

# Mire használható a kiterjesztett lineáris szigma modell?

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HUN-REN Wigner FK

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## Based on the paper: arXiv:2407.18348

arXiv &gt; hep-ph &gt; arXiv:2407.18348

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High Energy Physics - Phenomenology

[Submitted on 25 Jul 2024]

## Ordinary and exotic mesons in the extended Linear Sigma Model

Francesco Giacosa, Péter Kovács, Shahriyar Jafarzade

The extended Linear Sigma Model (eLSM) is a hadronic model based on the global symmetries of QCD and the corresponding explicit, anomalous, and spontaneous breaking patterns. In its basic three-flavor form, its mesonic part contains the dilaton/gluoball as well as the nonets of (pseudo)scalar and (axial-)vector mesons, thus chiral symmetry is linearly realized. In the chiral limit and neglecting the chiral anomaly, only one term – within the dilaton potential – breaks dilatation invariance. Spontaneous symmetry breaking is implemented by a generalization of the Mexican-hat potential, with explicit symmetry breaking responsible for its tilting. The overall mesonic phenomenology up to 2 GeV is in agreement with the PDG compilation of masses and decay widths. The eLSM was enlarged to include other conventional quark-antiquark nonets (pseudovector and orbitally excited vector mesons, (axial-)tensor mesons, etc.), as well as two nonets of hybrid mesons, the lightest one with exotic quantum numbers  $J^{PC} = 1^{-+}$  not allowed for  $\bar{q}q$  objects such as the resonance  $\pi_1(1600)$  and the recently discovered  $\eta_1(1855)$ . In doing so, different types of chiral multiplets are introduced: heterochiral and homochiral multiplets, which differ in the way they transform under chiral transformations. Moreover, besides the scalar gluoball, the tensor, the pseudoscalar and the vector gluoballs were coupled to the eLSM: the scalar  $f_0(1710)$  turns out to be mostly gluonic, the tensor gluoball couples strongly to vector mesons, and the pseudoscalar gluoball couples can be assigned to  $X(2370)$  or  $X(2600)$ . The eLSM contains chiral partners on an equal footing and is therefore well suited for studies of chiral symmetry restoration at nonzero temperature and densities. The QCD phase diagram and the location of the critical endpoint were investigated within this framework.

Comments: Review paper about the extended Linear Sigma Model. Prepared for Progress in Particle and Nuclear Physics. 118 pages, 47 tables, 14 figures

Subjects: High Energy Physics - Phenomenology (hep-ph); High Energy Physics - Lattice (hep-lat); High Energy Physics - Theory (hep-th)

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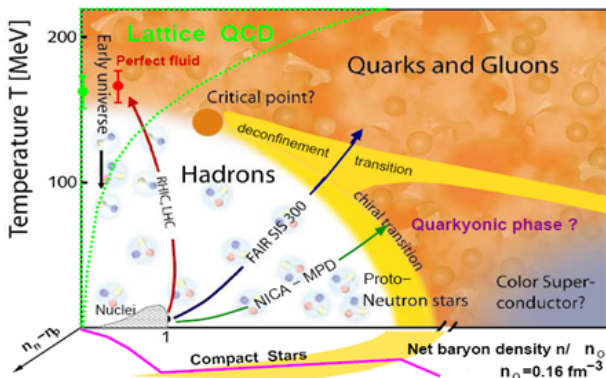
Conventional ( $\bar{q}q$ ) mesons

$L$	$S$	$\mathcal{P} = (-1)^{L+1}$	$\mathcal{C} = (-1)^{L+S}$	$J^{PC}$	$n^{2S+1}L_J$	Resonances
0	0	-1	+1	$0^{-+}$	$1^1S_0$	$\pi, K, \eta, \eta'$
1	1	+1	+1	$0^{++}$	$1^3P_0$	$a_0, K_0^*, f_{0,L}, f_{0,H}$
1	0	+1	-1	$1^{+-}$	$1^1P_1$	$b_1, K_{1,B}, h_{1,L}, h_{1,H}$
0	1	-1	-1	$1^{--}$	$1^3S_1$	$\rho, K^*, \omega_{1,L} = \omega, \omega_{1,H} = \phi$
1	1	+1	+1	$1^{++}$	$1^3P_1$	$a_1, K_{1,A}, f_{1,L}, f_{1,H}$
2	1	-1	-1	$1^{--}$	$1^3D_1$	$\rho_D, K_{1D}^*, \omega_D, \phi_D$
2	0	-1	+1	$2^{-+}$	$1^1D_2$	$\pi_2, K_2, \eta_2, \eta_2'$
1	1	+1	+1	$2^{++}$	$1^3P_2$	$a_2, K_2, f_{2,L} = f_2, f_{2,H} = f_2'$
2	1	-1	-1	$2^{--}$	$1^3D_2$	$\rho_2, K_2^*, \omega_{2,L} = \omega_2, \omega_{2,H} = \phi_2$
3	1	+1	+1	$2^{++}$	$1^3F_2$	$a_{2F}, K_{2F}, f_{2F,L}, f_{2F,H}$

Non-conventional mesons (like  $J^{PC} = 1^{-+}$  do not appear), such exotic quantum numbers can be realized by glueballs and hybrid mesons.

