l)}^{(k-1)}\rangle_m$$

$$\mathtt{rot}|\theta\rangle|0\rangle := \sin(\theta)|\theta\rangle|0\rangle + \cos(\theta)|\theta\rangle|1\rangle$$

Efficient ? Quantum query complexity

[Phys. Rev. E 103, 063302 (2021)]
[Quantum 5, 431 (2021)]
[Quantum 5, 463 (2021)]

QPE [arXiv:1912.05335]

Adiabatic state preparation
Arthur R.: Preparing Arbitrary Continuous Functions in Quantum Registers With Logarithmic Complexity
[arXiv:2205.00519]

Ground state preparation
[arXiv:2311.03347]
[arXiv:2303.11962]
[arXiv:2306.14993]
[PRX Quantum 3, 040305 (2022)]
[Quantum 4, 372 (2020)]

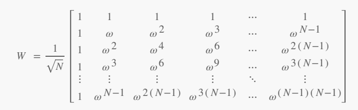
Quantum Walks
[arXiv:2405.20273]

Our third contribution!

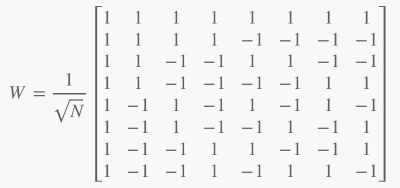
Building block to load the linear function

Discrete transforms

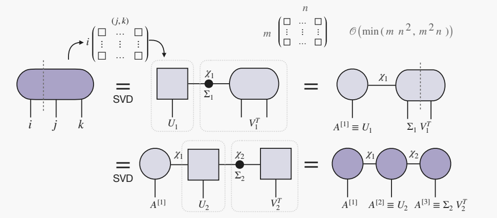
Fourier
[Quantum Science and Technology (Vol. 9, Issue 1, p. 015002) 2023]
[New J. Phys. 23 103022]



Walsh Hadamard
[arXiv:2307.10917]
[arXiv:2307.08384]



Matrix Product State
[npj Quantum Information volume,10, 15 (2024)]



For an arbitrary polynomial the maximum bond dimension is $\chi = d + 1$.

Block encoding

QSVT
[arXiv:2210.14892]
[arXiv:2309.09839]
[Quantum 8, 1297 (2024)]

Quantum Sampling
[arXiv:2405.11436]

[arXiv:2405.21058]

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Thank you!

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ikerbasque Basque Foundation for Science
Universidad del País Vasco Euskal Herriko Unibertsitatea
EHU QC
EHU Quantum Center

Quantum SPAIN
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UNIÓN EUROPEA Fondo Social Europeo
AGENCIA ESTATAL DE INVESTIGACIÓN
Quromorphic