(Note: If  $J^{PC} = 1^{-+} \implies L = 2k$  (from  $P$ ),  $S = 0$  (from  $C$  and  $S \in (0, 1)$ ), thus  $J = 2\ell$ , so  $J \neq 1$ )

# Envisaged phase diagram of QCD



Important details of the phase diagram is still unknown

Properties of the phase diagram especially at finite baryon densities/baryochemical potential can be well investigated with the help of effective field theories of QCD  $\rightarrow$  e.g. details of the phase boundary like existence and location of the CEP, in medium dependence of meson masses, or properties of compact stars etc.

# Chiral symmetry, chiral models

If the quark masses are zero (chiral limit)  $\implies$  QCD invariant under the following global symmetry transformation :

$$\begin{aligned} \mathcal{G}_{cl} &\equiv U(3)_L \times U(3)_R = U(1)_L \times U(1)_R \times \underbrace{SU(3)_L \times SU(3)_R}_{\mathcal{G}_{chiral}} \\ &= \underbrace{U(1)_V}_{\text{baryon number}} \times \underbrace{U(1)_A}_{\text{axial}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\mathcal{G}_{chiral}} \end{aligned}$$

$U(1)_V \longrightarrow$  baryon number conservation (exact symmetry of nature)

$U(1)_A \longrightarrow$  axial symmetry (connected to axial anomaly)

$SU(3)_L \times SU(3)_R \longrightarrow$  broken down to  $SU(3)_V$  if  $m_u = m_d = m_s \neq 0$ ;  
 or to  $SU(2)_V$  if  $m_u = m_d \neq m_s \neq 0$ ;  
 or to  $U(1)_V$  if  $m_u \neq m_d \neq m_s \neq 0$

Since QCD is very hard to solve  $\longrightarrow$  low energy effective models  $\longrightarrow$  reflecting the global symmetries of QCD  $\longrightarrow$  degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry  $\longrightarrow$  linear sigma model  
 (nonlinear representation  $\longrightarrow$  chiral perturbation theory (ChPT))

# Lagrangian of the eLSM model

$\mathcal{L}$  constructed based on linearly realized global  $U(3)_L \times U(3)_R$  symmetry and its **explicit breaking**

$$\begin{aligned}
 \mathcal{L} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
 & + c_1 (\det \Phi + \det \Phi^\dagger) + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
 & + \text{Tr} \left[ \left( \frac{m_1^2}{2} \mathbb{1} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
 & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
 & + \bar{\Psi} (i\gamma^\mu D_\mu - g_F(S - i\gamma_5 P)) \Psi - g_V \bar{\Psi} (\gamma^\mu (V_\mu + \gamma_5 A_\mu)) \Psi,
 \end{aligned}$$

$$\begin{aligned}
 \Phi &= S + iP \equiv \sum_{a=0}^8 (S_a \lambda_a + iP_a \lambda_a) \\
 D^\mu \Phi &= \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi], \\
 L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}, \\
 R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}, \\
 D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with } G^\mu = g_s G_a^\mu T_a.
 \end{aligned}$$

+ Polyakov loop potential (for  $T > 0$ )

# Particle content

- **Vector** and **Axial-vector** meson nonets

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{N+\rho^0}}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_{N-\rho^0}}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N+a_1^0}}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N-a_1^0}}{\sqrt{2}} & K_1^0 \\ K_1^- & K_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \rightarrow \rho(770)$ ,  $K^* \rightarrow K^*(894)$   
 $\omega_N \rightarrow \omega(782)$ ,  $\omega_S \rightarrow \phi(1020)$

$a_1 \rightarrow a_1(1230)$ ,  $K_1 \rightarrow K_1(1270)$   
 $f_{1N} \rightarrow f_1(1280)$ ,  $f_{1S} \rightarrow f_1(1426)$

- **Scalar** ( $\sim \bar{q}_i q_j$ ) and **pseudoscalar** ( $\sim \bar{q}_i \gamma_5 q_j$ ) meson nonets

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_{N+a_0^0}}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_{N-a_0^0}}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & K_0^{*0} & \sigma_S \end{pmatrix} \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_{N+\pi^0}}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_{N-\pi^0}}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix}$$

multiple possible assignments  
 mixing in the  $\sigma_N - \sigma_S$  sector

$\pi \rightarrow \pi(138)$ ,  $K \rightarrow K(495)$   
 mixing:  $\eta_N, \eta_S \rightarrow \eta(548)$ ,  $\eta'(958)$

Spontaneous symmetry breaking:  $\sigma_{N/S}$  acquire nonzero expectation values  $\phi_{N/S}$   
 fields shifted by their expectation value:  $\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S}$

In case of compact stars, also nonzero vector condensates



# Determination of the parameters

14 unknown parameters ( $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F$ )  $\longrightarrow$  determined by the **min. of  $\chi^2$** :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[ \frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

$(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$ ,  $Q_i(x_1, \dots, x_N) \longrightarrow$  from the model,  $Q_i^{\text{exp}} \longrightarrow$  PDG value,  $\delta Q_i = \max\{5\%, \text{PDG value}\}$

multiparametric minimalization  $\longrightarrow$  **MINUIT**

- ▶ PCAC  $\rightarrow$  2 physical quantities:  $f_\pi, f_K$
- ▶ Curvature masses  $\rightarrow$  16 physical quantities:  
 $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}$
- ▶ Decay widths  $\rightarrow$  12 physical quantities:  
 $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi},$   
 $\Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$
- ▶ Pseudocritical temperature  $T_c$  at  $\mu_B = 0$

# A usual (hybrid) approximation

- ▶ D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  - $u, d, s$  constituent quarks ( $m_u = m_d$ )
  - (Polyakov loop variables  $\Phi, \bar{\Phi}$  with  $\mathcal{U}_{\log}^{\text{YM}}$  or  $\mathcal{U}_{\log}^{\text{glue}}$ )

- ▶ **no mesonic fluctuations**, only fermionic ones

$$Z = e^{-\beta V \Omega(\mathcal{T}, \mu q)} = \int_{\text{PBC}} \prod_a \mathcal{D}\xi_a \int_{\text{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp \left[ - \int_{\mathbf{0}}^{\beta} d\tau \int_V d^3x \left( \mathcal{L} + \mu q \sum_f q_f^\dagger q_f \right) \right] \text{ approximated}$$

$$\text{as } \Omega(\mathcal{T}, \mu q) = \mathcal{U}_{\text{meson}}^{\text{tree}}(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(\mathcal{T}, \mu q) + \mathcal{U}_{\log}(\Phi, \bar{\Phi}), \quad \bar{\mu}q = \mu q - iG_4$$

$$e^{-\beta V \Omega_{\bar{q}q}^{(0)}} = \int_{\text{APBC}} \prod_{f,g} \mathcal{D}q_g \mathcal{D}q_f^\dagger \exp \left\{ \int_{\mathbf{0}}^{\beta} d\tau \int_x q_f^\dagger \left[ \left( i\gamma_{\mathbf{0}} \vec{\gamma} \cdot \vec{\nabla} - \frac{\partial}{\partial \tau} + \bar{\mu}q \right) \delta_{fg} - \gamma_{\mathbf{0}} \mathcal{M}_{fg} |_{\xi_a = \mathbf{0}} \right] q_g \right\}$$

- ▶ tree-level (axial)vector masses
- ▶ fermionic **thermal** fluctuations included in the (pseudo)scalar **curvature masses**
- ▶ 2 (or 4) coupled  $T/\mu_B$ -dependent field equations for the condensates  $\phi_N, \phi_S, (\Phi, \bar{\Phi})$  at  $N_c = 3$
- ▶ Polyakov-loops and **fermionic vacuum** fluctuations

# Introduction to Polyakov-loops/Polyakov-loop variables

Definition of Polyakov-loop

$$L(\vec{x}) = \mathcal{P} \exp \left\{ i \int_0^\beta A_4 dt \right\}$$

Polyakov-loop variables:

$$\Phi(\vec{x}) = \frac{1}{N_c} \text{Tr}_c L(\vec{x}), \text{ and } \bar{\Phi}(\vec{x}) = \frac{1}{N_c} \text{Tr}_c L(\vec{x})^\dagger, \text{ (non center } (C_n) \text{ symmetric)}$$

If  $\Delta F_{q/\bar{q}}$  is a change in the free energy, when an infinitely heavy quark (or antiquark) is added to the system, then

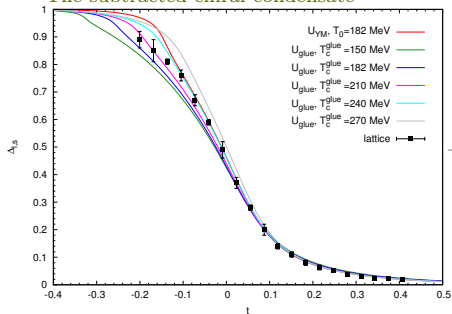
$$\langle \Phi(\vec{x}) \rangle_\beta = e^{-\beta \Delta F_q(\vec{x})}, \quad \langle \bar{\Phi}(\vec{x}) \rangle_\beta = e^{-\beta \Delta F_{\bar{q}}(\vec{x})} \quad \textit{Phys. Rev. D 24 (1981) 450}$$

- $C_n$  symm. phase  $\rightarrow \langle \Phi(\vec{x}) \rangle_\beta = 0 \rightarrow \Delta F_{q/\bar{q}} = \infty \rightarrow$  **confinement**
- $C_n$  NON symm. phase  $\rightarrow \langle \Phi(\vec{x}) \rangle_\beta \neq 0 \rightarrow \Delta F_{q/\bar{q}} < \infty \rightarrow$  **deconfinement**

Thus  $\Phi(\vec{x})$  and  $\bar{\Phi}(\vec{x})$  can be used as order parameters for confinement

## $t$ -dependence of the condensates compared to lattice results

### The subtracted chiral condensate



– subtracted chiral condensate:

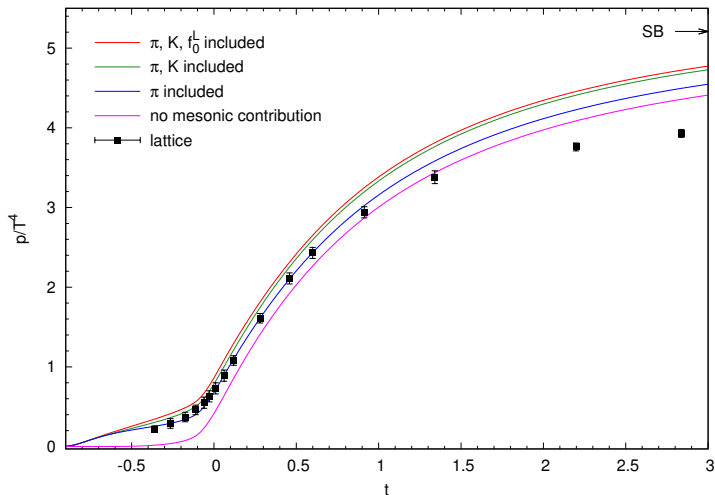
$$\Delta_{I,S} = \frac{\left( \phi_N - \frac{h_N}{h_S} \cdot \phi_S \right) \Big|_T}{\left( \phi_N - \frac{h_N}{h_S} \cdot \phi_S \right) \Big|_{T=0}}$$

–  $U_{\log}^{\text{glue}}$  with  $T_c^{\text{glue}} \in (210, 240)$  MeV gives good agreement with the lattice result of

*Borsányi et al., JHEP 1009, 073 (2010)*

here we use the reduced temperature:  $t = (T - T_c)/T_c$

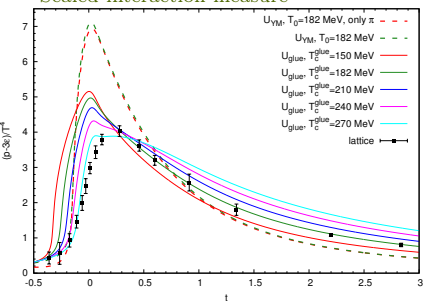
## Normalized pressure and the effects of meson contributions



- ▶ pions dominate the pressure at small  $T$
- ▶ contribution of the kaons is important
- ▶ at high  $T$  the pressure overshoots the lattice data of [Borsányi et al., JHEP 1011, 077 \(2010\)](#)

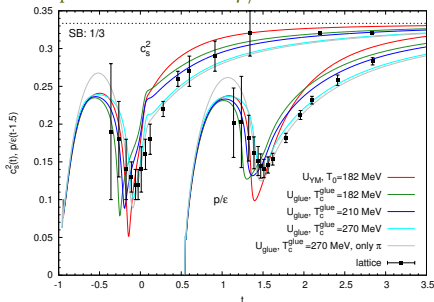
Scaled interaction measure, speed of sound and  $p/\epsilon$ 

## Scaled interaction measure



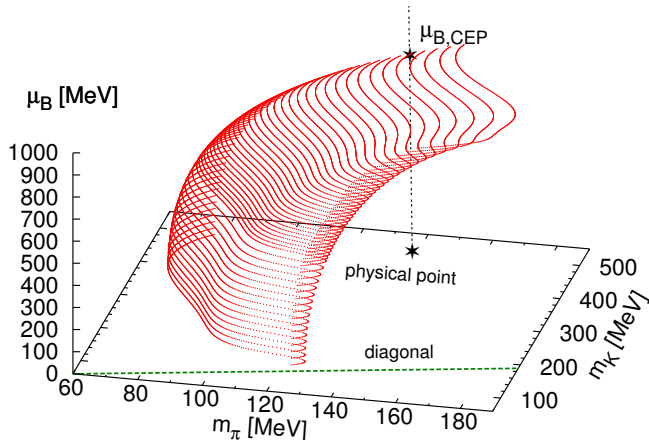
$$\Delta = \frac{\epsilon - 3p}{T^4}$$

By properly setting the  $T_C^{glue}$  parameter  $\rightarrow$  good agreement with lattice

Speed of sound and  $p/\epsilon$ 

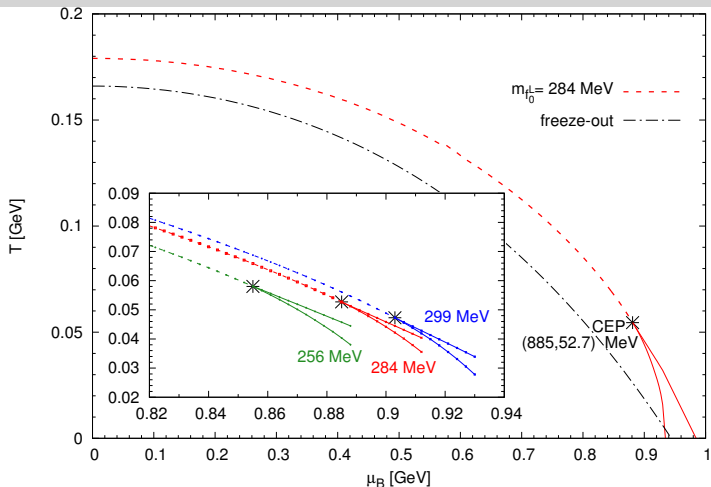
# Second order surface in $m_\pi - m_K - \mu_B$

ChPT for baryons used to obtain the value of  $m_{u,s}$  P. Kovács, Zs. Sz, PRD75 (2007) 025015



The surface bends towards the physical point  $\Rightarrow$  existence of CEP

# $T - \mu_B$ Phase Diagram



– we used  $U_{\log}^{\text{glue}}$  with  $T_c^{\text{glue}} = 210$  MeV

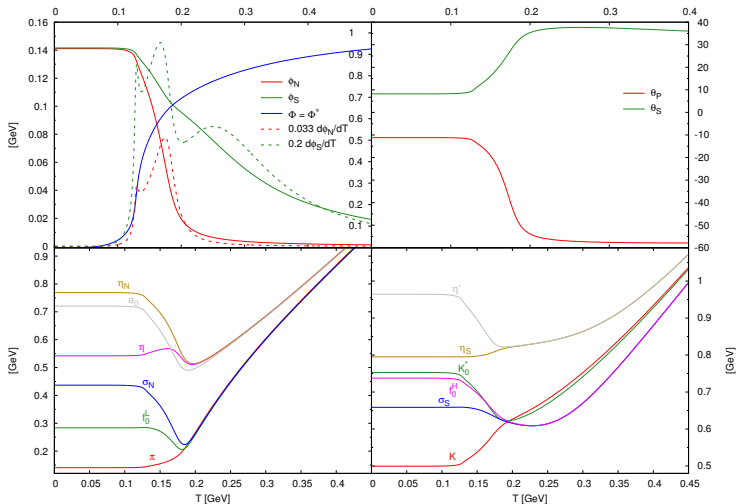
– freeze-out curve from Cleymans *et al.*, J.Phys.G 32, S165 (2006)

– curvature  $\kappa$  at  $\mu_B = 0$  obtained from the fit  $\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa \left( \frac{\mu_B}{T_c(\mu_B=0)} \right)^2$

$\kappa = 0.0193$  obtained, close to the lattice value  $\kappa = 0.020(4)$  of Cea *et al.*, PRD93, 014507

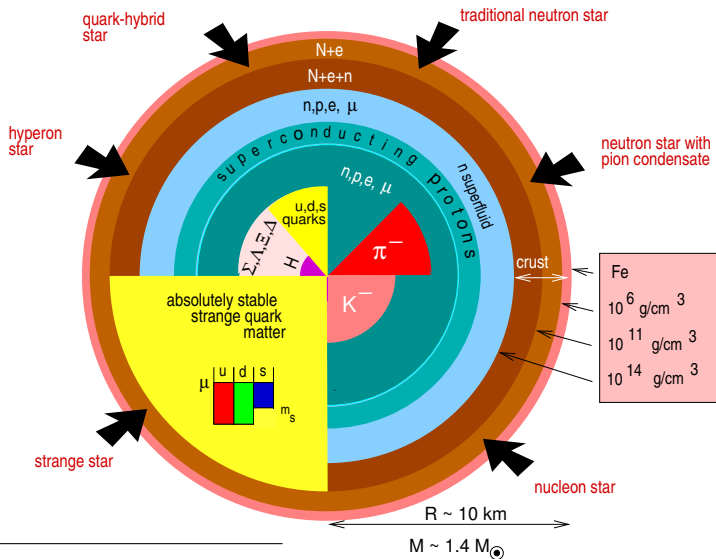


# T dependence of masses, condensates, mixing angles



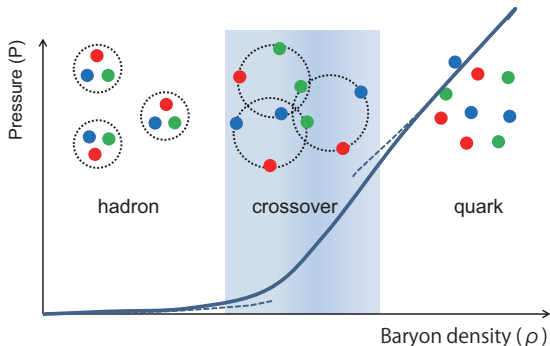
- ch. partners ( $\pi, f_0^L$ ), ( $\eta, a_0$ ) and ( $K, K_0^*$ ), ( $\eta', f_0^H$ ) become degenerate at high  $T$
- $U(1)_A$  not restored, axial partners ( $\pi, a_0$ ) and ( $\eta, f_0^L$ ) not become degenerate

## Structure of compact stars

Fig. from F. Weber, *J. Phys. G* **27**, 465 (2001)

# Hybrid EoS for hybrid stars

*K. Masuda et al., PTEP 2013 073D01*



In the crossover region hadrons starts to overlap  
 $\rightarrow$  both low and high  $\rho_B$  models loose their validity  
 Gibbs condition (extrapolation from the dashed lines) can be misleading

# EoS part I (hadronic part)

Hybrid stars also have a **hadronic crust and outer core**

- ▶ at low densities we use hadronic EoSs:
  - ▶ the **SFHo** EoS to represent soft hadronic EoSs
  - ▶ the **DD2** to represent stiff hadronic EoSs
- ▶ we apply a smooth connection between the two phases:  
 $\varepsilon(n_B)$  interpolation with polynomial

$$\varepsilon(n_B) = \sum_{m=0}^N C_m n_B^m, \quad n_{BL} < n_B < n_{BU},$$

the  $C_m$  coefficients given by the matching of the  $\varepsilon$  and its derivatives on the boundary

- ▶ we have 4 adjustable parameters:
  - ▶ 2 from the constituent quark model:  $m_\sigma, g_V$
  - ▶ 2 describing the concatenation:  $\bar{n}, \Gamma$

## EoS part II (quark part)

For large density the e(P)QM model is used

- ▶ nonzero scalar condensates:  $\phi_N = \langle \sigma_N \rangle$ ,  $\phi_S = \langle \sigma_S \rangle$
- ▶ nonzero vector condensates:  $\langle (\rho^0)^0 \rangle = \phi_\rho$ ,  $\langle (\omega)^0 \rangle = \phi_\omega$ ,  $\langle (\Phi)^0 \rangle = \phi_\Phi$
- ▶ free electron gas +  $\beta$ -equilibrium
- ▶ modified chemical potentials:

$$\mu_u = \mu_q - \frac{2}{3}\mu_e - \frac{1}{2}g_V(\phi_\omega + \phi_\rho)$$

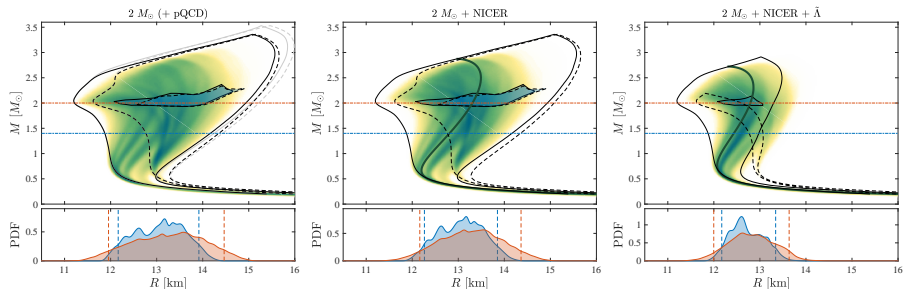
$$\mu_d = \mu_q + \frac{1}{3}\mu_e - \frac{1}{2}g_V(\phi_\omega - \phi_\rho)$$

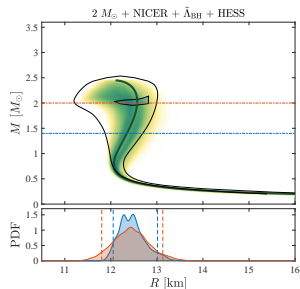
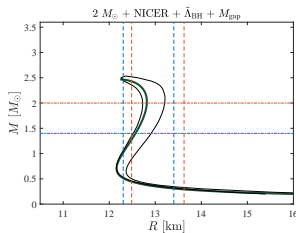
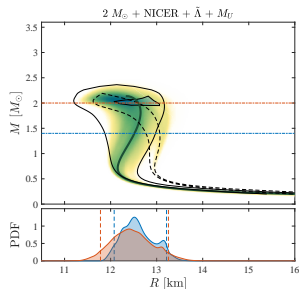
$$\mu_s = \mu_q + \frac{1}{3}\mu_e - \frac{1}{\sqrt{2}}g_V\phi_\Phi$$

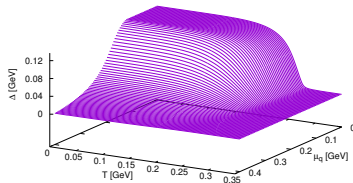
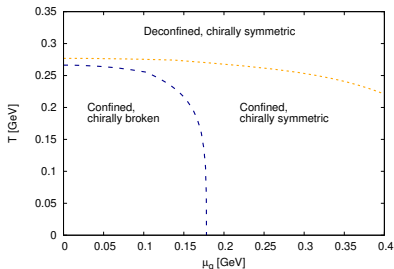
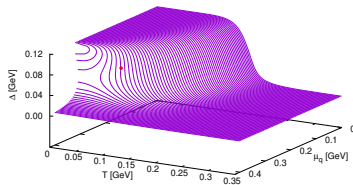
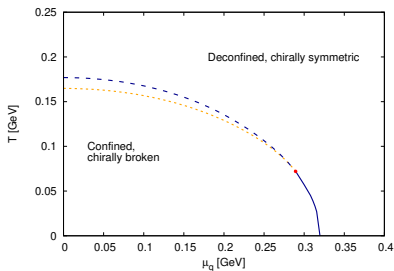
- ▶ charge neutrality:  $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$
- ▶ field equations (FEs):

$$\frac{\partial \Omega_{tot}}{\partial \phi_N} = \frac{\partial \Omega_{tot}}{\partial \phi_S} = \frac{\partial \Omega_{tot}}{\partial \phi_\rho} = \frac{\partial \Omega_{tot}}{\partial \phi_\omega} = \frac{\partial \Omega_{tot}}{\partial \phi_\Phi} = 0 \quad \rightarrow \text{p}(\varepsilon)\text{curve}$$

# Restriction of $M - R$ curves with Bayesian analysis I

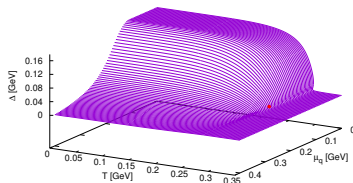
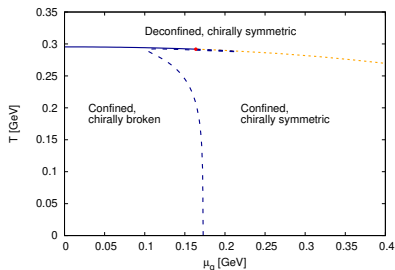


Restriction of  $M - R$  curves with Bayesian analysis II

Phase boundary for different  $N_c$ s I.

top:  $N_c = 3$ , CEP exist, crossover for small  $T$ ; bottom :  $N_c = 33$  crossover everywhere



Phase boundary for different  $N_c$ s II.

$N_c = 63$  CEP exist again, crossover for large  $T$

$$\Delta(T, \mu_q^{\text{fix}}) = \frac{(\phi_N - \frac{h_N}{h_S} \phi_S)|_{T, \mu_q^{\text{fix}}}}{(\phi_N - \frac{h_N}{h_S} \phi_S)|_{T=0, \mu_q^{\text{fix}}}} \quad (1)$$



# Conclusion

- ▶ eLSM can be used to study the properties of various meson nonnets, such as their masses or decay widths, based on the global chiral symmetries of QCD.
- ▶ eLSM can be used in the vacuum or at finite temperature and baryon chemical potentials, and it is free from the sign problem found on the Lattice.
- ▶ In recent years, more and more precise observations, e.g. mass, radius, tidal deformability, have been made, from which in-medium properties of strongly interacting matter can be inferred, which can be studied with eLSM.
- ▶ The  $M - R$  curves calculated so far are consistent with current observations when certain parameters of the model are constrained (e.g.  $2.5 < g_V < 4.3$ )
- ▶ The phase diagram at large  $N_c$  can also be studied within the eLSM. Three different scaling regions of the pressure can be found, while the CEP interestingly disappears along the  $\mu_B$  axis and reappears along the  $T$  axis as  $N_c$  is increased.
- ▶ For more details, see <https://arxiv.org/abs/2407.18348>

Thank you for your attention!

# Zimányi 2024



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## ZIMÁNYI SCHOOL 2024



L. Kassák: Image architecture

24th ZIMÁNYI SCHOOL  
 WINTER WORKSHOP  
 ON HEAVY ION PHYSICS

December 2-6, 2024

Budapest, Hungary



József Zimányi (1931 - 2006)

### Registration

**Registration closes on September 30, 2024**

Some special or accented characters might not appear correctly, please double check them.

**Name (title, first name, last name):**

**Email:**

# Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter  $\rightarrow$  TOV eqs.:

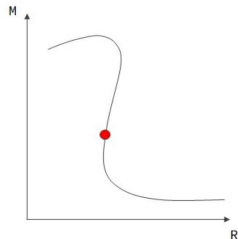
$$\frac{dp}{dr} = - \frac{[\rho(r) + \varepsilon(r)] [M(r) + 4\pi r^3 \rho(r)]}{r[r - 2M(r)]} \quad (2)$$

where

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific EoS ( $\rho(\varepsilon)$ ):

- ▶ For a fixed  $\varepsilon_c$  central energy density Eq. (2) is **integrated until  $\rho = 0$**
- ▶ Varying  $\varepsilon_c$  a series of compact stars is obtained (with given  $M$  and  $R$ )
- ▶ Once the maximal mass is reached, the stable series of compact stars ends



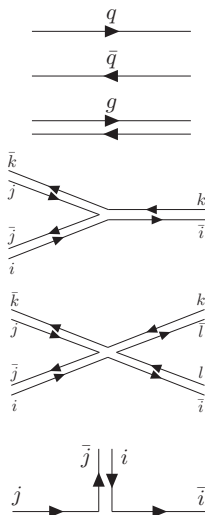
# Basics of Large $N_c$ I.

G. 't Hooft. (1974), Nucl. Phys. B 72:461

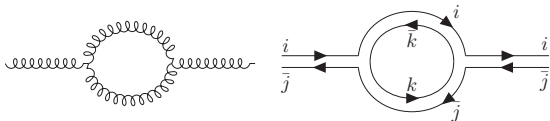
G. 't Hooft. (1974), Nucl. Phys. B 75:461–470

E. Witten. (1979), Nucl. Phys. B 160:57–115

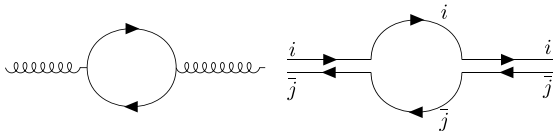
- ▶ No expansion parameter in QCD if  $m_{u/d/s} \approx 0 \rightarrow$  not so obvious expansion parameter:  $N_c$
- ▶  $SU(3) \rightarrow SU(N_c)$
- ▶ double line notation based on color structure of gluons:  $A_j^{\mu; i} \sim q^i \bar{q}_j$
- ▶ 3-coupling:  $A_{\mu; j}^i A_{\nu; k}^j \partial^\mu A_i^{\nu; k}$
- ▶ 4-coupling:  $A_{\mu; j}^i A_{\nu; k}^j A_l^{\mu; k} A_i^{\nu; l}$
- ▶ quark-gluon vertex:  $\bar{q}_i \gamma^\mu q^j A_{\mu; j}^i$



# Basics of Large $N_c$ II.



$N_c$  combinatorial factor due to closed color loop  $\implies g \sim \frac{1}{\sqrt{N_c}}$



Quark loops are  $1/N_c$  suppressed.

Leading diagrams are planar diagrams with minimum number of quark loops

Investigation of  $N$ -point functions of quark bilinears ( $J = \bar{q}q, \bar{q}\gamma^\mu q$ ) leads to the large  $N_c$  properties of mesons



# Properties of mesons and baryons for Large $N_c$

- ▶ mesons are free, stable, and non-interacting
- ▶ mesons are pure  $q\bar{q}$  states for large  $N_c$
- ▶ meson masses  $\sim N_c^0$
- ▶ meson decay amplitudes  $\sim 1/\sqrt{N_c}$
- ▶ for one meson creation:  $\langle 0|J|m \rangle \sim \sqrt{N_c}$
- ▶  $k$  meson vertex  $\sim N_c^{1-k/2}$ . Specifically, the three- and four-meson vertices are  $\sim 1/\sqrt{N_c}$  and  $\sim 1/N_c$ , respectively
- ▶ baryon masses  $\sim N_c$ . Consequently constituent quark masses  $\sim N_c^0$

# $N_c$ scaling of the Lagrange parameters

The parameters are:  $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, h_{N/S}$

- $m_0^2, m_1^2, \delta_s \sim N_c^0$ , because terms of tree level meson masses
- $g_1, g_2 \sim \frac{1}{\sqrt{N_c}}$ , three couplings
- $\lambda_2, h_2, h_3 \sim \frac{1}{N_c}$ , four couplings
- $\lambda_1, h_1 \sim \frac{1}{N_c^2}$ , four couplings **with different trace structure**
- $c_1 \sim \frac{1}{N_c^{3/2}}$   $U_A(1)$  anomaly term has extra  **$1/N_c$  suppression**
- $h_{N/S} \sim \sqrt{N_c}$ , Goldstone-theorem:  $m_\pi^2 \Phi_N = Z_\pi^2 h_N + \text{PCAC}$ :  $\Phi_N = Z_\pi f_\pi$
- $g_F \sim \frac{1}{\sqrt{N_c}}$ ,  $m_{u/d} = g_F \Phi_N$

We expect  $\Phi_{N/S} \sim \sqrt{N_c}$ , since  $\Phi_N = Z_\pi f_\pi$ ,  $f_\pi \sim \sqrt{N_c}$ , but have to check!

practically:  $g_1 \rightarrow g_1 \sqrt{\frac{3}{N_c}}$ ,  $h_{N/S} \rightarrow h_{N/S} \sqrt{\frac{N_c}{3}} \dots \text{etc.}$

Parameter sets at  $N_c = 3$ 

Parameter	Set A	Set B
$\phi_N$ [GeV]	0.1411	0.1290
$\phi_S$ [GeV]	0.1416	0.1406
$m_0^2$ [GeV <sup>2</sup> ]	$2.3925_{E-4}$	$-1.2370_{E-2}$
$m_1^2$ [GeV <sup>2</sup> ]	$6.3298_{E-8}$	0.5600
$\lambda_1$	-1.6738	-1.0096
$\lambda_2$	23.5078	25.7328
$c_1$ [GeV]	1.3086	1.4700
$\delta_S$ [GeV <sup>2</sup> ]	0.1133	0.2305
$g_1$	5.6156	5.3295
$g_2$	3.0467	-1.0579
$h_1$	37.4617	5.8467
$h_2$	4.2281	-12.3456
$h_3$	2.9839	3.5755
$g_F$	4.5708	4.9571
$M_0$ [GeV]	0.3511	0.3935

- ▶ Set A:  $m_\sigma = 290$  MeV  
from [Phys.Rev.D 93 \(2016\) 11, 114014](#)
- ▶ Set B: similar  $m_\sigma$  mass and additional constraint,  
 $3h_1 + 2h_2 + 2h_3 < 0$   
from [Phys.Rev.D 105 \(2022\) 10, 103014](#)

# Polyakov-loop variables at large $N_c$ (I.)

Polyakov gauge  $\rightarrow A_4$  is diagonal and time independent  
 $\rightarrow$  further simplification: homogeneous gluon field

$$L = e^{i\beta A_4} = \text{diag} (e^{iq_1}, \dots, e^{iq_{N_c}}), \quad q_j \in \mathbb{R}, \quad \sum_j q_j = 0$$

$N_c - 1$  independent diagonal  $SU(N_c)$  matrix  $\rightarrow \Phi$  and  $\bar{\Phi}$  above are not enough

$$\Phi_n = \frac{1}{N_c} \text{Tr}_c L^n, \quad \bar{\Phi}_n = \frac{1}{N_c} \text{Tr}_c L^{\dagger n}, \quad n \in \left(1, \dots, \lfloor \frac{N_c}{2} \rfloor\right), \quad \text{Phys. Rev. D 86, 105017 (2012)}$$

our approx.  $\rightarrow$  quarks prop. on a const. gluon background  $\rightarrow$  color dep.  
 chem. pot.

$$\Omega_{\bar{q}q}^{(0)}(T, \mu_q) = \Omega_{\bar{q}q}^{(0)\text{v}} + \Omega_{\bar{q}q}^{(0)\text{T}}(T, \mu_q),$$

$$\Omega_{\bar{q}q}^{(0)\text{T}}(T, \mu_q) = -2T \sum_f \int \frac{d^3 p}{(2\pi)^3} [\ln g_f^+(p) + \ln g_f^-(p)]$$

$$\ln g_f^+(p) \equiv \text{Tr}_c \ln \left[ \mathbb{1} + L^\dagger e^{-\beta(E_f(p) - \mu_q)} \right] = \ln \text{Det}_c \left[ \mathbb{1} + L^\dagger e^{-\beta(E_f(p) - \mu_q)} \right]$$

$$\ln g_f^-(p) \equiv \text{Tr}_c \ln \left[ \mathbb{1} + L e^{-\beta(E_f(p) + \mu_q)} \right] = \ln \text{Det}_c \left[ \mathbb{1} + L e^{-\beta(E_f(p) + \mu_q)} \right]$$